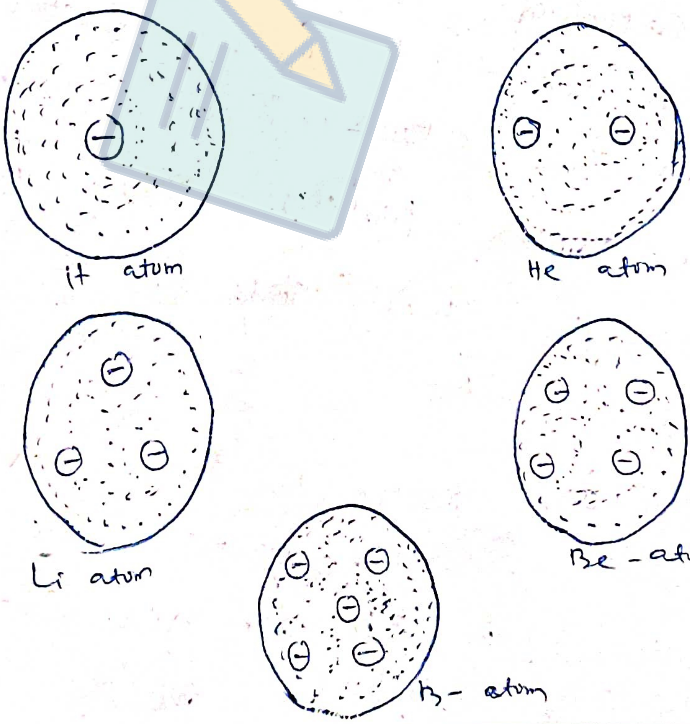
Atomic Physics

Models of the atom :

Different models were put forward by different scientists to explain the behaviour of atoms. The following 3 models will explain the gradual development of the present concepts regarding the atom.

Thomson's plum-pudding model

According to Thomson, all the +ve charges of the atom form a sphere with uniform charge distribution inside which electrons are embedded just like plums in a pudding. This model explains the neutral behaviour of an atom. But this model could not survive due to Rutherford's α-ray scattering experiment.



SCIENCE AND ENGINEERING NOTES

Rutherford's α- ray scattering experiment & Rutherford's model (Called planetary model of the atom)

The experiment involves a beam of α- particles starting from the source kept in a groove in a lead block. A very thin gold foil is kept in its path due to which the α- particles get scattered. A fluorescent screen coated with ZnS or Barium platino cyanide is kept to know the number of α- particles scattered at a particular angle θ. This number is denoted by $N(\theta)$

When a graph is plotted between $N(\theta)$ along Y-axis & θ along X-axis; it is seen that max^m number of α- particles are scattered at $\theta = 0°$. This indicates that there is a lot of empty space in between the atoms. Even at $\theta \to 180°$, there are some particles. This is called back scattering which could not be explained from thomson's model.

To explain the back scattering, Rutherford gave a new model which resembles the solar system. All the +ve charges of the atom are concentrated within a very

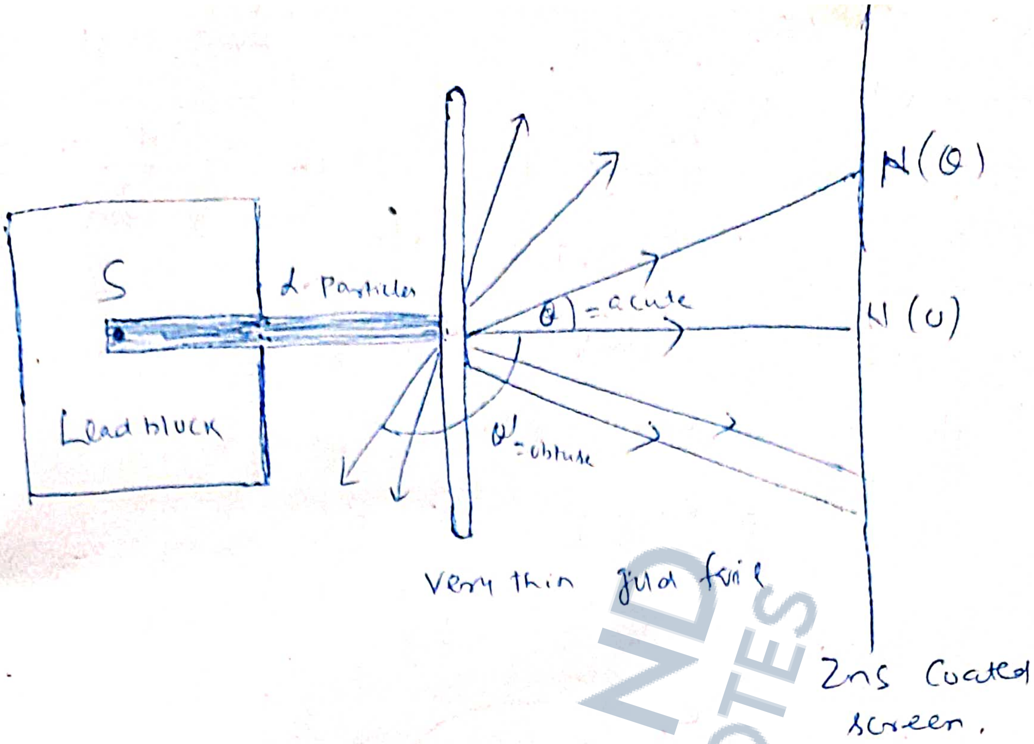


Fig - II

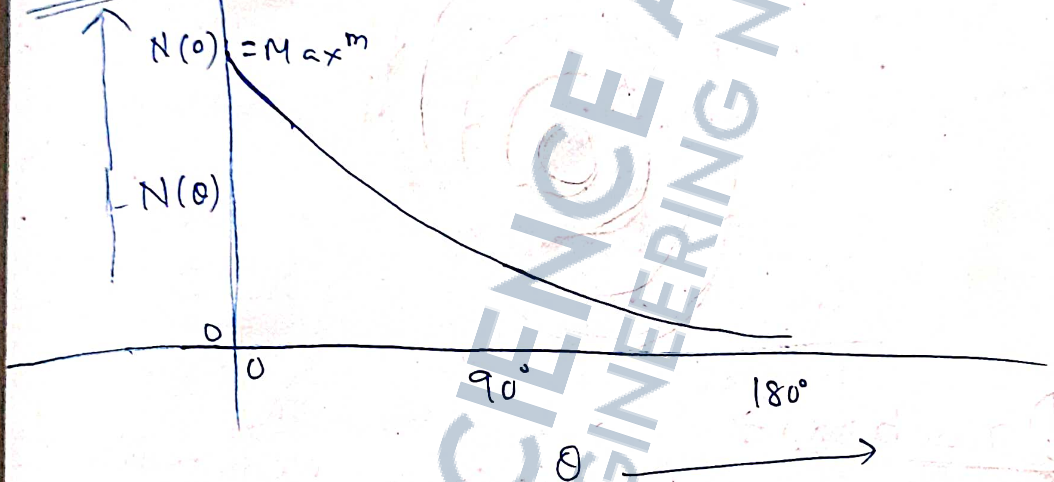
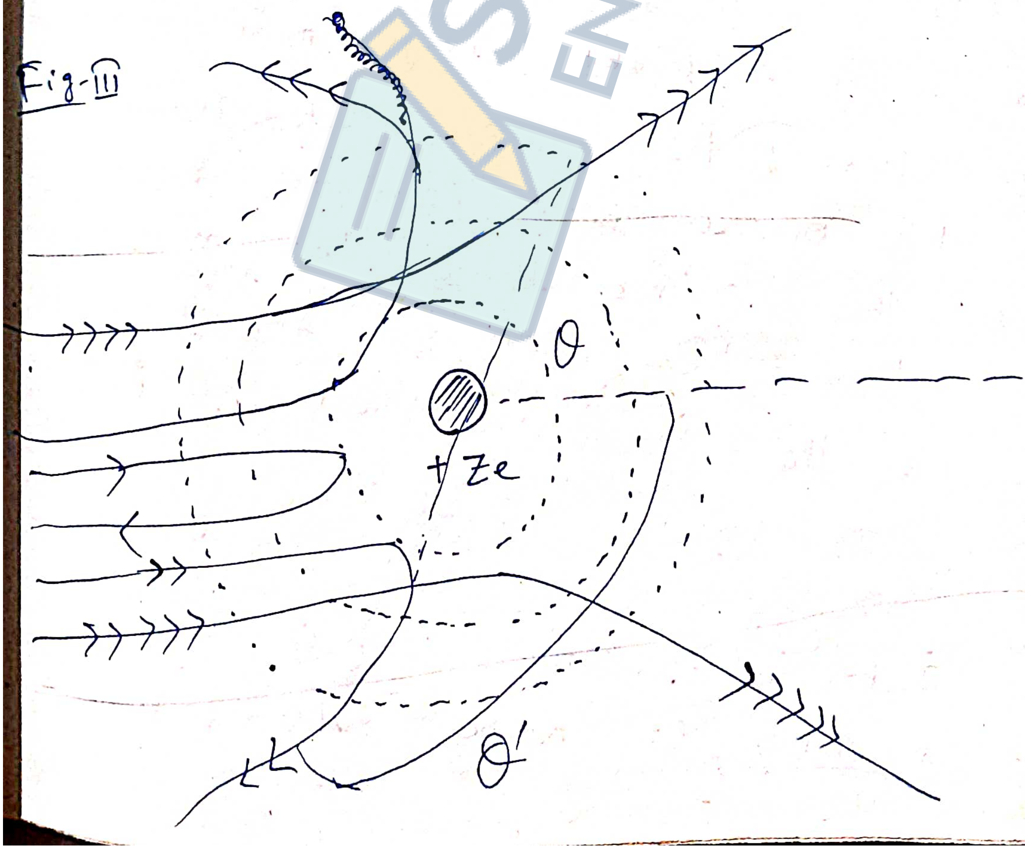
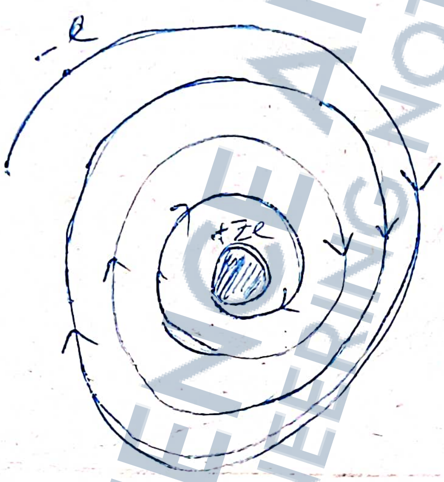


Fig-III



small space called nucleus. The electrons revolve around the nucleus to escape from the attraction of the nucleus. The centripetal force necessary for the circular motion is provided by the Colombic force of attraction.

ie $\frac{mv^2}{r} = K \cdot \frac{Ze \cdot e}{r^2}$



SCIENCE AND ENGINEERING NOTES

Drawbacks →

(i) According to classical electromagnetic theory, an accelerated charge must radiate energy. Here, the electrons while revolving around the nucleus causes centripetal accn. They must radiate energy towards the nucleus. Hence the orbit will shrink & then the electron will fall into the nucleus. Ultimately the atom will make the atom unstable. This will make the atom unstable.

But in reality atom is quite stable. This discrepancy could not be explained by Rutherford.

(i) When a gas is taken in a discharge tube at low pressure & excited by means of a high voltage, it gives a coloured light. When this light is analysed by grating spectrometer, several coloured lines appear. These coloured lines are the characteristics of the atoms present inside the tube.

Balmer could determined the wave lengths of all such lines of the hydrogen spectrum.

Rydberg could give a formula connecting these wave lengths like

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where R = Rydberg constant
$= 1.097 \times 10^{7}$ meter^{-1}

and $n = 3, 4, 5$ ....

Rutherford failed to derive this formula from his model of the atom.

Bohr's model

To remove the defects of Rutherford's model, of the atom, Bohr proposed a new model for atoms having only one electron. (H like atoms)

Ex → H, Deuterium, ~~Tritium~~ Tritium, He^{+} (Singly ionised Helium), Li^{++} (Doubly ionised ~~atom~~), etc

This model is based on the following

Postulates.

(i) The electron revolves in certain specified circular orbits where it does not radiate energy. These are called stationary orbits.

(ii) Angular momentum of electron in these stationary orbits is an integral multiple of a quantity $\frac{h}{2\pi}$.

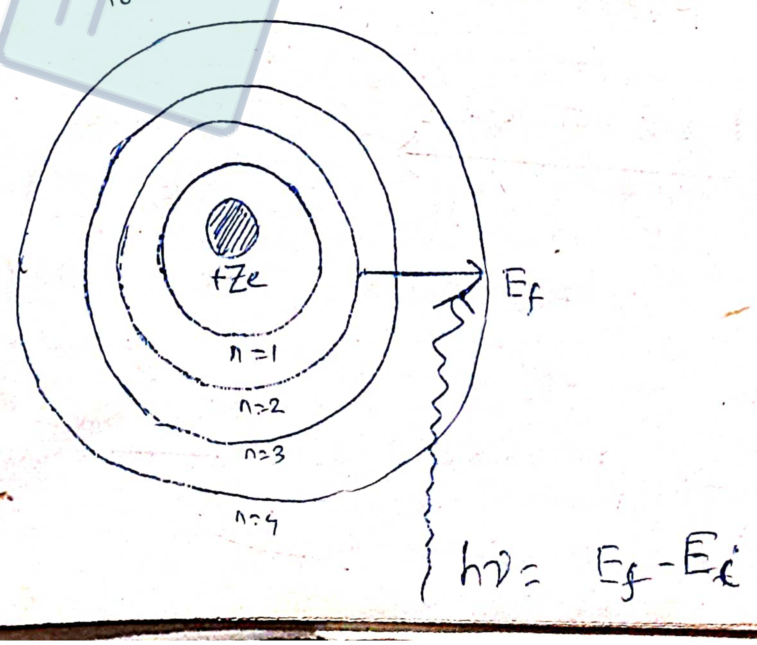
where h = Planks Constant
$= 6.63 \times 10^{-34}$ Joule. sec.

i.e $mvr = \frac{nh}{2\pi}$ where $n = 1, 2, 3$ ---

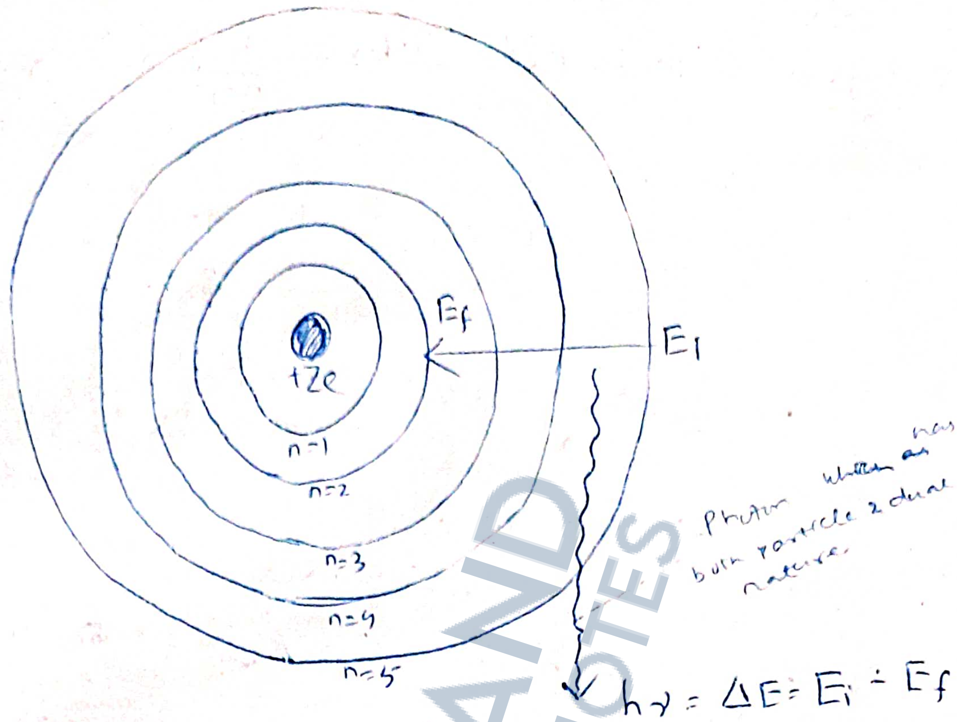
no electron can have angular momentum in between $\frac{h}{2\pi}$ & $\frac{2h}{2\pi}$

(iii) When an atom absorbs energy, the electron goes from the ~~inner~~ inner orbit to some higher orbit.

