* Switching Circuit / Algebra →

The variables in the system are switches which may be ON and OFF. Symbolically the states are represented by 1 and 0 respectively. It is important to ~~understand~~ understand that 1 & 0 are only symbols used to represent a logical state. Hence the set in switching algebra consists of two elements 0 & 1.

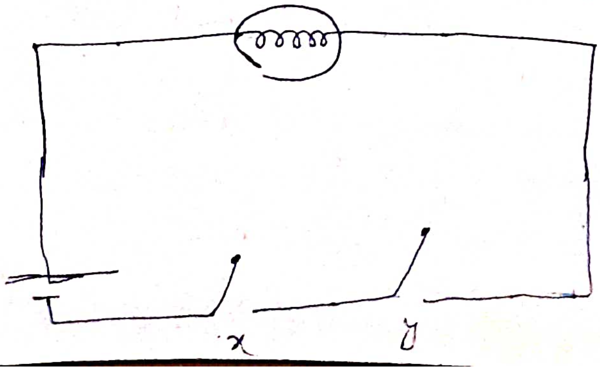
Logic Gates →

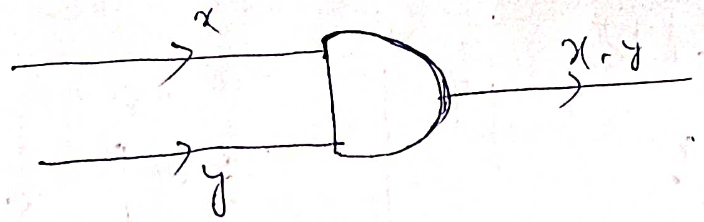
1. ~~And~~ AND Operation →

It is denoted by the symbol (·). Let x and y be the symbols of the set S(0, 1). The different values of x & y are shown by the following table

x	y	$x \cdot y$
0	1	0
1	0	0
0	0	0
1	1	1

The table shows that the resulting output of the switching circuit is ON only when x & y are ON and is OFF in all other cases.



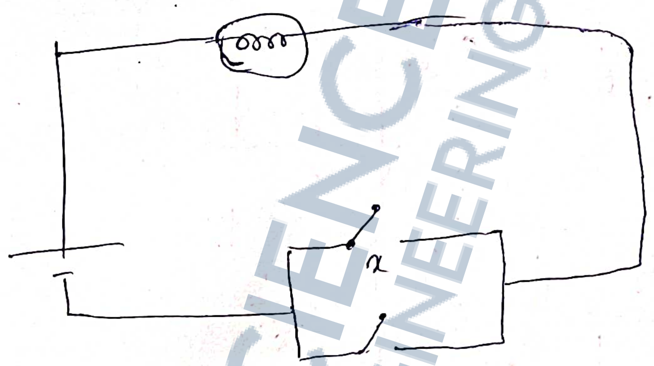


2. OR Operation →

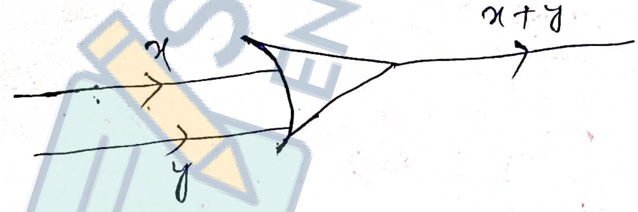
It is denoted by the symbol (+).

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

This table shows that the resulting output is OFF when x & y are both OFF and ON in all other cases.



OR Symbol

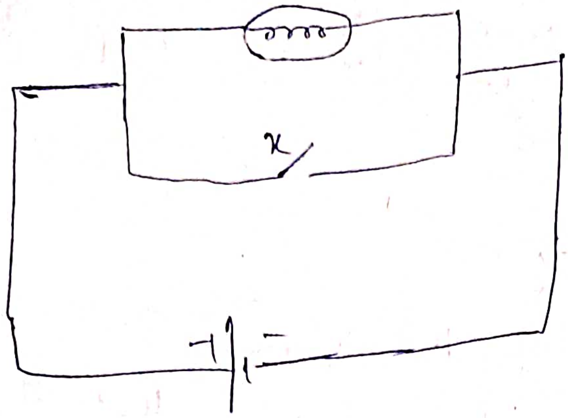


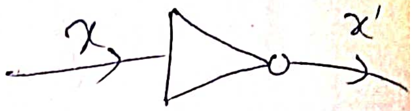
3. NOT Operation

It is denoted by (') sign

x	x'
0	1
1	0

When x is open OFF the bulb will glow and when it is closed (ON) the bulb will ~~glow~~ not glow. Hence this circuit represent a NOT operation.



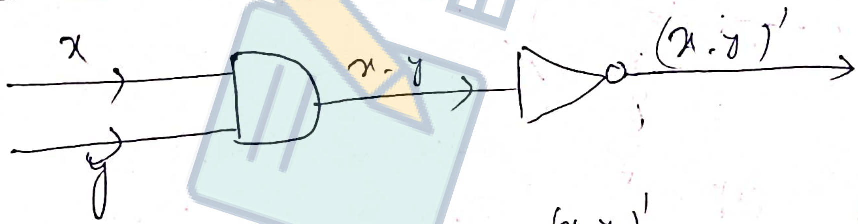


4. NAND OPERATION→

If we will connect the output of AND gate as the input of NOT gate, the whole circuit is called NAND gate.

x	y	$x.y$	$(x.y)'$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

NAND Symbol



or

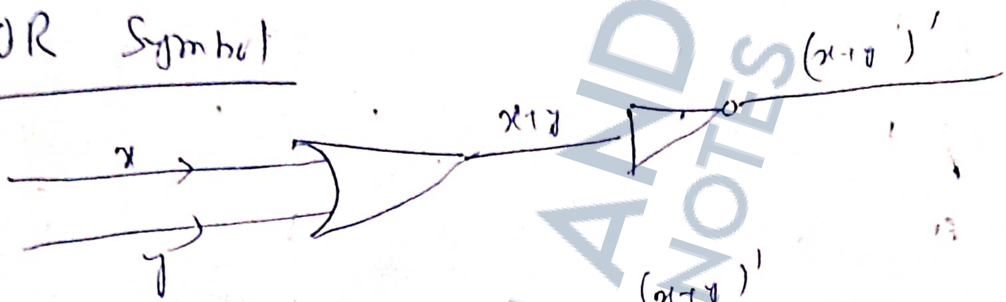
5. NOR Operation →

When the output of an OR gate is used as the input of a NOT gate, the circuit is called a NOR gate.

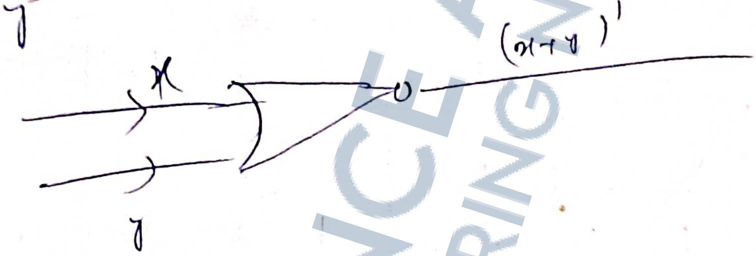
Truth table is

x	y	$x+y$	$(x+y)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

NOR Symbol

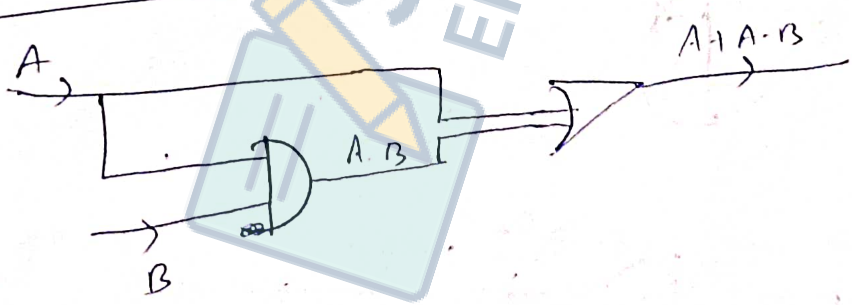


or



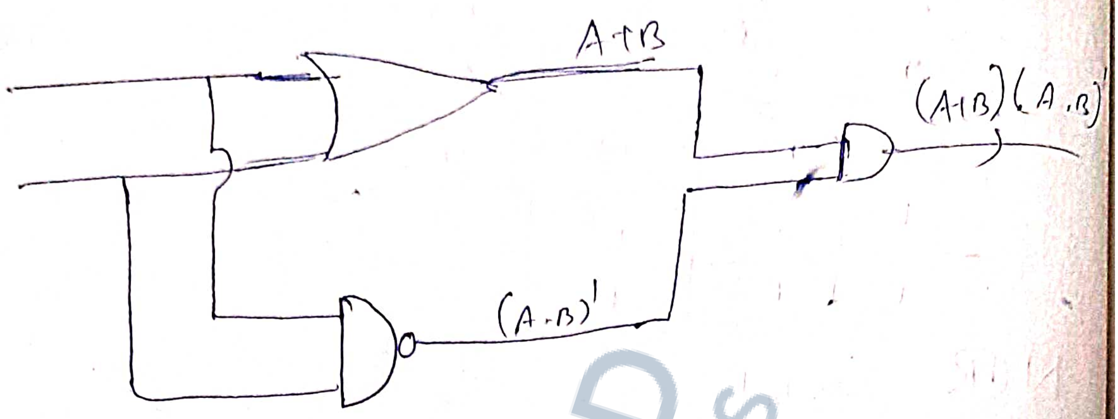
Problems →

1. Draw the symbol of $A+A.B$ along with a truth table



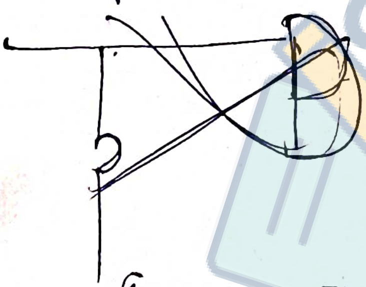
A	B	A.B	A+A.B
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

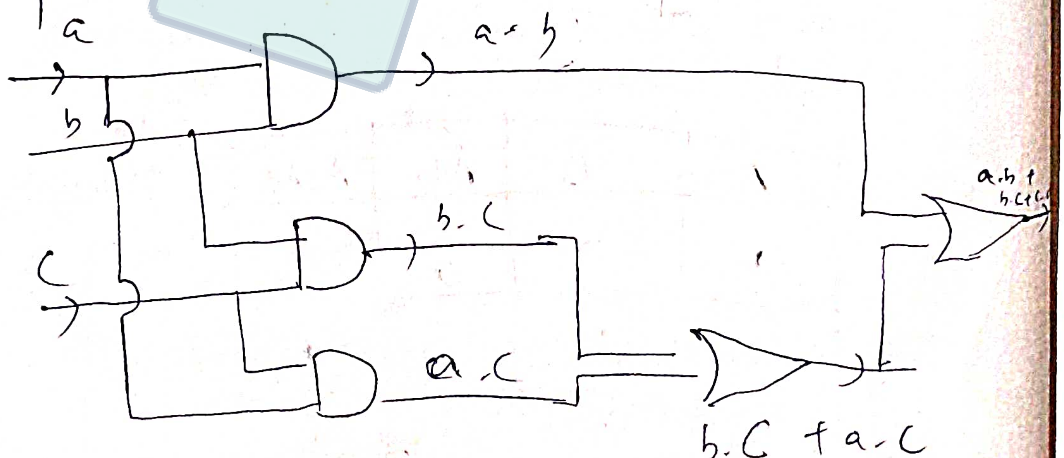
2. Draw $(A+B).(A.B)'$



A	B	A+B	A.B	(A.B)'	(A+B).(A.B)'
0	0	0	0	1	0
0	1	1	0	1	1
1	0	1	0	1	1
1	1	1	1	0	0

3. Draw the symbol for ~~A.B~~ a.b+b.c+c.a along with the truth table





a	b	c	a.b	b.c	c.a	a.b+b.c	a.b+b.c+c.a
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	0	0	1	0	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1
0	1	1	0	1	0	1	1

Ex-1

Draw the logic circuit for $x+y \cdot z'$

Ex-2

Draw the logic circuit for $(x+y) \cdot z'$

Ex-3

Draw the logic circuit for $a \cdot b + c'$

Ex-4

Draw logic circuit for $x' \cdot z + x \cdot y'$

Text Book:-

1) Digital Design:- M. Morris. Mano, PHI, 3rd ed, 2006.

Ref Book:-

1) An Engineering approach to digital design, Fletcher, PHI.

2) Digital Fundamentals :- Floyd, Pearson Education.

— o —

Binary Logic

→ Binary logic deals with variables that take on two discrete values and operations that assume logical meaning. The two values the ~~values~~ variables take may be called by different names (true and false, yes and no, etc.) but for our purpose, it is convenient to think in terms of bits and assign the values of 1 and 0.

→ Binary Logic consists of binary variables and logical operations. The variables are designated by letters of the alphabets such as A, B, C, x, y, z, etc., with each variable having two and only two distinct possible values: 1 and 0. There are 3 basic logical operations: AND, OR, and NOT

For example :-

OR: This operation is represented by a plus sign.

e.g, $x+y=Z$ is read "x OR y is equal to Z", meaning that $Z=1$ if $x=1$ or if $y=1$ or if both $x=1$ and $y=1$. If both $x=0$ and $y=0$, then $Z=0$.

→ Binary logic should not be confused with binary arithmetic. One should realize that an arithmetic variable designates a number that may consist of many digits. A logic variable is always either 1 or 0. e.g. in binary arithmetic, we have $1+1=10$ (read: "one plus one is equal to 2"), whereas in binary logic, we have $1+1=1$ (read: "one OR one is equal to one")

→ A truth table is a table of all possible combinations of the variables showing the relation between the values that the variables may take and result of the operation.

Ex

AND

x	y	x.y
0	0	0
0	1	0
1	0	0
1	1	1

NOT

x	x'
0	1
1	0

⇒ Logic gates:-

Logic gates are electronic circuits that operate on one or more input signals to produce an o/p signal

Electrical signals such as voltages or currents exist throughout a digital system in either of two recognizable values. Voltage-operated circuits respond to two separate voltage levels that represent a binary variable equal to logic 1 or logic 0.

Generally '1' is represented by higher voltage, which we will refer to as a HIGH and a '0' is represented by a lower voltage level, which we will refer to as LOW. This is called positive logic.

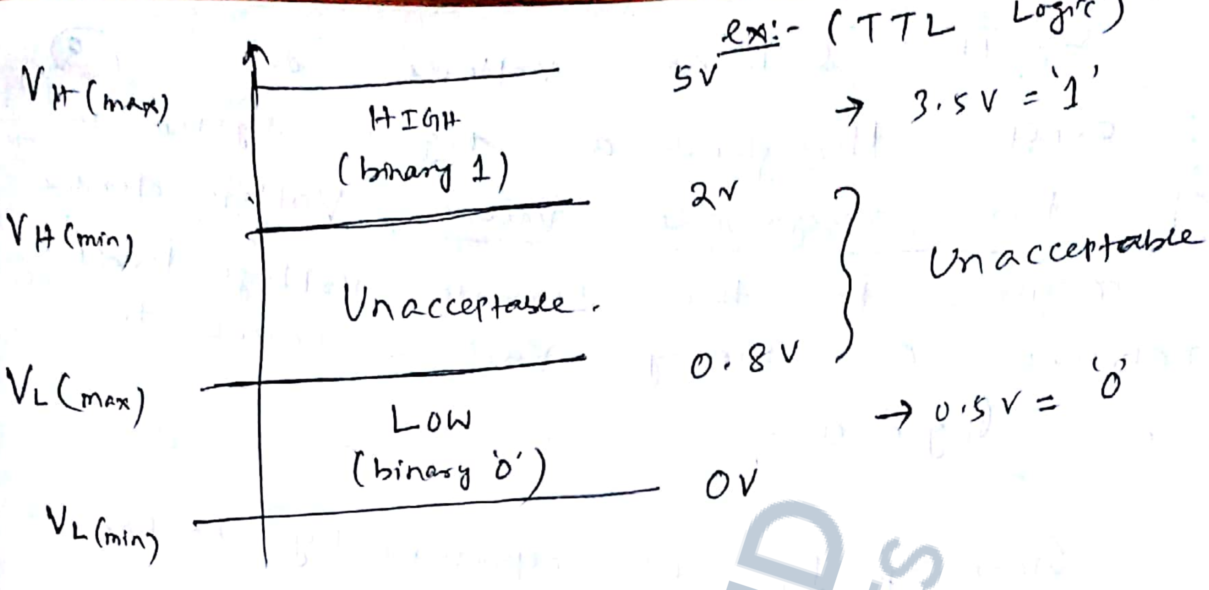
HIGH = 1, LOW = 0 (NPN) transistor

Another system in which a '1' is represented by a LOW and a '0' is represented by a HIGH is called negative logic.

LOW = 1, HIGH = 0 (PNP) transistor

→ Logic levels:-

The voltages used to represent a '1' and a '0' are called logic levels. In a practical digital circuit, a HIGH can be any voltage between a specified minimum value and a specified maximum value. Likewise, a LOW can be any voltage between a specified minimum and specified maximum. There can be no overlap between the accepted HIGH levels and the accepted LOW levels.

