'Jay Jagannath'

Ray Optics

Reflection of light

The phenomenon of a ray of light striking a plane surface and returning back into the same medium obeying certain laws is called reflection of light.

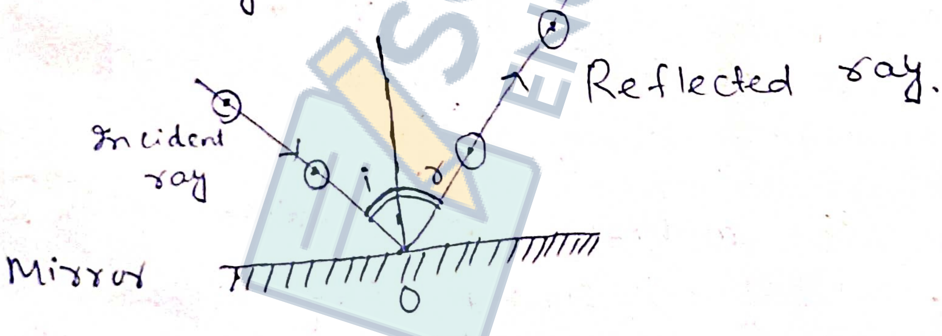
Laws of reflection

(1.) First law :-

The incident ray, the reflected ray and the normal at the point of incidence lie in one plane.

(2) Second law :-

The angle of incidence (i) = Angle of reflection (r)

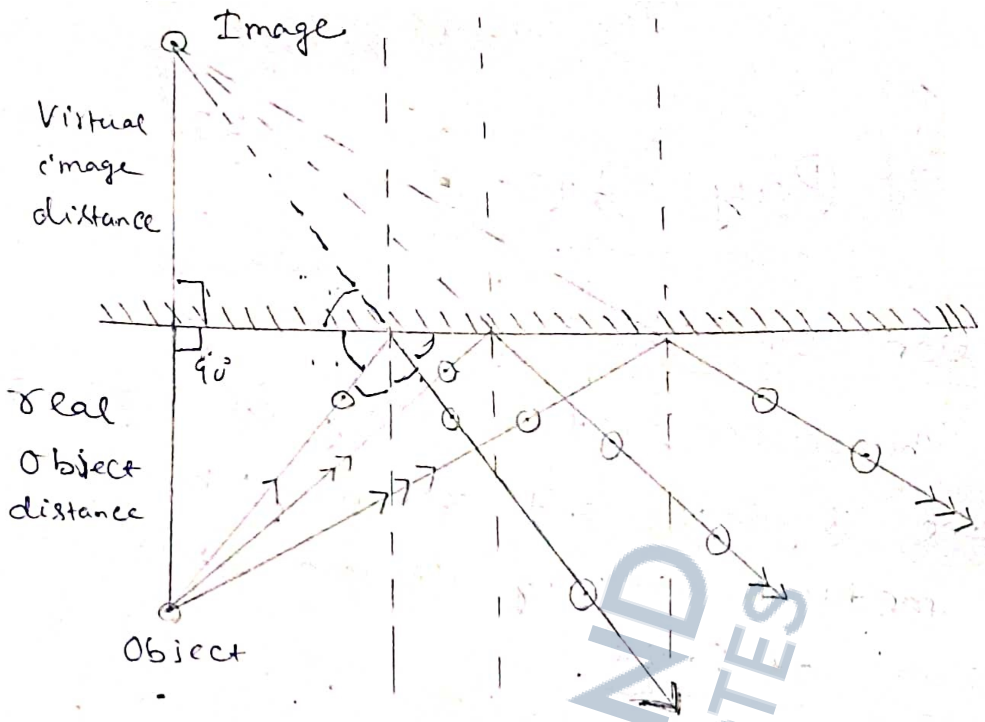


Applications

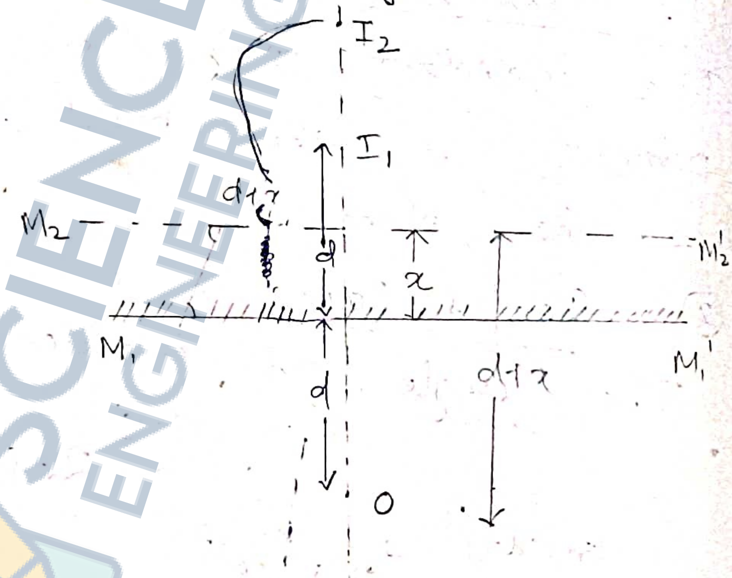
(1) Size of object = Size of image

and object distance = image distance.

Lateral inversion takes place. i.e left hand will appear as the right hand of image.



(2) If a mirror is displaced by x, the image will be displaced by $2x$.



$I_1 I_2$

$= OI_2 - OI_1$

$= 2(d+x) - 2d$

$= 2d + 2x - 2d$

$= 2x$

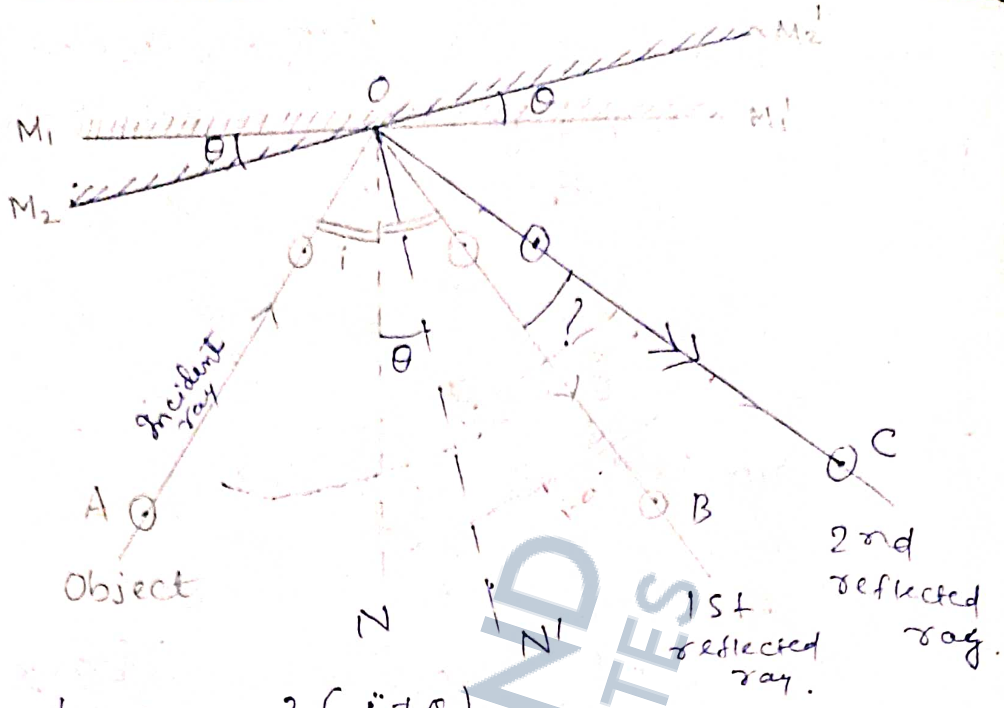
(3) If a mirror be rotated by an angle θ, the reflected ray rotates by an angle 2θ.

Let $\angle AON = i = \angle NOB$

$\therefore \angle AOB = 2i$

When the mirror is rotated by an angle θ, the normal also rotates by θ.

Angle of incidence $= \angle AON' = i + \theta = \angle N'OC$



$\therefore \angle AOC = 2(i+\theta)$

$\therefore \angle BOC = \angle AOC - \angle AOB$

$= 2(i+\theta) - 2i$

$= 2i + 2\theta - 2i$

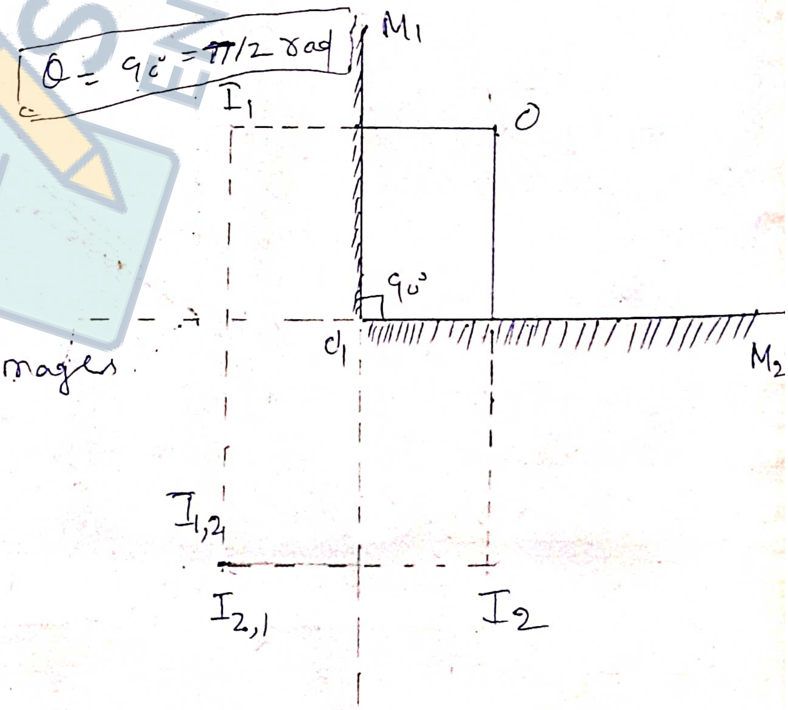
$= 2\theta$ (Proved)

(4) Number of images formed by two plane mirrors inclined at an angle θ

$= \left(\frac{2\pi}{\theta} - 1\right)$

Example-1

$\theta = 90^\circ = \pi/2$ rad



$I_{1,2}$ and $I_{2,1}$ coincide.

Hence No of images

$= \frac{2\pi}{\pi/2} - 1$

$= 4 - 1$

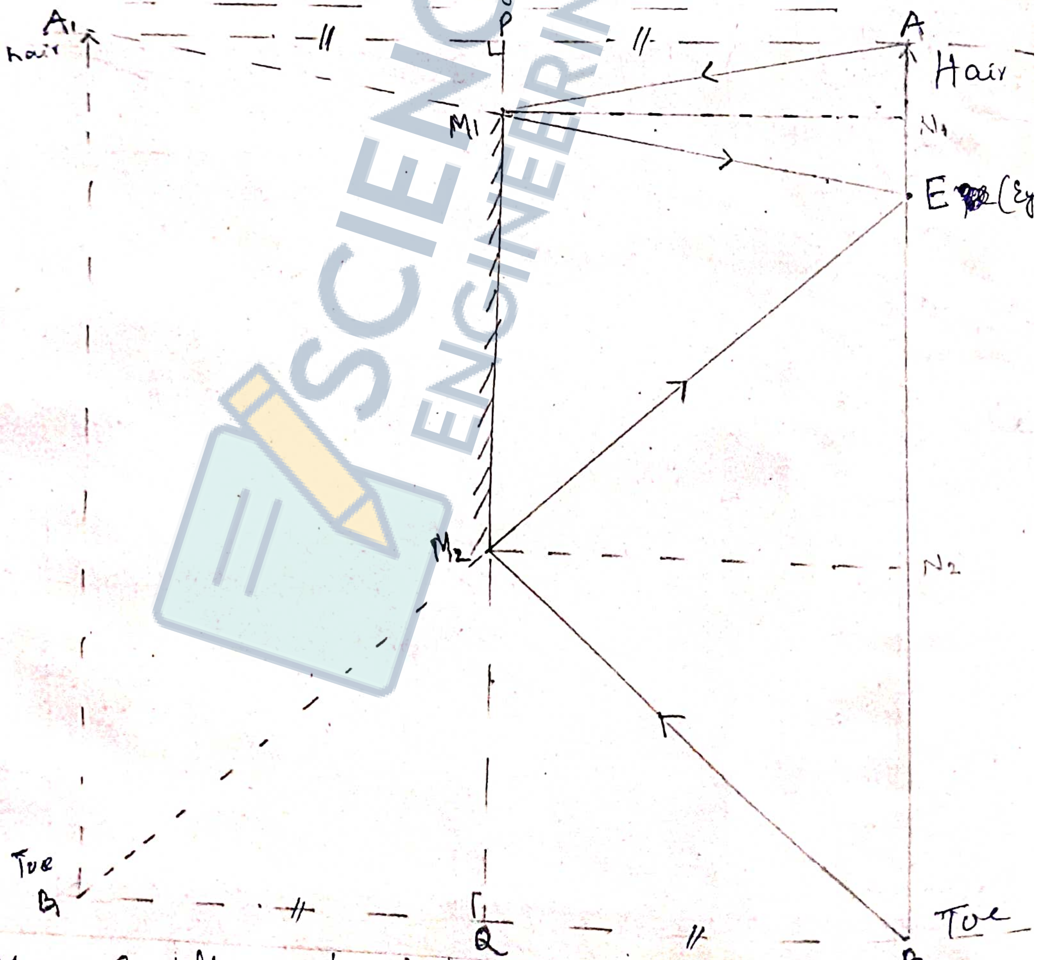
$= 3$

(Verified)

Ex-2 $\theta = 0^\circ$ i.e The object is placed in between two parallel plane mirror.

No of images $= \infty$ (Large)

(5) Min^m size of the mirror necessary to view the entire body of human being is half of the height of the human being.



M_1M_2 length $= \frac{1}{2} \times A'C' = \frac{1}{2} AC$.

because M_1 and M_2 are the midpoints of the two sides of the $\Delta A'B'C'$.

Proof → Let AB be the position of man in front of mirror M_1M_2 Rays of light starting from A and B enter the eye at E, after reflection from M_1 and M_2 respectively, thereby producing virtual images at A_1 and B_1. Thus A_1B_1 is the virtual image of the man.

$$\therefore AP = A_1P$$

In triangle A_1PM_1 and A_1AE

$$\left\{\begin{array}{l} \angle PA_1M_1 = \angle AA_1E \quad \text{(common angle)} \\ \angle A_1PM_1 = \angle A_1AE = 90^\circ \end{array}\right.$$

$\therefore$ The triangles are similar.

$$\therefore \frac{A_1P}{AA_1} = \frac{A_1M_1}{A_1E}$$

$$\Rightarrow \frac{1}{2} = \frac{A_1M_1}{A_1E}$$

$$\Rightarrow A_1E = 2A_1M_1$$

$$\therefore A_1M_1 = EM_1$$

$\therefore M_1$ is mid point of A_1E

Similarly M_2 is mid point of B_1E.

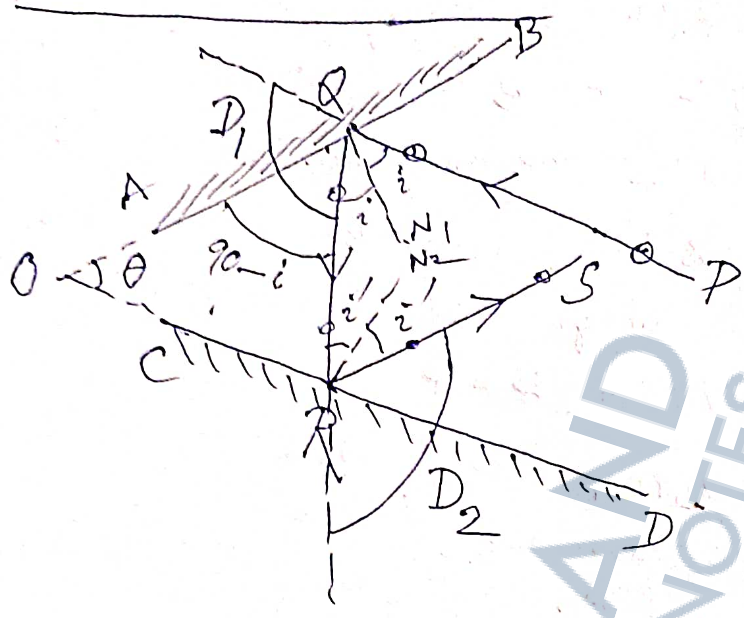
$$\therefore M_1M_2 = \frac{1}{2}A_1B_1$$

($\because$ The line joining the mid point of two sides of triangle is half of the base side)

$$\therefore M_1M_2 = \frac{1}{2}AB$$

Thus, the min^m size of mirror should be half the size of man.

⑥ Angle between two straight lines by Optical Method



D_1 = Angle of deviation of the ray PQ due to the mirror AB.

$= \pi - 2i$

D_2 = Angle of deviation of the ray QR due to the mirror CD.

$= \pi - 2i'$

$D_1 + D_2 = 2\pi - 2i - 2i' = 2[\pi - (i+i')]$

$\Rightarrow \frac{D_1 + D_2}{2} = \pi - (i+i') \quad \text{———} \quad (1)$

In the Δ QOR, the sum of 3 angles is 180°.

$\therefore \theta + \cancel{90^\circ} - i + \cancel{90^\circ} - i' = \cancel{180^\circ}$

$\Rightarrow \theta = i + i' \quad \text{———} \quad (2)$

Using eqn (2) in eqn (1), we get

$\frac{D_1 + D_2}{2} = \pi - \theta \quad \Rightarrow \quad \boxed{\theta = \pi - \left(\frac{D_1 + D_2}{2}\right)}$

1.09.2K1

Spherical mirror

These are of two types. Namely
(i) Concave mirror.
(ii) Convex mirror.

They are prepared out of hollow, transparent spheres. Usually a circular ~~apparature~~ aperture is chousen. If the bulging part be coated with HgO, then reflection of light will take place at the concave side and we call it a concave mirror.

If the concave side be coated with HgO, then reflection of light will take place on the bulging side and we call it a convex mirror.

Some defn associated with spherical mirrors

① Centre of Curvature (C)

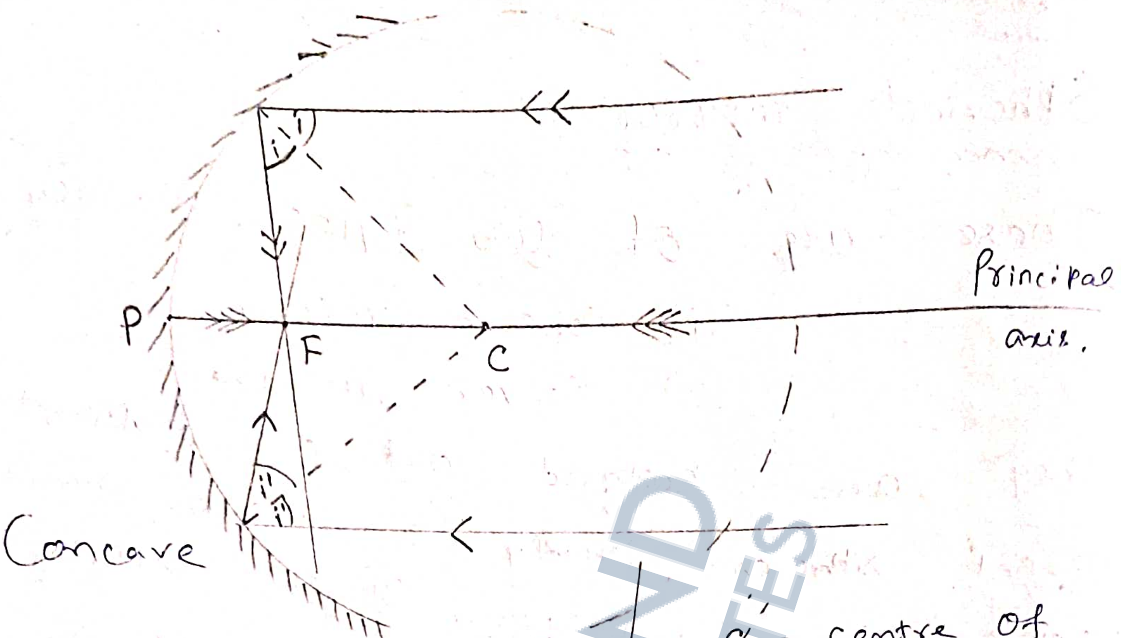
It is defined as the centre of the spherical surface of which the mirror forms a part.

② Radius of Curvature (r = PC)

It is defined as the radius of the spherical surface of which the mirror forms a part.

③ Pole (P)

It is defined as the centre of the circular



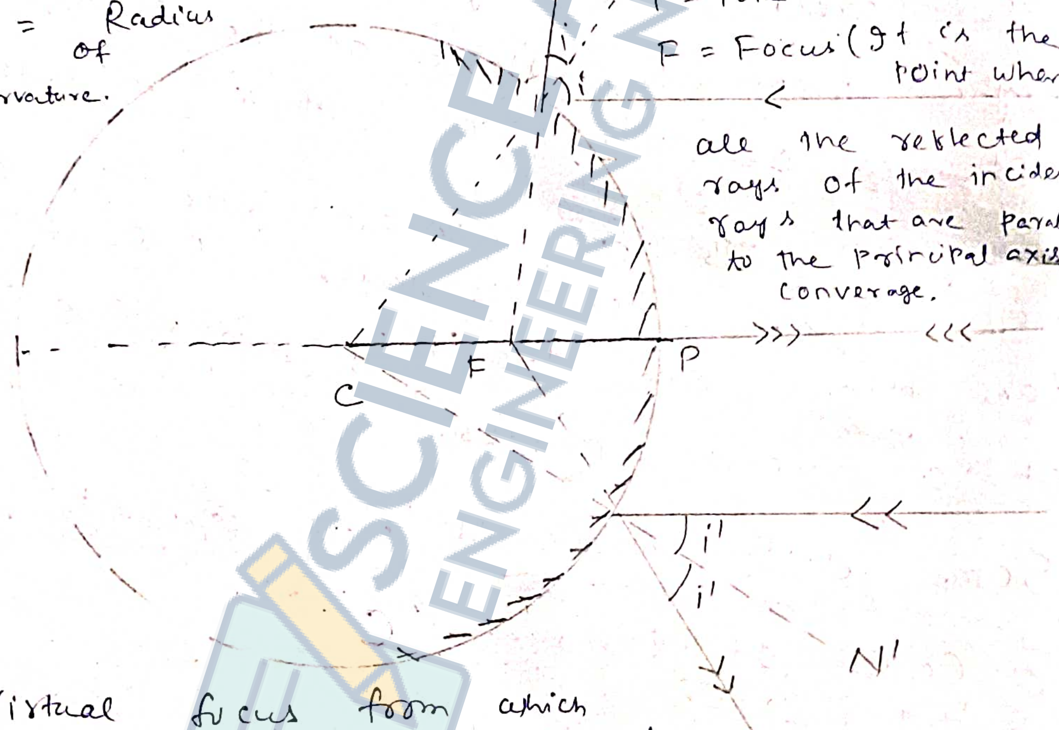
PF = Focal length

PC = Radius of Curvature.

C = centre of Curvature

P = Pole of the mirror.

F = Focus (It is the point where all the reflected rays of the incident rays that are parallel to the principal axis) converge.



F = Virtual focus from which the diverging rays appear to come from.

(*) Aperture that takes out the mirror from the hollow transparent sphere.

(4) Principal axis (PC extended)

It is the line joining the pole and the centre of curvature.

(5) Focus (F): When rays parallel to the

Principal axis are incident on a concave mirror, then, after reflection, the rays actually meet at a point on the principal axis which we call as the real focus.

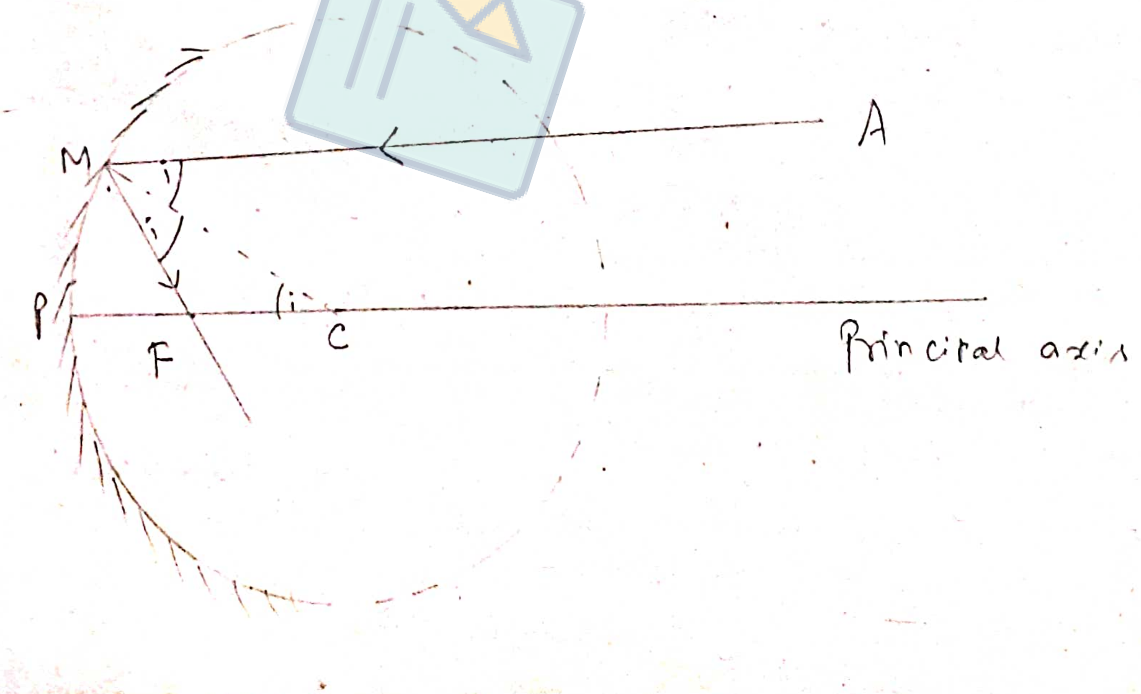
When such rays parallel to the principal axis are incident on a convex mirror, after reflection the rays diverge ([illegible]). When extended backwards they appear to come from a point on the principal axis called virtual focus.

Focal length ($f = PF$)

It is defined as the distance between the pole and the focus.

To prove that $r \simeq 2f$

Case-1 (Concave mirror)



Lets consider a ray AM parallel to the principal axis. MC is the normal at the point M.

From the laws of reflection, we can write

$$m\angle AMC = m\angle CMF = i = m\angle MCF$$

Thus MCF Δ is an isosceles triangle.

Thus $MF = FC$. Now $r = PC = PF + FC$

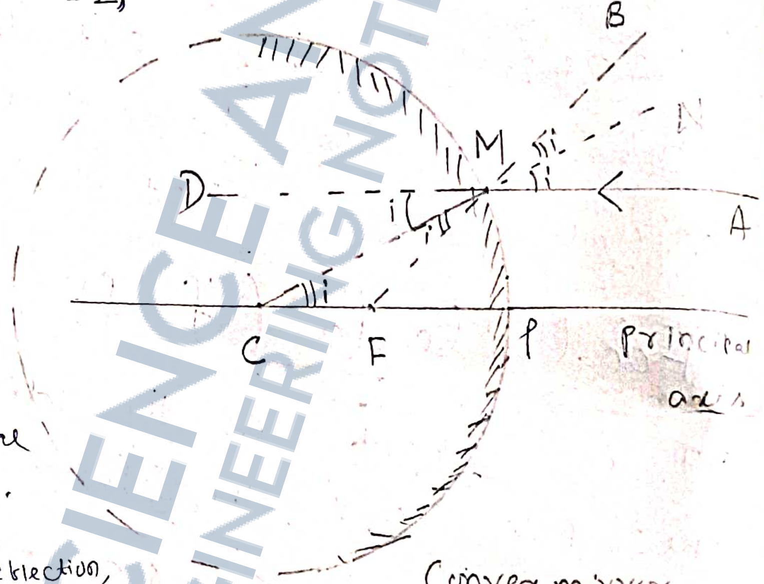
$= PF + MF$

If rays very near to the principal axis be considered then M will approach P and $MF \simeq PF$.

$\therefore r \simeq 2PF \simeq 2f$

Case-II

(Convex mirror)



Lets consider a ray AM parallel to the principal axis.

MC is the normal at the point M.

From laws of reflection, we can write

$$\angle AMN = \angle BMN = i$$

But $\angle AMN = \angle MCF = i$ (Corresponding angle)

$\angle BMN = \angle CMF = i$ (opposite angle)

$\therefore \angle MCF = \angle CMF = i$. The triangle CMF is isosceles

$\therefore CF = MF$

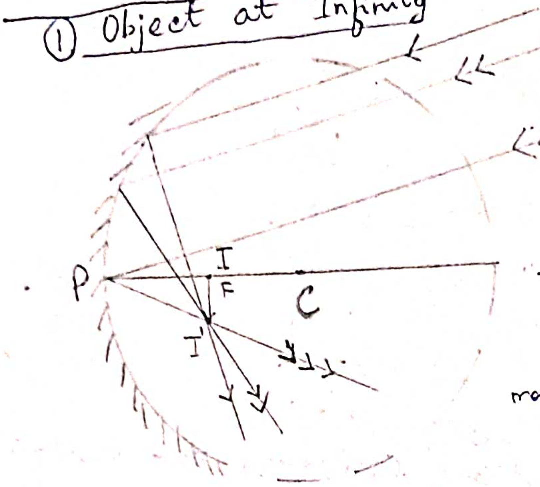
Now $PC = CF + PF = MF + PF$

If the rays are very near to the principal axis be considered then M will approach P and $MF \simeq PF$.

$\therefore r \simeq 2PF \simeq 2f$

Formation of image in a Concave mirror for various position of the Object

① Object at Infinity



$u = \infty$

focal length = $-\infty$

Formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{\infty} = -\frac{1}{f}$

$\Rightarrow v = -f$

Thus image is obtained at the focus of focal plane. Linear magnification, $m = \frac{v}{u} = \frac{-f}{\infty} = 0$.

Image is real, inverted, very much diminished and formed on the focus.

② Object between C and Infinity

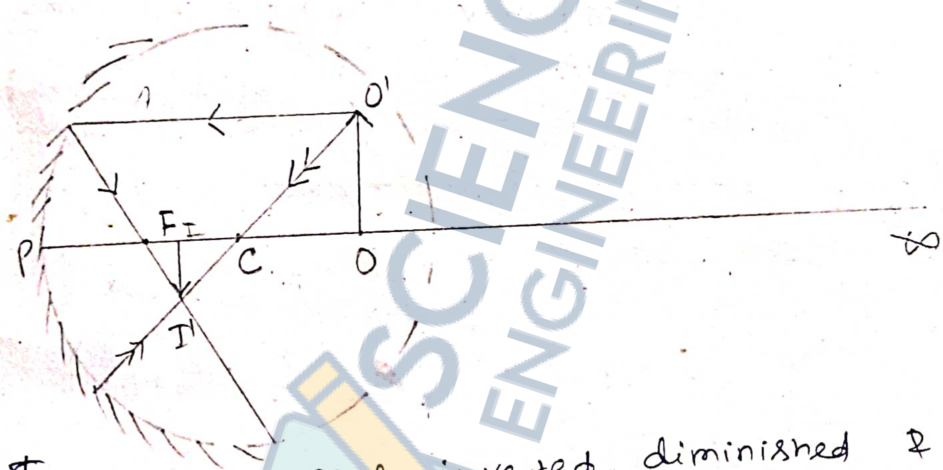


Image is real, inverted, diminished & formed between F and C.

③ Object at C

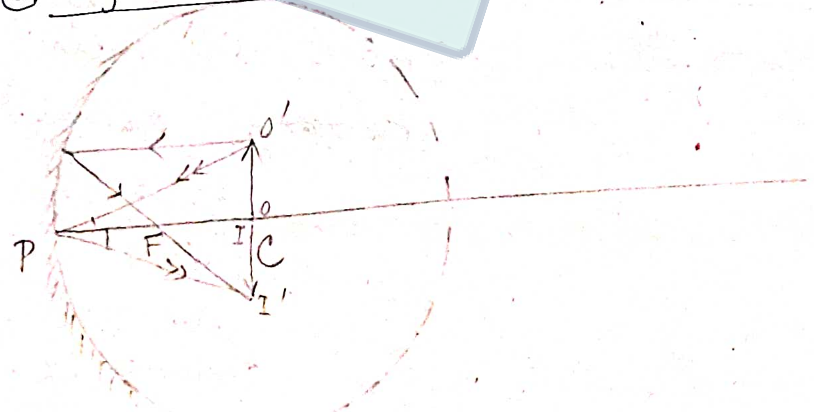


Image is real, inverted, of the same size as the object & formed at C.

(4) Object between F and C

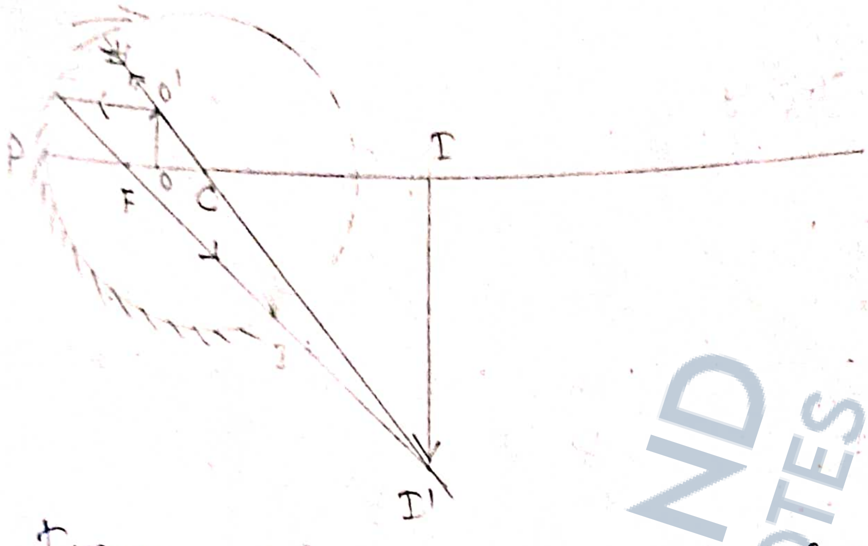
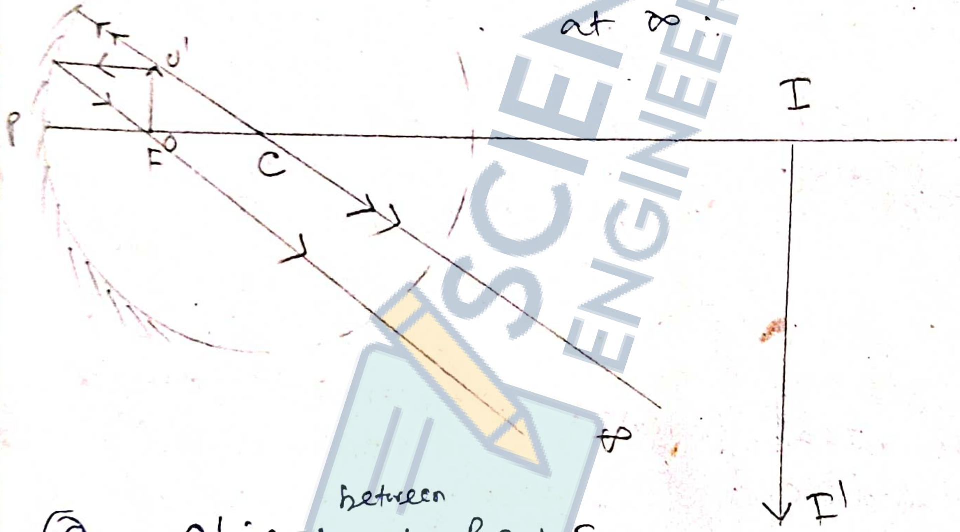


Image is real, inverted, enlarged and formed between C and ∞.

(5) Object at F

Image is real, inverted, very much enlarged and formed at ∞.



(6) Object ~~at~~ between P and F

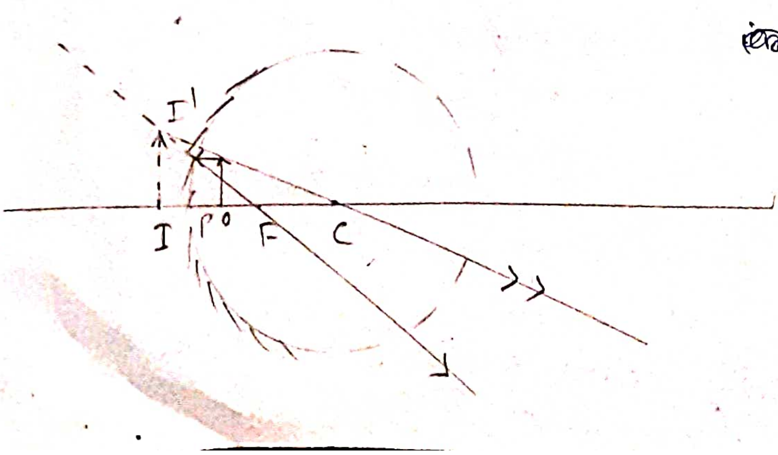


Image is virtual, ~~imaginary~~ errect, enlarged, formed on the other side of the mirror

Image formation by a convex mirror

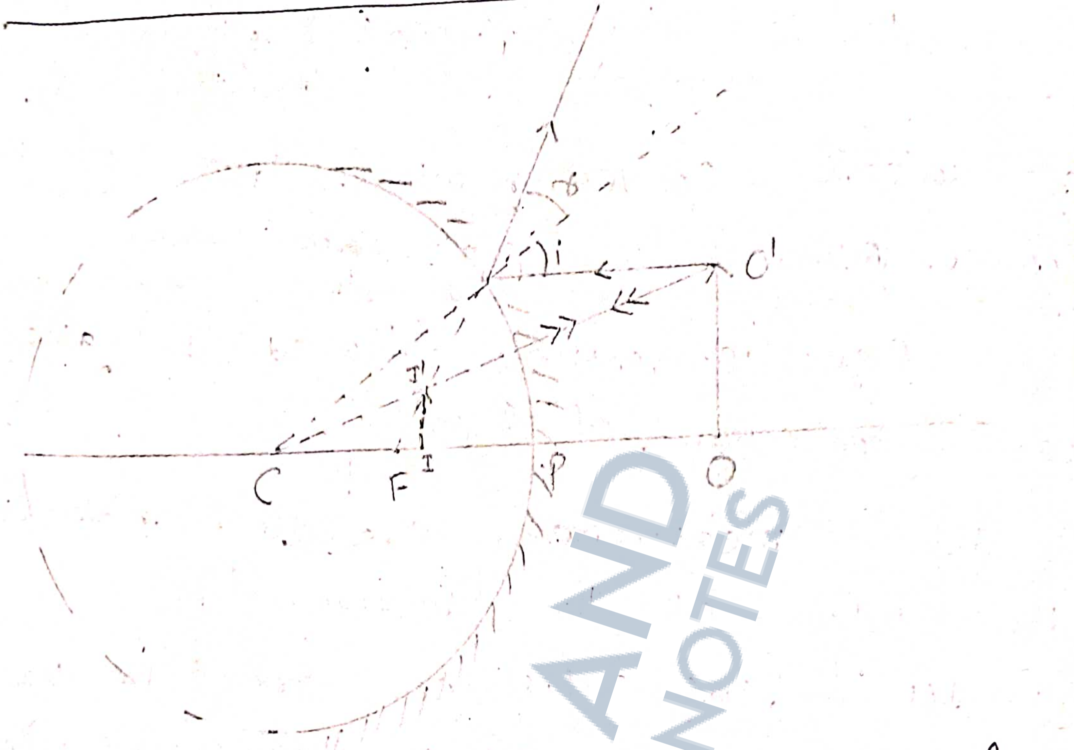
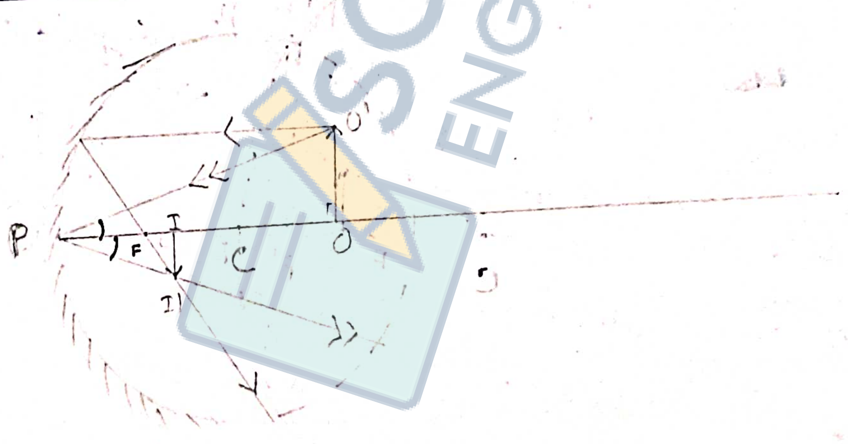


Image is virtual, erect, diminished, ~~formed~~ formed on the otherside of the mirror between F and P

Magnification (m)



Magnification is defined as the ratio of the size of image to the size of object.

OO'P and II'P are two similar right angled triangles.

$$\frac{II'}{OO'} = \frac{PI}{PO} = \frac{v}{u} = m = \text{Magnification}$$

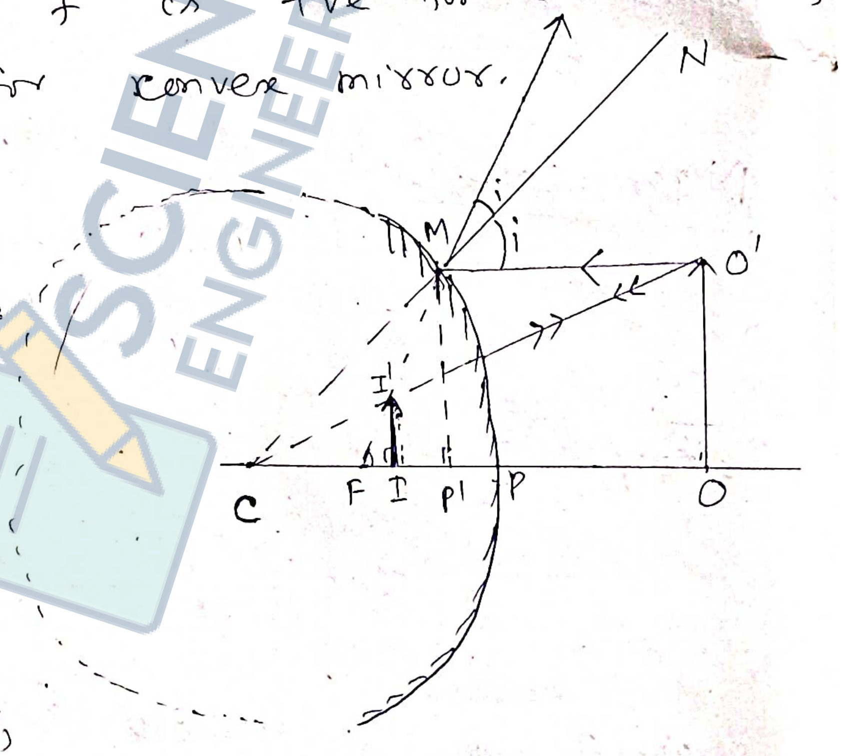
To derive the formula $\boxed{\frac{1}{v}+\frac{1}{u}=\frac{1}{f}}$ for a ~~Concave~~ Convex mirror producing virtual image.

