Unit - 5

Relativity

Some defns

1. Observer :-

The observer may be a person or an instrument like microscope, telescope, satellite

2. Event :-

An occurence or happening is called an event. ex: A fruit falling from a tree, An accident etc.

3. Frame of reference

It is the co-ordinate system from which the observer makes observations for different events. Ex -1 :- Suppose a car & a motorcycle are moving with almost same speed in the same direction on a road. To an observer inside the car, the motorcyclist appears to be stationary or moving with a very ~~small~~ slow speed.

On the other hand to an observer standing on the roadside, the motor cyclist appears to move with high speed. This shows that description of the motion changes to a great extent with the change of the co-ordinate system.

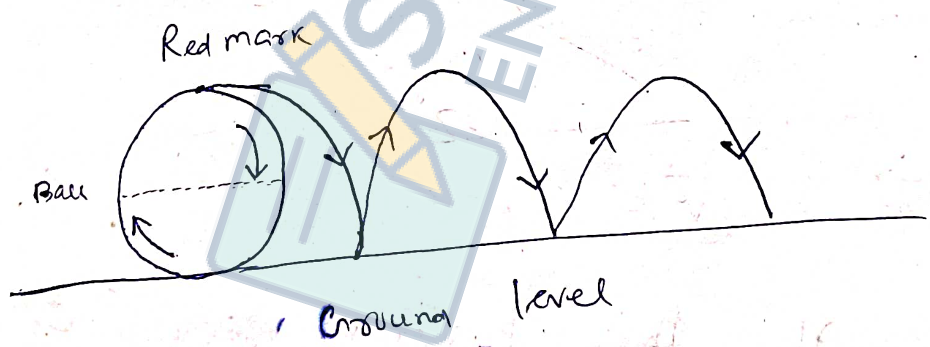
Ex- 2

Suppose a ball with a red mark be rolling on the ground. To an outside observer, the red mark appears to make a cycloid type of motion.

The same red mark appears to be stationary when observed by an observer placed at some distance on the surface of the ball.

The red mark appears to make a circular motion when observed from the centre of the ball.

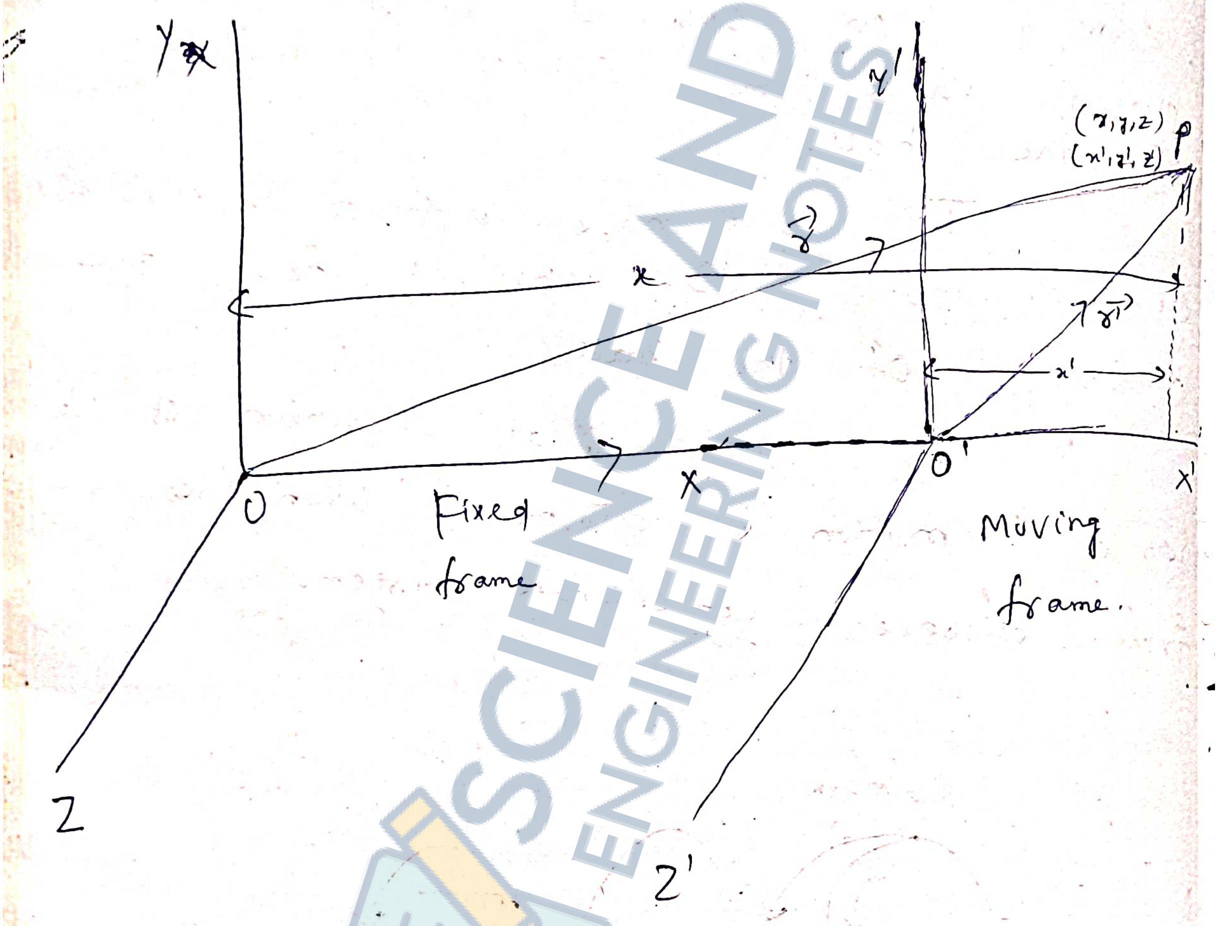
This shows that description of the same motion is different when observed from different co-ordinate system.



'Galilean transformation' [Long question]

Let there be two frames of reference which coincide at $t = 0$. One of the frames be allowed to move with constant speed v w.r.t the fixed frame. In a time t sec, the moving fram has moved through a distance vt.

Lets observe a moving particle P from these two frames. The instantaneous position co-ordinates be (x, y, z) with respect to the fixed frame and (x', y', z') w.r.to the moving frame O'.



$$\vec{OO'} = \vec{v}t$$

If the position vectors of the moving particle P w.r.to the fixed frame and the moving frame be $\vec{r}$ and $\vec{r'}$ respectively, then a relation between them can be established with the help of $\triangle$ law of vectors.

$$\vec{OO'} + \vec{O'P} = \vec{OP}$$

$$\Rightarrow \vec{v}t + \vec{r}' = \vec{r} \quad \text{——— (i)}$$

Writing each vector in component form, we get

$$\hat{i}\,vt + (\hat{i}x' + \hat{j}y' + \hat{k}z') = \hat{i}x + \hat{j}y + \hat{k}z$$

$$\Rightarrow \hat{i}(vt + x') + \hat{j}y' + \hat{k}z' = \hat{i}x + \hat{j}y + \hat{k}z$$

$$\Rightarrow vt + x' = x, \quad y' = y, \quad z' = z$$

$$\Rightarrow \boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}}$$

These are called Gallilean transformation eqns.

