'Jay Jagannath'

(Counting) ⟷ **Permutation** (How to count without counting)

Fundamental principle of Counting (F.P.C)

Or Fundamental Rule.

A number of multiple choices are to be made. There are m_1 possibilities for the 1st choice, m_2 possibilities for the 2nd choice, m_3 for the third etc. If these choices can be combined freely then total number of possibilities for the whole set of choices is equal to $m_1 \times m_2 \times m_3 \times \ldots$

Example:

Bread or rice — 2 possibilities choose one

H. Dal, M. Dal, Dalma — 3 possibilities choose one.

V. Curry, F. Curry, E. Curry, Ch. Curry — 4 possibilities choose one.

In how many ways one can choose his dinner.

Ans: $2 \times 3 \times 4 = 24$ (By F.P.C)

There are ten true-false questions. How many sequence of answers are possible?

Ans → For each question there are 2 possibilities true or false. The total number of possibilities for the ten question

$= 2 \times 2 \times 2 - - - - - - -$ 10 times (By F.P.C)

$= 2^{10}$ (Ans)

Q → ② In howmany ways can 5 women draw water from 5 taps if no tap remains unused?

$5 \times 4 \times 3 \times 2 \times 1$

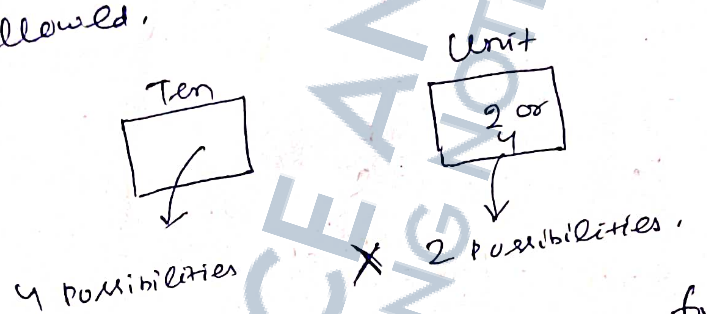
Ans:- For the 1st tap any one among 5 women can draw water. For this there are 5 possibilities. One woman being kept at 1st tap, there are 4 women left for the 2nd tap, so there are 4 possibilities for the 2nd tap. After keeping one woman at 2nd tap there are 3 women left for the 3rd tap and there are 3 possibilities for the 3rd tap. Similarly 2 possibilities for the 4th tap. And 1 possibilities for the 5th tap.

∴ The total number of ways 5 women can drawn water ~~is~~ in

$= 5 \times 4 \times 3 \times 2 \times 1$
$= 120$ ways (By F.P.C)

Q→3 : How many 2-digits even numbers can be formed from the digits 1, 2, 3, 4 and 5 if repitation of digits is not allowed.

Ans:→



2-digit even numbers can be formed using 1, 2, 3, 4, & 5. For the unit place any one of 2 or 4 can be placed. So there are 2 possibilities for the unit place.

Keeping any one of 2 or 4 in unit place the ten place can be filled up by rest 4 digits in 4 different ways.

∴ The total number of even numbers formed = $2 \times 4 = 8$ (By F.P.C)

Q→4 How many ways can a President a secy and a treasurer be chosen out of 20 peoples?

Ans: $20 \times 19 \times 18$ (by F. F. C)

Factorial n ($\underline{|n}$) or n-factorial ($n!$)

The product of 1st n natural numbers is called factorial n or n-factorial. It is denoted by $\underline{|n}$ or $n!$

i.e. $\underline{|n}$ or $n! = 1 \times 2 \times 3 \times 4 \cdots\cdots n$

$\underline{|5} = 1 \times 2 \times 3 \times 4 \times 5 = 120$

$\underline{|1} = 1$

$\underline{|2} = 1 \times 2 = 2$

we define $\underline{|0} = 1$

Q→ Prove that $\underline{|0} = 1$

we know

$\underline{|n} = 1 \times 2 \times 3 \cdots\cdots \times n$

$= 1 \times 2 \times 3 \cdots\cdots (n-r)(n-r+1)(n-r+2) \cdots\cdots (n-1)\, n$

$= \underline{|(n-r)} \cdot (n-r+1)(n-r+2) \cdots\cdots (n-1)\, n$

$\Rightarrow \dfrac{\underline{|n}}{\underline{|n-r}} = (n-r+1)(n-r+2) \cdots\cdots (n-1)\, n$

for $r = n$, we have

$\dfrac{\underline{|n}}{\underline{|0}} = 1 \times 2 \cdots\cdots \times n = \underline{|n}$

$\Rightarrow \underline{|0} = \dfrac{\underline{|n}}{\underline{|n}} = 1$ (Proved)

Imp. **Q** Prove that $(2n)! = 2^n \cdot n! \{1.3.5 \cdots (2n-1)\}$

Proof: $(2n)! = 1 \times 2 \times 3 \cdots (2n-2)(2n-1)\, 2n$

$= \{1 \times 3 \times 5 \cdots (2n-1)\}\{2 \times 4 \times 6 \cdots \times 2n\}$

$= \{1 \times 3 \times 5 \times \cdots (2n-1)\}\{(2 \times 1)(2 \times 2) \cdots (2 \times n)\}$

$= \{1 \times 3 \times 5 \times \cdots (2n-1)\}\, 2^n \{1 \times 2 \times 3 \cdots n\}$

$= \{1 \times 3 \times 5 \cdots 2n-1\}\, 2^n \cdot n!$ (Proved)

Defination of permutation definite (Arrangement in a order)

A permutation of given set of objects in an arrangement of these objects in a definite order taking some or all at a time.

The number of permutations of n different objects taken r at a time $(r \leq n)$ is defined by ^nP_r or $P(n,r)$ or $(n)_r$.