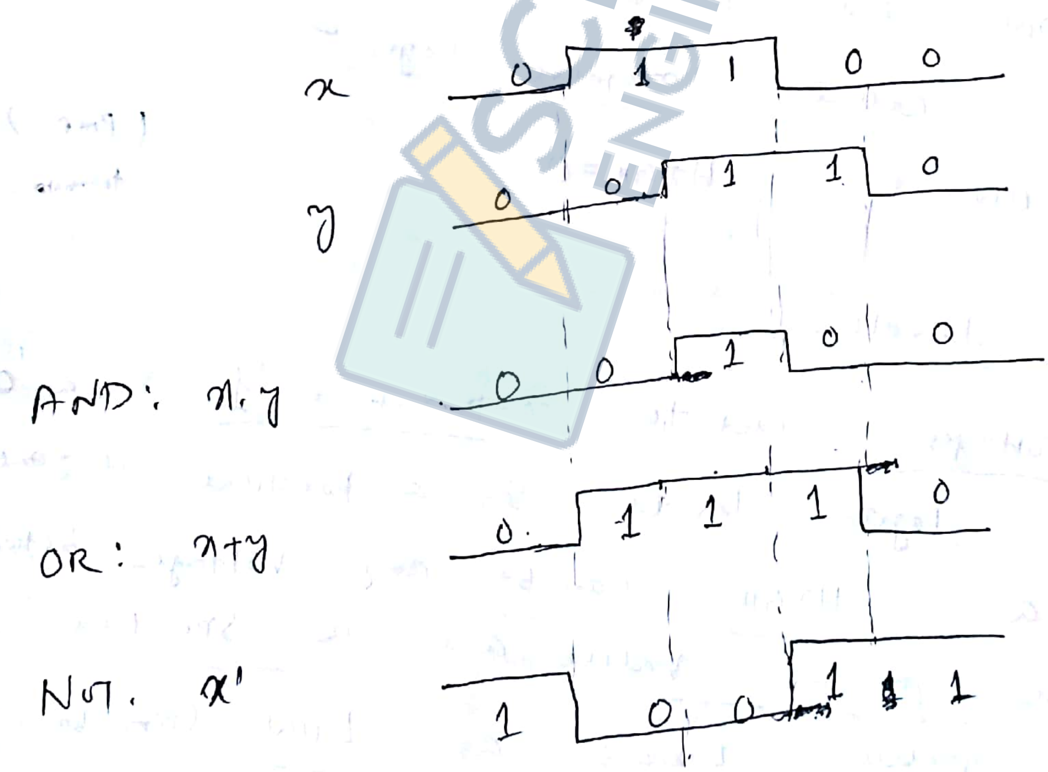


$V_L(min)$ to $V_L(max)$ = LOW = '0'

$V_H(min)$ to $V_H(max)$ = HIGH = '1'

$V_L(max)$ to $V_H(min)$ → Unacceptable

I/P – O/P Signals for ~~gates~~ Gates



Logic Gates & Boolean Algebra

↳ Perform logical operation.

→ Logic gates are the basic building blocks of digital system. Basically there are 7 gates are available in digital electronics, those are discussed in this section.

→ Boolean algebra, named after its pioneer George Boole, is the algebra of logic which has 2 states (0 & 1). Any digital ckt can be described using this algebra.

→ Logic Gates:-

Logic gates are basic building blocks of any digital ckts. They can have one or more i/p and only one o/p ckt symbol.

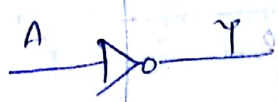
→ NOT, AND, OR } Basic gates

NAND, NOR } Universal gates

X-OR, X-NOR } Special gates

NOT Gate:- IC (7404) or 74LS04

→ One i/p one o/p

→ 

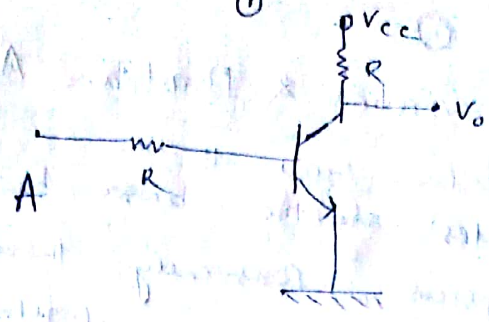
Truth table

A	$Y = \bar{A}$
0	1
1	0

→ $Y = \bar{A}$

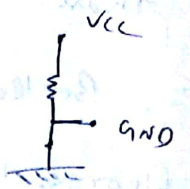
→ If i/p is low o/p will be high. If i/p is high o/p is low.

①



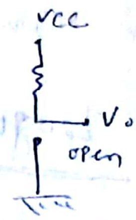
<u>If A = 1</u>, transistor conducts, o/p short-circted,

V_0 = GND. potential = 0.



If $\boxed{A = 1, \quad Y = 0}$

<u>If A = 0</u>, transistor is off,



$V_0 = V_{CC} = 1$

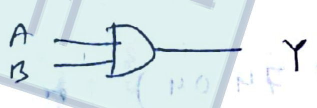
$\boxed{A = 0, \quad Y = 1}$

<u>AND gate :-</u> (74 LS 08)

→ Many i/p → one o/p.

→ $Y = A \cdot B$

→

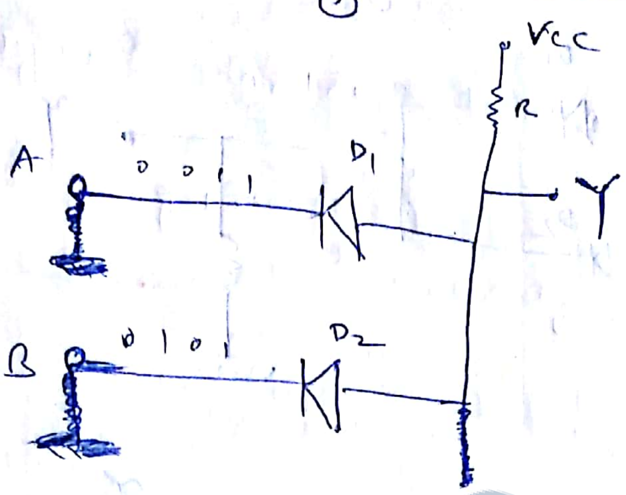


→

A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

→ The o/p of AND gate is high if all i/ps are high.

①



A	B	D1	D2	Y
0	0	on	on	0
0	1	on	off	0
1	0	off	on	0
1	1	off	off	1

OR Gate :- (74LS 32)

→ Many i/p → one o/p.

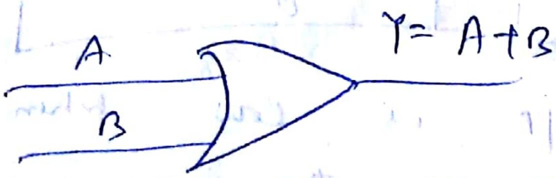
→ Y = A + B

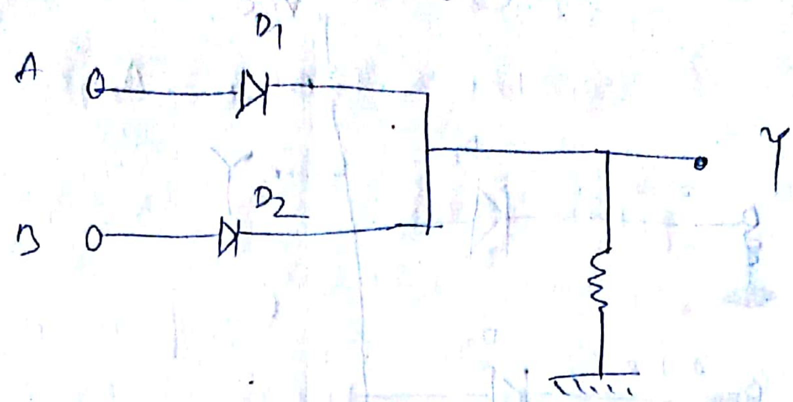
→

A	B	Y = A+B
0	0	0
0	1	1
1	0	1
1	1	1

→ The o/p of OR gate is high if any or all i/ps are high.

→





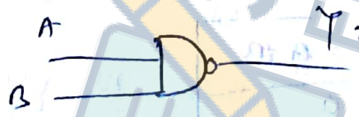
A	B	D_1	D_2	Y
0	0	off	off	0
0	1	off	on	1
1	0	on	off	1
1	1	on	on	1

NAND gate :- (74LS00)

→ Many i/p one o/p.

→ $Y = \overline{AB}$

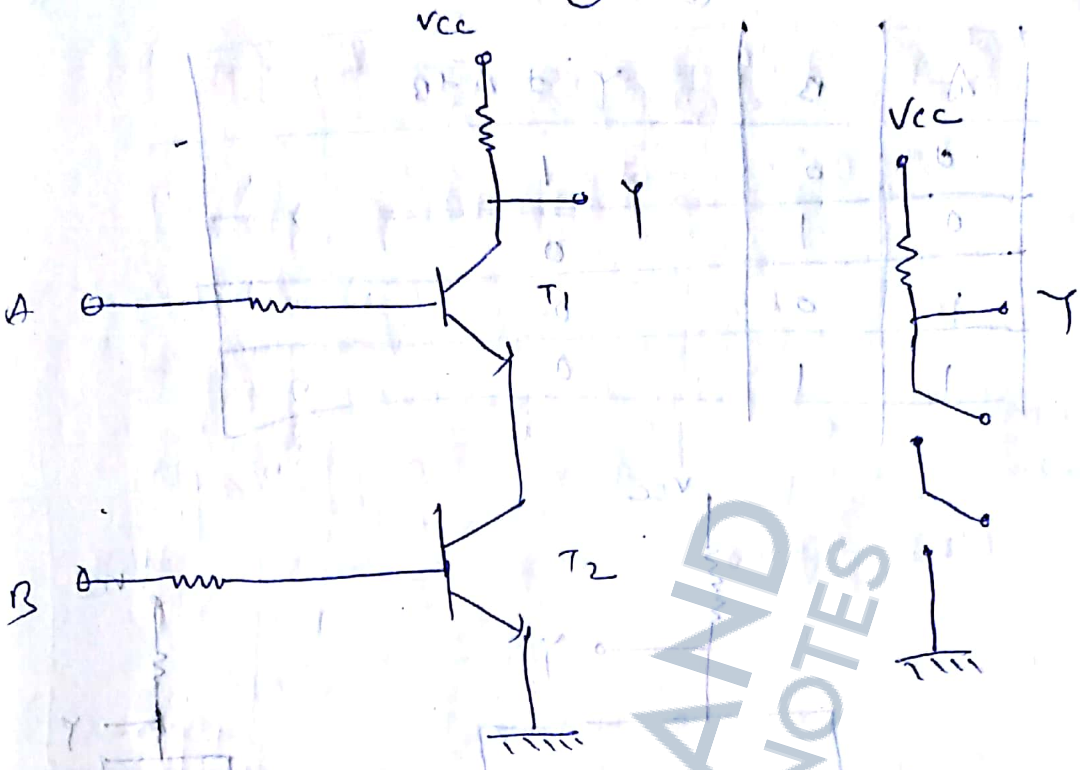
→



$Y = \overline{AB}$

A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

→ The o/p is low when all i/ps are high, otherwise the o/p is high.



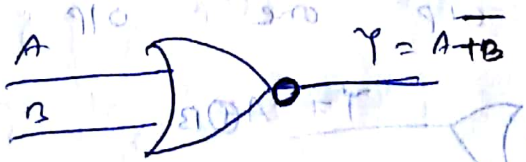
A	B	T_1	T_2	Y
0	0	off	off	1
0	1	off	on	1
1	0	on	off	1
1	1	on	on	0

NOR Gate (74 LS 02)

→ Many i/p ⟶ one o/p.

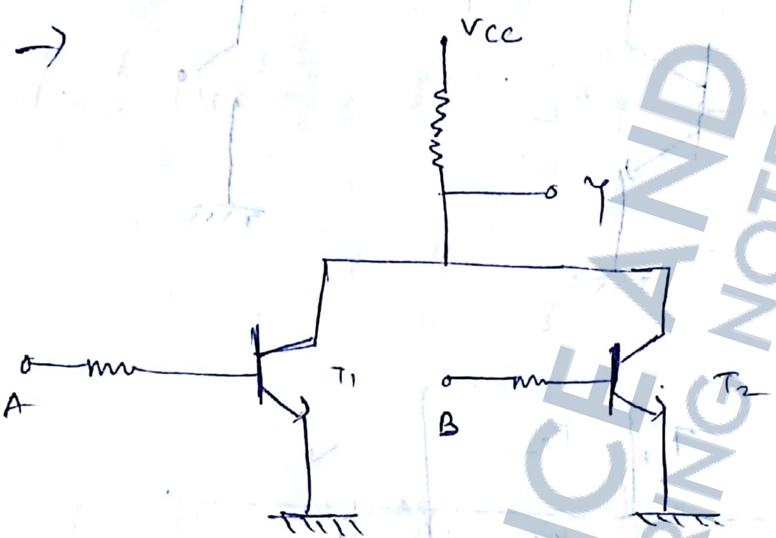
→ $Y = \overline{A+B}$

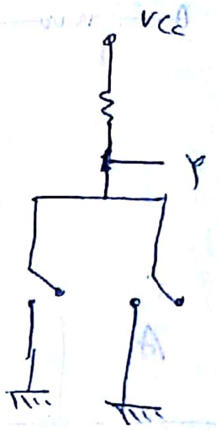
→



A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

→





A	B	T_1	T_2	Y
0	0	off	off	1
0	1	off	on	0
1	0	on	off	0
1	1	on	on	0

EX-OR Gate :- (74LS86)

→ Many I/P one O/P

→

→ $Y = A \oplus B = A\bar{B} + \bar{A}B$

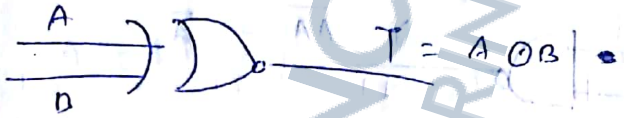
→ ⊙

A	B	A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

→ The gate produces o/p high when i/ps have odd no. of 1s.

Ex-NOR (74LS266)

→ Many i/p one o/p

→ 

→ $A \odot B = \cancel{AB + \overline{AB}}\ AB + \bar{A} \cdot \bar{B}$

→

A	B	Y = A⊙B
0	0	1
0	1	0
1	0	0
1	1	1

→ The o/p of X-NOR is high when i/p have even no. of zeros or even no. of 1s.

→ X-NOR gate is also known as "Equivalence function".

Universal Gates :-

→ NAND & NOR gates are called universal gates, because both can be used to implement any logic gate or any logic expression.

NAND as universal gate :-

A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

(i) NOT gate

$Y = \overline{A}$

Boolean Algebra should be taught before showing universal gates. Because the o/p of any gate can be simplified by boolean algebra.

Boolean Algebra :- (Mathematics of digital system)

✓ Duality principle :-

It states that any boolean expression has a dual form which can be obtained as following steps.

(a) Replace OR operation by AND operation.

(b) Replace AND operation by OR operation.

(c) Replace 1 by 0 & 0 by 1.

ex :- Find dual of $F = 1 . A + \bar{B} . 0 + C\bar{A} + 1$

Ans :- $F = (0 + A) . (\bar{B} + 1) . (C + \bar{A})$

Laws of Boolean Algebra

1) Commutative law

$$A + B = B + A$$
$$A . B = B . A$$

2) Associative law

$$A + (B + C) = (A + B) + C$$
$$A . (B . C) = (A . B) . C$$

3) Distributive law

Distributive law of addition

$$A . (B + C) = AB + AC$$

Distributive law of multiplication

$$A + BC = (A + B) . (A + C)$$

Thm's / Rules of Boolean Algebra :-

Thm 1 (a) $0 . A = 0$ (AND)

(b) $1 + A = 1$ (OR)

(b) is dual form of (a).

Note :-
- $\cdot \rightarrow$ Intersection
- $+ \rightarrow$ Union.
- $0 \rightarrow \phi$ = Null set
- $1 \rightarrow U$ = Universal set.

Thm 2 (a) $1 . A = A$ AND

$1 . 0 = 0$
$1 . 1 = 1$
$1 . A = A$

(b) $0 + A = A$ OR

Thm 3 (a) $A . A = A$

(b) $A + A = A$

. → intersection
+ → union

Thm 4 (a) $A . \bar{A} = 0$

4 (b) $A + \bar{A} = 1$

Thm 5 (a) $A + AB = A$

5 (b) $A . (A + B) = A$

Proof

5 (a) L.H.S $A + AB$

$= A(1 + B)$

$= A . 1$

$= A$ (R.H.S)

(b) $A . (A + B)$

$= A . A + A . B$

$= A + AB$

$= A(1 + B)$

$= A . 1$

$= A$

Thm

6 (a) $A + BC = (A + B)(A + C)$

R.H.S

$(A+B)(A+C)$

$= A.A + A.C + B.A + B.C$

$= A + \underbrace{A.C + B.A}_{A(B+C)} + B.C$

$= A[1 + (B+C)] + BC$

$= A.1 + BC$ | $\because 1+(B+C) = 1$

$= A+BC$

(b) $A.(B+C) = AB+AC$ (Distributive law)

7. $\overline{\overline{A}} = A$

Demorgan's Th^m

(i) $\overline{A+B} = \overline{A}.\overline{B}$

(ii) $\overline{AB} = \overline{A}+\overline{B}$

i.e (i) The complement of sum of 2 or more ~~varia~~ variables is equal to the product of the complement of the variable

(ii) The complement of the product of 2 or more variables is equal to the sum of the complements of the variables.

The above th^m can be proved by the help of truth table.

A	B	$\bar{A}$	$\bar{B}$	AB	A+B	$\overline{AB}$	$\bar{A}+\bar{B}$	$\overline{A+B}$	$\bar{A}\cdot\bar{B}$
0	0	1	1	0	0	1	1	1	1
0	1	1	0	0	1	1	1	0	0
1	0	0	1	0	1	1	1	0	0
1	1	0	0	1	1	0	0	0	0

From the above table, we proved that

(i) $\overline{A+B} = \bar{A}\cdot\bar{B}$

(ii) $\overline{AB} = \bar{A}+\bar{B}$

SL No	Thm s.	Dual Rel^n
1	$A+0=A$	$A\cdot 1=A$
2	$A+\bar{A}=1$	$A\cdot\bar{A}=0$
3	$A+A=A$	$A\cdot A=A$
4	$A+1=1$	$A\cdot 0=0$
5	$\bar{\bar{A}}=A$	$\bar{\bar{A}}=A$
6	$A+B=B+A$	$A\cdot B=B\cdot A$
7	$A+(B+C)=(A+B)+C$	$A\cdot(BC)=(A\cdot B)\cdot C$
8	$A(B+C)=AB+AC$	$A+BC=(A+B)(A+C)$
9	$\overline{A+B}=\bar{A}\cdot\bar{B}$	$\overline{AB}=\bar{A}+\bar{B}$
10	$A+AB=A$	$A\cdot(A+B)=A$

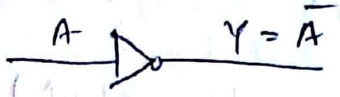
Universal gates :

→ NAND & NOR gates are called universal gates because both can be used to implement any logic gates or any logic expression.