Ans: This formula is a relation connecting object distance, image distance, focal length of a spherical mirror, Sign convention followed is called new convention of signs

(1) All measurements are to be made from the pole of the mirror.

(2) All real distances are to be taken as +ve and all virtual distances are to be taken as -ve.

Then f is +ve for concave mirror, and -ve for convex mirror.



Here

OO' = object

Co = Principal axis.

II' = Image.

PF = focal length.

C = Centre of curvature

PC = Radius of curvature

F = Focus.

P = Pole

M = Point of incidence.

PO = object distance (U)

PI = Image distance (V)

The two right angled triangles OO'C and II'C are similar.

$$\therefore \frac{OO'}{II'}=\frac{CO}{CI} \quad \text{———(i)}$$

The two right ~~to~~ angled triangles MP'F and I'IF are similar.

$$\therefore \frac{MP'}{II'} = \frac{FP'}{FI}$$

or $$\frac{OO'}{II'} = \frac{FP'}{FI} \quad \text{--- (2)}$$

Equating R.H.S' of eq^{n}s (i) and (ii), we get

$$\frac{CO}{CI} = \frac{FP'}{FI} \quad \text{--- (3)}$$

If ~~[illegible]~~ we will consider only rays that are nearer to be principal axis, then the ⊥lar from M will fall on P i.e P' coincides with P.

Hence, eq^n (3) becomes

$$\frac{CO}{CI} = \frac{FP}{FI}$$

where

u = object distance

v = image distan

or $$\frac{CP+PO}{CP-PI} = \frac{FP}{FP-PI}$$

or $$\frac{-r+u}{-r-(-v)} = \frac{-f}{(-f)-(-v)}$$

or $$\frac{-2f+u}{-2f+v} = \frac{-f}{v-f}$$

or $$(u-2f)(v-f) = (-f)(v-2f)$$

or $$uv - uf - 2vf + \cancel{2f^2} = -fv + \cancel{2f^2}$$

$$\Rightarrow uv = uf + vf$$

Dividing by uvf throughout, we get

$$\frac{uv}{uvf} = \frac{uf}{uvf} + \frac{vf}{uvf}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{(Proved)}$$

$$\Rightarrow \frac{1}{R/2} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{2}{R} = \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Task → Derive this formula for a concave mirror producing (a) real image (b) virtual image

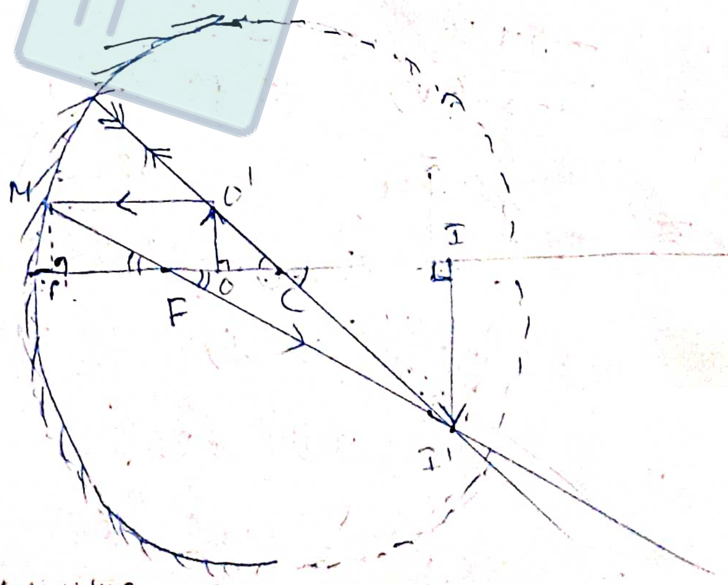
It is a relation connecting object distance, image distance and focal length of a spherical mirror.

(a) Real image

Sign convention is followed is called new convention of signs

1. All measurement are to be made from the pole of the mirror.
2. All the real distances are to be taken as +ve and all virtual distance are to be taken as −ve.

Then f is +ve for concave mirror and −ve for convex mirror.



Here

OO' = object

II' = Image

PE = Principal axis extended.

PF = Focal length.

C = Centre of curvature

PC = radius of curvature.

F = Focus

P = Pole, M = Point of incidence, PO = object distance, PI = Image distance.

The two right angled triangles $OO'C$ and $\cancel{o}\ II'C$

$$\frac{II'}{OO'} = \frac{CI}{CO} \quad ----- (i)$$

The ~~two triangle~~ two right angled triangles $MP'F$ and $II'F$ are similar

$$\frac{II'}{\cancel{IO}\ MP'} = \frac{IF}{P'F} \quad \cancel{------ ii}$$

$$\quad ---------- (ii)$$

or $$\frac{II'}{OO'} = \frac{IO}{P'F}$$

Equating $\cancel{k}$ l.h.s of eqn (i), (ii), we get

$$\frac{CI}{CO} = \frac{IF}{P'F} \quad ----- (iii)$$

If we will consider only rays nearer the principal axis, then the ⊥ drawn to the from the point M will fall on P i.e. P' coincides with P.

Hence eqn (iii) becomes

$$\frac{CI}{CO} = \frac{IF}{PF}$$

$$\Rightarrow \frac{PI - PC}{PC - PO} = \frac{PI - PF}{PF}$$

$$\Rightarrow \frac{v - r}{r - u} = \frac{v - \cancel{2}f}{\cancel{2}f}$$

$$\Rightarrow \frac{V - 2f}{2f - u} = \frac{V - f}{f}$$

$\Rightarrow f(v-2f) = (2f-u)(v-f)$

$\Rightarrow vf - \cancel{2f^2} = 2fv - \cancel{2f^2} - uv + uf$

$\Rightarrow uv = vf + uf$

Dividing both the sides by uvf, we get

$\Rightarrow \frac{uvf}{uvf} = \frac{vf}{uvf} + \frac{uf}{uvf}$

$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (Proved)

$\Rightarrow \frac{2}{2f} = \frac{1}{u} + \frac{1}{v}$

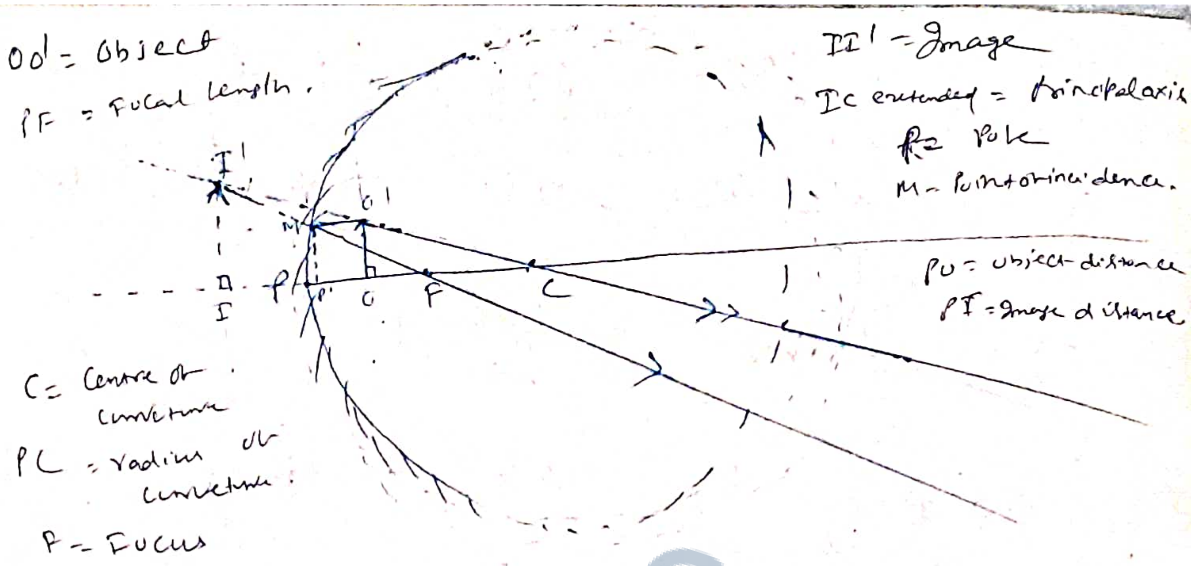
$\Rightarrow \frac{2}{r} = \frac{1}{u} + \frac{1}{v}$

(b) When a object is placed between the Pole and focus of Concave mirror, a virtual, errect, magnified image is formed. Sign Convention is followed is called new convention of signs.

(i) All the measurement are to be made from the Pole of the mirror.

(ii) All the real distances are to be taken as +ve and all virtual distances are to be taken as −ve. Then f is +ve for Concave mirror and −ve for Convex mirror.

The Two right angled triangle II'C and OO'C



are similar

$$\frac{II'}{OO'} = \frac{IC}{OC} \quad \text{---(i)}$$

The two right angled triangle MP'F and II'F are similar.

$$\frac{II'}{MP'} = \frac{IF}{P'F}$$

or

$$\frac{II'}{OO'} = \frac{IF}{P'F} \quad \text{——(ii)}$$

Equating L.H.S of eqn (i) and (ii), we get

$$\frac{IC}{OC} = \frac{IF}{P'F} \quad \text{---(iii)}$$

If we will consider any rays that are nearer to the principal axis, then clear from M will fall on P i.e P' coincides with P.

Hence eqn (3) becomes,

$$\frac{CI}{CO} = \frac{IF}{PF}$$

$$\Rightarrow \frac{PC+IP}{CP-PO} = \frac{PF+IP}{PF} \Rightarrow \frac{r-v}{r-u} = \frac{f-v}{f}$$

$$\Rightarrow \frac{2f-v}{2f-u} = \frac{f-v}{f}$$

(where u is object distance, v is image distance)

$\Rightarrow (2f-v)(f) = (2f-u)(f-v)$

$\Rightarrow 2f^2 - vf = 2f^2 - 2fv - uf + uv$

$\Rightarrow uv = uf + vf$

Dividing by uvf throughout, we get

$$\Rightarrow \frac{uv}{uvf} = \frac{uf}{uvf} + \frac{vf}{uvf}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \frac{2}{2f} = \frac{1}{v} + \frac{1}{u} \quad \text{(Proved)}$$

$$\Rightarrow \frac{2}{r} = \frac{1}{v} + \frac{1}{u}$$

Problems

1. How far from a concave mirror of radius 2 ft would you place the object to get an image magnified 3 times?

Ans: $\frac{4}{3}$ ft → For real magnification, $\frac{2}{3}$ ft → for virtual "

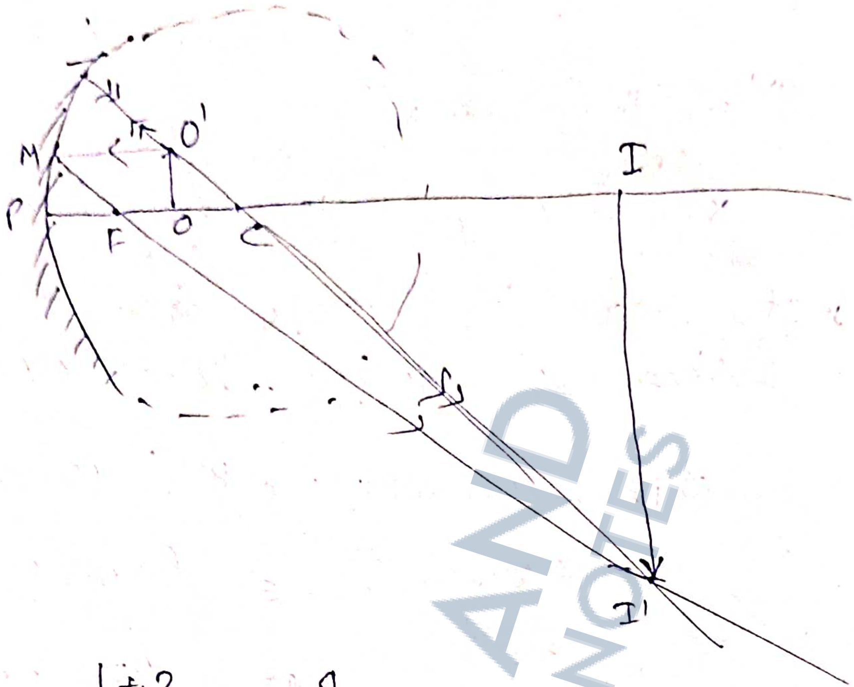
Ans → Magnification $\frac{v}{u} = 3$

$\therefore v = 3u$

For real image formation, the object has to be placed in between F and C

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{3u} + \frac{1}{u} = \frac{1}{1}$$



$$\Rightarrow \frac{1+3}{3u} = 1$$

$$\Rightarrow 4 = 3u \Rightarrow u = \frac{4}{3}$$

For virtual magnification the object must be in between the focus and the pole.

$v = -3u$

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \Rightarrow \frac{1}{-3u} + \frac{1}{u} = \frac{1}{1} = 1$$

$$\Rightarrow \frac{1-3}{-3u} = 1 \qquad \Rightarrow \frac{-2}{-3u} = 1$$

$$\Rightarrow 2 = 3u \Rightarrow u = \frac{2}{3}$$

2. The image of a gas flame standing at a distance of 6 ft from the screen should be magnified 3 times. Where would you hold the mirror and what

sort of mirror would you required?

Ans:- $u = 3\,ft$, $f = 2.25\,ft$, Concave mirror.

3). <u>Newton's formula</u>

If x and y be the distances of a object and its image from the focus of a spherical mirror, prove $xy = f^2$

<u>Proof</u> Object distance $= PO = PF + FO$

$= f + x = u$

Image distance $= PI = PF + FI$

$= f + y = v$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{f+y} + \frac{1}{f+x} = \frac{1}{f} \Rightarrow \frac{f+x+f+y}{(f+y)(f+x)} = \frac{1}{f}$$

$$\Rightarrow f(2f+x+y) = f^2 + fx + fy + xy$$

$$\Rightarrow 2f^2 + \cancel{fx} + \cancel{fy} = f^2 + \cancel{fx} + \cancel{fy} + xy$$

$$\Rightarrow 2f^2 - f^2 = xy$$

$$\Rightarrow f^2 = xy.$$

4. The Image formed by a Convex mirror is only $\frac{1}{3}$ of the size of the object. If the focal length of the mirror is 1 ft, where is the image?

Ans :- $-\frac{2}{3}$ ft

5. An image produced by a convex mirror is $\left(\frac{1}{n}\right)^{th}$ of the size of the object. Prove that the latter must be at a distance "$(n-1)f$" from the mirror where f is focal length.

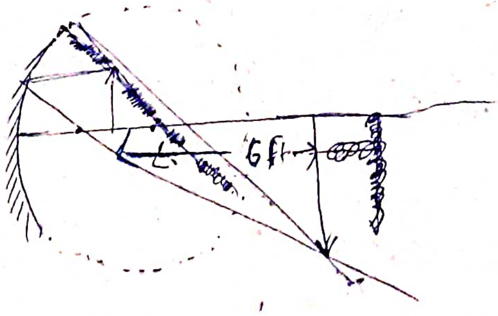
6. A concave mirror forms a real image on a screen which is magnified two times. The screen and the object are moved. So that the image magnified 3 times the object is formed. If the screen is shifted by 25 cm, Calculate the shift of the object and focal length of the mirror.

Ans :- $-\frac{25}{6}$ cm, 25 cm

Answers

~~2. $v-u=6f$~~ ~~$v-u=6ft$~~

$\cancel{\frac{v}{u}=3}$ $\Rightarrow \cancel{\frac{6}{u}=3}$ $\Rightarrow \cancel{u=2}$



2. $v-u=6ft$

$\Rightarrow v=u+6ft$

But $\frac{v}{u}=3 \Rightarrow \frac{u+6}{u}=3$

$\Rightarrow u+6=3u \Rightarrow 2u=6$

$\Rightarrow u = \frac{6}{2} = 3$ ft

$u = 6+3 = 9$ ft

10/27 (2..

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow f = \frac{uv}{u+v} = \frac{3 \times 9}{12} = \frac{27}{12}$$

$$= 2.25 \text{ ft}$$

Since the object is magnified, the mirror is concave.

Q. 4. $\frac{v}{u} = \frac{1}{3}$, $f = 1$ ft

$\Rightarrow u = 3v$

$$f = \frac{u(-v)}{u(-v)} = \frac{3v \cdot v}{3v+v} = \frac{3v^2 v}{34v}$$

$$\Rightarrow 1 = \frac{3v}{4}$$

$\Rightarrow 3v = 4 \quad \Rightarrow v = \frac{4}{3}$

Since the image is virtual $v = -v$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow f = \frac{uv}{u+v} = \frac{u(-v)}{u-v}$$

$$= \frac{u(-3u)}{u-3u}$$

$$= \frac{-3u^2}{4u} = -\frac{3u}{4}$$

Since the object is virtual $u = -v$

$\therefore \quad u = -3v$

$\cancel{\frac{1}{u} - \frac{1}{3v} = \frac{1}{f}}$ $\quad \cancel{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$

$\Rightarrow \quad \Rightarrow \cancel{\frac{1}{-3v} + \frac{1}{v} = \frac{1}{f}}$

$\Rightarrow \cancel{f = \frac{3v(-v)}{-3v+v} = \frac{3v^2}{2v}}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \frac{1}{-\frac{u}{3}} + \frac{1}{u} = \frac{1}{-1} = -1$

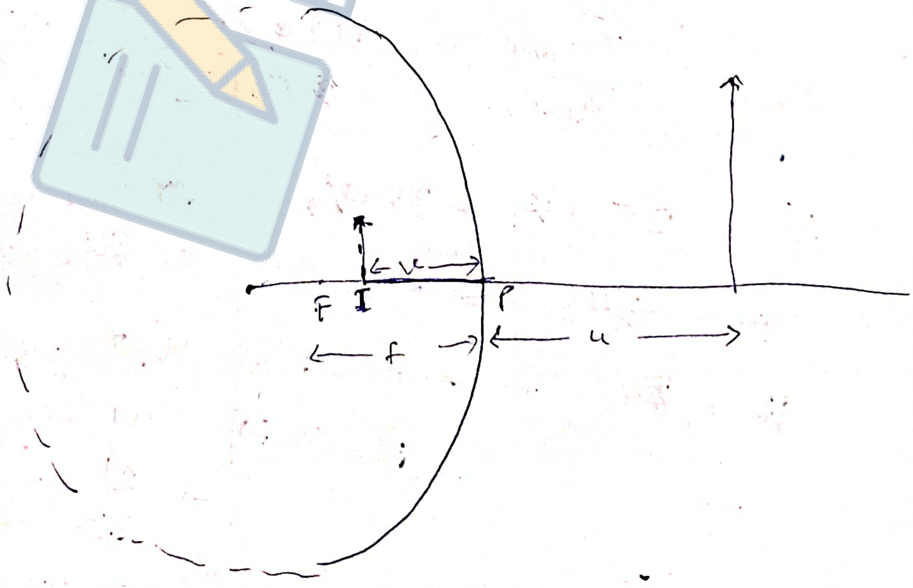
$\Rightarrow \quad -\frac{3}{u} + \frac{1}{u} = -1$

$\Rightarrow \quad \frac{-2}{u} = -1$

$\Rightarrow u = 2$.

$v = -\frac{u}{3} = -\frac{2}{3}$ The image is virtual in nature.

5.



$\frac{v}{u} = \frac{1}{n}$, $\quad u = ?$ $\quad$ focal length $= f$

$\Rightarrow v = \frac{u}{n}$

$$\frac{1}{u} + \frac{1}{-v} = \frac{-1}{f}$$

$$\Rightarrow \frac{1}{u} + \frac{1}{-\frac{u}{n}} = \frac{-1}{f}$$

$$\Rightarrow \frac{1}{u} - \frac{n}{u} = \frac{-1}{f}$$

$$\Rightarrow \cancel{\frac{u - n}{u} = \frac{-1}{f}}$$

$\Rightarrow$

$$\Rightarrow 1 - n = \frac{-u}{f}$$

$$\Rightarrow u = -f(1-n) = f(n-1)$$

6. $v_2 - v_1 = 25$ ——(i), $\frac{v_1}{u_1} = 2 \Rightarrow v_1 = 2u_1$

$v_1 - 2u_1 = 0$ ——(ii)

$v_2 - 3u_2 = 0$ ——(iii)

$$f = \frac{u_1 v_1}{u_1 + v_1}, \quad f = \frac{u_2 v_2}{u_2 + v_2}$$

$$\therefore \frac{u_1 v_1}{u_1 + v_1} = \frac{u_2 v_2}{u_2 + v_2} \quad \text{——(iv)}$$

$v_2 = 25 + v_1 = 25 + v_1$

$\Rightarrow 3u_2 = 25 + v_1$

$\Rightarrow 3u_2 = 25 + 2u_1$

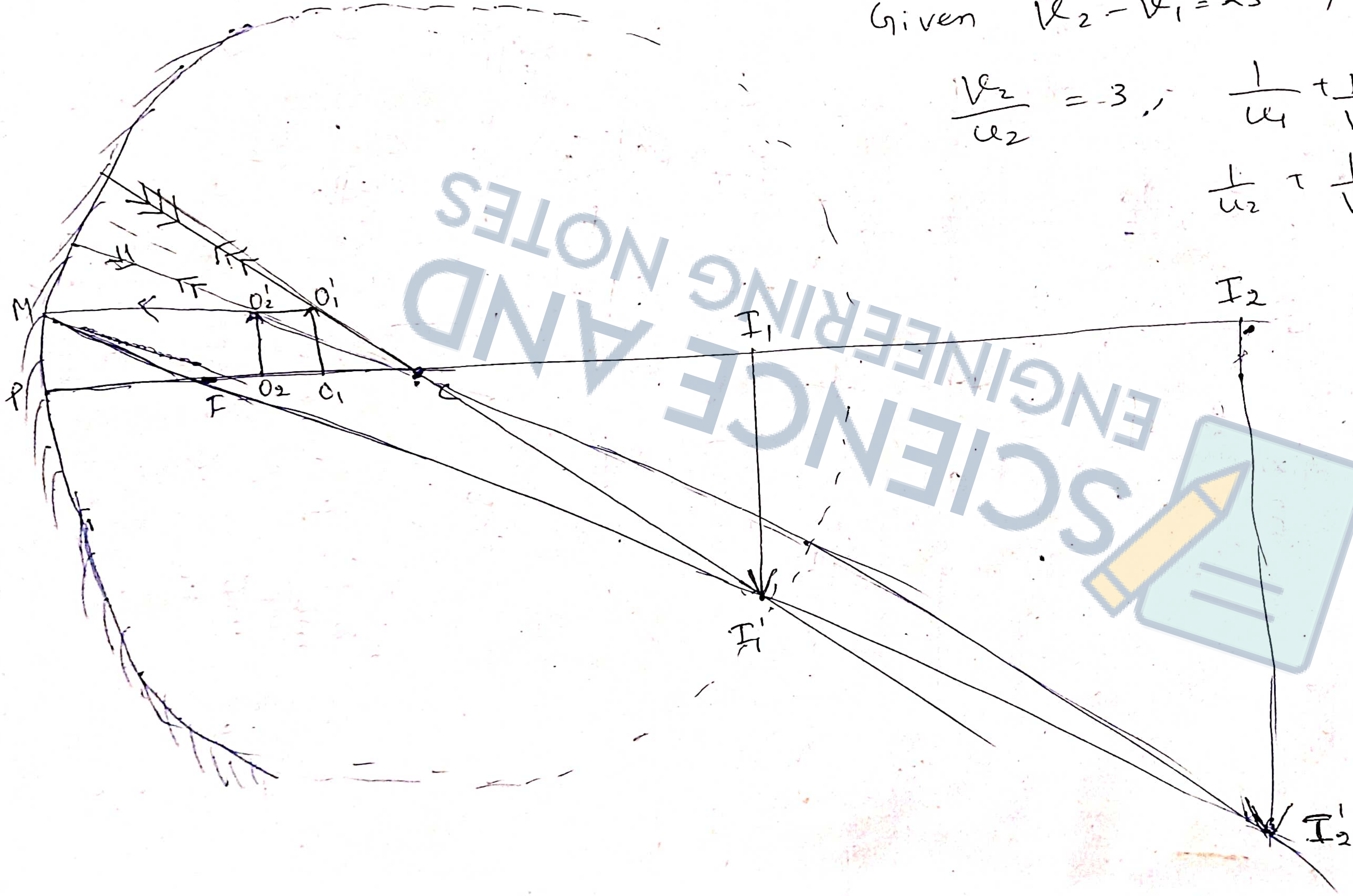
$\cancel{v_2 = 3u_2 \Rightarrow 2}$

$\cancel{v_2 = 3u_2 \Rightarrow 25 + v_1 = 3u_2}$

From (iv) $\rightarrow$

$$\frac{u_1 \cdot 2u_1}{u_1 + 2u_1} = \frac{u_2 \cdot (25 + 2u_1)}{u_2 + (25 + 2u_1)}$$

$$\Rightarrow \frac{2u_1}{1 + u_1} = u_2$$



$u_1 - u_2 = ?$ $\quad f = ?$

Given $v_2 - v_1 = 25$, $\dfrac{v_1}{u_1} = 2$

$\dfrac{v_2}{u_2} = -3$, $\dfrac{1}{u_1} + \dfrac{1}{v_1} = \dfrac{1}{f}$

$\dfrac{1}{u_2} + \dfrac{1}{v_2} = \dfrac{1}{f}$

or $\frac{v_1}{u_1} = 2$ $\therefore v_1 = 2u_1$

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{2u_1} + \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1+2}{2u_1} = \frac{1}{f}$$

$$\Rightarrow \frac{3}{2u_1} = \frac{1}{f} \Rightarrow f = \frac{2u_1}{3} \quad \text{—— (i)}$$

$\frac{v_2}{u_2} = 3$ $\therefore v_2 = 3u_2$

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{3u_2} + \frac{1}{u_2} = \frac{1}{f}$$

$$\Rightarrow \frac{1+3}{3u_2} = \frac{1}{f} \Rightarrow \frac{4}{3u_2} = \frac{1}{f}$$

$$\Rightarrow f = \frac{3u_2}{4} \quad \text{—— (ii)}$$

Equating (i) and (ii)

$$\frac{2u_1}{3} = \frac{3u_2}{4}$$

$$\Rightarrow 8u_1 = 9u_2$$

$$\Rightarrow u_2 = \frac{8}{9}u_1$$

It is given that

$$v_2 - v_1 = 25$$

$$\Rightarrow 3u_2 - 2u_1 = 25$$

$$\Rightarrow \cancel{9}\cdot\left(\frac{8}{\cancel{9}_3}u_1\right) - 2u_1 = 25$$

$$\Rightarrow \frac{8u_1 - 6u_1}{3} = 25 \Rightarrow 2u_1 = 75$$

$$\Rightarrow u_1 = \frac{75}{2}$$

$$u_2 = \frac{8}{9} \cdot u_1 = \frac{8^4}{9} \cdot \left(\frac{75}{2}\right) = \frac{300}{9} = \frac{100}{3}$$

$$= \cancel{33} \cdot \frac{100}{3}$$

$$u_1 - u_2 = \frac{75}{2} - \frac{100}{3} = \frac{225 - 200}{6} = \frac{25}{6}\ \text{cm}$$

$$\delta = \cancel{\ \ }\ \frac{2u_1}{3} = \frac{2}{3} \cdot \left(\frac{75}{2}\right) = 25\ \text{cm}.$$

Uses of Concave mirror

1. A concave mirror is used as a shaving mirror or make-up mirror.
2. Doctors use concave mirror for focussing light on ear, nose, throat for their close examination.
3. Concave mirrors, when used as reflectors, can be employed for constructing reflecting telescopes.

Use of Convex mirror (Uly Convex mirror is used in automobiles)

① A convex mirror can be used as a rear view mirror in automobiles. The image obtained with a convex mirror is diminished but the mirror covers a large field of view. Hence, the objects situated behind the automobile and on both sides of road can easily seen by the driver. A convex mirror is used also as street light reflector

Refraction of light

When light is incident at a surface separating two different medias, it is found to bent towards the normal inside the denser medium or bends away from the normal in the rarer medium. This is called refraction of light.

Laws of refraction

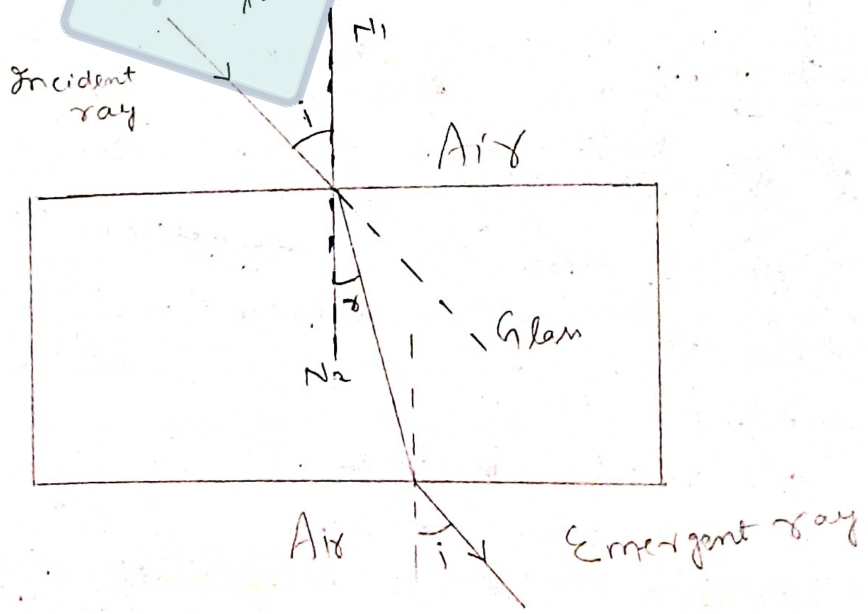
1) The incident ray, the refracted ray and the normal at the point of incidence lie in one plane.

2) **Snell's law**

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant called refractive index for the two media.

i.e, $\frac{\sin i}{\sin r} = \text{constant}, = {}_{a}\mu_{g}$

= Refractive index of glass w.r.t to air.



Refractive index of medium 2 w.r.t to medium 1 ${}^1\mu_2$

$$= \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}$$

i.e ${}^1\mu_2 = \frac{v_1}{v_2}$.

Naturally $= {}^2\mu_1 = \frac{v_2}{v_1} = \left(\frac{1}{\frac{v_1}{v_2}}\right) = \frac{1}{{}^1\mu_2}$

Absolute refractive index of a medium

$$= \frac{\text{Velocity of light in vacuum}}{\text{velocity of light in medium}}$$

$\therefore \mu_1 = \frac{c}{v_1}, \mu_2 = \frac{c}{v_2}$

${}^1\mu_2 = \frac{v_1}{v_2} = \left(\frac{c}{v_2}\right) / \left(\frac{c}{v_1}\right) = \frac{\mu_2}{\mu_1}$

Problem)

1) If refractive index of glass w.r.t air is 1.5 and water w.r.t air $\frac{4}{3}$, Calculate ${}^w\mu_g$ and ${}^g\mu_w$.

Ans: We know ~~that~~ the relation

${}^a\mu_w \cdot {}^w\mu_g \cdot {}^g\mu_a = 1$

$\Rightarrow \frac{4}{3} \cdot {}^w\mu_g \cdot \frac{1}{{}^a\mu_g} = 1$

$\Rightarrow \frac{4}{3} \cdot {}^w\mu_g \cdot \frac{1}{1.5} = 1$

$\Rightarrow {}^w\mu_g = \frac{4.5}{4} = \frac{45}{40} = \frac{9}{8}$

${}^g\mu_w = \frac{8}{9}$

7. Calculate the critical angle for glass having ${}^a\mu_g = 1.5$ and then for diamond having ${}^a\mu_d = 2.24$.

We know that

$$\mu = \frac{1}{\sin C}$$

$$1.5 = \frac{1}{\sin c} \Rightarrow \sin c = \frac{1}{1.5}$$

$$\Rightarrow c = \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$= \sin^{-1}(.6666)$$

$$= 41.8^\circ$$

$$2.24 = \frac{1}{\sin c} \Rightarrow \sin c = \left(\frac{1}{2.24}\right)$$

$$\Rightarrow c = \sin^{-1}(.4464)$$

$$= 26.5^\circ$$

224) 1000 (.4464
896
1040
1120

9. Given $V_e = 0.8 \times V_a$

$${}^a\mu_e = \frac{V_a}{V_e} = \frac{V_e}{0.8 \times V_e} = \frac{1}{.8} = 1.25$$

8. Given $\lambda = 5893\ A^\circ$

$$= 5893 \times 10^{-10} \text{ metre}$$

$$= 589.3 \times 10^{-9} \text{ metre}$$

Frequency remains constant when a wave passes from one medium to another medium. $c = \nu\lambda$

3×10^8 m/sec $= \nu \times 5893 \times 10^{-10}$

$\Rightarrow \nu = \dfrac{3 \times 10^8 \times 10^{10}}{5893} = \dfrac{30{,}000 \times 10^{14}}{5893}$

$= 5.09 \times 10^{14}$

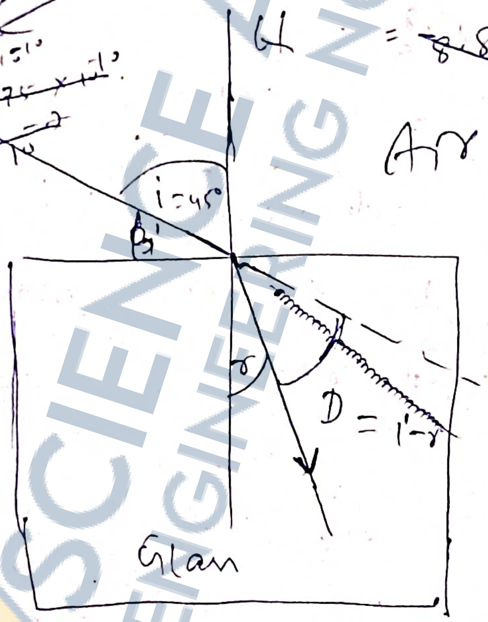
$\frac{4}{3} = \mu = \frac{V_a}{V_w} \Rightarrow V_w = \frac{3}{4} \times c = \frac{3}{4} \times 3 \times 10^8$ m/sec

$V = \nu \lambda' \Rightarrow \lambda' = \frac{V_w}{\nu}$

$= \cancel{4.5 \times 10^3 \times \frac{58{,}930}{30{,}000}}$

$= \cancel{8.82 \times 10^3}$

22.



$\dfrac{\sin i}{\sin r} = 1.52 \Rightarrow \dfrac{\sin 45}{\sin r} = 1.52$

$\Rightarrow \dfrac{.707106}{1.52} = \sin r$

$\Rightarrow r = \sin^{-1}(\quad) = 27.72°$

Angle of deviation $= i - r$

$= 45 - 27.72$

$= 17.3°$

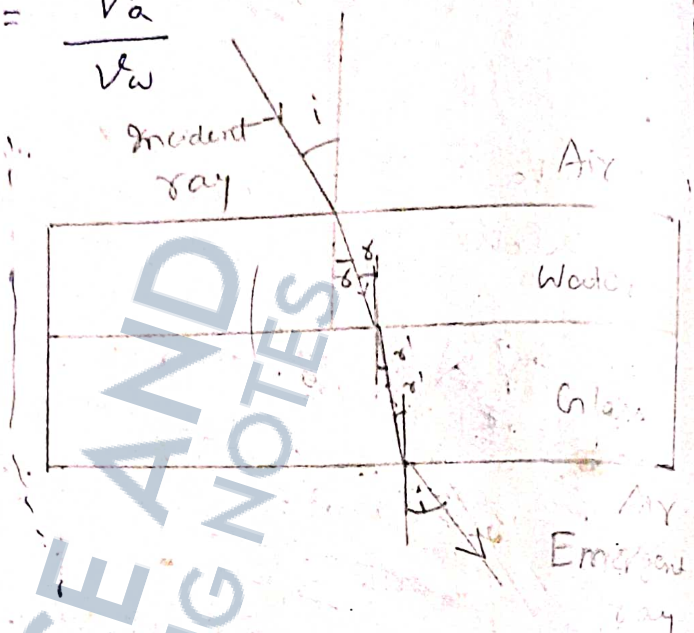
Relation among different R.I (Refractive index) when a ray passes through several media

$$\mu^a_w = \frac{\sin i}{\sin r} = \frac{V_a}{V_w}$$

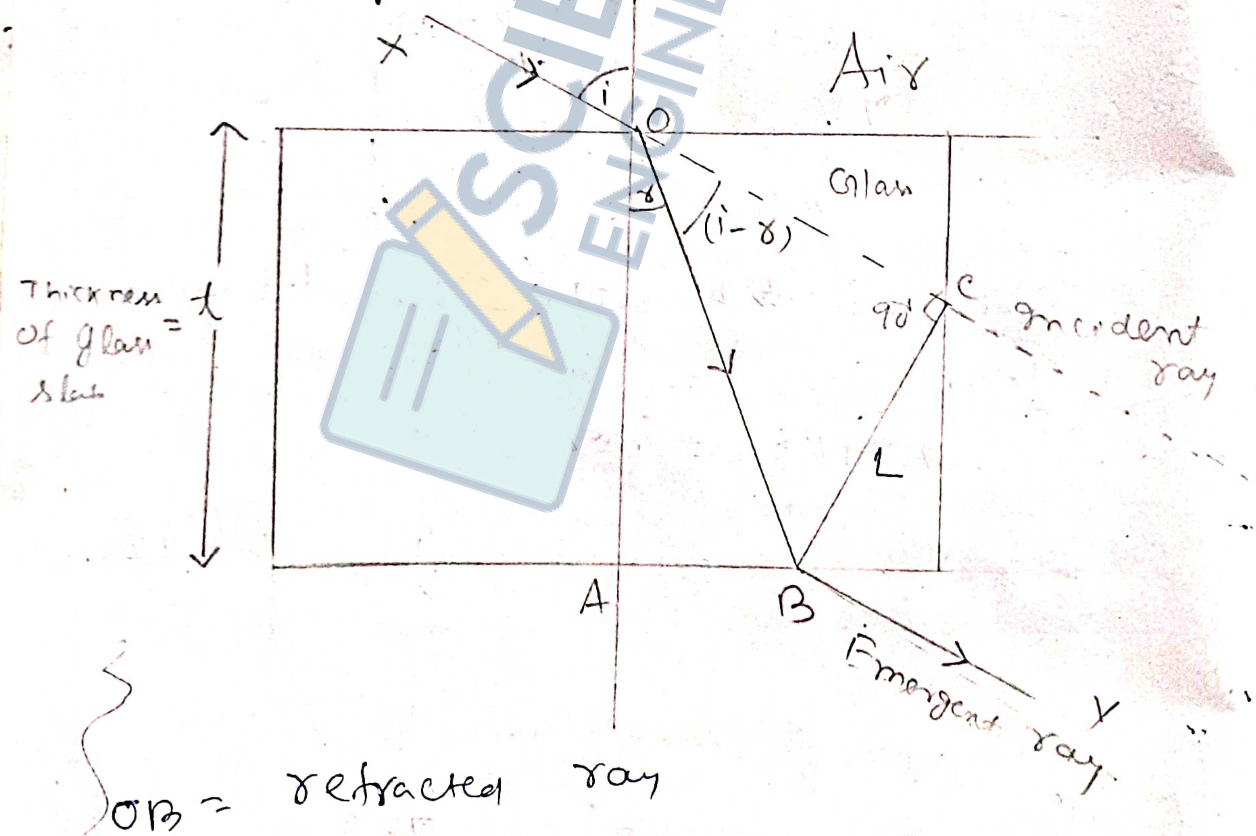
$$\mu^w_g = \frac{\sin r}{\sin r'} = \frac{V_w}{V_g}$$

$$\mu^g_a = \frac{\sin r'}{\sin i} = \frac{V_g}{V_a}$$

$$\mu^a_w \times \mu^w_g \times \mu^g_a = 1$$



Lateral displacement (L) is defined as the $\perp$ distance between the incident and emergent rays.