$\therefore E_i + h\nu = E_f$



(iv)



An atom can radiate energy only when the electron jumps from some higher orbit to some lower orbit. The difference of these two energies is radiated out as a photon.

$$h\nu = E_i - E_f$$

Bohr's Theory

(a) Expression for Bohr's ~~radii~~ Radii

When an electron revolves around the nucleus in an circular orbit, the centripetal force is provided by the electrostatic force of attraction by the nucleus.

$$\therefore \frac{mv^2}{r} = K\frac{Ze \cdot e}{r^2} \quad \text{———— (i)}$$

$$\Rightarrow mv^2 = \frac{KZe^2}{r} \quad \text{———— (ii)}$$

Angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$

$$\therefore \quad mvr = n.\frac{h}{2\pi} \quad \text{——— (iii)}$$

$$\Rightarrow \quad mv = n.\frac{h}{2\pi r}$$

$$\Rightarrow \quad m^2v^2 = n^2 \frac{h^2}{4\pi^2 r^2} \quad \text{——— (iv)}$$

Dividing eqn (ii) by eqn (iv), we get

$$\frac{1}{m} = \frac{K Ze^2}{\cancel{r}} \times \frac{4\pi r^{\cancel{2}} r}{n^2h^2}$$

$$\frac{1}{m} = \frac{K\ Ze^2\ 4\pi\ r}{n^2h^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{m K Ze^2 4\pi^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2 K Z me^2}$$

$$\Rightarrow \quad \boxed{r_n = \frac{n^2h^2}{4\pi^2 K Z me^2}}$$

= Radius of the n^{th} orbit for the electron.

$$\therefore \quad r_1 = \frac{h^2}{4\pi^2 K Z me^2}$$ = Radius of the ~~electron~~ first orbit.

$$\therefore \quad \boxed{r_n = n^2 . r_1}$$

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In the special case for hydrogen in the MKS system $Z = 1$ & $K = \frac{1}{4\pi\varepsilon_0}$

$$\therefore (r_1)_H = \frac{h^2}{\cancel{4}\pi^{\cancel{2}} \cdot \frac{1}{\cancel{4\pi}\varepsilon_0} \cdot 1 \cdot me^2}$$

$$= \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Calculation of radii

$$(r_1)_H = \frac{h^2 \varepsilon_0}{\pi m e^2} = \frac{(6.63 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{3.14 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} \text{ metre}$$

$$= \frac{(6.63)^2 \times 8.85}{3.14 \times 9.1 \times (1.6)^2} \times 10^{-11} \text{ metre}$$

$$= 5.3 \times 10^{-11} \text{ meter}$$

$$= 5.3 \times 10^{-11} \times 10^{10} \text{ A}^\circ$$

$$= 0.53 \text{ A}^\circ$$

$$\therefore (r_1)_{He^+} = \frac{(r_1)_H}{Z} = \frac{0.53 \text{ A}^\circ}{2} = 0.265 \text{ A}^\circ$$

$$(r_1)_{Li^{++}} = \frac{(r_1)_H}{Z} = \frac{0.53 \text{ A}^\circ}{3} = 0.176 \text{ A}^\circ$$

$$(r_2)_{Li^{++}} = 2^2 \cdot (0.176 \text{ A}^\circ)$$

$$= 4 \times (0.176)$$

$$= .704 \text{ A}^\circ$$

SCIENCE AND ENGINEERING NOTES

(b) Energy of the electron in a Bohr Orbit

The electron possesses both K.E as ~~also~~ well as electrostatic ~~pot.~~ potential energy.

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{KZe^2}{r} \quad \left(\text{From eqn ii}\right)$$

Electrostatic P.E of the electron at a point on the orbit is defined as the amount of workdone to bring the electron from infinity upto a point on the orbit.

Electrostatic P.E

$= \text{Workdone}$

$= Q \cdot \Delta V$

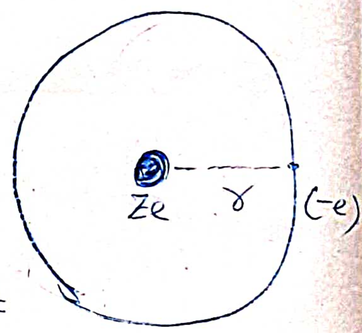
$= (-e) \cdot (V - V_\infty)$

$= -e\left(\frac{KZe}{r} - 0\right)$

$= -\frac{KZe^2}{r}$

$V = \frac{KQ}{r}$

$= \frac{KZe}{r}$



Total energy of the electron in the orbit

$= E = K.E + P.E$

$= \frac{1}{2}\frac{KZe^2}{r} - \frac{KZe^2}{r}$

SCIENCE AND ENGINEERING NOTES

$$= \frac{-1}{2} \frac{KZe^2}{r}$$

$$\therefore E_n = \frac{-1}{2} \frac{KZe^2}{r_n}$$

$$= \frac{-1}{2} \frac{KZe^2}{\frac{n^2h^2}{4\pi^2 KZme^2}}$$

$$= \frac{-1}{\cancel{2}} \frac{KZe^2 \times \overset{2}{\cancel{4}}\pi^2 KZme^2}{n^2h^2}$$

$$= -\frac{2\pi^2 K^2 Z^2 me^4}{n^2h^2}$$

$$\therefore E_1 = -\frac{2\pi^2 K^2 Z^2 me^4}{h^2}$$

$$\therefore \boxed{E_n = \frac{E_1}{n^2}}$$

In the special case for hydrogen in the M.K.S system, we have

$$Z = 1,\ K = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore (E_1)_H = -\frac{2\pi^2 \cdot \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot 1 \cdot me^4}{h^2}$$

$$= -\frac{2\cancel{\pi^2}\, me^4}{16\cancel{\pi^2}\varepsilon_0^2 h^2} = \frac{-me^4}{8\varepsilon_0^2 h^2}$$

Calculation of energies

$$(E_1)_H = \frac{-me^4}{8\epsilon_0^2 h^2}$$

$$= -\frac{9.1\times10^{-31}\times(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2\times(6.63\times10^{-34})^2} \text{ Joule}$$

$$= -\frac{9.1\times(1.6)^4}{8\times(8.85)^2\times(6.63)^2}\times10^{-15} \text{ Joule}$$

$$= -0.002176\times10^{-15} \text{ Joule}$$

$$= -\frac{0.002176\times10^{-15}}{1.6\times10^{-19}} \text{ eV}$$

$$= -13.6 \text{ eV}$$

$$(E_2)_H = \frac{(E_1)_H}{2^2} = \frac{-13.6}{4} = -3.4 \text{ eV}$$

$$(E_3)_H = \frac{(E_1)_H}{3^2} = \frac{-13.6}{9} = -1.51 \text{ eV}$$

$$(E_\infty)_H = \frac{(E_1)_H}{(\infty)^2} = 0 \text{ eV} = \text{Max}^m$$

$$(E_1)_{He^+} = Z^2 (E_1)_H = 2^2.(-13.6)$$

$$= -54.4 \text{ eV}$$

$$(E_1)_{Li^{++}} = Z^2 (E_1)_H = 3^2.(-13.6) = -122.4 \text{ eV}$$

$$(E_2)_{Li^{++}} = \frac{(E_1)_{Li}}{n^2}$$

$$= \frac{-122.4}{4}$$

$$= -30.6\ eV$$

Spectral series of hydrogen (Hydrogen Spectrum)

From Bohr's theory, the energy of an electron in the nth orbit of Hydrogen atom is found to be

$$E_n = \frac{E_1}{n^2} = -\frac{me^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

If the electron jumps from some higher orbit to some lower orbit, the difference of the energies is radiated out as a photon

$$E_i - E_f = h\nu$$

$$\Rightarrow -\frac{me^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n_i^2} + \frac{me^4}{8\epsilon_0^2 h^2} \frac{1}{n_f^2} = h\nu$$

$$\Rightarrow \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = h\frac{c}{\lambda}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{me^4}{8\epsilon_0 h^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\Rightarrow \boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

SCIENCE AND ENGINEERING NOTES

where $R = \frac{me^4}{8\epsilon_0 h^3 c}$ = Rydberg. Constant

$= 1.097 \times 10^7 \, m^{-1}$

Thus Bohr arrived at a formula similar to that of Rydberg formula & perfectly matches with it only when ~~nf = 2~~ $n_f = 2$.

The prediction of Bohr for $n_f = 1, 3, 4, 5$ ~~could b~~ could be experimentally verified. This is a great success of Bohr's model

① Lyman series

If the electron jumps from any higher orbit to the first orbit, then the wavelength are found to lie emitted by the hydrogen atom in the ultra violet region.

i.e $n_f = 1$ & $n_i = 2, 3, 4 ----$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$$

② Balmer ~~by~~ Series

If the electron jumps from any higher orbit to the 2nd orbit, then the wavelengths are found to lie emitted by the hydrogen atom in the visible region.

i.e $n_f = 2$, & $n_i = 3, 4, 5$ ------

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

(3) Paschen's series

If the electron jumps from any higher orbit to the 3rd orbit, then the wave length emitted by the hydrogen atom are found to lie in the infrared region.

i.e $n_f = 3$, & $n_i = 4, 5, 6$, ------

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$$

(4) Brackett series

If the electron jumps from any higher orbit to the 4th orbit, then the wave length emitted by the hydrogen atom are found to lie in the infrared region

i.e $n_f = 4$, & $n_i = 5, 6, 7$ ------

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$$

(5) Pfund series

If the electron jumps from any higher orbit to the 5th orbit, then the wavelength emitted by by the hydrogen atom are found to be lie in the infrared region

i.e $n_f = 5$, & $n_i = 6, 7, 8$ ———

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$$

Problem

1. Wave length of the first line of Balmer series is 6563 A°, Find

(a) λ of the 2nd, 3rd, 5th & last line of the Balmer series.
4340.6, 3798 A°, 3646.1

(b) Shortest & longest wavelength of the Lyman series. (Ans 912 A°)
1216 A°

(c) Shortest & longest wavelength of the Paschen series (8204 A°) (18752 A°)

— o —

(a) For the Balmer series wave lengths are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where $n_i = 3, 4, 5$ ———

For the first line, $n_i = 3$, and similarly

$$\therefore \frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) \text{ (i)}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{16}\right) \text{ — (ii)}$$

Dividing eqn (i) by eqn (ii), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{\frac{5R}{36}}{\frac{3R}{16}} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27}$$

$$\Rightarrow \lambda_2 = \frac{20}{27}\lambda_1$$

$$= \frac{20}{27} \times 6563 \text{ A}^\circ$$

$$= 4861.48 \text{ A}^\circ$$

Continued →

(b) For the Lyman series the electron will jump from ∞ orbit to the first orbit ΔE = Max^m & frequency will be max^m but wave length will be min^m

$$\therefore \frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$= R$$

Using this value of R in eq^n (i), we get

$$\frac{1}{\lambda_1} = \frac{1}{\lambda_{min}} \times \frac{5}{36}$$

$$\Rightarrow \lambda_{min} = \frac{5}{36}\lambda_1 = \frac{5}{36} \times 6563 \text{ A}^\circ$$

$$= 911.527 \text{ A}^\circ$$

= Shortest wavelength

For the Lyman series, when the electron will jump from 2nd orbit to the first orbit ΔE is min^m, frequency will be min^m, wave length will be max^m

$$\therefore \frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= \frac{3R}{4} \quad (10)$$

Dividing eqn (1) & by eqn (10), we get

$$\frac{\lambda_{max}}{\lambda_1} = \frac{5R \times 4}{36 \times 3R} = \frac{5}{27}$$

$$= \frac{5}{27} \times 6563\ A^\circ$$

$$= 1215.37\ A^\circ$$

SCIENCE AND ENGINEERING NOTES

Continued → Problem 1

For the Balmer series wave lengths are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where $n_i = 3, 4, 5, 6, 7$ ------

For the first line $n_i = 3$, ~~2nd line~~

3rd line $n_i = 5$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R \times \frac{5}{36} \quad \text{—(i)}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = R \times \frac{21}{100} \quad \text{—(ii)}$$

Dividing, $\frac{\lambda_2}{\lambda_1} = \frac{5}{\cancel{36}_{9}} \times \frac{\cancel{100}^{25}}{21} = \frac{125}{189}$

$\Rightarrow \lambda_2 = \frac{125}{189} \times \lambda_1 = \frac{125}{189} \times 6563 A^\circ$

$= 4340.6 A^\circ$.

