'Jay Jagannath' Computing → (Computer arithmetic)

Decimal Numbers :-

Decimal number system has a base 10 with digits 0, 1, 2, 3 - - - - - - - 9. A decimal number is expressed as a polynomial in 10.

Eg - $(335)_{10} = 3\times10^2 + 3\times10^1 + 5\times10^0$

$(4987.625)_{10} = 4\times10^3 + 9\times10^2 + 8\times10^1 + 7\times10^0 + 6\times10^{-1} + 2\times10^{-2} + 5\times10^{-3}$

Binary numbers :-

Binary number system has a base 2 called as binary with digits 0 & 1. A binary number is digits (or bits). expressed as a polynomial in 2.

Eg : $(101)_2 = 1\times2^2 + 0\times2^1 + 1\times2^0 = (5)_{10}$

Octal numbers :-

Octal number system has a base 8 with digits 0, 1, 2, 3, 4, 5, 6, and 7. An octal number is expressed as a polynomial in 8.

Eg = $(15)_8 = 1\times8^1 + 5\times8^0 = 8+5 = (13)_{10}$

Hexadecimal numbers :-

The hexadecimal number system has a 16. with digits 0, 1, 2, 3 - - - - - - 9 and A, B, C, D, E, F for 10, 11, 12, 13, 14, 15 respectively. A hex is also called hex. It is expressed as a polynomial in 16.

E.g. $(63)_{16} = 6\times16^1 + 3\times16^0 = (99)_{10}$

Conversion of binary to decimal

Q. 1. Convert $(101101)_2$ to decimal

Ans : $(101101)_2 = 1\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$$= (45)_{10}$$

Q - 2 Convert $(0.1010)_2$ to decimal

Ans : $(.1010)_2 = 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} + 0\times2^{-4}$

$$= \frac{5}{8} = (0.625)_{10}$$

Q - 3 : Convert $(111.011)_2$ to decimal

Ans : $(111.011)_2 = 1\times2^2 + 1\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}$

$$= 7 + \frac{3}{8} = (7.375)_{10}$$

Conversion of decimal to binary

Q → 4 Convert $(294)_{10}$ to binary.

Ans: Here ~~binary~~ divide successively the given number by 2 & record the remainders which are 0 and 1. The process terminates as soon as the quotient is 0.

		Remainder
2	294	0
2	147	1
2	73	1
2	36	0
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

Then write the last remainder 1st, last but one remainder next & finally the 1st remainder last.

$\therefore (294)_{10} = (100100110)_2$

<u>Q-5</u> Convert $(58)_{10}$ to binary

<u>Ans</u>: $(111010)_2$ ✓

<u>Q-6</u> Convert $(.859375)_{10}$ to binary

Ans:- Here we multiply the fractional part by 2 & the integral part which is 0 or 1 is taken out. Then fractional part of the product is again multiplied by 2 and the integral part is taken out. The process is repeated ~~full~~ till all the digits in the fractional part becomes 0.

$$
\begin{array}{rl}
 & .859375 \\
\times & 2 \\
\hline
1 & .718750 \quad \longrightarrow 1 \\
\times & 2 \\
\hline
1 & .437500 \quad \longrightarrow 1 \\
\times & 2 \\
\hline
0 & .875000 \quad \longrightarrow 0 \\
\times & 2 \\
\hline
1 & .750000 \quad \longrightarrow 1 \\
\times & 2 \\
\hline
1 & .500000 \quad \longrightarrow 1 \\
\times & 2 \\
\hline
1 & .000000 \quad \longrightarrow 1
\end{array}
$$

The 1st integral part is written 1st after fixing a binary point. Then next integral part & so on.

$\therefore (.859375)_{10} = (.110111)_2$

Imp Q-7 . Convert $(0.7)_{10}$ to binary

$$
\begin{array}{l}
0.7 \times 2 = 1.4 \rightarrow 1 \\
0.4 \times 2 = 0.8 \rightarrow 0 \\
0.8 \times 2 = 1.6 \rightarrow 1 \\
0.6 \times 2 = 1.2 \rightarrow 1 \\
0.2 \times 2 = 0.4 \rightarrow 0 \\
0.4 \times 2 = 0.8 \rightarrow 0 \\
0.8 \times 2 = 1.6 \rightarrow 1 \\
0.6 \times 2 = 1.2 \rightarrow 1 \\
0.2 \times 2 = 0.4 \rightarrow 0
\end{array}
$$

Since the binary fraction is non-terminating the process is ~~stoped~~ stopped after a certain or step

$\therefore (0.7)_{10}$

$= \cancel{(0.10110)_2}$

$= (0.1\overline{0110})_2$

Conversion of binary to octal

While converting binary to octal we group them in 3; to the right and left of binary point by adding sufficient zeros to complete groups and replace each group by its octal equivalent.