NAND as universal gate: $\left[Y = \overline{AB}\right]$

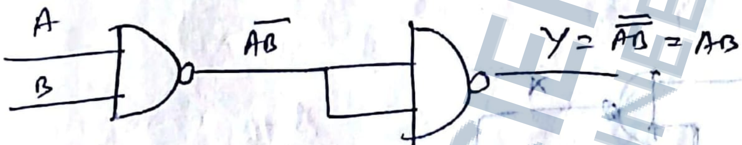
(i) NOT Gate

$$Y = \overline{A}$$

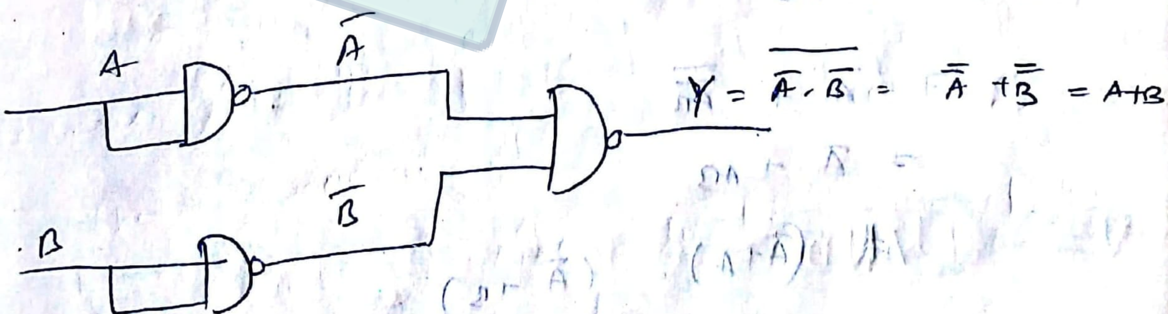


(ii) AND Gate:-

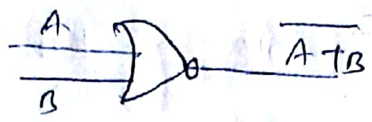
Trick:-

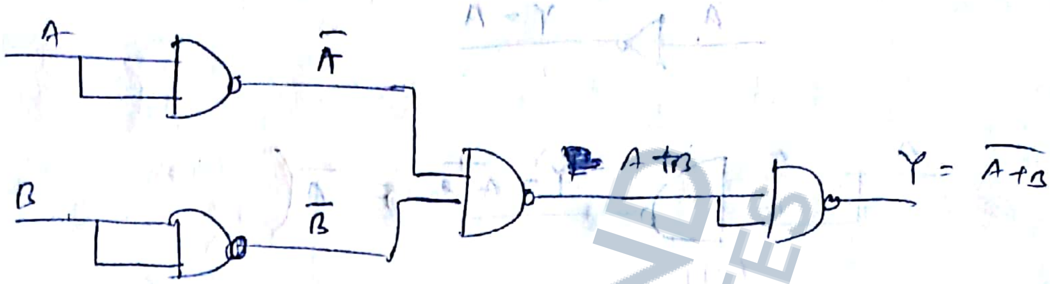


(iii) OR Gate

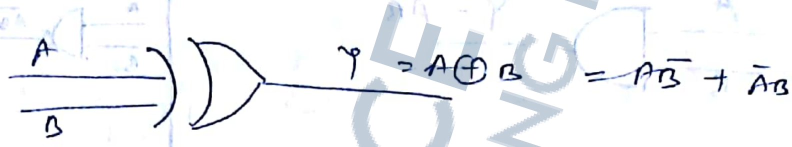


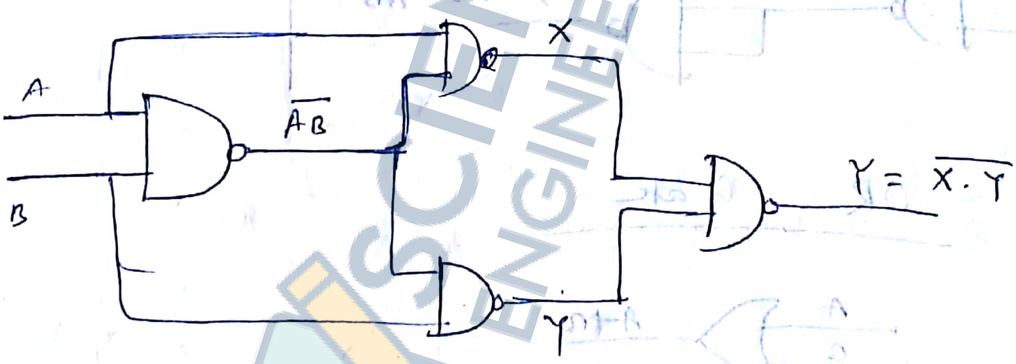
(iv) NOR gate





(v) EX-OR gate





$$X = \overline{A \cdot \overline{AB}}$$

$$= \bar{A} + \overline{\overline{AB}}$$

$$= \bar{A} + AB$$

$$= (\bar{A} + A) \cdot (\bar{A} + B)$$

$$= 1 \cdot (\bar{A} + B)$$

$$X = \bar{A} + B$$

$$Y = \overline{\overline{AB} \cdot B}$$

$$= \overline{\overline{AB}} + \overline{B}$$

$$= AB + \overline{B}$$

$$= \overline{B} + AB$$

$$= (\overline{B} + A)(\overline{B} + B)$$

$$= (\overline{B} + A) \cdot 1$$

$$= \overline{B} + A$$

$$Y = \overline{X \, Y}$$

$$= \overline{(\overline{A} + B) \cdot (\overline{B} + A)}$$

$$= \overline{\overline{A}\overline{B} + \cancel{\overline{A}A}^{0} + \cancel{B\overline{B}}^{0} + AB}$$

$$= \overline{\overline{A}\,\overline{B} + AB}$$

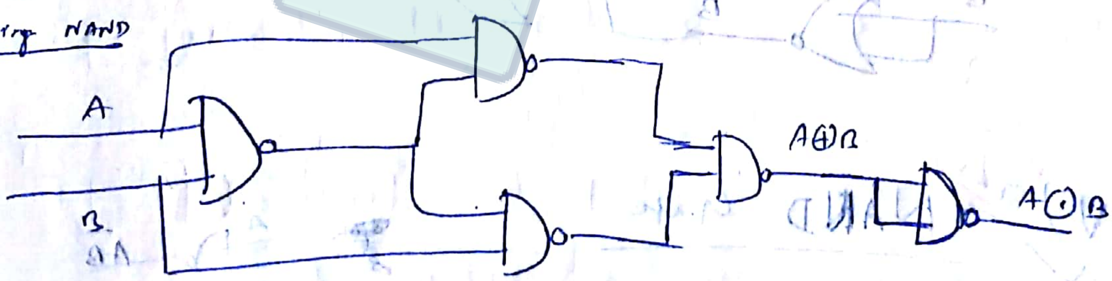
$$= \overline{A \odot B}$$

$$Y = A \oplus B$$

(Proved)

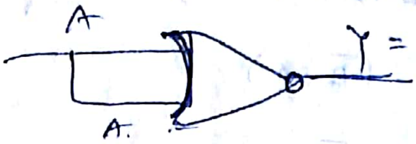
(Vi) X-NOR gate

Using NAND



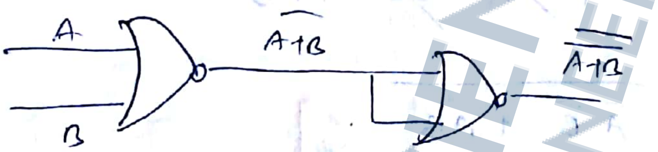
NOR as universal gate :-

(i) NOT gate :-



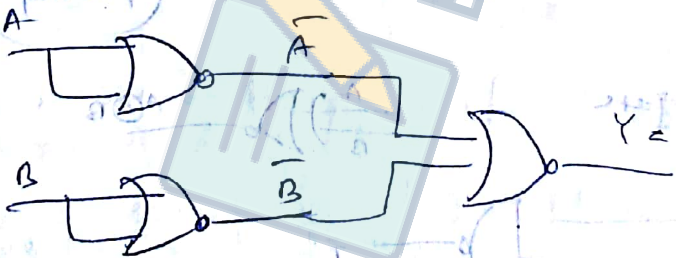
$Y = \overline{A+A} = \bar{A}$

(ii) OR Gate :-



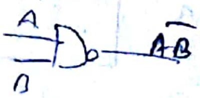
$\overline{\overline{A+B}} = A+B$

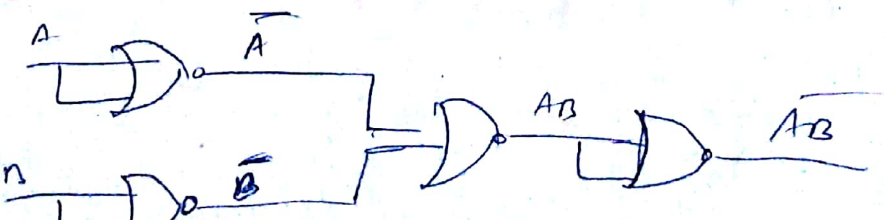
(iii) AND gate :-



$Y = \overline{\bar{A}+\bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = AB$

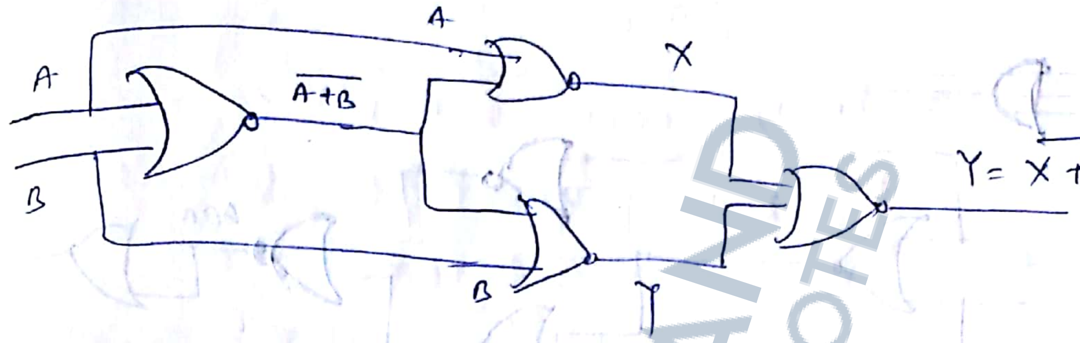
(iv) start NAND Gate





(v) Ex-Nor gate

$A \odot B = AB + \overline{A}\,\overline{B}$



$$X = \overline{A + \overline{A+B}}$$

$$= \overline{A} \cdot \overline{\overline{A+B}}$$

$$= \overline{A} \cdot (A+B)$$

$$= \overline{A} \cdot A + \overline{A}B$$

$$= \overline{A}B$$

$$Y = \overline{B + \overline{A+B}}$$

$$= (\overline{B}) \cdot (\overline{\overline{A+B}})$$

$$= (\overline{B}) \cdot (A+B)$$

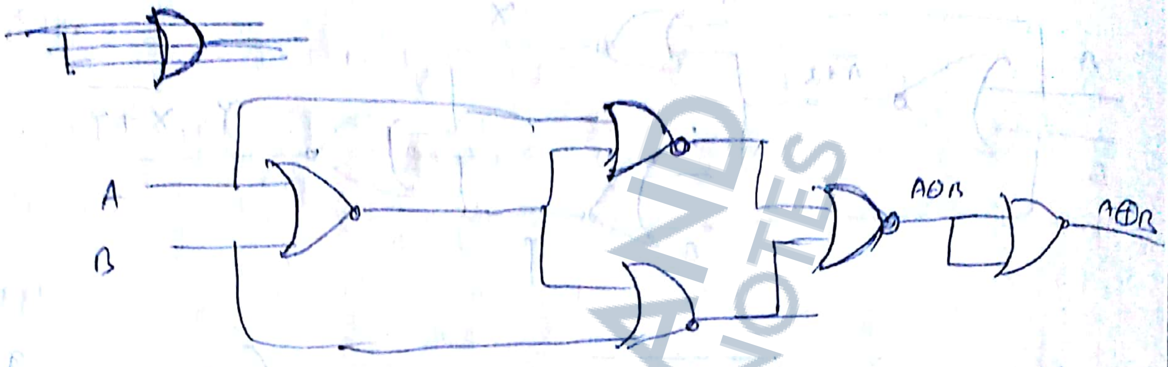
$$= \overline{B}A + \overline{B} \cdot B$$

$$= \overline{B}A$$

$$Y = \overline{X+Y} = \overline{\overline{A}B + \overline{B}A} = \overline{A \oplus B} = A \odot B$$

(Proved)

(vi) Ex-OR Gate



Q. Draw X-OR gate using basic gates

$Y = A \oplus B = A\bar{B} + \bar{A}B$

Standard Forms of Boolean Expression:-

There are 2 forms of Boolean expressions; These 2 forms are

(i) SOP (Sum of the Product) form.

(ii) POS (Product of the Sum) form.

SOP form

This is an expression in which AND terms (product) ORed (summed) together.

ex:- $F = \bar{A}BC + ABC + A\bar{B}C + BC$

POS form :-

This is an expression in which OR terms (sum) ANDed (product) together.

Ex:- $F = (A+B)(\bar{C}+D)(\bar{D}+A)$

Conversion of a General expression to SOP form

Any logic expression can be changed to SOP form by applying Boolean Algebra technique

Ex-1, $A(B+CD) = AB + ACD$

Ex-2 $AB + B(CD+EF)$

$= AB + BCD + BEF$

Ex-3: $\overline{\overline{(A+B)} + C}$

$= \overline{\overline{(A+B)}} \cdot \bar{C}$

$= (A+B) \cdot \bar{C}$

$= A\bar{C} + B\bar{C}$

The Standard SOP form:- [Cannonical form]

A standard SOP expression is one in which all the variables in the domain appear in the each product term.

Step-I :- Multiply each non standard product term by a term made up of sum of a missing variable and its complement. This results in 2 product terms.

Step-2 :- Repeat step I, until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form.

Ex:- $A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$

we know
$$A + \bar{A} = 1$$
$$B + \bar{B} = 1$$
$$C + \bar{C} = 1$$
$$D + \bar{D} = 1$$

$$\therefore \quad A\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C}).(D+\bar{D}) + AB\bar{C}D$$

$$\cancel{= A\bar{B}CD + \bar{A}\bar{B}\,C\bar{D} +}$$

Consider

(1) $A\bar{B}C(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$

(2) $\bar{A}\bar{B}(C+\bar{C})(D+\bar{D})$

$= (\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C})(D+\bar{D})$

$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

(3) $A B \bar{C} D$ → It contains 4 variables.

∴ Finally,

$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$

$= A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$
$+ \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

Binary representation of a standard SOP form :-

Ex :- If $A\bar{B}.C.\bar{D} = 1$, what is $ABCD$?

Ans :- $A\bar{B}.C\bar{D} = 1$,

Since there product is 1, Individual terms must be 1. If any one would have zero, then their product would be zero.

$A = 1$

$\bar{B} = 1 \Rightarrow B = 0$

$C = 1$

$\bar{D} = 1 \Rightarrow D = 0$

$\boxed{AB.CD = 1010}$

Imp

A SOP expression is equal to 1 (True) only if one or more of the product terms in the expressions is equal to 1.

(Because $A \cup 1 = 1$) [At least one expression = 1
ex: $0 \cup 1 = 1$, $0 \cup 1 \cup 1 = 1$]

The Standard POS form (Cannonical POS form)

A Standard POS expression is one in which all the variables in the domain appear in each sum term in the expression.

Steps

1. Add to each nonstandard product term a term made up of the product of the missing variable and its complement. This results in 2 sum terms. (because we can add 0 to anything without changing its value).

2. Apply the rule $A+BC = (A+B)(A+C)$

3. Repeat step 1 until all resulting sum terms contain all variables in the domain in either complemented or uncomplemented form.

Ex: Convert the following Boolean expression into standard POS form:

$$(A+\bar{B}+C)(\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$$

Ans:

$$A+\bar{B}+C = \underbrace{A+\bar{B}+C}_{A}+\underbrace{D\bar{D}}_{BC}$$

$\because D\bar{D}=0$, we can add.

$$= (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$$

$A+BC = (A+B)(A+C)$

$$\bar{B}+C+\bar{D} = A\bar{A}+\bar{B}+C+\bar{D}$$

⊙

$$= \underline{\bar{B}+C+\bar{D}} + \underline{A\bar{A}}$$

$$= (\bar{B}+C+\bar{D}+A)\ (\bar{B}+C+\bar{D}+\bar{A})$$

→ $(A+\bar{B}+\bar{C}+D)$ it contains all the variables.

∴ <u>Finally</u>

$(A+\bar{B}+C)\ (\bar{B}+C+\bar{D})\ (A+\bar{B}+\bar{C}+D)$ Can be converted into standard POS form as:

$$= (A+\bar{B}+C+D)\ (A+\bar{B}+C+\bar{D})\ (\bar{B}+C+\bar{D}+A)(\bar{B}+C+\bar{D}+\bar{A})\ (A+\bar{B}+\bar{C}+D)$$

$$= (A+\bar{B}+C+D)\ (A+\bar{B}+C+\bar{D})\ (A+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})\ (A+\bar{B}+\bar{C}+D)$$

Converting Standard SOP to Standard POS

<u>Step I</u>:- Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product term.

<u>Step 2</u>:- Determine all of the binary number not included in the evaluation of Step I.