Differenciating both the sides of eqn (i) w.r.t time, we get

$$\frac{d}{dt}(\vec{v}t) + \frac{d}{dt}\vec{r}' = \frac{d\vec{r}}{dt}$$

$$\Rightarrow \vec{v}\frac{dt}{dt} + \vec{u}' = \vec{u}$$

$$\Rightarrow \vec{v} + \vec{u}' = \vec{u} \quad \text{——— (ii)}$$

Where $\vec{u}'$ = velocity of the moving particle as observed by the moving frame.

$\vec{u}$ = Velocity of the moving particle P as observed by the fixed frame

Eqn (ii) shows that the velocity of the moving particle as observed by the fixed frame and the moving frame are differene.

Differenciating both the sides of eqn (ii) w.r.to time, we get

$$\frac{d\vec{V}}{dt} + \frac{d\vec{u'}}{dt} = \frac{d\vec{u}}{dt}$$

$$\Rightarrow 0 + \vec{a'} = \vec{a}$$

$$\Rightarrow \vec{a'} = \vec{a} \quad \text{———— (iii)}$$

This shows that the acclns of the moving particle as observed by the fixed frame and the moving frame are the same.

Multiplying by the mass of the moving particle (m) to both the sides of eqn (3), we get

$$m\vec{a'} = m\vec{a}$$

$$\Rightarrow \vec{F'} = \vec{F} \quad \text{———— (iv)}$$

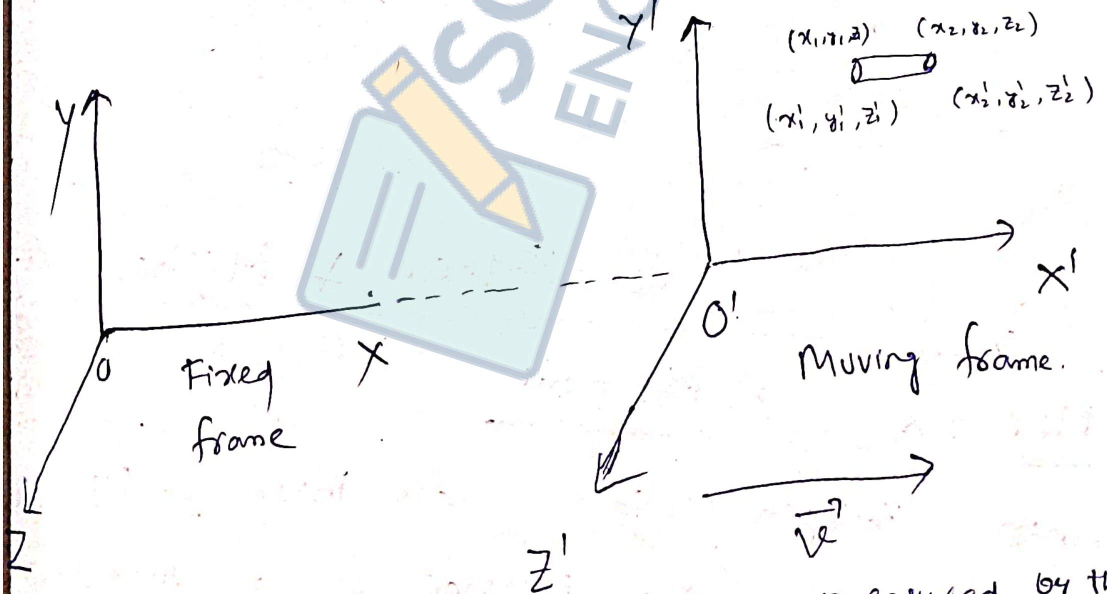
Thus the force on the particle P as measured by the fixed frame & the moving frame are the same.

Inertial frames of reference

These are the co-ordinate systems where Newton's first law & 2nd law hold good.

Ex → The fixed frame O and the moving frame O' which moves with constant speed w.r.to O, are the inertial frames of reference.

To prove that the length of a rod does not change under Galilean transformation



Length of the rod as measured by the fixed frame $= l = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$,

Length of the rod as measured by the

moving frame

$$= l_0' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$

From Galilean transformation eqn, we know that

$$x_1' = x_1 - vt, \quad y_1' = y_1$$

$$z_1' = z_1$$

$$x_2' = x_2 - vt, \quad y_2' = y_2$$

$$z_2' = z_2$$

Putting these eqn in the expression for l' we get

$$l' = \sqrt{(x_2 - \cancel{vt} - x_1 + \cancel{vt})^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= l$$

Concept of ether & Michelson's Morley's experiment

When Huygen gave wave theory of light, he imagined the presence of a medium called ether. He said that ether was present everywhere in the universe, even inside a glass, water vacuum.

After nearly 250 years, Michelson & Morley tried to verify the presence of ether. They used Michelson's interferometer which is a very accurate instrument. They did the experiment at many places of the earth during day & night & during different seasons of the year. Yet they failed to notice the trace of ether. The following conclusions can be draw from their experiment.

① Ether is just a conception and it has no reality.

② Velocity of light in vacuum is the highest attainable velocity by a material object i.e no speed can be higher than the speed of light in vacuum.

Postulates of special theory of relativity.

~~Postulates of special theory of relativity~~

To develope the special theory of relativity, the following 2- postulates are necessary

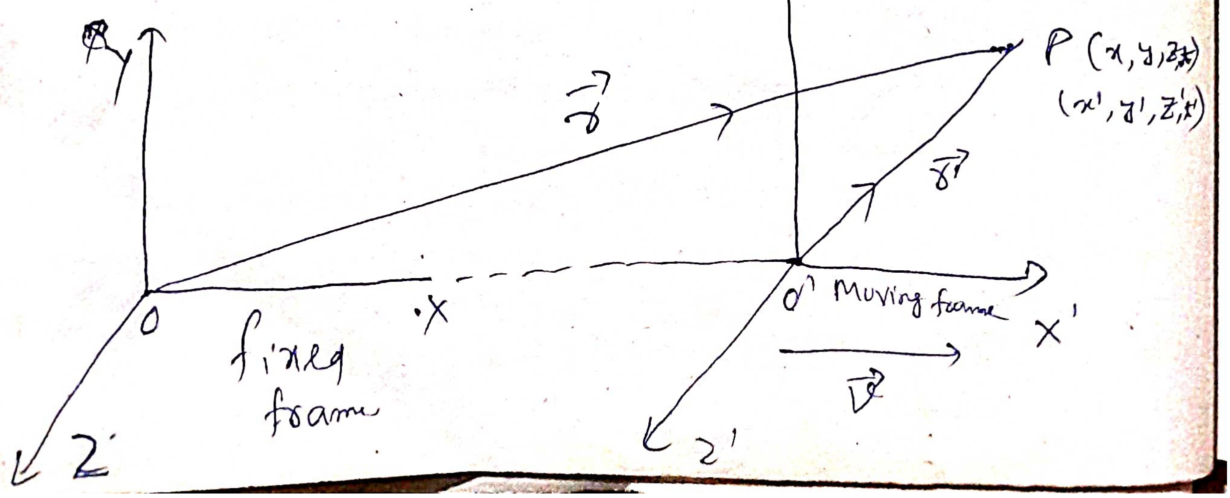
(i) All the laws of physics remains the same in all the inertial frames of reference.