Imp **Theom-1:** The number of permutations of n different objects taken r at a time $r \leq n$ is $\frac{n!}{(n-r)!}$ i.e $P(n,r) = \frac{n!}{(n-r)!}$

Proof: The number of permutations of n different objects taken r at a time is the same as the number of different

Arrangements of n objects on r places in a row.

1st	2nd	3rd	- - - - - - - - - -	rth
$n \times$	$n-1 \times$	$n-2$		$\times\, n-(r-1) = n-r+1$

The 1st place can be filled up by anyone of n objects in n different ways. Keeping one object in 1st place there are $(n-1)$ objects ~~left~~ left for the 2nd place. So the 2nd place can be filled up in $(n-1)$ ways.

$\therefore$ The 1st two places can be filled up in $n(n-1)$ different ways (By F.P.C)

After the 1st two places are filled up there are $(n-2)$ objects left for the 3rd place. So the 3rd place can be filled up in $(n-2)$ different ways.

$\therefore$ The 1st 3 places can be filled up $n(n-1)(n-2)$ different ways (By F.P.C)

Similarly continuing up to r^{th} place the r^{th} place can be filled up in $n-r+1$ different ways. So all the r places can be filled up in $n(n-1)(n-2)$. - - - - - $(n-r+1)$ different ways. (By F.P.C)

Hence the total no. of ways of filling all the r places.

$$= n(n-1)(n-2) \cdots (n-r+1)$$

i.e $P(n,r) = n(n-1)(n-2) \cdots (n-r+1)$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)(n-r-1)\cdots 1}{(n-r)(n-r-1)\cdots 1}$$

$$= \frac{n!}{(n-r)!}$$

$\therefore P(n,r) = \frac{n!}{(n-r)!}$ (Proved)

Corollary : The no of permutations of n different objects taken all at a time is $P(n,n) = n!$

$\underset{n}{\square}\ \underset{n-1}{\square} \cdots \underset{1}{\square}$

Proof is similar.

Th^m :2 : The no. of permutation of n different objects taken r at a time in which a particular object always occures is $r \times P(n-1, r-1)$

Proof : Let the n different objects be $a_1, a_2, a_3 \cdots\cdots a_n$

A particular object ~~may~~ say a_1 always occures. So we are to choose $(r-1)$ object from the rest $(n-1)$ objects & this can be done in $P(n-1, r-1)$ different ways.

In each time of these $P(n-1, r-1)$ permutations, the particular object a_1 can be arranged in r different ways.

$\therefore$ The total no of permutations

$$= r\,P(n-1, r-1)$$

(Proved)

Th^m - 3

The no. of permutations of n different objects taken r at a time in which one particular object never occures is

$$P(n-1, r)$$

Proof :- Here there are n different objects & we have to take r at a time and a particular object never occures. So we are to arrange the rest $(n-1)$ objects taken r at a time. This can be done in $P(n-1, r)$ different ways.

$\therefore$ The no. of permutations of n different objects taken r at a time when particular object occures is

$P(n-1, r)$ (Proved)

Imp for problems

Thm - 4 : The no of permutations of n objects taken all at a time in which m are of one kind and the rest $(n-m)$ are of another kind is $\dfrac{n!}{m!\,(n-m)!}$

let be x arrangement

○○○○○○ (6 white ball, $6!$) ●●●● (4 black ball, $4!$)

Each time x : $6!\,4!$

$x \cdot 6!\,4! = 10!$

Proof : There are n ~~digits~~ objects where m are of one kind and $n-m$ are of another kind. We are to arrange them taking all at a time.

Let the required no. of permutations be x.

Consider any one of these x permutations. In this particular ~~permut~~ permutation if the m objects are different & also different from remaining $n-m$ objects then there are $m!$ permutations of these m objects.

Again if each of the $(n-m)$ objects is considered distinct and different from the m objects of 1st kind then we have $(n-m)!$ permutations.

Hence this particular one of x permutations will give rise to $m!\,(n-m)!$ permutations if all objects are distinct.

Hence if all objects are distinct

The x permutation will give ~~[illegible]~~ rise to

x m! (n-m)! permutations.

But if all n objects are distinct

then the no of permutations = n!

∴ $x\, m!\,(n-m)! = n!$

Calculus

$\Rightarrow x = \dfrac{n!}{m!\,(n-m)!}$

Ex: In how many different ways

Can the ~~[illegible]~~ letters of the word

MATHEMATICS be arranged?

Ans → The word MATHEMATICS contains

11 ~~[illegible]~~ letters. Out of which 2 M's,

2 A's, 2 T's, one H, one E, one I, one

C and one S.

∴ The no. of ~~[illegible]~~ permutation

$= \dfrac{11!}{2!\,2!\,2!\,1!\,1!\,1!\,1!\,1!}$

= 4989600

The no of permutations of n objects taken r at a time when each object may be repeated up to r times in any arrangement

Then-5 is n^r

Proof: The number of permutation of n objects taken r at a time when each object may be repeated upto r times is same as the no. of different arrangements of n objects in r places in a row.

1st 2nd 3rd - - - - - - - - rth

$n \times n \times n$ - - - - - - $\times n$

The 1st place can be filled up by any one of n objects in 'n' different ways.

Since each object is repeated up to 'r' times, the 2nd place can be filled up in 'n' different ways.

Hence the 1st ~~...~~ two places can be filled up in $n \times n = n^2$ (By F.P.C) ways.

The 3rd place can be filled up in n ways so the 1st 3 places can be filled up in $n \times n \times n = n^3$ ways By (F.P.C)

Proceeding like wise the rth place can be filled up in n ways. So all the r places can be filled up in $n \times n \times n$ ---r times ie n^r ways (By F.P.C).