For the 8 the line $n_i = 10$

Eqn (iii) becomes

$$\frac{1}{\lambda_3} = R\left(\frac{1}{2^2} - \frac{1}{10^2}\right) = R \times \frac{24}{100}$$

Dividing eqn (i) by eqn (iii), we get

$$\frac{\lambda_3}{\lambda_1} = \frac{5}{\cancel{36}_{9}} \times \frac{\cancel{100}^{25}}{24} = \frac{125}{216}$$

$$\Rightarrow \lambda_3 = \frac{125}{216} \times 6563 A^\circ$$

$$= 3798.03 A^\circ$$

Similarly (iv) Eqn will be

$$\frac{1}{\lambda_4} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

Dividing eqn (1) by eqn (4) we get

$$\frac{\lambda_4}{\lambda_1} = \frac{5}{\cancel{36}_{9}} \times \frac{\cancel{4}}{1} = \frac{5}{9}$$

$$\Rightarrow \lambda_4 = \frac{5}{9} \times \lambda_1 = \frac{5}{9} \times 6563 A^\circ$$

$$= 3646.11 A^\circ$$

Problems from book $\boxed{\text{Beiser}}$

Q.6. $f = \dfrac{1}{\text{Time period}}$

$$\text{Time} = \frac{2\pi r}{v}$$

We know that centripetal force

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$\Rightarrow \dfrac{mv^2}{} = \dfrac{KZe^2}{r^2}$ ——— (1)

but $mvr = \dfrac{nh}{2\pi}$

$\Rightarrow mv = \dfrac{nh}{2\pi r}$ ——— (11)

Dividing $v = \dfrac{KZe^2}{\cancel{r}} \times \dfrac{2\pi \cancel{r}}{nh}$

$= \dfrac{KZe^2\,2\pi}{nh}$

Time $= \dfrac{2\pi r}{v} = \dfrac{2\pi r}{\dfrac{KZe^2\,2\pi}{nh}} = \dfrac{\cancel{2\pi} r \times nh}{KZe^2\,\cancel{2\pi}}$

frequency $= \dfrac{KZe^2}{rnh} = \dfrac{\frac{1}{4\pi\varepsilon_0} Ze^2}{\left(\dfrac{n^2h^2}{4\pi^2 KZme^2}\right) nh}$

$\dfrac{Ze^2}{4\pi\varepsilon_0} \times \dfrac{4\pi^2 K Z me^2}{n^3h^3}$

(a) $= \dfrac{4\pi^2 m z^2 e^4}{(4\pi\varepsilon_0)^2 n^3h^3}$ $= \dfrac{Ze^2}{4\pi\varepsilon_0} \times \dfrac{\cancel{4\pi^2 K Z me^2}}{2\pi\, n^2h^2\, nh}$

(b) $= \dfrac{KZ^2me^4}{2n^3h^3} = \dfrac{Z^2me^4}{\left(\frac{1}{4\pi\varepsilon_0}\right) 8\pi\varepsilon_0\, 2n^3h^3}$

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10. $f = \dfrac{1}{T} = \dfrac{1}{\left(\dfrac{2\pi r}{v}\right)} = \dfrac{v}{2\pi r}$

But $mvr = \dfrac{nh}{2\pi}$

$\therefore v = \dfrac{nh}{2\pi mr}$

$\therefore f_n = \dfrac{v_n}{2\pi r_n} = \dfrac{nh}{2\pi m r_n \times 2\pi r_n} = \dfrac{nh}{(2\pi)^2 m r_n^2}$

15.

$$f_n = \frac{nh}{4\pi^2 m \left(\frac{n^2h^2}{4\pi^2 k z m e^2}\right)^2}$$

$$= \frac{nh \cdot 16\pi^4 k^2 z^2 m^2 e^4}{4\pi^2 \cdot m \cdot n^4 h^4}$$

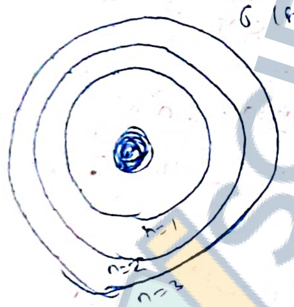
$$= \frac{4\pi^2 k^2 z^2 m e^4}{n^3 h^3}$$

$$= \frac{4\pi^2 z^2 m e^4}{(4\pi\epsilon_0)^2 n^3 h^3} \quad \text{Ans}$$

Expression $V_n, T_n, f_n \longrightarrow$

V_1, V_n ... $(V_1 ...), (V_1 ...)$ 6.1×10^5 1×54.4

10.



E_3 or $H = -1.51$ eV

Energy required

$= E_\infty - E_3$

$= 0 - (-1.51)$

$= 1.51$ eV

15. Ionization Potential is energy required to remove an electron from its orbit.

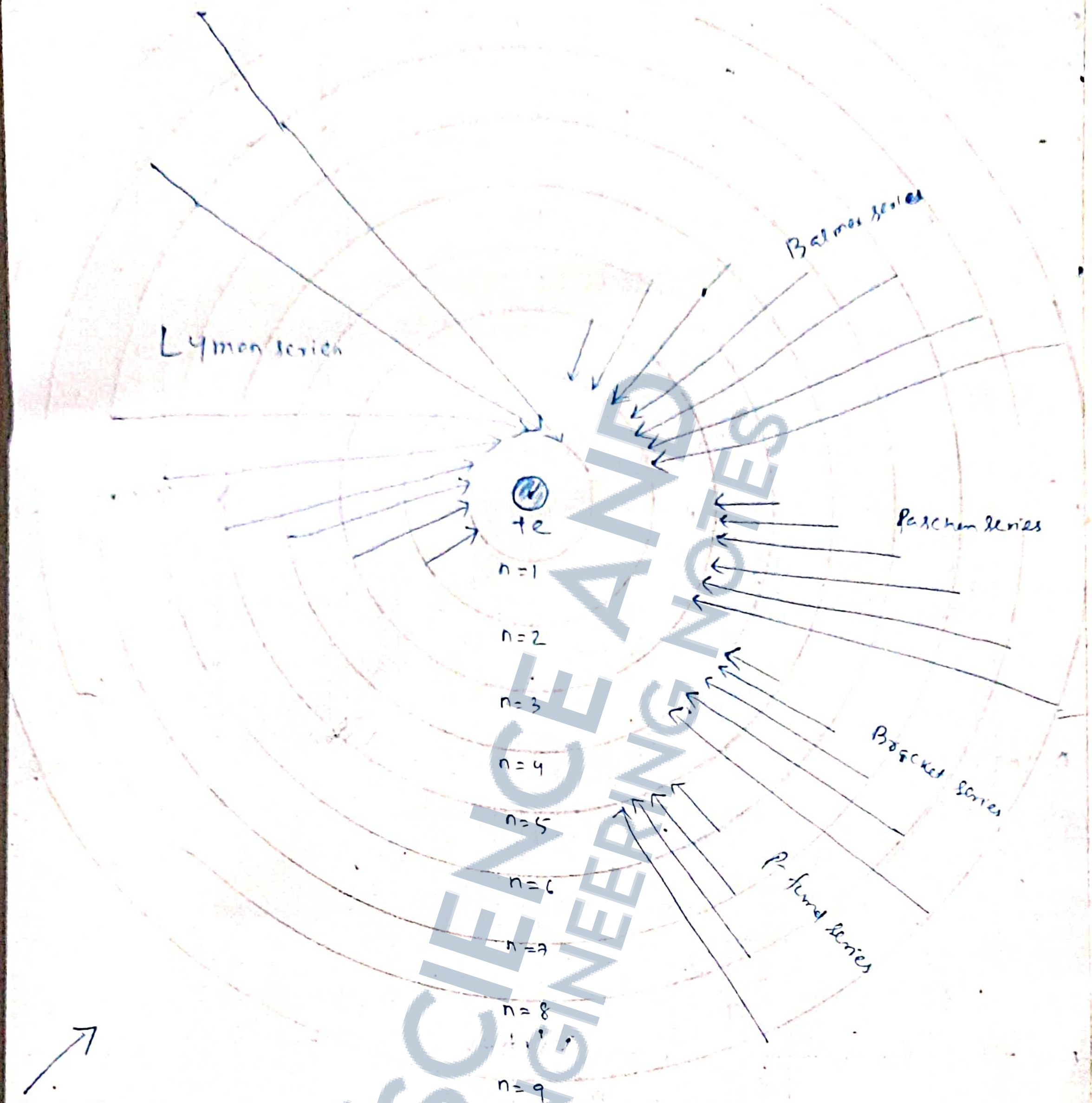
Or requirement

$E_\infty - E_2$

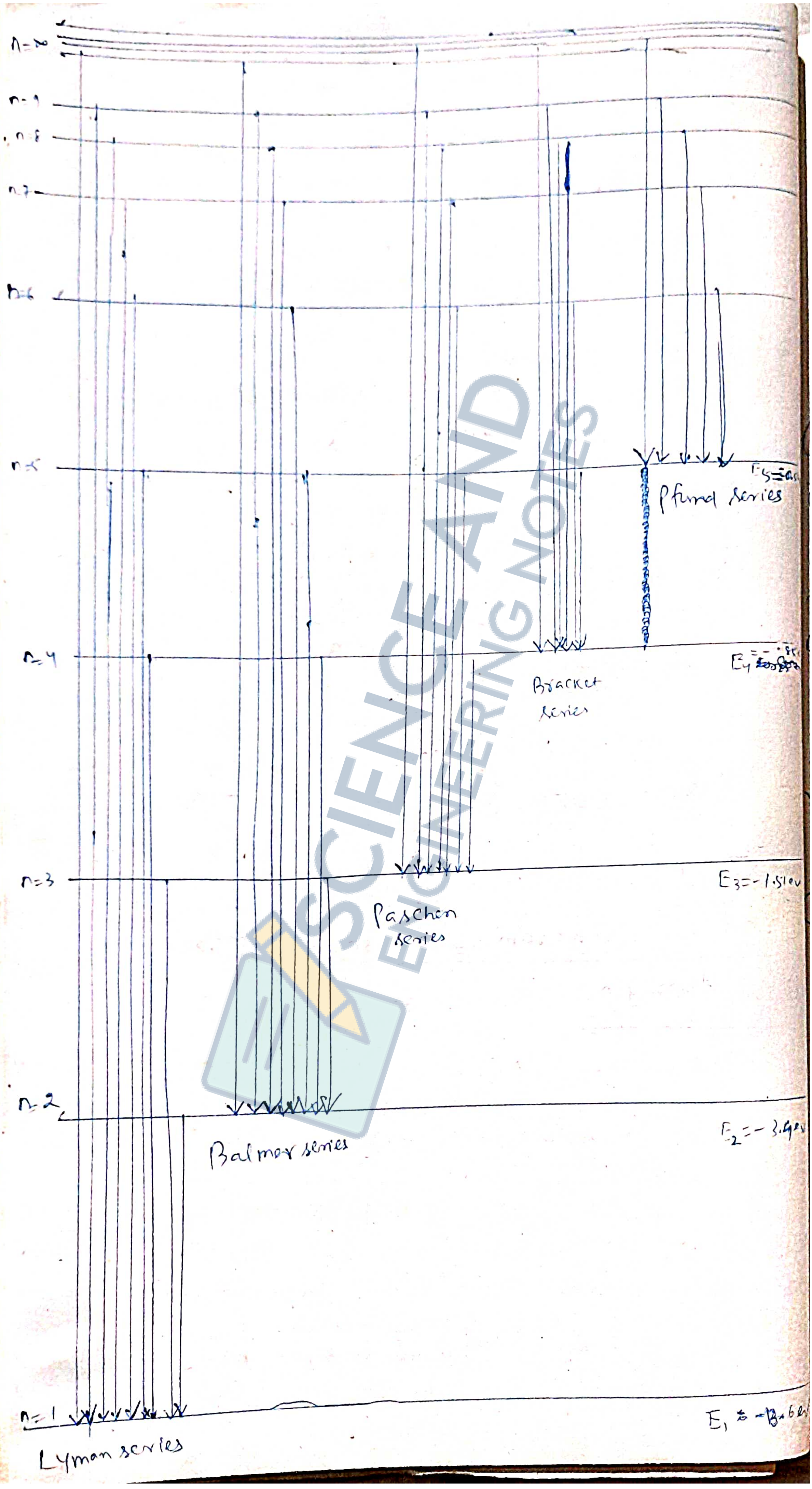
$= 0 - (-54.4)$

$= 54.4$ eV

SCIENCE AND ENGINEERING NOTES



Orbit diagram to show spectral series of Hydrogen



n=∞
n=9
n=8
n=7
n=6
n=5
n=4
n=3
n=2
n=1
Pfund series
Bracket series
Paschen series
Balmer series
Lyman series
$E_3 = -1.51 eV$
$E_2 = -3.4 eV$

Defects of Bohr Model

Although Bohr's Theory has been very successful in explaining the hydrogen spectrum & in giving valuable information about the atomic structure, it has the following defects.

1. An individual line of the hydrogen spectrum when examined under a high resolving spectroscope is found to consists of a number of faint lines. This is called fine structure which could not be explained from Bohr's theory. It can be explained by taking into consideration the relativistic variation of mass of electron & the ~~spin~~ spin of the electron.

2. Bohr's theory can not explain the variation in the intensity of the spectral line of an element. This can be explained by quantum mechanics.

3. Bohr's ~~theo~~ theory is applicable to the atom having only one electron. such as hydrogen, He^+, Li^{++} etc. It can not explain the spectra of complex atoms.

4. Electron configuration of atoms requires 4 quantum numbers. Bohr's theory provides only one quantum number.

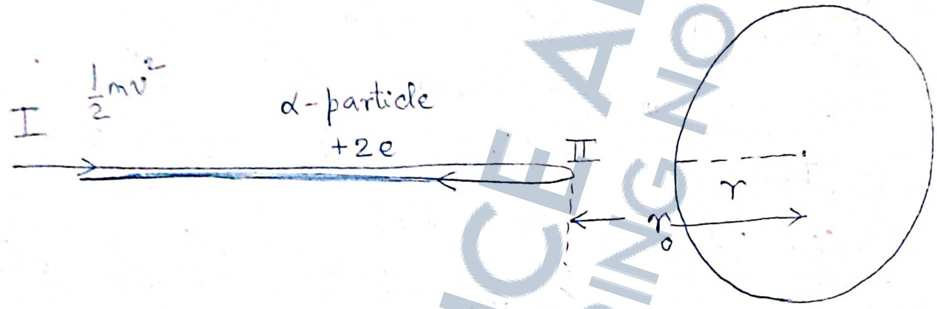
(5) Bohr's theory can not explain (irregular) anomalous Zeeman effect.

(6) Bohr's model failed to explain Heisenberg's uncertainty principle.

Distance of Closest approach

r = Radius of the nucleus

r_0 = Distance of closest approach.



$r < r_0$ $+Ze$

$Z = 79$ for Gold nucleus.

Knowledge of r_0 helps us to estimate the radius of the nucleus.

To derive & an expression for r_0, lets calculate the total energy of the α particles at position I & II.

At position I, the α particle is far away from the nucleus & electrostatic potential energy = 0 & K.E = $\frac{1}{2}mv^2$

Total energy at position I = $\frac{1}{2}mv^2$ —(i)

At position II, the α-particle is momentarily at rest so that K.E = 0 the

Electrostatic Potential energy = ΔW

$= Q . \Delta V$

$= +2e . (V - V_\infty)$

$= +2e \left(\frac{KZe}{r_0} - 0 \right)$

Total energy at position II $= \frac{2KZe^2}{r_0}$ ——— (ii)

From the principle of conservation of energy, we can write

Total energy at position (I) = Total energy at position II

$\therefore \frac{1}{2} m v^2 = \frac{2KZe^2}{r_0}$ ——— (iii)

$\Rightarrow r_0 = \frac{4KZe^2}{mv^2}$ ——— (iv)

Prob

1. Estimate the radius of a Gold nucleus when an α particle of energy 6MeV is incident on its centre & get scattered through 180°. (Ans: $\leq$ 37.92 Fermi; 1 fermi = 10^{-15} metre)

Ans: Using eqn (iii), we get

$\frac{1}{2} m v^2 = \frac{2KZe^2}{r_0}$

(5) P.

$\Rightarrow 6 \times 1.6 \times 10^{-13} = \dfrac{2 \times 9 \times 10^{9} \times 79 \times (1.6 \times 10^{-19})^2}{r_0}$

$\Rightarrow r_0 = \dfrac{\cancel{2} \times \overset{3}{\cancel{9}} \times 79 \times (1.6)^2}{\cancel{6} \times 1.6} \times \dfrac{10^{9} \times 10^{-38}}{10^{-13}}$

$= \dfrac{3 \times 79 \times \overset{1.6}{\cancel{2.56}}}{\cancel{1.6}} \times 10^{-16}$

$= 379.2 \times 10^{-16}$

$= 37.92 \times 10^{-15}$ meter

$= 37.92$ fermi.