OB = refracted ray

In the right angled Δ OBC,

$$\sin(i-r) = \frac{BC}{OB} = \frac{L}{OB} \quad \text{——(i)}$$

In the right angled $\triangle$ OAB,

$$\cos r = \frac{OA}{OB} = \frac{t}{OB} \quad \text{——(2)}$$

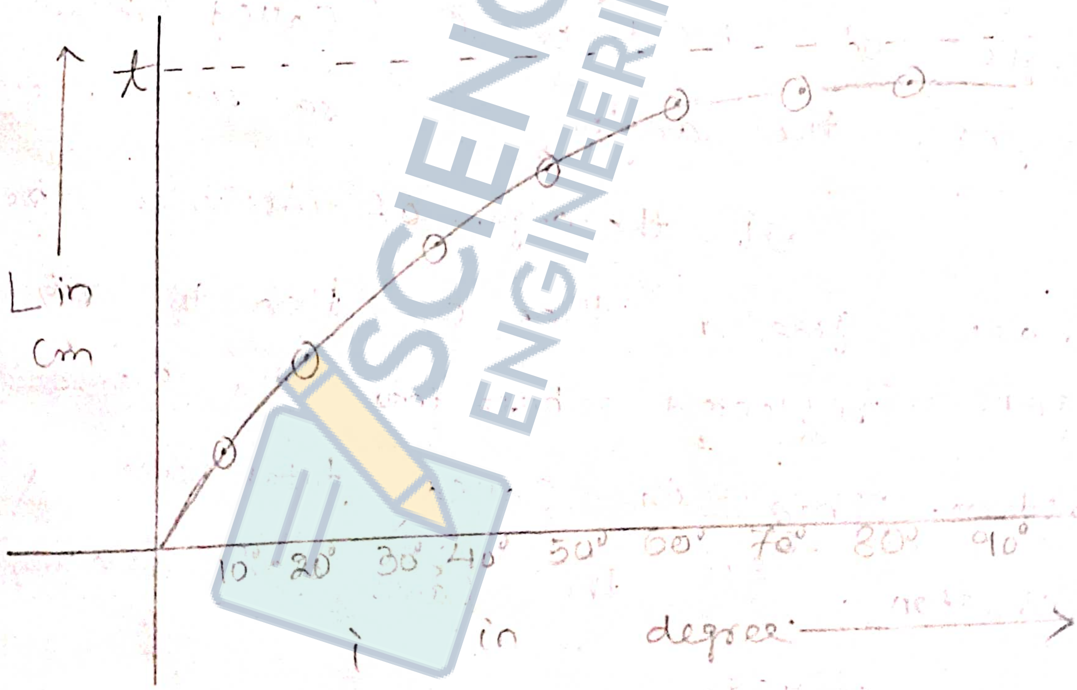
Dividing eqn (i) by eqn (2), we get

$$\frac{\sin(i-r)}{\cos r} = \frac{L}{t}$$

or $$L = t\,\frac{\sin(i-r)}{\cos r}$$

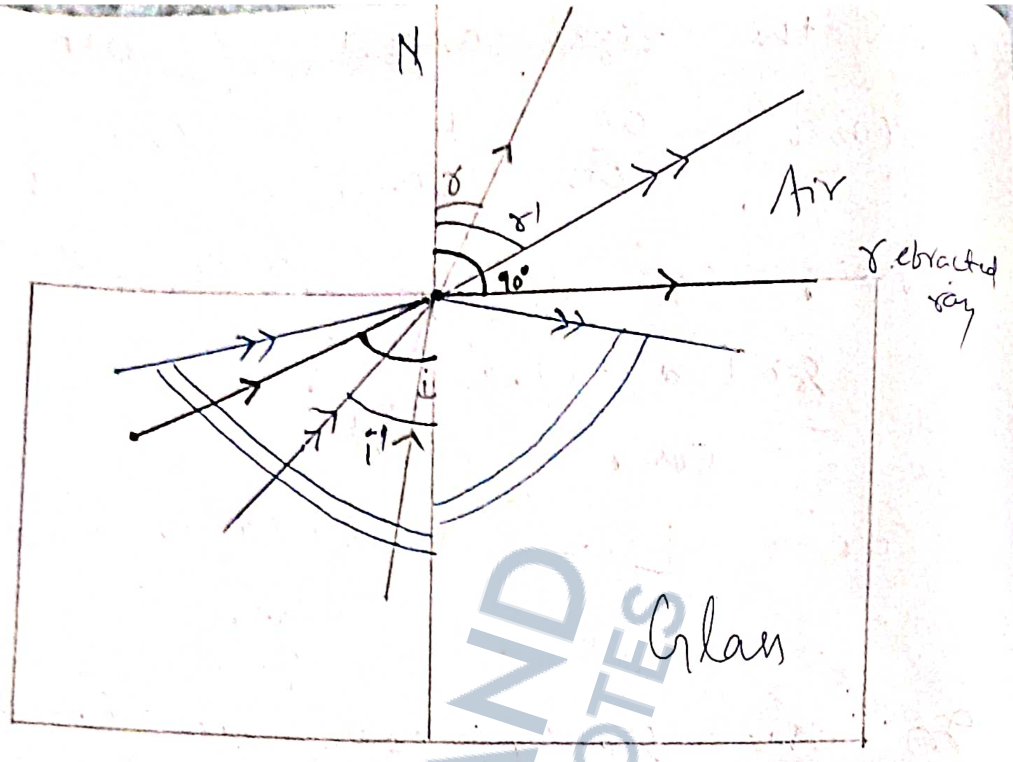
In the limiting case when $i \to 90°$

$$L \to t \cdot \frac{\sin(90° - r)}{\cos r} = t$$



Critical angle and Total internal Reflection

When a ray of light inside a denser medium is incident at the surface of separation with a rarer medium, then the ray bends away from the normal, i.e. $r > i$, $r' > i'$



If we go on increasing the angle of incidence, for a particular angle of incidence c (called critical angle), the angle of refraction is 90°.

If the angle of incidence be made greater than c, then the ray gets reflected back into the same medium. This is called total internal reflection. All the laws of reflection are obeyed.

For $i \leq c$, there is refraction and for $i > c$, there is reflection.

That is why c is called critical angle. For $i = c$, the refracted ray grazes the surface of separation.

From Snell's law of refraction,

$$\mu_a^g = \frac{\sin C}{\sin 90^\circ}$$

$$\Rightarrow \frac{1}{\mu_g^a} = \frac{\sin C}{1}$$

$$\Rightarrow \boxed{\mu_g^a = \frac{1}{\sin C}}$$

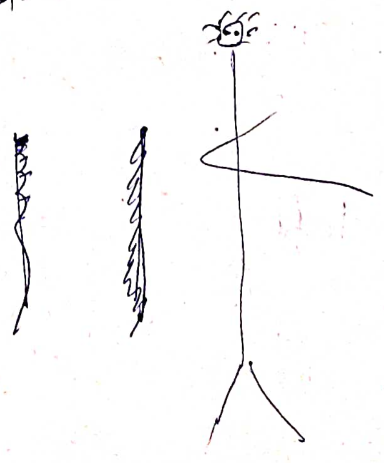
(1) A observer walks towards a plane mirror at a speed of 12 ft/sec. At what speed does he approach his image?

(2) How can a ray be reflected back along the same steps by a ~~mirror~~

Problems on reflection.

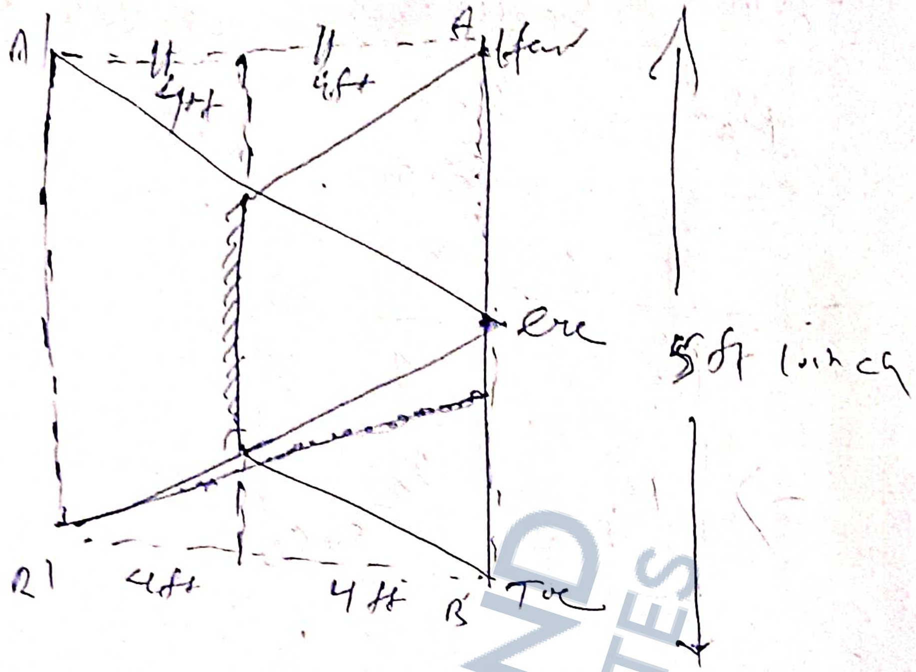
2. A mirror is needed to reflect the whole body of man should half of its height. So 1m mirror is needed.

4. a



(a) The mirror's height is more than half of man's height. It could reflect the man's whole body. So the size of image 5ft when

(b)



(b) 8ft because AA' = 8, BB' = 8 ft

(c) ~~5ft~~ = 1 ft = 12 inch

5 ft = 60 inch

+ 10 inch

70 inch

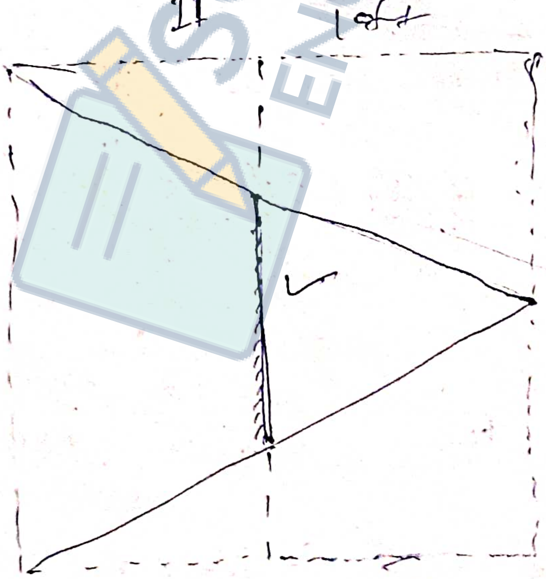
Minm height of mirror = 35 inch

12) 35 (2 — 2 ft and 11 inch.

24

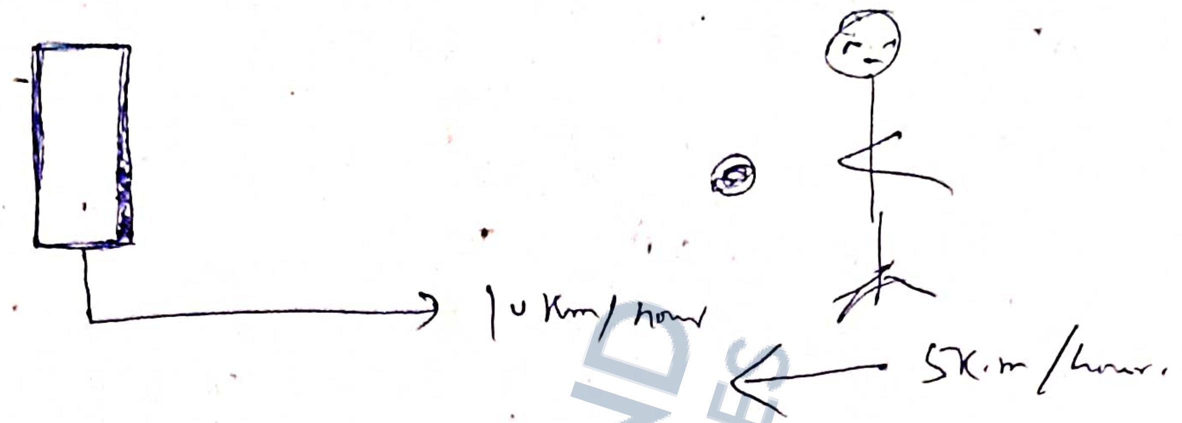
11

(d)



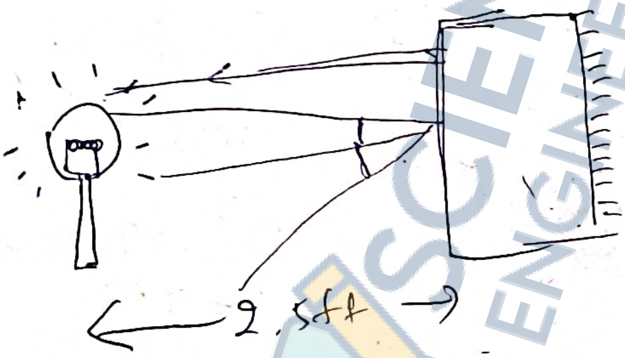
Length of mirror is sufficient, if it is half of height of man i.e 2 ft 11 inch or is independent of distance of man.

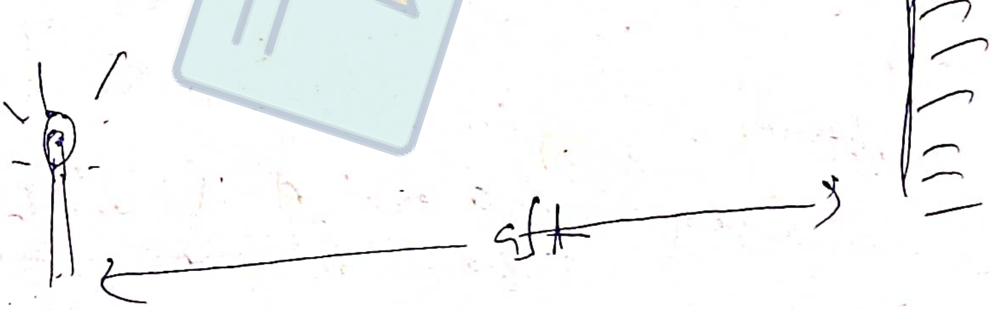
6.



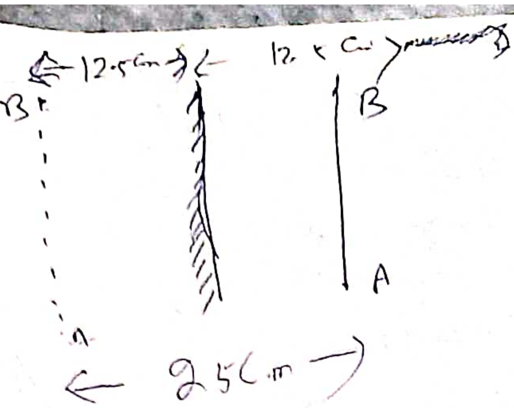
Relative velocity = $(10+5)$ Km/hour = 15 K.m/hour

8.



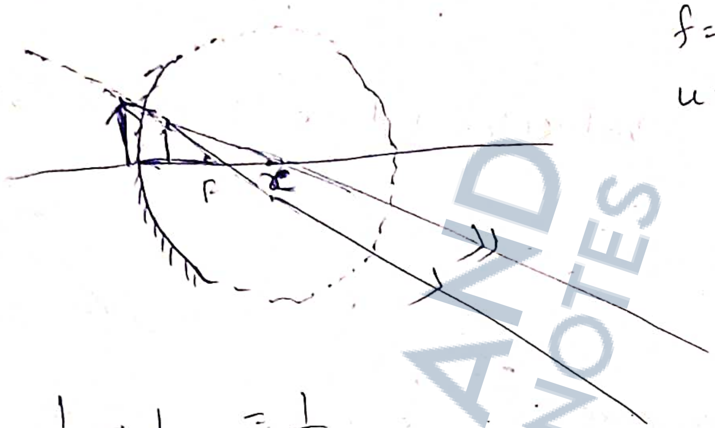


10.



12.

$r = 60$ cm
$f = 30$ cm.
$u = 20$ cm.



$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{20} + \frac{1}{v} = \frac{1}{30}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = \frac{-1}{60}$$

$$\Rightarrow v = -60 \text{ cm}$$

The ~~object~~ image is virtual, erect, enlarged and formed on the other side of the mirror.

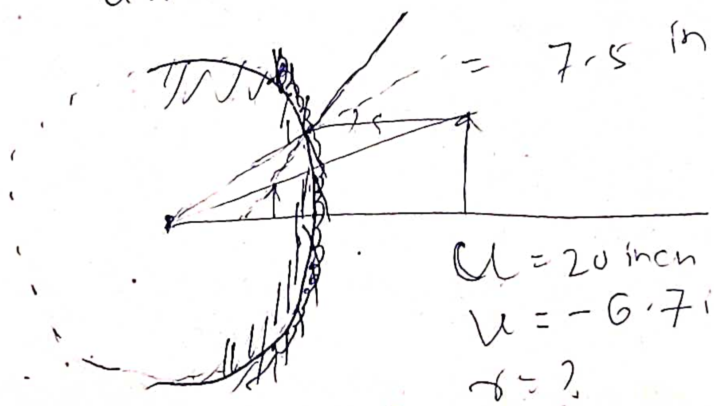
14. $f = ?$, $\frac{v}{u} = 5 \Rightarrow v = 5u$

$u = 9$ inch

$$f = \frac{uv}{u+v} = \frac{9\times(5\times9)}{9+(45)} = \frac{9\times5\times9}{54}$$

$= 7.5$ inch.

16.



$u = 20$ inch
$v = -6.7$ inch
$r = ?$

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}$$

$$\Rightarrow \frac{1}{20}-\frac{1}{6.7}=\frac{2}{r}$$

$$\Rightarrow \frac{6.7-20}{20\times 6.7}=\frac{2}{r}$$

$$\Rightarrow \frac{r}{2}=-\frac{\cancel{20}^{10}\times 6.7}{13.3}$$

$$\Rightarrow r=-20 \text{ inch}$$

$$r=18 \text{ cm}$$

18. $\frac{v}{u}=3$, $\Rightarrow v=3u$

$\therefore$ $\cancel{u=\frac{v}{3}}$

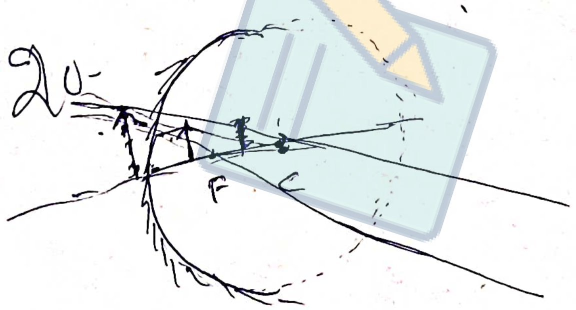
$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$$

$$\Rightarrow \frac{1}{3u}+\frac{1}{u}=\frac{2}{18}$$

$$\Rightarrow \frac{1+3}{3u}=\frac{2}{18}$$

$$\Rightarrow \cancel{3}u=\frac{\cancel{18}^{6}\times 4}{\cancel{2}}^{2}=12 \text{ Cm.}$$

20.



$r=6$ Cm

$u=2$ Cm.

$$\frac{2}{r}=\frac{1}{u}+\frac{1}{v}$$

$$\Rightarrow \frac{\cancel{2}}{\cancel{6}_3}=\frac{1}{2}+\frac{1}{v}$$

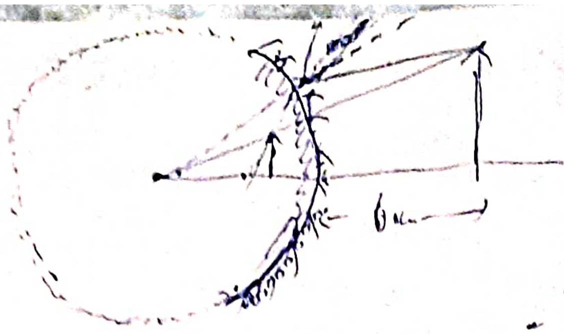
$$\Rightarrow \frac{1}{v}=\frac{1}{3}-\frac{1}{2}=\frac{2-3}{6}=-\frac{1}{6}$$

$$\Rightarrow v=-6 \text{ cm}$$

$$M=\left|\frac{v}{u}\right|$$

$$=\left|\frac{6}{2}\right|=3 \text{ times}$$

22.



$f = 15\text{cm}$

$u = 60\text{ cm}$

$\frac{1}{-f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow -\frac{1}{15} = \frac{1}{60} + \frac{1}{v}$

$\Rightarrow \frac{1}{v} = -\frac{1}{60} - \frac{1}{15}$

$= \frac{-1-4}{60} = -\frac{5}{60}$

$v = -12\text{ cm}$

$\Rightarrow \frac{1}{v} = \frac{1-4}{60}$

$\Rightarrow v = 12\text{ cm}$

But $v = -12\text{ cm}$

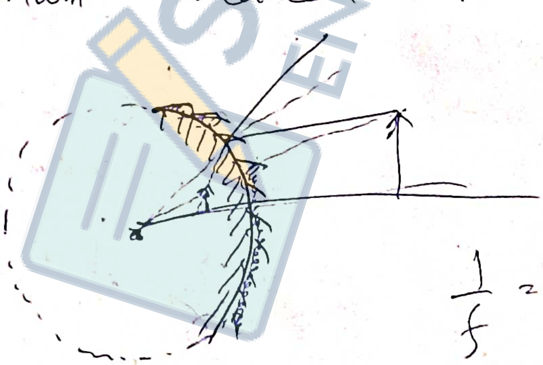
$u = 60\text{ cm}$

$m = \frac{v}{u} = \frac{12}{60} = \frac{1}{5}$

So the image is $\frac{1}{5}$ times of object

If the object has height 10 cm then, it must reduced to 2 cm.

24.



$u = 30\text{ cm}$

$f = 10\text{ cm}$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow f = \frac{uv}{u+v} = \frac{uv}{u-v}$

$10 = \frac{30 \times v}{30-v}$

$\Rightarrow 3v = 30 - v$

$\Rightarrow 4v = 30$

$\Rightarrow v = 7.5\text{ cm}$

The image is virtual behind the mirror. $\Rightarrow$

$m = \frac{v}{u} = \frac{7.5}{30} = \frac{1}{4}$

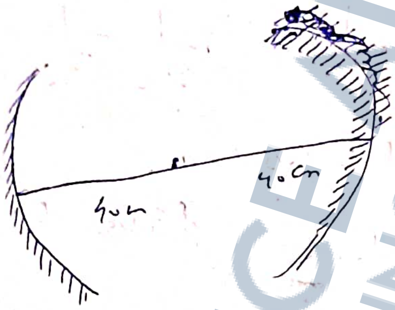
26. $f = 20\,\text{cm}$, $-\frac{v}{u} = 2$ $\Rightarrow v = -2u$

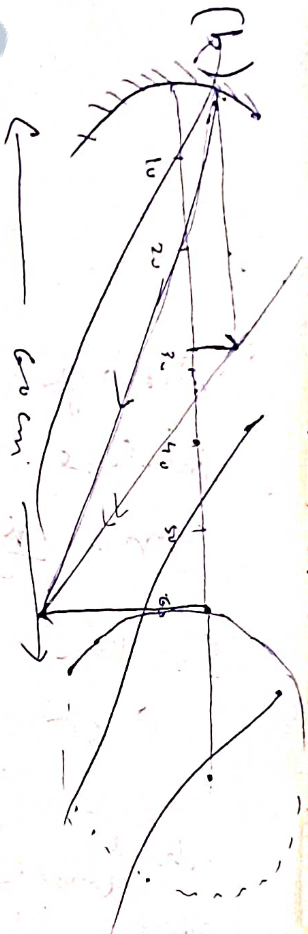
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{u} - \frac{1}{2u}$$

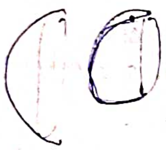
$$\Rightarrow \frac{1}{20} = \frac{2-1}{2u} = \frac{1}{2u}$$

$\Rightarrow u = 10\,\text{cm}$

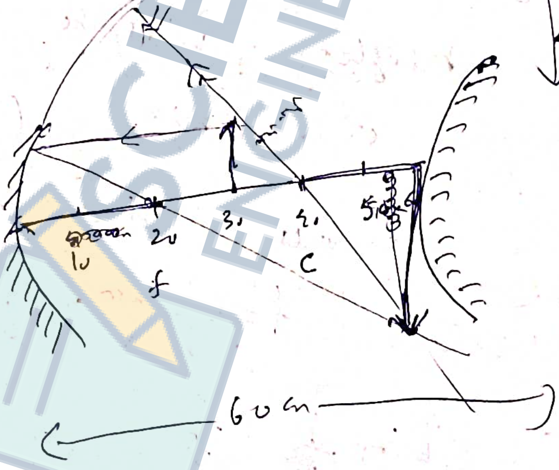
28.



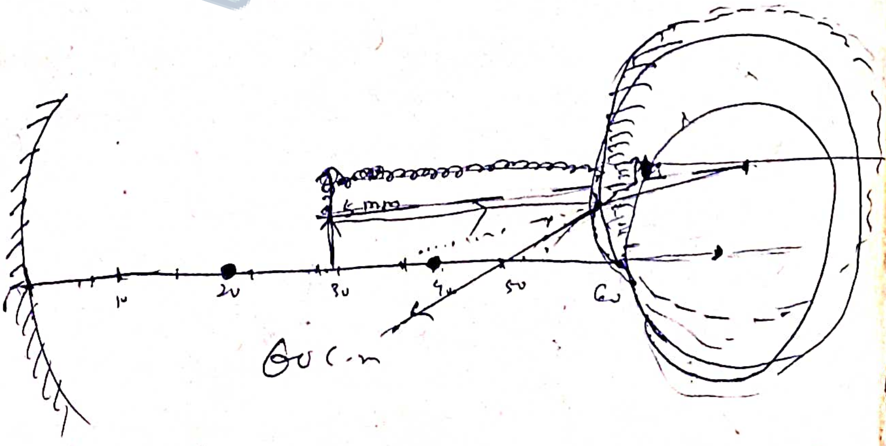




(a)



(b)...



(a) Or first reflection, takes place at concave surface

Then radius curvature $r = 40$ c.m
$f = 20$ c.m

$v = ?$ $u = 30$ c.m,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{30} + \frac{1}{v} = \frac{1}{20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$$

$$\Rightarrow v = 60 \text{ c.m}$$

∴ Image will fall on the Convex surface.

$$m = \frac{v}{u} = \frac{60}{30} = 2$$

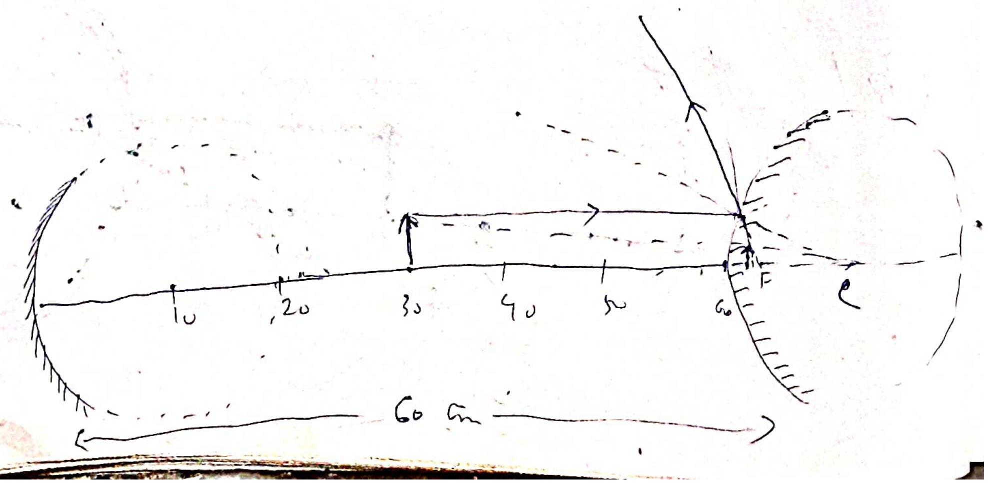
So the height will be magnified by 2 times.

i.e it new height $(5mm \times 2) = 10$ mm.

when it will fall on Convex surface

Now object distance $u = 0$; $f = 20$ c.m,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$



(b) If the reflection first takes place at the convex mirror first

then $r = -40$ cm, $f = -20$ cm

$u = 30$ c.m.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{30} + \frac{1}{v} = \frac{-1}{20}$$

$$\Rightarrow \frac{1}{v} = -\frac{1}{20} - \frac{1}{30} = \frac{-3-2}{60} = \frac{-5}{60} = -\frac{1}{12}$$

$$\Rightarrow v = -12 \text{ cm}.$$

i.e the image will form out side of the mirror.

$$m = \frac{v}{u} = \left|\frac{12}{30}\right| = \left|\frac{6}{15}\right| = \frac{2}{5}$$

So the size of the image is $\frac{2}{5}$ times of the 5 mm, i.e $5 \times \frac{2}{5} = 2$ mm.

Now this is object for concave mirror,

$u = 60 + 12 = 72$ c.m

$f = 20$ cm.

$r = 20$ cm.

$$\frac{1}{72} + \frac{1}{v} = \frac{1}{20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{72} = \frac{72-20}{72 \times 20} = \frac{\cancel{52}^{\cancel{26}\,13}}{\cancel{72 \times 20}_{36\ 10}}$$

$$\Rightarrow v = \frac{360}{13} = 27.7 \text{ c.m.}$$

$\simeq 28$ c.m.

The imag will at a distance 28 cm from Concave mirror.

$$m = \frac{v}{u} = \frac{28}{72}$$

So the image has size =

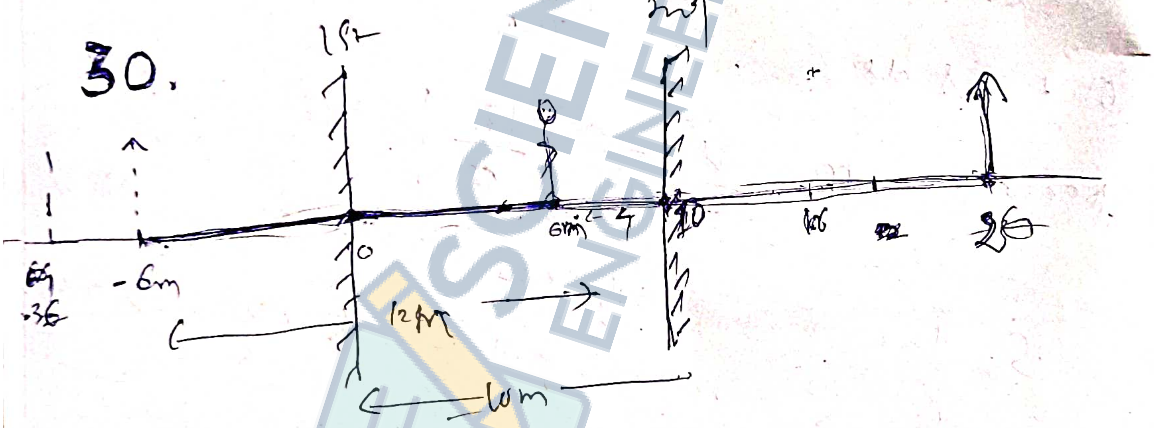
$= 2\,mm \times \frac{\cancel{28}\ 14\ 7}{\cancel{72}\ \cancel{36}\ 18}$

$= \frac{14}{18}$

$= .78\ mm$

18) 140 (.78
126
140
144

30.



① 12m, because the image is at the back of mirror at a distance 6cm.

$6m - (-6m) = 12m$

② ~~Another~~ It is object for 2nd mirror. It is at a distance 10m from 2nd mirror. Its image will form 16m from the mirror behind. Its position is 26 metre.

It is a distance from the person

$26 - 6 = 20$ metre

(iii) It is 36 m away from the first mirror, so from the person it is at a distance $36+6 = 42$ metre.

32.1 (a)

(a) If first reflection takes place on the plane mirror, then its image will be at a distance $\boxed{30\text{ cm back}}$ for concave mirror.
Now it is direct

~~magnification~~ $\cancel{\frac{v}{u} = 90}$

$u = 90$, $v = ?$, $f = 40$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow f = \frac{u \cdot v}{u+v}$$

$$\Rightarrow 40 = \frac{90 \times v}{90+v}$$

$$\Rightarrow 360 + 4v = 9v$$

$$\Rightarrow 5v = 360$$

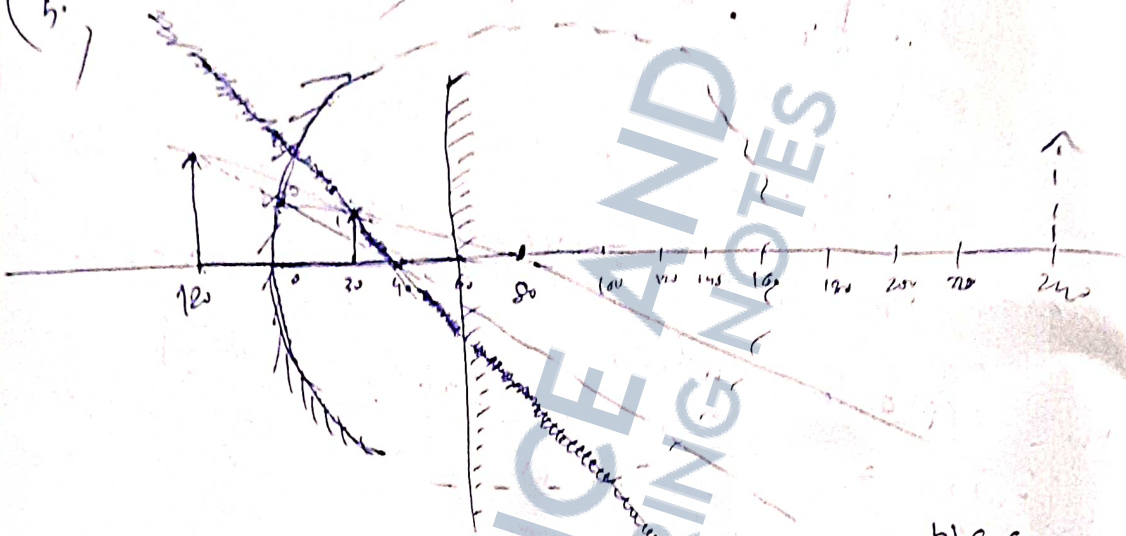
$$\Rightarrow v = 72\text{ cm}$$

5/40 · 8

$$m = \frac{v}{u} = \frac{72}{90} = \frac{4}{5} = 0.8$$

(B)

(5.)



Or First reflection takes place at the concave mirror.

$f = 40, \quad u = 30$ cm.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{30} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{40} - \frac{1}{30} = \frac{1}{v}$$

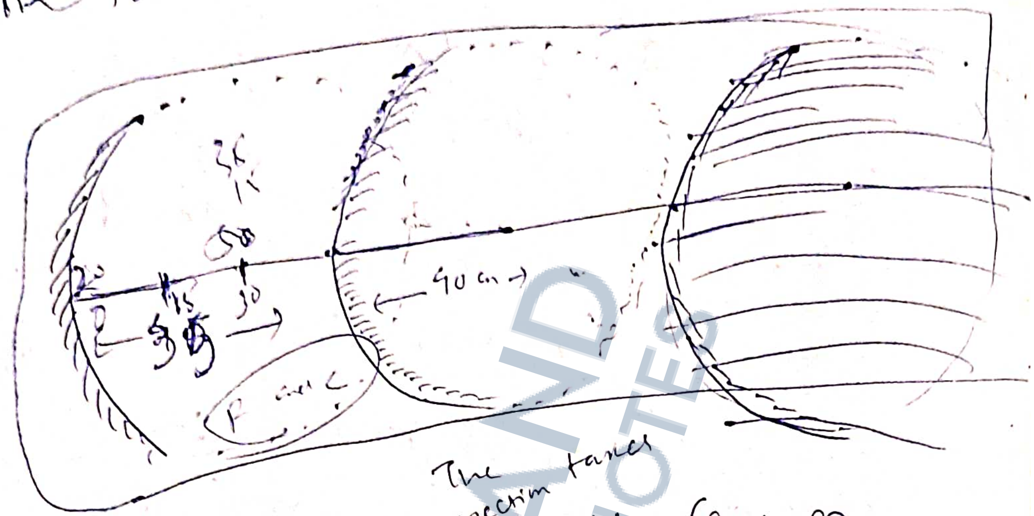
$$\Rightarrow \frac{1}{v} = \frac{30 - 4}{120} = -\frac{1}{120}$$

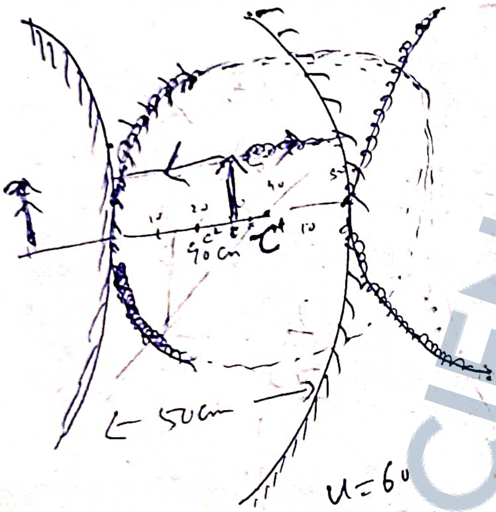
$$\Rightarrow v = -120 \text{ cm.}$$

$$m = \frac{120}{30} = 4$$

This is object for plane mirror.

Image will at a distance 180 cm from the plane mirror.

34. 



The reflection takes place first on Concave mirror.

$u = 20\,cm$, $r = 30\,cm$. $f = 15\,cm$.

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow \frac{1}{15} = \frac{1}{20} + \frac{1}{v}$

$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{20} = \frac{4-3}{60} = \frac{1}{60}$

$\Rightarrow v = 60\,cm$.

$u = 60$

$f = -20$

$f = \frac{uv}{u+v}$

$\Rightarrow -20 = \frac{60 \times v}{60+v}$

$\Rightarrow -1200 - 20v = 60v$

$80v = -1200$

$\Rightarrow v = -\frac{1200}{80}$

Or reflection takes place at Convex mirror

$u = 30\,cm$,

$f = -20\,cm$.

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow -\frac{1}{20} = \frac{1}{30} + \frac{1}{v}$

$\Rightarrow \frac{1}{v} = -\frac{1}{20} - \frac{1}{30} = \frac{-3-2}{60}$

$= \frac{-5}{60}$

$\Rightarrow v = -12$

Then for Concave mirror

$u = 62, f = 15$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

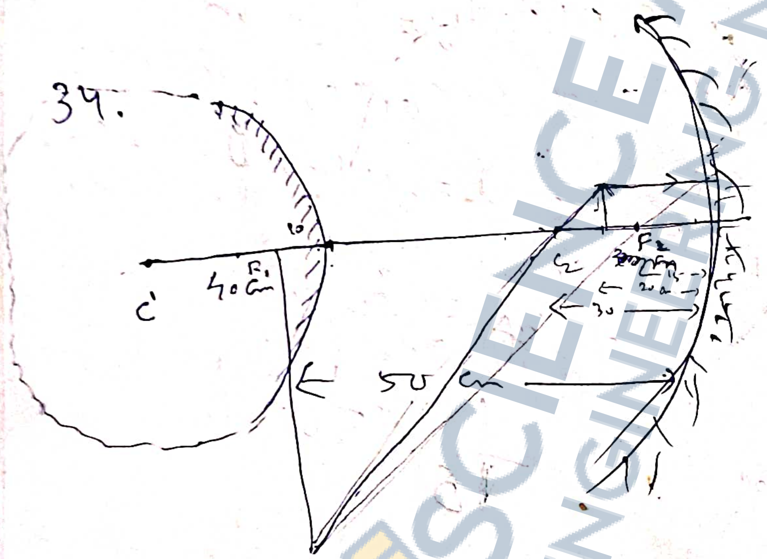
$$\Rightarrow \frac{1}{15} = \frac{1}{62} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{62} = \frac{62-15}{62 \times 15}$$

$$= \frac{47}{62 \times 15}$$

$$\Rightarrow v = \frac{62 \times 15}{47}$$

34.



$u = 20$ cm.

$r = 30$ cm.

$v = ?$

$$\frac{2}{r} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{2}{30} = \frac{1}{20} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{15} - \frac{1}{20} = \frac{1}{v}$$

$$\Rightarrow \frac{1}{v} = \frac{4-3}{60} = \frac{1}{60}$$

$\Rightarrow v = 60$ cm.

$u = -10, f = -20$ cm.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{-20} = \frac{1}{10} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{v} = \frac{-1}{20} - \frac{1}{10} = \frac{1-2}{20} = \frac{-1}{20}$$

To prove that $\boxed{\frac{\text{Real depth}}{\text{Apparent depth}} = \mu}$

OC = Real depth

PC = Apparent depth.

In the △ triangle AOC,

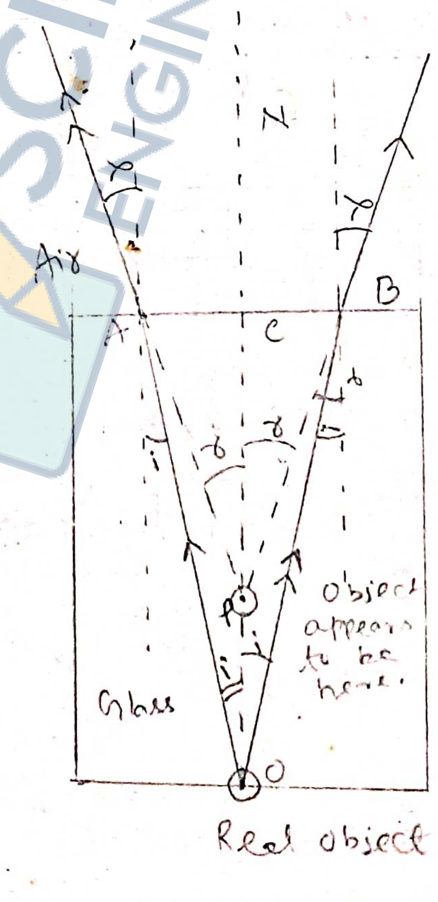
$\sin i = \frac{AC}{AO} \simeq \frac{AC}{OC}$ (Because, $i \to 0$ and $\sin i \simeq i = \tan i$)

In the △ APC,

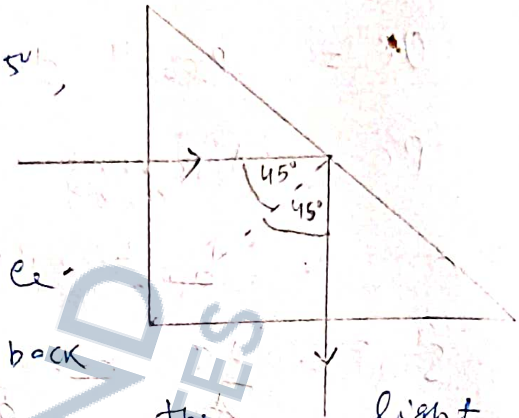
$\sin r = \frac{AC}{AP} \simeq \frac{AC}{PC}$ (Because, $r \to 0$ and $\sin r \simeq \tan r$)

$\therefore \frac{\sin i}{\sin r} = \frac{PC}{OC}$ $\quad \therefore {}^g\mu_a = \frac{PC}{OC}$

$\Rightarrow \frac{1}{{}^a\mu_g} = \frac{PC}{OC}$ $\Rightarrow {}^a\mu_g = \frac{OC}{PC} = \frac{\text{Real depth}}{\text{Apparent depth}}$.