$\therefore$ The no. of permutations $= n^r$ (Proved)

Circular permutations

Q→① In how many ways can 5 boys form a ring ?

Ans:- Let A, B, C, D and E be the 5 boys. Let us make one of them say A fixed and then find the no. of arrangements of remaining 4 boys taken all together. This can be done in $P(4,4) = 4! = 24$ different ways. Hence the required no of permutations = 24.

Note →

Arrangements in a row are called linear permutations. Arrangements in a circle are called as circular permutations.

In a circular permutation there will be no ends to set. Hence the no. of permutations of n different object taken all at a time in linear permutation is $n!$ but in circular permutation it is $(n-1)!$

Q→2 In how many 5 beads of unlike colours are threaded on a necklace?

Ans :-

Case-I

There are 5 beads of unlike colours. In this case arrangements, if the necklace containing beads in any order be turned over on its other side, the clockwise & anti-clockwise arrangements become identical.

Hence the no. of permutation $= \frac{4!}{2} = 12$

Case-II

Suppose the clockwise & anti-clockwise arrangements are not identical then the no. of permutations $= 4!$

Q -> 3 In how many ways can 5 persons be seated at a round table?

Ans -> Case-I : If the ~~person~~ position of persons with respect to the table is fixed then no. of permutation $= 5!$

Case-II If the position of persons w.r.to the table is not fixed then the no. of permutations $= 4!$

Q -> Imp In how many ways 5 letters can be posted in 4 letter boxes?

Ans: Each ~~B~~ letter may be posted in 4 ways. Since there are 5 letters, the total no. of ways

$= 4 \times 4 \times 4 \cdots 5 \text{ times} = 4^5 = 1024.$

Problem page ~~117-119~~ 69

8. (3(b) Ex) $P(m+n, 2) = 56$

$\Rightarrow P(x, 2) = 56$ where $x = m+n$

$\Rightarrow \frac{x}{\lfloor x-2} = 56$

$\Rightarrow (x-1)x = 56$

$\Rightarrow x^2 - x - 56 = 0$

$\Rightarrow x = 8$ or $x = -7$

But x cannot be $-ve$.

$\therefore x = 8 \Rightarrow m+n = 8$ —(i)

$P(m-n, 2) = 12$

$\Rightarrow P(y, 2) = 12$ where $y = m-n$

$\Rightarrow \frac{\lfloor y}{\lfloor y-2} = 12$ $\Rightarrow y(y-1) = 12$

$\Rightarrow y^2 - y - 12 = 0$

$\Rightarrow y = 4$ or -3

But y is not $-ve$.

$\therefore y = 4 \Rightarrow m-n = 4$ —(ii)

$m+n = 8,\ m-n = 4 \Rightarrow 2m = 12$

$\Rightarrow m = 6$

$\therefore n = 2$.

Q → In how many different ways can 5 boys and 3 girls sit in rows, so that no 2 girls are together?

5 boys can be seated and within them there are 6 places. The 3 girls can be seated in 6 places in $P(6,3)$ ways. Again each time the 5 boys can be arranged among themselves in $5!$ ways.

$\therefore$ The total no. of ways required $= P(6,3)\, 5!$
$= 14400$.

Combination

A combination of a given set of objects is a selection of these objects taking some or all at a time where the order of selection of the objects is immaterial.

The no. of combination of n different objects taken r at a time $(r \leq n)$ is denoted by.

$^{n}C_{r}$ or $C(n,r)$ or $\binom{n}{r}$

Imp

Theorem-6

The no of combinations of 'n' different objects taken r at a time is $\frac{n!}{r!(n-r)!}$ i.e $C(n,r) = \frac{n!}{r!(n-r)!}$

Proof →

Let the no. of combinations of n different objects taken r at a time be x.

Consider any one of these x combination, In this particular combination, if we arrange among the r objects we get $r!$ permutations.

Hence each combination gives rise to $r!$ permutations.

∴ x combinations gives rise to $x r!$ permutations. But no. of permutations of 'n' different objects taken r at a time is $P(n,r)$.

$$\therefore x r! = P(n,r)$$

$$\Rightarrow x r! = \frac{n!}{(n-r)!}$$

$$\Rightarrow x = \frac{n!}{r!(n-r)!}$$

$$\text{i.e } C(n,r) = \frac{n!}{r!(n-r)!}$$

(Proved)

Note: $C(n,r)\, r! = P(n,r)$

i.e The no of permutations of n different objects taken r at a time is r! more than the no of combinations of these n objects taken r at a time.

Deduction:

(i) $C(n,r) = C(n, n-r)$

Proof: $C(n, n-r) = \dfrac{n!}{(n-r)!\,(n-(n-r))!}$

$= \dfrac{n!}{(n-r)!\cdot r!} = \dfrac{n!}{r!\cdot (n-r)!}$

$= C(n,r)$ (proved)

Note: $C(n,r)$ & $C(n, n-r)$ are called complementary combinations.