Hence $r < 39.92$ fermi.

(2). Calculate the I.P for Li atom when the electron was in the 2nd orbit.

Ans: 30.6 eV

$\Delta E = E_\alpha - E_2$

$= 0 - (-30.6 \text{ eV})$

$= 30.6 \text{ eV}$

8 24

8. The energy of the electron is

2.42×10^{-13} erg

$= 2.42 \times 10^{-20}$ Joule.

Energy of the electron on the 1st orbit of Hydrogen atom $= -13.6\ eV$

$= -13.6 \times 1.6 \times 10^{-19}$ Joule

$= -21.76 \times 10^{-19}$ Joule

13.6
1.6
21.76

$\Delta E = E - E_1$

$= 2.42 \times 10^{-20} - (-21.76 \times 10^{-19})$

$= 2.42 \times 10^{-20} + 21.76 \times 10^{-19}$

$= 10^{-19}(.242 + 21.76)$

$= 22.002 \times 10^{-19}$

21.76
$.242$
22.002

But $\Delta E = h\nu$

$\Rightarrow 22.002 = 6.63 \times 10^{-34} \times \nu$

$\Rightarrow \nu = \dfrac{22.002 \times 10^{-19} \times 10^{34}}{6.63}$

$= 3.318 \times 10^{15}$ hz

13. For the absorption of energy by an atom, Bohr's model gives

$E_i + h\nu = E_f$

$\Rightarrow E_1 + 12.02\ eV = E_n = \dfrac{E_1}{n^2}$

$\Rightarrow -13.6 + 12.2\ eV = E_n$

$\Rightarrow -1.4 = E_n$

$\Rightarrow \frac{E_1}{n^2} = -1.4$

$\Rightarrow \frac{-13.6}{n^2} = -1.4$

$\Rightarrow n^2 = \frac{13.6}{1.4} = 9.7$

$\Rightarrow n = 3.116$

$\simeq 3$

$\therefore$ Orbit number can not be fractional it must be near to 3.116 ie 3.

$\therefore$ Electron will go to the 3rd orbit after absorbing an energy of amount 12.2 eV.

Problems on Photoelectric effect

Einstein photo electric eqn is

$$h\nu = W_0 + \frac{1}{2} m v_{max}^2$$

where W_0 = work function $= h\nu_0$

ν_0 = Threshold frequency

= Minm frequency needed to just liberate an electron from the surface of the metal.

1. Calculate the threshold frequency of photons which can remove photoelectrons from (a) Cs (b) Ni surfaces

Given W_0 for $C_s = 1.8\ eV$

" " $Ni = 5.9\ eV$

Ans: 4.3×10^{14} hz, 1.42×10^{15} hz

2. If the speed of a photo electron is 10^4 metre/sec, what should be the frequency of the incident radiation on a K metal Ans: 5.5×10^{14} hz

W_0 of $K = 2.3\ eV$

(3) The threshold wavelength $\left(\lambda_0 = \frac{c}{\nu_0}\right)$ of photoelectric emission in a metal is 2300 A°. What wavelength of light must be used in order that electron with max^m energy of 1.5 eV are to be ejected (Ans: 1800 A°).

— o —

3. We know, the Einstein photo electric eqn

$$h\nu = W_0 + \frac{1}{2} m V_{max}^2$$

$$\Rightarrow h.\frac{c}{\lambda} = h\nu_0 + 1.5\ ev$$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{\nu_0}{c} + \frac{1.5 \times 1.6 \times 10^{-19}}{hc} \quad \left(\because \lambda_0 = \frac{c}{\nu_0}, \ \frac{\nu_0}{c} = \frac{1}{\lambda_0} \right)$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{1}{\lambda_0} + \frac{1.5 \times 1.6 \times 10^{-19}}{hc}$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{1}{2300 \times 10^{-10}} + \frac{2.40 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$

$= \frac{10^8}{23} + \frac{2.4}{6.63 \times 3} \times 10^{-19} \times 10^{34} \times 10^{-8}$

$= \frac{10^8}{23} + \frac{.8}{6.63} \times 10^{7}$

$= 10^7 \left(\frac{10}{23} + \frac{.8}{6.63} \right)$

$= 10^7 \left(.43 + \frac{80}{663} \right)$

$= 10^7 (.43 + .12)$

$= .55 \times 10^7$

$\Rightarrow \lambda = 10^{-7} \times \frac{1}{.55} = 1.8 \times 10^{-7} = 1800 \times 10^{-10}\,m = 1800\,A^\circ$

2. $V_{max} = 10^4$ metre/sec

$\nu = ?$

$W_0 = 2.3\ eV$

~~We know that $W_0 = h\nu_0$~~

~~$\Rightarrow 2.3 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times \nu_0$~~

$h\nu = W_0 + \frac{1}{2} m V_{max}^2$

$\Rightarrow 6.63 \times 10^{-34} \times \nu = 2.3 \times 1.6 \times 10^{-19} + \frac{1}{2} (9.1 \times 10^{-31}) 10^{8}$

$= 3.68 \times 10^{-19} + \frac{1}{2}\, 4.55 \times 10^{-23}$

$\Rightarrow 6.63 \times 10^{-34} \times \nu = 10^{-19}\left(3.68 + .000455\right)$

$\Rightarrow \nu = \dfrac{10^{-19} \times (3.680455)}{6.63 \times 10^{-34}}$

$= 10^{15} \times \dfrac{3.680455}{6.63}$

$= 10^{14} \times 36.80455$

$= 5.55 \times 10^{14}$ hz

1. We know that

work function $= h\nu_0$

For Cs, $W_0 = 1.8$ eV

$= 1.8 \times 1.6 \times 10^{-19}$ Joule

$\Rightarrow h\nu_0 = 1.8 \times 1.6 \times 10^{-19}$

$\Rightarrow 6.63 \times 10^{-34} \times \nu_0 = 1.8 \times 1.6 \times 10^{-19}$

$\Rightarrow \nu_0 = \dfrac{1.8 \times 1.6}{6.63 \times 10^{-34}} \times 10^{-19}$

$= \dfrac{1.8 \times 1.6}{6.63} \times 10^{15} \rightarrow 4.3 \times 10^{14}$ hz

For Ni, $W_0 = 5.9 \times 1.6 \times 10^{-19}$ Joule

$\Rightarrow h\nu_0 = 5.9 \times 1.6 \times 10^{-19}$

$\Rightarrow \nu_0 = \dfrac{5.9 \times 1.6}{6.63 \times} \times 10^{-19} \times 10^{34}$

SCIENCE AND ENGINEERING NOTES

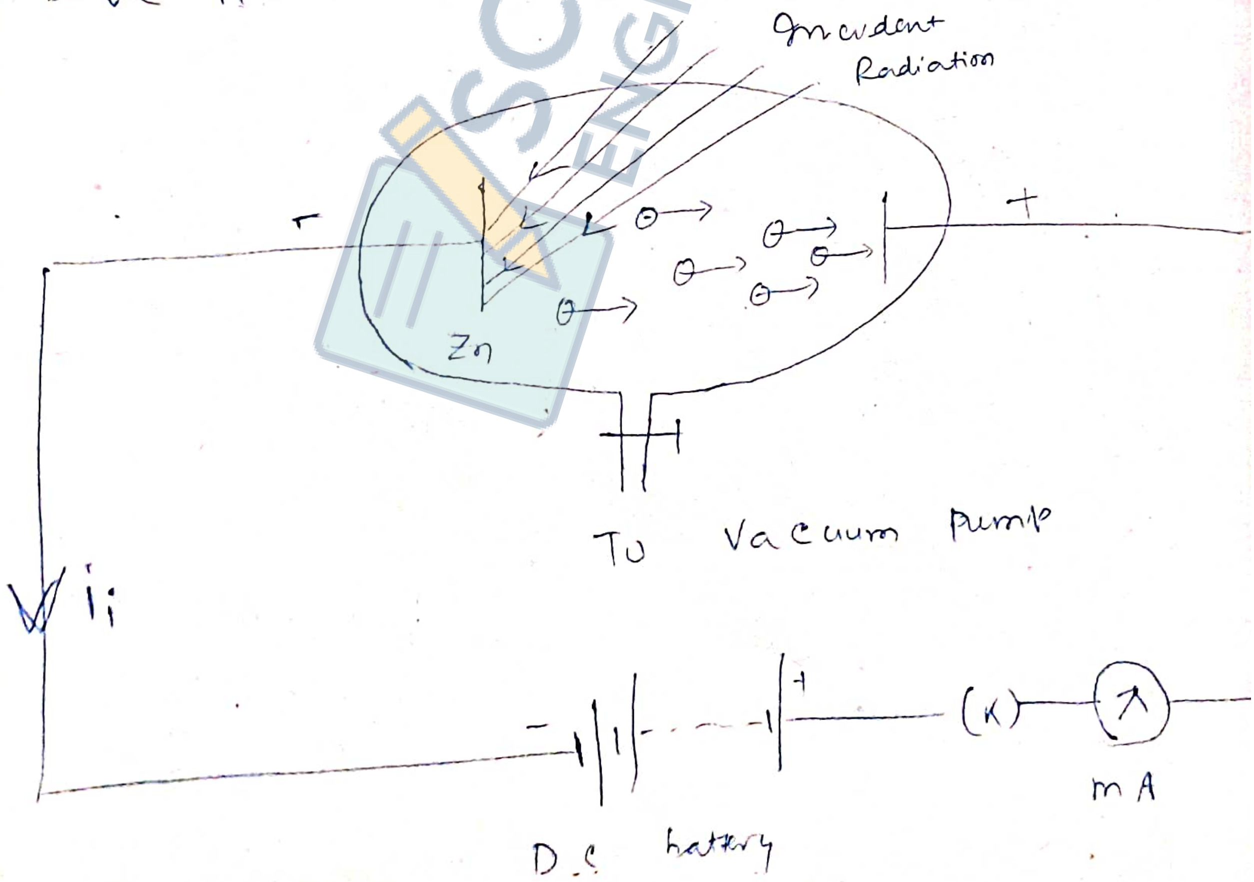
$\therefore \nu_0$ for Ni $= 1.42 \times 10^{15}$ Hz

The Phenomenon of emission of electrons from the surfaces of certain substances, mainly metals, when of shorter wavelength is is incident upon them, is called Photo electric effect.

Photo electric effect

When visible, ultra violet or X-ray is allowed to fall on certain substances like Sodium, K, Zn etc, electron are found to be ejected from the surface of ~~them~~ these metals. These electrons are called Photo electrons. The current aries out of these electrons is called Photo electric current.

To make the emission easier, the incident radiation is allowed to fall on the -ve plate as shown in the diagram.



SCIENCE AND ENGINEERING NOTES

Laws of Photoelectric emission

Many scientists like Millikan, Lenard, Hertz etc have worked on photo electric effect. Their results can be expressed by the following 3 laws.

(i) Photo electric emission is an instantaneous process. i.e there is no time lag between the falling of light & the emission of photo electron. (10^{-8} sec)

(ii) Different photo electrons are found to possess different velocities. The max^m velocity of the photo electrons is regulated by the frequency of the incident light.

Explanation: Suppose a particular metal is being radiated with a 40 watt green bulb in turn. red bulb & a 40 watt

It will be found that the photo current remains the same in both the cases.

But max^m velocity of the photo electrons for the green bulb will be higher than that for the red bulb.

because $\lambda_{green} < \lambda_{red}$

$\Rightarrow \nu_{green} > \nu_{red}$

(iii) Number of photo electrons emitted by a metal is directly proportional to the intensity of the incident light.

Explanation :

Suppose a particular metal be radiated with yellow light by 60W bulb & 100W bulb in turn. It will be found that the photo current will be higher in case of 100W bulb than that for the 60W bulb. But ν_{max} of photoelectrons in both the will be the same.

Tasks

1. Find r_n, V_n, T_n, f_n, $(V_1)_H$

Relation between V_n & V_1, T_n & T_1, f_n & f_1

	T	F
$(V_1)_H$ = ✓		
$(V_2)_H$ =		
$(V_n)_H$ =		
$(V_1)_{He^+}$ →		
$(V_1)_{Li^{2+}}$ → ✓		

Ans (i) → V_n (family)

SCIENCE AND ENGINEERING NOTES

We know that

$$m v_n r_n = \frac{nh}{2\pi}$$

$$\Rightarrow v_n = \frac{nh}{2\pi m r_n}$$

$$= \frac{nh}{2\pi m\left(\frac{n^2 h^2}{4\pi^2 k^2 Z m e^2}\right)}$$

$$= \frac{\cancel{n} \cancel{h} \cdot \cancel{4\pi^2}^{2\pi} k Z \cancel{m} e^2}{\cancel{2\pi} \cancel{m} \cancel{n^2} \cancel{h^2} nh}$$

$$= \frac{2\pi k Z e^2}{hn}$$

$$v_1 = \frac{2\pi k Z e^2}{h}$$

$$\therefore \boxed{v_n = \frac{v_1}{n}}$$

In the special case for hydrogen in the MKS unit we have

$$Z = 1, \quad k = \frac{1}{4\pi\varepsilon_0}$$

$$(v_1)_H = \frac{2\pi \cdot 1 \cdot 1 \cdot e^2}{4\pi \varepsilon_0 \cdot h} = \frac{e^2}{2\varepsilon_0 h}$$

Calculation of velocity

$$(v_1)_H = \frac{e^2}{2\varepsilon_0 h} = \frac{(1.6\times10^{-19})^2}{2\times(8.85\times10^{-12})\times(6.63\times10^{-34})}$$

$$= \frac{1.6\times1.6}{2\times8.85\times6.63} \times 10^{-38} \times 10^{34} \times 10^{12}$$

$$= 0.21814 \times 10^{8} \text{ m/sec}$$

$$= 2.184 \times 10^{6} \text{ m/sec.}$$

$$(v_2)_H = \frac{(E_1)_H}{2} = \frac{2.1814}{2} \times 10^6 \text{ m/sec}$$
$$= 1.0907 \times 10^6 \text{ m/sec.}$$

$$(V_\infty)_H = \frac{(E_1)_H}{\infty} = 0$$

$$(V_1)_{He^+} = 2(V_1)_H = 2 \times 2.1814 \times 10^6 \text{ m/sec}$$
$$= 4.3628 \times 10^6 \text{ m/sec.}$$

$$(V_1)_{Li^{++}} = 3(v_1)_H = 3 \times 2.1814 \times 10^6 \text{ m/sec}$$
$$= 6.5442 \times 10^6 \text{ m/sec}$$

$$(v_2)_{Li^{++}} = \frac{(V_1)_{Li^{++}}}{2} = \frac{6.5442 \times 10^6}{2} \text{ m/sec}$$
$$= 3.2721 \times 10^6 \text{ m/sec.}$$