(ii) Velocity of light measured by the fixed frame & moving frame are the same and equal to the speed of light in vacuum.

Lorentz transformation

According to Lorentz the time is regarded as the 4th co-ordinate which is different when measured by the fixed frame and by the moving frame. This can be explained easily if we will accept the second postulate of special theory of relativity.

Ex→ [illegible] length [illegible] we measure length [illegible]



$$\frac{OP}{c} = t \; ; \quad \frac{O'P}{c} = t'$$

$$\therefore \quad t > t'$$

The transformation eqns are

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{\left(t - \frac{v}{c^2} x\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Consequences of relativity

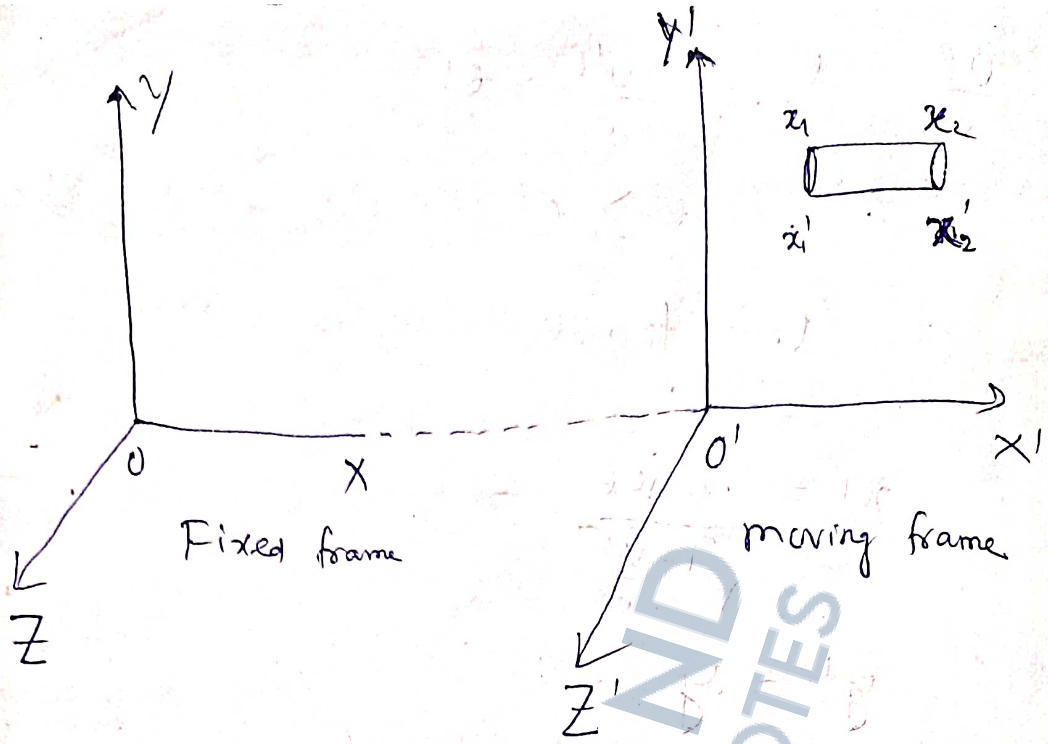
1. Lorentz - Fitzgerald Contraction of length

Suppose there is a 'rod' present in the moving frame. The length of the rod as measured by the moving frame is called proper length (l_0) because the rod is w.r.to the moving frame at rest.

$$\therefore \quad x_2' - x_1' = l' = l_0 = \text{Proper length.}$$

Using Lorentz transformation eqns, we can write

$$x_2' - x_1' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$\Rightarrow \quad l_0 = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \quad x_2 - x_1 = l_0 \sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \quad \boxed{l = l_0 \sqrt{1-\frac{v^2}{c^2}}}$$

Where l = length of the rod as measured from the fixed frame

But $v < c$

$\frac{v}{c} < 1$

$\Rightarrow \frac{v^2}{c^2} << 1$

$1 - \frac{v^2}{c^2}$ is a fraction.

$\sqrt{1-\frac{v^2}{c^2}}$ is another "

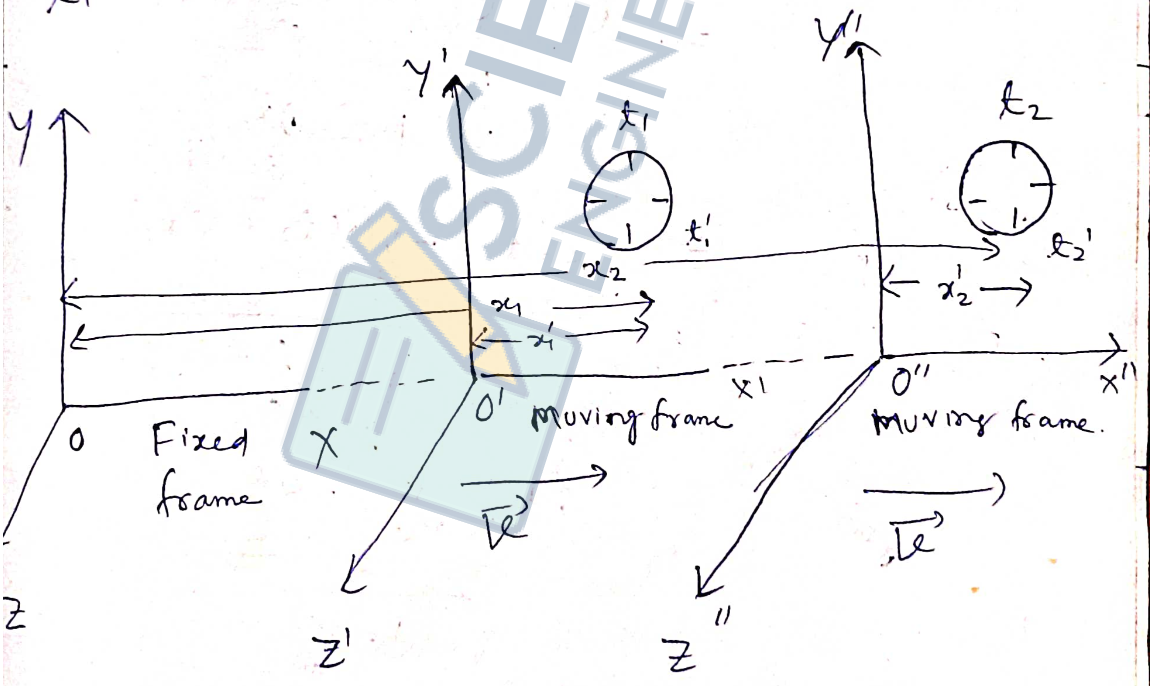
$l = l_0 \cdot$ a fraction.

$\Rightarrow \quad l < l_0$

i.e why the length of a rod appears to be contracted when observed from a fixed frame.

② Time dilation

Let the two events occure at times t_1 & t_2 when observed by the fixed frame. The same event when observed from the moving frame are at times t_1' & t_2' respectively.