Right angled prism is better reflector than a plane mirror

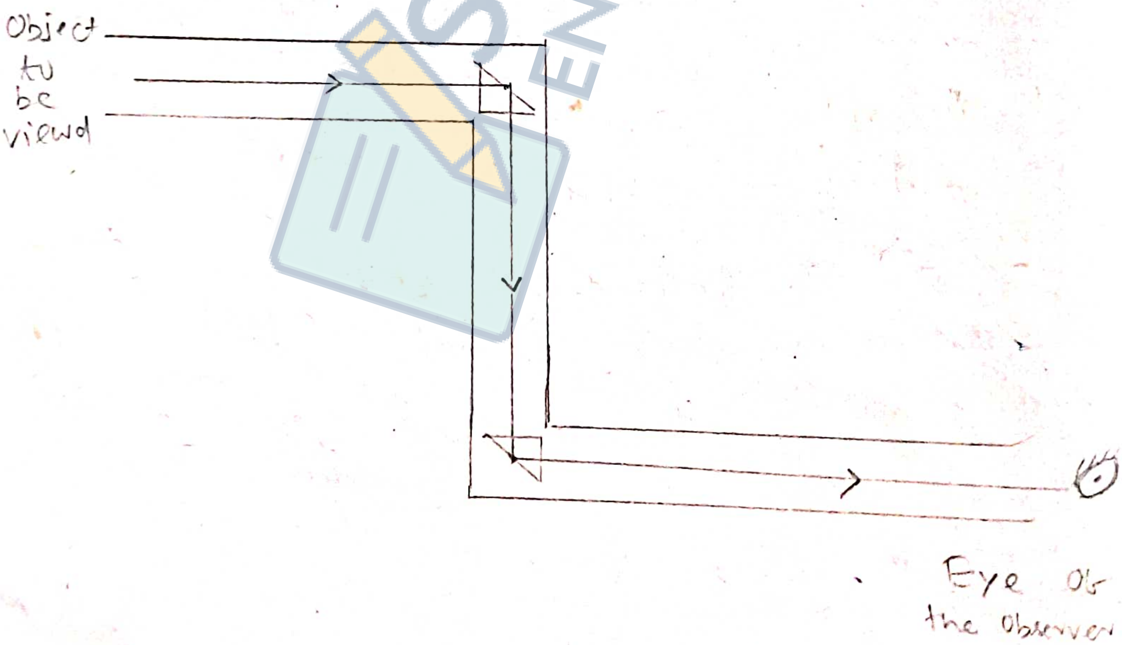


Since $C = 42°$ & $i = 45°$,

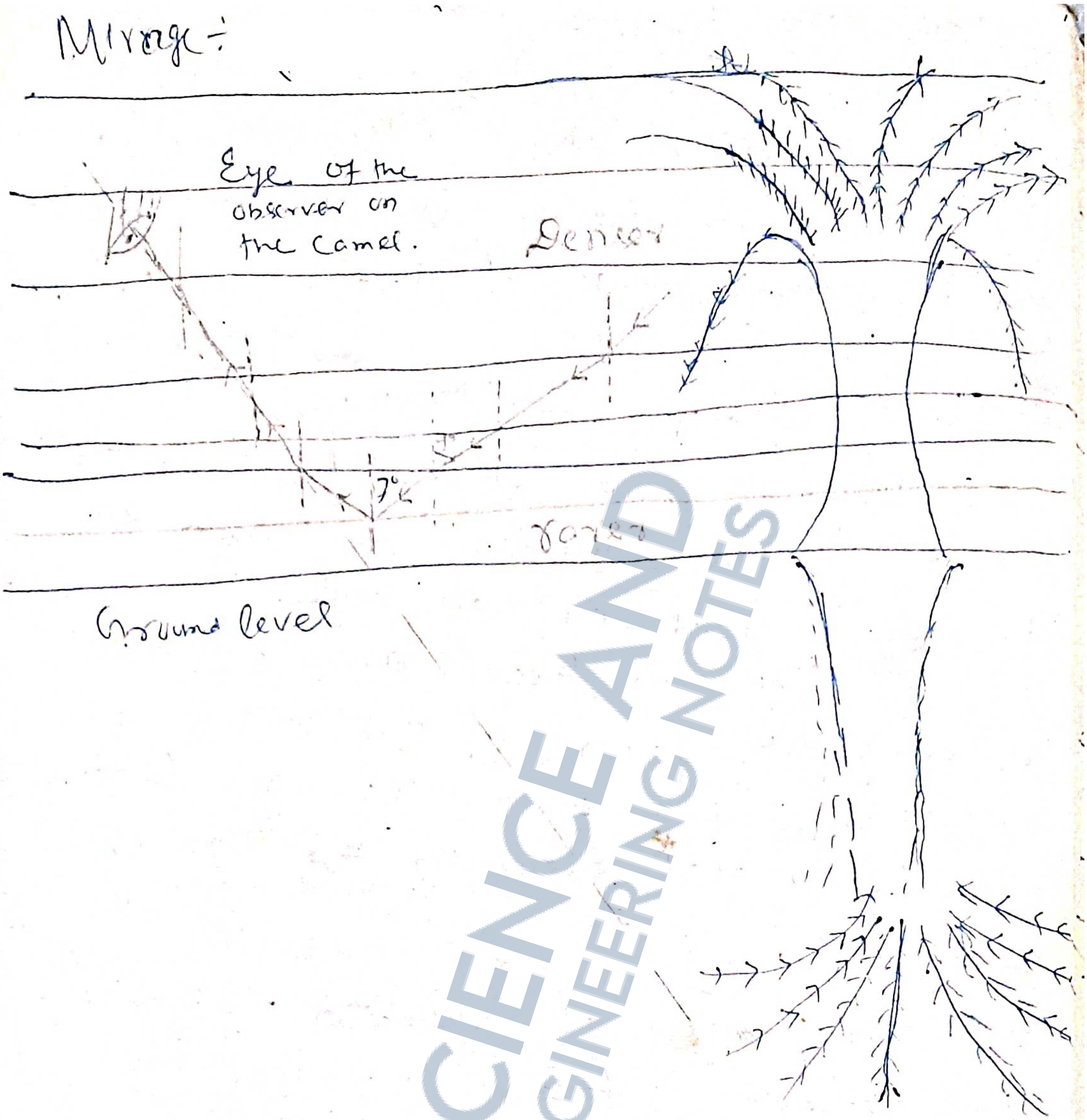
total internal reflection takes place.

In the mirror the back side coating absorbs the light to some extent but here only the glass medium absorbs.

Periscope is device used by submarines & short sighted persons to view the object at a great height.



Mirage :-



~~Mirage~~ :-

It is the optical illusion which is normally encountred in deserts. A thirsty person finds the image of the tree as inverted and thinks that a pond is near by, but after reaching the tree he get disappointed.

This happens because of total internal reflection. Due to extreme heat in deserts, the lower layers of air has density less than the uper layers as shown in the diagram, the ray bends and ultimately suffers total internal reflection. The observer looks straight and finds the virtual inverted image of the tree

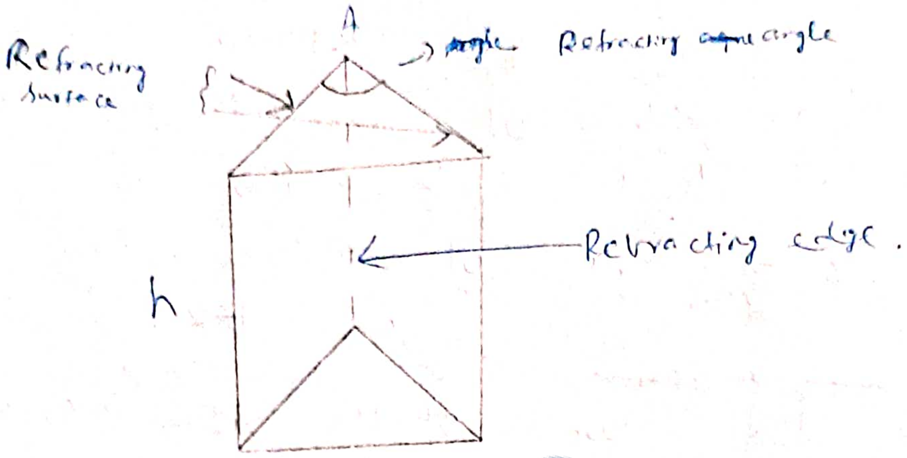
Refraction through a prism.

A prism is a portion of a transparent medium lying between two ~~prism~~ plane faces inclined at an angle.

The two faces are called refracting ~~b~~ faces of the prism and the line along which the two faces meet is called refracting ~~age~~ edge of the prism.

The angle of inclination between the two refracting face is called the refracting angle of the prism.

Any section of the prism made by a plane ⊥ to the edge of the prism is called principal section of the prism. This principal section is triangular in shape. It contains incident ray, normal at the point of incidence, refracted ray, normal at the point where the refracted ~~ray~~ ray strikes the second surface and the emergent ray.



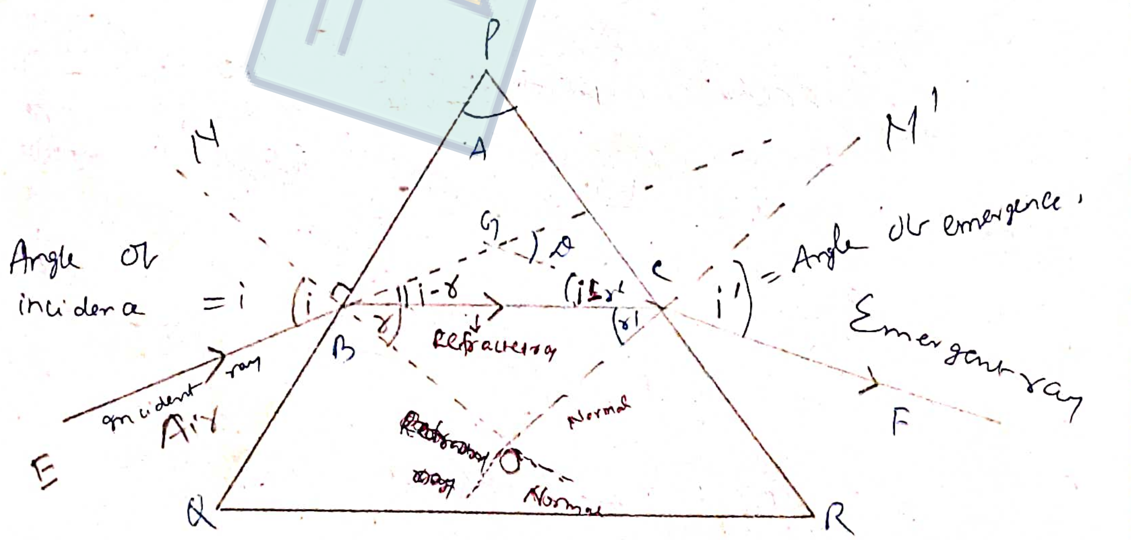
Derivation of the formula $\mu_g = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)}$,

Where A = Angle of the prism.

δ_m = Angle of $\min^m$ deviation.

PQR represents the principal section of a prism having refracting angle A, Angle of deviation (δ) is defined as the angle between the incident ray and the emergent ray.

In the triangle GBC, the side BG has been extended. The exterior angle is equal



EB = Incident ray, CF = Emergent ray

BC = Refracted ray, δ = Angle of deviation.

i = Angle of incidence.

i' = Angle of imergence.

r = Angle of refraction for the first surface PQ

r' = Angle of incidence for the second surface ER.

To the sum of the two interior opposite angles

$$\therefore D = \angle GBC + \angle GCB$$

$$= i - r + i' - r'$$

$$= (i + i') - (r + r') \quad \text{——— (i)}$$

In the triangle BCO sum of the three angle is $180°$.

$$\therefore \quad r + r' + \angle BOC = 180° \quad \text{——— (ii)}$$

In the quadrilateral PBOC, the sum of 4 angle is $360°$, out of which

$\angle PBO = 90° = \angle PCO$

$$\therefore \quad \angle A + \angle BOC = 180° \quad \text{—— (iii)}$$

Comparing the L.H.S of eqns (ii) and (iii), we get

$$A = r + r' \quad \text{——— (4)}$$

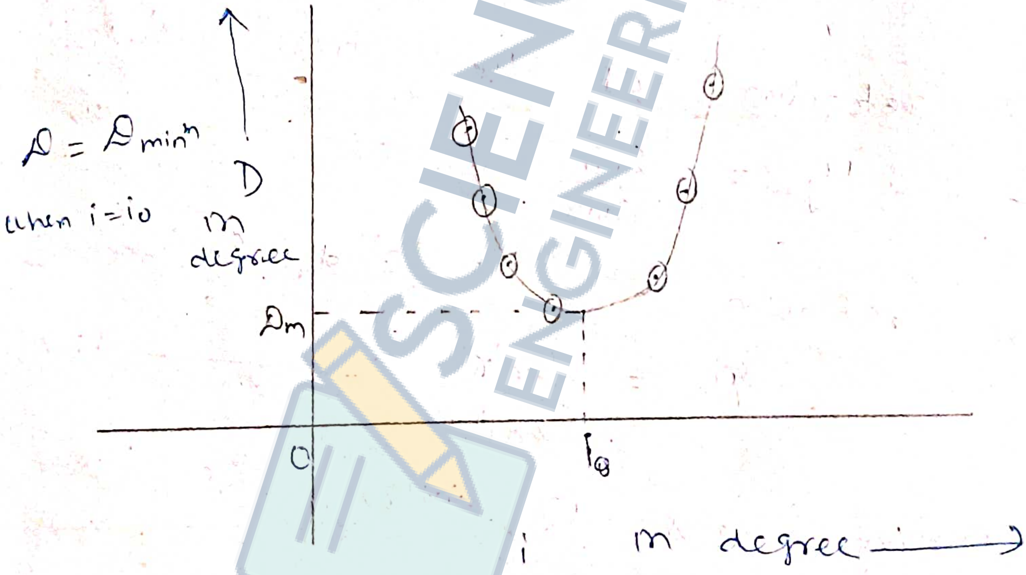
Using eqn (4) in eqn (i), we get

$$D = (i + i') - A \quad \text{——— (5)}$$

For different angles of incidence like 30°, 35°, 40°, 45°, ----- 60°; angles of deviation can be ~~for~~ found out.

A graph can be plotted between i along X-axis and D along Y-axis. The U-shaped curve thus obtained is called $i-D$ curve. From which D_m can be found out.

Angle of deviation is min^m when the angle of incidence is equal to the angle of emergence.

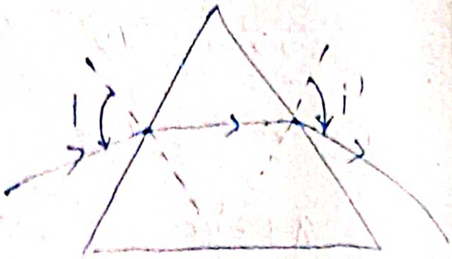


Differenciating both the sides of eq^n (5), w.r.t. to i, we get

$$\frac{dD}{di} = 1 + \frac{di'}{di} - 0$$

$$\Rightarrow 0 = 1 + \frac{di'}{di} \quad (\text{when } D = D_m)$$

$$\Rightarrow \frac{di'}{di} = -1$$



$\Rightarrow di' = -di$

$\Rightarrow i' = -i$

The $-$ve sign appears because i and i' are measured in opposite sense.

Putting this condn in eqn (5), we get

$$\Rightarrow D_m = i + i - A$$

$$D_m = 2i - A$$

$$\Rightarrow 2i = A + D_m$$

$$\Rightarrow i = \frac{A + D_m}{2} \quad \text{———} (6)$$

For refraction at the first surface PQ, ${}^a\mu_g = \frac{\sin i}{\sin r}$ ——— (7)

For refraction at the 2nd surface PR

$${}^g\mu_a = \frac{\sin r'}{\sin i'} \quad \text{———} (8)$$

Multiplying eqns (7) and (8), we get

$$1 = \frac{\sin i}{\sin r} \cdot \frac{\sin r'}{\sin i'}$$

$$\Rightarrow \sin r = \sin r' \quad (\text{when } i = i')$$

$$\Rightarrow r = r' \quad \text{———} (9)$$

Using eqn (9) in eqn (9), we get

$$A = r + r = 2r$$

$\Rightarrow \quad r = \frac{A}{2}$ ——— (10)

Using eqn s (6) and (10) in eqn (7), we get

$$ {}^a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \text{(Proved)} $$

Special Case

① For a thin prism

Since A is small, δ_m will also be small

we can use the approximation $\sin\theta \simeq \theta$ and expressed in radian, provided θ is small

$$ \therefore \; {}^a\mu_g \simeq \frac{\frac{A+\delta_m}{2}}{\frac{A}{2}} = 1 + \frac{\delta_m}{A} $$

$$ \Rightarrow \; {}^a\mu_g - 1 = \frac{\delta_m}{A} $$

$$ \Rightarrow \; \delta_m = ({}^a\mu_g - 1)A $$

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12. Given ${}^a\mu_g = \frac{3}{2}$, ${}^a\mu_w = \frac{4}{3}$

We know the chain rule

$$ {}^a\mu_w \times {}^w\mu_g \times {}^g\mu_a = 1 $$

$$ \Rightarrow \frac{4}{3} \times {}^w\mu_g \times \frac{2}{3} = 1 $$

$$ \Rightarrow {}^w\mu_g = \frac{9}{8} $$

$$\therefore \quad {}^{g}\mu_w = \frac{1}{{}^{w}\mu_g} = \frac{1}{\frac{9}{8}} = \frac{8}{9} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \frac{8}{9} = \frac{\sin 30^\circ}{\sin r} = \frac{1}{2\sin r}$$

$$\Rightarrow \cancel{8}\,16 \sin r = 9$$

$$\Rightarrow \sin r = \frac{9}{16}$$

18.

$$\frac{\text{Real depth}}{\text{Apperant depth}} = \mu = \frac{4}{3}$$

$$\Rightarrow \frac{\text{Real depth}}{\frac{2}{3}} = \frac{4}{3}$$

$$\Rightarrow \frac{3}{2} \cdot \text{Real depth} = \frac{4}{3}$$

$$\Rightarrow \text{Real depth} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

It is actually $\frac{8}{9}$ th part full.

3. $\mu = \sqrt{2}$, for an equilateral prism.

$i = 45^\circ$. Calculate i' and δ, δ_{min}.

Given ~~that ${}^{a}\mu_g = \sqrt{}$~~

For refraction at at first surface PQ, Snell's law gives.

$${}^{a}\mu_g = \frac{\sin i}{\sin r}$$

$$\Rightarrow \sqrt{2} = \frac{\sin 45^\circ}{\sin r} = \frac{1}{\sqrt{2}\sin r}$$

$$\Rightarrow 2 \sin r = 1$$

$\Rightarrow \sin r = \frac{1}{2}$

$\Rightarrow r = 30^\circ$.

But $A = r + r'$ $\Rightarrow r' = A - r = 60^\circ - 30^\circ = 30^\circ$

for the second surface PR

$${}^{g}\mu_a = \frac{\sin r'}{\sin i'}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sin 30^\circ}{\sin i'} = \frac{1}{2\sin i'}$$

$$\Rightarrow 2\sin i' = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow i' = 45^\circ.$$

~~Since $i = i'$~~

$$D = i + i' - A = 45^\circ + 45^\circ - 60 = 30^\circ.$$

Since $i = i'$, thus $D = D_m = 30^\circ$.

4. $\mu = 1.517$ for a prism with $A = 60^\circ$, Find D_m. Ans: $38^\circ 40'$

Ans: We know that

$${}^{a}\mu_g = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow 1.517 = \frac{\sin\left(\frac{60^\circ + D_m}{2}\right)}{\sin 30^\circ} = 2\sin\left(\frac{60^\circ + D_m}{2}\right)$$

$$\Rightarrow \sin\left(\frac{60^\circ + D_m}{2}\right) = \frac{1.517}{2} = 0.7585$$

$$\Rightarrow \frac{30^\circ + D_m}{2} = \sin^{-1}(.7585)$$

$\therefore 30° + \frac{D_m}{2} = 49.3333$

$\Rightarrow \frac{D_m}{2} = \begin{array}{r} 49.3333 \\ 30.0000 \\ \hline 19.3333 \end{array}$

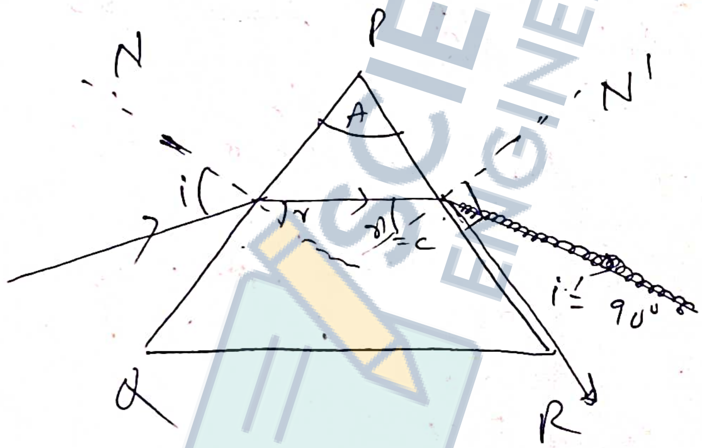
$\Rightarrow D_m = 38°.6666$

$= 38°\ 40'$

$\cancel{60°}\ 2/3$ 5. $\mu = \sqrt{\frac{7}{3}} = 1.5275$

What is the limiting angle of incidence of a ray that will be just transmitted through the equilateral prism.

(Ans = 30°)



We know that refractive index and critical angle are related by the eqn

$\mu = \frac{1}{\sin c}$

$\Rightarrow 1.5275 \cdot \sin c = 1$

$\Rightarrow \sin c = \frac{1}{1.5275} = .65466$

$\Rightarrow c = \sin^{-1}(.65466)$

$= 40^\circ.9^\circ = r'$

$\therefore r + r' = A = 60^\circ$

$\Rightarrow r = 60^\circ - 40^\circ.9 = 19.1^\circ$

Using Snell's law for the surface PQ

$$\frac{\sin i}{\sin r} = \mu = \frac{a}{\mu_g} = 1.5275\ldots$$

$$\Rightarrow \frac{\sin i}{\sin(19.1^\circ)} = \cancel{0.038730}\ 1.5275$$

$\Rightarrow \sin i = (.3273) \times (1.5275)$

$\Rightarrow i = \sin^{-1}(0.4999)$

$= 29.9$

$\simeq 30^\circ$

LENS

A lens can be defined as a transparent refracting medium bounded by two surfaces which may be spherical, cylindrical, one of them may be plane etc.

P.T.O

Classifications

- Convex or Converging
  - Double Convex or Biconvex
  - Plano Convex
  - Concavo Convex
- Concave or Diverging
  - Double Concave or Biconcave
  - Plano-Concave
  - Convexo Concave

Some defn associated with lens

① **Principal axis:-**

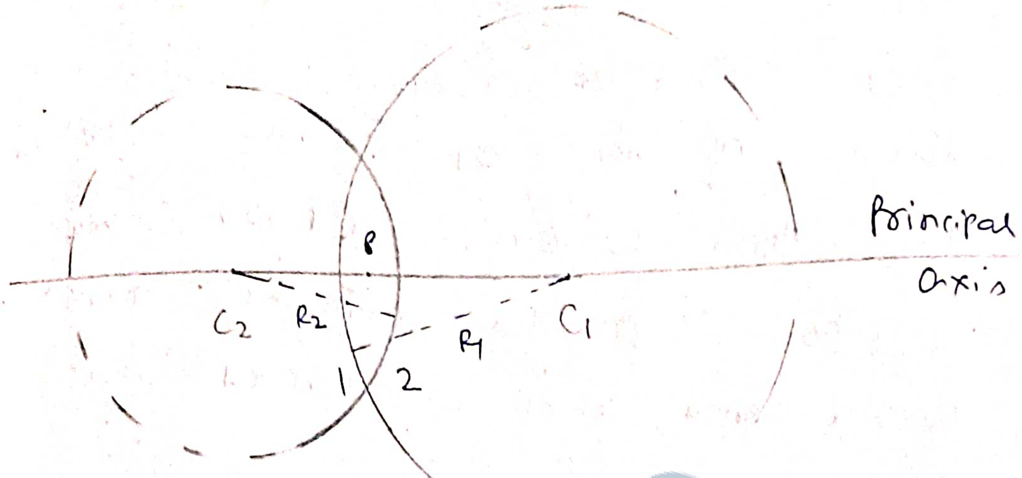
The line joining the two centres of curvature is called principal axis (C_1C_2 extended)

② **Principal section:-**

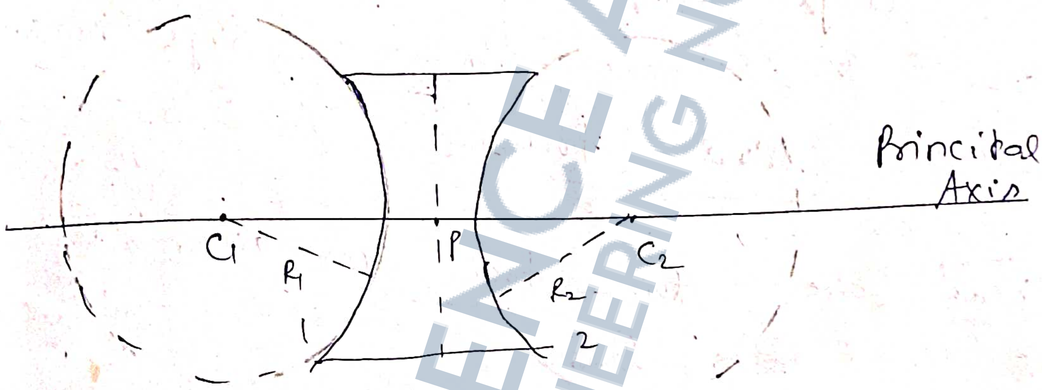
A section of a lense made by a plane passing through the principal axis called principal section.

③ **Optical Centre (P)**

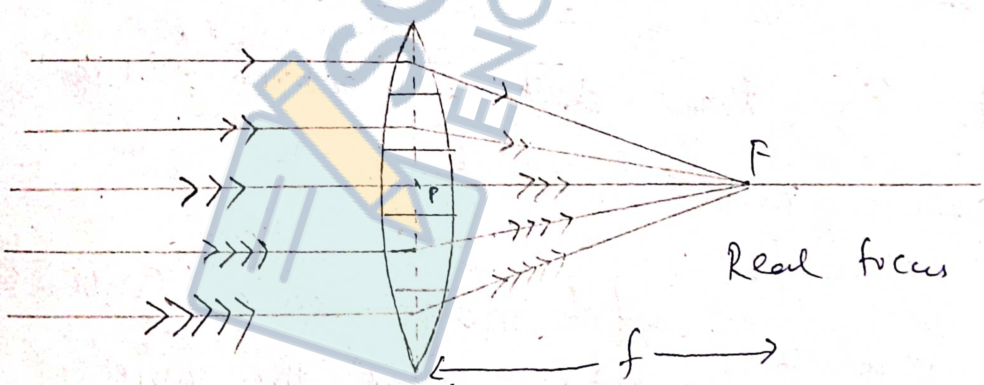
It is a point that lies inside the lens such that any ray passing through it does not deviate



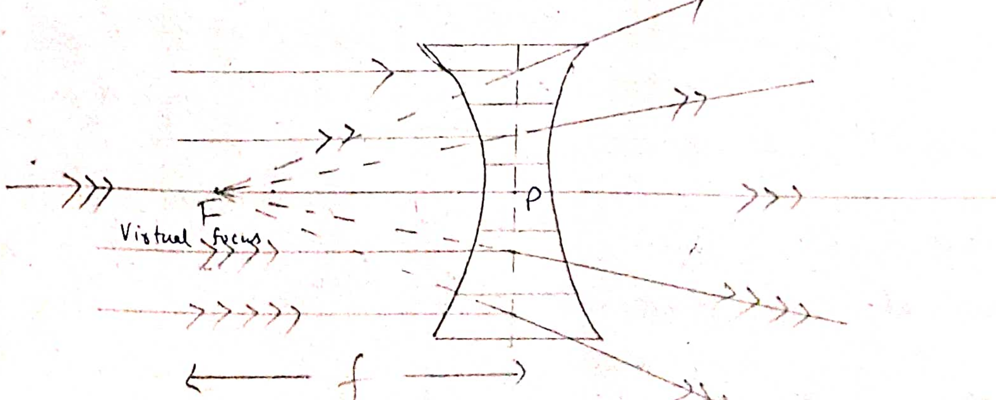
Convex
Lens



Concave lense



Convex lens



Con~~vex~~cave lens

(4) Focus (F)

A lense can be regarded as a combination of large number of prisms. Rays ∥ to the principal axis actually converge (tend to meet at a point) to a point on the principal axis for convex lense which is called real focus.

In case of concave lense, the rays after refraction diverge and appear to come from a single point called virtual focus.

There are two foci on either side of the ~~line~~ lens.

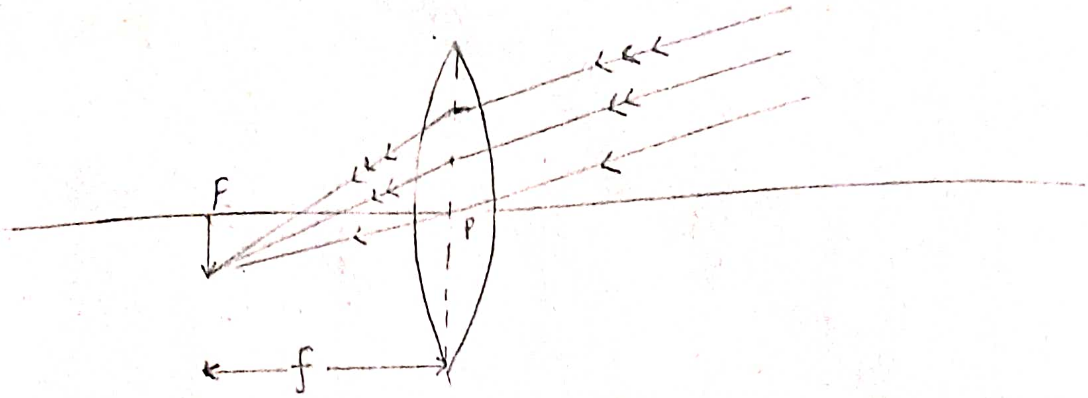
~~Formation~~ (5) Focal length (f)

It is the distance between the optical centre and the focus.

Formation of image in a convex lens for various position of the object

(1) Object at infinity

Image is real, inverted, very much diminished, and formed on the focus of the other side of the lens.



② Object between 2F and infinity

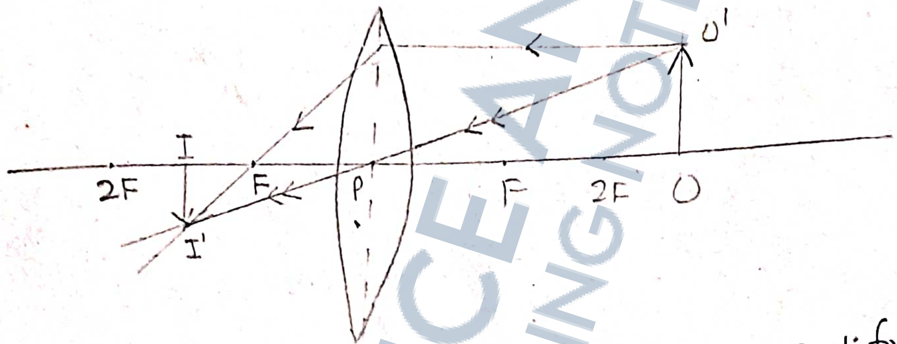


Image is real, inverted, diminished and formed on the other side of the between F and 2F on the other side of the lense.

③ Object at 2F

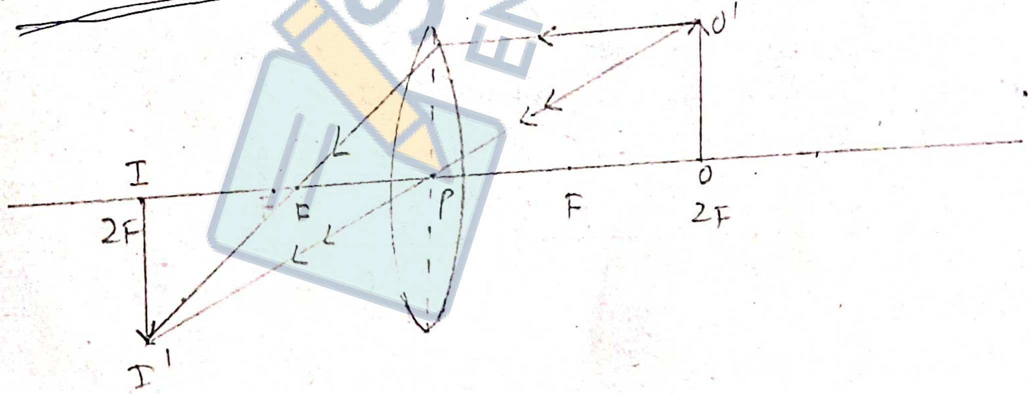
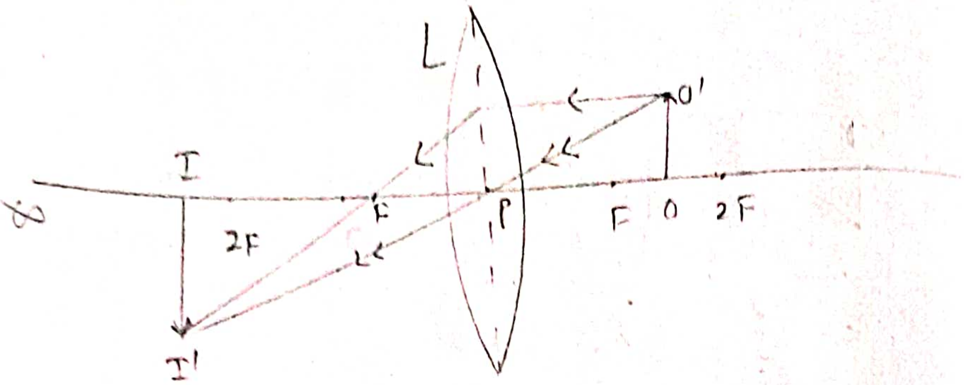


Image is real, inverted, of the same size as the object and formed on the other side of the lens at 2F.

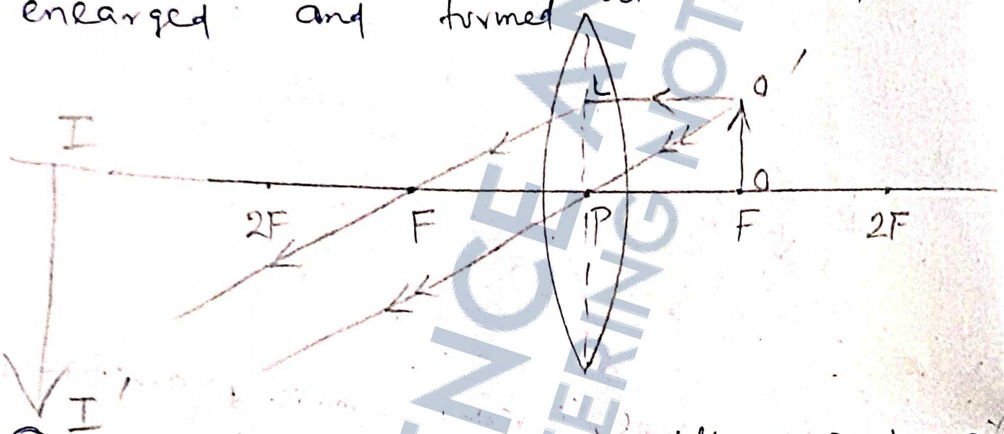
④ Object between F and 2F

Image is real, inverted, enlarged, and formed between 2F and ∞ on the other side of the lense.



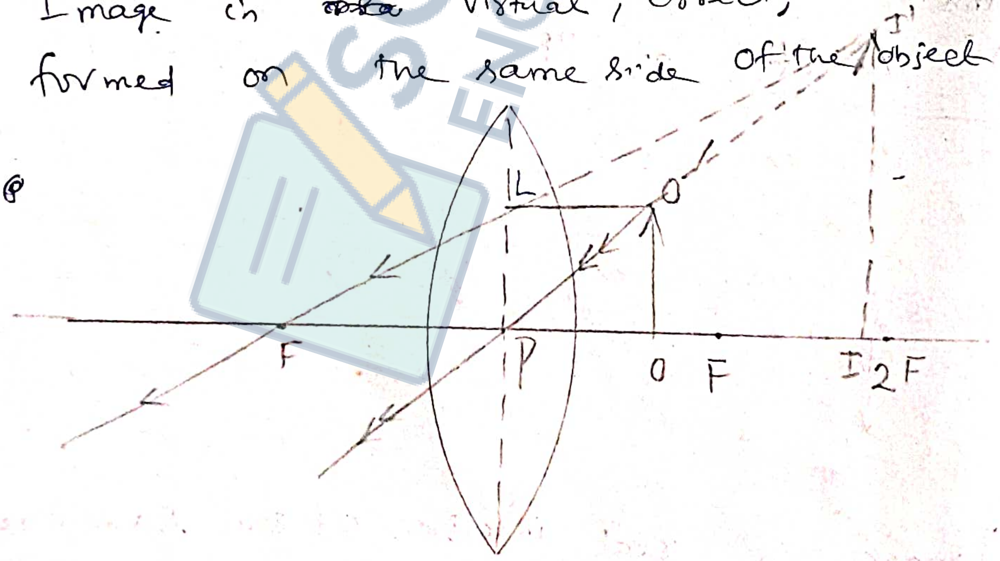
⑤ Object at the focus

The image is real, inverted, very much enlarged and formed at infinity.



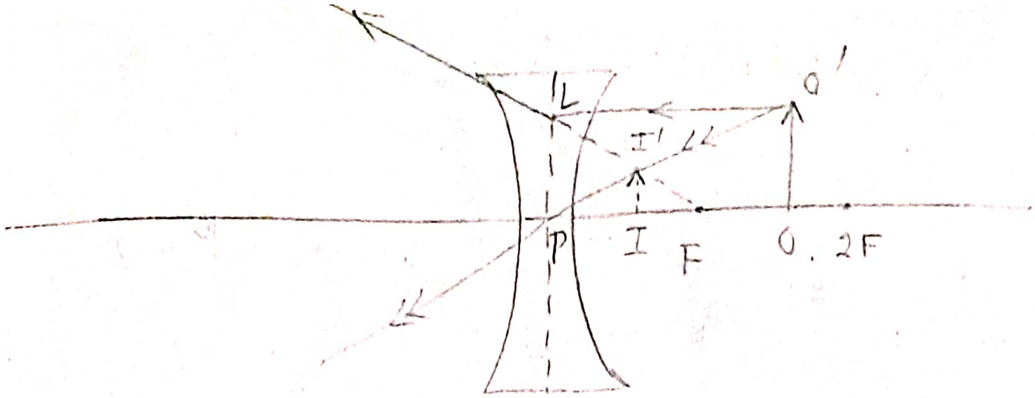
⑥ Object between the optical centre and the focus

Image is ~~real~~ virtual, erect, enlarged and formed on the same side of the object.



⑦ Formation of image in a ~~convex~~ Concave lens

Image is virtual, erect, diminished and formed on the same side of the lens as that of the object. in between



the optical centre and the focus.

Task : ~~Derive of~~ Derivation of lens eqn $\frac{1}{v} \cancel{+} \frac{1}{u} = \frac{1}{f}$ for a convex lense
(a) producing real image
(b) producing virtual image.

and for a concave lense producing virtual image..

Problems on refraction

2. $i = 40^\circ$, $r = 29^\circ$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 40^\circ}{\sin 29} = \frac{.6428}{.4848}$$

1607
~~3214~~
~~2424~~
1212

$= 1.3259$

4. $_a\mu_w = \frac{V_a}{V_w} = \frac{V_a}{(0.8) V_a} = \frac{1}{0.8} = \frac{10}{8}$

$= 1.25$

6. $r = 22^\circ$, $\mu = \frac{4}{3}$

$$\frac{4}{3} = \frac{\sin i}{\sin 22^\circ} \Rightarrow \sin i = \frac{4 \times .3740}{3}$$

$= .4994$

$\Rightarrow i = \sin^{-1}(.4994) = 29.96$

8. $\lambda = 5893 A^0$. , ${}^a\mu_w = 1.33$.

$f = ?$ $\lambda_w = ?$

$V_w = f\lambda \Rightarrow \frac{V_a}{\cancel{\mu}} = f.(5893 \times 10^{-10} m)$

$\Rightarrow \cancel{\frac{3\times 10^8}{1.33 \times 5893 \times 10^{-10}}} = f$ $\cancel{= \frac{30,000 \times 10^{14}}{1.33 \times 5893}}$

$\cancel{\Rightarrow f = \frac{3\times 10^{18}}{1.33 \times 5893}} =$

$\Rightarrow \frac{3\times 10^8 \times 10^{10}}{5893} = f$

$\Rightarrow \frac{30,000 \times 10^{14}}{5893} = f$

$\Rightarrow f = 5.09 \times 10^{14}$ per sec

But ${}^a\mu_w = \frac{V_a}{V_w} \Rightarrow \frac{4}{3} = \frac{3\times 10^8}{V_w}$

$\Rightarrow V_w = \frac{9}{4} \times 10^8$ M/sec.

But $V_w = f. \lambda'$

$\Rightarrow \frac{9 \times 10^8}{4} = 5.09 \times 10^{14} \times \lambda'$

$\cancel{\Rightarrow \lambda' = \frac{9 \times 10^8 \times 10^4}{4 \times 5.09 \times 10^{14}}}$

$\Rightarrow \lambda' = \frac{9 \times 10^8}{4 \times 5.09 \times 10^{14}} 10^6$ $\cancel{= \frac{90}{4 \times 5.09} \times 10^{-3}}$

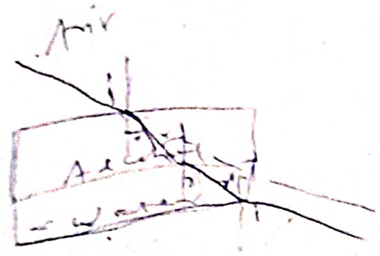
$= 4.42 \times 10^{-7} m$. $\cancel{= 4.42 \times 10^{-3}}$ meter

$= 44.2 \times 10^{-8}$

$= 44.2 A^0$

10. $\mu = 1.354$,

We know the chain rule



$$ {}^{a}\mu_{alcohol} \times {}^{Alcohol}\mu_{w} \times {}^{w}\mu_{a} = 1 $$

$$ \Rightarrow 1.354 \times {}^{al}\mu_{w} \times \frac{1}{1.33} = 1 $$

$$ \Rightarrow {}^{alcohol}\mu_{w} = \frac{1.33}{1.354} = \cancel{1.1078} $$

$$ \frac{\sin i}{\sin r} = {}^{a}\mu_{alcohol} $$

$$ \Rightarrow \frac{\sin 30^\circ}{\sin r} = 1.354 $$

$$ \Rightarrow \frac{1}{2\sin r} = 1.354 $$

$$ \Rightarrow 2 \cdot \sin r \cdot 1.354 = 1 $$

$$ \Rightarrow \sin r = \frac{1}{2.708} = .3692762 $$

$$ \Rightarrow r = \sin^{-1}(\quad) = 21.67^\circ. $$

$$ \frac{\sin r}{\sin r'} = {}^{Alcohol}\mu_{water} = \cancel{1.1078} = \frac{1.33}{1.354} $$

$$ \Rightarrow \frac{\sin 21^\circ.67^\circ}{\sin r'} = \cancel{1.1078} = \frac{1.33}{1.354} $$

$$ \Rightarrow \frac{\sin .3692762}{\sin r'} = 1.1078 = \frac{1.33 \cdot 4}{1.354 \; 3 \times 1.354} $$

$$ \Rightarrow \sin r' = \frac{\sin 21.7 \times 1.354}{1.33} $$

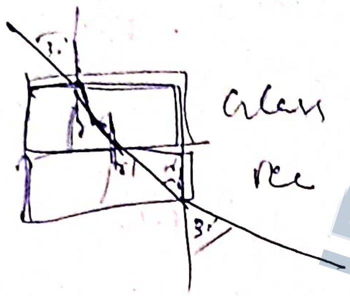
$$ \Rightarrow r' = \sin^{-1}(\quad) = 22.1^\circ $$

Angle of incidence = 30° : 0

Angle of refraction at water surface = 22.7°

Deviation .. 7.3°

12.