< T_n family)

$$T_n = \frac{nhr_n}{KZe^2} = \frac{nh}{KZe^2} \cdot \frac{n^2h^2}{4\pi^2 KZme^2} = \frac{n^3h^3}{4\pi^2K^2Z^2me^4}$$

$$T_1 = \frac{h r_1}{KZe^2} = \frac{h^3}{4\pi^2K^2Z^2me^4}$$

$$\therefore \boxed{T_n = n^3 . T_1}$$

In the special case for hydrogen atom in M.K.S unit, we have $Z = 1$, $K = \frac{1}{4\pi\varepsilon_0}$

$$(T_1)_H = \frac{hr}{\frac{1}{4\pi\varepsilon_0} \cdot 1 \cdot e^2} = \frac{4\pi\varepsilon_0 \, hr}{e^2}$$

<u>Calculation</u> $$(T_1)_H = \frac{4 \times 3.14 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34} \times 0.53 \times 10^{-10}}{(1.6 \times 10^{-19})^2}$$

$= \frac{4 \times 3.14 \times 8.85 \times 6.63 \times 0.53}{(1.6)^2} \times 10^{-12} \times 10^{-34} \times 10^{-11} \times 10^{38}$

$= 152.57 \times 10^{-18}$

$= 1.5257 \times 10^{-16}$ sec

$(T_2)_H = 2 \times (1.5257) \times 10^{-16}$ sec

$= 3.0514 \times 10^{-16}$ sec

$(T_\infty)_H = \infty \times (T_1)_H = \infty$

$(T_1)_{He^+} = \frac{(T_1)_H}{2} = \frac{1.5257 \times 10^{-16}}{2}$

$= .76285 \times 10^{-16}$ sec

$= 7.6285 \times 10^{-17}$ sec

$(T_1)_{Li^{++}} = \frac{(T_1)_H}{3} = \frac{1.5257 \times 10^{-16}}{3}$

$= .508566 \times 10^{-16}$ sec

$= 5.08566 \times 10^{-17}$ sec

$(T_2)_{Li^{++}} = 2 \cdot (T_1)_{Li^{++}} = 2 \times 5.08566 \times 10^{-17}$ sec

~~$= 16.25632$~~

$= 10.17132 \times 10^{-17}$ sec

$\langle f_n$ family $\rangle$

$f_n = \frac{\cancel{K} \cancel{Z} e^2}{\cancel{n} h^3} = \frac{1}{T_n}$

$f_1 = \frac{\cancel{K} \cancel{Z} e^2}{\cancel{h}} = \frac{1}{T_1}$

$\therefore \frac{f_n}{f_1} = \frac{T_1}{T_n} = \frac{T_1}{n^3 T_1} \Rightarrow f_n = \frac{f_1}{n^3}$

$\boxed{f_n = \frac{f_1}{n^3}}$

In the special case for hydrogen in MKS system, we have

$$Z = 1, \quad K = \frac{1}{4\pi\varepsilon_0}$$

$$(f_1)_H = \frac{1 \cdot 1 \cdot e^2}{4\pi\varepsilon_0 \cdot h \gamma}$$

$$= \frac{e^2}{4\pi\varepsilon_0 h \gamma}$$

Calculation for frequency

$$(f_1)_H = \frac{(1.6\times10^{-19})^2}{4\times3.14\times8.85\times10^{-12}\times6.63\times10^{-34}\times0.53\times10^{-10}}$$

$$= \frac{(1.6)^2}{4\times3.14\times8.85\times6.63\times0.53} \times 10^{-38}\times10^{12}\times10^{34}\times10^{10}$$

$$= 6.55\times10^{-3} \times 10^{18}$$

$$= 6.55\times10^{15}\ hz$$

$$(f_2)_H = \frac{(f_1)_H}{2} = \frac{6.55\times10^{15}}{2}$$

$$= 3.275\times10^{15}$$

$$(f_2)_\infty = \frac{(f_1)_H}{\infty} = 0$$

$$(f_1)_{He^+} = 2\times(f_1)_H = 2\times6.55\times10^{15}\ hz$$

$$= 13.10\times10^{15}\ hz$$

$$(f_1)_{Li^{++}} = 3\times(f_1)_H = 3\times6.55\times10^{15}$$

$$= 19.65\times10^{15}\ hz$$

$$(\nu_2)_{Li^{++}} = \frac{(\nu_1)_{Li^{++}}}{2} = \frac{19.65 \times 10^{15}\ Hz}{2} = 9.825 \times 10^{15}\ Hz$$

1. What is the momentum of an electron if its wavelength is 2A°?

Ans:
$$\lambda = \frac{h}{p}$$

$$\Rightarrow 2 \times 10^{-10}\ \text{meter} = \frac{6.63 \times 10^{-34}\ \text{Joule. sec}}{p}$$

$$\Rightarrow p = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.3 \times 10^{-24}\ \text{Kg m/sec}$$

2. Calculate the de-Broglie wave length for electrons and protons if their speed is 10^5 m/sec.

Ans:
$$v = 10^5\ \text{m/sec.}$$

$$h = 6.63 \times 10^{-34}\ \text{Joule. sec}$$

$$\lambda_p = \frac{h}{p}$$

$$= \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^5}$$

$$= 3.9 \times 10^{-12}\ \text{meter}$$

$$\lambda_e = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^5}$$

$$= .725 \times 10^{-8}$$

$$= 7.25 \times 10^{-9} \text{ meter}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \qquad \left(\because E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} \right)$$

3. $K.E = 100\,eV = 100 \times 1.6 \times 10^{-19}$

$$= 1.6 \times 10^{-17} \text{ Joule}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-17}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 1.6} \quad 10^{-24}}$$

$$= \frac{6.63}{5.396} \times 10^{-34} \times 10^{24}$$

$$= 1.22 \times 10^{-10} \text{ metre}$$

$$\cancel{= 1.22 A^0} = 1.228 A^0$$

4. $v = \frac{3 \times 10^8}{2}$

$$= 1.5 \times 10^8 \text{ m/sec.}$$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.5 \times 10^8}$$

$$= 0.48 \times 10^{-11}$$

$= .048 \times 10^{-10}$ meter

$= .048$ A°

8. 6 1 ton = 2000 lb / 32 8 lbs

4000 - 11 5 (32) 15 lbs = 15 kg

4. From Bragg's law we know that

$$2d \sin\theta = n\lambda$$

where d = Distance between 2 adjacent layers of a crystal lattice.

$\boxed{\text{Task } 3}$

θ = Grazing angle of incidence.
= Angle made by the incident beam with the surface of the the crystal.

n = Order of diffraction.
= 1, 2, 3, - - - - - - - -.

λ = ~~free~~ wave length of the monocromatic X-ray beam incident on the crystal.

For the first order diffraction, we have

$$2d \sin\theta_1 = 1.\lambda$$

$\Rightarrow 2 \times 2.81 A° \times \sin\theta_1 = .721 A°$

$\Rightarrow \sin\theta_1 = .721/2 \times 2.81 = .1282 \Rightarrow \theta_1 = 7.4°$

For the 2nd order deflection we have

$$2d \sin\theta_2 = 2\lambda$$

$\Rightarrow 2 \times 2.81 \times \sin\theta_2 = 2 \times .721$

$\Rightarrow \sin\theta_2 = \frac{2 \times .721}{2 \times 2.81} = .2565 \Rightarrow \theta_2 = 14.8°$

SCIENCE AND ENGINEERING NOTES

Q → Prove that the energy equivalent of 1 a.m.u is 931 meV.

Soln → 1 amu = ~~1.6 × 10^{-27} Kg~~

$= \frac{1}{12}$ part of a Carbon nucleus $({}_{6}C^{12})$

$= 1.66 \times 10^{-27}$ Kg.

From Einstein's mass energy relation we know that

$$E = mc^2$$

$$= (1.66 \times 10^{-27}) \times (3 \times 10^8)^2$$

$$= 9 \times 1.66 \times 10^{-27} \times 10^{16}$$

$$= 14.94 \times 10^{-11} \text{ Joule}$$

1.60×10^{-13} Joule = 1 meV

1 " = $\frac{1}{1.66 \times 10^{-13}}$ meV

14.94×10^{-11} Joule = $\frac{14.94 \times 10^{-11}}{1.60 \times 10^{-13}}$ meV

$= 9.33 \times 10^{2}$

$= 933$ MeV

$\simeq 931$ meV.

Charge of the electron $1.6021892 \times 10^{-19}$ Coulomb

2. How many nucleons are there in 12 gm of $_6C^{12}$? (Ans: 7.22×10^{24})

Ans: We know that 1 gm atom of any substance contain Avogadro number of atoms.

12 gm of Carbon contain $(6.023 \times 10^{23}) \times 12$ atom.

1 atom carry = 12 nucleon.

$(6.023 \times 10^{23}) 12$ " " $= 144 \times 6.023 \times 10^{23}$

3. How many neutron are there in 1 gm of (a) $_1H^1$ (b) $_{92}U^{235}$ Ans: (0, 3.7×10^{23})

Ans: (a) Hydrogen atom carry no neutron.

(b) 235 gm Uranium contains $= 6.023 \times 10^{23}$ atom.

1 gm " " $= \frac{6.023 \times 10^{23}}{235}$ atom.

1 atom contains $= \begin{array}{r} 235 \\ -92 \\ \hline 143 \end{array} = 143$ neutron.

$\frac{6.023 \times 10^{23}}{235}$ atom contains $= \frac{6.023 \times 10^{23}}{235} \times 143$

$= 3.665 \times 10^{23}$

3. A neutron has energy 20 e.V, find its velocity. Ans $= 6.18 \times 10^{4}$ m/sec.

SCIENCE AND ENGINEERING NOTES

Mass of neutron = 1.6749×10^{-27} kg

Ans:

$$E = \frac{1}{2} m v^2$$

$$\Rightarrow 20 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 1.6749 \times 10^{-27} v^2$$

$$\Rightarrow v^2 = \frac{40 \times 1.6 \times 10^{-19}}{1.6749 \times 10^{-27}}$$

$$= \sqrt{38.21 \times 10^{8}}$$

$$= 6.18 \times 10^4 \text{ m/sec}$$

(Ans)

4. What is the period of a deuteron in a magnetic field of $1\,wb/m^2$. Ans: 1.295×10^{-7} s

Hints → Centripetal force necessary for the circular motion is provided by the Lorentz force.

$$\therefore \frac{mv^2}{r} = QVB \sin 90^\circ = QVB$$

$$\Rightarrow \frac{mv}{r} = QB$$

$$\Rightarrow \frac{V}{r} = \frac{QB}{m}$$

~~⇒ T =~~

T = Time period of revolution = $\frac{2\pi r}{V}$

$$= \frac{2 \times 22/7}{\frac{V}{r}} = \frac{2 \times 22/7}{\frac{QB}{m}}$$

$$= \frac{2 \times 22/7 \times m}{Q \times B}$$

$$= \frac{2 \times 22.7 \times (2 \times 1.66 \times 10^{-27})}{1.6 \times 10^{-19} \times (1)} \left(\frac{1r}{1n}\right)$$

$= .9$

$= 1.304 \times 10^{-7}$ ...

(5) 4 hydrogen atoms forming a helium atom & 2 positrons each of mass 0.000549 amu. Calculate the energy released. Ans: 25.7 meV

Mass of 1 H atom = 1.0081 a.m.u.

mass of a He atom = 4.0039 a.m.u.

Fusion reaction is

$$4\,{}_1H^1 \xrightarrow[\text{very high temp}]{\Delta t} {}_2He^4 + 2\,{}_{+1}e^0 + \text{energy}.$$

(6.) Calculate the amount energy liberated when 1 kg of ${}_{92}U^{235}$ is fissioned as per the nuclear reaction

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + \text{energy}.$$

Express the energy released in meV, Joule, KWh, Calory.

Given → mass of Uranium = 235.0439 a.m.u

Mass of a neutron = 1.0087 amu.
Mass of Barium = 140.9139
" " Kr = 91.8973 amu.

Ans ÷ 5.137×10^{26} mev, 8.2195×10^{13} Joule, 2.283×10^{7} kwh, 1.957×10^{13} cal

5.

$4\,_{1}H^{1} \longrightarrow \,_{2}He^{4} + 2\,_{+1}e^{0}$ + energy.

~~Energy~~ Mass on the left hand side = 1.0081
4
4.0324

Mass on the right hand side = 4.003900
2×(.000549)
~~4.004999~~
= 4.003900
.001098
4.004998

Mass defect = 4.032400
− 4.004998
.027402 amu.

1 amu = 931 MeV.

.027451 " = 931 × (.027402). ev.

= ~~26.022 MeV.~~

= 25.511 meV

6.

$_{92}U^{235} + \,_{0}n^{1} \rightarrow \,_{56}Ba^{141} + \,_{36}Kr^{92} + 3\,_{0}n^{1}$ + energy

Total mass in left hand

= 235.0439
+ 1.0087
236.0526

Total energy on the right hand side

$$\begin{array}{r} 140.9139 \\ 91.8973 \\ 3.0261 \\ \hline 235.8373 \end{array} \qquad \begin{array}{r} 1.0087 \\ \times 3 \\ \hline 3.0261 \end{array}$$

$$\text{Mass defect} = \begin{array}{r} 236.0526 \\ 235.8373 \\ \hline 0.2153 \end{array} \text{ gm.}$$

235 gm Uranium contains 6.023×10^{23} atom

1 " " $= \frac{6.023 \times 10^{23}}{235}$ atom.

1000 gm " " $= \frac{6.023}{235} \times 10^{26}$ atom

$= 0.025629 \times 10^{26}$ atom

$= 2.5629 \times 10^{24}$ atom.

1 atom causes mass defect $= .2153$ gm.

2.5629×10^{24} " " " $= .2153 \times 2.5629 \times 10^{24}$ gm.

1 amu $= 1.6 \times 10^{-24}$ gm.

1 gm $= \frac{1}{1.6 \times 10^{-24}}$ a.m.u

$.2153 \times 2.5629 \times 10^{24}$ gm $= \frac{.2153 \times 2.5629 \times 10^{24}}{1.6 \times 10^{-24}}$ amu

$= .3448 \times 10^{48}$ amu

$= 3.448 \times 10^{47}$ a.m.u

$1 \text{ amu} = 931 \text{ meV}$

$3.448 \times 10^{42} \text{ amu} = 931 \times 3.448 \times 10^{47}$

$=$

$= 5.137 \times 10^{26} \text{ MeV}.$

(b) $1 \text{ meV} = 1.6 \times 10^{-13}$ Joule

$5.137 \times 10^{26} = (5.137 \times 1.6) \times 10^{26} \times 10^{-13}$

$= 8.2195 \times 10^{13}$ Joule

(c) $1 \text{ KWH} = 36 \times 10^{5}$ Joule.