Here $\Delta t = t_2 - t_1$
= Time interval between the two events as observed by the fixed frame

$\Delta t' = t_2' - t_1'$

$\Delta t'$ = Time interval between the same 2 events as observed by the moving frame

= proper time.

Relation between Δt & $\Delta t'$ can be obtained by Lorentz transformation eqns

$$\Delta t' = t_2' - t_1'$$

$$= \frac{t_2 - \frac{v}{c^2} \cdot x_2}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1 - \frac{v}{c^2} \cdot x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\Delta t - \frac{v}{c^2}(v t_2 - v t_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\Delta t - \frac{v^2}{c^2} \cdot \Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\Delta t \left(1 - \frac{v^2}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \Delta t \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow \boxed{\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Since $\sqrt{1-\frac{v^2}{c^2}}$ is a fraction,

$$\therefore \Delta t = \frac{\Delta t'}{\text{fraction}}$$

$$\Rightarrow \Delta t > \Delta t'$$

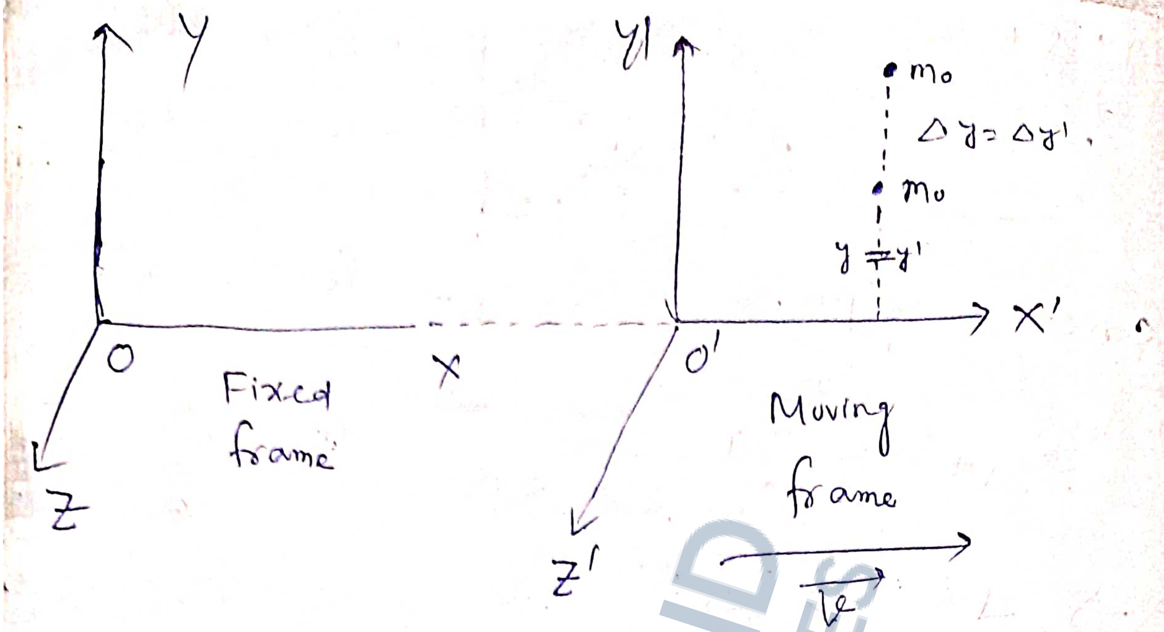
Variation of mass with velocity

Lets consider a body of rest mass m_0 in the moving frame. Its position be specified by $y = y'$. Its it will be displaced along the Y direction by an amount $\Delta y = \Delta y'$, then the y-component of the velocity as measured by the fixed frame $= v_y = \frac{\Delta y}{\Delta t}$.

whereas the y component of the velocity as measured by the moving fram

$$v'_y = \frac{\Delta y'}{\Delta t'}$$

$$\therefore v'_y = \frac{\Delta y}{(\Delta t)\left(\sqrt{1-\frac{v^2}{c^2}}\right)} = \frac{v_y}{\sqrt{1-\frac{v^2}{c^2}}}$$



Multiplying by m_0 to both the sides of the above eqn, we get

$$m_0 v'_y = \frac{m_0 v_y}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{——— (i)}$$

From the first postulate of the special theory of relativity, we know that all the laws of physics must have identical form in all the inertial frames of reference. Hence, the conservation of linear momentum should also hold good.

$$p'_y = p_y$$

$$\Rightarrow m_0 v'_y = m \cdot v_y \quad \text{——— (ii)}$$

where m is called 'moving mass' which is obtained by comparing eqn (i) and (ii)

$$\therefore \boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Since $v < c$ & $\sqrt{1-\frac{v^2}{c^2}}$ is a fraction,

We must have $m > m_0$.

i.e mass of a body increases with velocity when observed ~~frame~~ from a fixed frame.

(4) Einstein's mass-energy relation

According to Einstein mass & energy are interrelated by a relation

$$\boxed{\Delta E = \Delta m . c^2}$$

where c = Speed of light in vacuum
= 3×10^8 m/sec.

Δm = Mass i.e destroyed by which energy of amount ΔE is developed.

(5) Einstein's velocity addition formula:-

Einsteins had shown that ordinary laws ~~are~~ of addition are no more applicable when velocity of objects are near about the velocity of light.

He gave a formula for the addition of velocities as shown below.

$$\boxed{u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} \cdot u'_x}}$$

where u_x = x- component of the velocity of the moving object as measured by the fixed frame

u'_x = x - component of velocity as measured by the moving frame.

v = Constant velocity with which the moving frame moves along the x-axis

Ex-1

Suppose a ray of light starts from a rocket moving with velocity (v) along the x- direction.

$\therefore$ ~~$u'x = c$~~

$\therefore$ $u'x = c$

Hence $u_x = \dfrac{c + v}{1 + \frac{v}{c}} = \dfrac{c+v}{c+v} \cdot c$

$= c$ (As expected from the 2nd Postulate of the special theory of relativity.)

Problems :-

6. ~~The~~ A rocket moves above the laboratory (গবেষণা [illegible]) with a speed of 0.8 C. A flying saucer passes the rocket with the speed of 0.8 c. Find the speed of the flying saucer as determined at the laboratory.

<u>Ans</u> : 0.976 c

given $u'_x = 0.8\,c$

$v = 0.8\,c$.

From Einstein's velocity addition formula we know, that

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} \cdot u'_x}$$

$$\Rightarrow u_x = \frac{0.8c + 0.8c}{1 + \frac{0.8 \cancel{c}}{\cancel{c^2}} \cdot 0.8 \cancel{c}}$$

$$= \frac{(1.6)\,c}{(1.64)}$$

$$= 0.976\,c$$

7. A rocket travels with a constant velocity of 0.5 c from the earth ~~two~~ to a star 5 light years away. Calculate

(a) The time taken for the trip according to estimate made on the earth.

(b) The time according to a passanger.

(c) The distance from the earth to a star according to a passanger.

(d) The ~~of~~ velocity of earth & the star as measured by the passanger during the trip.

Ans: 1 light year = Distance covered by a light in 1 year

= $c \times 1$ year

Distance to be covered = 5 light years
= $5 \times c \times 1$ year.