Q-8 Convert $(1101001.1110011)_2$ to octal

Ans ÷ $(1101001.1110011)_2$

$= (\underline{001}\ \underline{101}\ \underline{001}.\underline{111}\ \underline{001}\ \underline{100})_2$

$(151.714)_8$

Conversion of binary to hexadecimal

For conversion to hex, we group in 4 & replace each group by its hex equivalent.

Q → 9 Convert $(1101001.111011)_2$

Soln $(1101001.111011)_2$

$= (0110\ 1001\ .\ 1110\ 0110)_2$

$= (69.E6)_{16}$

Conversion of hex to binary

Q - 10. Convert $(AB2)_{16}$ to binary

Ans ÷ $(AB2)_{16}$

hex → decimal → binary → adjust

Now $A = 10 = (1010)_2$

$B = 11 = (1011)_2$

$2 = 2 = (10)_2 = (0010)_2$

$\therefore (AB2)_{16} = (101010110010)_2$

Conversion of octal to binary

Q - 11 → Convert $(314)_8$ to binary

Ans ÷ $(314)_8$

Here $3 = (11)_2 = (011)_2$

$1 = (1)_2 = (001)_2$

$4 = (100)_2$

$\therefore (314)_8 = (011\ 001\ 100)_2 = (11001100)_2$

Conversion of hex to decimal

Q→13 Convert $(3A.12)_{16}$ to decimal

$$(3A.12)_{16} = 3\times16^1 + A\times16^0 + 1\times16^{-1} + 2\times16^{-2}$$
$$= 48 + 10 + \frac{1}{16} + \frac{2}{256}$$
$$= 58 + \frac{18}{256}$$
$$= (58.0703)_{10}$$

Conversion of octal to decimal

Q→12 Convert $(701.56)_8$ to decimal

$$(701.56)_8 = 7\times8^2 + 0\times8^1 + 1\times8^0 + 5\times8^{-1} + 6\times8^{-2}$$
$$= 448 + 0 + 1 + \frac{5}{8} + \frac{6}{64}$$
$$= 449 + \frac{46}{64} = (449.71875)_{10}$$

$64 \times 7 = 448$

imp Decimal to octal

Q→14 :- Convert $(57.75)_{10}$ to octal

Ans:-

$$\begin{array}{r|l} 8 & 57 \\ \hline 8 & 7 \quad \longrightarrow 1 \\ \hline & 0 \quad \longrightarrow 7 \end{array}$$

Remainder ↑

$\therefore (57)_{10} = (71)_8$

$$\begin{array}{r} 0.75 \\ \times\ 8 \\ \hline 6.00 \end{array} \longrightarrow 6$$

↓

$\therefore (0.75)_{10} = (.6)_8$

$\therefore (57.75)_{10} = (71.6)_8$

Decimal to hex

Q ÷ 15 Convert $(57.75)_{10}$ to hex

Ans:-

$$\begin{array}{r|l} 16 & 57 \\ \hline 16 & 3 \quad \longrightarrow 9 \\ \hline & 0 \quad \longrightarrow 3 \end{array}$$

Remainder ↑

$\therefore (57)_{10} = (39)_{16}$

$$\begin{array}{r} .75 \\ 16 \\ \hline (12).00 \end{array} \longrightarrow 12 = C \downarrow$$

$$\therefore (.75)_{10} = (.C)_{16}$$

$$\therefore (57.75)_{10} = (39.C)_{16}$$

① Notes ÷ To convert hex to octal we first convert hex to binary and then binary to octal.

② To convert octal to hex we first convert octal to binary and binary to hex.