We know the chain formula

$${}^{a}\mu_{g} \times {}^{g}\mu_{ice} \times {}^{ice}\mu_{air} = 1$$

$$\Rightarrow \frac{3}{2} \times {}^{g}\mu_{ice} \times \frac{1}{\frac{4}{3}} = 1$$

$$\Rightarrow {}^{g}\mu_{ice} = 1 \times \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

$$\frac{\sin r}{\sin i} = \frac{8}{9}$$

$\Rightarrow \sin r$

14. Apparent depth = 3 m depth.

$$\mu = \frac{4}{3}$$

$$\therefore \frac{R.d}{A.d} = \frac{4}{3} \Rightarrow R.d = 3 \times \frac{4}{3} = 4 \text{ meter}$$

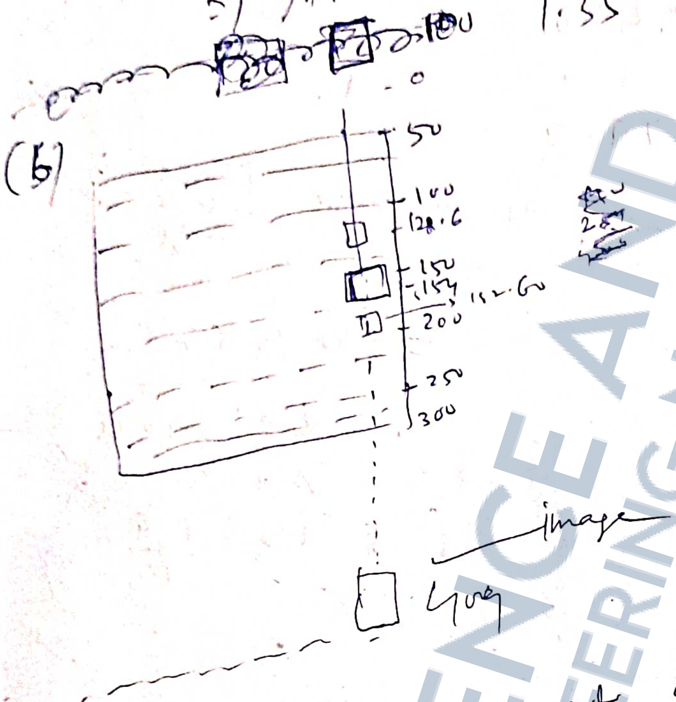
Yes it is deep enough, 4 meter height water is in the pool

16. $^a\mu_{water} = 1.35$

Real depth = 308 cm

$\frac{R.d}{A.d} = 1.35 \Rightarrow \frac{308}{A d} = 1.35$

$\Rightarrow A.d = \frac{308}{1.35} = 228.1$ cm



154.00
25.40
28.60

154.00
28.60
182.60

The mirror is at a height = 154 cm.

It appear to be at height = $\frac{154 \text{ cm}}{1.35}$

(Apparent height) = 114.07 cm.

Object is suspend at 25.4 cm above the mirror, so image will form below the mirror at 25.4 cm

But appears to $\left(\frac{25.4}{1.35}\right)$ 18.81 cm from the mirror.

So image will appear to at a distance

114.07
18.81
132.88 cm

from the observer

Or When mirror is at a ~~height~~ depth 154 cm and object is 25.4 cm above him, so image is 25.4 cm before the mirror so at a height

$$\begin{array}{r} 154.0 \\ 25.4 \\ \hline 179.4 \text{ cm.} \end{array}$$

But it appears at a height $\frac{179.4}{1.35}$

$= 132.88$ cm.

$\left(\because \frac{R.d}{A.d} = 1.35\right)$

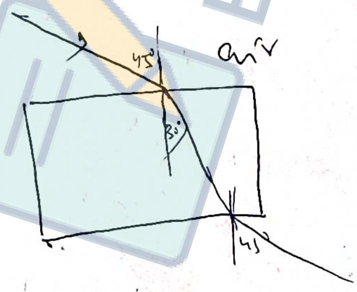
$\left(\text{So } A.d = \frac{R.d}{1.35}\right)$

18. $A.d = \frac{2}{3}$, $\mu = \frac{4}{3}$

$R.d = A.d \times \frac{4}{3} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

20. Sodium vapor

1.6 km = (mile



$${}^{a}\mu_{e} = \frac{\sin 45°}{\sin 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414$$

Speed, $\cancel{\frac{\mu_a}{\mu_e}} = \frac{V_a}{V_e} = 1.414$

$\Rightarrow V_e = \frac{V_a}{1.414} = \frac{3 \times 10^{8} \text{ m/sec}}{1.414}$

$$= \frac{3 \times 10^5}{1.609 \times 1.414} \text{ mile/sec.}$$

$$= 1.318 \times 10^5 \text{ mile/sec}$$

$$= \cancel{132000000}$$

$$= 131,800 \text{ mile/sec.}$$

$$ {}^{a}\mu_{g} = \frac{1}{\sin c}$$

$$\Rightarrow 1.414 = \frac{1}{\sin c}$$

$$\Rightarrow \sin c = \frac{1}{1.414} = \left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

22. $i = 45^\circ$, $\mu = 1.52$

$$\frac{\sin i}{\sin r} = 1.52$$

$$\Rightarrow \frac{\sin 45}{1.52} = \sin r$$

$$= \sin r$$

$$\Rightarrow \frac{1}{\sqrt{2}\cdot(1.52)}$$

$$\Rightarrow r = \sin^{-1}(.4652) = 27.7$$

Angle of deviation $= i - r$

$$= 45^\circ - 0 - 27.7 = 17.3^\circ$$

24. $A = 60^\circ$, $\mu = 1.64$

We know the formula that

$$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow 1.64 = \frac{\sin\left(\cancel{60}\ 30^\circ + \frac{D_m}{2}\right)}{\frac{1}{2}}$$

$$\Rightarrow .82 = \sin\left(30^\circ + \frac{D_m}{2}\right)$$

$$\Rightarrow 30^\circ + \frac{D_m}{2} = \sin^{-1}(.82) = 55.08$$

$$\Rightarrow \frac{D_m}{2} = 55.08 - 30^\circ.00 = 25.08$$

$$\Rightarrow D_m = 50.16^\circ$$

26. $A = 60^\circ$, $\left(30^\circ + \frac{D_m}{2}\right) = \sin^{-1}\left(\frac{1.643}{2}\right)$

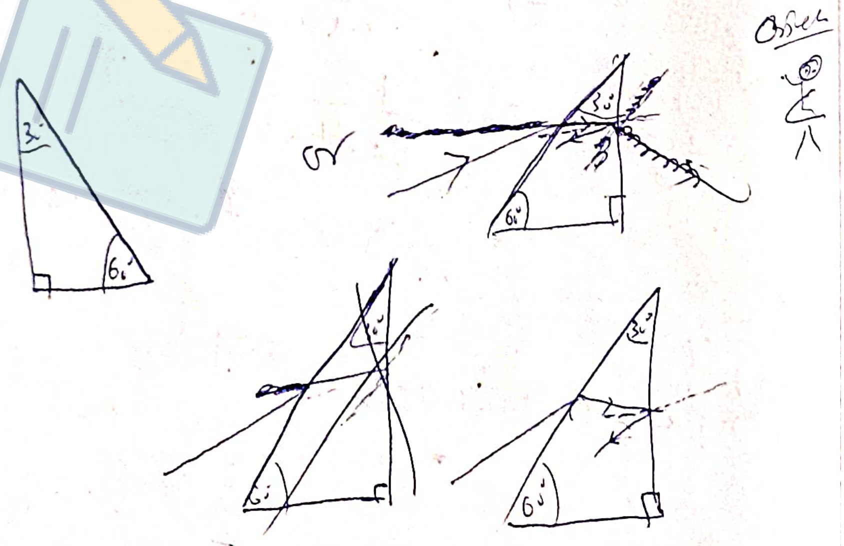
$$= \sin^{-1}(.8215)$$

$$\Rightarrow \frac{D_m}{2} = 55.23 - 30^\circ.00 = 25.23$$

$.665 / 1.33$

$$\Rightarrow D_m = 50.46$$

28.



30.

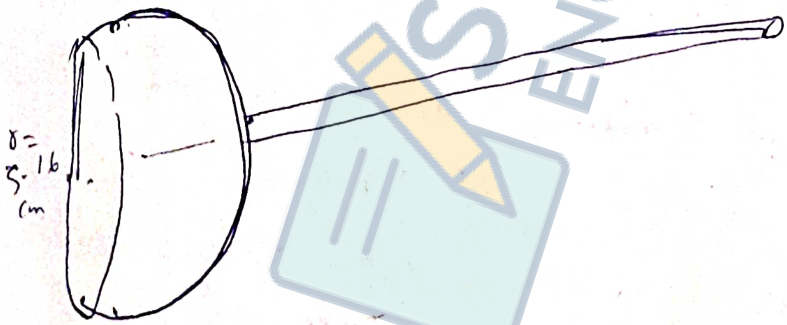
$$\mu = 1.33 = 2 \sin\left(30^\circ + \frac{\delta_m}{2}\right)$$

$$\Rightarrow 30^\circ + \frac{\delta_m}{2} = \sin^{-1}(.665)$$

$$\Rightarrow \frac{\delta_m}{2} = 41.68 - 30^\circ = 11.68$$

$$\Rightarrow \delta_m = 23.36$$

32.



Derive the lens formula $\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$ for a Concave lense producing

~~(i) Real image,~~

This formula is a relation connecting focal length of the lens with the distance of objects and images.

Let OO' be an object situated on the principal axis of lens. Consider two rays O'M and O'C. First ray goes parallel to the principal axis while second goes through the optical centre. If the object is situated between f and 2F of a convex lense, a real image II' is formed as shown in the figure. (i)

If the object is placed between P and F of a convex lense a virtual image II' is formed, as shown in fig (ii)

Sign convention followed is called new convention of signs.

(i) All measurements are to be made from the optical centre of the lense.

(ii) All real distances are to be taken as +ve and all ~~real~~ virtual distances are to be taken as -ve

Then f is +ve for convex lense and f is -ve for concave lens.

Fig (i)

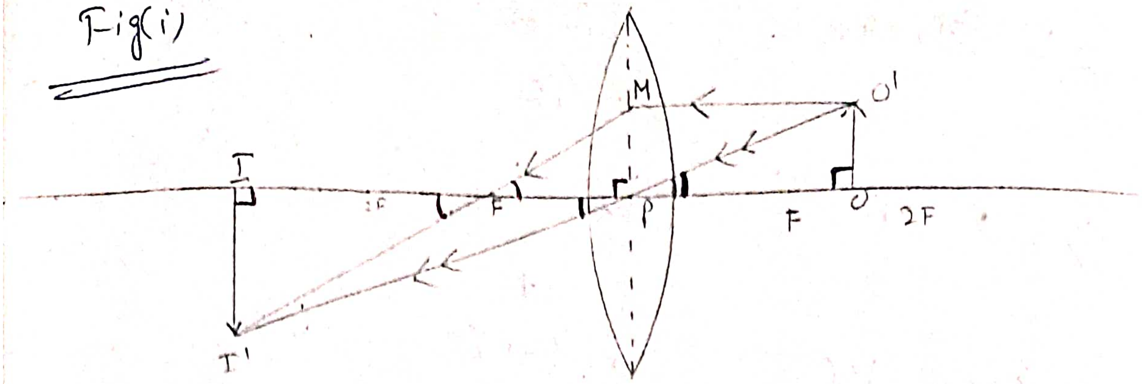
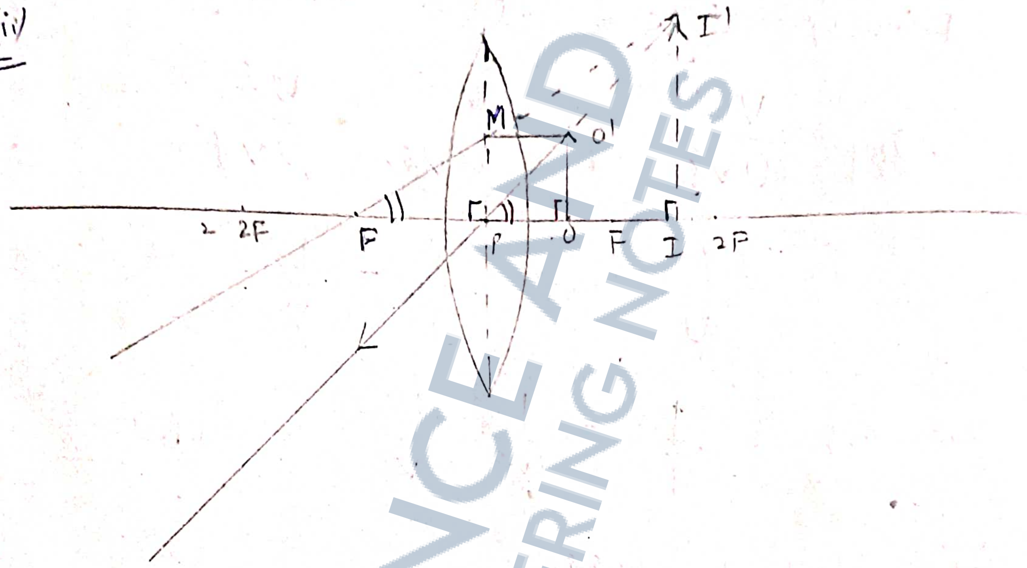


Fig (ii)



(i) Real image

In fig (i) $\triangle$ Triangles $OO'P$ and $II'P$ are similar

$$\therefore \frac{II'}{OO'} = \frac{IP}{OP} \text{ ——— (i)}$$

The $\triangle$ triangles MPF and $II'F$ are also similar

$$\frac{II'}{MP} = \frac{IF}{PF}$$

or

$$\frac{II'}{OO'} = \frac{IF}{PF} \text{ ——— (ii)}$$

Equating L.H.S of eqn (i), (ii), we get

$$\frac{IP}{OP} = \frac{IF}{PF}$$

$$\Rightarrow \frac{IP}{OP} = \frac{IP - PF}{PF}$$

$$\Rightarrow \frac{v}{u} = \frac{v - f}{f}$$

$$\Rightarrow vf = vu - uf$$

Dividing ~~...~~ uvf through out the eqⁿ, we get

$$\Rightarrow \frac{vf}{uvf} = \frac{vu}{uvf} - \frac{uf}{uvf}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \text{(proved)}$$

(ii) Virtual image

In fig (ii) △ triangles $II'P$ and $OO'P$ are similar.

$$\therefore \frac{II'}{OO'} = \frac{PI}{PO} \quad \text{—— (i)}$$

The △ triangles $II'F$ and MPF are similar.

$$\frac{II'}{MP} = \frac{IF}{PF}$$

$$\text{or } \frac{II'}{OO'} = \frac{IF}{PF} \quad \text{—— (ii)}$$

Equating the L.H.S of eqⁿ (i) and (ii), we get

$$\therefore \frac{PI}{PO} = \frac{IF}{PF}$$

$$\Rightarrow \frac{-v}{u} = \frac{IP - PF}{f} = \frac{-v + f}{f}$$

$$\Rightarrow -vf = -uv + uf$$

$\Rightarrow$ Dividing uvf through out the eqn, we get.

$$\Rightarrow -\frac{1}{u} = -\frac{1}{f} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \text{(proved)}$$

(b) For a concave mirror producing virtual image

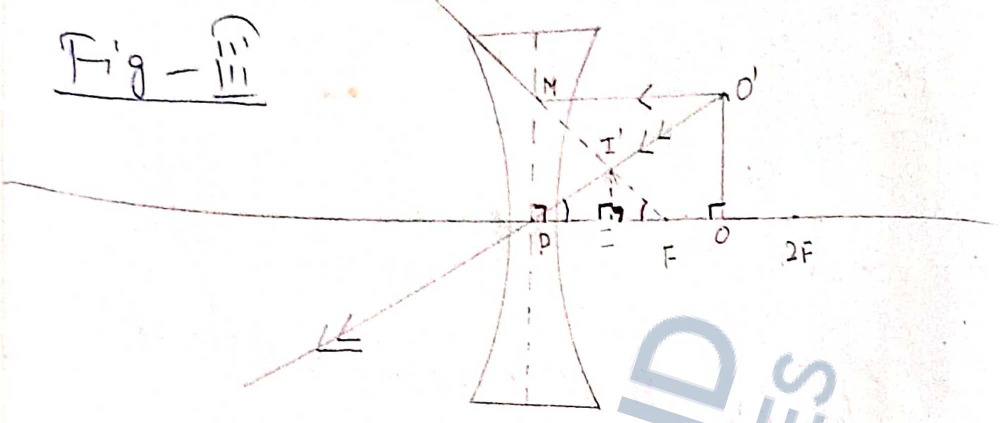
Let OO' be an object situated on the principal axis of concave lens. consider two rays O'M an O'C. First ray goes parallel to the principal axis while second goes through the optical centre. Image is virtual, erect, diminished and formed on the same side of the lens as that of object as shown in fig (iii)

Sign convention followed is called new convention of signs.

(i) All measurement are to be made from the optical centre of the lens.

(ii) All real distances are to be taken as +ve and all virtual distances are to be taken as -ve. So f is -ve for ~~convex~~ concave lens.

Fig - iii



In fig (iii) the right angled triangles $II'P$ and $OO'P$ are similar.

$$\therefore \quad \frac{OO'}{II'} = \frac{PO}{PI} \quad \text{——(i)}$$

The $\triangle MPF$ and $\triangle II'F$ are similar:

$$\frac{MP}{II'} = \frac{PF}{IF} \quad \text{——(ii)}$$

$$\text{or} \quad \frac{OO'}{II'} = \frac{PF}{IF} \quad \text{——(ii)}$$

Equating the L.H.S of eq^n (i) and (ii), we get

$$\frac{PO}{PI} = \frac{PF}{IF}$$

$$\Rightarrow \frac{u}{-v} = \frac{-f}{PF - PI} = \frac{-f}{-f-(-v)} = \frac{-f}{-f+v}$$

$$\Rightarrow -uf + uv = vf$$

Dividing uvf through out the eq^n, we get

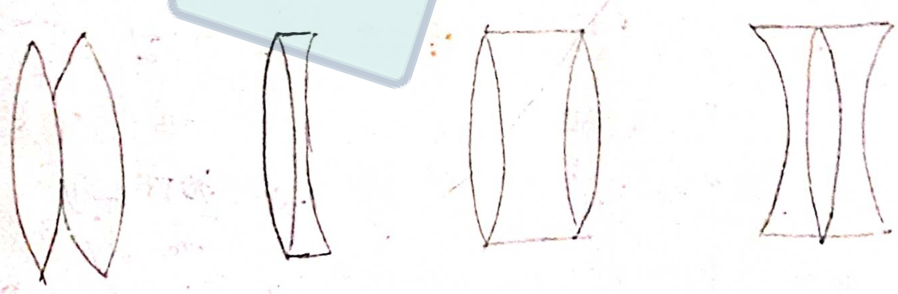
$$\Rightarrow -\frac{1}{v} + \frac{1}{f} = \frac{1}{u}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Power of a lense.

It is expressed by dioptre and given by the reciprocal of the focal length expressed in meter or 100/f when f is expressed in cm.

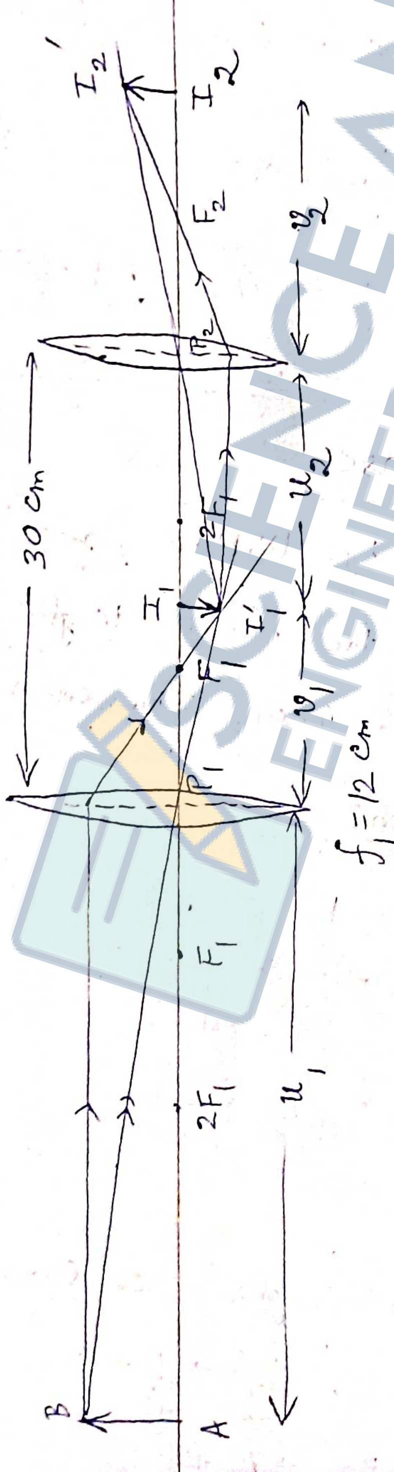
Thin lenses in contact



Equivalent focal length of combination (f) is given by $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \cdots$

Hence $P = P_1 + P_2 + P_3 + \cdots$

10.



For the first lens

$f_1 = 12$ cm

$u_1 = 60$ cm.

$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$

$\Rightarrow \frac{1}{12} = \frac{1}{60} + \frac{1}{v_1}$

$\Rightarrow \frac{1}{v_1} = \frac{1}{12} - \frac{1}{60}$

$= \frac{5-1}{60} = \frac{4}{60}$

$\Rightarrow v_1 = \frac{60}{4} = 15$ cm.

Magnification $= \frac{15}{60} = \frac{1}{4}$

$= \frac{I_1 I_1'}{AB} = \frac{1}{4}$

$\Rightarrow \frac{I_1 I_1'}{10} = \frac{1}{4}$

$\Rightarrow I_1 I_1' = \frac{10}{4} = \frac{5}{2}$

For the second lense

$u_2 = 30 - v_1$, $f_2 = 10$ cm.

$= 30 - 15$

$= 15$ cm.

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{15} + \frac{1}{v_2}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{10} - \frac{1}{15} = \frac{15-10}{150} = \frac{\cancel{5}}{\cancel{150}} = \frac{1}{30}$$

$\Rightarrow v_2 = 30$ cm

magnification $= \dfrac{v_2}{u_2} = \dfrac{30}{15} = 2$.

(because $I_1 I_1'$ is object for 2nd lense)

net magnification $= \dfrac{1}{4} \cdot 2 = \dfrac{1}{2}$.

$= m_1 \cdot m_2$

$\therefore$ The image is erect, real and 30 cm from the 2nd lense and net magnification $\frac{1}{2}$

Or Magnification $= \dfrac{\text{Final image}}{\text{Initial object}} = \dfrac{5}{10} = \dfrac{1}{2}$

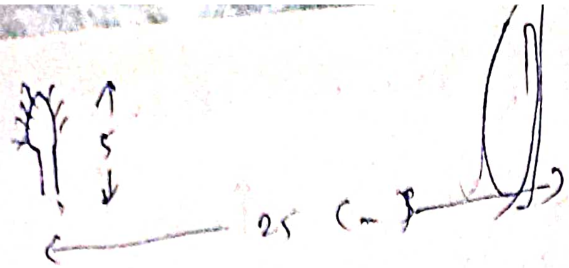
(because $\dfrac{v_2}{u_2} = 2$

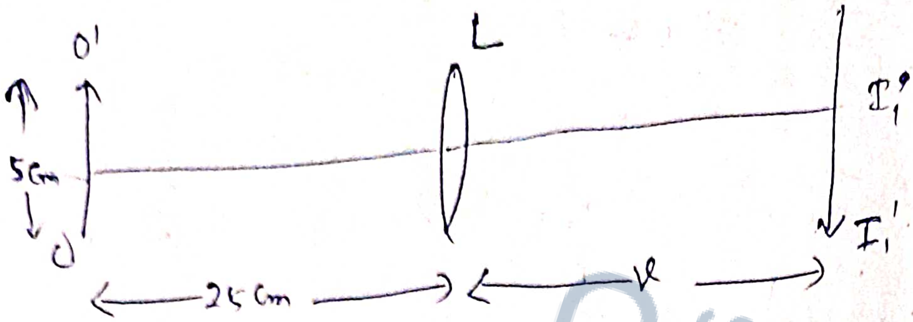
$I_1 I_1' = \frac{5}{2}$,

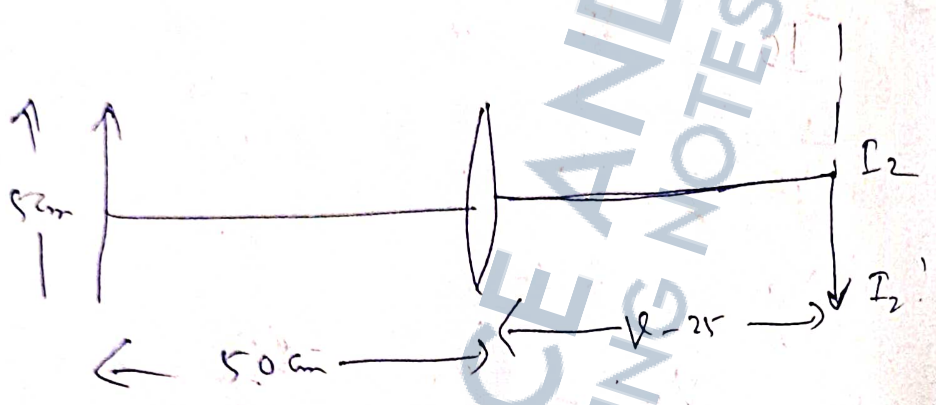
magnification $= 2$

$\therefore I_2 I_2' = 5$)

19.







For the first figure $u = 25$
$v = ?$
$f = ?$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{25} + \frac{1}{v} \text{ ——— (i)}$$

for the second figure

$$\frac{1}{f} = \frac{1}{50} + \frac{1}{v-25} \text{ ——— (ii)}$$

Equating L.H.S, we get

$$\frac{1}{25} + \frac{1}{v} = \frac{1}{50} + \frac{1}{v-25}$$

$$\Rightarrow -\frac{1}{v} + \frac{1}{v-25} = \frac{1}{25} - \frac{1}{50} = \frac{2-1}{50} = \frac{1}{50}$$

$$\Rightarrow \frac{\cancel{-v} + 25 + \cancel{v}}{(v-25)v} = \frac{1}{50}$$

$\Rightarrow v^2 - 25v - \cancel{500v}\ 1250 = 0$

$\Rightarrow v = \dfrac{25 \pm \sqrt{625 - 4.(1).(-1250)}}{2.(1)}$

$= \dfrac{25 \pm \sqrt{625 + 5000}}{2}$

$= \dfrac{25 \pm \sqrt{5625}}{2} = \dfrac{25 \pm 75}{2}$

$= 50, -25$

The image is formed on screen, so it is real so $v \neq -25$.

$\therefore v = 50$ cm.

$\dfrac{1}{f} = \dfrac{1}{25} + \dfrac{1}{50} = \dfrac{2+1}{50} = \dfrac{3}{50}$.

$\Rightarrow f = \dfrac{50}{3}$ cm

~~Ob~~
Distance between object and screen

$25 + v = 25 + 50 = 75$ cm

for the first case $u = 25$; $v = 50$ cm.

magnification $= \dfrac{50}{25} = 2$.

$\therefore \dfrac{II'}{OO'} = 2$

$\Rightarrow \dfrac{II'}{5} = 2$

$\Rightarrow II' = 10$ cm.

for the second Case

$u = 50$

$v = v - 25$
$= 50 - 25 = 25$

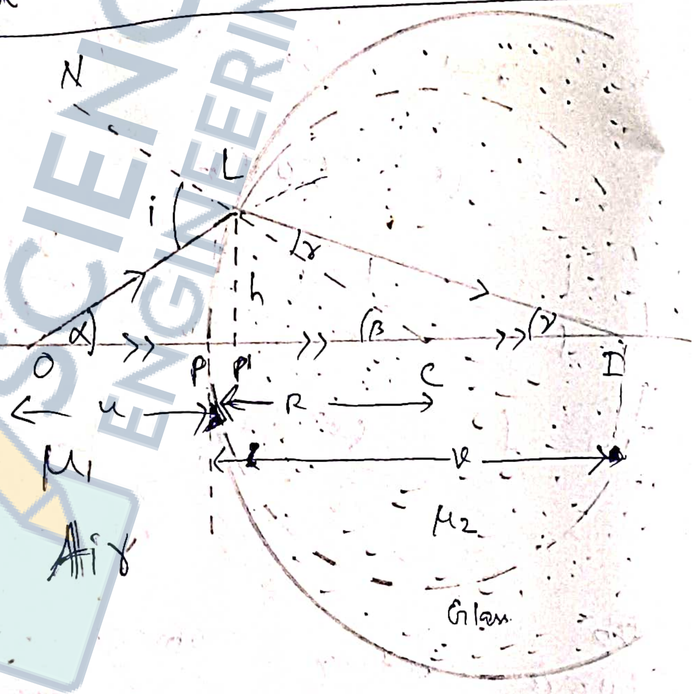
$$m = \frac{v}{u} = \frac{25}{50} = \frac{1}{2}$$

$$\therefore \frac{\text{Image size}}{\text{object size}} = \frac{1}{2}$$

$$\Rightarrow \frac{\text{Image}}{5\text{ cm}} = \frac{1}{2}$$

$\Rightarrow$ Image size $= 2.5$ cm

Refraction at a spherical surface



Lets have two media separated by a spherical surface. O is a point object situated on the principal axis which forms a real image I on the principal axis.

$\mu_1 \equiv$ Absolute refractive index of the first medium or incident medium

μ_2 = Absolute refractive index of the second medium (Refractive medium)

In the triangle $\angle OC$, the side CL has been extended upto N.

The exterior angle = ~~...~~ Sum of the two interior angles

$$\therefore \quad i = \alpha + \beta \qquad \text{——— (i)}$$

In the triangle LCR, The side IC has been extended.

$$\therefore \beta = \gamma + \gamma'$$

$$\Rightarrow \gamma = \beta - \gamma' \qquad \text{——— (ii)}$$

From the Δs LOP', LP'C and LP'I, we get

where Δ = P'P

$$\left.\begin{aligned} \tan\alpha &= \frac{h}{u+\Delta} \\ \tan\beta &= \frac{h}{R-\Delta} \\ \tan\gamma' &= \frac{h}{v-\Delta} \end{aligned}\right\} \qquad \text{——— (iii)}$$

If we will consider those rays that are nearer to the principal axis, then α, β, γ will be very small

$\therefore$ Δ ~~at~~ will tend to zero. We can use the approximations $\tan\theta \simeq \theta$

(when θ is ~~radian~~ expressed in radian)

Then eqn (iii) becomes

$$\alpha = \frac{h}{u}, \quad \beta = \frac{h}{R}, \quad \gamma = \frac{h}{v} \quad \text{---(iv)}$$

Using eqn (4) in eqns (i) and (ii), we get

$$i = \frac{h}{u} + \frac{h}{R} = h\left(\frac{1}{u} + \frac{1}{R}\right) \quad \text{---(v)}$$

$$r = \frac{h}{R} - \frac{h}{v} = h\left(\frac{1}{R} - \frac{1}{v}\right) \quad \text{---(vi)}$$

Snell's law on Refraction is

$$\frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \mu_1 \sin i = \mu_2 \sin r \quad \text{---(vii)}$$

When α, β, γ are small angles expressed in radian,

i and r will also become small

Then we can use approximation like

$\therefore \sin i \simeq i$ and $\sin r \simeq r$

Using this approximations in eqn (7), we get

$$\Rightarrow \mu_1 i = \mu_2 r \quad \text{---(viii)}$$

Using eqns (5) and (6) in eqn (8), we get

$$\Rightarrow \mu_1 \cdot \cancel{h}\left(\frac{1}{u} + \frac{1}{R}\right) = \mu_2 \cdot \cancel{h} \cdot \left(\frac{1}{R} - \frac{1}{v}\right)$$

$$\Rightarrow \frac{\mu_1}{u} + \frac{\mu_1}{R} = \frac{\mu_2}{R} - \frac{\mu_2}{v}$$

$$\Rightarrow \boxed{\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Problem

1. A ray of light goes from air to a median of refractive index μ. If i and r are the angles of incidence and refraction respectively, prove that the deviation δ suffered by the refracted ray is given by $\tan\frac{\delta}{2} = \frac{\mu-1}{\mu+1}\tan\left(\frac{i+r}{2}\right)$

Hint: $\mu = \frac{\sin i}{\sin r}$, ~~Combin~~ Apply Componedo and dividendo and put $\delta = i - r$. (See the before figure)

Answer

We know that

$$\frac{\mu}{1} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \frac{\mu-1}{\mu+1} = \frac{\sin i - \sin r}{\sin i + \sin r} \quad \text{(by Componedo and dividendo)}$$

$$\Rightarrow \frac{\mu-1}{\mu+1} = \frac{\cancel{2}\cos\left(\frac{i+r}{2}\right)\sin\left(\frac{i-r}{2}\right)}{\cancel{2}\cdot\sin\left(\frac{i+r}{2}\right)\cdot\cos\left(\frac{i-r}{2}\right)}$$

$$\Rightarrow \frac{\mu-1}{\mu+1} = \frac{1}{\tan\left(\frac{i+r}{2}\right)}\cdot\tan\left(\frac{i-r}{2}\right)$$

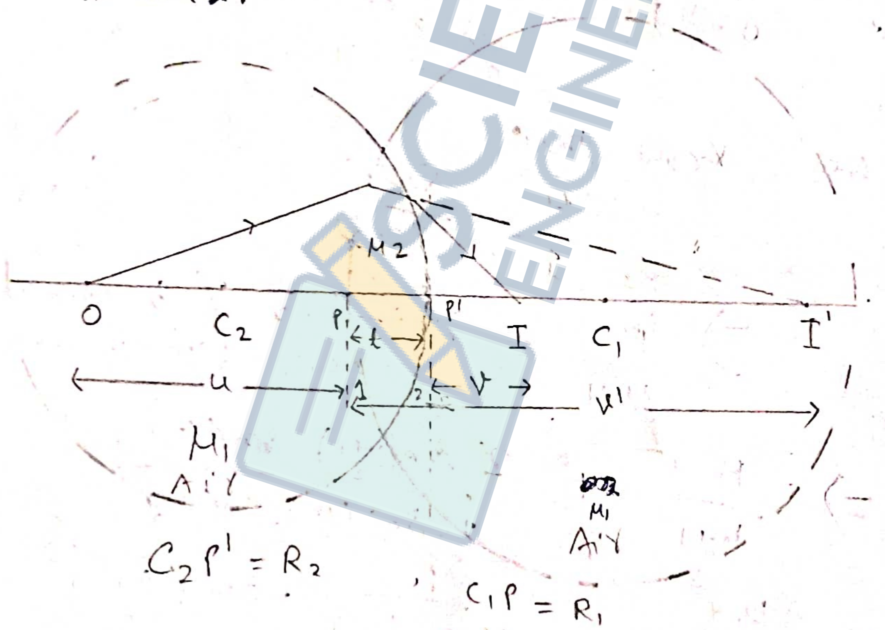
$$\Rightarrow \frac{\mu-1}{\mu+1} = \frac{1}{\tan\left(\frac{i+r}{2}\right)}\cdot\tan\left(\frac{\delta}{2}\right)$$

$$\Rightarrow \tan\frac{\delta}{2} = \frac{\mu-1}{\mu+1}\cdot\tan\left(\frac{i+r}{2}\right)$$

(proved)

Derivation of the thin lens equation: (Lens maker formula) $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Let us consider a point object on the principal axis which form a point image at I after being refracted at the two surfaces 1 and 2:



We can treat this case separately for the refraction at surface 1 and refraction at surface 2. For refraction at surface 1, a real image can be imagined to have been formed in glass at I'.

$$\therefore \frac{\mu_1}{u} + \frac{\mu_2}{v'} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{———(i)}$$

Now this image I' in glass can be regarded as the virtual object for the 2nd surface forming a real image at I in air.

$$\frac{\mu_2}{-(v'-t)} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R_2} \quad \text{———(ii)}$$

For a thin lense $t \to 0$ so that eqn (ii) becomes

$$\frac{\mu_2}{-v'} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R_2} \quad \text{———(iii)}$$

Adding eqn's (i) and (iii), we get

$$\frac{\mu_1}{u} + \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_1 - \mu_2}{R_2}$$

$$\Rightarrow \mu_1\left(\frac{1}{u} + \frac{1}{v}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the object be at infinity, then $u \to \infty$ and $v \to f$, so that the above eqn becomes

$$\frac{1}{\infty} + \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

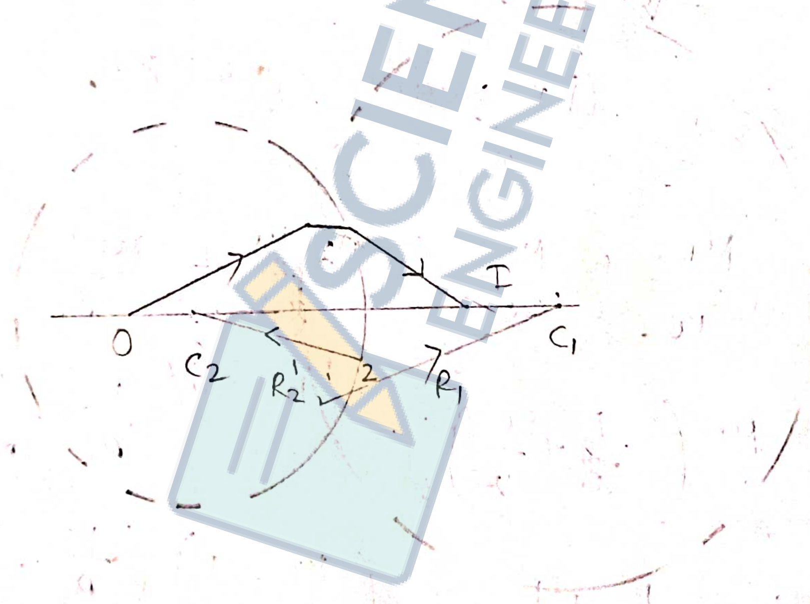
Special cases

A sign convention for the radius of Curvature can be developed like

(i) R must be measured from the curve towards the centre of curvature.

(ii) If the direction of measurement of R along the direction of the ray, it will be regarded as +ve and if it is opposite, then R will be −ve

Case - 1 (Convex lense)



Here $R_1 = +ve$, $R_2 = -ve$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= (+ve)\ (+ve)$$

$$= +ve$$

$\Rightarrow f = \frac{1}{+ve} = +ve$ (In conformity with earlier sign convention.)

Case-II (Concave lens)

$R_1 = -ve$

$R_2 = +ve$

$$\cancel{\text{Here}}\ \therefore \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left\{-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right\}$$

$$= -ve$$

$\Rightarrow f = -ve$ (In conformity with the earlier sign convention)

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28. $\frac{1}{f}$ = Power = 12.5

$$\Rightarrow f = \frac{1}{12.5} = 0.08\text{m} = 8\text{ Cm.}$$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_1}\right)$$

$$\Rightarrow \frac{1}{8} = \left(\frac{1.5}{1} - 1\right)\left(\frac{2}{R_1}\right)$$

$$\Rightarrow \frac{\cancel{2}}{R_1} = \frac{1}{8} \times 2 = \frac{\cancel{2}}{8} \Rightarrow R_1 = 8\text{ Cm.}$$

22.(a) For a convex lens, we know that

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{u+v}{uv} = \frac{1}{f}$$

$$\Rightarrow uv = (u+v) f \quad \text{———— (i)}$$

Squaring both the sides, we get

$$u^2v^2 = (u+v)^2 f^2 = (u^2+v^2+2uv) f^2$$

Subtracting $4uvf^2$ from both the sides of the above eqn, we get

$$u^2v^2 - 4uvf^2 = (u^2+v^2+2uv) f^2 - 4uvf^2$$

$$\Rightarrow uv(uv - 4f^2) = f^2(u^2+v^2+2uv - 4uv)$$

$$= f^2 (u-v)^2 \quad \text{———— (ii)}$$

$$\Rightarrow uv(uv - 4f^2) \geq 0$$ (because R.H.S is a squared quantity)

Since $u \neq 0, v \neq 0$ we must have

$$uv - 4f^2 \geq 0$$

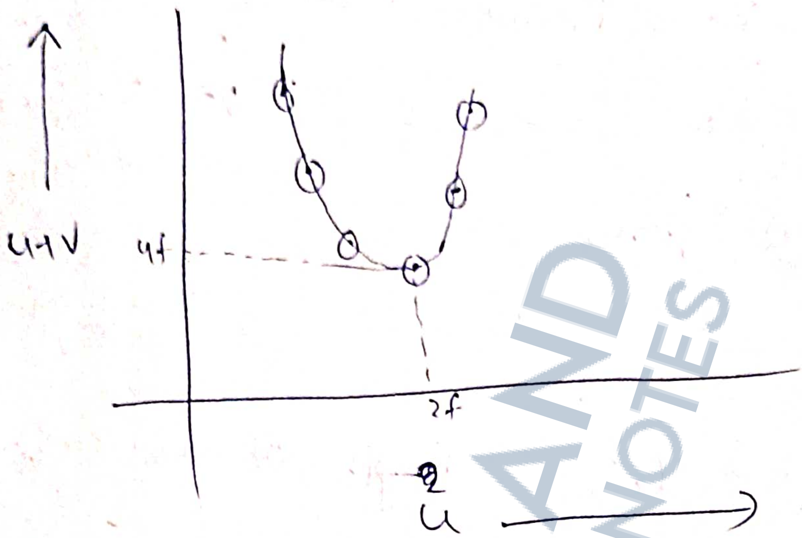
$$\Rightarrow uv \geq 4f^2 \quad \text{———— (iii)}$$

Using eqn (i) in eqn (iii), we get

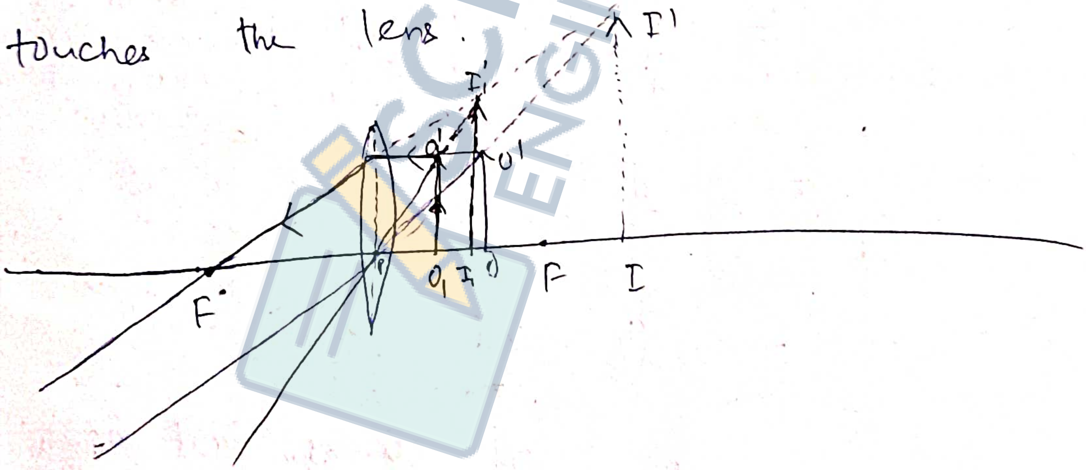
$$(u+v) f \geq 4f^2$$

$\Rightarrow u+v \geq 4f$

$\Rightarrow (u+v)_{minm} = 4f$



(b) The distance between the object and the virtual image decreases and ultimately it will be zero, when the object just touches the lens.