<u>Step 3</u>:- Write the equivalent sum term for each binary number from step 2 and expression in

POS form. ⓪

Ex:- $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$

Ans: SOP 000 + 010 + 011 + 101 + 111

i.e 0, 2, 3, 5, 7

So left 1, 4, 6

001, 100, 110

$\cancel{(\bar{A}+B+C)}$

POS $(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

(In POS form $0 \rightarrow A$, $1 \rightarrow \bar{A}$

In SOP form $0 \rightarrow \bar{A}$, $1 \rightarrow A$)

Q) Convert POS → SOP $(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C) \rightarrow$? SOP

Q) Convert $\bar{A}B + A\bar{B}$ = ? into POS.

→ (A POS expression is equal to 0 only if one or more of sum terms in the expression is equal to 0)

Ex) Determine the binary value of the variables for which following POS expression is equal to 0.

$(A+B+C+D)(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

Ans :- The total expression = 0, if any one of them is 0.

If $A+B+C+D = 0 \Rightarrow A=0, B=0, C=0, D=0$

If $A+\overline{B}+\overline{C}+D = 0 \Rightarrow A=0, B=1, C=1, D=0$

If $\overline{A}+\overline{B}+\overline{C}+\overline{D} = 0 \Rightarrow A=1, B=1, C=1, D=1$

Min term :-

~~Min~~ → Minterm is a product terms which contain all the variables either in true form or complement form.

$$F(A,B,C) = ABC + \overline{A}BC + B\overline{C} + AC$$

min term.

N.B. - For n I/P variable, we have 2^n min terms.

$4 \rightarrow 2^4 \rightarrow 16$ minterms.

Minimal SOP form :- It is an SOP expression containing min product term which can't be further simplified.

Max term:-

Maxterm is sum term which contains all the variables given in the expression either in true form or in complement form.

ex. $F(A,B,C) = (A+B+C)(B+\bar{C})$

Minimal POS form :- It is a POS expression containing minm sum term which can't be further simplified.

Min term & max term representation

For 3 i/p binary variable function, we have $2^3 = 8$ min terms as well as max terms.

x y z	min term (m_r)	Max term (M_r)
0 0 0	$\bar{x}\bar{y}\bar{z}$ (m_0)	$(x+y+z)$ M_0
0 0 1	$\bar{x}\bar{y}z$ (m_1)	$(x+y+\bar{z})$ M_1
0 1 0	$\bar{x}y\bar{z}$ (m_2)	$(x+\bar{y}+z)$ M_2
0 1 1	$\bar{x}yz$ (m_3)	$(x+\bar{y}+\bar{z})$ M_3
1 0 0	$x\bar{y}\bar{z}$ (m_4)	$(\bar{x}+y+z)$ M_4
1 0 1	$x\bar{y}z$ (m_5)	$(\bar{x}+y+\bar{z})$ M_5
1 1 0	$xy\bar{z}$ (m_6)	$(\bar{x}+\bar{y}+z)$ M_6
1 1 1	xyz (m_7)	$(\bar{x}+\bar{y}+\bar{z})$ M_7

Imp

→ For n input variables we have 2^n min as well as max term.

→ Min term is a product term whose value equal to 1 i.e denoted by

$$\sum_{i=0}^{2^n-1} m_i = 1$$, i.e either of them 1.

Given:

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

e.g

$$f(A,B,C) = \Sigma(0,2,4,6)$$

i.e

$$= 000 + 010 + 100 + 110$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$

→ Find the truth table ?

→ Max term is a sum term whose value equal to 0, i.e denoted by

$$\prod_{i=0}^{2^n-1} M_i = 0$$, i.e either of them is 0.

Given:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Find the truth table.

e.g

$$F(A,B,C) = \Pi(0,2,4,7)$$

$$= (000)\,(010)\,(100)\,(111)$$

$$= \cancel{(\bar{A}+\bar{B}+\bar{C})}$$

$$= (A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$$

→ Min term & Max term are complement to each other.

$$\Sigma(0,2,4,6) = \Pi(1,3,5,7)$$

→ Minimization,

Ex - 1 (Simplify the following expression)

(i) $AB C + A\bar{B}C + AB\bar{C}$

Ans:

$$ABC + A\bar{B}C + AB\bar{C}$$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB \cdot 1 + A\bar{B}C$$

$$= AB + A\bar{B}C$$

$$= A(B + \bar{B}C)$$

$$= A[(B+\bar{B})(B+C)]$$

$$= A[(1)(B+C)]$$

$$= A(B+C)$$

$$= AB + AC$$ (Ans) → (Minimized expression)

formula

$A + BC = (A+B)(A+C)$

(ii) $(A+\bar{B}+C)(\bar{A}B\bar{C}D)$

$$= A.(\bar{A}B\bar{C}D) + \bar{B}\,\bar{A}B\bar{C}D + C\cdot\bar{A}B\bar{C}D$$

$$= 0 + 0 + 0$$

$$= 0$$

$A\bar{A} = 0$

$B\bar{B} = 0$

$C\bar{C} = 0$

(iii) $\bar{x}\bar{y}z + yz + xz$

$$= \bar{x}\bar{y}z + z(x+y)$$

$$= z[x+y + \bar{x}\bar{y}]$$

$$= z[(x+y) + \overline{x+y}]$$

$= z\,[\cancel{\ldots}]$ $\quad\because A + \bar{A} = 1$

$= z \cdot [1]$

$= z$ (Ans)

(iv) $AB + \bar{B}C + AC$

~~$A(B+C) + \bar{B}C$~~

$= AB + \bar{B}C + AC \cdot 1$

$= AB + \bar{B}C + AC(B + \bar{B})$

$= AB + \bar{B}C + ABC + A\bar{B}C$

$= AB(1 + C) + \bar{B}C(1 + A)$

$= AB + \bar{B}C$

See $AB + \bar{B}C + AC = AB + \bar{B}C$,

one term is eliminated (i.e. AC)

Trick :- See one term is complemented i.e. $\bar{B}C$ for ans. Keep the complemented term & its corresponding uncomplemented term, eliminate the other one.

$AB + \bar{B}C + AC$

$\bar{B}C \longrightarrow$ Complemented term

$AB \longrightarrow$ Corresponding uncomplemented term.

Keep these two.

Ans $\rightarrow AB + \bar{B}C$.

$$\cancel{= Z\,[\ldots]}$$

$$= Z \cdot [1]$$

$$= Z \quad \text{(Ans)}$$

$\because A + \bar{A} = 1$

(iv)

$$AB + \bar{B}C + AC$$

$$\cancel{A(B+C) + \bar{B}C}$$

$$= AB + \bar{B}C + AC \cdot 1$$

$$= AB + \bar{B}C + AC(B + \bar{B})$$

$$= AB + \bar{B}C + ABC + A\bar{B}C$$

$$= AB(1 + C) + \bar{B}C(1 + A)$$

$$= AB + \bar{B}C$$

See $AB + \bar{B}C + AC = AB + \bar{B}C$,

one term is eliminated (i.e. $\underline{AC}$)

Trick:- See one term is complemented i.e. $\bar{B}C$ for ans. Keep the complemented term & its corresponding uncomplemented term, eliminate the other one.

$AB + \bar{B}C + AC$

→ Complemented term

→ Corresponding uncomplemented term.

Keep these two.

Ans → $AB + \bar{B}C$.

(v) Try

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

Ans:

$$AB + \bar{A}C + BC \cdot 1$$

$$= AB + \bar{A}C + BC(A + \bar{A})$$

$$= AB + \bar{A}C + ABC + \bar{A}BC$$

$$= AB(1 + C) + \bar{A}C(1 + B)$$

$$= AB + \bar{A}C$$

(v) Try

$$AB + B\bar{C} + A\bar{C} = B\bar{C} + AC$$

(vi) Prove that

$$ABCD + AB\bar{C}\bar{D} + ABC\bar{D} + AB\bar{C}D + ABCDE + AB\bar{D}E$$

$$= AB$$

Ans:- $ABCD + AB(\bar{C}\bar{D}) + ABC\bar{D} + AB\bar{C}D$

$+ ABCDE + AB\bar{D}E$

$$= ABC(D + \bar{D}) + AB(\bar{C} + \bar{D}) + AB\bar{C}D$$

$+ ABCDE + AB\bar{D}E$

$$= ABC + AB\bar{C} + AB\bar{D} + AB\bar{C}D + ABCDE + AB\bar{D}E$$

$$= ABC + AB\bar{C}(1 + D) + AB\bar{D}(1 + E) + ABCDE$$

$$= ABC + AB\bar{C} + AB\bar{D} + ABCDE$$

$= ABC(1+DE) + AB\bar{C} + AB\bar{D}$

$= ABC + AB\bar{C} + AB\bar{D}$

$= AB(C+\bar{C}) + AB\bar{D}$

$= 1 \cdot AB + AB\bar{D}$

$= AB(1+\bar{D})$

$= AB$ (Ans).

Prove that

(ii) $\overline{AB+BC+CA} = \cancel{\bar{A}\bar{B}+\bar{B}\bar{C}+\bar{A}\bar{C}}$

$= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}$

Proof:

$\overline{AB+BC+CA}$

$= \overline{AB} \cdot \overline{BC} \cdot \overline{CA}$ (Demorgan's thm $\overline{A+B} = \bar{A}\cdot\bar{B}$)

$= (\bar{A}+\bar{B}) \cdot (\bar{B}+\bar{C}) \cdot (\bar{C}+\bar{A})$ (Again Demorgan's thm $\overline{A \cdot B} = \bar{A}+\bar{B}$)

multiply

$= (\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\cdot\bar{B} + \bar{B}\bar{C})(\bar{A}+\bar{C})$

$= (\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B} + \bar{B}\bar{C})(\bar{A}+\bar{C})$

$\cancel{= (\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{B}(1+\bar{C})}$

$= [\bar{B}(1+\bar{A}+\bar{C}) + \bar{A}\bar{C}](\bar{A}+\bar{C})$

$= [\bar{B} + \bar{A}\bar{C}](\bar{A}+\bar{C})$

$= \bar{B}\bar{A} + \bar{B}\bar{C} + \bar{A}\bar{C}\bar{A} + \bar{A}\bar{C}\bar{C}$

$= \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}\bar{C}$

$A \cdot A = A$

$A + A = A$

①

$\rightarrow \overline{AB+BC+CA} = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

$\because A+A=A$

(proved)

Alternative Method for Converting SOP → POS

1) Take the complement of given SOP expression & expand using Demorgan's th^m.

2) Simplify the above expression using th^m of Boolean algebra.

3) Take once again complement of the simplified expression obtained in step 2, to get the POS form.

Ex :- $F = A\bar{B} + \bar{A}B$

Step I

$\bar{F} = \overline{A\bar{B} + \bar{A}B}$

$= \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

$= (\bar{A} + \bar{\bar{B}})(\bar{\bar{A}} + \bar{B})$

$= (\bar{A} + B)(A + \bar{B})$

Step II

$\bar{F} = \cancel{\bar{A}\cdot A}^{\,0} + \bar{A}\bar{B} + AB + \cancel{B\bar{B}}^{\,0}$

$= AB + \bar{A}\bar{B}$

Step 3

$\bar{\bar{F}} = \overline{AB + \bar{A}\bar{B}}$

$= \overline{AB} \cdot \overline{\bar{A}\bar{B}}$

$A = A \cdot A$

$A = A + A$

$= (\bar{A} + \bar{B})(\bar{\bar{A}} + \bar{\bar{B}})$

$\Rightarrow \boxed{F = (\bar{A}+\bar{B})(A+B)}$ POS form

POS → SOP

Step 1 : Expand the POS expression

Step 2 : Simplify using theorems of Boolean algebra.

Ex : $F = (\bar{A}+\bar{B})(A+B)$

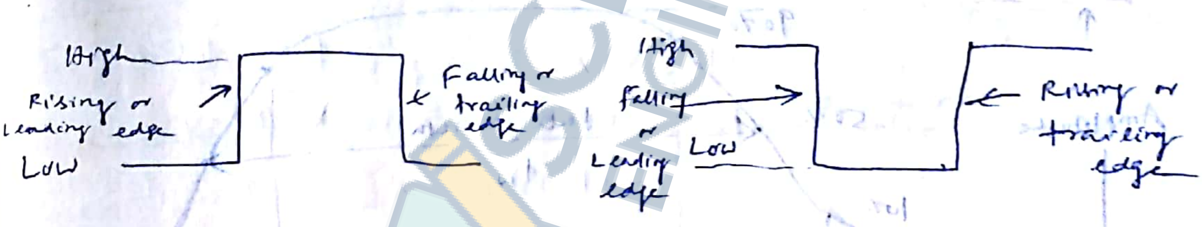
Step 1 :

$F = \cancel{\bar{A}.A}^{0} + \bar{A}.B + \bar{B}.A + \cancel{B\bar{B}}^{0}$

Step 2 : $= \bar{A}B + A\bar{B}$ (SOP form)

Q) Design $A+B\bar{C}$ using NAND / NOR.

Digital waveform :-



(a) +ve going pulse (b) -ve going pulse.

→ A positive going pulse is generated when the voltage (current) goes from its normally low level to its high & then backs to low level.

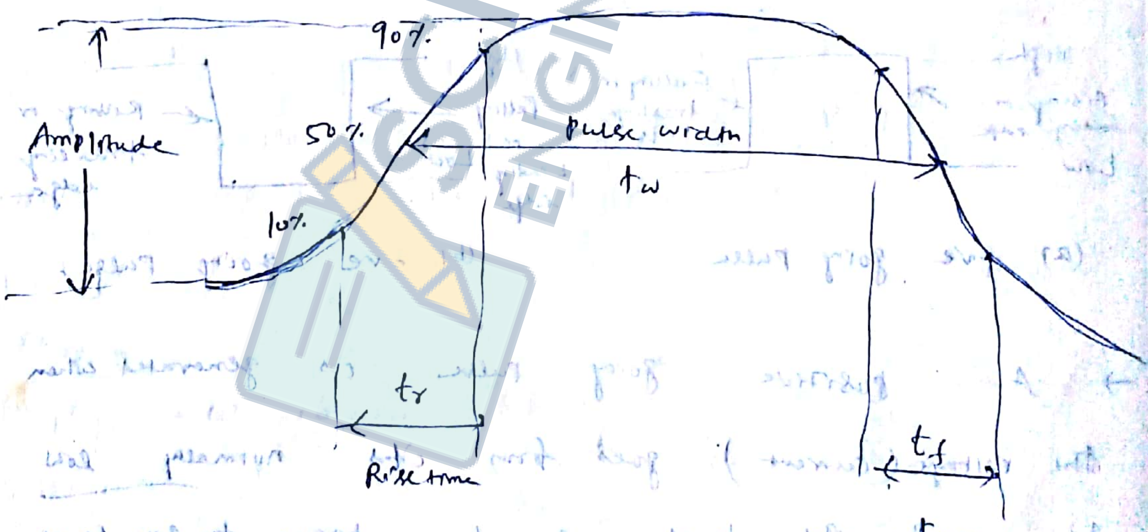
→ A -ve going pulse is generated when the voltage goes from its normally High level to its low level & back to high level

t_r (Rise time)

The time required for the pulse to go from its low level to high level is called rise time (t_r). Practically, it is the measure of time from 10% of pulse amplitude to 90% of pulse amplitude.

t_f (Fall time)

The time required for the transition from High level to Low level is called the fall time (t_f). Practically it is the measure of time from 90% to 10% of pulse amplitude.



$$\text{Duty cycle} = \frac{t_w}{T} = \frac{\text{Pulse width}}{\text{Time period}}$$

$f = \frac{1}{T}$ i.e. $\text{frequency} = \frac{1}{\text{Time period}}$

Pulse width:-

It is a measure of the duration of the pulse and is often defined as the time interval between 50% point on the rising & falling edge.

20

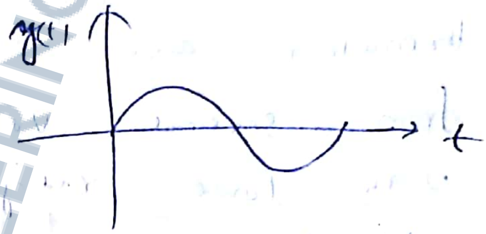
2010 Back

1) (a) Analog, digital, discrete time signal ?

Analog:-

The analog signal is that type of signal which varies continuously with certain interval of time.

e.g $y(t) = \sin t$



Digital:-

Digital signal is that type of signal which represented as sequence of numbers i.e magnitude at an instant of time.

e.g :- Pulse train shown.

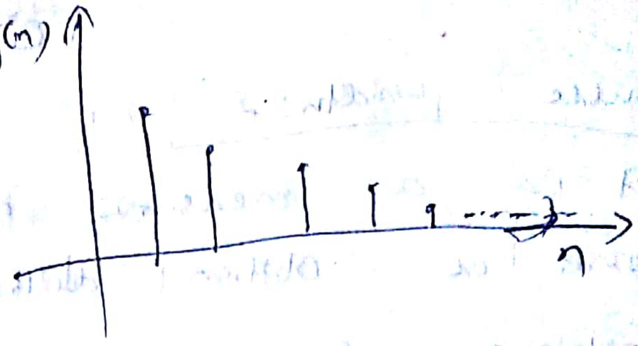
High = +5V (called logic 1)

low = 0V (called logic 0)

Discrete-Time Signal:-

It is defined at certain specific values of time. The time instant need not be equidistant in practice. They are usually taken as equally ~~interval at~~ spaced intervals for computational convenience and mathematical help.

$$y(n) = \begin{cases} 0.2^n, & \text{for } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



n = integer.

b) P-N junction diode made up of which material (Si, Ge, GaAs) will have highest thermal stability? why?

Ans :- P-N junction diode made up of Si material will have highest thermal stability, because Silicon diodes operated in Avalanche breakdown are available with maintaining voltages from several volts to several hundred volts with power rating up to 50 watts. They can operates in high temperature.

Internal 2 - 4) An amplifier operating from ±5 V supplies provides a 3.2 V peak sine wave across 50Ω load, when provided with 0.4 V peak i/p from which 2.0 mA peak is drawn. The avg current in each supply is measured to be 30 mA. Find voltage gain, current gain & power gain expressed in dB. as well as supply power, amplifier dissipation & amplifier efficiency.