~~8.2195 Joule~~ ~~8.2195~~

$1 \text{ Joule} = \frac{1}{36 \times 10^{5}} \text{ KWH}$

$8.2195 \times 10^{13} \text{ Joule} = \frac{8.2195 \times 10^{13}}{36 \times 10^{5}} \text{ KWH}$

$= .2283 \times 10^{8}$

$= 2.283 \times 10^{7} \text{ KWH}$

(d) $W = JH$

$\Rightarrow H = \frac{W}{J} = \frac{8.2195 \times 10^{13} \text{ Joule}}{4.2}$

$= 1.957 \times 10^{13} \text{ Cal}.$

0.2588 × 6.06

155528

155528 ×

1568328

3.03 × 2 = 6.06

Task

3. We know $n\lambda = 2d \sin\theta$

$\Rightarrow 1 \times \lambda = 2 \times (3.03) A^{\circ} \times \sin 15$

$\Rightarrow \lambda = \frac{2 \times 3.03 \times 0.2588}{1} = 1.56 A^{\circ}$

SCIENCE AND ENGINEERING NOTES

Qus →

~~K.E. = 100 eV~~

~~$= 100 \times 1.6 \times 10^{-19}$ Joule~~

~~$= 160 \times 10^{-19}$ Joule.~~

6. (a) $K.E = 45 eV = 45 \times 1.6 \times 10^{-19}$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-37}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 45 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 45 \times 1.6}\ \sqrt{10^{-50}}}$$

$$= \frac{6.63}{36.19} \times 10^{-9}$$

$$= 0.183 \times 10^{-9} = 1.83 \times 10^{-10}\ \text{meter}$$

$$= 1.83\ A^{\circ}.$$

m of α = 4 amu = $4 \times 1.66 \times 10^{-27} = 6.64 \times 10^{-27}$ kg

(b) $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.64 \times 10^{-27} \times 36 \times 10^{-13}}} = \frac{6.63}{\cancel{2.1.4.6}\ 6.} \times 10^{-17}$$

(c) Sand has mass = 1.6 mg.

$$= \frac{1.6}{10^3}\ \text{gm} = \frac{1.6}{10^6}\ \text{Kg}$$

$$= 1.6 \times 10^{-6}\ \text{kg}$$

1.6

$$v = \frac{25\ \text{mile}}{\text{hours}} = \frac{25 \times 1.6\ \text{Km}}{3600\ \text{sec}} = \frac{40}{3600_{90}} = \frac{1}{90}\ \text{Km/sec.}$$

$$= \frac{10^3}{90}\ \text{meter}$$

$$= \frac{25 \times 1.6 \times 10^3}{3600}\ \frac{m}{sec} = \frac{100}{90}\ m/sec$$

$$= \frac{100}{9}\ m/sec.$$

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\times 9}{1.6\times10^{-6}\times 100}$$

$$= 37.29\times10^{-30}$$

$$= 3.729\times10^{-29} \text{ meter.}$$

8. $K.E = 20\,eV = 20\times1.6\times10^{-19}$ Joule

(a) $\lambda = \frac{h}{\sqrt{2ME}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times20\times1.6\times10^{-19}}}$

$$= \frac{6.63}{\sqrt{2\times9.1\times20\times1.6}}\times10^{-34}\times10^{25}$$

$$= .27\times10^{-9}$$

$$= 2.7\times10^{-10} \text{ meter. } 2.7\,\mathring{A}$$

(b)

Neutron has mass 1.6749×10^{-27} kg

$E = 10\times1.6\times10^{-13}$ Joule

$$\lambda = \frac{h}{\sqrt{2ME}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.6749\times10^{-27}\times16\times10^{-13}}}$$

$$= \frac{6.63}{\sqrt{2\times1.6749\times16}}\times10^{-34}\times10^{20}$$

$$= .905\times10^{-14}$$

$$= 9.05\times10^{-15} \text{ meter.}$$

(c) $m = 5\times10^{-6}$ gm. $= 5\times10^{-9}$ kg

$v = \frac{2 \text{ cm}}{\text{sec}} = \frac{.02 \text{ meter}}{\text{sec}} = 2\times10^{-2}$ m/sec

SCIENCE AND ENGINEERING NOTES

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{5\times10^{-9}\times 2\times10^{-2}}$

$= .663\times10^{-23}$

$\cancel{= 6.63\times10^{-24}}$ ~~meter~~

$= 6.63\times10^{-25}$ meter

(d)

1 ton = 2000 lb

2 ton = 4000 lb

$= \frac{4000}{32}$ slug. $\left(Mg = 4000 \rightarrow m = \frac{4000}{32}\right)$

1 slug = 14.594 kg

$\frac{4000}{32}$ slug $= \frac{4000}{32} \times 14.594$ kg

$= 1824.25$ kg.

$v = \frac{30 \text{ mile}}{3600 \text{ sec}} = \frac{30\times1.6}{3600} = \frac{48}{3600} = \frac{1}{75}$ km

$= \frac{1000}{75}$ meter/sec

$= 13.33$ meter/sec.

$\therefore \lambda = \cancel{\frac{h}{\sqrt{2mr}}}$ $\Rightarrow \lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{1824.25\times13.33}$

$= 2.725\times10^{-4}\times10^{-34}$

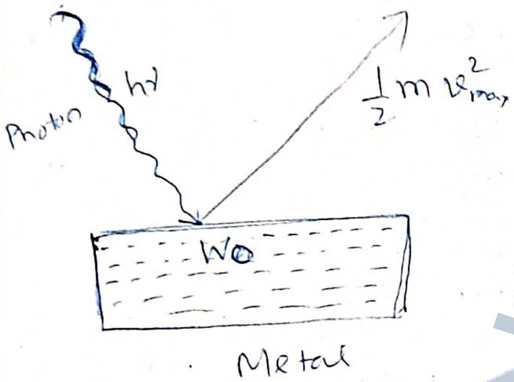
$= 2.725\times10^{-38}$ meter

SCIENCE AND ENGINEERING NOTES

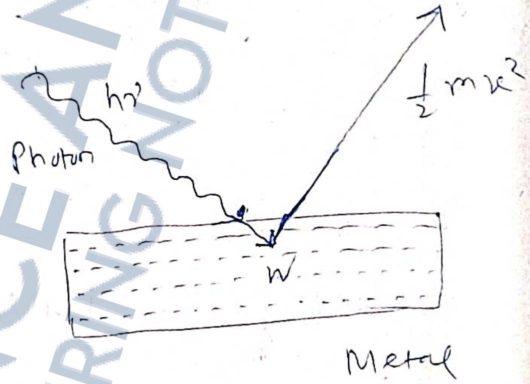
Einstein's explanation of Photoelectric emission

Huygen's wave theory could not explain Photo electric effect. Therefore, emission of photo electron was explained by Einstein with the help of Planck's Quantum theory.

According to him an incident photon may strike an electron on the surface of a metal. The electron will be excited and it will try to liberate itself from the surface of the metal.



(Fig i)



(Fig ii)

The amount of workdone by the electron to liberate itself from the surface of the metal is called work function (W_0). Naturally W_0 must be depending on the nature of the metalic surface (polish or not) and the nature of the metal. The rest energy appears as kinetic energy. Thus from the principle of conservation of energy, one can write

$$\boxed{h\nu = W_0 + \frac{1}{2} m v_{max}^2}$$

This is called Einstein's photo electric eqn and it is shown in fig (i).

If the incident photon does not bring an electron on the surface of the metal, then it enters into the metal & gives all the energy to an electron. This electron has to spend more energy to liberate itself as shown in fig (ii). Therefore, the Kinetic energy of such an electron is less than the ~~per~~ previous Kinetic energy. Hence, the speed also decreases.

$$\therefore \boxed{h\nu = W + \frac{1}{2} m v^2}$$

Since $W > W_0$, $v < v_{max}$.

If the incident frequency be gradually decreased, the velocity of the electron will decrease. For a particular frequency called threshold frequency (ν_0), the photo electron will be emitted with zero velocity.

$$\therefore \boxed{h\nu_0 = W_0}$$

Naturally ν_0 must be depending on the nature of the metals. For $\nu < \nu_0$, there will be no photo electric emission.

SCIENCE AND ENGINEERING NOTES

If the radiation be allowed to fall on the +ve metalic plate, then photo electrons are omitted as before, but they will be dragged backwards. If the voltage will be small, then the slow moving photo electrons will be held back, but the fastest moving electron manage to reach the opposite plate & give rise to a small current. When the applied voltage is gradually increased, a time comes when the fastest moving photo electron is also held back. This is known from the zero current of the milliammeter present in the circuit.

This voltage which can stop the fastest moving photo electron is called stopping potential (V_0)

Knowledge of V_0 helps us to calculate the max^m velocity of a photo electron as shown below.

$$\Delta W = \Delta E_K$$

$$\Rightarrow q \cdot \Delta V = \frac{1}{2} m v_{max}^2 - \frac{1}{2} m \cdot 0^2$$

$$\Rightarrow \cancel{(e) \cdot V}$$

$$\Rightarrow e \cdot V_0 = \frac{1}{2} m v_{max}^2$$

$$\Rightarrow v_{max}^2 = \frac{2 e V_0}{m}$$

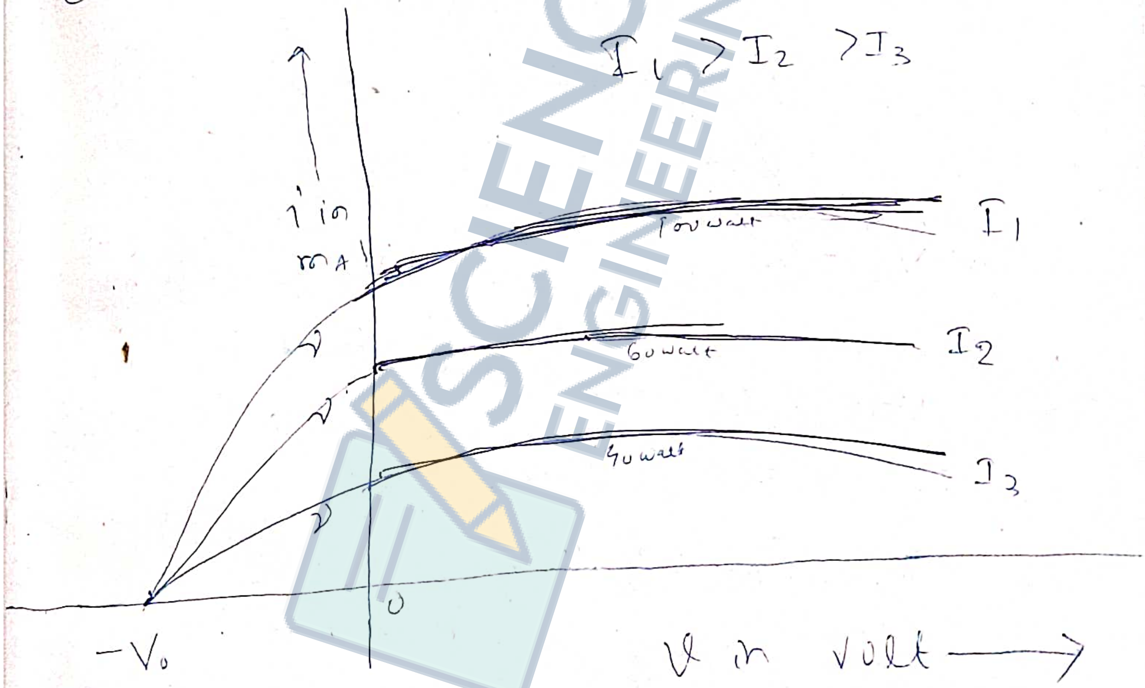
$$\Rightarrow \boxed{v_{max} = \sqrt{\frac{2 e V_0}{m}}}$$

Naturally V_0 must be dependent on the nature of the material.

More experiments on Photoelectric effect

(1) Frequency kept constant, intensity varied

A particular photo sensitive metal be subjected to light from 40W, 60W, 100W bulbs having the same colour. Gradually the applied voltage is changed and the corresponding photo current (i) be measured. The light should be allowed to fall on the −ve plate. The current ~~so~~ so obtained indicates that there is a common stopping potential as expected from



Einstein's photo electric equation.

$$h\nu = W_0 + \frac{1}{2} m v_{max}^2$$

$$\Rightarrow \text{constant} = W_0 + e\,V_0$$

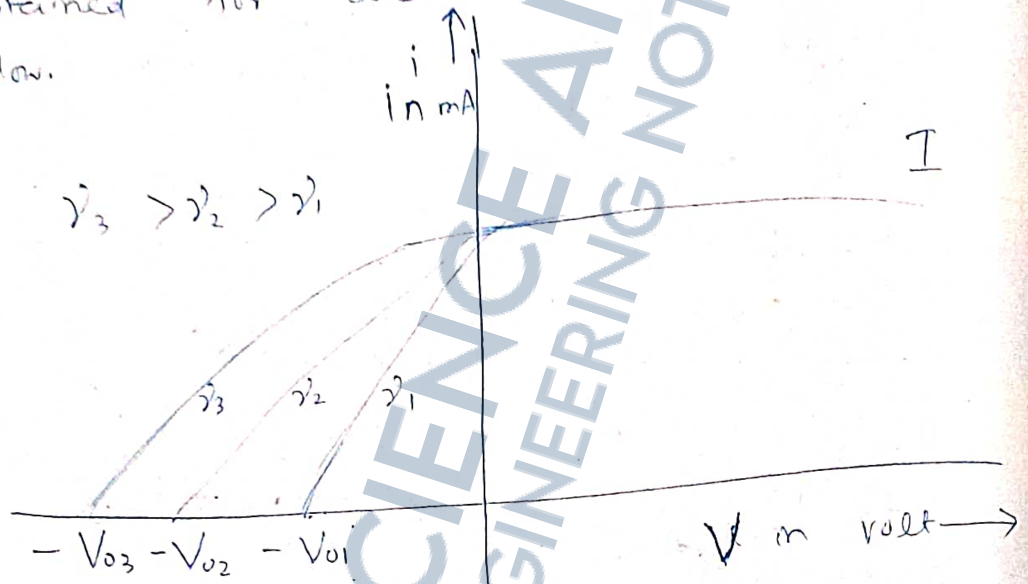
$$\Rightarrow e\,V_0 = \text{constant} - W_0$$

$$\Rightarrow V_0 = \frac{\text{constant} - W_0}{e} = \text{Another constant for a particular metal.}$$

SCIENCE AND ENGINEERING NOTES

(2) Intensity Kept Constant frequency varying

A particular photo sensitive metal is subjected to light from a 60 W bulb having provision for different colour filters. The applied voltage of the circuit be gradually changed & corresponding photo currents be measured. The curves so obtained for different frequencies are drawn below.