Speed of the rocket = $0.5c$

Δt = ~~5x6~~ = Time taken by the rocket as calculated by the fixed frame

$$= \frac{\cancel{5}^{10} \times c \times 1\,\text{year}}{\cancel{0.5}\,\cancel{c}}$$

$$= 10 \text{ year}$$

(b) From the ~~t~~ time dillation formula we know that

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow 10 \text{ year} = \frac{\Delta t'}{\sqrt{1-\frac{.25\cancel{c^2}}{\cancel{c^2}}}}$$

$$\Rightarrow \Delta t' = 10 \text{ year} \sqrt{.75}$$

$$= \cancel{10}^{5} \text{ year} \frac{\sqrt{3}}{\cancel{2}}$$

$$= 5 \times (1.732)$$

$$= 8.660 \text{ year}$$

(c) Distance covered according to the passenger $= \Delta t' \times 0.5c$

$$= 8.66 \text{ year} \times \frac{5}{10} c$$

$$= 4.33 \text{ light year}$$

(d) When the rocket is moving towards the star with a speed of $0.5c$, it appears as if the star is approaching the rocket with $0.5c$ & the ~~rocket~~ earth appears to move away with a speed of 0.5c

8. Calculate how much of the sun's mass reaches the earth in one year. Average rate of radiant energy received from the sun $= 1.4 \times 10^3$ watt/m2.

Ans → 1.25×10^8 kg

$R = 6.37 \times 10^6$ meter.

Hints → Surface area of the earth that receives the sun $= 2\pi r^2$

9. In fusion, 4 Hydrogen atom combine into a helium atom. Rest masses of hydrogen & helium are 1.0081 & 4.0039 a.m.u. respectively. Calculate the energy released in this process in MeV.

Ans: 26.61 MeV, 1 a.m.u $= 1.66 \times 10^{-27}$ kg.

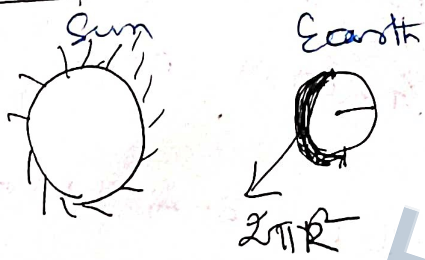
Δm = Mass defect.

= Total mass of all the constituents of the nucleus — mass of the nucleus formed.

10. How much of water is to be heated from 0°C to 100°C so as to increase its mass by 1 gm

(Ans: 2.1×10^8 kg)

Ans

8. 

During the revolution of earth's one part gets light & another part remains in dark.

So the surface gets light $= 2\pi R^2$

Energy received from Sun $= 1.4 \times 10^3$ watt/m²

1 m² Area in 1 sec receives energy $= 1.4 \times 10^3$ Joule

$2 \times 3.14 \times (6.37 \times 10^6)^2$ m² " $= 1.4 \times 10^3 \times 2 \times 3.14 \times (6.37)^2 \times 10^{12}$ Joule

$= 356.93 \times 10^{15}$ Joule.

In $(365 \times 24 \times 3600$ sec) it receives energy $= 11256144.8 \times 10^{17}$ Joule.

$E = mc^2$

$\Rightarrow m = \frac{E}{c^2} = \frac{11256 \times 10^{17}}{3^2 \times 10^{16}}$

$= 125{,}058272$

$= 1.25058272 \times 10^8$ kg

9. Mass of 1 hydrogen atom

$= 1.0081$ a.m.u

$= \underline{1.0081} \times \underline{1.66} \times 10^{-27}$

$= 1.673446 \times 10^{-27}$ kg.

Mass of 4 hydrogen atom

$= 6.693784 \times 10^{-27}$ kg

Mass of 1 helium atom

$= 4.0039$ amu

9.

Mass of 1 H atom $= 1.0081$ amu

" 4 " "

$$\begin{array}{r} 1.0081 \\ \times \quad 4 \\ \hline 4.0324 \end{array}$$

Mass of 1 helium atom $= 4.0039$

Loss mass $=$
$$\begin{array}{r} 4.0324 \\ 4.0039 \\ \hline .0285 \end{array}$$ amu.

$\because$ 1 amu $= 1.66 \times 10^{-27}$ kg.

$.0285$ " $= .0285 \times 1.66 \times 10^{-27}$ kg.

$= 0.047310 \times 10^{-27}$ kg.

Energy released $E = mc^2$

$= (0.04731) \times 10^{-27}$ kg $\times 9 \times 10^{16}$

$= .42579 \times 10^{-11}$ Joule.

1 MeV $= 10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-13}$ Joule

1.6×10^{-13} Joule $\equiv$ 1 eV

1 " $= \dfrac{1}{1.6\times10^{-13}}$ MeV

$.42579 \times 10^{-11}$ " $= \dfrac{.42579\times10^{-11}}{1.6\times10^{-13}}$

$= \dfrac{4.2579\times10^{-12}\times10^{13}}{1.6}$

$= 2.661 \times 10^{1}$

$= 26.61$ MeV.

3. To increase the mass of water by 1 gm energy required

$E = mc^2$

$= \left(\dfrac{1}{10^3}\right) kg \times 9\times10^{16}\ m^2/sec^2$

$= 9 \times 10^{13}$ ~~m²/sec²~~ Joule.

But $W = JH$

$\Rightarrow H = \dfrac{W}{J} = \dfrac{9\times10^{13}\ \text{Joule}}{4.2\ \text{Joule/cal}}$

$= 2.14\times10^{13}$ Cal

~~42)9(~~

42)90(2.14
84
60
42
180
168

$M.S.\Delta\theta = 2.14\times10^{13}$ cal

$\Rightarrow M\times1\times100 = 2.14\times10^{13}$ cal

$\Rightarrow M = \dfrac{2.14\times\cancel{10^{13}}\ 10^{11}}{\cancel{10^2}}$ gm

$= 2.14\times10^{8}$ kg

11. An air plane moves around the earth at a speed of 250 m/sec How many years would elapse before a clock on the earth & one on the air plane would differ by one day.

Ans:- 7.89×10^{9} years

Ans $\rightarrow$

If the time interval measured by the fixed from be Δt and that by the moving from is $\Delta t'$, then as per the question

$$\Delta t - \Delta t' = 1 \text{ day.}$$

From the time dilation formula, we know that

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \Delta t' = \Delta t \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

Since $\frac{v}{c} = \frac{250 \, m/sec}{3\times10^8 \, m/sec}$ = very small

binomial extension can be made

$\therefore \Delta t' \simeq \Delta t \left(1 - \frac{1}{2}\cdot\frac{v^2}{c^2}\right)$

$\Rightarrow \Delta t' = \Delta t - \frac{\Delta t \cdot v^2}{2c^2}$

$\Rightarrow \Delta t - \Delta t' = \frac{\Delta t \, v^2}{2c^2}$

$\Rightarrow 1 \text{ day} = \frac{\Delta t}{2} \cdot \frac{(250)^2}{(3\times10^8)^2}$

$\Rightarrow \Delta t = \frac{2\times(3\times10^8)^2}{(250)^2}$ day

$= \frac{2\times(3\times10^8)^2}{(250)^2\times 365}$ years

$= 7.89\times10^9$ years.