Logic Gates & Boolean Algebra

1) BPUT 2018

Prove that

(b) $\overline{AB+AC} + A\bar{B}C = \bar{A}+\bar{B}$

Ans ⇒

$\overline{AB+AC} + A\bar{B}C$

$= \overline{AB}\cdot\overline{AC} + A\bar{B}C$ ∵ $\overline{A+B} = \bar{A}\cdot\bar{B}$

$= (\bar{A}+\bar{B})(\bar{A}+\bar{C}) + A\bar{B}C$

∵ $\bar{A}\cdot\bar{A} = \bar{A}$

$= \bar{A}\cdot\bar{A} + \bar{A}\cdot\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{B}C$

$= \underbrace{\bar{A} + \bar{A}\bar{C}} + \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{B}C$

$= \bar{A}(1+\bar{C}) + \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{B}C$

$= \underbrace{\bar{A} + \bar{A}\bar{B}} + \bar{B}\bar{C} + A\bar{B}C$

$= \bar{A}(1+\bar{B}) + \bar{B}\bar{C} + A\bar{B}C$

$= \bar{A} + \bar{B}\bar{C} + A\bar{B}C$

$= \bar{A} + \bar{B}(\bar{C}+AC)$ ∵ $A+BC = (A+B)(A+C)$

$= \bar{A} + \bar{B}\left[(\bar{C}+A)(\bar{C}+C)\right]$ $\bar{C}+C=1$

$= \bar{A}+\bar{B}(\bar{C}+A)$

$= \bar{A}+\bar{B}\bar{C} + A\bar{B}$

$= (\bar{A}+A)(\bar{A}+\bar{B}) + \bar{B}\bar{C}$

$= 1\cdot(\bar{A}+\bar{B}) + \bar{B}\bar{C} = \bar{A}+\bar{B}+\bar{B}\bar{C}$

$= \cancel{\bar{A} + (\bar{B}+\bar{B})(\bar{B}+\bar{C})}$

$\overline{AB+AC} + A\bar{B}C$

$= \bar{A} + \bar{B} + \bar{B}C$

$= \bar{A} + \bar{B}(1+\bar{C})$ $\quad | \quad 1+\bar{C} = 1$

$= \bar{A} + \bar{B}$

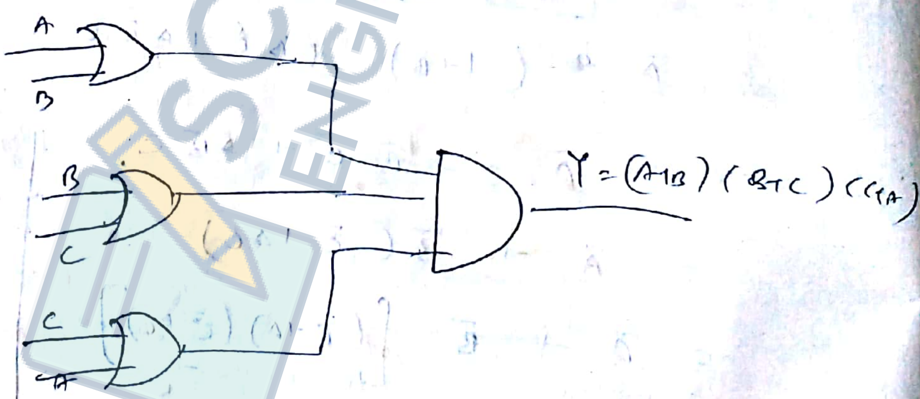
(proved)

2) (a) Draw the logic diagram for the boolean expression

$Y = (A+B)(B+C)(C+A)$

(b) Simplify the above equation & draw the logic diagram for simplified expression.

Ans :-



(ii) $(A+B)(B+C)(C+A)$

$= (AB + AC + B.B + BC)(C+A)$

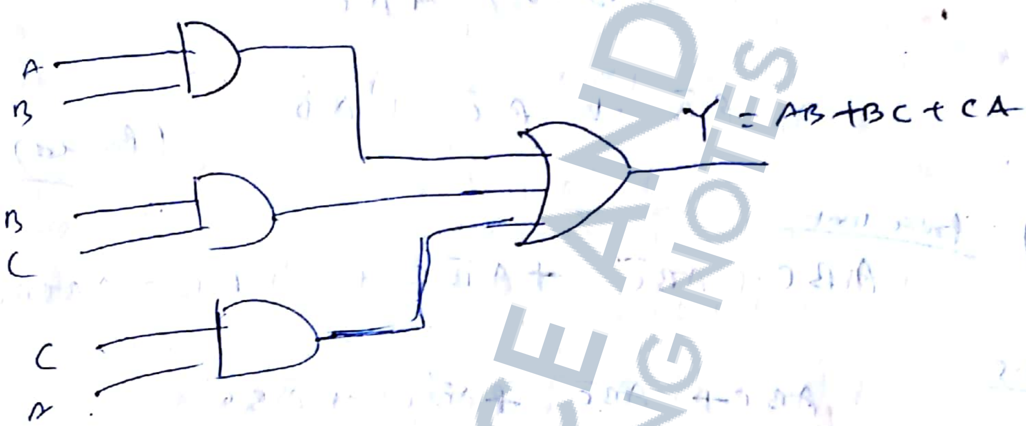
$= (AB + AC + B + BC)(C+A)$

$= (B(1+A+C) + AC)(C+A)$

$$(A+B)(B+C)(C+A) = (B+AC)(C+A)$$

$$= BC + BA + A \cdot C \cdot C \quad \mid \quad C \cdot C = C$$

$$= AB + BC + AC$$



3) Prove that

$$A\bar{B} + AB\bar{D} + AB\bar{C}D = A\bar{B} + A\bar{C} + A\bar{D}$$

Proof

$$A\bar{B} + AB\bar{D} + AB\bar{C}D$$

$$= A\bar{B} + AB[\bar{D} + \bar{C}D]$$

$$= A\bar{B} + AB[(\bar{D}+\bar{C})(\bar{D}+D)]$$

$$= A\bar{B} + AB[(\bar{C}+\bar{D})]$$

$$= A\bar{B} + AB\bar{C} + AB\bar{D}$$

$$= A(\bar{B} + B\bar{C}) + AB\bar{D}$$

$$= A[(\bar{B}+B)(\bar{B}+\bar{C})] + AB\bar{D}$$

$$= A[\bar{B}+\bar{C}] + AB\bar{D}$$

$$= A\bar{B} + A\bar{C} + AB\bar{D}$$

$$= A(\bar{B} + B\bar{D}) + A\bar{C}$$

=)

$A\bar{B} + AB\bar{D} + AB\bar{C}D$

$= A(\bar{B} + B\bar{D}) + A\bar{C}$

$= A\left[(\bar{B}+B)(\bar{B}+\bar{D})\right] + A\bar{C}$

$= A(\bar{B}+\bar{D}) + A\bar{C}$

$= A\bar{B} + A\bar{C} + A\bar{D}$ (Proved)

4) Prove that

$ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC = AB + BC + CA$

L.H.S

$ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC$

$= AB(C+\bar{C}) + A\bar{B}C + \bar{A}BC$

$= AB + A\bar{B}C + \bar{A}BC$

$= A(B + \bar{B}C) + \bar{A}BC$

$= A\left[(B+\bar{B})(B+C)\right] + \bar{A}BC$

$= A(B+C) + \bar{A}BC$

$= AB + AC + \bar{A}BC$

$= AB + C(A + \bar{A}B)$

$= AB + C(A+\bar{A})(A+B)$ $\quad \because A+\bar{A}=1$

$= AB + C(A+B)$

$= AB + AC + BC$

$=$ R.H.S (Proved)

①

1 Find the complement of the function given below and implement using logic gates.

$$F = \bar{x}(\bar{y}+\bar{z})(x+y+\bar{z})$$

$$\bar{F} = \overline{\bar{x}(\bar{y}+\bar{z})(x+y+\bar{z})}$$

$$= \bar{\bar{x}} + \overline{(\bar{y}+\bar{z})} + \overline{(x+y+\bar{z})}$$

$$= \cancel{x +} (\bar{\bar{y}} . \bar{\bar{z}}) + (\bar{x} . \bar{y} . \bar{\bar{z}})$$

$$= x + (\bar{\bar{y}} . \bar{\bar{z}}) + (\bar{x} . \bar{y} . \bar{\bar{z}})$$

$$= x + yz + (\bar{x}\bar{y}z)$$

$$= x + z(y + \bar{x}\bar{y})$$

$$= x + z[(y+\bar{x})(y+\bar{y})]$$

$$= x + yz + \bar{x}z$$

$$= (x+\bar{x})(x+z) + yz$$

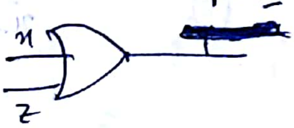
$$= (x+z) + yz$$

$$= x + z(1+y)$$

$$\bar{F} = x + z$$

$\overline{ABC} = \bar{A}+\bar{B}+\bar{C}$

$\overline{A+B} = \bar{A}.\bar{B}$



6) Simplify the following expression

$$F(x,y,z) = (x+y)\left[\overline{\bar{x}(\bar{y}+\bar{z})}\right] + \overline{xy} + \bar{x}.\bar{z}$$

$$(x+y)\left[\overline{\bar{x}(\bar{y}+\bar{z})}\right] + \overline{xy} + \bar{x}\cdot\bar{z}$$

$$= (x+y)\left[\bar{\bar{x}} + \overline{(\bar{y}+\bar{z})}\right] + \overline{xy} + \bar{x}.\bar{z}$$

$\overline{AB} = \bar{A}+\ldots$

$$= (x+y)(x+\bar{\bar{y}}.\bar{\bar{z}}) + \overline{xy} + \bar{x}.\bar{z}$$

$$= (x+y)(x+yz) + \overline{xy} + \bar{x}.\bar{z}$$

$$= (x.x + xyz + xy + yyz) + \overline{xy} + \bar{x}.\bar{z}$$

$$= (x + xy + xyz + yz) + \overline{xy} + \bar{x}.\bar{z}$$

$$= [x(\underbrace{1+y+yz}_{1}) + yz] + \overline{xy} + \bar{x}.\bar{z}$$

$$= (x+yz) + \overline{xy} + \bar{x}.\bar{z}$$

$$= x + \bar{x}\bar{z} + yz + \overline{xy}$$

$$= (x+\bar{x})(x+\bar{z}) + yz + \overline{xy}$$

$$= x + \bar{z} + yz + \bar{x} + \bar{y}$$

$$= \bar{x} + \bar{y} + \bar{z} + yz$$

$$= \bar{x} + \bar{y} + (\bar{z}+z).(\bar{z}+y)$$

$$= \bar{x} + \bar{y} + \bar{z} + y$$

$$= 1 + \bar{x} + \bar{z} = 1 \quad \text{(Ans)}$$

7) If $\bar{A}B + A\bar{B} = C$

Then prove $\bar{A}C + A\bar{C} = B$

Proof: $A\bar{C} + \bar{A}C$

$= A\left(\overline{\bar{A}B + A\bar{B}}\right) + \bar{A}\left(\bar{A}B + A\bar{B}\right)$ | $\because C = \bar{A}B + A\bar{B}$

$= A\left(\overline{\bar{A}B} \cdot \overline{A\bar{B}}\right) + \bar{A}\cdot\bar{A}\cdot B + \cancel{\bar{A}\cdot A}\cdot\bar{B}$ 0

$= A\left((\bar{\bar{A}} + \bar{B})(\bar{A} + \bar{\bar{B}})\right) + \bar{A}B$

$= A\left((A + \bar{B})(\bar{A} + B)\right) + \bar{A}B$

$= A\left[\cancel{A\bar{A}}^{0} + AB + \bar{B}\bar{A} + \cancel{\bar{B}B}^{0}\right] + \bar{A}B$

$= A\left[AB + \bar{A}\bar{B}\right] + \bar{A}B$

$= AAB + \cancel{A\bar{A}}^{0}\bar{B} + \bar{A}B$

$= AB + \bar{A}B$

$= B(A + \bar{A})$

$= B$ (proved)

8) Simplify the logic expression & draw the logic diagram for the simplified expression.

$(\bar{x} + xy\bar{z}) + (\bar{x} + xy\bar{z})(x + \bar{x}\bar{y}z)$

$$(\bar{x} + xy\bar{z}) + (\bar{x} + xy\bar{z})(x + \bar{x}\bar{y}z)$$

$$= (\bar{x} + xy\bar{z})\left[1 + (x + \bar{x}\bar{y}z)\right]$$

$$= \underset{A}{\bar{x}} + \underset{B}{xy}\,\underset{C}{\bar{z}}$$

$$= (\bar{x} + x)(\bar{x} + y\bar{z})$$

$$= \cancel{\bar{x} + y\bar{z}}$$

$$= \bar{x} + y\bar{z} \quad \text{(Ans)}$$

9) Given $F = A(B+\bar{C}) + D$

express as

(a) Minimal SOP (ii) minimal POS

Ans:

$$A(B+\bar{C}) + D$$

$$= AB + A\bar{C} + D$$

The above expression is itself minimal SOP, since further reduction is not possible.

∴ (i) Minimal SOP $= AB + A\bar{C} + D$.

(ii)

$$F = A(B+\bar{C}) + D$$

$$\bar{F} = \overline{A(B+\bar{C}) + D}$$

$$= \overline{AB + A\bar{C}} \cdot \bar{D}$$

$$\Rightarrow \bar{F} = \overline{AB} \cdot \overline{A\bar{C}} \cdot \bar{D}$$

$$= (\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{\bar{C}}) \cdot \bar{D}$$

$$= (\bar{A} + \bar{B})\,(\bar{A} + C) \cdot \bar{D}$$

$$= (\bar{A}\cdot\bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}C)\,\bar{D}$$

$$= (\bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}C)\,\bar{D}$$

$$= [\bar{A}(1 + C + \bar{B}) + \bar{B}C] \cdot \bar{D}$$

$$= [\bar{A} + \bar{B}C]\,\bar{D}$$

$$= \bar{A}\bar{D} + \bar{B}C\bar{D}$$

$$\bar{\bar{F}} = \overline{\bar{A}\bar{D} + \bar{B}C\bar{D}}$$

$$\Rightarrow F = \overline{\bar{A}\bar{D}} \cdot \overline{\bar{B}C\bar{D}}$$

$\overline{A+B} = \bar{A}\cdot\bar{B}$

$$= (\bar{\bar{A}} + \bar{\bar{D}})(\bar{\bar{B}} + \bar{C} + \bar{\bar{D}})$$

$$F = (A + D)(B + \bar{C} + D)$$

= Minimal POS form.

Q) Minimize the following expression

ex: $F = \Sigma(4, 5, 6, 7)$

Ans:

P.T.O

$F = \Sigma(4,5,6,7)$

$= \cancel{0100 + 0}$

$= 100 + 101 + 110 + 111$

$= \underbrace{A\bar{B}\bar{C} + A\bar{B}C} + \underbrace{AB\bar{C} + ABC}$

$= A\bar{B}(C+\bar{C}) + AB(C+\bar{C})$

$= A\bar{B} + AB$

$= A(B+\bar{B})$

$\boxed{F = A}$

11) Convert following expression into POS form.

$F = \Sigma(0,1,2,6)$

Ans : $F = \pi(3,4,5,7)$

(∵ $F = m_0 + m_1 + m_2 + m_6$

$\cancel{\bar{F} = \overline{m_0 + m_1 + m_2 + m_6}}$

$\cancel{= \bar{m}_0 \cdot \bar{m}_1 \cdot \bar{m}_2 \cdot \bar{m}_6}$

$\bar{F} = m_3 + m_4 + m_5 + m_7$

$\bar{\bar{F}} = \overline{m_3 + m_4 + m_5 + m_7}$

$= \bar{m}_3 \cdot \bar{m}_4 \cdot \bar{m}_5 \cdot \bar{m}_7$

$F = M_3 \cdot M_4 \cdot M_5 \cdot M_7$

$F = 011 \cdot 100 \cdot 101 \cdot 111$

$F = (A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})$

12) Simplify the following boolean expression

$F = \Sigma(0, 2, 4, 6)$

~~$F = 000 + 0010 + 0100 +$~~

$F = 000 + 010 + 100 + 110$

$= \underbrace{\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}} + \underbrace{A\bar{B}\bar{C} + AB\bar{C}}$

$= \bar{A}\bar{C}(\bar{B} + B) + A\bar{C}(\bar{B} + B)$

$= \bar{A}\bar{C} + A\bar{C}$

$= \bar{C}(A + \bar{A})$

$= \bar{C}$ (Ans).

13) Realize the following expression using

(a) Basic logic gates.

(b) Universal logic gates.