Note: If $C(n,r) = C(n,p)$ then $r = p$ or $r = n-p$

Q → If $C(8,r) = C(8,3)$ & $r \neq 3$, then find r

Ans: $r = 5$

(2) $\dfrac{C(n,r)}{C(n,r-1)} = \dfrac{n-r+1}{r}$

Proof: $\dfrac{C(n,r)}{C(n,r-1)}$

Proof :

$$\frac{C(n,r)}{C(n,r-1)} = \frac{\dfrac{\lfloor\underline{n}}{\lfloor\underline{r}\cdot\lfloor\underline{n-r}}}{\dfrac{\lfloor\underline{n}}{\lfloor\underline{r-1}\;\lfloor\underline{n-r+1}}}$$

$$= \frac{\cancel{\lfloor\underline{n}}}{\lfloor\underline{r}\,\lfloor\underline{n-r}} \times \frac{\lfloor\underline{r-1}\cdot\lfloor\underline{n-r+1}}{\cancel{\lfloor\underline{n}}}$$

$$= \frac{1}{\cancel{\lfloor\underline{r-1}}\cdot r \cdot \cancel{\lfloor\underline{n-r}}} \times \cancel{\lfloor\underline{r-1}}\cdot\cancel{\lfloor\underline{n-r}}\,(n-r+1)$$

$$= \frac{n-r+1}{r} \quad \text{(Proved)}$$

③ I.V. imp

$$C(n,r) + C(n,r-1) = C(n+1,r)$$

L.H.S $\quad C(n,r) + C(n,r-1)$

$$= \frac{\lfloor\underline{n}}{\lfloor\underline{r}\cdot\lfloor\underline{n-r}} + \frac{\lfloor\underline{n}}{\lfloor\underline{r-1}\cdot\lfloor\underline{n-r+1}}$$

$$= \frac{\lfloor\underline{n}}{\lfloor\underline{r-1}\cdot r\cdot\lfloor\underline{n-r}} + \frac{\lfloor\underline{n}}{\lfloor\underline{r-1}\cdot\lfloor\underline{n-r}\cdot(n-r+1)}$$

$$= \frac{\lfloor\underline{n}}{\lfloor\underline{r-1}\cdot\lfloor\underline{n-r}}\left\{\frac{1}{r} + \frac{1}{n-r+1}\right\}$$

$$= \frac{\lfloor\underline{n}}{\lfloor\underline{r-1}\cdot\lfloor\underline{n-r}}\left\{\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right\}$$

$$= \frac{\lfloor\underline{n}\;(n+1)}{\lfloor\underline{r-1}\cdot r\cdot\lfloor\underline{n-r}\cdot(n-r+1)} = \frac{\lfloor\underline{n+1}}{\lfloor\underline{r}\cdot\lfloor\underline{n-r+1}}$$

$$= \frac{\lfloor\underline{n+1}}{\lfloor\underline{r}\cdot\lfloor\underline{n+1-r}} = C(n+1,r) = \text{R.H.S}$$

□

Them - 7 → The no. of combinations of n different objects taken r at a time in which

(i) One particular object always occures in $C(n-1, r-1)$

(ii) One particular object never occures in $C(n-1, r)$

Proof ÷ (i) Since one particular object is to occure always, so we are to select $(r-1)$ objects from the rest $(n-1)$ objects in $C(n-1, r-1)$ ways.

(ii) Since one particular object never occures, so we are to select r objects from the rest $(n-1)$ objects in $C(n-1, r)$ ways (proved)

Note ÷ Arrangement, ordering, line up etc denote permutations. Selection, subset, Committee, group etc denote combinations.

4.1.m

Practical ex $C(n,r) = C(n, n-r)$

Q → From 5 consonants and 4 vowels how many words can be constructed using 3 consonants and 2 vowels?

Soln → 3 Consonants can be chosen out of 5 consonants in $C(5,3)$ ways. Each time the 2 vowels can be chosen out of 4 vowels in $C(4,2)$ ways.

∴ 3 Consonants & 2 vowels can be chosen out of 5 consonants & 4 vowels in $C(5,3) \cdot C(4,2)$ ways.

Now 3 consonants & 2 vowels can be arranged among themselves in $5!$ ways. (my words: 3 consonant & 2 vowels are 5 distinct letters, so they are arranged in 5! ways)

∴ The no of words formed

$= C(5,3)\, C(4,2)\, 5!$ (Ans)

=

Q → In how many ways a committee of 4 gentlemen and 3 ladies can be formed out of 8 gentleman and 6 ladies. How many different committees of 7 can be formed with at least 3 ladies?

Sol^n: $C(8,4) \cdot C(6,3)$ calculate →

2nd part: $3l + 4g \rightarrow C(6,3)\,C(8,4) +$

$4l + 3g \longrightarrow C(6,4)\ C(8,3) +$

$5l + 2g \longrightarrow C(6,5)\ C(8,2) +$

$6l + 1g \longrightarrow C(6,6)\ C(8,1)$

Add

Q-1 How many triangles can be formed by joining the angular points of a decagon & how many ~~diagram~~ diagonals is it has?

$\boxed{\text{Try on rectangle}}$

Soln: In a decagon there are 10 angular points & 10 sides. For a triangle we require 3 angular points at a time

$\therefore$ For forming triangles we have to select 3 pts out of 10 points $C(10,3)$ ways

(Here for a triangle the order of angular points is immaterial)

$\therefore$ No. of triangles formed $= C(10,3)$
$= 120$ (Ans)

2nd Part: In a decagon there are 10 angular points and 10 sides. ~~For~~ a diagonal we require 2 pts at a time.

The no. of straight lines formed $= C(10,2)$

But there are 10 sides.

$\therefore$ The no of diagonals $= C(10,2) - 10$

$= 35$ (Ans)

Q → A bag contains 4 red, 3 white, & 2 black balls. 3 balls are drawn at random. Determine the no of ways of selecting at least one white ball in the selection.

Soln ÷

4 red	3 white	2 black	
0 +	3 +	0	$\rightarrow C(4,0)\,C(3,3)\,C(2,0)$
1 +	2 +	0	$\rightarrow C(4,1)\,C(3,2)\,C(2,0)$
0 +	2 +	1	$\rightarrow C(4,0)\,C(3,2)\,C(2,1)$
1 +	1 +	1	$\rightarrow C(4,1)\,C(3,1)\,C(2,1)$
0 +	1 +	2	$\rightarrow C(4,0)\,C(3,1)\,C(2,2)$
2 +	1 +	0	$\rightarrow C(4,2)\,C(3,1)\,C(2,0)$

$^4C_3 - 4$

Add

Imp Q → Prove that the product of p consecutive natural numbers is divisible by p!