12. Monochromatic X-rays is reduced to $\frac{1}{3}$ of its original intensity in passing through a gold foil (Z=79) of 3 mm thickness. Calculate the absorption coefficient for the X-ray.

(Ans: $3.7 \, cm^{-1}$)

Soln → If I_0 be the intensity of the X-ray beam when it entres the gold foil of thickness 0.3 cm &

I is the intensity of the X-ray when it comes out, then these are related by the formula $$I = I_0 e^{-\mu x}$$

Here $I = \frac{I_0}{3}$,

$$\Rightarrow \frac{I_0}{3} = \frac{I_0}{e^{\mu x}}$$

$$\Rightarrow e^{\mu x} = 3$$

Taking logarithm of both the sides

$$\Rightarrow \ln 3 = \mu x$$

$$\Rightarrow 2.3026 \times \log_{10} 3 = \mu \times (0.3)$$

$$\Rightarrow \mu = \frac{2.3026 \times 0.4771}{.3 \text{ cm}}$$

$$= 3.6619 \text{ cm}^{-1}$$

$$\simeq 3.7 \text{ cm}^{-1}$$

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Sol

Minm Wavelength of the Continuous X-ray spectrum is given by

$$\lambda_{min} (\text{in } A^0) = \frac{12400}{V (\text{in volt})}$$

$$\Rightarrow \lambda \quad '' = \frac{12,400}{500 \times 10^3}$$

$$= \frac{124}{500} = 0.248 A^0$$

~~500) 1240 (.24 1000 2400~~

Formulae On Relativity :-

1. **Contraction of length** $l = l_0\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$

where l_0 = Proper length
= length of the body when observed by a frame moving along with it.

l = Length of the moving body ~~by~~ when observed by a fixed frame.

2. **Time dilation**

$$\Delta t = \frac{\Delta t'}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

where Δt = Time interval as measured by the fixed frame.

$\Delta t'$ = Time interval as measured by the moving frame

3. **Variation of mass with velocity**

$$m = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

where m = moving mass
m_0 = Rest mass.

4. $E = mc^2$, c = Velocity of sound = 3×10^8 m/sec.

1. A rocket is 100 m long on the ground. If its length decreases to 99 m to an observer on the ground during flight, Calculate its speed. (Ans 4.23×10^{7} m/sec)

Ans :

From the ~~Contraction~~ Contraction of length

$$l = l_0 \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

Here given $l_0 = 100$ m.

$l = 99$ metre

$c = 3 \times 10^8$ m/sec

$$99 = 100\left(1 - \frac{v^2}{9 \times 10^6}\right)^{\frac{1}{2}}$$

$$\Rightarrow 99 \times 99 = 100 \times 100 \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \cancel{99 \times 99} = \cancel{100 \times 100 \left(\frac{9 \times 10^6 - v^2}{9 \times 10^6}\right)}$$

$$\Rightarrow \cancel{\frac{99 \times 99}{9 \times 10^6} \times \frac{1}{100 \times 10}} = 9 \times 10^6 - v$$

$$(99)^2 = (100)^2 - (100)^2 \times \frac{v^2}{c^2}$$

$$\Rightarrow (100)^2 \frac{v^2}{c^2} = (100)^2 - (99)^2 = (100 + 99)(100 - 99)$$

$$= 199 \times 1$$

$$\therefore v^2 = \frac{199 \times c^2}{(100)^2}$$

$$v = \sqrt{\cdots} = \frac{14.1 \times 3 \times 10^8}{100} \text{ m/s}$$

2. Calculate the energy equivalant of a proton & an electron in M e.V

$m_e = 9.1 \times 10^{-31}$ kg.

$m_p = 1.6725 \times 10^{-27}$ kg.

1 MeV $= 10^6$ eV

$= 10^6 \times 1.6 \times 10^{-19}$ Joule.

(Ans ÷ 940.73, 0.511 MeV)

Ans: We know that

$E = mc^2$

$= \cancel{9.1 \times 10^{-31}} \times$

$= 1.6725 \times 10^{-27}$ kg $\times (3 \times 10^5)^2$

$= 15.0525 \times 10^{-17}$ Joule

But 1.6×10^{-13} Joule $= 1$ MeV

1 Joule $= \dfrac{1}{1.6 \times 10^{-13}}$

15.0525
~~5.0175~~ $\times 10^{-22}$ Joule $= \dfrac{15.0525 \times 10^{-17}}{1.6 \times 10^{-13}}$

9.407×10^{-4}

$= 940.7 \times 10^{-6}$ MeV

(ii) $E = 9.1 \times 10^{-31}$ kg $\times (3 \times 10^5)^2$ Joule

$= 9.1 \times 9 \times 10^{-21}$ Joule

$= \dfrac{9.1 \times 9 \times 10^{-21}}{1.6 \times 10^{-13}}$ MeV

$= 51.1875 \times 10^{-8}$

$= .511875 \times 10^{-6}$ MeV

$\boxed{= 42.3 \times 10^6 \text{ m/s}}$

3. Calculate heat developed in Calory from the energy Converted fully from 1 gm of mass

(Ans: 2.1×10^{13} Cal)

$$E = mc^2$$

$$= \left(\frac{1\ kg}{10^3}\right) \times (3 \times 10^8)^2$$

$$= 10^{-3} \times 9 \times 10^{16}$$

$$= 9 \times 10^{13} \text{ Joule}$$

But $W = JH$

$$\Rightarrow H = \frac{W}{J} = \frac{9 \times 10^{13} \text{ Joule}}{4.2 \text{ Joule/Cal}}$$

$$= 2.1 \times 10^{13} \text{ Cal}$$

4 Long At what speed the mass of an object will be doubled of its value at rest?

(Ans: 2.598×10^8 m/sec)

Ans:

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$\Rightarrow 2\cancel{m_0} = \frac{\cancel{m_0}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$\Rightarrow 4 = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\Rightarrow 4 - 4\frac{v^2}{c^2} = 1$$

$$\Rightarrow \frac{4v^2}{c^2} = 3$$

$$\Rightarrow \frac{v^2}{c^2} = \frac{3}{4}$$

$$\Rightarrow \frac{v}{c} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow v = \frac{1.732}{\cancel{2}} \times \cancel{3}^{1.5} \times 10^8$$

$$= 2.598 \times 10^8 \text{ m/sec}$$

5: Twin Paradox

Out of two twins A & B of age 20 years each, A takes a round trip to a star with velocity $v = 0.99c$. The star is 30 light years away from the ~~year~~ earth. Compare the ages of A & B when A comes back to B on earth.

(Ans: Age of B = 80.6 years,
A = 28.5 years)

1 light year = Distance covered by light during 1 year

$= c \times 1$ year.

$= 3 \times 10^8$ m/sec $\times 365 \times 24 \times 3600$ sec

$= 9.46 \times 10^{15}$ meter.

Distance to be covered by the rocket as calculated by B

$= 60$ light years.

$= 60 \times c \times 1$ year.

Speed of the rocket $0.99c$,

Time taken by the rocket

$$= \frac{\text{Distance}}{\text{speed}}$$

$$= \frac{60 \times \cancel{c} \times 1 \text{ year}}{.99 \times \cancel{c}}$$

$= 60.6$ year

$\therefore$ Age of B when the rocket will return back to earth

$= 20 + 60.6$ years.