Addⁿ of binary numbers

Q-16 Find 1+11+111+1111

Ans ÷

$$\begin{array}{r} 1 \\ 11 \\ 111 \\ 1111 \\ \hline 11010 \end{array}$$

Subtraction of binary numbers

Q→17 Find 11110 - 10111

Ans ÷

$$\begin{array}{r} 11110 \\ 10111 \\ \hline 00111 \end{array}$$

$(111)_2$ (Ans).

$$\begin{array}{r} 1101 \\ 1011 \\ \hline 0010 \end{array}$$

$= (10)_2$

Multiplication of binary numbers

Q-18 Find 1010 × 11

Ans ÷

$$\begin{array}{r} 1010 \\ 11 \\ \hline 1010 \\ 1010 \\ \hline 11110 \end{array}$$

Division of binary numbers:

Q→19 Find 10010 ÷ 101

```
         ________
101 )  10010   | 11.1001
       101
       ------
        1000
         101
       ------
          10
         101
       ------
       00.1000
           101
       ------
          0110
           101
       ------
```

∴ 10010 ÷ 101 = 11.1001

Complement

1's complement of 1 = 0
" " " 0 = 1

In a binary number if we replace each 1 by 0 & each 0 by 1 which is we get another binary no. called one's complement of the 1st no.

The 2's complement is obtained adding 1 to its 1's complement

E.g ÷ Suppose the no = $(10101)_2$
Its 1's Complement = $(01010)_2$
Its 2's Complement = 01010 + 1
= 01011

Substraction using 1's Complement

Rule ÷
1) Firstly make each no. of same no. of digits.
(2) Find the 1's Complement of the number

to be subtracted.

3) Add this to the other no.

4) If there is final carry of 1, add it to obtain the result.

If there is no final carry, recomplement the sum & attach a -ve sign to get the result.

E.g ÷ Subtract $(100011)_2$ from $(010010)_2$ using 1's complement

$$\begin{array}{r} 010010 \\ 011100 \\ \hline 101110 \end{array}$$

(No carry)

Its 1's complement = 010001 & attach -ve sign.

∴ Result = $-(010001)_2$

E.g ÷ Subtract $(11101)_2$ from $(11111)_2$

Ans ÷ 1's compliment of $(11101)_2$ is $(00010)_2$

$$\begin{array}{r} 11111 \\ +00010 \\ \hline (1)\,00001 \\ 1 \\ \hline (00010)_2 \end{array}$$

∴ Result = $(00010)_2 = (10)_2$

Subtract using 2's complement

Rule

(1) Firstly make each number of same no

of digits.

(2) Find 2's complement of the no to be subtracted.

(3) Add this to the other no.

(4) If there is final carry then drop it & it is the result.

If there is no final carry and then find its 2's complement and attach -ve sign.

E.g - Subtract $(1111)_2$ from $(11101)_2$ (using 2's complement)

Ans: 1's complement of $(11111)_2 = (00000)_2$

2's complement $= (00001)_2$

$$
\begin{array}{r}
11101 \\
+\ 00001 \\
\hline
11110
\end{array}
$$

(No carry)

Its 1's complement = 00001

Its 2's complement = 00001 + 1
= 00010

∴ Result = $-(00010)_2$

E.g: Subtract $(11101)_2$ from $(11111)_2$

Ans: 1's complement of $(11101)_2$ is $(00010)_2$

2's complement = 0011

$$
\begin{array}{r}
11111 \\
+\ 00011 \\
\hline
(1)00010
\end{array}
$$

dropped

Ans: $(00010)_2$

Ex- 17(C)

4. (i) $(6AE)_{16} + (1FA)_{16} = (8A8)_{16}$

Ans ÷

$$\begin{array}{r} 6AE \\ 1FA \\ \hline 8A8 \end{array}$$

(iv) $(7253)_8 + (6017)_8 = (15272)_8$

$$\begin{array}{r} 7253 \\ 6017 \\ \hline 15272 \end{array}$$

5. (i) $(6F)_{16} - (2F)_{16} = (40)_{16}$

$$\begin{array}{r} 6F \\ -2F \\ \hline 40 \end{array}$$

6. (i)

$$\begin{array}{r} A18 \\ 4C \\ \hline 7920 \\ 2860 \\ \hline 2FF20 \end{array}$$

(xiv) $(216.476)_8 \div (67.06)_8$

$(21647.6)_8 \div (6706)_8$

=

$$\begin{array}{r|l|l} 6706 & 21647.6 & 2.45 \\ & 15614 & \\ \hline & 40336 & \\ & 33430 & \\ \hline & 47060 & \end{array}$$