~~$F = ABC + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}$~~

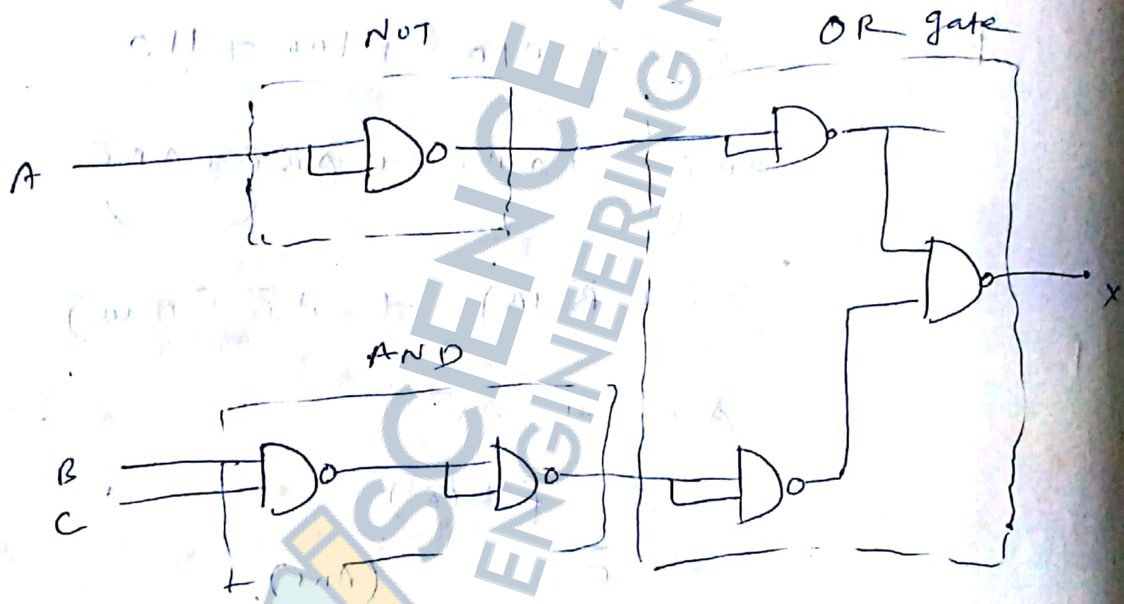
$F = \bar{A} + BC$

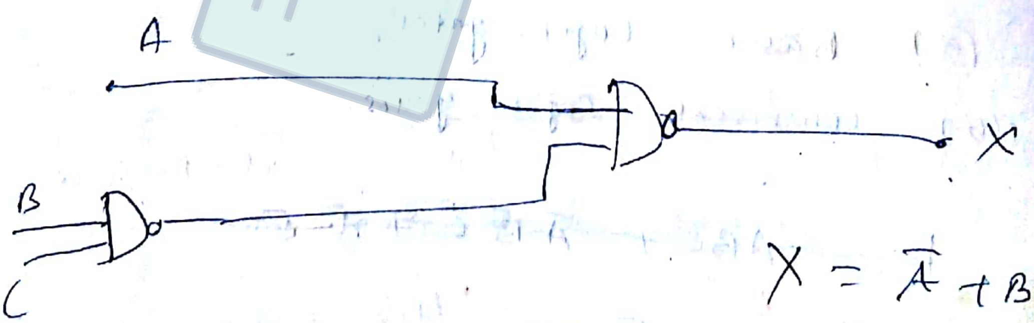
(a) using Basic logic gates:-

(i)

$F = \bar{A} + BC$

Replacing each gate by corresponding NAND gate

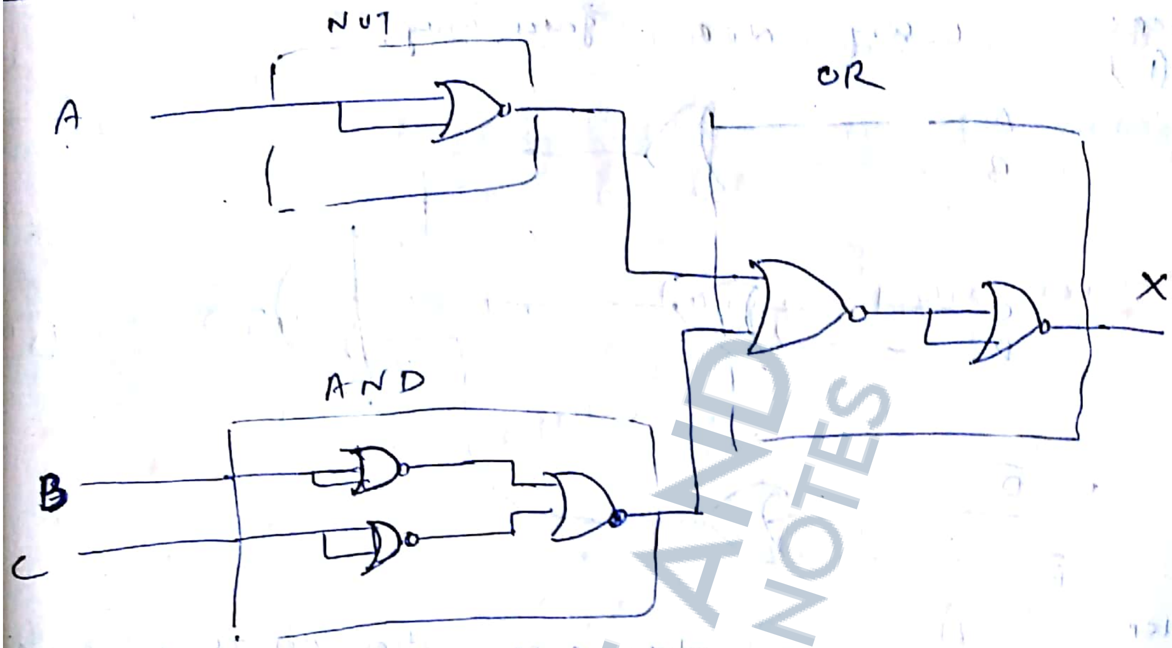




$X = \bar{A} + BC$

($\because$ 2 Not gates cancel each other $\bar{\bar{A}} = A$)

Replacing each gate by NOR gate



$X = \bar{A} + BC$

Note:-

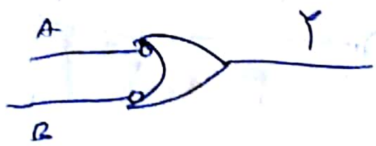
→ Bubbled AND = NOR

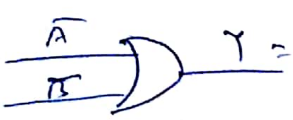
→ Bubbled OR = NAND

Bubbled AND :-

$Y = \bar{A} \cdot \bar{B} = \overline{A+B} = NOR$

Bubbled OR



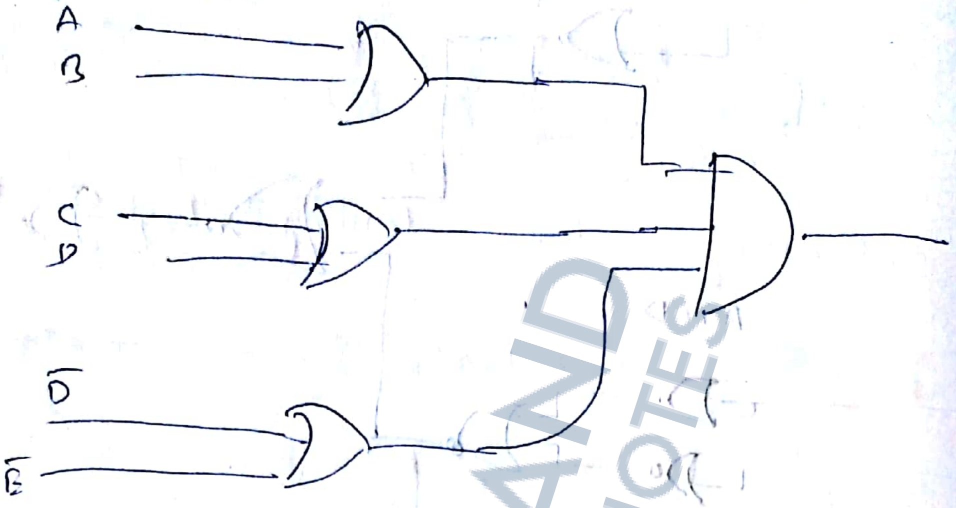


$Y = \bar{A} + \bar{B} = \overline{AB} = NAND.$

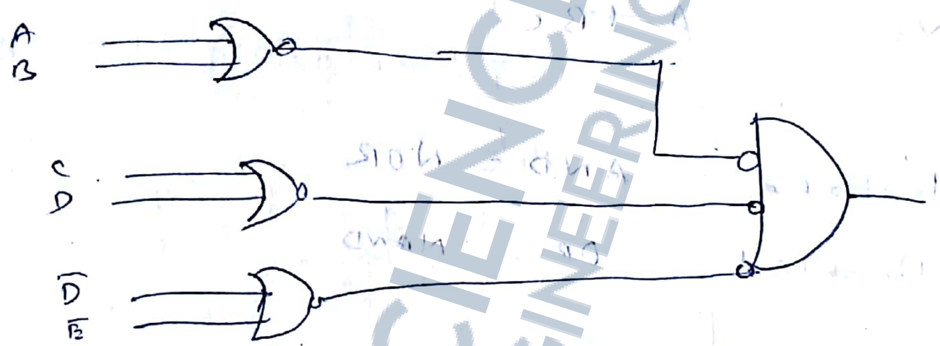
(4) Implement

$F = (A+B)\cdot(C+D)\cdot(\bar{D}+E)$.

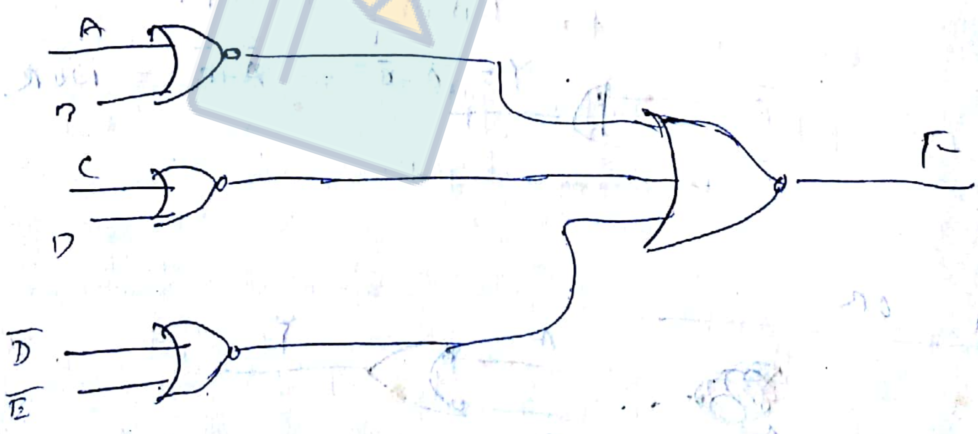
Step (1) Using NOR gates only!



Step (2) Adding ~~NOT~~ 2 NOT gate does not change ($\bar{\bar{A}} = A$)



(3) Bubbled AND = NOR



Another Method of Using NAND and NOR

Q) Implement $A+B\bar{C}$ Using Minm NAND gates & Minimum NOR gates.

Ans :

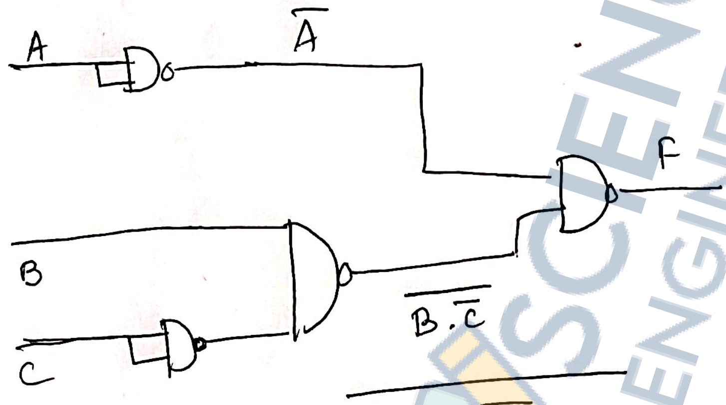
Minimum NAND gates

(express in product term bar)

$$F = A + B\bar{C}$$

$$F = \bar{\bar{F}} = \overline{\overline{A+B\bar{C}}}$$

$$\Rightarrow F = \overline{\bar{A} \cdot \overline{(B\bar{C})}}$$



$$F = \overline{\bar{A} \cdot \overline{B\bar{C}}}$$

Minimum NOR gates

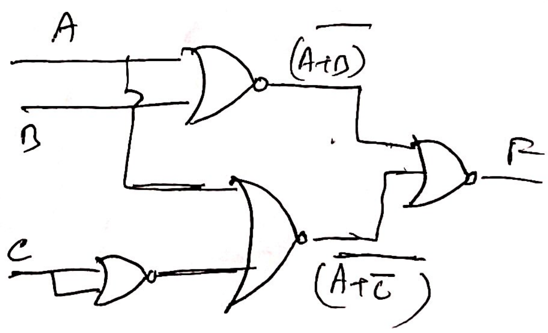
(express in sum term bar)

$$F = A + B\bar{C}$$

$$F = \bar{\bar{F}} = \overline{\overline{A+B\bar{C}}}$$

$$\Rightarrow F = \overline{\overline{(A+B)(A+\bar{C})}}$$

$$= \overline{\overline{(A+B)} + \overline{(A+\bar{C})}}$$



$$F = \overline{\overline{(A+B)} + \overline{(A+\bar{C})}}$$

Minimization of Boolean function

1) Using Karnaugh Map or (K-Map)

The Karnaugh map is used for simplifying Boolean expressions to their minimum form. A minimized SOP expression contains the fewest possible terms with fewest possible variables per term.

Mapping a Standard SOP expression

Step 1:- Determine the binary value of each product term in the standard SOP expression.

Step 2:- As each product term is evaluated, place 1 on the K-map in the cell having the same value as the product term.

Ex:-1 Map the following SOP expression on a K-Map.

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$$

Ans:- $\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$

Binary values → 001 010 110 111

AB \ C	0	1
00		1
01	1	
11	1	1
10		

001 i.e $\bar{A}\bar{B}C$

010 i.e $\bar{A}B\bar{C}$

111 i.e ABC

110 i.e $AB\bar{C}$

fig:- 3 Variable K-Map

2) <u>Map</u>

$\bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$

Ans:- 00 11   0100   1101   1111   1100   0001   1010

$\overrightarrow{CD}$, $\downarrow AB$

AB \ CD	00	01	11	10
00		1	1	
01	1			
11	1	1	1	
10				1

fig:- 4 Variable K-Map

<u>Note</u>:- 1) The cell in a K-Map are arranged so that there is only a single variable change between adjacent cells. <u>Adjacency</u> is defined by a single variable change. Cells that differ by only <u>one</u> variable are <u>adjacent</u>.

ex:- 010 cell is adjacent to <u>000, 110</u> — <u>Reason</u> 1 ~~bit~~ bit change

[fig 3] but not " 001, 111 — 2 bit change.

010

010 → 000, 010 → 110 (1 bit change)

010 ↔ 001, 010 → 111 (2 bit change)

2) A Boolean expression must <u>first be in standard form</u> before it is used for a K-Map. If an expression is <u>not in standard form</u>, then it must be converted <u>to standard form</u>. ~~by the~~

Ex-2

Map $\bar{A} + A\bar{B} + AB\bar{C}$ on a K-Map.

A:- First convert the expression into standard SOP form.

$$\bar{A} + A\bar{B} + AB\bar{C}$$

$$= \bar{A}(B+\bar{B})(C+\bar{C}) + A\bar{B}(C+\bar{C}) + AB\bar{C}$$

$$= \underbrace{\bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}}_{(1)} + A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$$

2nd method

Given SOP expression $\bar{A} + A\bar{B} + AB\bar{C}$.

To convert into standard SOP form.

for $\bar{A}$ → the missing terms are B & C.

So first write the binary value of the variable $\bar{A}$; then attach all possible values for missing variables B & C. as follows

$\bar{A}$	B	C	
0	0	0	→ $\bar{A}\bar{B}\bar{C}$
0	0	1	→ $\bar{A}\bar{B}C$
0	1	0	→ $\bar{A}B\bar{C}$
0	1	1	→ $\bar{A}BC$

← (2)

(1) & (2) are equal.

Similarly

AB	C	
1 0	0	$\rightarrow A\bar{B}\bar{C}$
1 0	1	$\rightarrow A\bar{B}C$

— (3)

Combining (2) & (3), we have

$$\bar{A} + A\bar{B} + AB\bar{C} = \underbrace{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC} + \underbrace{A\bar{B}\bar{C} + A\bar{B}C} + AB\bar{C}.$$

Mapping

$\bar{A}$ + $A\bar{B}$ + $AB\bar{C}$

i.e.

$\bar{A}$	$A\bar{B}$	$AB\bar{C}$
0 0 0	1 0 0	1 1 0
0 0 1	1 0 1	
0 1 0		
0 1 1		

AB \ C	0	1
00	1	1
01	1	1
11	1	
10	1	1

Ex: 3) Mapping the following SOP expression to K-Map.

$$\bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

AB \ CD	00	01	11	10
00	1	1		
01				
11	1	1		
10	1	1	1	1

$\bar{B}\bar{C} + A\bar{D} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$

1010 0001 1011

0 | 00 | 0   10 | 00   11 | 0 | 0
0 | 00 | 1   10 | 01   110 | 1
1 | 00 | 0   10 | 10
1 | 00 | 1   10 | 11

K-Map Simplification of SOP expression

After an SOP expression has been mapped, there are 3 steps in the process of obtaining a minimum SOP expression: (A) Grouping the 1s, (B) determining the product term for each group, and (C) Summing the resulting product term.

(A) Grouping of 1's

You can group 1s on K-Map according to the following rules by enclosing those adjacent cells containing 1s. The goal is to maximize the size of the groups and minimize the number of groups.

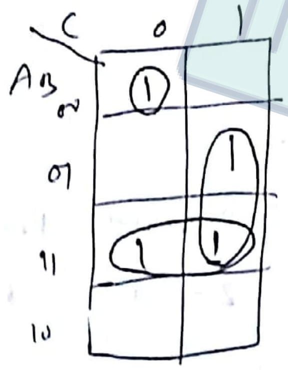
1. A group must contain either 1, 2, 4, 8 or 16 cells, which are all power of two. In the case of a 3-variable map, $2^3 = 8$ is the maximum group.

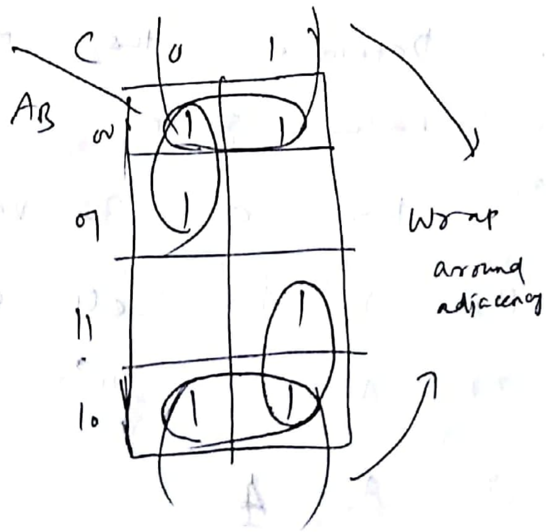
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group don't have to be adjacent to each other.