$= 80.6$ years

From the time dilation formula we know that

$$\Delta t = \frac{\Delta t'}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$\Rightarrow 60.6 \text{ years} = \frac{\Delta t'}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}}$$

$$\Rightarrow \left(60.6 \text{ years}\right)^2 = \frac{\Delta t^2 \cancel{\text{years}^2}}{\left(1^2 - (.99)^2\right)}$$

$$\Rightarrow 60.6 \times 60.6 = \frac{\Delta t^2 \cancel{y}}{(1.99) \times (.01)}$$

$$\Rightarrow \Delta t = \sqrt{(60.6)^2 \times (1.99) \times (.01)}$$

$$= 8.54 \text{ years}$$

Age of A $= 20 + 8.54 = 28.54$ years.

Problems on atomic Physics

1. What is the frequency of a photon whose energy is 75 eV

Ans : 18×10^{15} Hz

Energy of photon is give by

$$E = h\nu$$

$$\Rightarrow \nu = \frac{E}{h} = \frac{75 \times 1.6 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J sec}}$$

$$= 18.09 \times 10^{15} \text{ Hz}$$

2 Calculate the energy of a Photon whose (a) frequency is 1000 K cycles/sec (Radio waves)

(b) Wavelength = 6000 A° (Yellow light)

(c) wave length = 0.6 A° (X-rays) $= 6\times10^{-1}\times10^{-10}$

Ans: (a) $E = h\nu$

$= 6.63\times10^{-34}$ Joule.sec $\times 1000\times10^{3}$ cycle/sec

$= 6.64\times10^{-28}$ Joule.

(b) $E = \dfrac{hc}{\lambda}$

$= \dfrac{6.63\times10^{-34}\times3\times10^{8}}{6\times1000\times10^{-10}\ \text{m}}$

$= 3.315\times10^{-26}\times10^{7}$

$= 3.315\times10^{-19}$ Joule.

(c) $E = \dfrac{hc}{\lambda}$

$= \dfrac{6.63\times10^{-34}\times3\times10^{8}}{6\times10^{-11}}$

$= 3.315\times10^{-15}$ Joule

3. Potential difference across the electrodes of a cathode ray gun is 500 volt. Calculate the

(a) Energy gained by the electron

(b) Speed of the electron.

(c) Momentum of the electron.

(Ans: (b) 1.3259×10^{7} m/sec (c) 1.2×10^{-23} kg.m/sec.
(a) 8×10^{-17} Joule)

Ans → (a) Energy gained by the electron = $\Delta W = q \cdot \Delta V$

$= 1.6 \times 10^{-19} \times 500$

$= 8 \times 10^{-17}$ Joule.

(b) But $\Delta W = \Delta E_K$

$= \frac{1}{2} m v^2 - \frac{1}{2} m \cdot 0^2$

$\Rightarrow v^2 = \frac{2 \Delta W}{m} = \frac{2 \times 8 \times 10^{-17}}{9.1 \times 10^{-31}}$

$= 1.7582 \times 10^{14}$

$\Rightarrow v = 1.3259 \times 10^{7}$ m/sec

(c) moment $= mv = 9.1 \times 10^{-31} \times 1.3259 \times 10^{7}$

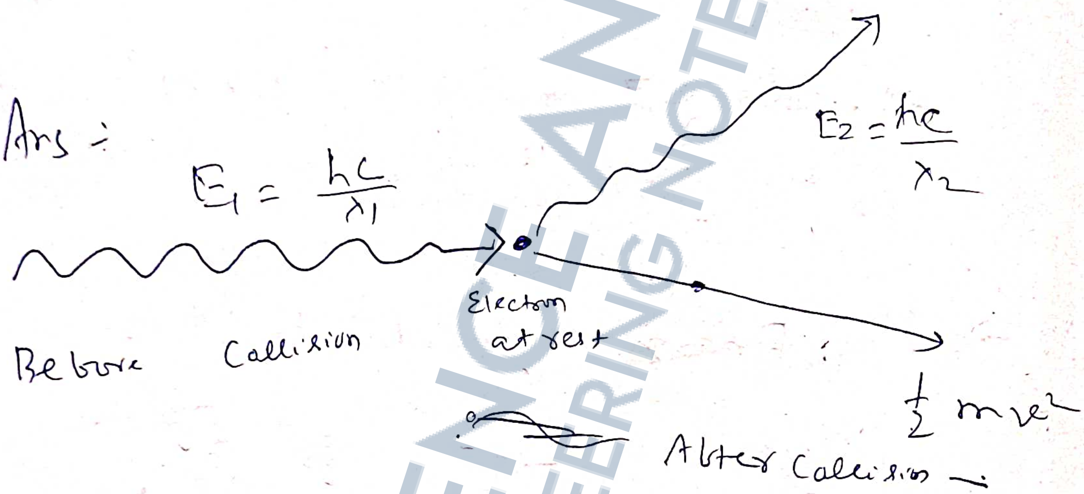
$= 12.06 \times 10^{-24}$ kg m/sec

$= 1.206 \times 10^{-24}$ kg m/se.

4. Wave length of photon $\lambda = 1.4\,A^\circ$ It Collides of with an electron at rest. After this Collision, wave length becomes $2A^\circ$. Calculate the energy of the scattered electron.

$(4.2589 \times 10^{-16}$ Joule$)$

Ans :



$\lambda_1 = 1.4\,A^\circ$,

$$\cancel{E_1 = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.4 \times 10^{-10}} = 14.2 \times 10^{-18}}$$

$$\text{Energy} = E_1 - E_2$$

$$= \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$= hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= hc\left(\frac{1}{1.4 \times 10^{-10}\,m} - \frac{1}{2 \times 10^{-10}\,m}\right)$$

$$= hc\left(\frac{10^{10}}{1.4} - \frac{10^{10}}{2}\right)$$

$$= hc \times 10^{10}\left(\frac{2-1.4}{2\times 1.4}\right)$$

$$= \frac{6.63\times 10^{-34} \times 3\times 10^{8} \times 10^{10} \times \cancel{6}^{3} \times 10^{-1}}{\cancel{2}\times 1.4}$$

$$= 42.589 \times 10^{-17} \text{ Joule}$$

$$= 4.2589 \times 10^{-16} \text{ Joule}$$

5. Calculate De-Broglie's wavelength of a beam of electron which traverses a p.d 300 volt. Assume the initial speed of electron to be zero.