Problem

1) A bi-convex lens of focal length f is split into two part so that two plano convex lenses are formed. Find the focal length of each. Ans:- '2f'

(2) 2 watch glasses each of radius of curvature 20 cm are cemented together to form an air lense. Find its focal length when placed inside water. $\left(\mu_w = \frac{4}{3}\right)$ What is the nature of this lense) (Ans: -40 cm diverging)

Answers

1) The focal length of the biconvex lense is f.

Let the focal length of the plano convex is f_1.

We know = $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_1}$ [Since the biconvex lense is split into 2 parts]

$\Rightarrow \frac{1}{f} = \frac{2}{f_1}$

$\Rightarrow f_1 = 2f$

2. The radius of curvature of each lense is 20 c.m

given that ${}^{a}\mu_w = \frac{4}{3} = \frac{\mu_w}{\mu_a}$

We know the expression,

$\frac{1}{f} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \left(\frac{3}{4} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right)$

$= -\frac{1}{4} \times \frac{2}{20} = -\frac{1}{40} \Rightarrow f = -40$.

$\therefore$ The lense is diverging

Defects Of vision:

The following types of defects are observed in the human eye.

1. Long sight or Hypermetropia.

(3) Short sight or Myopia.

(4) Astigmatism.

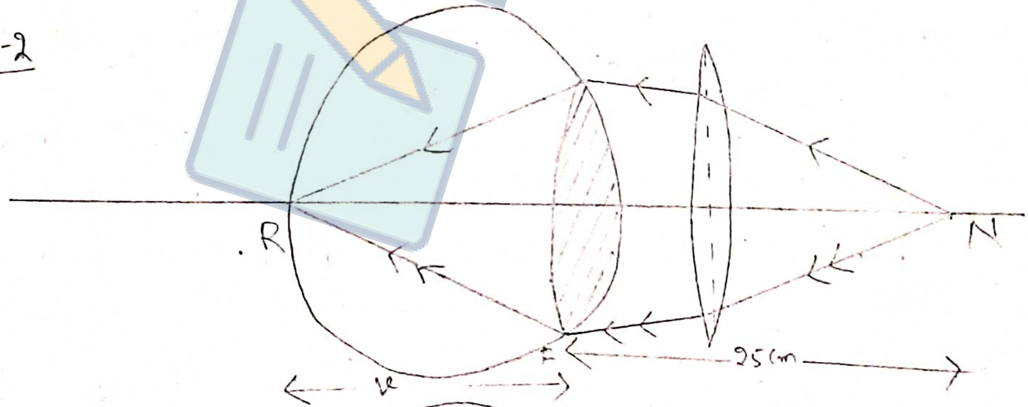
(2) Presbyopia or Far sight.

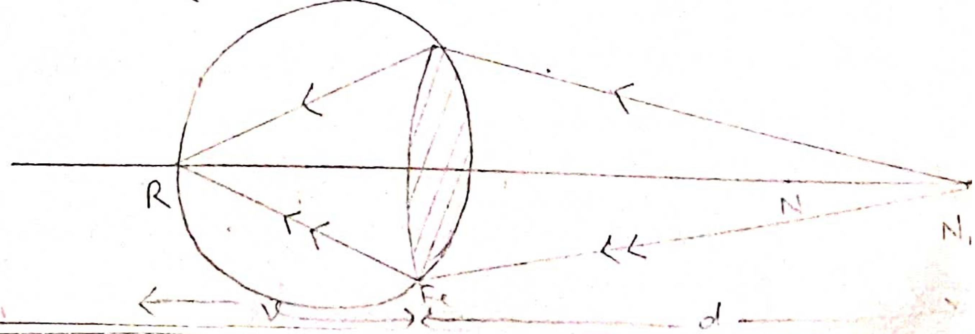
1. Long sight

For a normal eye, the near point is at 25 cm and for a long sighted person it is greater. A long sighted eye can not see near objects distinctly, but there is no difficult about distant objects

Fig-1

Fig-2





The rays from the normal near point are brought to a focus behind the retina. The causes are

(a) The eye-ball is too small.

(b) The focal length of the eye lens is elongated.

As shown in the diagrams, a convex lense has to be used in between the eye and the object so that the image will be formed on the retina.

From fig(iii), we can write

$$\frac{1}{v_e} + \frac{1}{d} = \frac{1}{f_e} \quad \text{--- (1)}$$

From fig-2, we can write

$$\frac{1}{v_e} + \frac{1}{25} = \frac{1}{F} = \frac{1}{f_e} + \frac{1}{f} \quad \text{--- (2)}$$

where f_e = focal length of the eye lense.

f = focal length of the convex lens used.

d = Near point of for the defective eye.

25 = L.D.D.V = Least distance of distinct vision.

F = Combined focal length of eye and ~~common~~ convex lense

Subtracting eqn (1) from eqn (2), we get

$$\frac{1}{25} - \frac{1}{d} = \frac{1}{f}$$

Since $d > 25\,cm$, $\frac{1}{f} = +ve$

$\Rightarrow f = +ve$

i.e A Convex lens has to be used having +ve power.

2. Presbyopia or far sight

This is another form of long sight which is due to old age. The biological lenses of eyes lose elasticity gradually with age and the accomodating power of the ciliary muscles decreases. Thus a short sighted eye in childhood tends to become normal in later years, but the defect of long sighted ness is sure to increase.

3. Short sight or Myopia:-

A short sighted person cannot see distant objects distinctly. The rays from a distant object ~~are brought~~ are brought to a focus in front of the retina R. The causes of the defect may be

(a) The eye-ball is too elongated.
(b) The focal length of the eye lense is too short.

This type of defect can be avoided by using a concave lens. So that the rays will diverge to some extend and the eye lens will focus them at retina.

From fig ②, we can write

$$\frac{1}{u}+\frac{1}{D}=\frac{1}{f_e} \quad \text{——— (1)}$$

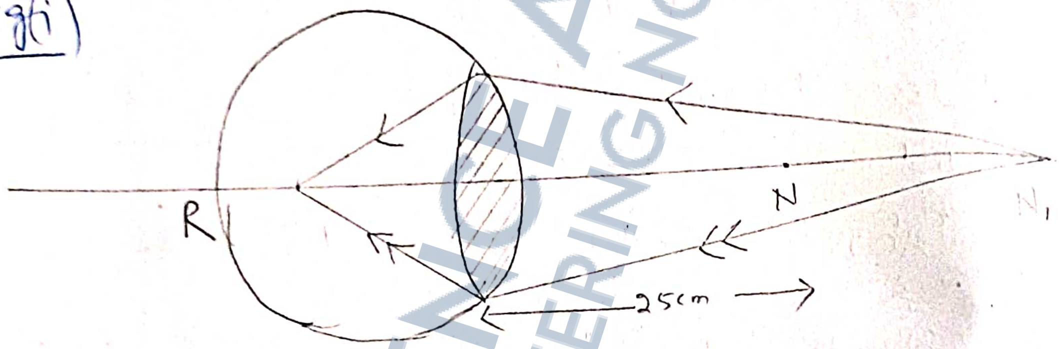
From fig ③, we can write

$$\frac{1}{u}+\frac{1}{d}=\frac{1}{F}=\frac{1}{f_e}+\frac{1}{f} \quad \text{——— (2)}$$

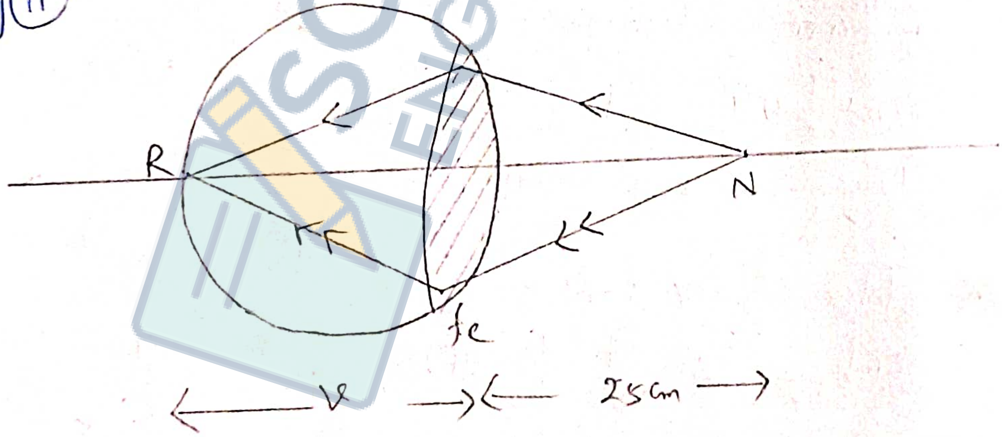
Subtracting eqn ② from eqn ①, we get

$$\frac{1}{D}-\frac{1}{d}=-\frac{1}{f} \quad \text{——— (3)}$$

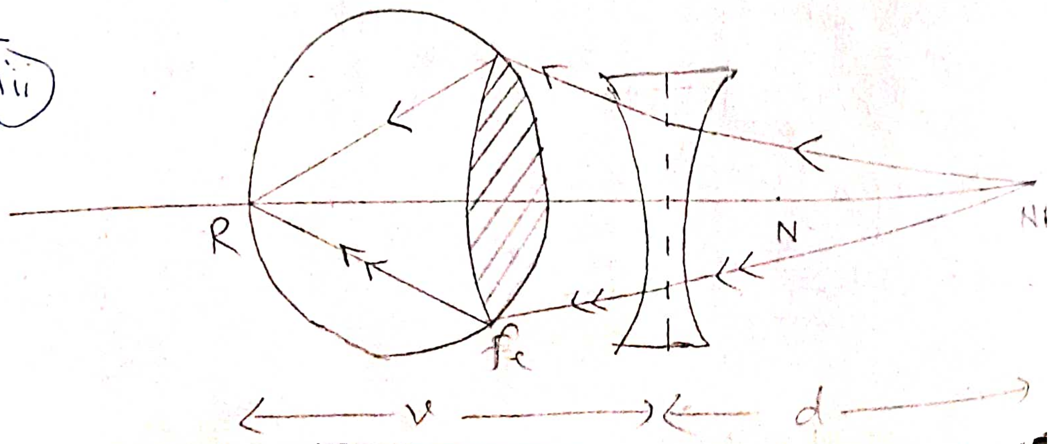
Fig(i)



Fig(ii)



Fig(iii)



Since $d > D$, $\frac{1}{D} > \frac{1}{d}$

$\therefore$ LHS = +ve = $-\frac{1}{f}$ of eqn (3)

$\Rightarrow f = -ve$

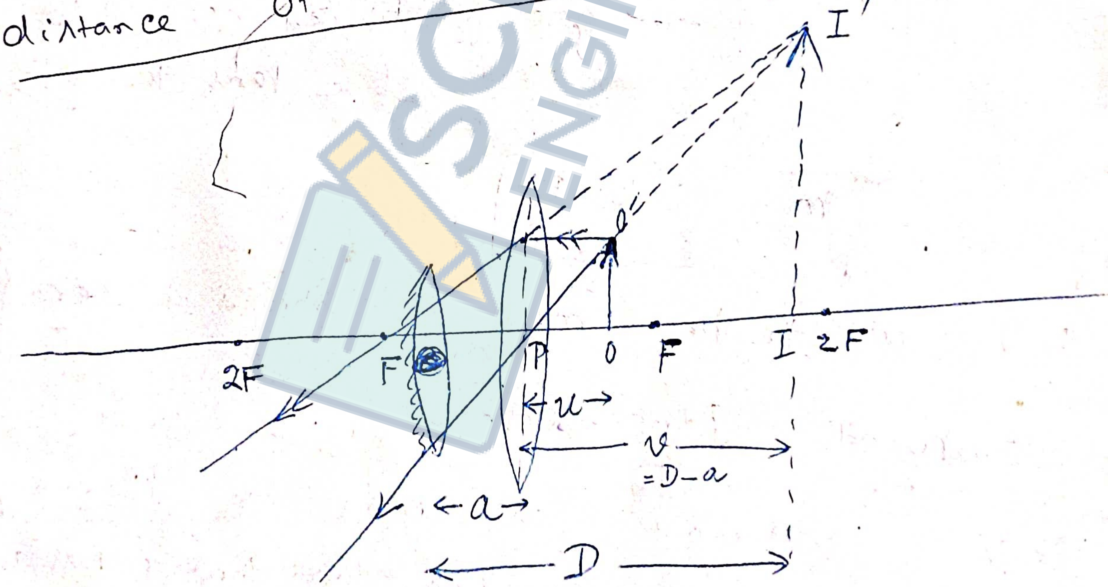
This shows that a concave lens has to be placed very near to the eye for distant object to form an image at retina.

(4) Astigmatism :-

This defect of eye is ~~done~~ due to irregularity in the curvature of the vertical and horizontal sections of the cornea, the curve generally is more pronounced in the vertical section than in the horizontal with the result that the vertical line at the horizontal and will not be in the same distance focus at the same time. Such an ~~eye~~ eye when looking at a network may be able to see clearly the horizontal wires while vertical wires may be indistinct or curved. A cylindrical or spherocylindrical lens is used to remove this defect. The defect may differ in degree in the two eyes.

Simple microscope :-

It is just a convex lens with a holder by which small object can be magnified. The object is to be placed in between the optical centre and the focus. A magnified erect & virtual image is formed on the same side of the object. For distinct vision, the image should be formed at 25cm away from the eye which should be kept very near to the lens. This distance is called least distance of distinct vision (LDDV)



From the figure we see that

$PO = u = +ve$.

$v = PI = -ve$ (because image is virtual)

$v = -(D-a)$ where a = Distance between the Eye & the optical centre

From the lense eq^n we know that

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-(D-a)} + \frac{1}{u} = \frac{1}{f}$$

Multiplying $(D-a)$ to both the sides

$$\Rightarrow -1 + \frac{D-a}{u} = \frac{D-a}{f}$$

$$\Rightarrow \frac{D-a}{u} = 1 + \frac{D-a}{f}$$

m = magnification $= \left|\frac{V}{u}\right| = \frac{D-a}{u} = 1 + \frac{D-a}{f}$

$$\boxed{m = 1 + \frac{D-a}{f}}$$

If the eye is placed closed to the lens then $a = 0$

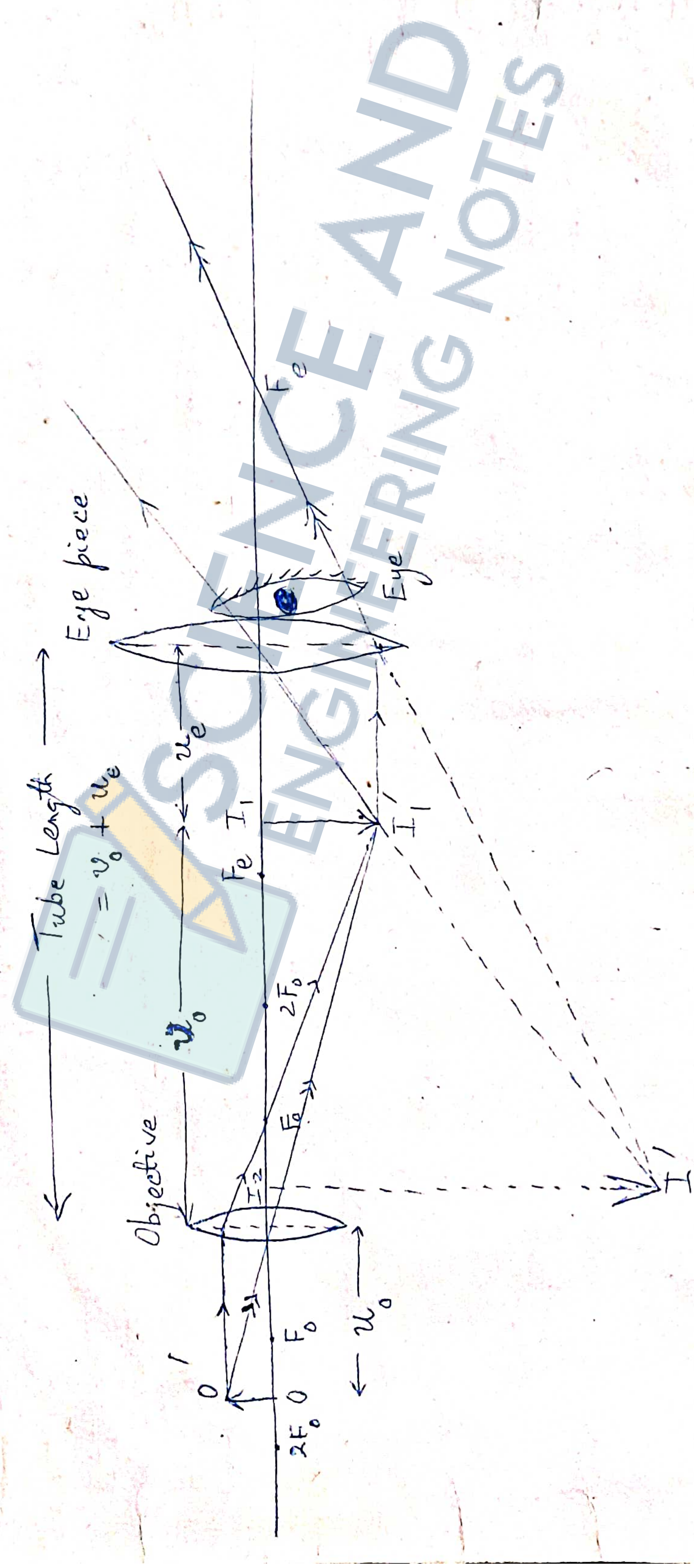
$$\boxed{m = 1 + \frac{D}{f}}$$

For m to be max 'a' should be Zero.

Compound Microscope

This instrument is used for producing much greater magnification than that is possible with a simple microscope.

It consists of two convex lenses placed in a tube. That convex lense which lies ~~between~~ towards the object is called Objective lens. It has a short aparature and short focal length. The convex lens which lies nearer to eye is called <u>eye piece</u>. It has also short focal length but large apprarature to receive more rays. There is provision to alter the distance between the objective and the eye piece



Magnification - $m_o m_e = \frac{V_o}{U_o}\left(1+\frac{D}{f_e}\right)$

Tube length = $V_o + U_e$

Net magnification by the compound microscope

$$= \frac{\text{Size of the final image}}{\text{Size of the Object}} = \frac{I_2 I_2'}{OO'}$$

$$= \frac{I_2 I_2'}{I_1 I_1'} \times \frac{I_1 I_1'}{OO'} = m_2 \times m_1 = \left(1+\frac{D}{f_e}\right)$$

Telescope →

These are instruments used to view distant objects. These are mainly of 2 types.

1. Refracting 2. Reflecting.

Referacting telescopes are of three types.

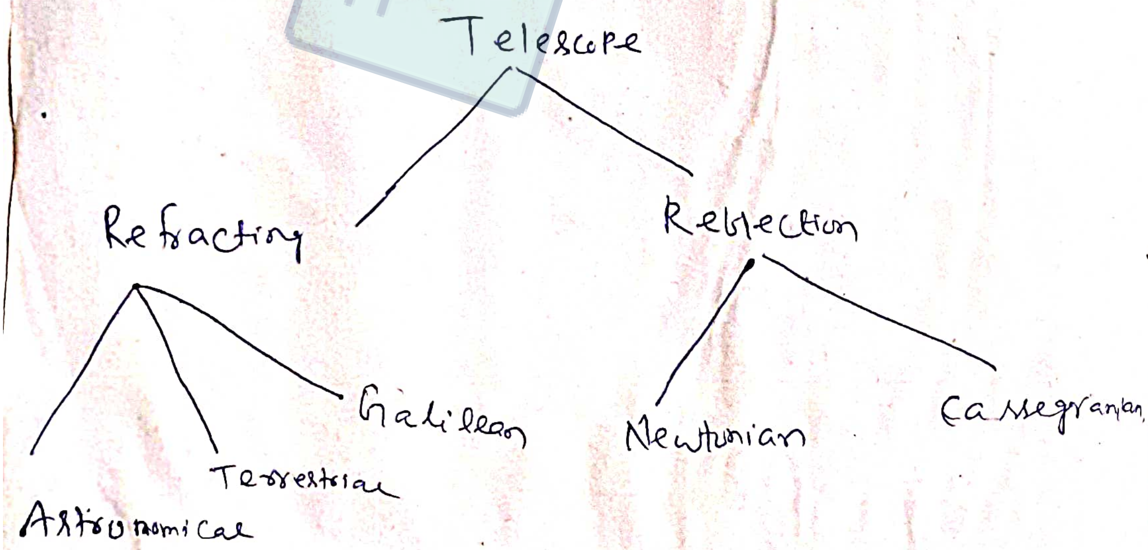
(i) Astronomical (ii) Terrestrial

(iii) Galilean.

Reflecting telescopes are of 2 types.

① Newtunian.

② Cassegranian.



The Astronomical Telescope

It is a refracting type of telescope having two convex lenses mounted co-axially in a tube. The objective 'O' has a large focal length and large diameter so that it can collect more rays from distant objects. The eye-piece has a short focal length. It should not have large aperature due to which some of the rays may be lost. This can be used in two different ways.

(1) The telescope be focused to infinity

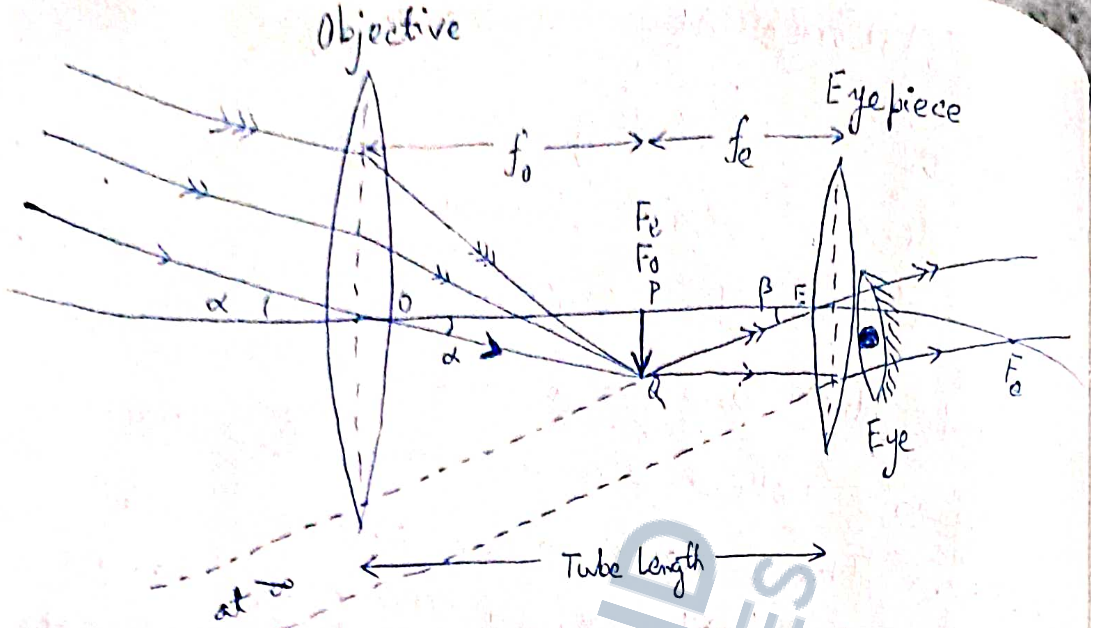
i.e the image is very much enlarged and formed at infinity. This is made possible when the focus of the objective and that of the eye piece coincide.

Angular magnification is defined as the ratio of the angle subtended at the eye by the image to that subtended by the object at the eye (objective)

$$\therefore m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha}$$ ($\because \alpha$ and β are both very small angles and expressed in radian)

$$\tan\beta = \frac{PQ}{PE}, \quad \tan\alpha = \frac{PQ}{PO}$$

$$\therefore m = \frac{\tan\beta}{\tan\alpha} = \frac{PQ}{PE} \times \frac{PO}{PQ} = \frac{f_o}{f_e}$$



Tube length = $f_o + f_e$

The telescope be focussed to LDDV.

(2.) The telescope be adjusted so that the final image will be formed at the least distance of distinct vision.

Magnification by the telescope

$$= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\cancel{PQ}}{PE} \times \frac{PO}{\cancel{PQ}}$$

~~For the eye piece~~

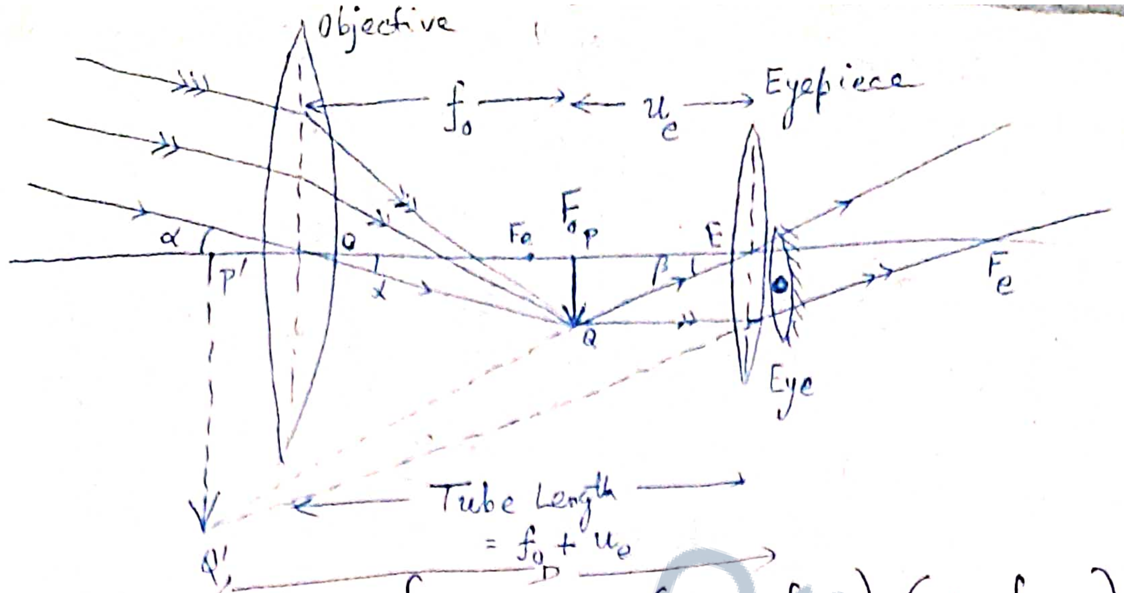
$$= \frac{PO}{PE} = \frac{f_o}{u_e}$$

For the eye piece

But $\frac{1}{v_e} + \frac{1}{u_e} = \frac{1}{f_e}$

$$\Rightarrow \frac{1}{-D} + \frac{1}{u_e} = \frac{1}{f_e}$$

$$\therefore \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{D + f_e}{f_e \cdot D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$



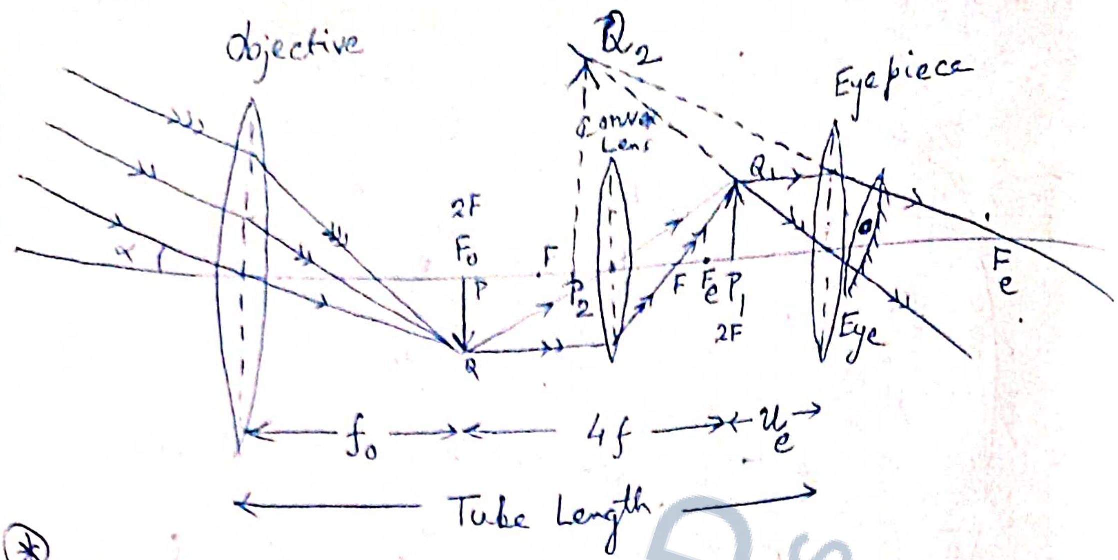
$$\therefore\ m = \frac{f_o}{u_e} = \left(+\frac{f_o}{f_e}\right)\left(1+\frac{f_e}{D}\right)$$

B. Terrestrial Telescope

One of the defects of astronomical telescope is that the final image is inverted w.r.t object. This can be made erect by using another convex lens of suitable focal length placed in between the objective and the eye-piece.

But the tube length increases and the image becomes fainter due to absorption of light by the three lenses. Some light is reflected and lost at each glass surfaces. Cross wires can be fitted at the site of PQ both in the astronomical and terrestrial telescope.

Erect image is essential if the instrument is to be used by navigators, surveyors etc.

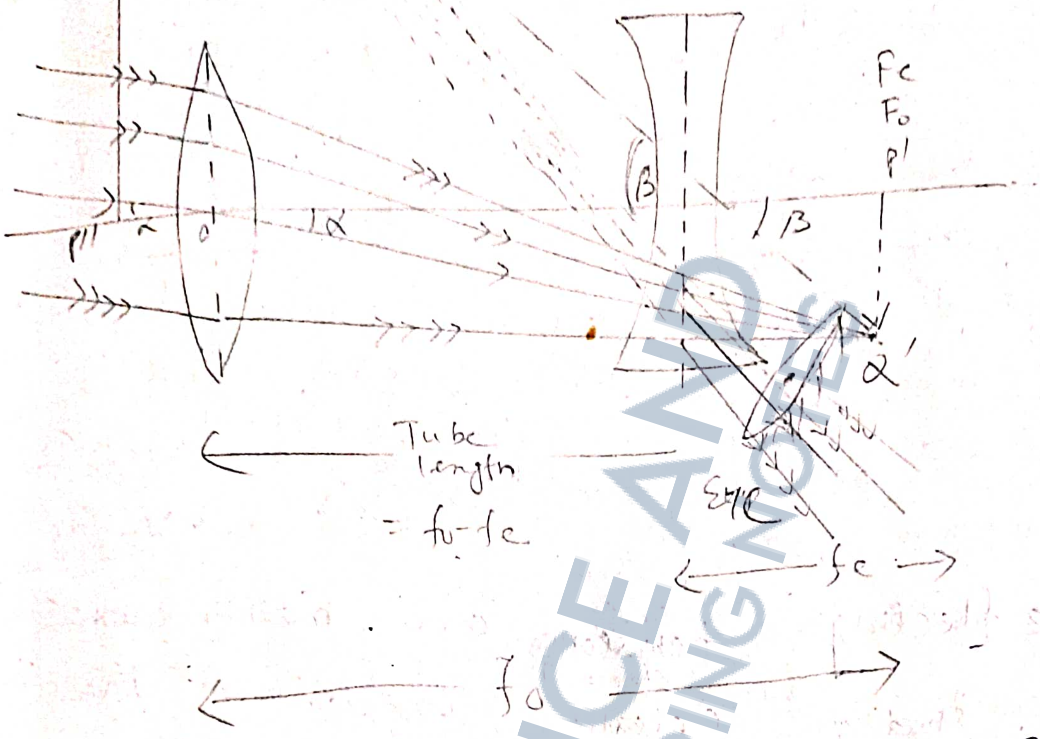


(*) C. Galilean Telescope

The disadvantage of the great length of a terrestrial telescope is avoided in Galileo's telescope. It consists of a convex lens as the objective of large focal length and a concave lense as the eye piece. A real, inverted image P'Q' of the object would have been formed at the focal plane of the objective, but the concave lens intercepts the rays before they reach the focus and the rays emerge from it as a parallel pencil. The eye sees a virtual erect and magnified image P''Q'' at infinity. The eye should be placed as close as possible to the eye piece, since the rays diverge.

$$\text{Magnification} = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} = \left(\frac{P'Q'}{P'O'}\right) \Big/ \left(\frac{P'Q'}{P'O}\right)$$

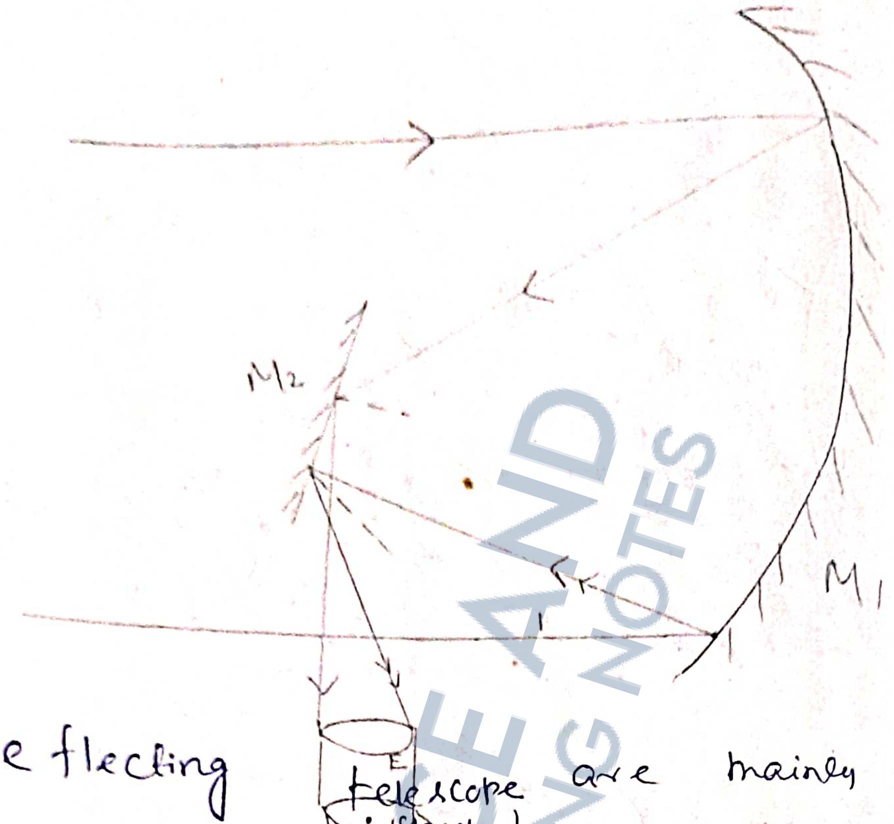
$$\alpha' \quad m = \frac{P'O}{P'O'} = \frac{f_o}{f_e}$$



Comparision between Astronomical and Galilean telescope

Astronomical	Galilean
① The final image is inverted and Virtual.	① The final image is erect and Virtual.
② Tube length = $f_o + f_e$ when focussed to infinity	2. Tube length $= f_o - f_e$ when focussed to infinity.
③ The final image is sharper compared to the image of the galillan telescope	3. The final image is fainter compared to that of the astronomical telescope because many rays escape the eye
4. Cross wire can be fitted at the position of the first image by which measurements can be done.	4. Cross wires can not be fitted in the ~~gal~~ Galilean telescope and measurements cannot be taken.

D. Newtonian type of Reflecting Telescope



Reflecting telescope are mainly used for studing celestial (star etc.), objects and the main advantages of the reflecting telescope over the refracting telescope are

1. The image is sharper because of less absorption of light

2. There is no chromatic aberration. (defect)

3. Reflecting mirrors of ~~an~~ large diameter can be prepared. On the otherhand, lenses of large size ~~are~~ are difficult to make and the absorption of light will be more.

4. By using a paraboloidal mirror, spherical aberration can also be

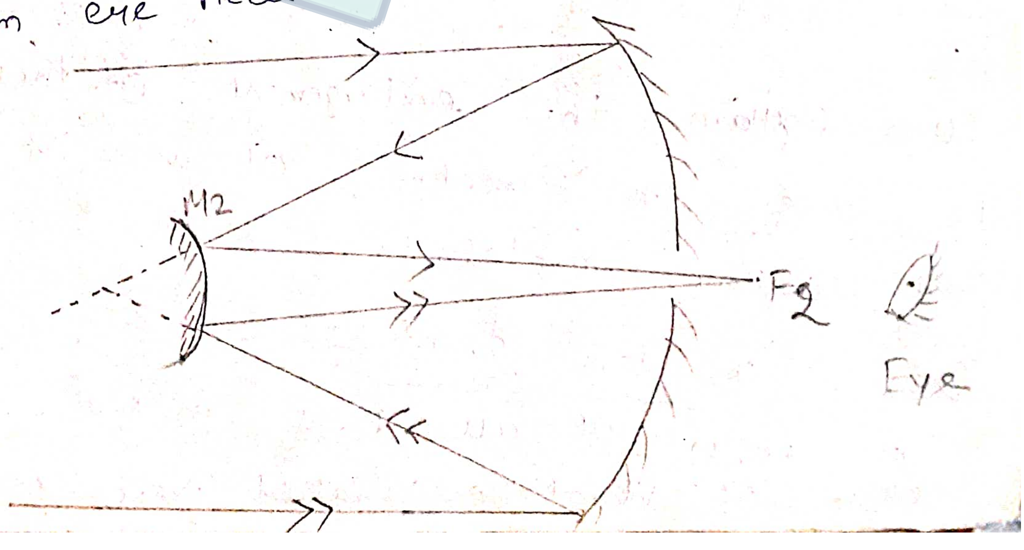
avoided.

A parallel beam of light from a distant object falls on a Concave mirror M1 and after reflection, real, inverted & diminished image would have been formed at its focal plane, but before reaching it, the rays are intercepted by a small plane mirror M2 inclined at 45° to the axis of the instrument by which the image is shifted to the side tube where it is viewed by means of a the eye piece E.

The final image is virtual and magnified and formed at infinity.

(E.) Cassegranian type of reflecting telescope

A beam of light coming from the object is received on a paraboloidal objective mirror M1 but before converging at F1, (the focus of the objective). It is intercepted by a convex hyperboloidal mirror M2 whereby the image is formed at F2 as shown in figure through a hole in the objective. It can be viewed by means of an eye piece.

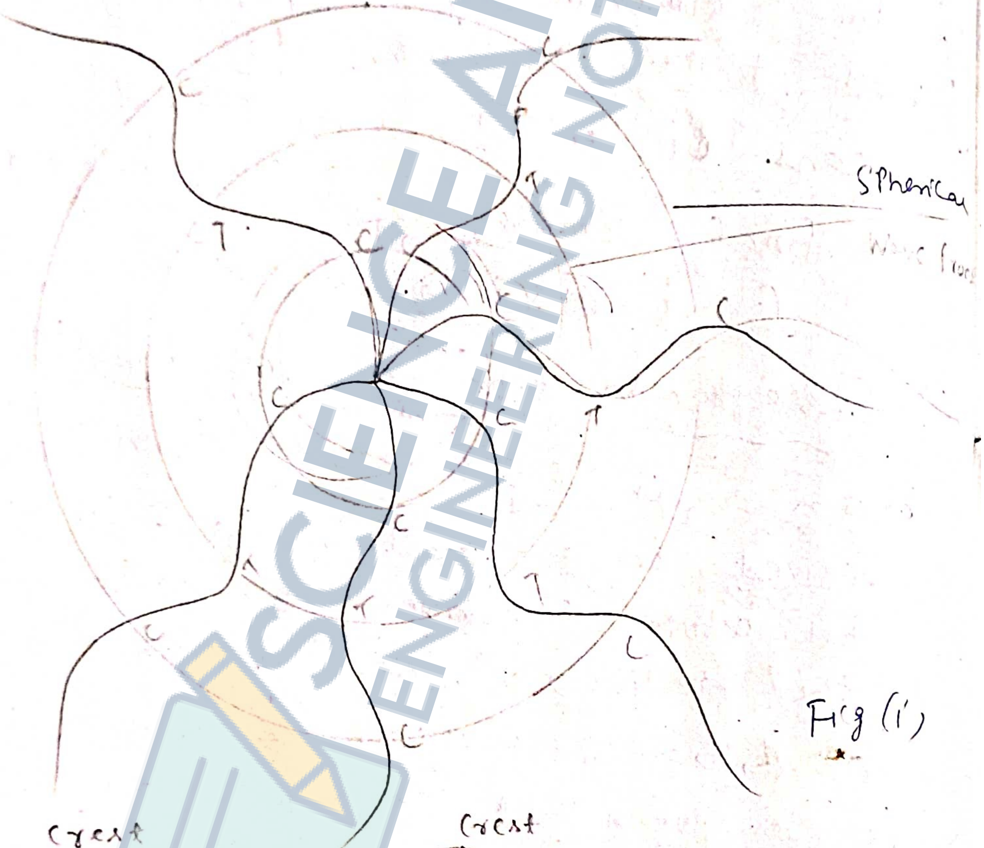


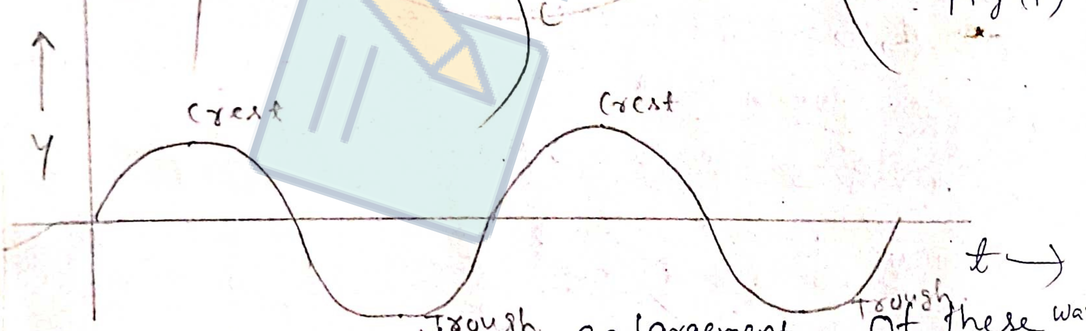
Jay Jagannath

Wave optics

Huygen's principle : In 1678 Huygen gave wave theory of light.

According to him point source of light gives spherical waves, where as linear source gives out cylindrical waves. With the passage of time these waves enlarges and ultimately become plane





To explain the enlargement of these waves he gave the concept of wave front and secondary wavelets.