3. Always include the largest possible number of 1s in a group in accordance with rule 1.

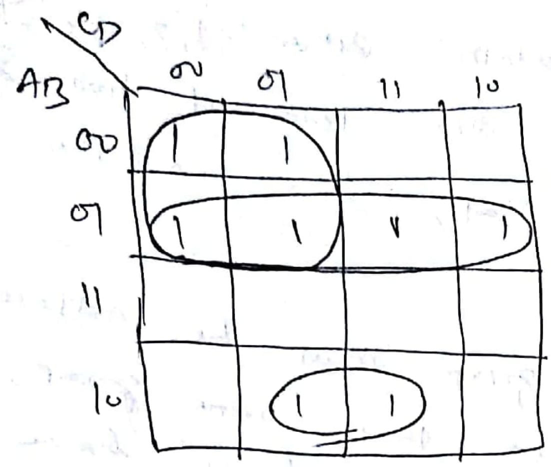
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include non common 1s.

Ex 4:.

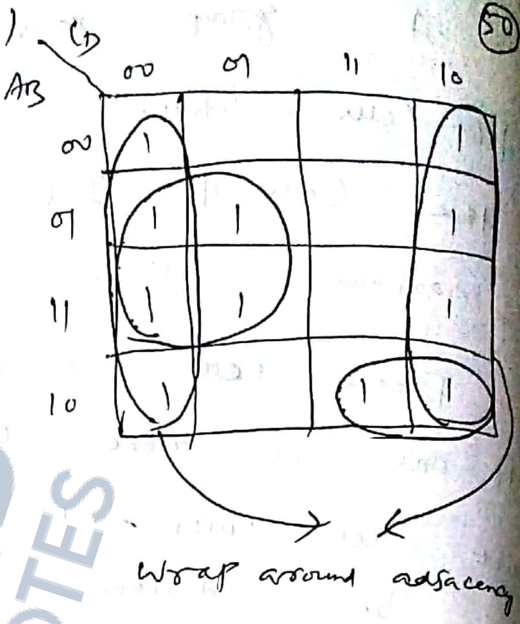




6)



7)



(B) Determining the minimum SOP expression from the map

1. Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occures in only one form (either uncomplemented or complemented) within the group. Variables that occure both uncomplemented and complemented within the group are eliminated. These are called Contradictory variables.

2. Determine the minimum product term for each group.

(a) For a 3-variable map:

(1) A 1-cell group yields 3-variable product term.

(2) A 2- " " " 2- " " "

(3) A 4- " " " 1 " " "

(4) A-8 " " " a value of '1' for the expression.

(b) For a 4-variable Map

(1) A 1 - cell group yields a 4-variable product term.

(2) A 2 - cell group yield 3-variable product term.

(3) A 4 - cell " " 2 " "

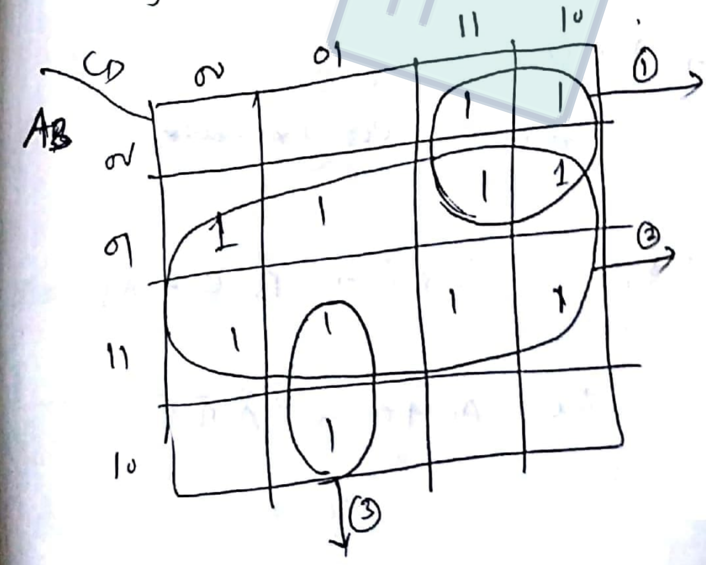
(4) A 8 - cell " " 1 " "

(5) A 16 " " a value of '1' for the expression.

(C) Summing the resulting product term

When all the minimum product terms are derived from K-map, they are summed to form the minm SOP expression.

Ex : 8 : Determine the product term of the K-map as shown below.



Ans :- $\Sigma m(2,3,4,5,6,7,9,12,13,14,15)$

①

For this group D is changing 1 → 0

B is changing 0 → 1

A = const = 0 = $\bar{A}$

C = const = 1 = C

∴ product term = $\bar{A}C$

For group ②

A is changing $0 \to 1$

C is " .

D " " .

only constant is $B = 1 = B$.

$\therefore$ Product term is B.

< since 8 cells for a 4 variable map we got 1-variable term >

For group ③

$\bar{C}D$ is const., B is changing $1 \to 0$

A is constant.

$\therefore$ Product term $\underline{A\bar{C}D}$.

$\therefore$ The resulting minimum SOP expression is the sum of these product terms:

$$B + \bar{A}C + A\bar{C}D.$$

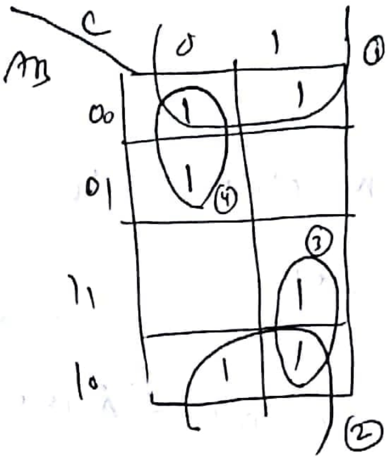
Ex :- 9 Determine the minm SOP expression.

AB \ C	0	1
00	1	
01		1
11	1	1
10		

Ans :- $\bar{A}\bar{B}\bar{C} + BC + AB$

i.e $AB + BC + \bar{A}\bar{B}\bar{C}$

Ex-10



① & ② Corners combined betn $\bar{A}\bar{B}$ & $A\bar{B}$

$\bar{B}$ is const.
A is changing

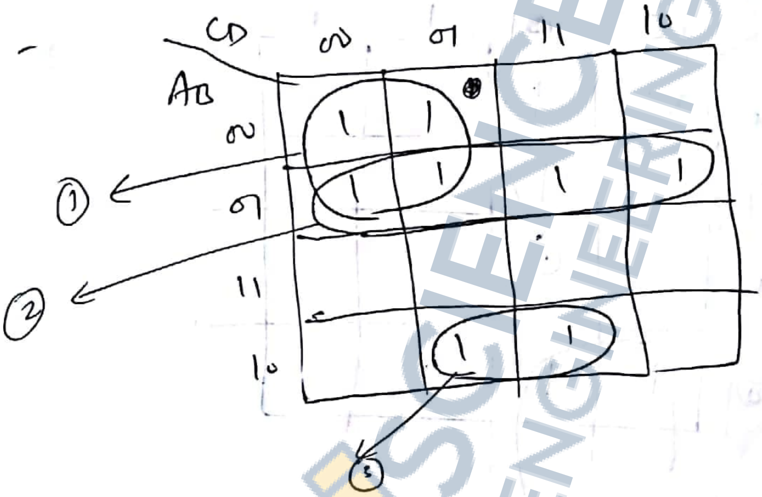
① & ② → $\bar{B}$

③ → AC

④ → $\bar{A}\bar{C}$

∴ $SOP = \bar{B} + AC + \bar{A}\bar{C}$

ex-11:-



Group

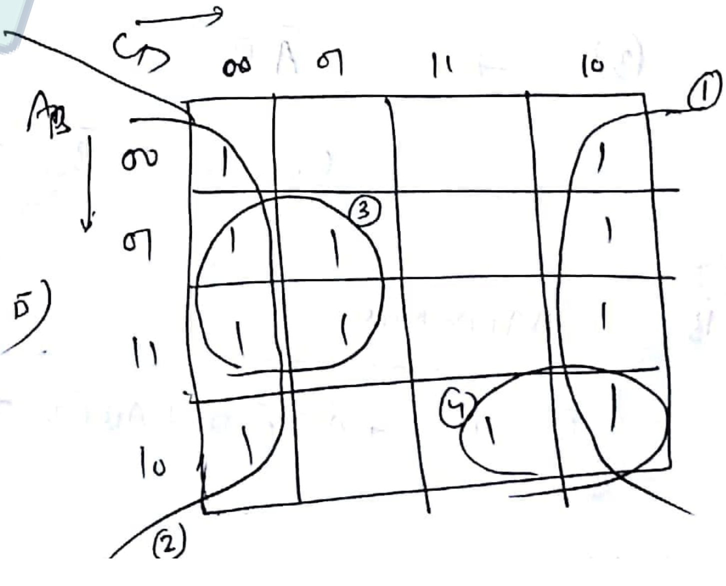
① → $\bar{A}\bar{C}$

② → $\bar{A}B$

③ → $A\bar{B}D$

$SOP = \bar{A}\bar{C} + \bar{A}B + A\bar{B}D$

ex-12



Group

① & ② → $\bar{D}$ (∵ $\bar{C}\bar{D}$ & $C\bar{D}$)

③ → $B\bar{C}$

④ → $A\bar{B}C$

Sop → $\bar{D} + B\bar{C} + A\bar{B}C$

Ex :- 14) Use K-Map to minimize the Standard SOP.

Ans. $A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

Ans : The binary values of the expression are

101 + 011 + 001 + 000 + 100

AB \ C	0	1
00	1	1
01		1
11		
10	1	1

① & ② → $\bar{A}\bar{B}$ & $A\bar{B}$ → $\bar{B}$

③ → $\bar{A}C$

∴ Sop = $\bar{B} + \bar{A}C$

ex 15) Minimize

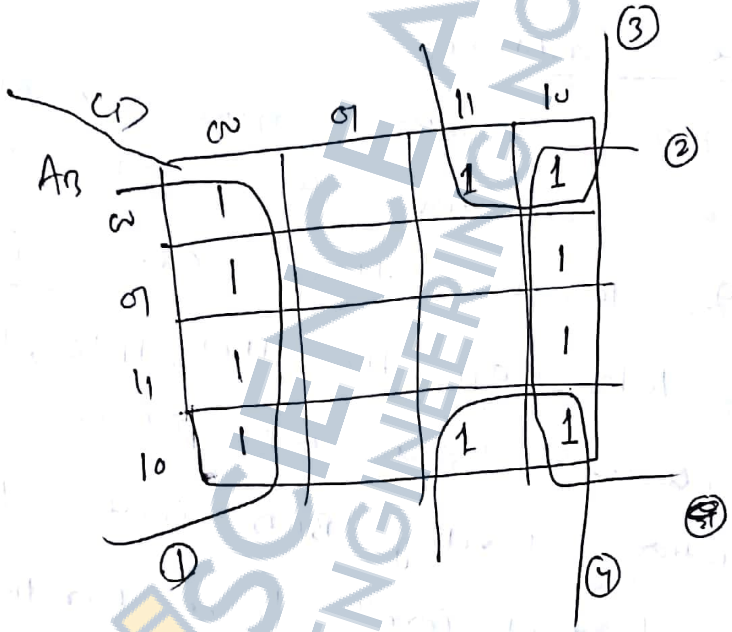
$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$

Ans:- Converting $\bar{B}\bar{C}\bar{D}$ to Standard SOP

0	0 0 0	→	$\bar{A}\bar{B}\bar{C}\bar{D}$
1	0 0 0	→	$A\bar{B}\bar{C}\bar{D}$

So Binary numbers corresponding to SOP are

0000, 1000 ($\bar{B}\bar{C}\bar{D}$), 0100, 1100, 0011, 1011, 0010, 0110, 1110, 1010



① & ② → $C\bar{D}$ & $C\bar{D}$ → $\bar{D}$

③ & ④ → $\bar{A}\bar{B}C$ & $A\bar{B}C$ → $\bar{B}C$

∴ SOP = $\bar{D} + \bar{B}C$

Mapping Directly from truth table.

The ones in the o/p column of a truth table can be mapped directly onto K-Map into the cells corresponding to the values of the associated i/p variable combinations.

Ex:

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

→

AB \ C	0	1
00	1	
01		
11	1	1
10	1	

Don't Care Conditions:-

Sometimes a situation arises in which some i/p variable combinations are not allowed. For example, In BCD there are 6 invalid combinations: 1010, 1011, 1100, 1101, 1110, 1111. Since these unallowed states will never occur in an application involving BCD code, they can be treated as "don't care" terms w.r. to their effect on the o/p. That is, for these 'don't care' terms either a 1 (~~SOP~~) or a 0 (~~POS~~) may be assigned to the o/p; it really does n't matter since they will never occur.

The 'don't care' terms can be used to advantage on the K-map. Figure 1(b) shows that for each 'don't care' term, an X is placed in the cell. When grouping the 1s, the Xs can be treated as 1s to make a larger grouping or as 0's if they can't be used to advantage. The larger a group, the simpler the resulting term will be.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

fig 2 (a)

AB \ CD	00	01	11	10
00				
01			1	
11	X	X	X	X
10	1	1	X	X

fig 1 (b)

① → A

② → BCD

Note:- If taking don't care helps in minimizing take it Otherwise don't take them as they add an extra term.

∴ SOP → A + BCD

If don't is not taken → $A\bar{B}\bar{C} + \bar{A}BCD$

i.e the advantage of using don't care terms to get simplest expression.

K-Map POS Minimization

For a POS expression in standard form, a '0' is placed on the K-map for each sum term in the expression. e.g. the sum term $(A+\bar{B}+C)$, a '0' goes in the 010 cell on a three variable map.

Steps

1. Determine the binary value of each sum term in the standard POS expression.
2. As each sum term is evaluated, place a '0' on the K-map in corresponding cell.

Ex-1) Map $(A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(\bar{A}+B+\bar{C})$

AB \ C	0	1
00	0	
01	0	
11	0	
10		0

Binary Values

$A+B+C \rightarrow 000$

$A+\bar{B}+C \rightarrow 010$

$\bar{A}+\bar{B}+C \rightarrow 110$

$\bar{A}+B+\bar{C} \rightarrow 101$

Ex 2) Use K-Map to minimize the following standard POS

$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

Ans :- The combinations of binary values of the expression are

$(0+0+0)\ (0+0+1)\ (0+1+0)\ (0+1+1)\ (1+1+0)$

AB \ C	0	1
00	0	0
01	0	0 ②
11	0 ①	
10		

① → $(\bar{B}+C)$

② → A

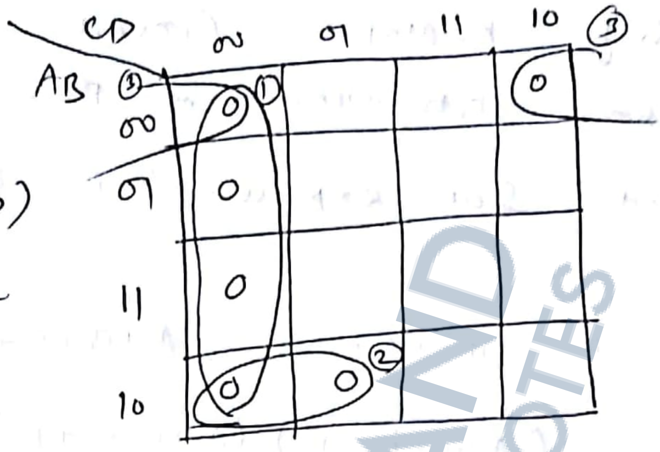
POS → $A(\bar{B}+C)$

8) Minimize POS

$(B+C+D)\ (A+B+\bar{C}+D)\ (\bar{A}+B+C+\bar{D})\ (A+\bar{B}+C+D)\ (\bar{A}+\bar{B}+C+D)$

0 0 1 0   1 0 0 1   0 1 0 0   1 1 0 0

Ans:-



Other terms

~~1 1 0~~

0 0 1 0
1 0 0 1
0 1 0 0
1 1 0 0

First convert $(B+C+D)$ to standard POS form.

A B C D

0 ~~0 0 0~~ → 0 0 0 0 → $A\,B\,C\,D$

1 0 0 0 → 1 0 0 0 → $\bar{A}\,B\,C\,D$

Mapping 6 terms in the K map, we have.

grouping the 0's in the cells, we have.

① → $(C+D)$

② → $(\bar{A}+B+C)$

③ → $(A+B+D)$

∴ POS minimized → $(C+D)\ (A+B+D)\ (\bar{A}+B+C)$

Converting POS → SOP, SOP → POS using K-Map.

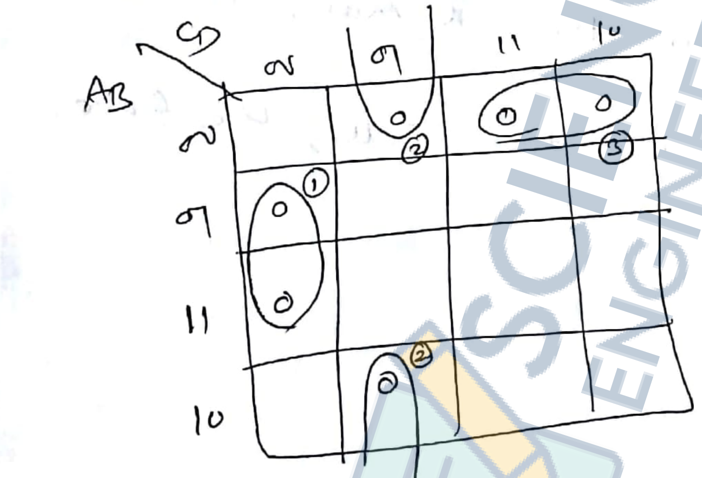
For a POS expression, all the cells that don't contain 0s contain 1s, from which the SOP expression is derived.

Likewise, for an SOP expression, all the cells that don't contain 1s contain 0s, from which the POS expression is derived.

Ex:- 4) Using K-Map convert the following POS expression into (i) minimum POS expression, (ii) Standard SOP expression (iii) minimum SOP expression.

$$(\bar{A}+\bar{B}+C+D)\,(A+\bar{B}+C+D)\,(A+B+C+\bar{D})$$
$$(A+B+\bar{C}+\bar{D})\,(\bar{A}+B+C+\bar{D})\,(A+B+\bar{C}+D)$$

Ans :- First map the POS into the K-Map.