Proof ÷ Let the greatest among p consecutive natural numbers be n.

$\therefore$ The p consecutive natural numbers are;

$n, n-1, n-2, \text{------------} n-p+1$

Their product

$= n \times (n-1)(n-2) \cdot \text{---------} (n-p+1)$

$$= \frac{n(n-1)(n-2) \cdot \text{----} (n-p+1)(n-p) \cdots 2\times1}{(n-p)\times \text{----} 2\times1}$$

$$= \frac{n!}{(n-p)!} = \frac{n!}{(n-p)!\,p!}\,p! = C(n,p)\,p!$$

Here $C(n,p)$ is the no of combinations of 'n' different objects taken ~~at~~ p at a time which is an integer.

$\therefore$ The product = (Some integer) $p!$

$\therefore$ The product is divisible by $p!$ (Proved)

16 Pg. 399 Topics $\rightarrow$ How many divisors are there of the number 115500 excluding 1 and the number itself.

Ans: $115500 = 2^2 \times 3^1 \times 5^3 \times 7 \times 11$

For finding divisors we may take 2 ~~ones~~ 0, 1 or 2 times i.e 2 can be taken in 3 different ways.

Similarly 3 can be taken in 2 different ways.

5 " " " " 4 " "

7 " " " " 2 " "

11 " " " " " " "

∴ The no. of divisors

$= 3\times2\times4\times2\times2 = 96$ by (F.P.C)

Excluding the two divisors, 1 and the no of itself $= 96-2 = 94$

Q-) What is the ~~no~~ rank of the word 'Mother' when its letters are arranged in the dictionary.

Ans → There are 6 places to be filled up with letters M, O, T, H, E, R such that all the words formed come before the word ~~mother~~ "MOTHER" as in the dictionary.

Suppose at the 1st place either E or H then all words formed will be before "MOTHER".

$\square\ \underbrace{\square\ \square\ \square\ \square\ \square}_{5!}$

Number of such words $= 2\times5! = 240$

If the 1st place is occupied by M, then the 2nd place can be filled up by any of E and H which are before O.

$\overset{M}{\underset{1}{\square}}\ \underset{2}{\square}\ \underbrace{\square\ \square\ \square\ \square}_{4!}$

Number of such words $= 2\times4! = 48$

Suppose the 1st place is M and ~~B~~ 2nd place in O & the 3rd place can be filled up by any one of E, H, R which are before T. $\boxed{M}\,\boxed{O}\,\underset{3}{\boxed{\ }}\,\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }}_{3!}$

$\therefore$ Number of such words $= 3 \times 3! = 18$

Similarly 1st 2nd and 3rd places are filled up by M, O, T respectively & 4th places can be filled up by E which is before H. $\boxed{M}\,\boxed{O}\,\boxed{T}\,\underset{1}{\boxed{E}}\,\underbrace{\boxed{\ }\boxed{\ }}_{2!}$

No of such words $= 1 \times 2! = 2$

There is no other word before MOTHER. $\boxed{M\,O\,T\,H\,E\,R}$ Hence the $= 240 + 48 + 18 + 2$

$= 308$ No before Mother

$= \cancel{309}$.

Mother has rank $\boxed{309}$

No - 25 (Higher secondary page - 164)

A boat's ~~crew~~ crew is to be manned by 8 sailors of whom 3 can only row on the stroke-side & 2 can only row on the bow-side. In how many ways can the crew be arranged?

Soln: let the 8 sailors be A, B, C, D, E, F, G, H.

Suppose A, B, C remain on stroke-side & D & E remain on the bow-side. It is given as below.

Stroke	Bow
A	Q
B	E
C	

Since 4 men will remain on each side, of the remaining 3, one must be placed on stroke side & other two will be on bow side. Now one can be chosen out of 3 in ~~3~~ 3 different ways.

Again ~~each~~ each time of these selecting the 4 men on stroke side can be arranged among themselves in 4! ways & on bow side the 4 men can be arranged in 4! ways.

∴ The required no. of ways

$= 3 \times 4! \times 4!$ (By F.P.C)

$= 3 \times 24 \times 24$

$= 1728$ (Ans)

(Q7) A fruit basket contains 4 oranges, 5 apples & 6 mangoes. In how many ways can a person make a selection of fruits from among the fruits in the basket?

Soln → Case - I

Fruits of same kind are all taken to be of the same shape.

Out of 4 oranges ~~mangoes~~ we select 0, 1, 2, 3 or 4 at a time in 5 different ways. Out of 5 apples we select 0, 1, 2, ----- 5 at a time in 6 different ways.

Out of 6 mangoes, we select 0, 1, 2 ---- 6 at a time in 7 different ways.

Total no. ways of selection

$= 5 \times 6 \times 7$ (By F. P. C)

$= 210$

Excluding that zero fruit choice, we have the total no of selected ways $= 210 - 1 = 209$.

Case II

If fruits of same kind are different shapes.

$2^4 - 1 \times 2^5 - 1 \times 2^6 - 1$

The Binomial Theorem

Binomial :- An expression containing 2 terms is called a Binomial.

Ex:- $a+x$, $x+y$ etc.

$(a+x)^0 = 1$

$(a+x)^1 = a+x$

$(a+x)^2 = a^2 + 2ax + x^2$

$(a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$

$(a+x)^4 = a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4$.