The stopping potentials are found to be different as expected from the eqn:

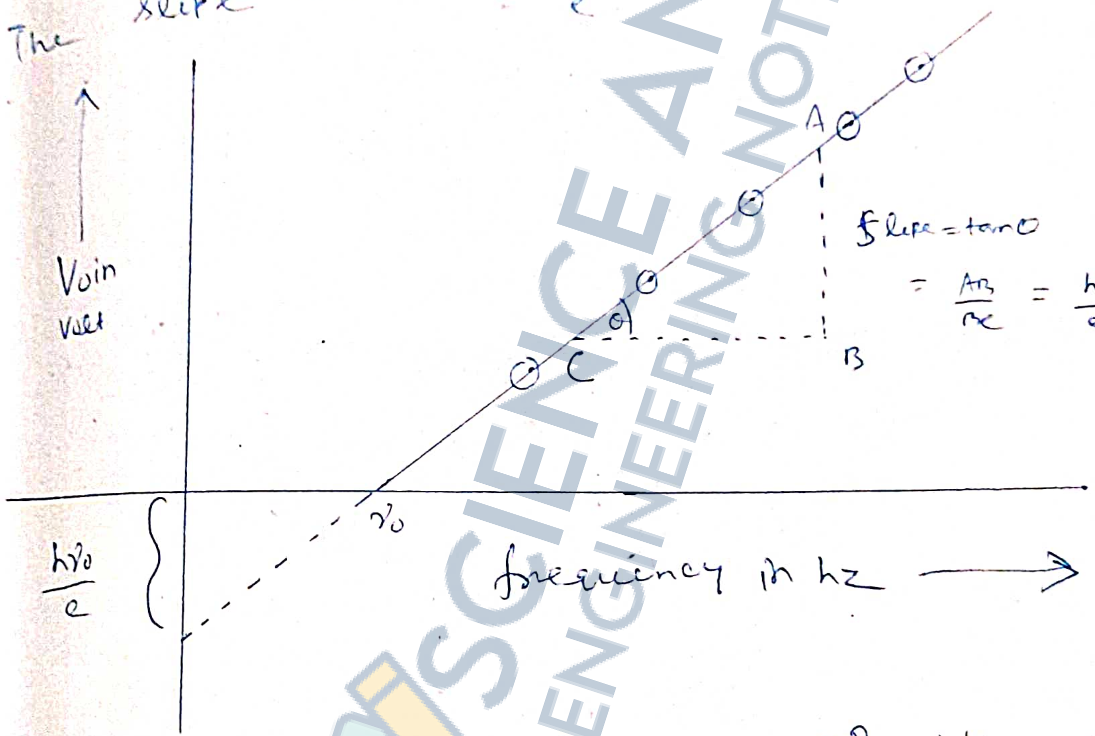
$$h\nu = W_0 + eV_0$$

But intensity controls the photo current. Hence the same for Max^m photo current remains the same for all types of coloured light.

(3) Experimental determination of Planck's Constant in the laboratory

Light of different colours with frequencies determined by a frequency meter are allowed to fall on a photo cell with a closed circuit having a milliammeter. The voltage is gradually increased till zero current is produced. This voltage gives the stopping potential.

A graph be plotted with stopping potential along the y-axis & frequency of the incident light along the x-axis. The slope provides $\frac{h}{e}$ as shown below.



From Einstein's photo electric eqn, we know that

$$h\nu = W_0 + \frac{1}{2} m v_{max}^2$$

$$= h\nu_0 + eV_0$$

$$\Rightarrow eV_0 = h\nu - h\nu_0$$

$$\Rightarrow V_0 = \frac{h}{e}\nu - \frac{h\nu_0}{e}$$

The above eqn resembles $y = mx + c$ where

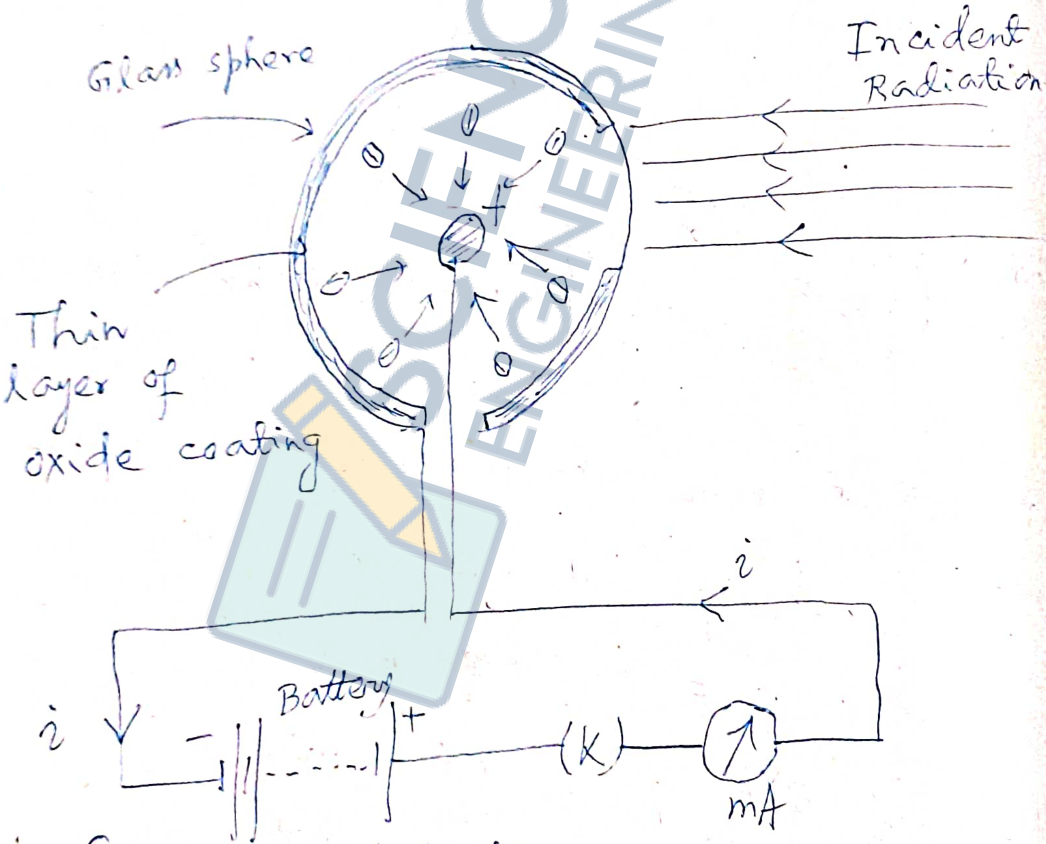
$$m = \frac{h}{e} \quad \& \quad c = \frac{h\nu_0}{e}$$

SCIENCE AND ENGINEERING NOTES

Applications of Photoelectric effect

① Photo cell

It is a device to have photoelectric effect readily. The incident light is made to fall on a hollow glass sphere or cylinder having oxide coating at the inner surface. The emitted electrons proceed towards the centre which is connected to the +ve terminal of the battery. A small current of the order of milliamperes is produced.



② Smoke detector

In high buildings having several storages there is constant fear from fire. Therefore photo cells are kept in each room, which become active there is a fire. The amplified curr reaches the Control room at the down stairs action Can be taken.

&

③ Burglar ~~alaram~~ alarm

Photo cells are kept ready near a sleeping person. When a thief with a torch light enters the room the Photo cells becomes active and sound is produced. Then the thief is readily caught.

Even ultraviolet rays Can be used, to excite the Photo cell. Then rays are invisible to our eyes & Circuits are prepared in such a way that, only when the light is blocked sound is produced. The thief entering the room in darkness. Can be caught by this method.

④ Automatic Opening & Closing of doors & windows

Photo electric Circuits are prepared in such a manner that obstruction of light will help in Opening the doors. This is made possible because of the connection of circuits with the hinges.

In the Case of windows the opposite thing happens. The falling of light on the window makes them open. When the light is off, the windows again get closed.

Dual nature of light, matter waves.

Newton was the first scientist to gave the
(particle)
Corpuscular theory of light. He said that
light consists of small packets of energy
called corpuscles. He could explain the
phenomena of reflection & refraction. But
his theory failed to explain the phenomena
like interference, diffraction & polarisation.

Huygen gave wave theory of light by
which reflection, refraction, interference, diffraction
& polarisation could be explained. But wave theory
of light failed to explain the photo electric effect
& blackbody radiation.

In 1900, Max Planck gave the quantum
theory of light. This theory resembles the
corpuscular theory and explains the photo electric
effect & blackbody radiation. But phenomenon like
interference, diffraction, polarisation could not be
explained.

From the above discussion we comes to the
conclusion that no single theory can explain all
the phenomena exhibited by light. We have to
consider light sometimes behaving as wave &
sometimes as ~~a like~~ a particles. This is called
dual nature of light.

De-Broglie argued that small

SCIENCE AND ENGINEERING NOTES

particles like electrons, protons, α-particles should possesses wave nature at times. This is from an analogy (similarity) or with the light waves which sometimes behave as particles. The wave associated with the particles is known as matter waves. De-Broglie derived an expression for the wave length associated with such waves as

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}} \qquad K.E = \frac{p^2}{2m}$$

Davisson & Germer experimentally verified the concept of matter waves. They allowed a beam of electrons to strike a nickle chloride crystal. The wave length of the electron waves are found to be equal to that calculated from Bragg's law.

Bragg's law is expressed as

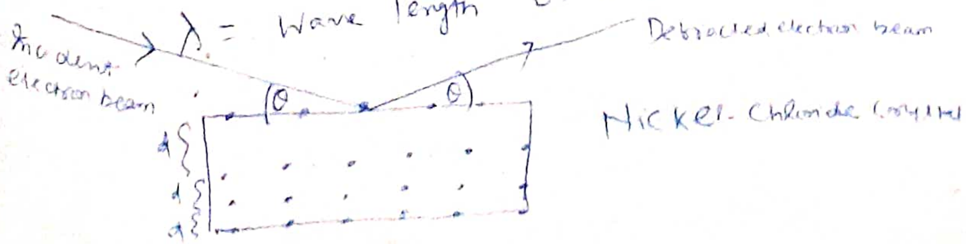
$$\boxed{2d\sin\theta = n\lambda}$$

where d = Spacing between two adjacent layers of a solid

θ = Angle between the incident beam and the surface of the crystal.

n = Order of diffraction.
= 1, 2, 3 - - - - -

λ = Wave length of the incident beam.



Discovery of electron

After the discovery of the discharge tube phenomena, electrons were discovered. A gas at low pressure is taken inside a hard glass tube & a high voltage is applied to it. There is arrangement to decrease the pressure gradually

At a pressure of the order of 5 mm of mercury, the entire tube is found to glow, the colour of the glow depending on the nature of the gas taken inside.

Ex:- CO_2 gas $\xrightarrow{\text{give}}$ green light

H_2 gas $\rightarrow$ Pink coloured light.

Hg gas $\rightarrow$ Silver white light.

Na vapour $\rightarrow$ golden yellow light.

When the pressure is further reduced to about 2 mm of mercury, then the +ve glow is reduced in size, the cathode is seemed to glow with bluish light & a dark space is found in between the two glows which is called Faraday's dark space.

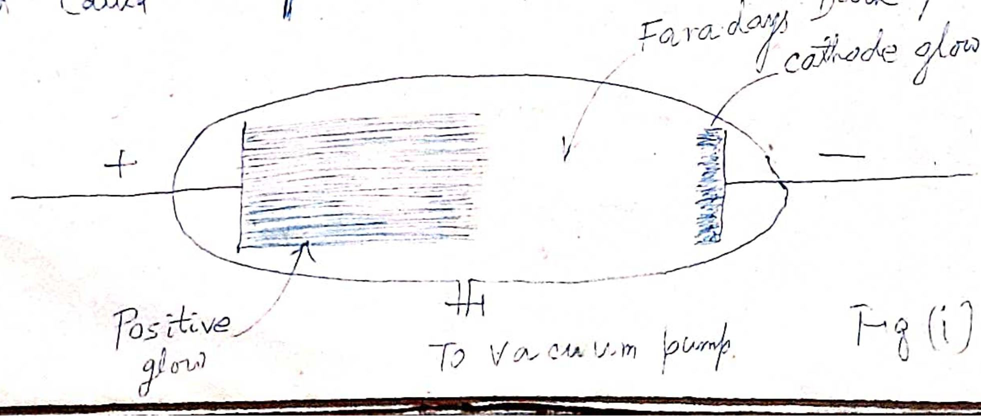
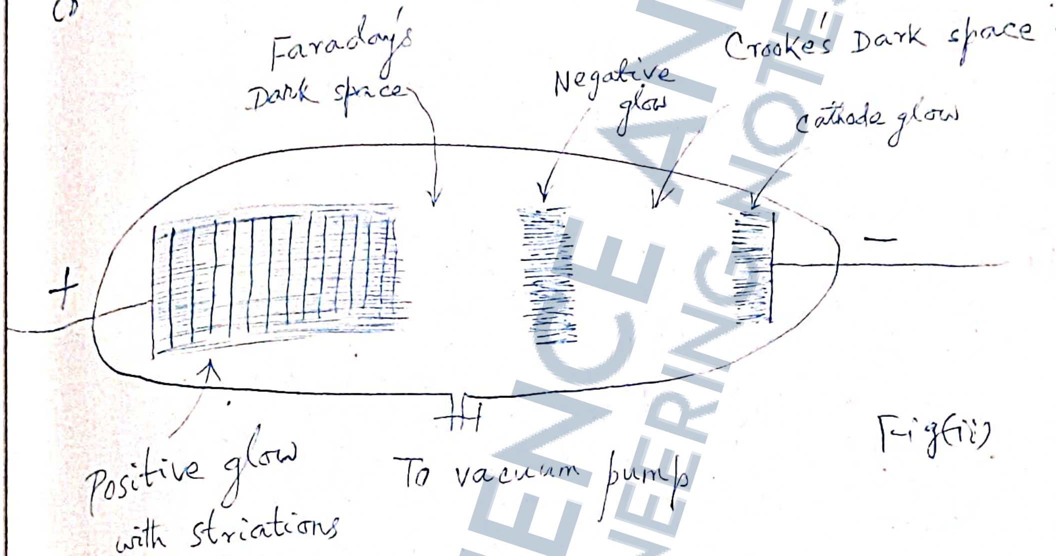


Fig (i)

SCIENCE AND ENGINEERING NOTES

If the pressure be further reduced to about 1 mm of mercury, then the +ve glow is found to reduce in size, with some brightness found in it at regular intervals called striations. A part of the cathode glow gets deatached which is called negative glow. The dark space present in between cathode & -ve glow is called Crooke's dark space.



Fig(ii)

Thus the ~~en~~entire tube is devided into 5 distinct regions as shown in fig(ii).

If the pressure be reduced further, then the +ve glow is reduced in size & ultimately vanishes. The -ve glow also moves towards the anode & disappears. At a pressure about 0.001 mm of mercury the entire tube is filled with Crooke's dark space. When this darkspace was analysed it was found to consists of large number of electrons running from the cathode up to the anode. Then high speed electrons are called cathode rays. Some of the properties of cathod rays are given below

(i) Cathode ray travel in straight line which is proved from the fact that they produce shadow of obstacles placed on their ways.

(ii) Cathode rays have momentum & Kinetic energy. This has been proved by placing Mica-Vane (ছোট পাখা) on its way which is rotated by the impact of electrons.

(iii) Cathode rays are highly penetrating by nature.

(iv) Cathode rays produce fluorescence and phosphorescence.

(v) The manner of deflection of Cathode rays ~~& magnet~~ by electric & magnetic field proves that they are -vely charged particles.

(vi) On striking a dense surface like tungsten, Molybdenum, X-rays is produced.

(vii) They affect photographics plates

(viii) They can ionise a gas through which they pass.