Ans = 0.709 A°.

$$\boxed{\begin{array}{l} h\nu = \frac{hc}{\lambda} \\ mc^2 = \frac{hc}{\lambda} \end{array}}$$

$$\lambda = \frac{h}{p}$$

Ans → $\Delta W = q\,\Delta V$

$$= 1.6\times 10^{-19} \times 300 \text{ volt}$$

$$= 4.8 \times 10^{-17} \text{ Joule}.$$

$h = \frac{p^2}{2m}$

$$\therefore E_K = \frac{1}{2}mv^2 - \frac{1}{2}m{v_i}^2$$

$$4.8\times 10^{-17} = \frac{1}{2}mv^2 = \frac{1}{2}\times 9.1\times v^2$$

$$\Rightarrow v^2 = \frac{9.6\times 10^{-17}}{}$$

$$K.E = \frac{p^2}{2m} = \frac{p^2}{2 \times 9.1 \times 10^{-31}}$$

$$\Rightarrow 4.8 \times 10^{-17} = \frac{p^2}{2 \times 9.1 \times 10^{-31}}$$

$$\Rightarrow p^2 = 9.6 \times 9.1 \times 10^{-48}$$

$$\Rightarrow p = 9.34 \times 10^{-24}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{9.34 \times 10^{-24}} = \frac{66.3}{9.34} \times 10^{-35} \times 10^{24}$$

$$= 7.09 \times 10^{-11}$$

$$\lambda = (0.709) \times 10^{-10} \text{ meter} = .709 \, A^{\circ}$$

824 → 1, 2, 5, 11, 12

1	2	5	11	12
41.4 A°	0.02428	4.4×10^{-40}	3.38×10^{-19}	1.0527×10^{-13} J

---

1. $V = 300 \, V$

$$\Delta W = q \, \Delta V$$

$$= 1.6 \times 10^{-19} \times 300 \text{ Joule}$$

$$= E$$

$$E = h\nu$$

$$\Rightarrow \nu = \frac{E}{h} = \frac{1.6 \times 10^{-19} \times 300}{6.63 \times 10^{-34}} = \frac{48}{6.63} \times 10^{-18} \times 10^{34}$$

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 300}$$

100

$= 4.14 \times 10^{-9}$

$= 41.4 \times 10^{-10}$ meter

$= 41.4$ A°

2. $m = 9.1 \times 10^{-31}$ Kg

$q = 1.6 \times 10^{-19}$ Columb.

$\lambda = ?$

$h = 6.63 \times 10^{-34}$ Joule. sec

$C = 3 \times 10^{8}$ m/sec.

$$E = mc^2$$

$= (9.1 \times 10^{-31}) \times (3 \times 10^{8})^2$

$= 9.1 \times 9 \times 10^{-31} \times 10^{16}$

$= 81.9 \times 10^{-15}$

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{81.9 \times 10^{-15}}$$

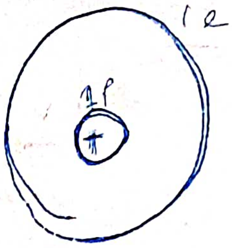
$= \frac{19.89}{8.19} \; 10^{-34} \times 10^{8} \times 10^{+14}$

$= 2.428 \times 10^{-12}$

$= .02428 \times 10^{-10}$

$= .02428$ A°

5.



M_1 = mass of proton
$= 1.6725 \times 10^{-27}$ Kg.

$q_1 = 1.6 \times 10^{-19}$ coloumb.

m_2 = mass of electron = 9.1×10^{-31} Kg.

$q_2 = 1.6 \times 10^{-19}$ Coloumb.

$r = .53 \times 10^{-10}$ meter $= 5.3 \times 10^{-11}$ meter

Gravitational force of attraction

$$= \frac{m_1 m_2}{r^2} = \frac{1.6725 \times 10^{-27} \times 9.11 \times 10^{-31}}{(5.3)^2 \times (10^{-11})^2}$$

$$= \frac{15.2197}{28.09} \times \frac{10^{-58}}{10^{-22}}$$

$= .5418 \times 10^{-36}$ Newton.
$= 5.418 \times 10^{-37}$ Newton.

Electrostatic force of attraction

$$\frac{q_1 q_2}{r^2} = \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{28.09 \times (10^{-11})^2}$$

$$= \frac{2.56 \times 10^{-38}}{28.09 \times 10^{-22}}$$

$$= \frac{256}{28.09} \times 10^{-38} \times 10^{22} \times 10^{-2}$$

$= 9.11 \times 10^{-18}$

$$= \frac{4.704 \times 10^{-35} \times 10^{22}}{28.09}$$

$$= \frac{47.04 \times 10^{-35+22}}{28.09}$$

$= 1.67 \times 10^{-14}$

Ration between

$$\frac{E.F}{G.F} = \frac{9.11 \times 10^{-18}}{5.418 \times 10^{-37}}$$

$$= 4.4 \times 10^{40}$$

$$\frac{G.F}{E.F} = \frac{5.418 \times 10^{-37}}{1.67 \times 10^{-14}} = 4.4 \times 10^{-40}.$$

11. Difference between the energy of 2 Orbits in the form of bright yellow light.

$$E_1 - E_2 = \frac{hc}{\lambda}$$

$$\Rightarrow \frac{hc}{\lambda_1} - \frac{hc}{\lambda}$$

∴ change in energy

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5876 \times 10^{-10}}$$

$$= 3.38 \times 10^{3} \times 10^{-37} \times 10^{10} \times 10^{8}$$

$\frac{e}{m} = 1.7599 \times 10^{8}$

$\frac{e}{} = 1.672 \times 10^{8}$

$= 3.38 \times 10^{-19}$ Joule

12. $\dfrac{hc}{\lambda_1} - \dfrac{hc}{\lambda_2}$

$= hc\left(\dfrac{1}{4861\times10^{-10}} - \dfrac{1}{6583\times10^{-10}}\right)$

$= hc\times10^{10}\left(\dfrac{6583-4861}{4861\times6583}\right)$

$= \dfrac{6.63\times10^{-34}\times3\times10^{8}\times1722}{4861\times6583}\times10^{10}$

$= 1.0611 \times 10^{-3}\times10^{-34}\times10^{8}\times10^{10}$

$= 1.06\times10^{-19}$ Joule

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5 $\rightarrow$ Proton $\longrightarrow$ charge 1.6×10^{-19}

Ans $\longrightarrow$?

$G = 6.67\times10^{-11}$

$r = 5.3\times10^{-11}$

---

5. Gravitational force of attraction

$= F = \dfrac{G\, m_e m_p}{r^2}$

$\Rightarrow F = \dfrac{6.67\times10^{-11}\times9.11\times10^{-31}\times1.6725\times10^{-27}}{(5.3\times10^{-11})^2}$

$= \dfrac{\ \ }{(5.3)^2}\times10^{-11}\times10^{-31}\times10^{-27}\times10^{22}$

$= 3.613 \times 10^{-47}$ Newton.

Electrostatic force of attraction

$$F = k \frac{q_1 q_2}{r^2}$$

$$= 9\times10^{9} \frac{\text{Newton meter}^2}{(\text{coulomb})^2} \times \frac{1.6\times10^{-19} \times 1.6\times10^{-19} (\text{coulomb})^2}{(5.3\times10^{-11})^2 \text{ meter}^2}$$

$$= \frac{9\times1.6\times1.6}{(5.3)^2} \times 10^{-29} \times 10^{22}$$

$$= \cancel{23.04\times10^{-7}}$$

$$= .82 \times 10^{-7}$$

$$= 8.2\times10^{-8}$$

Ratio of the two forces

$$\frac{\text{Gravitational force}}{\text{Electrostatic force}}$$

$$= \frac{3.613\times10^{-47}}{8.2\times10^{-8}}$$

$$= \frac{36.13}{8.2} \times \frac{10^{-48}}{10^{-8}}$$

$$\simeq 4.4\times10^{-40}$$