The Pascal Triangle

$$
\begin{array}{ccccccccccc l}
 & & & & & 1 & & & & & & \to n=0 \\
 & & & & 1 & & 1 & & & & & \to n=1 \\
 & & & 1 & & 2 & & 1 & & & & \to n=2 \\
 & & 1 & & 3 & & 3 & & 1 & & & \to n=3 \\
 & 1 & & 4 & & 6 & & 4 & & 1 & & \to n=4 \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 & \to n=5
\end{array}
$$

V. Imp. The Binomial Theorem for +ve integral index (power)

Statement :- Let $a, x \in R$ and n is any +ve integers, then

$(a+x)^n = C(n,0)\,a^n + C(n,1)\,a^{n-1}x + C(n,2)\,a^{n-2}x^2$

$+ C(n,3)\,a^{n-3}x^3 + \cdots\cdots + C(n,r)\,a^{n-r}x^r + \cdots$

Proof:-

Let $a, x \in R$ and n is any +ve integers.

Let P_n be the statement.

$(a+x)^n = C(n,0)\,a^n + C(n,1)\,a^{n-1}x + C(n,2)\,a^{n-2}x^2$

$+ C(n,3)\,a^{n-3}x^3 + \cdots\cdots + C(n,r)\,a^{n-r}\cdot x^r$

$+ \cdots\cdots + C(n,n)\,x^n$

Then proceed → P_1 → To prove P_1 is true

$= 1\cdot a + 1\cdot 1\cdot x = a+x$

$= (a+x)^1 =$ L.H.S of P_1

$\therefore P_1$ is true

Let us assume that P_K be true i.e

$(a+x)^K = C(K,0)\,a^K + C(K,1)\,a^{K-1}x + C(K,2)\,a^{K-2}x^2$

$+ C(K,3)\,a^{K-3}x^3 + \cdots\cdots\cdots$

$+ C(K,r-1)\,a^{K-r+1}\cdot x^{r-1} + C(K,r)\,a^{K-r}x^r$

$+ \cdots\cdots + C(K,K)\,x^K$ is true

To prove that P_{K+1} is true

Now P_{K+1} is

$(a+x)^{K+1} = C(K+1,0)\,a^{K+1} + C(K+1,1)\,a^{(K+1)-1}\cdot x$

$+ C(K+1,2)\,a^{(K+1)-2}\cdot x^2 + \cdots\cdots$

$+ C(K+1,r)\,a^{K+1-r}\cdot x^r + \cdots + C(K+1,K+1)\,x^{K+1}$

Now L.H.S of P_{K+1} :

$$= (a+x)^{K+1} = (a+x)(a+x)^K$$

$$=(a+x)\left\{ C(K,0)a^K + C(K,1)a^{K-1}x + C(K,2)a^{K-2}x^2 + C(K,3)a^{K-3}x^3 + \cdots + C(K,r-1)a^{K-r+1}x^{r-1} + C(K,r)a^{K-r}x^r + \cdots + C(K,K)x^K \right\}$$

By Induction hypothesis

$$= C(K,0)a^{K+1} + C(K,0)a^K x + C(K,1)a^K x + C(K,1)a^{K-1}x^2 + C(K,2)a^{K-1}x^2 + C(K,2)a^{K-2}x^3 + C(K,3)a^{K-2}x^3 + C(K,3)a^{K-3}x^4 + \cdots + C(K,r-1)a^{K-r+2}x^{r-1} + C(K,r-1)a^{K-r+1}x^r + C(K,r)a^{K-r+1}x^r + C(K,r)a^{K-r}x^{r+1} + \cdots + C(K,K)ax^K + C(K,K)x^{K+1}$$

$$= C(K,0)a^{K+1} + \left(C(K,0)+C(K,1)\right)a^K x + \left(C(K,1)+C(K,2)\right)a^{K-1}x^2 + \left(C(K,2)+C(K,3)\right)a^{K-2}x^3 + \cdots + \left(C(K,r)+C(K,r-1)\right)a^{K-r+1}x^r + \cdots + C(K,K)x^{K+1}$$

$$= C(K+1,0)a^{K+1} + C(K+1,1)a^{K+1-1}x + C(K+1,2)a^{K+1-2}x^2 + C(K+1,3)a^{K+1-3}x^3$$

$+ \cdots + C(k+1, r)\, a^{k+1-r} x^{r} + \cdots$

$+ C(k+1, k+1)\, x^{k+1}$

$=$ R.H.S of P_{k+1}

$\left[\begin{array}{l} \because C(k,0) = C(k+1,0) = 1 \\ C(n,r) + C(n,r-1) = C(n+1, r) \\ C(k+1, k+1) = C(k,k) = 1 \end{array}\right.$

$\therefore P_{k+1}$ is true.

$\therefore P_n$ is true $\forall\ n = 1, 2, 3 \cdots$

by method of induction. (Proved)

Notes :-

① If the exponent (power) in Binomial Theorem is n then there are $n+1$ terms in the ~~extression~~ expansion.

② The sum of the powers of a and x in $(a+x)^n$ in each term is n.

③ In the ~~expression~~ expansion $(a+x)^n$, the powers of 'a' gradually decreases & powers of x gradually increases.