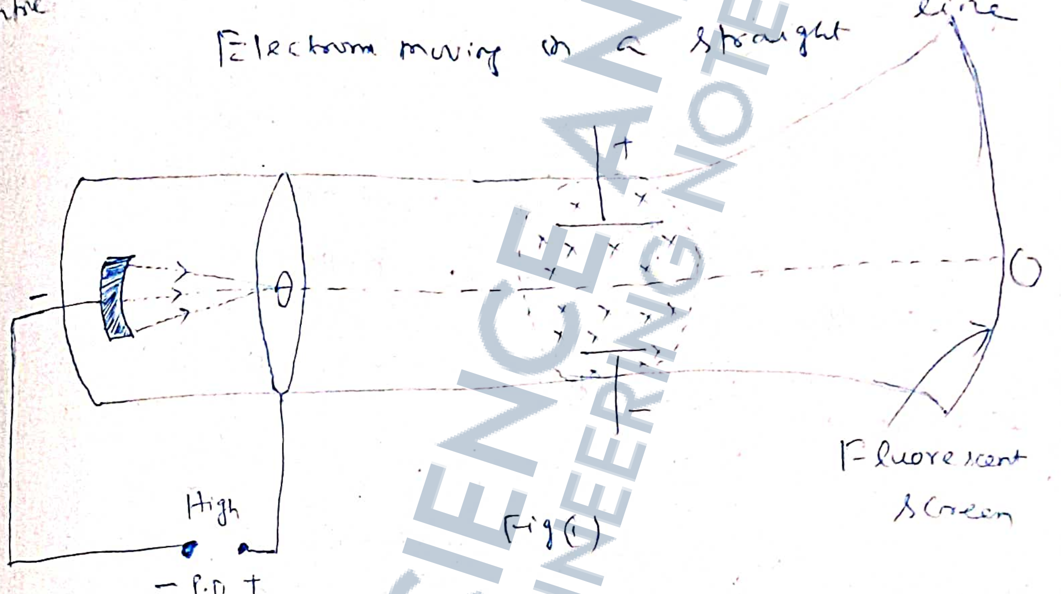
+ve rays

Goldstein in 1886 discovered a new type of rays from a hole made in the cathode. Analysis of +ve rays or Canal rays shows that they are actually +ve ~~ions~~ ions of the gas ~~taken~~ taken in the discharge tube.

SCIENCE AND ENGINEERING NOTES

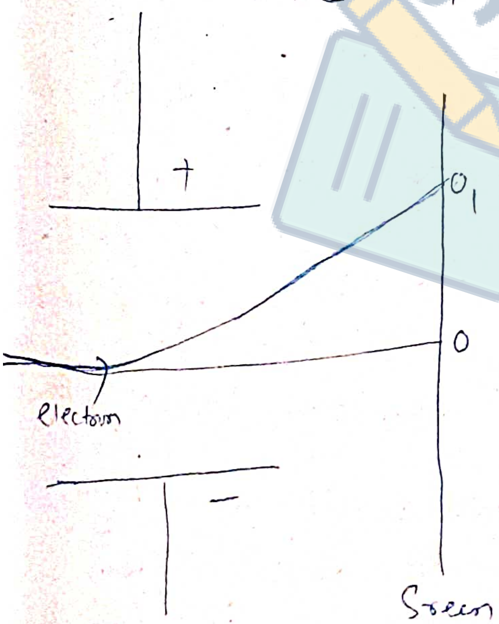
Thomson's method of measuring $\frac{e}{m}$ of electrons

After the discovery of cathode rays in 1870, J.J. Thomson could determined the ratio of charge & mass of an electron by using an apparatus shown in fig(i). There is provision to produce cathode rays & to accelerate them by making a small hole at the centre of the anode plate.



fig(i)

through a hole are allowed to pass, others are blocked. Then electrons are then subjected to a combined electric & magnetic field such that there is no deflection.



Fig(ii)

If the electric field & magnetic field are absent, then the cathode rays moves straight to form a greenish light at O. This is shown in fig(i). If

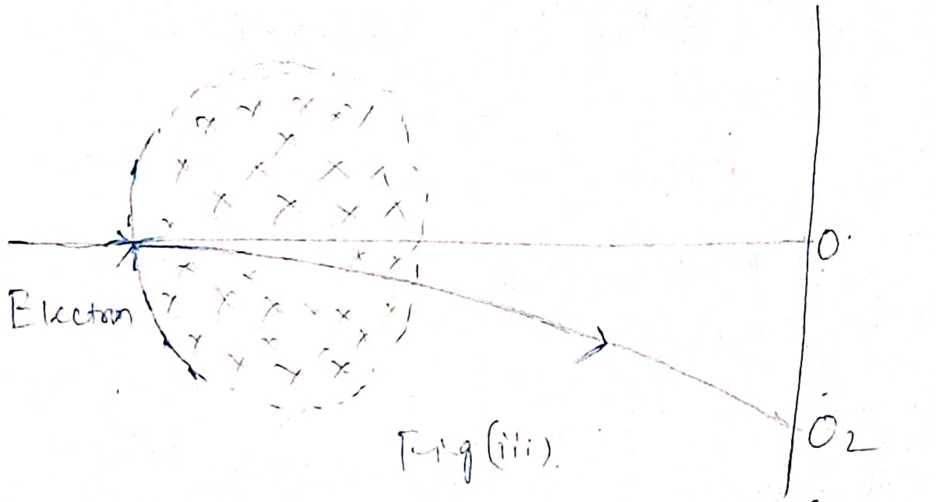


Fig (iii).

Only electric field will be applied, then electron will proceed to the point O_1 as shown in fig (ii).

If the only magnetic field be applied in to the plane of the paper, indicated by crosses, then the electrons proceed towards a point O_2 as shown in fig (iii). This bending can be explained with the help of Lorentz force, i.e. $\vec{F} = q(\vec{V} \times \vec{B})$

Here $q = -e$

θ = Angle between $\vec{V}$ & $\vec{B}$

$= 90^\circ$

$\therefore |\vec{F}| = -e\, VB \sin 90^\circ$

$= -eVB$

The force on the electron due to the electric field having intensity $\vec{E}$ is given by

$$\vec{F} = q\vec{E}$$

$$\Rightarrow |\vec{F}| = -e \cdot E$$

For no deflection of the electron beam, the force due to the electric field will be equal to the force due to the magnetic field.

$$-eE = -evB$$

$$\Rightarrow v = \frac{E}{B} \quad \text{—— (i)}$$

If the potential difference applied between the cathode & the anode be V volt, then the K.E gained by the electron must be equal to the amount of workdone on it.

$$\Delta E_k = \Delta W$$

$$\Rightarrow \frac{1}{2} m v^2 - \frac{1}{2} m \cdot 0^2 = q \cdot \Delta V$$

$$\Rightarrow \frac{1}{2} m v^2 = e \cdot V$$

$$\Rightarrow v^2 = \frac{2eV}{m}$$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}} \quad \text{—— (ii)}$$

Equating these two expressions (i) & (ii), for the velocity of the electron, we get

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

Squaring both the side, we get

$$\frac{E^2}{B^2} = \frac{2eV}{m}$$

SCIENCE AND ENGINEERING NOTES

$\Rightarrow$ $$\boxed{\frac{e}{m} = \frac{E^2}{2VB^2}}$$

The original experimental determination of Thumson was 0.77×10^{11} Columb/Kg

Later on more refined experiments have given the value of e/m as 1.76×10^{11} Columb/Kg.

Determination of the mass of the electron

From the Thumson's experiment, $\frac{e}{m}$ was found to be 1.759×10^{11} Columb/Kg.

Charge of the electron was found to be 1.602×10^{-19} Columb.

$$\therefore \quad m = \frac{e}{\frac{e}{m}} = \frac{1.602 \times 10^{-19}}{1.759 \times 10^{11}} \text{ kg}$$

$$= 9.1 \times 10^{-31} \text{ Kg.}$$

Determination of Charge of an electron by Millikan's Oil drop experiment

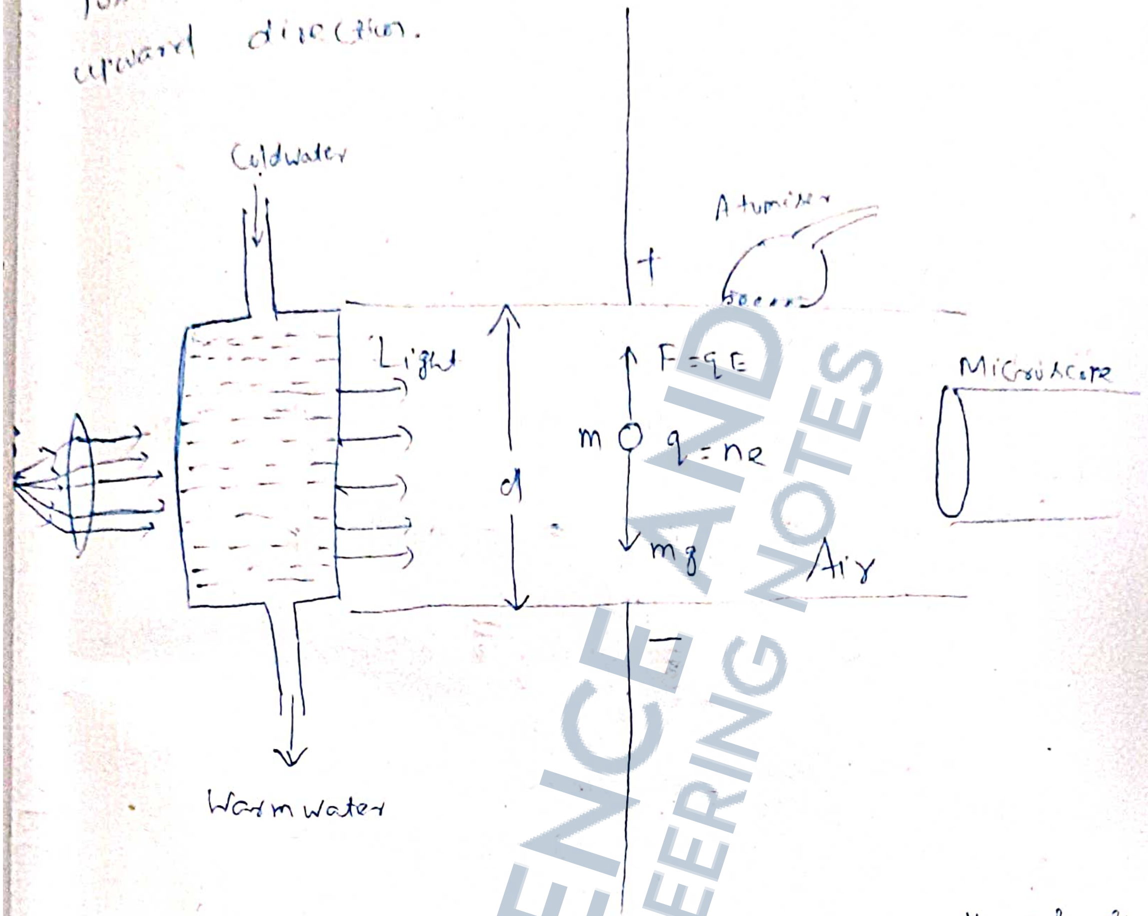
Phenomena $\xrightarrow{GM}$ sca $\rightarrow$ • or —

The experimental arrangement has been shown in the diagram. Very small oil drops are produced by means of atomiser. They are allowed to move between the space bounded by 2 horizontal plates charged oppositely. Due to friction, the oil drops acquire some amount of charge which must be multiples

SCIENCE AND ENGINEERING NOTES

of the charge of the electron.

By using proper voltage it is possible to make the oil drops move in the upward direction.



By means of a microscope the velocity of the oil drop can be measured provided the cross wires are marked properly. To see properly, light is passed into the space in between the two plates. The heat associated with the light may vaporise the oil drops. Hence it is filtered out by passing the light through the container having water.

<u>Theory</u> → Let the radius of 1 small drop be 'a', its mass be 'm' & charge carried by it be 'q'.

If the viscosity of air be η, then net force on the oil drop in the absence of an electric field $mg - 6\pi\eta a v_g = m.0$

acceleration is zero it moves with uniform terminal velocity.

$$\Rightarrow mg = 6\pi\eta a v_g \quad \text{—(i)}$$

where v_g = Terminal speed of the oil drop under the action of gravity only.

If the electric field intensity $\vec{E}$ be applied in between the two plates $\left(E = \frac{V}{d}\right)$ net force on the oil drop will be

$$qE - mg - 6\pi\eta a v_e = m.0 = 0$$

$$\Rightarrow qE = mg + 6\pi\eta a v_e \quad \text{—(ii)}$$

Where v_e = Terminal speed when electric field is present.

Adding eqn (i) & (ii), we get

$$qE = 6\pi\eta a (v_g + v_e)$$

$$\Rightarrow q = \frac{6\pi\eta a (v_g + v_e)}{E} \quad \text{—(iii)}$$

Using eqn (i) in eqn (iii), we get

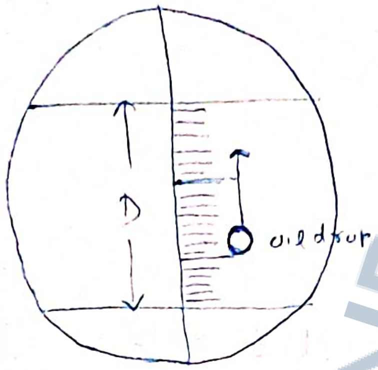
$$q = \frac{mg}{v_g} \cdot \frac{v_g + v_e}{E} \quad \text{—(iv)}$$

Let D be the separation between two horizontal cross wires of the microscope.
If t_g & t_e be time taken by the droplet to cover the distance D under

SCIENCE AND ENGINEERING NOTES

free fall & electric field E, then

$$V_g = \frac{D}{t_g} , \quad V_e = \frac{D}{t_e}$$



Eqn (iv) becomes

$$q = \frac{mg}{\frac{D}{t_g}} \left(\frac{\frac{D}{t_g} + \frac{D}{t_e}}{E} \right)$$

$$= \frac{t_g \, mg}{\cancel{D}} \cdot \cancel{D} \left(\frac{\frac{1}{t_g} + \frac{1}{t_e}}{E} \right)$$

$$= \frac{mg \, t_g}{E} \left(\frac{1}{t_g} + \frac{1}{t_e} \right) \text{ ——— (V)}$$

To find the mass of the oil drop, we have to find the radius of the oil drop because

$$m = \frac{4}{3} \pi a^3 \cdot \rho \text{ ——— (vi)}$$

Where ρ = Density of the oil.

From eqn (i), we have

$$\frac{2\cancel{4}}{\cancel{3}} \cancel{\pi} a^{\cancel{3}\,2} \rho g = \cancel{6}^{3} \cancel{\pi} \eta a v_g$$

$$3 \eta v_g = \frac{2}{3} a^2 \rho g$$

$$\Rightarrow a^2 = \frac{9 \eta v_g}{2 \rho g}$$

SCIENCE AND ENGINEERING NOTES

$$\Rightarrow a = \frac{3}{\sqrt{2}} \sqrt{\frac{\eta v_g}{\sigma g}} \qquad \text{—— (VII)}$$

Using eqn (VI) & (VII) in eqn (V), ~~we~~ one can get q which must be some multiple of e.

$\therefore q = ne$

Using $e = 1.6 \times 10^{-19}$ Coulombs, we get

$$n = \frac{q}{e} = \text{nearest whole number.}$$

Dividing q by this whole number charge of an electron can be found out.

Average of several such reading will give the value of an charge of an electron, Millikan got the result as

$$e = 1.6021892 \times 10^{-19} \text{ Coulombs}$$

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