Defn of wave front :
The locus of all points in the same state of vibration, is called wave front. As

shown in fig(i) the crest of all the waves coming out from S be joined to give rise to a spherical wave front. All the troughs be joined to give rise another spherical wave ~~form~~ front.

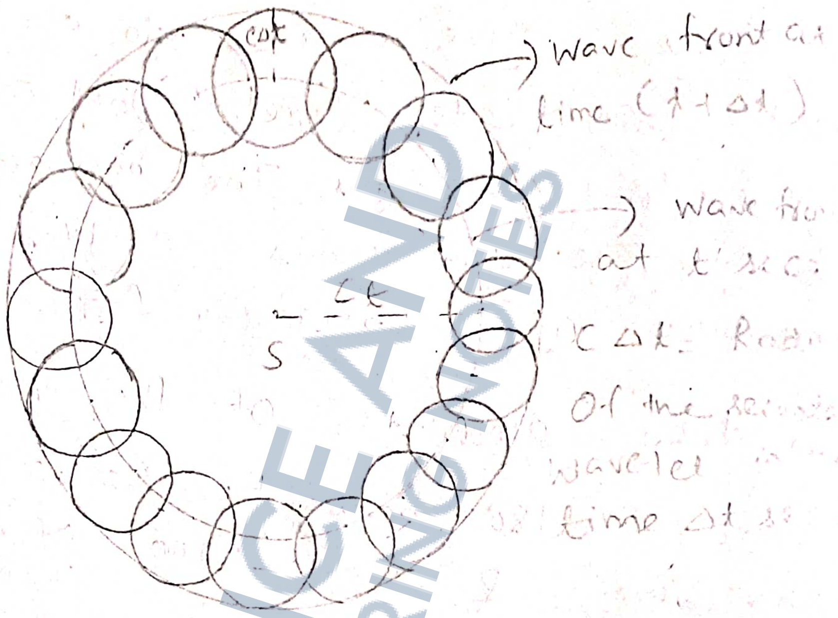
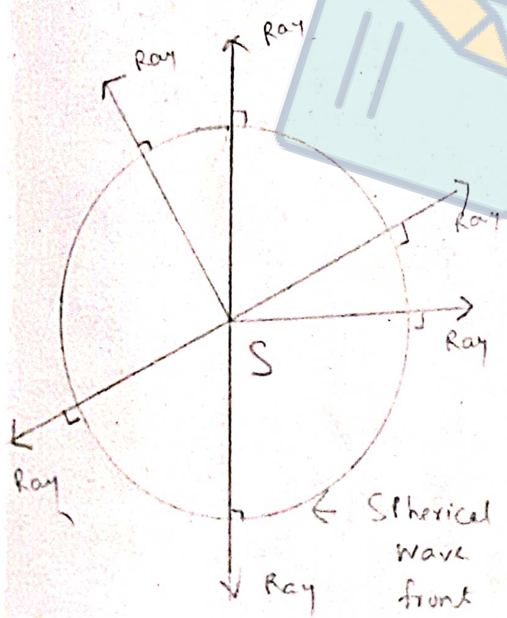


Fig-2



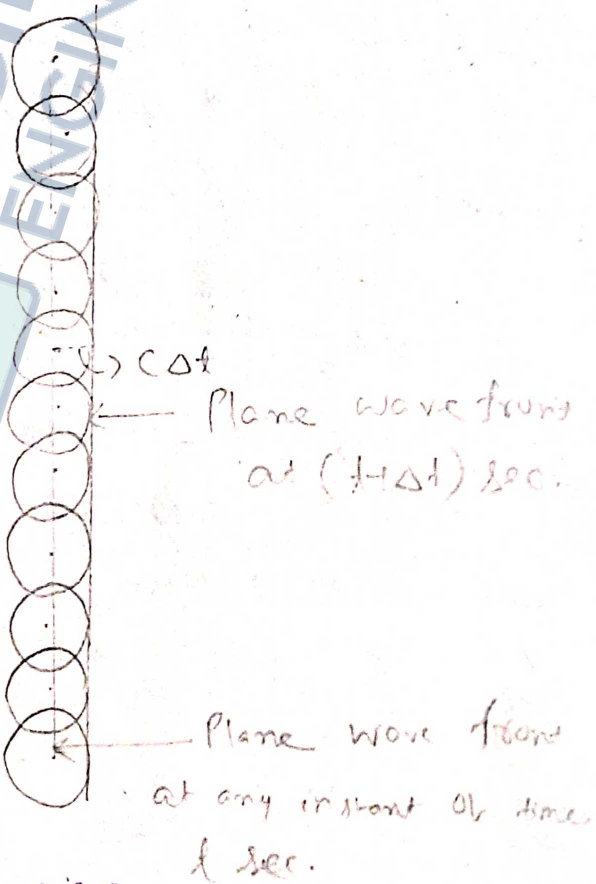


Fig-3

Fig-4

~~By~~ Huygen gave the idea of secondary wavelets which originate from each point of the wave front. In fig (ii) and (iii) secondary wavelets ~~[illegible]~~ have been shown. The envelope of ~~[illegible]~~ all these secondary wavelets give rise to a new wavefront at a later time. In this way the wave front enlarged. The backward motion of the secondary waves is prevented by the introduction of obliquity factor $(1+\cos\theta)$ in the amplitude of the vibrations. where $\theta = 180°$ between the actual direction of motion or the observation & the direction of secondary waves.

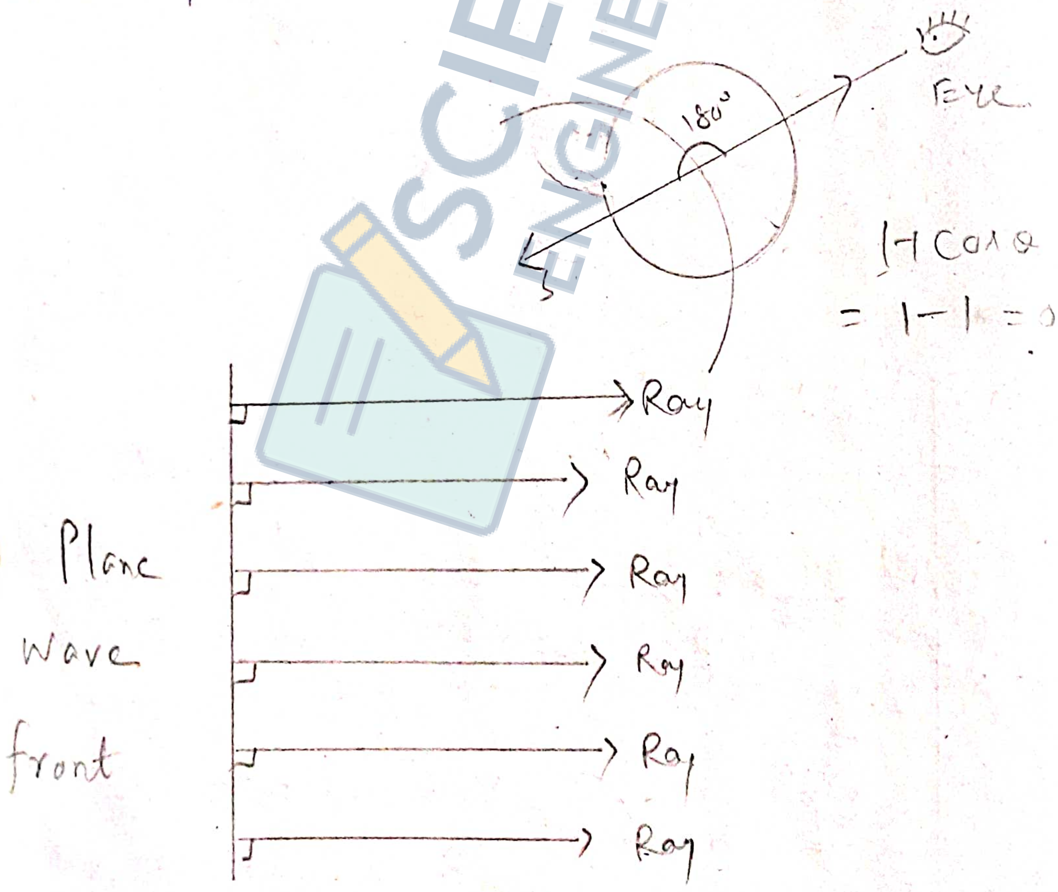


Fig-5

Defn of ray

A ray is a direction obtained by joining the source of light to any point on the wave front. As shown in fig (iv) the rays are always ⊥ to the wave front.

This idea that rays are ⊥ to the wave front explains the rays coming from as as ∥ to one another. This is shown in fig (v).

Long question

Interference

The occurrence of alternate and equidistant bright & dark spot on a screen due to super position of monochromatic and coherent light waves coming from two narrow slits on a Card board is called interference.

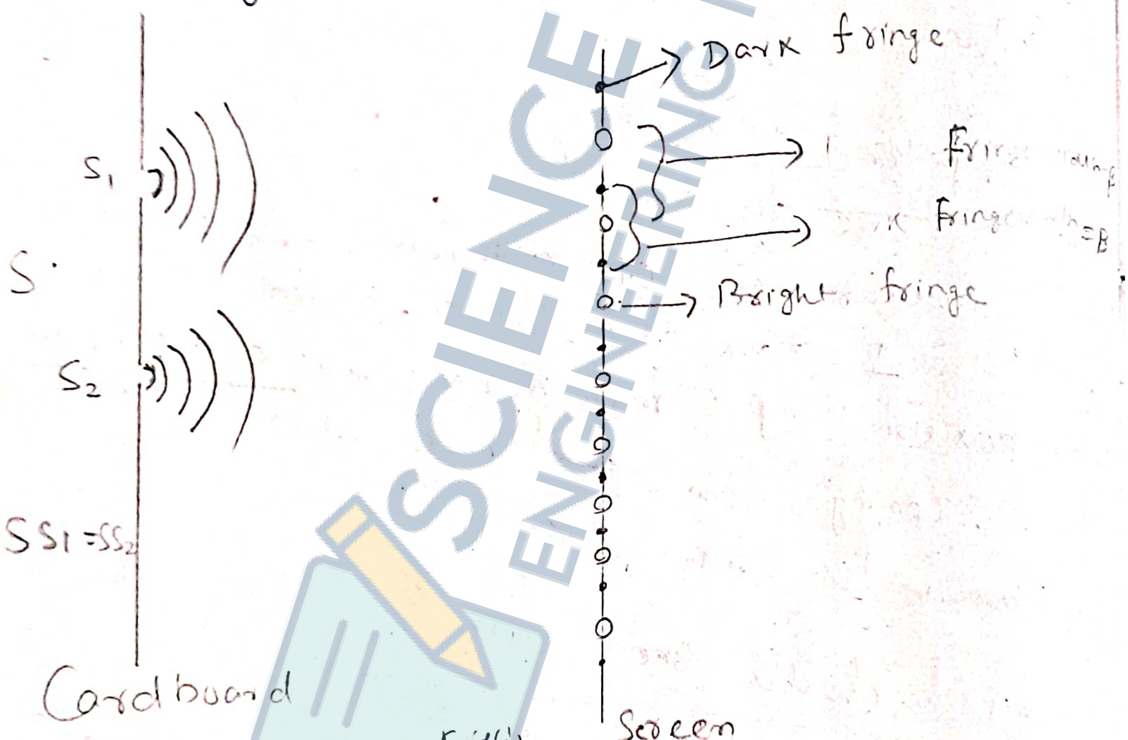
Young's double-~~street~~ slit experiment

In 1800, Young did a simple experiment by which a new phenomenon called interference was discovered. The experimental arrangement is shown in fig (i). A monochromatic source of light 'S' is kept at equal distance from the two small holes made in a card board. A screen is placed at some distance away. Alternate and equidistant bright and dark spots are fring on the screen.

The distance between consecutive bright spots or between two consecutive dark spots is called fringe width.

Explanation

Newton's 'corpuscular theory' fails to explain the occurence of bright and dark spots. This failure has been shown in fig (ii)



Fig(i)

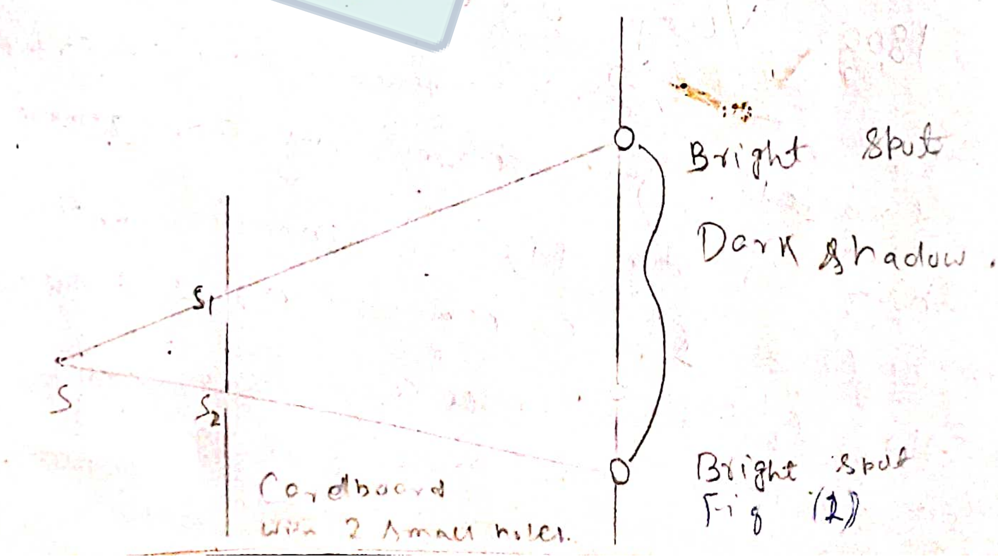


Fig (2)

Young had to use Huygen's wave theory to explain interference. According to him, a point on the screen will be a bright spot only when the ~~xx~~ crest of one wave from S_1 will fall on the crest of another wave from S_2 - or trough of one wave from S_1 will fall on the trough of the other wave from S_2. As shown in graph (i), the resultant amplitude will be $A_1 + A_2 = Max^m$.

We know that intensity is directly proportional to the square of the amplitude

$\therefore \; I_1 \propto A_1^2$

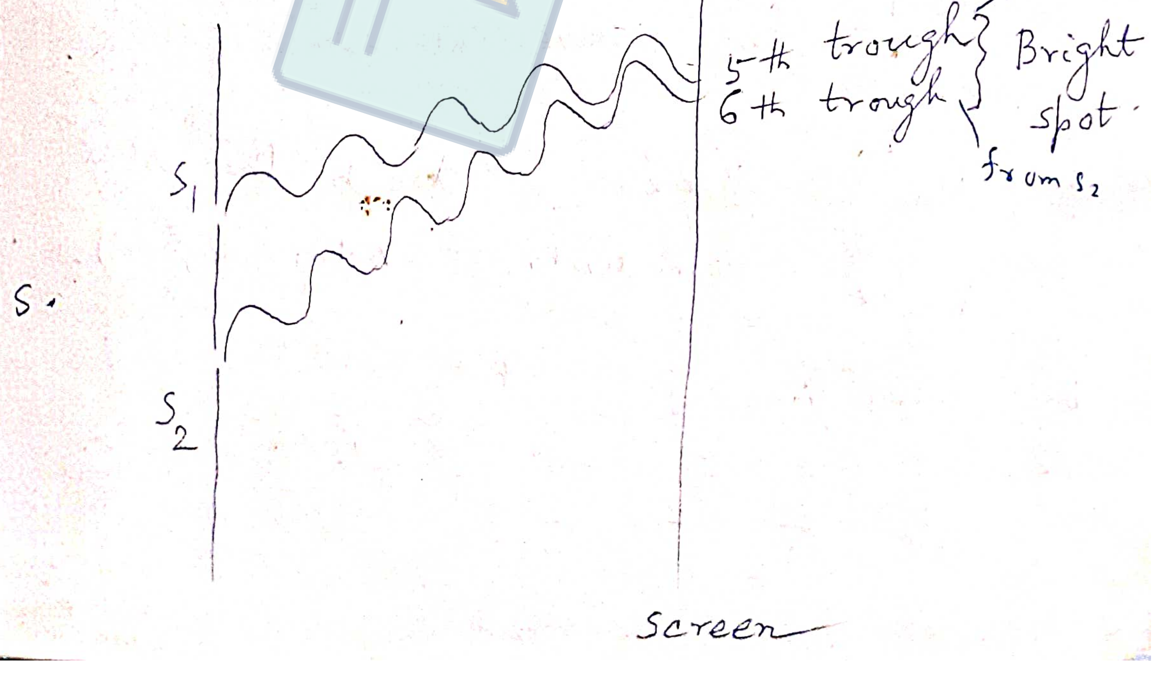
$\Rightarrow I_1 = K A_1^2$ ------ (i)

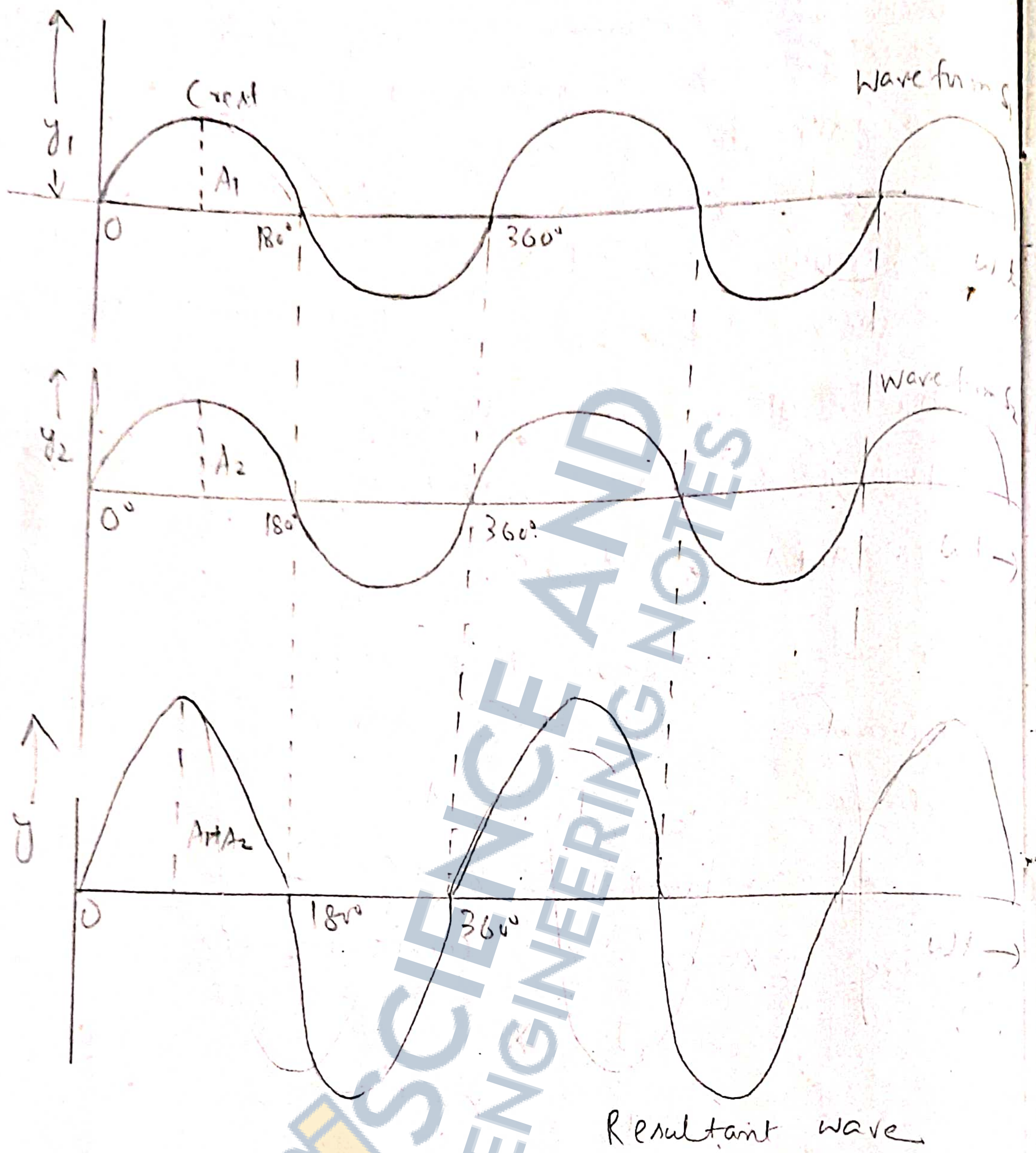
where K is a constant.

$I_2 \propto A_2^2$

$\Rightarrow I_2 = K A_2^2$ -------- (ii)

where I_1 & I_2 are the intensities of light





Graph-1 Constructive Interference.

On the screen due to the slit S_1 & S_2 respectively. Dividing eqn (i) by eqn (ii), we get

$$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = r^2 \quad \text{——— (iii)}$$

where $r = \frac{A_1}{A_2}$ = Amplitude ratio.

When crest falls on the crest,

resultant amplitude $= A_{Max}$

Hence the intensity will also be max^m.

$\therefore \quad I_{max} \propto A_{max}^2$

$$\Rightarrow I_{max} = K A_{max}^2 = K(A_1+A_2)^2 \quad \text{—— (iv)}$$

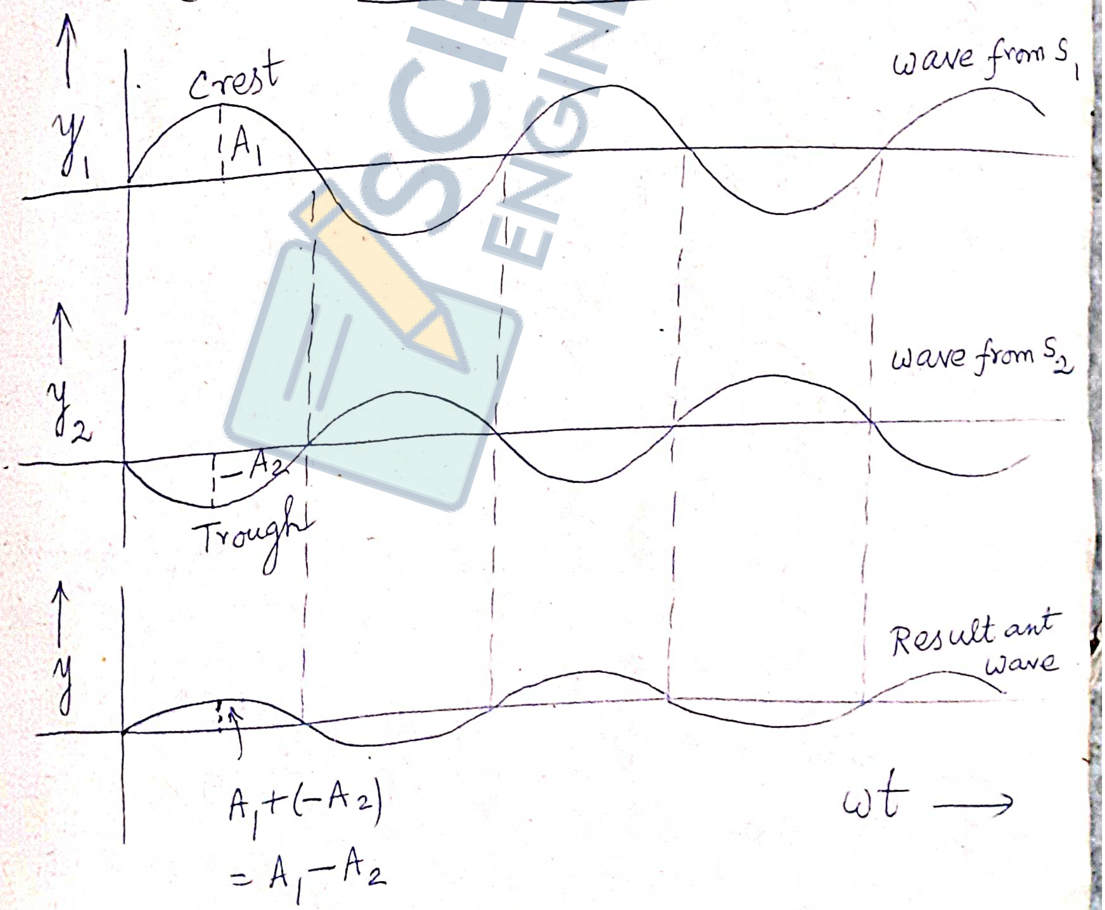
Dividing eq^n (4) by eq^n (i), we get

$$\frac{I_{max}}{I_1} = \frac{(A_1+A_2)^2}{A_1^2}$$

If $A_1 = A_2$, then : $\frac{I_{max}}{I_1} = \frac{4}{1}$

$$\Rightarrow I_{max} = 4I_1$$

i.e the bright spots of the interference pattern have intensities 4 times the intensity due to light coming from S_1 only. This is called <u>constructive interference</u>.



<u>Graph No-2</u> - Destructive Interference.

If the crest of one wave from S_1 will fall on the trough of the other wave from S_2 on the screen then that point will be dark spot.

The amplitude becomes minm

$= A_{min} = A_1 - A_2$ (As shown in graph II),

$\therefore I_{min} \propto A_{min}^2$

$\Rightarrow I_{minm} = K A_{min}^2 = K (A_1 - A_2)^2$ —— (V)

If $A_1 = A_2$

then $I_{minm} = 0$ and perfect dark spot is formed and we call it destructive interference.

Dividing eqn (iV) by eqn (V)

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2} = \frac{(r+1)^2}{(r-1)^2} \quad \text{—— (Vi)}$$

—·—

1. If the intensity ratio of the light obtained from 2 slits be 16 : 9, find the amplitude ration & the ration of the maxm & minm intensities.

(Ans :- 4 : 3 , 49 : 1)

Ans : Given $\frac{I_1}{I_2} = \frac{16}{9} = \frac{A_1^2}{A_2^2}$

$\Rightarrow \frac{A_1}{A_2} = \frac{4}{3} = \gamma$ = Amplitude ratio.

$$\frac{I_{max}}{I_{min}} = \frac{(\gamma+1)^2}{(\gamma-1)^2} = \frac{\left(\frac{4}{3}+1\right)^2}{\left(\frac{4}{3}-1\right)^2}$$

$$= \left(\frac{7}{3}\right)^2 \times \frac{9}{1} = \frac{49}{1}$$

Condition of interference

(i) The following 3 cond^{n}s must be satisfied to get interference fringes.

(i). The source of light must be mono-chromatic, i.e it should have a single colour & single wavelength.

Ex : Sodium vapour lamp

It gives golden yellow coloured light which contains only 2 lines of wavelengths $\lambda_1 = 5890 A^\circ$ & $\lambda_2 = 5896 A^\circ$

The average wavelength is $5893 A^\circ$ which is used for all calculations.

(ii) The light waves starting from the two slits must be coherent. i.e there should not be any ~~initi~~ initial phase difference between the light waves starting from S_1 and S_2. This is satisfied taking only one source & placing it at equal distances from S_1 & S_2 i.e

$SS_1 = SS_2$

If two monochromatic sources be used at S_1 & S_2 then interference fringes will not be visible because the light waves emitted by the two sources are not at the same time. Hence, the secondary waves which start from S_1 & S_2 may not be in the same phase.

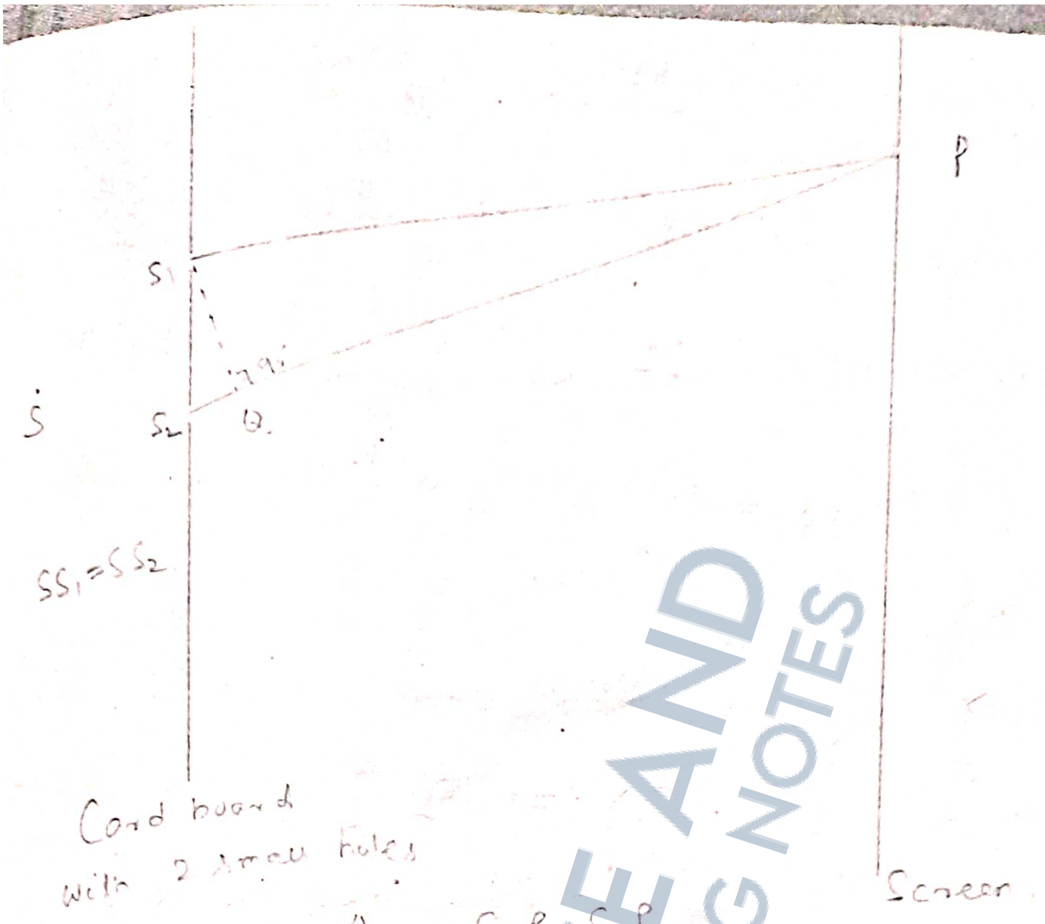
(iii) The path difference between light waves causing interference must be an even multiple of $\frac{\lambda}{2}$ for the occurence of a bright spot

The path difference between the light waves must be an odd multiple of $\frac{\lambda}{2}$ for the occurence of a dark spot

* Derivation of the third condn

For path difference λ, phase difference $= 2\pi$

" " " 1, " " $= \frac{2\pi}{\lambda}$



$S_2Q = \Delta = S_2P - S_1P$

Vector diagram.

For path difference Δ, phase difference $= \frac{2\pi}{\lambda} \cdot \Delta$

$= \phi$ (say)

If we will represent the wave from S_1 as $y_1 = A_1 \sin\omega t$, then the wave from S_2 will be represented by

$$y_2 = A_2 \sin(\omega t - \phi)$$

The resultant amplitude at a point on the screen will be obtained from the vector diagram,

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

When $\cos\phi = 1$, $A = max^m = A_1 + A_2$

$\Rightarrow \cos\phi = \cos 2n\pi$, where $n = 0, 1, 2, 3 ....$

$\Rightarrow \phi = 2n\pi$

$\Rightarrow \frac{\cancel{2\pi}}{\lambda} \cdot \Delta = \cancel{2}n \cancel{\pi}$

$\Rightarrow \Delta = n\lambda = 2n \cdot \frac{\lambda}{2}$

$=$ Even multiple of $\frac{\lambda}{2}$

This is called the cond^n for <u>constructive interference</u>.

When $\cos\phi = -1$, $A = Min^m = A_1 - A_2$

$\Rightarrow \cos\phi = \cos(2n+1)\pi$, where $n = 0, 1, 2, 3 ---$

$\Rightarrow \phi = (2n+1)\pi$

$\Rightarrow \frac{2\cancel{\pi}}{\lambda} \cdot \Delta = (2n+1)\cancel{\pi}$

$\Rightarrow \Delta = (2n+1)\frac{\lambda}{2}$

$=$ Odd multiple of $\frac{\lambda}{2}$.

This is also called the cond^n for <u>destructive interference</u>.

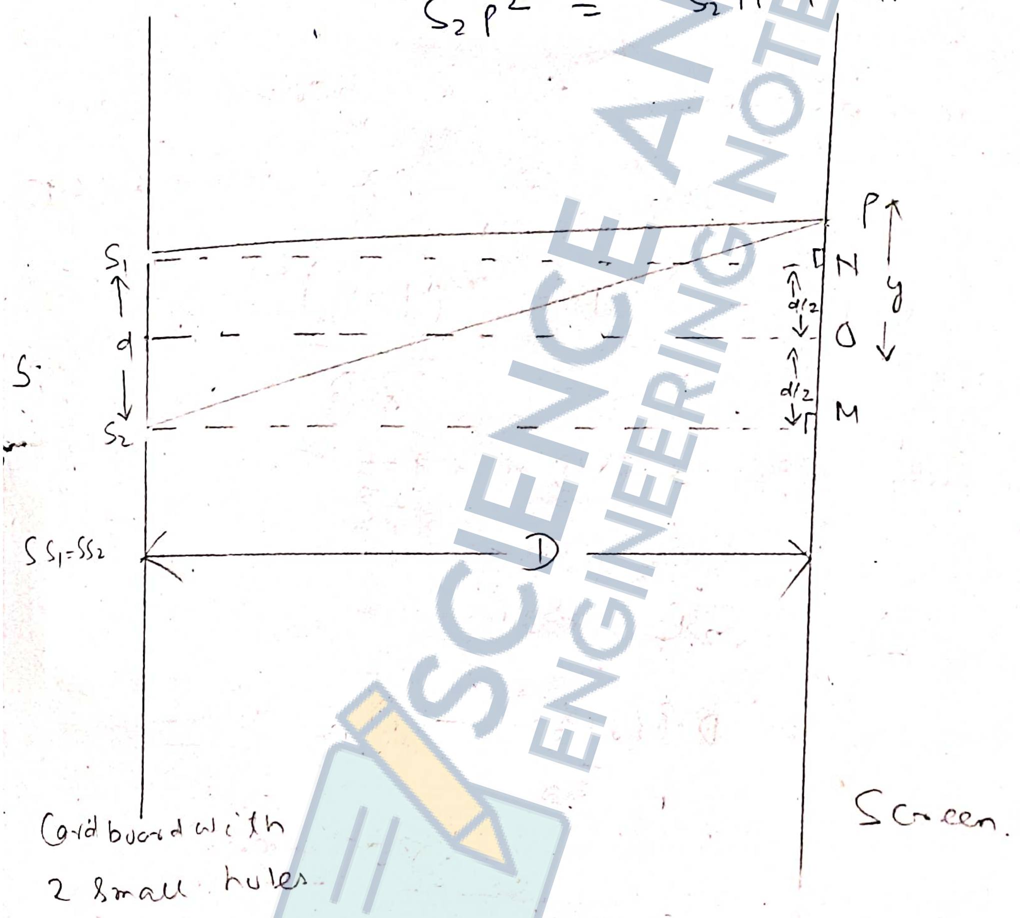
Expression for fringe width

The distance between two consecutive bright spots or between two consecutive dark spots on the screen is called **fringe width**.

Using Pythagoras theorem, we can write

$$S_1P^2 = S_1N^2 + NP^2$$

$$S_2P^2 = S_2M^2 + MP^2$$



Or D = Distance between the card board (having S_1 & S_2) & the screen

d = Distance between two slits.

y = Distance of any point on the screen from the point O. (O is the foot of the ⊥ dropped from S on the screen)

$$\therefore S_1P^2 = D^2 + (y - d/2)^2$$

$$S_2P^2 = D^2 + (y+d/2)^2$$

Now $$S_1P = \left\{D^2 + \left(y-\frac{d}{2}\right)^2\right\}^{\frac{1}{2}}$$

$$= \left\{D^2\left(1+\frac{(y-\frac{d}{2})^2}{D^2}\right)\right\}^{\frac{1}{2}}$$

$$= D\left\{1+\frac{(y-\frac{d}{2})^2}{D^2}\right\}^{\frac{1}{2}}$$

$$\simeq D\left(1+\frac{1}{2}\,\frac{(y-\frac{d}{2})^2}{D^2}\right)$$

where we have use Binomial expansion

$(1+x)^n \simeq 1+nx$ where $x << 1$.

$$S_1P = D + \frac{(y-\frac{d}{2})^2}{2D}.$$

Similarly $$S_2P = D + \frac{(y+\frac{d}{2})^2}{2D}$$

$$\therefore \Delta = S_2P - S_1P$$

$$= \cancel{D} + \frac{(y+\frac{d}{2})^2}{2D} - \cancel{D} - \frac{(y-\frac{d}{2})^2}{2D}$$

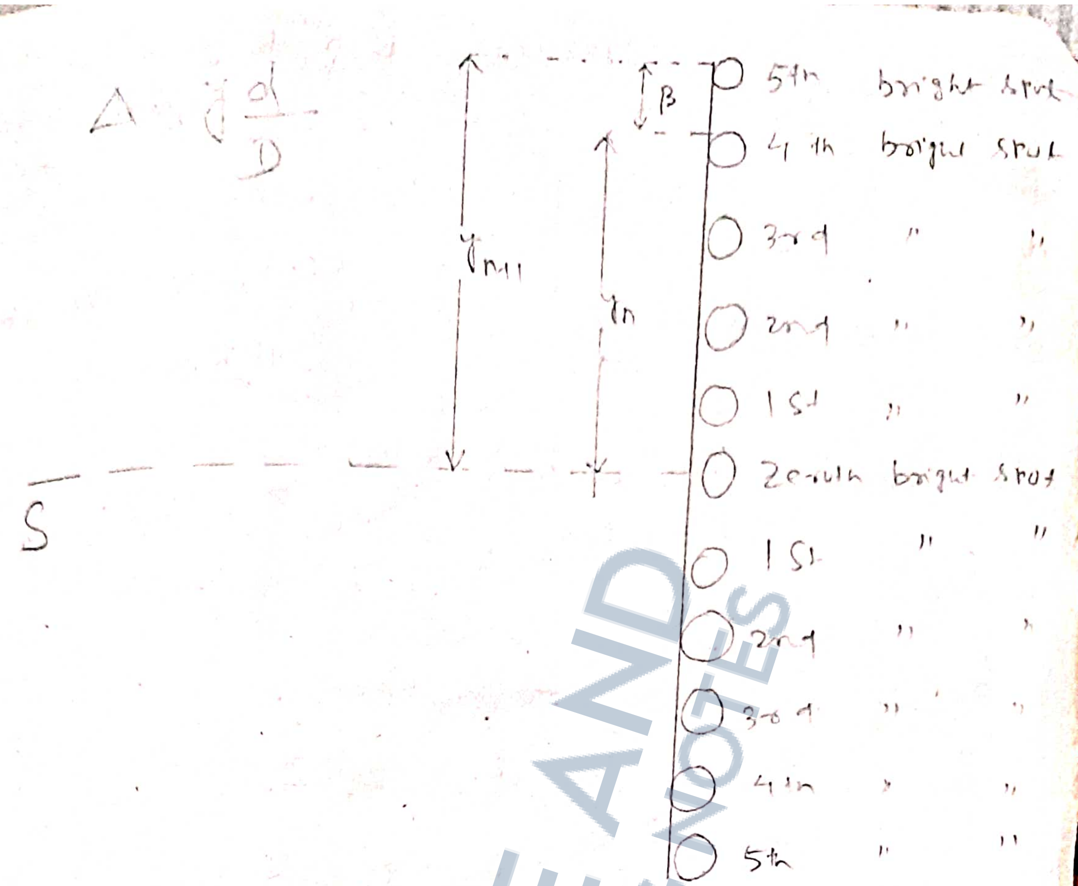
$$= \frac{(y+\frac{d}{2})^2 - (y-\frac{d}{2})^2}{2D}$$

$$= \frac{4\,\cancel{2}\,y\cdot\frac{d}{\cancel{2}}}{\cancel{2}D}$$

$$\Delta = \frac{yd}{D}$$

Let the point P be a bright spot, say, the nth bright spot.

Then $\Delta = \frac{y_n d}{D} = 2n\frac{\lambda}{2}$



$$\Rightarrow \quad y_n = \frac{n\lambda D}{d}$$

The distance of the $(n+1)$th bright spot from the point O will be

$$y_{n+1} = \frac{(n+1)\lambda D}{d}$$

Thus β = Fringe width

$$= y_{n+1} - y_n$$

$$= \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$= \frac{D\lambda}{d}\left(\cancel{n}+1-\cancel{n}\right)$$

$$\boxed{\beta = \frac{D\lambda}{d}}$$

Thus $\beta \propto D$, when λ, d are kept constants

ie as the screen is shifted away, the fringe width will increase.

$\beta \propto \lambda$, when D & d are kept constants

i.e $\beta_{red} > \beta_{green}$ because $\lambda_{red} > \lambda_{gr}$

$\beta \propto \frac{1}{d}$, when D & λ are kept constants

i.e β increases when d decreases.

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3. Given $d = 0.2$ mm $= 0.02$ cm.

$D = 80$ cm.

$\lambda = 6,000$ A°

$= 6,000 \times 10^{-8}$ cm.

$= 6 \times 10^{-5}$ cm.

$\therefore \beta = \frac{D\lambda}{d} = \frac{80 \times \cancel{6}^{3} \times 10^{-5} \times 100}{\cancel{2}}$

$= 240 \times 10^{-3}$

$= .24$ cm.