④ We denote,

$$C_0 = C(n,0) = \frac{\underline{|n}}{\underline{|0}\,\underline{|n}} = 1$$

$$C_1 = C(n,1) = \frac{\underline{|n}}{\underline{|1}\,.\,\underline{|n-1}} = n$$

$$C_2 = C(n,2) = \frac{\underline{|n}}{\underline{|2}\,.\,\underline{|n-2}} = \frac{n(n-1)}{\underline{|2}}$$

$$C_3 = C(n,3) = \frac{\underline{\lfloor n}}{\underline{\lfloor 3} \cdot \underline{\lfloor n-3}} = \frac{n(n-1)(n-2)}{\underline{\lfloor 3}}$$

— — — — — — — — — — — — —

— — — — — — — — — — — — —

— — — — — — — — — — — — —

$$C_r = C(n,r) = \frac{\underline{\lfloor n}}{\underline{\lfloor r} \cdot \underline{\lfloor n-r}} = \frac{n(n-1)(n-2)\cdots\cdots(n-r+1)}{\underline{\lfloor r}}$$

— — — — — — — — — — — — —

— — — — — — — — — — — — —

— — — — — — — — — — — — —

$$C_{n-1} = C(n, n-1) = \frac{\underline{\lfloor n}}{\underline{\lfloor n-1} \cdot \underline{\lfloor 1}} = n$$

$$C_n = C(n,n) = 1$$

Here C_0, C_1, C_2 — — — C_n are called Binomial Co-efficients.

(5) The general term ~~...~~ in $(a+x)^n$

$= (r+1)^{th}$ term

$= C(n,r)\, a^{n-r} x^r$

$C_0 = C_n$
$C_1 = C_{n-1}$
$C_2 = C_{n-2}$

Problems, ~~Ex-4, page-146~~ $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Q→ Simplify the 7^{th} term of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Ans:- $7th$ term of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

$$= C(9,6)\left(\frac{4x}{5}\right)^{9-6}\left(-\frac{5}{2x}\right)^6$$

$$= \frac{\lfloor 9}{\lfloor 6 \; \lfloor 3} \times \frac{(4x)^3}{5^3} \times \frac{(5)^6}{(2x)^6}$$ ($7 \times 8 \times 7$)

$$= 84 \times \frac{1}{x^3} \times 125 = 10500 x^{-3}$$

Q-) Determine the co-efficient of y^9 in $(5-2y)^{11}$.

Ans → General term in $(5-2y)^{11}$

$= (r+1)$th term in $(5-2y)^{11}$

$= C(11, r)\, 5^{11-r} (-2y)^r$

$= C(11, r) \cdot 5^{11-r} (-2)^r \cdot y^r$

To find the coefficient of y^9 we have to take $r=9$

$\therefore$ 10th term $= C(11, 9) \cdot 5^{11-9} \cdot (-2)^9 \cdot y^9$

$\therefore$ Coefficient of $y^9 = C(11,9) \cdot 5^{11-9} \cdot (-2)^9$

$$= \frac{-\lfloor 11}{\lfloor 9 \cdot \lfloor 2} \cdot 5^2 \cdot 2^9$$

$$= -55 \times 25 \times 512$$

$$= -704000$$

Q-) Find the ~~daxen~~ term independent of x in $\left(x^2 + \frac{1}{x}\right)^{12}$

Soln: $\left(x^2 + \frac{1}{x^2}\right)^{12}$

= General term in $(x^2+\frac{1}{x})^{12}$
= $(r+1)$ th term in $(x^2+\frac{1}{x})^{12}$

$$= C(12,r)\,(x^2)^{12-r}\left(\frac{1}{x}\right)^r$$

$$= C(12,r)\,x^{24-2r}\cdot\frac{1}{x^r}$$

$$= C(12,r)\,x^{24-3r}$$

To find the term independent of x, we have to take $24-3r=0$
$\Rightarrow r=8$

$\therefore$ Term independent of x = 9th term
$= C(12,8)\,x^0$
$= C(12,8)$
$= \frac{\lfloor 12}{\lfloor 8 \cdot \lfloor 4} = 495$

Middle terms

Suppose the Binomial expansion of $(a+x)^n$ is considered. The no. of terms in the expansion $= n+1$

Case-I Suppose n is even

Then $(n+1)$ is odd.
i.e The no. of terms is odd.
$\therefore$ There is only one middle term:

The middle term is $\left(\frac{n+2}{2}\right)^{th}$ term.

i.e. $\left(\frac{n}{2}+1\right)^{th}$ term.

$\therefore$ Middle term $= \left(\frac{n}{2}+1\right)^{th}$ term.

$$= C\left(n, \frac{n}{2}\right) \cdot a^{n-\frac{n}{2}} \cdot x^{\frac{n}{2}}$$

$$= \frac{\lfloor n}{\lfloor n/2 \lfloor n/2} a^{\frac{n}{2}} \cdot x^{\frac{n}{2}}$$

Case-II Suppose n is odd; then

n+1 is even.

ie the no. of terms is even.

$\therefore$ There are ~~two~~ middle terms.

The 1st middle term is $\left(\frac{n+1}{2}\right)^{th}$ term,

ie $\left(\frac{n-1}{2}+1\right)^{th}$ term.

$\therefore$ The 1st middle term $= \left(\frac{n-1}{2}+1\right)^{th}$ term

$$= C\left(n, \frac{n-1}{2}\right) a^{n-\frac{n-1}{2}} \cdot x^{\frac{n-1}{2}}$$

$$= \frac{\lfloor n}{\lfloor \frac{n-1}{2} \lfloor \frac{n+1}{2}} a^{\frac{n+1}{2}} \cdot x^{\frac{n-1}{2}}$$

The 2nd middle term is $\left(\frac{n+1}{2}+1\right)^{th}$ term

$\therefore$ The 2nd middle term $= \left(\frac{n+1}{2}+1\right)^{th}$ term

$$= C\left(n, \frac{n+1}{2}\right) a^{n-\frac{n+1}{2}} x^{\frac{n+1}{2}}$$

$$= \frac{\lfloor n}{\lfloor \frac{n+1}{2} \cdot \lfloor \frac{n-1}{2}} \cdot a^{\frac{n-1}{2}} \cdot x^{\frac{n+1}{2}}$$

Q → Find the middle term(s) of $(1-2x+x^2)^n$

Sol^n → $(1-2x+x^2)^n = (1-x)^{2n}$

Here the exponent is $2n$ and the no. of terms in the expansion is $2n+1$ which is odd.