Map

11	00
01	00
00	01
00	11
10	01
00	10

① → $(\bar{B}+C+D)$

② → $(B+\bar{C}+\bar{D})$

③ → $(A+B+\bar{C})$

Minimized POS → $(A+B+\bar{C})\,(\bar{B}+C+D)\,(B+C+\bar{D})$

(ii) To get the Standard SOP, Place the 1s in the cells that don't contain 0s.

AB \ CD	00	01	11	10
00	1			
01		1	1	1
11		1	1	1
10	1		1	1

Standard SOP

$$\bar A \bar B \bar C \bar D + \bar A B \bar C D + \bar A B C D + \bar A B C \bar D$$
$$+ A B \bar C D + A B C D + A B C \bar D$$
$$+ A \bar B \bar C \bar D + A \bar B C D + A \bar B C \bar D$$

(iii) Minimum SOP

AB \ CD	00	01	11	10
00	1			
01		1	1	1
11		1	1	1
10	1		1	1

1 → AC

2 → BC

3 → BD

4 → $\bar B \bar C \bar D$

SOP → $AC + BC + BD + \bar B \bar C \bar D$

Ans.

Note :-

1) Pair → A group of two logical 1's in K-map.
It eliminates one variable in the o/p expression.

Quad → A group of 4 logical 1's in K-map.
It eliminates 2 variables in o/p expression.

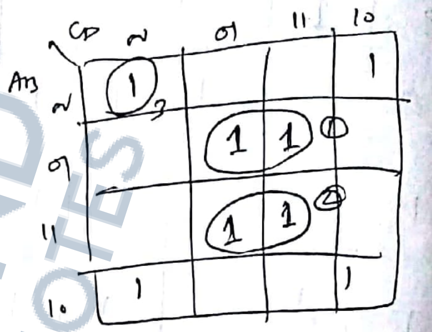
~~Octect~~ OCtet → group of '8', eliminates 3 variables.

Implicant

~~A group of 1~~

An implicant is a product term obtained by combining 1 or 2 or 4 or 8 adjacent squares in the map.

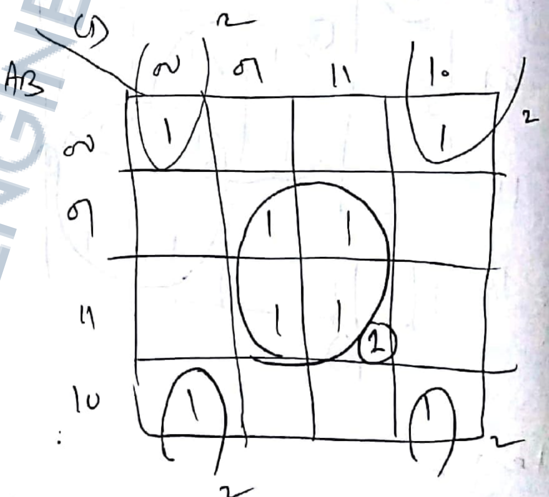
Ex:-



① & ②, ③ are called implicants.

Prime implicant:-

A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.

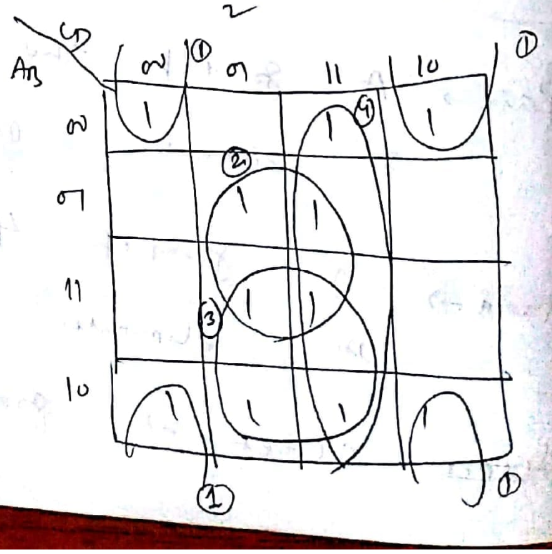


① → BD

② → $\bar{B}\bar{D}$

are called the prime implicants.

Ex:- 5



① → Combining the corners $\bar{B}\bar{D}$

② → BD

③ → AD

④ → CD

$F = BD + \bar{B}\bar{D} + CD + AD$ ✓

Taking different groups, we have

$F = BD + \bar{B}\bar{D} + CD + A\bar{B}$

$= BD + \bar{B}\bar{D} + \bar{B}C + AD$

$= BD + \bar{B}\bar{D} + \bar{B}C + A\bar{B}$

Essential Prime Implicants

The prime implicant which contains at least one '1' which can't be covered by any other prime implicants is called an essential prime implicant (EPI).

The prime implicant whose each 1 is covered by at least one EPI is called redundant prime implicant (RPI).

A prime implicant which is neither an (EPI) or (RPI) is called a selective prime implicant (SPI).

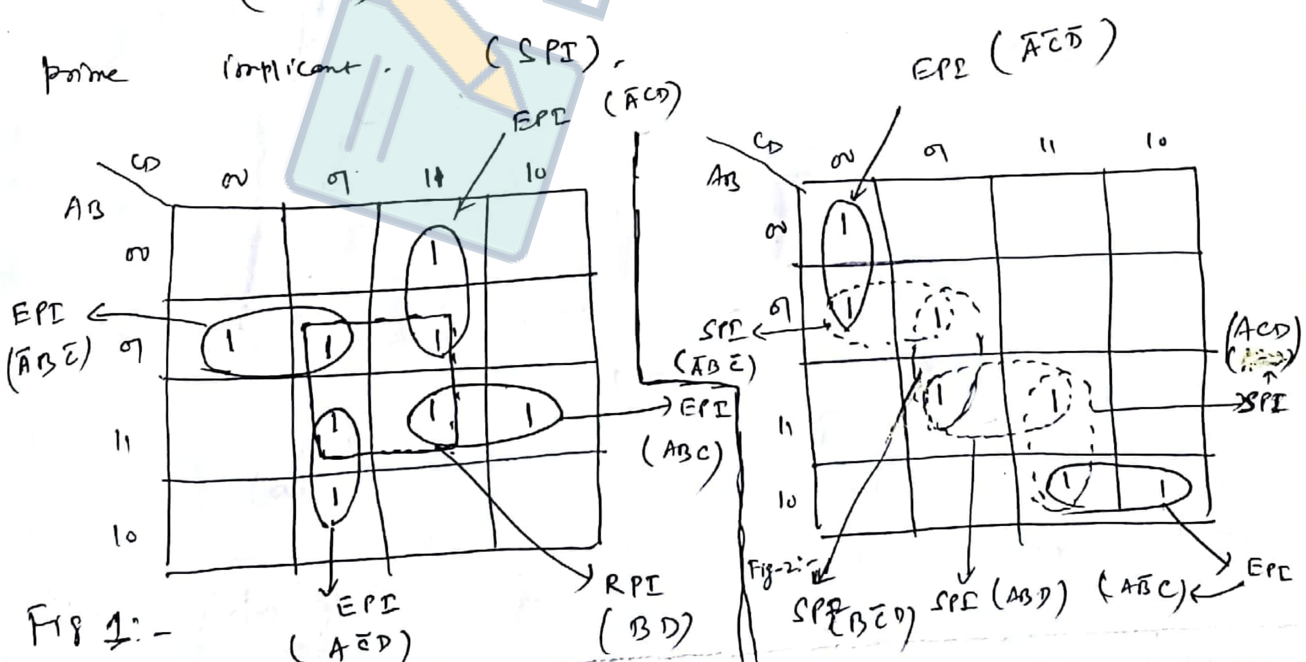


Fig 1:-

In figure 2 :- EPI → $\bar{A}\bar{C}\bar{D}$, $A\bar{B}C$

SPI → $\bar{A}BC$, $B\bar{C}D$, ABD , ACD

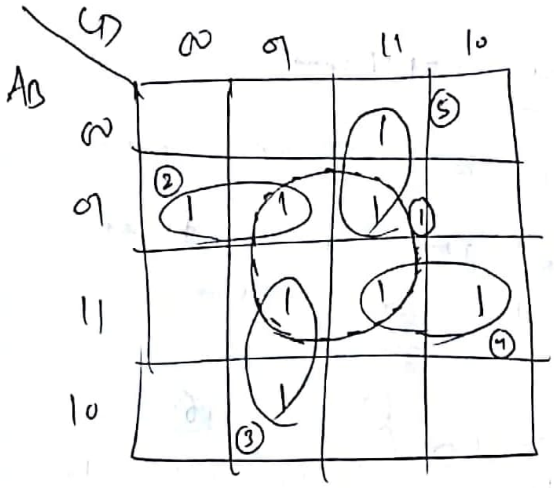
Redundant group. (More detail)

options

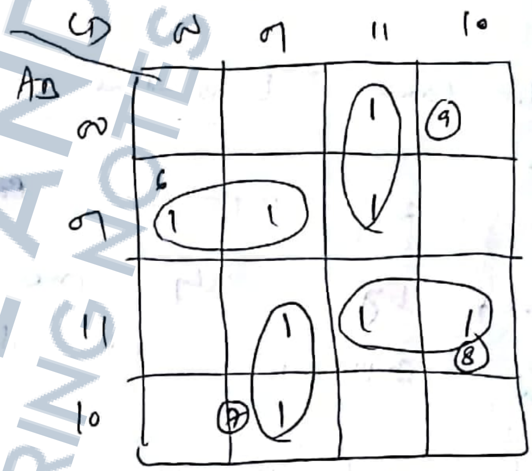
SOP = $\underbrace{\bar{A}\bar{C}\bar{D} + A\bar{B}C}$ + $\underbrace{\bar{A}B\bar{C} + ABD}$

or $\underbrace{\bar{A}\bar{C}\bar{D} + A\bar{B}C}$ + $\underbrace{B\bar{C}D + ABD}$

or $\underbrace{\bar{A}\bar{C}\bar{D} + A\bar{B}C}_{(EPI)}$ + $\underbrace{B\bar{C}D + ACD}_{(SPIs)}$



Case - I



Case - II

In Case I, suppose group 1 is taken, then group 2, 3, 4, 5 are taken.

So ~~often~~ we have 5 terms in the SOP expression.

But in Case II, group 6, 7, 8, 9 cover all the 1s. So no need of quad.

→ So group 1, is a redundant group and it is ~~that~~ unnecessary.

Note :- 1) In selective prime implicant case we have options either of the case we can take.

But in redundant group case we don't have any option. We have to eliminate the redundant group.

Five Variable Map

Two 4-variable maps (16 cells each) are used to construct a 5-variable map.

A=0

BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

A=1

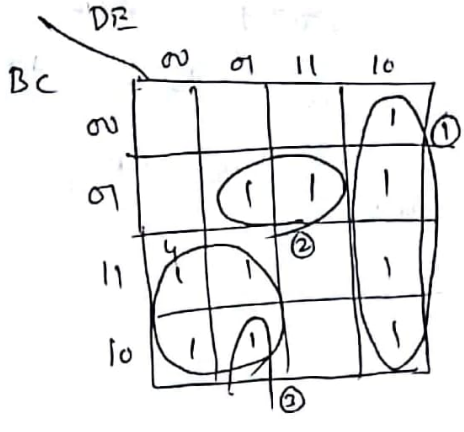
BC \ DE	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

Because A B C D E
0 11 01 → 13

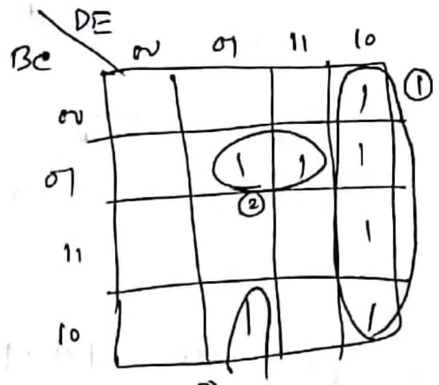
Because A B C D E
1 1 0 0 0 → 24

The Best way to visualize cell adjacencies is to imagine the two 16-cell maps between the map map is placed on the top that A=0 of the A=1 map. Each cell in the A=0 map is adjacent to the cell directly below it in the A=1 map.

Ex :- 1)



A = 0



③ A = 1

① → ~~DE~~ $D\bar{E}$

② → $\bar{B}CE$

③ → $B\bar{C}\bar{D}E$

④ → $\bar{A}B\bar{D}$

The total SOP expression

$$\boxed{SOP = \cancel{B}\,D\bar{E} + \bar{B}CE + \bar{A}B\bar{D} + \cancel{BCDE} + B\bar{C}\bar{D}E}$$

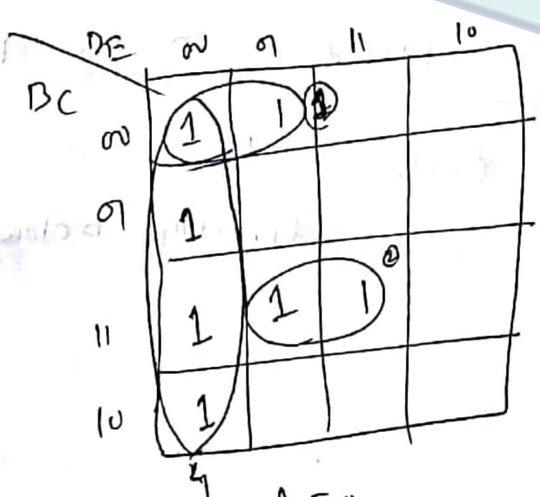
* Don't care example in extra note

2) Use K-Map to minimize

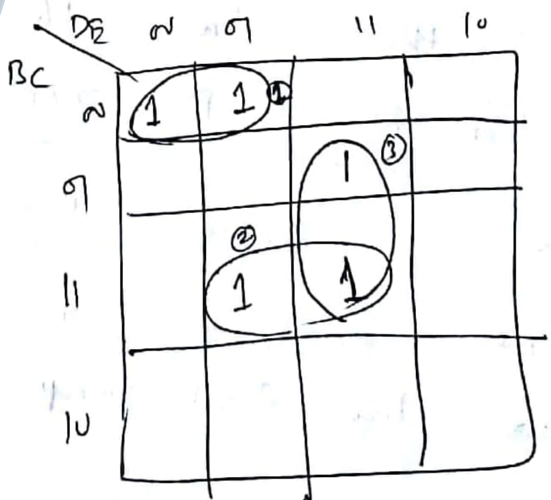
$$X = \underset{00000}{\bar{A}\bar{B}\bar{C}\bar{D}\bar{E}} + \underset{00100}{\bar{A}\bar{B}C\bar{D}\bar{E}} + \underset{01100}{\bar{A}BC\bar{D}\bar{E}} + \underset{01000}{\bar{A}B\bar{C}\bar{D}\bar{E}}$$

$$+ \underset{00001}{\bar{A}\bar{B}\bar{C}\bar{D}E} + \underset{01101}{\bar{A}BC\bar{D}E} + \underset{01111}{\bar{A}BCDE} + \underset{10000}{A\bar{B}\bar{C}\bar{D}\bar{E}}$$

$$+ \underset{10001}{A\bar{B}\bar{C}\bar{D}E} + \underset{11101}{ABC\bar{D}E} + \underset{11111}{ABCDE} + \underset{10111}{A\bar{B}CDE}$$



A = 0



A = 1

① → $\bar{B}\bar{C}\bar{D}$

② → ~~AB~~ BCE

③ → $ACDE$

④ → $\bar{A}\bar{D}\bar{E}$

∴ SOP = $\bar{A}\bar{D}\bar{E} + \bar{B}\bar{C}\bar{D} + BCE + ACDE$

Simplify

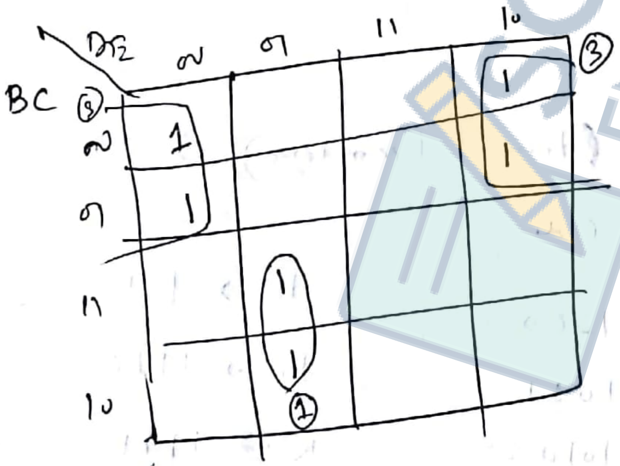
$F(A,B,C,D,E) = \Sigma(0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31)$

∴ 00000, 00010, 00100, 00110, 01001, ~~11001~~ 01101

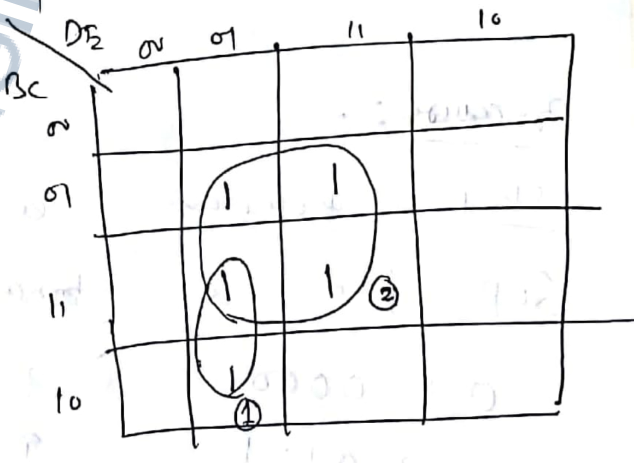
10101, 10111, 11001, 11101, 11111

A = 0

A = 1



A = 0



A = 1

① → $B\bar{D}E$

② → ACE

③ → $\bar{A}\bar{B}\bar{E}$

SOP → $F = \bar{A}\bar{B}\bar{E} + B\bar{D}E + ACE$

(Ans)