(a) The distance of the 2nd dark band from the central max $= \frac{3\beta}{2}$

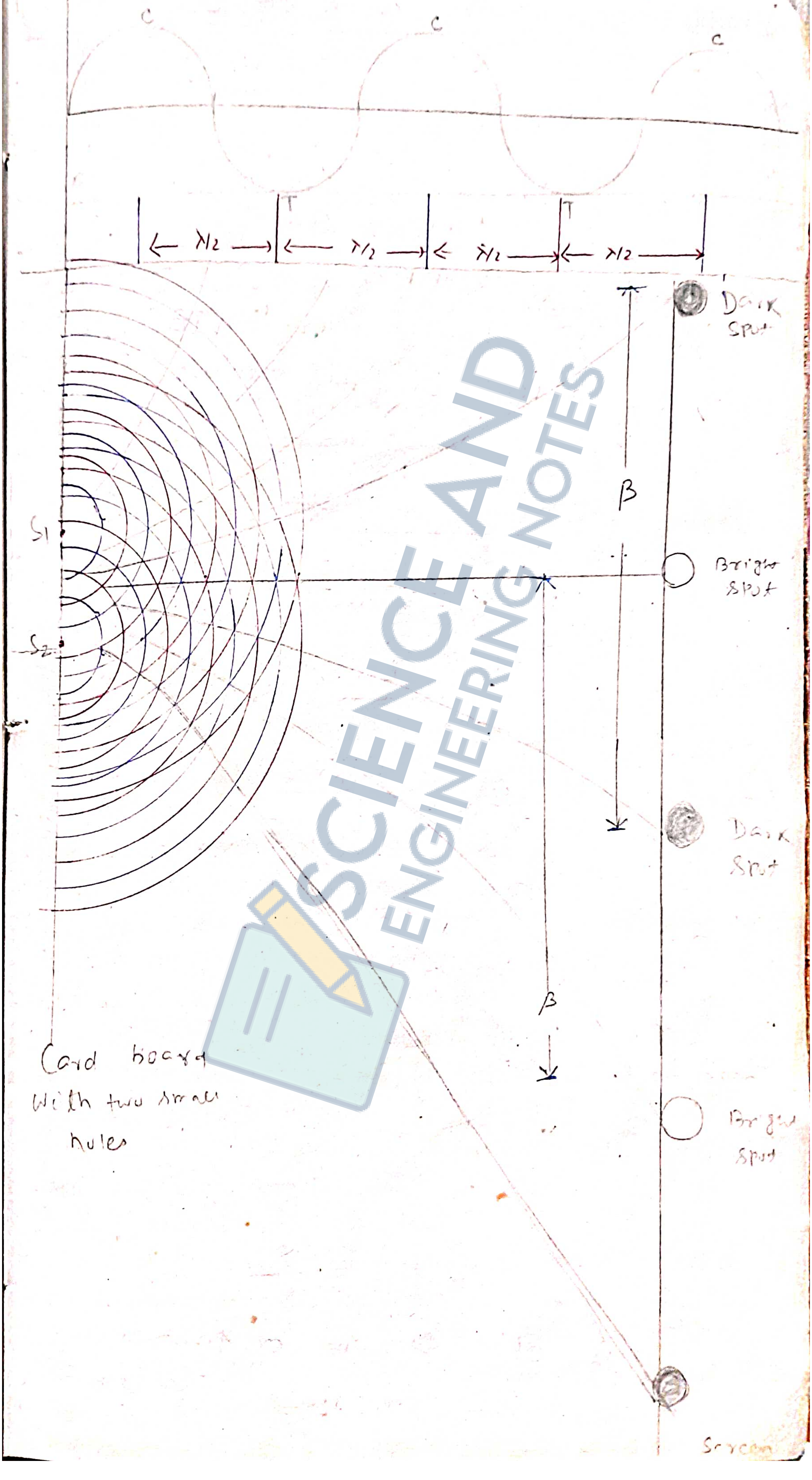
$= \frac{\cancel{3}^{1.5} \times 0.24}{\cancel{2}}$

$= .36$ cm.

Second ~~bri~~ light band from the central maxima

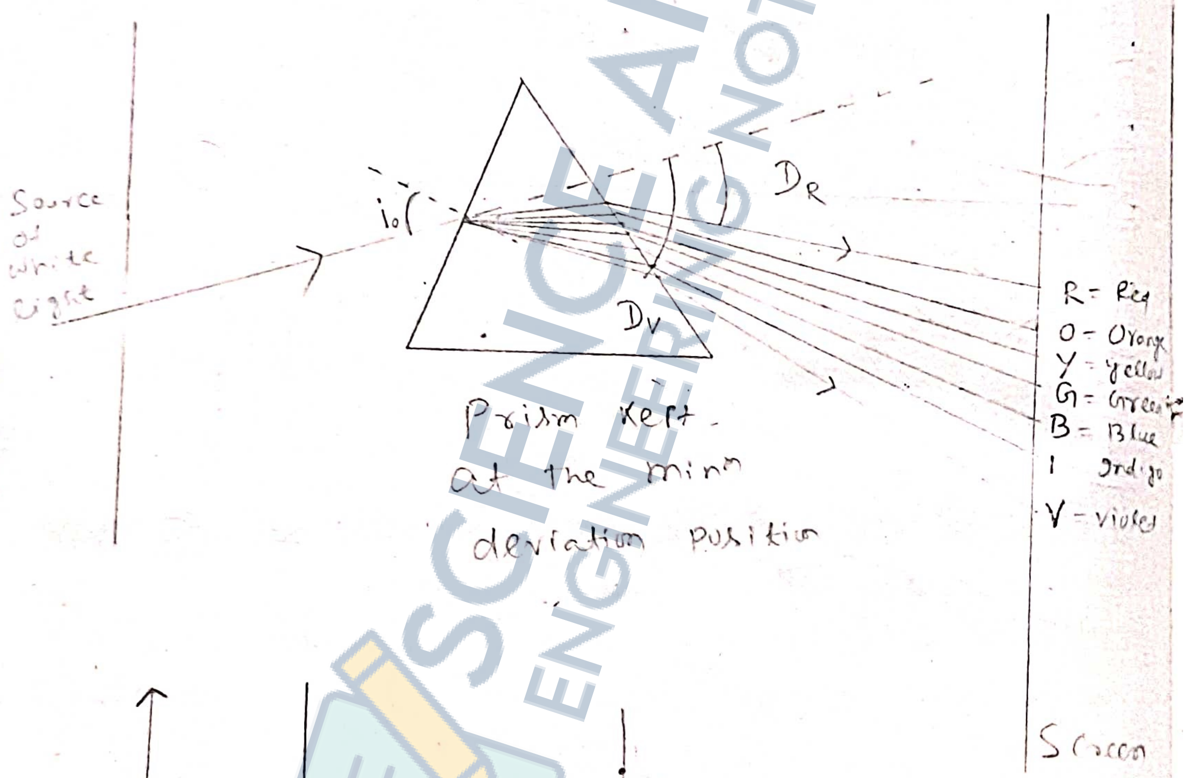
$= 2\beta = 2 \times .24 = 0.48$ cm

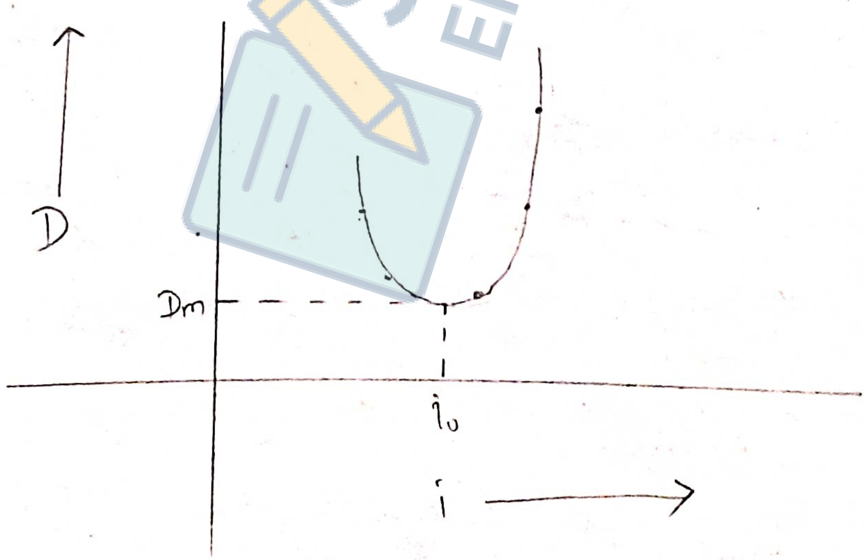
Geometrical construction to show interference



Dispersion of light

When a pencil of white light is incident on a prism kept at the minm ~~devit~~ deviation position the emergent light is found to consists of seven colours arranged in the order "VIBGYOR" from the buttom of the screen.





The angle of deviation ~~of~~ for red colour is minm and that for violet colour is maxm.

i.e why refractive index for violet colour is more than for the refractive index for the red colour.

i.e ~~why~~ $\therefore \mu_V > \mu_R$

$$\Rightarrow \frac{c}{v_V} > \frac{c}{v_R}$$

$$\Rightarrow \frac{1}{v_V} > \frac{1}{v_R}$$

$$\Rightarrow v_R > v_V$$

i.e speed of light is highest for red colour and lowest for the violet colour.

This display of various colours on the ~~screen~~ screen obtained through dispersion is called spectrum.

Dispersive power : (ω)

Before defining this property of an optically transparent medium, let's take the expression for the angle of min^m deviation due to a thin prism.

$$D_m = (\mu - 1) A$$

For different coloured lights D_m changes due to change of μ (Since A is same for all colours). For the violet coloured light $D_V = (\mu_V - 1) A$

For the red Coloured light

$$D_R = (\mu_R - 1)A$$

Thus $$D_V - D_R = (\mu_V - \cancel{1} - \mu_R + \cancel{1})A$$

$$= (\mu_V - \mu_R)A \quad \text{———(i)}$$

For the mean Color of light which is yellow, ~~$D_y = \mu_y - 1$~~

$$D_y = (\mu_y - 1)A \quad \text{———(ii)}$$

Dividing eqn (i) by eqn (ii), we get

$$\frac{D_V - D_R}{D_y} = \frac{\mu_V - \mu_R}{\mu_y - 1} = \frac{\Delta\mu}{\mu_y - 1} = \omega$$

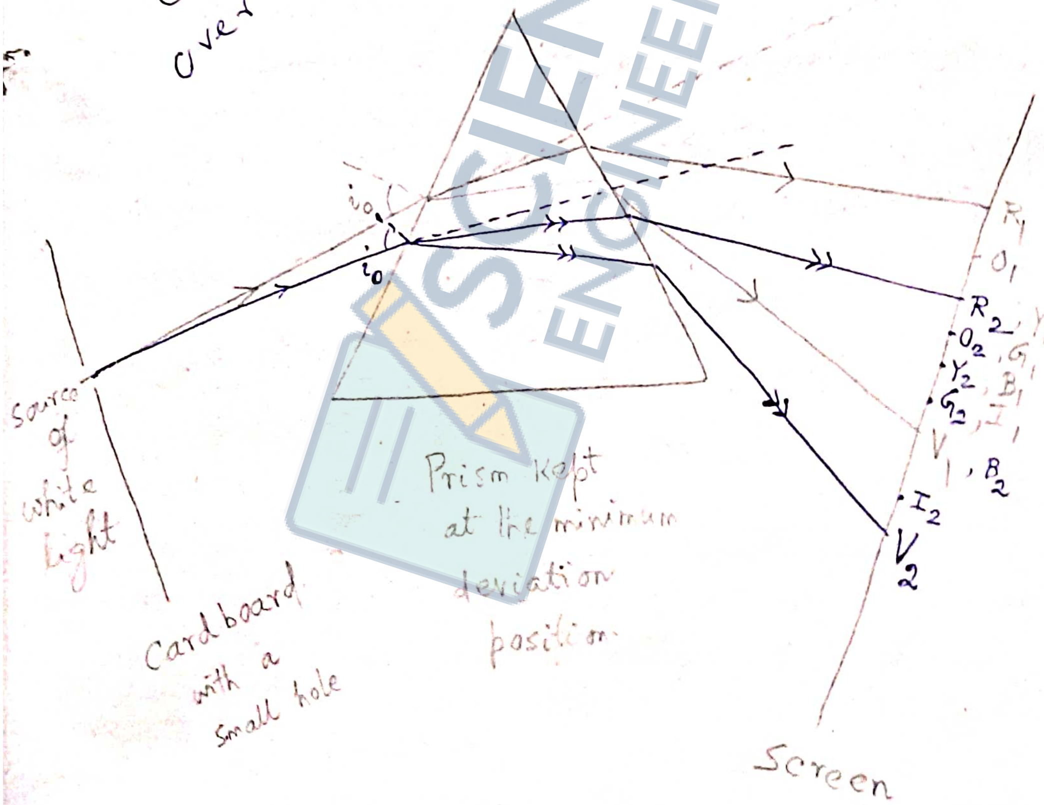
Thus, dispersive power of an optically transparent medium can be defined as the ratio of change in the angle of minm deviation between the two extreme colours of light and the angle of minm deviation for the mean colour of light (yellow).

Impure Spectrum

However small hole be made in the Cardboard placed near the white light source, one can not get a single ray of light. In the diagram, only 2 rays have been

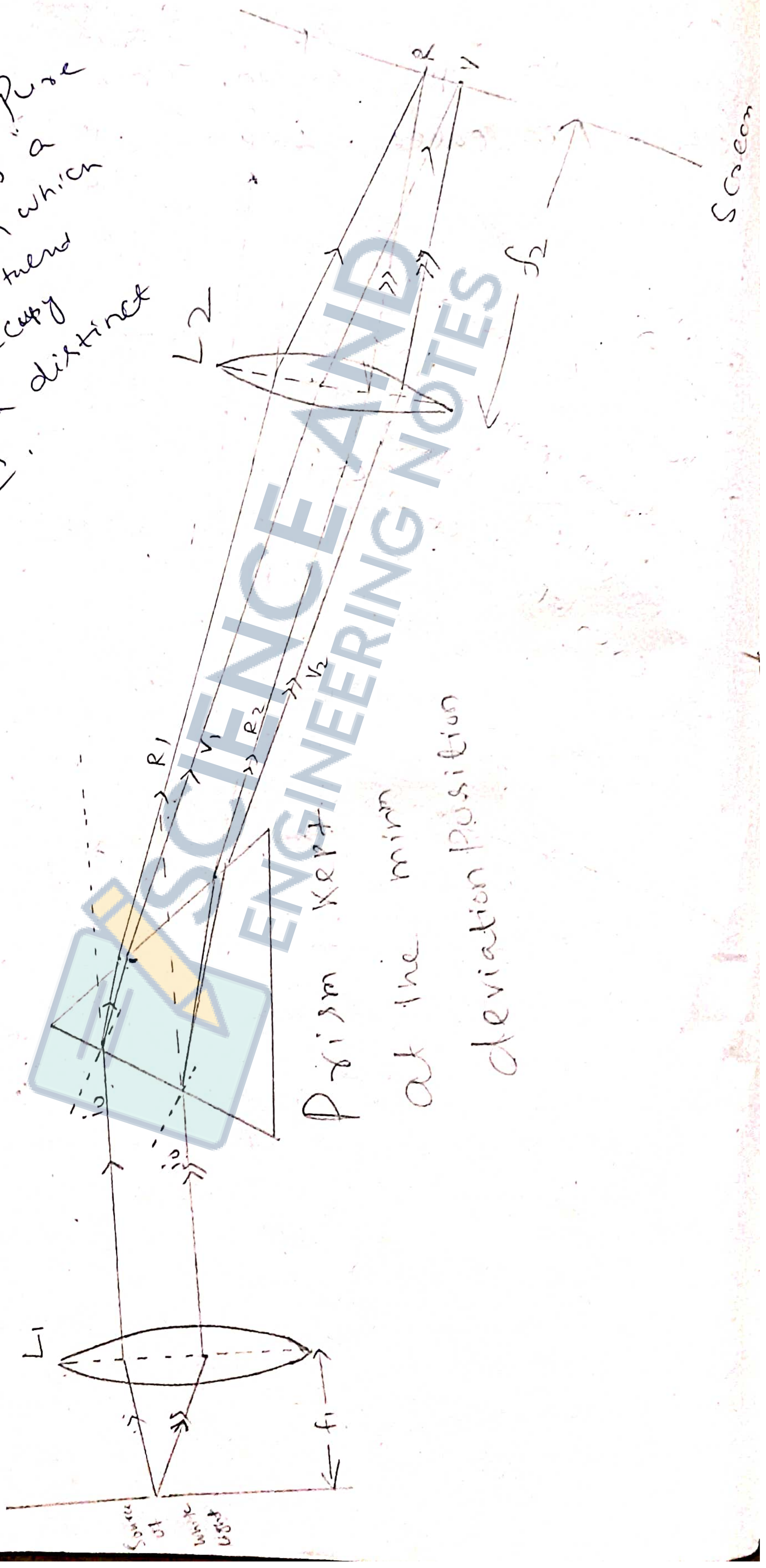
Show there is already superposition of colours. If large number of rays be considered, then there will be large scale superposition and one cannot get distinct colours on the screen. This type of spectrum is called impure spectrum.

Defn → Impure spectrum is a spectrum in which the constituent colours overlap each other.



Pure Spectrum

Defn → Pure spectrum is a spectrum in which all constituent colours occupy different & distinct positions.



It is a spectrum where different colours of whitelight donot overlap & they are found separately on the screen.

The condn. for the formation of pure spectrum are

(i) The slit should be narrow.

(ii) The prism should be placed in the minm deviation position for the mean colour (yellow).

(iii) An achromatic convex lense (The lense which is free from colour defects) is to be placed near the slit such that the slit will be the focus. Then the rays will be parallel when they are incident on the prism.

(iv) The colours obtained after dispersion are to be incident on another convex lense having the screen as it focal plane

The spectrometer :-

This an instrument by which wave length Of unknown lines of the spectrum Can be measured. For this purpose, grating is to replace the prism. A refractive index Of the material of a prism for different colours Can be measured accurate

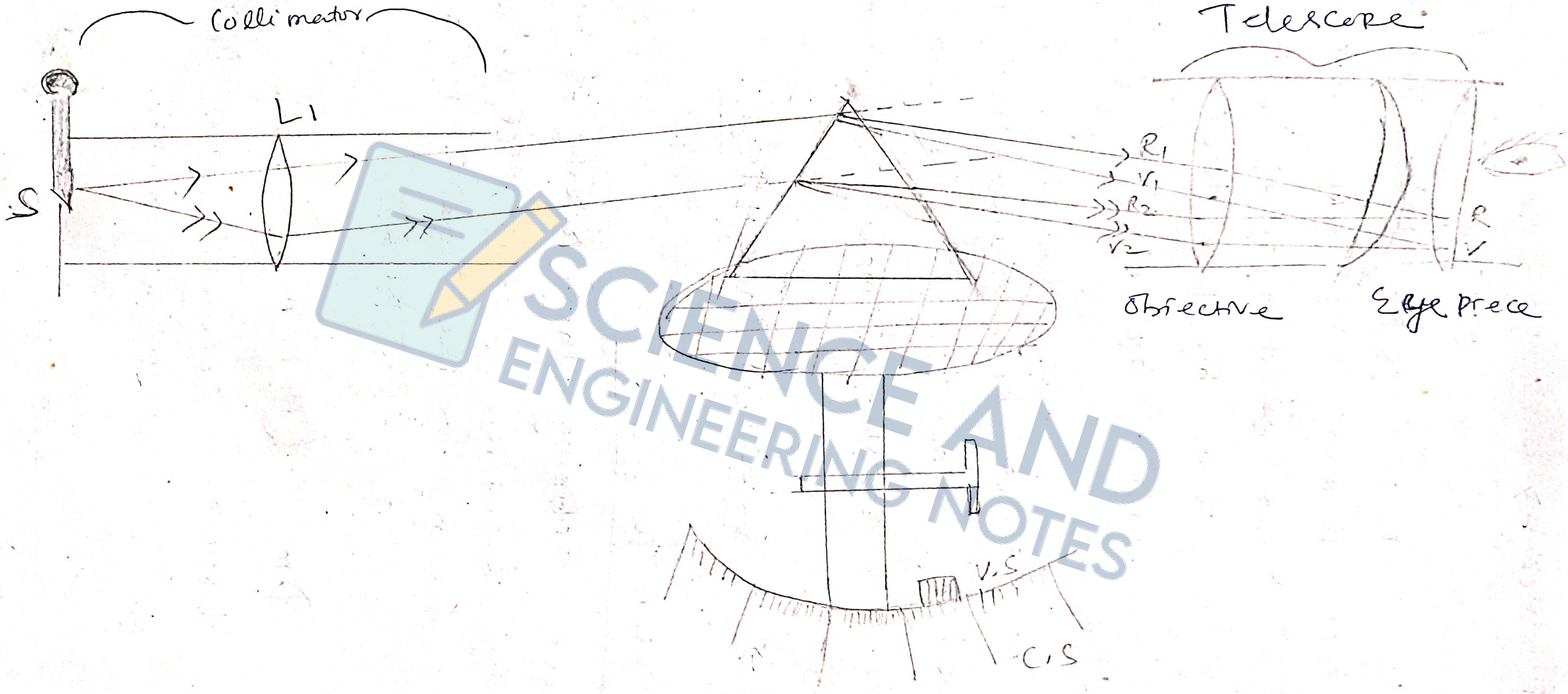
The main parts of the spectrometer are (i) Collimator (ii) Prism table (iii) Telescope.

(i) Collimator →

It is a metallic tube having an adjustable slit at one end which faces the source of light. There is a convex lense inside it which focus is at the slit. There by the rays become parallel.

(ii) Prism table →

It is a horizontal platform having circular shape provided with screws. Its height Can be adjusted. The prism or grating is placed on it. There is a circular scale at the bottom of the prism table which reads from 0° to 360°. Two verniers are ~~provided~~ provided having least Count 30 second (30″)



Collimator
Telescope
S
L1
R1
V1
R2
V2
R
V
Objective
Eye piece
V.S
C.S

Prism table

(iii) Telescope →

It consists of an objective, convex lens and a compound eye ~~piece~~ piece. The focusing of the parallel rays after dispersion by the prism or diffraction by the grating is done by the telescope. Cross wire is fitted by which reading can be taken.

The entire instrument is kept in position by means of 3 screws and these are to be adjusted to make the instrument horizontal.

Fig → See the (next+1) page.

Dispersive power (ω)

Before defining this property of an optically transparent medium, let us take the expression for the angle of min^m deviation due to a thin prism.

$$D_m = (\mu - 1)A$$

For different coloured lights, D_m changes due to change of μ. (Since A is same for all)

$\therefore (D_m)_v = (\mu_v - 1)A$, ~~where~~ for the violet coloured light.

$(D_m)_r = (\mu_r - 1)A$, for red coloured light.

Thus $(D_m)_v - (D_m)_r = (\mu_v - 1)A - (\mu_r - 1)A$
$= (\mu_v - \mu_r)A$ ——— (1)

For the mean colour of light which is yellow, $(D_m)_y = (\mu_y - 1)A$ ——— (2)

Dividing eqn (1) by eqn (2), we get

$$\frac{(D_m)_v - (D_m)_r}{(D_m)_y} = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)} = \frac{\Delta\mu}{(\mu_y - 1)} = \omega$$

The dispersive power of an optically transparent medium can be defined as the ratio of change in the angle of min^m deviation between the two extreme colours of light and the angle of min^m deviation for the mean colour of light (yellow).

— o —

Kinds of spectra

- Emission spectra
  - Line spectra
  - Band spectra
  - Continuous spectra
- Absorption spectra
  - Line spectra
  - Band spectra

Line emission spectra →

When a gas or vapour in atomic form is taken in a discharge tube and excited by a high voltage (when pressure of the gas is of the order of 1 mm of mercury pressure), the gas or vapour emits a light which depends on the nature of the gas or vapour. For example, Helium gives red coloured light, sodium vapour gives golden yellow light, mercury vapour gives silver white coloured light etc.

This light when analysed by a grating spectrometer is found to consists of many sharp coloured lines which characterize the gas or vapour.

For ex, sodium vapour contains

Only two yellow coloured lines with wave lengths $\lambda_1 = 5890 A^0$ and $\lambda_2 = 5896 A^0$

Similarly for other elements, the number of lines are also fixed.

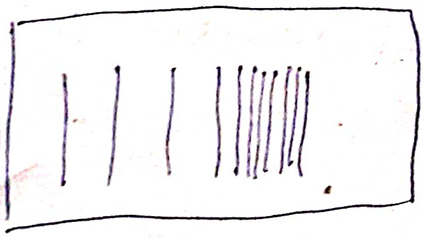
Band emission spectrum (Molecular spectrum)

When a gas or vapour in molecular form is taken in a discharge tube and excited by a high voltage when the pressure of the gas is low, about 1 mm pressure, then the gas or vapour emits a light characteristic of the gas or vapour.

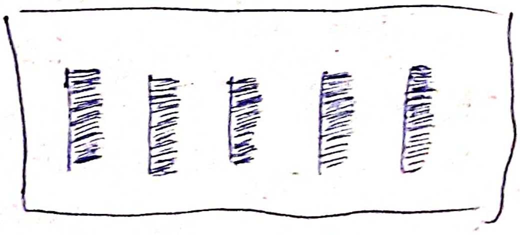
For ex, H_2 gas gives pink coloured light etc.

CO_2 gas gives greenish light

The light when analysed by a grating spectrometer is found to consist of several coloured bands with dark spaces between. These bands are sharp on one side and smeared on the other side. These bands characterize the gas or vapour in molecular form.



Line spectrum



Band spectrum

Continuous emission spectrum →

When a metallic wire is heated, at first it acquires red colour which gradually changes to orange and then ultimately to yellow and then becomes white. This white light when analysed by a grating spectrometer is found to consists of several coloured bands placed side by side without any discontinuity.

Line absorption spectrum →

When white light is passed through a glass chamber containing a gas in atomic form, the emergent light when analysed is found to consists of a few dark lines on a lighted back ground. Study of these dark lines indicates the type of gas present inside the box.

Band absorption spectrum

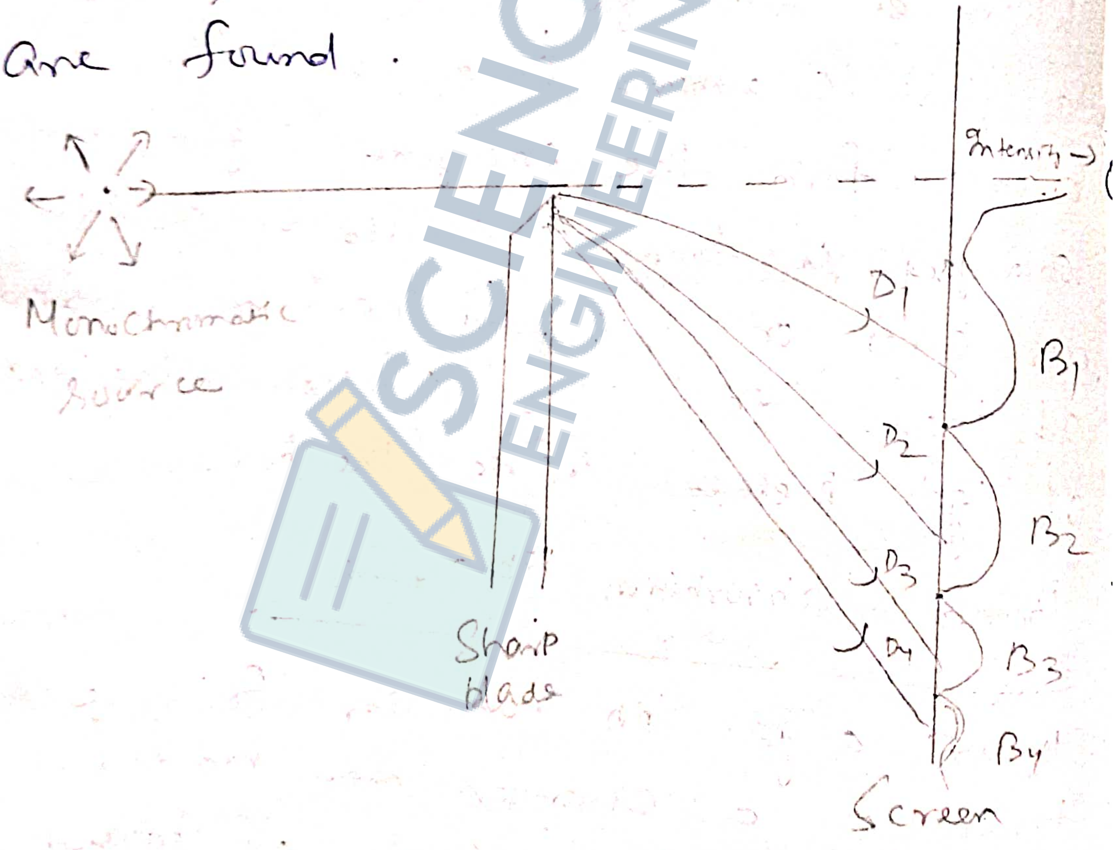
When a gas in molecular form be taken in a chamber ~~maa~~ made out of glass and white light is passed through it, then the emergent light when analysed is found to consists of a few dark bands in a lighted background. These bands ~~between~~ characterize the type of gas present inside the container.

Diffraction of light

The bending of light waves around an obstacle is called diffraction. The bending is clearly visible when the size of the obstacle is of the ~~form~~ same order as that of the wavelength of light.

EX → Diffraction at a sharp edge

In the ~~diag~~ region of geometrical shadow, alternate bright, dark spots are found.



But diffraction fringes differ from interference fringes as they are not equidistant and intensity of fringes go on decreasing.

This kind of dibbraction phenomena are called fresnel type of dibbraction because the source and the screen are at finite distances from the obstacle.

Another kind of dibbraction phenomena is called ~~F~~ Fraunhufer type of dibbraction are observed when the source and the screen are ~~ett~~ effectively at infanite ~~o~~ distances from the obstacle.

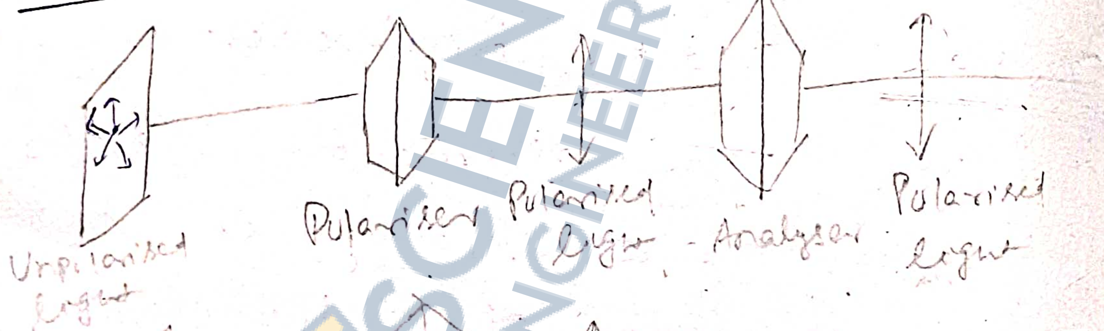
Here also the fringes are not equispace and the brightnesses are not the same. ~~The~~ dibbraction due to the 'N' slits (called grating) is a Fraunhofer type of diffraction.

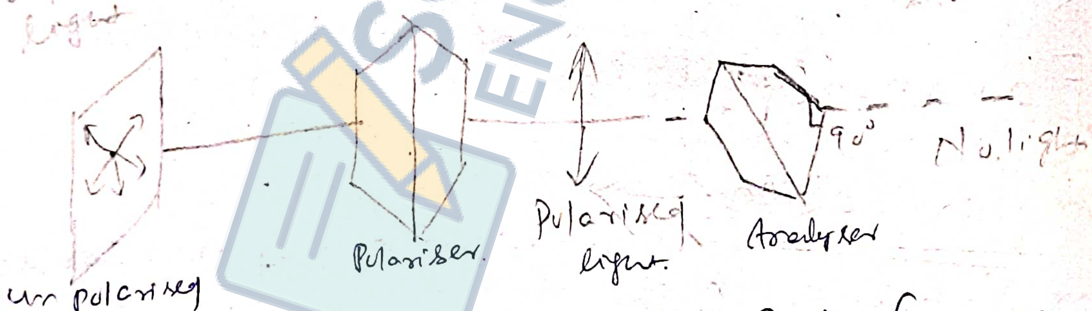
Polarisation →

Ordinary light is called unpolarised. Because it contains vibrations in all possible directions which are perpendicular to the direction of propagation.

Tourmaline is a naturally occuring crystal having a peculiar property of allowing only those vibrations to pass through it which are parallel to its axis of symmetry. The emergent light is called polarised. This can be tested by another tourmaline crystal called analyser. When the analyser is parallel to the polariser, there is maxm brightness. When the axes are $\perp$, ~~lig~~ light is completely cut off. At another orientations, there is some amount of light

Fig →

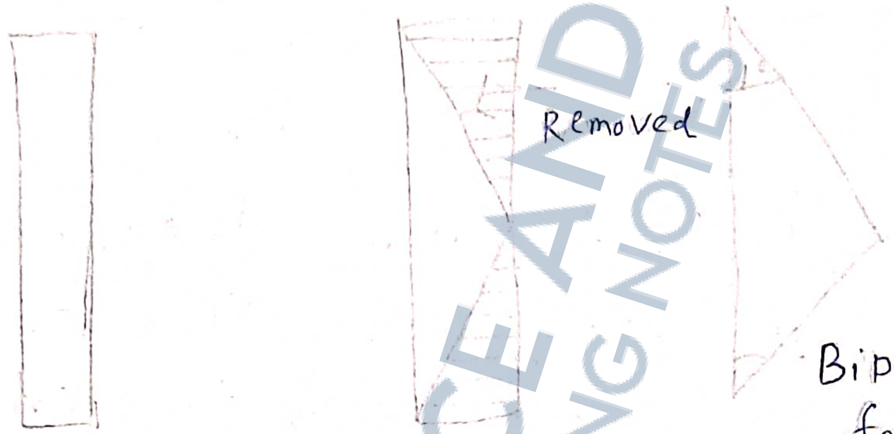




Methods of producing Interference

1. Fresnel's biprism :-

It is a device to produce interference fringes on a screen. The biprism is made out of a uniform glass plate by polishing such that angle of the prism become nearly equal to 1°. Due to refraction of light waves coming from the monochromatic source due to the biprism, two coherent, virtual sources S_1 & S_2 develop and interference fringes are formed on the screen. the shaded portion of the screen. This portion gets light both from S_1 & S_2.



Fringe width is given by

$$\beta = \frac{D\lambda}{d} = \frac{(a+b)\lambda}{d}.$$

where a = distance between the source of light & the biprism.

b = distance between the biprism and the screen.

λ = wave length of the ~~light~~ monochromatic source of light.

For Sodium, $\lambda = 5893\ A^\circ$

d = distance between the two virtual co-herent sources S_1 & S_2

Since it is not possible to locate S_1 & S_2, convex lens of suitable focal length is to be placed in between the biprism &

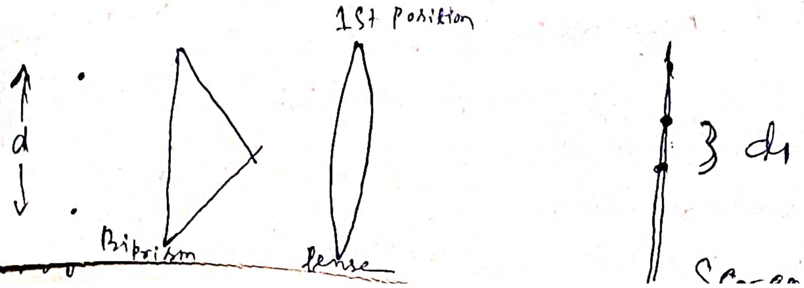
Screen.

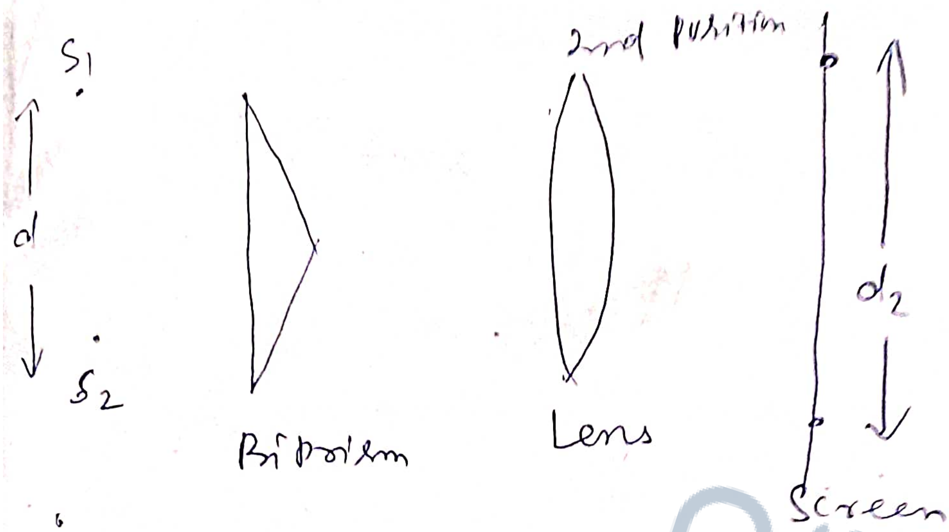
For two such positions of the convex lens, two real images (not very sharp) of S_1 & S_2 are found on the screen as shown on the figure. It can be proved that $d = \sqrt{d_1 d_2}$

where d_1 = width of the image on the screen for the 1st position of the convex lens.

d_2 = width of the images on the screen for the 2nd position of convex lens.

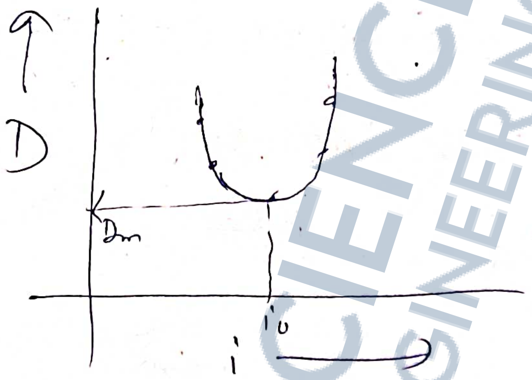
$$\therefore \beta = \frac{(a+b)\lambda}{\sqrt{d_1 d_2}}$$





Extra Theory for the Biprism

For a prism, refractive index of the material of the prism is given by

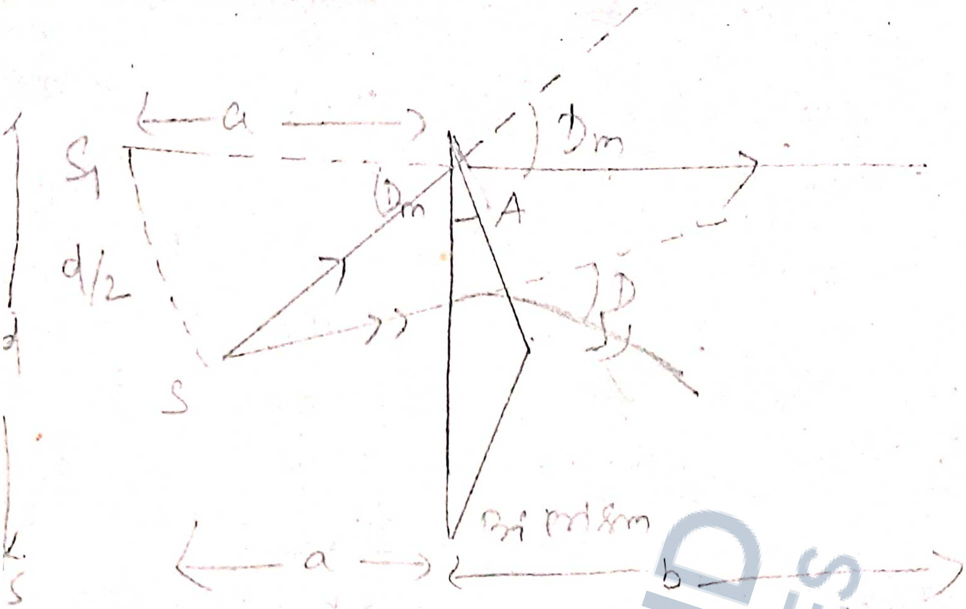


the expression $\mu = \dfrac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$

For a thin prism A is very small & D_m will also small. When expressed in radian, we can approximate $\sin\theta \simeq \theta$

Thus $\mu = \dfrac{\frac{A+D_m}{2}}{\frac{A}{2}} = 1 + \dfrac{D_m}{A}$

or $\mu - 1 = \dfrac{D_m}{A}$



or ~~for~~ $D_m = (\mu-1)A$

where $A \rightarrow$ Angle of the prism expressed in radian.

D_m in radian $= \dfrac{\text{Arc length}}{\text{Radius}}$

$\therefore D_m = \dfrac{S_1 S}{a} = \dfrac{d/2}{a} = \dfrac{d}{2a}$

$\therefore d = D_m \cdot 2a = (\mu-1)A \cdot 2a$

$\therefore \beta =$ Fringe width $= \dfrac{D\lambda}{d}$

$= \dfrac{D\lambda}{d} = \dfrac{(a+b)\lambda}{(\mu-1)A \cdot 2a}$

Problem $\rightarrow$

1. A Fresnel's biprism having an angle of 1° A & refractive index 1.5 forms interference fringes on a screen placed 80cm from the biprism. If the distance between

the source and the biprism is 20 cm
(a) Find the fringe separation when the wavelength of light used is 6900 A°
(b) 4600 A°.

Ans We know that $180^\circ = \pi$ rad
$1^\circ = \pi/180$ rad
$\alpha = A$

$a = 20$ cm , $\lambda = 6900$ A° $= 6900 \times 10^{-8}$ cm
$b = 80$ cm , $\mu = 1.5$

(a) Fringe width

$$\beta = \frac{(a+b)\lambda}{(\mu-1)A \cdot 2a}$$

$$= \frac{(20+80) \times 6900 \times 10^{-8}}{(1.5-1) \times \pi/180 \times 2 \times 20}$$

$$= \frac{100 \times 6900 \times 10^{-8}}{0.5 \times \pi/180 \times 2 \times 20} = \frac{5 \times 69 \times 10^{-6} \times 10}{5 \times \frac{\pi}{180} \times 2}$$

$$= \frac{69 \times 180 \times 10^{-5}}{2 \times 3.14} = 1977.707 \times 10^{-5} \text{ cm}$$

(b) Fringe width

$$= \beta = \frac{(a+b)\lambda}{(\mu-1)A \cdot 2a} = \frac{(20+80) \times 4600 \times 10^{-8}}{(1.5-1)\frac{\pi}{180} \times 2 \times 20}$$

$$= \frac{100 \times 46 \times 10^{-6}}{0.5 \times \frac{\pi}{180} \times 2 \times 20} = \frac{46 \times 10^{-5} \times 180}{2 \times 3.14} = 1318.4713 \times 10^{-5} \text{ cm}$$

∴ Fringe widths are 1977.707×10^{-5} cm and

β 18.4713 $\times 10^{-5}$ cm (Ans).

Second Application →

Lloyd Mirror →

It is device to produce interference fringes by the method of reflection. A monochromatic broad source of light is kept near a plane mirror PQ. The light waves coming directly from the source 'S' & that are reflected from the mirror PQ (All of which appear to come from S'), superpose on the screen to produce alternate and equidistant bright and dark spots are formed on the screen. These interference fringes are formed in the shaded region, because it gets light both from S and S'.

$$\beta = \frac{D\lambda}{d}$$

where D → Distance between 'S' & the screen

d → distance between two slits S_1 and S_2 = SS' (in our fig

λ → Wave length of the monochromatic source.

If the screen be gradually shifted towards the plane mirror, ultimately touching it at Q, then one expects a bright spot at Q, because $SQ = S'Q$ and $\Delta = 0$. But actually a dark ~~spo~~ spot is seen at Q - This can be explained only if we will accept stoke's idea that there occures an abrupt change of phase by π due to reflection from a surface backed by denser medium.

For path difference λ, phase difference $= 2\pi$ radian.

Thus, for path difference $\lambda/2$, phase difference $= \pi$ radian.

Hence a phase difference of π rad introduces a path difference $\lambda/2$

$= 1 \cdot \frac{\lambda}{2}$ = odd multiple of $\lambda/2$.

Thus, the formation of dark spot at O is explained.

<u>Lloyd's</u> experiment can be regarded as a visible demonstration of the truthfulness of Stokes idea