$\therefore$ There is only one middle term.

The middle term $= \left(\frac{2n+1+1}{2}\right)^{th} = (n+1)^{th}$

$$= C(2n, n)\, 1^{2n-n} (-x)^n$$

$$= \frac{\lfloor 2n}{\lfloor n \cdot \lfloor n} (-1)^n x^n$$

Q → Find the middle term(s) of $\left(x-\frac{1}{x}\right)^{13}$

Sol^n: $\left(x-\frac{1}{x}\right)^{13}$

The no. of terms $= 14$ which is even.

$\therefore$ There are two middle terms.

1st middle term $= \left(\frac{14}{2}\right)^{th} = 7^{th}$ term

$$= C(13,6)\, x^{13-6} \left(-\frac{1}{x}\right)^6$$

$$= \frac{13!}{6!\,7!} \cdot x^7 \cdot \frac{1}{x^6}$$

$$= 1716x$$

2nd middle term $= 8^{th}$ term

$$= C(13,7)\, x^{13-7} \left(-\frac{1}{x}\right)^7$$

$$= \frac{13!}{7!\,6!} \times x^6 \left(-\frac{1}{x^7}\right)$$

$$= -\frac{1716}{x}$$

V. imp

Properties Of Binomial Co-efficients

(1) $C_0 + C_1 + C_2 + \cdots + C_n = 2^n$ i.e

Sum of all Binomial Co-efficients is 2^n

Proof: We know from Binomial theorem

$$(a+x)^n = C_0 a^n + C_1 a^{n-1} x + C_2 a^{n-2} x^2 + \cdots + C_n x^n$$

Putting $a=1$, we get

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$$

Putting $x=1$, we get

$$(1+1)^n = C_0 + C_1 + C_2 + \cdots + C_n$$

i.e $2^n = C_0 + C_1 + C_2 + \cdots + C_n$

Proved.

(2) $C_0 + C_2 + C_4 + \cdots = C_1 + C_3 + C_5 + \cdots$

$= 2^{n-1}$

Proof: We know ~~that~~ from Binomial Theorem

$$(a+x)^n = C_0 a^n + C_1 a^{n-1} x + C_2 a^{n-2} x^2 \cdots + C_n x^n$$

Putting $a = 1$, we get

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 \cdots + C_n x^n \quad \text{(i)}$$

Putting $x = 1$, we get

$$2^n = C_0 + C_1 + C_2 \cdots + C_n \quad \text{(ii)}$$

Again putting $x = -1$ in eqn (i), we get

$$0 = C_0 - C_1 + C_2 - C_3 \cdots + (-1)^n C_n$$

$$\Rightarrow C_1 + C_3 + C_5 \cdots = C_0 + C_2 + C_4 \cdots = K \text{ (say)}$$

From eqn (ii)

$$2^n = (C_0 + C_2 \cdots) + (C_1 + C_3 + \cdots)$$

$$= K + K = 2K$$

$$\Rightarrow K = \frac{2^n}{2} = 2^{n-1}$$

$$\therefore C_1 + C_3 + \cdots = C_0 + C_2 + \cdots = 2^{n-1}$$

(Proved)

Q → Prove that

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} \cdots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$$

Proof: 1st method: →

L.H.S

$$= C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_n}{n+1}$$

$$= 1 + \frac{n}{2} + \frac{n(n-1)}{\lfloor 2 \times 3} + \cdots + \frac{1}{n+1}$$

$$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n}{\lfloor 2} + \frac{(n+1)(n-1)n}{\lfloor 3} + \cdots + 1\right]$$

$$= \frac{1}{n+1}\left[\left\{1 + (n+1) + \frac{(n+1)n}{\lfloor 2} + \frac{(n+1)n(n-1)}{\lfloor 3} + \cdots + 1\right\} - 1\right]$$

$$= \frac{1}{n+1}\left[\left\{C(n+1,0) + C(n+1,1) + C(n+1,2) + \cdots + C(n+1,n+1)\right\} - 1\right]$$

$$= \frac{1}{n+1}\left[2^{n+1} - 1\right] = \text{R.H.S} \quad \text{Proved}$$

2nd method

We know Binomial Theorem

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$$

Integrating both sides w.r.to x within limits 0 to 1, we get

$$\int_0^1 (1+x)^n\,dx = \int_0^1 (C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n)\,dx$$

$$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[C_0 x + C_1 \frac{x^2}{2} + C_2 \frac{x^3}{3} + \cdots + C_n \frac{x^{n+1}}{n+1}\right]_0^1$$

$$\Rightarrow \frac{2^{n+1}}{n+1} - \frac{1}{n+1} = \left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} ------ + \frac{C_n}{n+1}\right) - 0$$

$$\Rightarrow \frac{2^{n+1} - 1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ---- + \frac{C_n}{n+1}$$

Q-) How many different rectangles that can be found in a chess board.

Ans:- There are 9 horizontal and 9 vertical lines in a chess board.

For a rectangle we select any 2 horizontal and any 2 vertical lines

Out of 9 horizontal lines we select 2 lines and it can be done in C(9, 2) ways. Similarly 2 vertical lines in C(9, 2) ways.

∴ 2 horizontal & 2 vertical lines can be selected in C(9, 2) · C(9, 2)

= 36 × 36

= 1296 ways.

∴ 1296 different rectangles can be formed.

Q-) V. imp

$$C_0^2 + C_1^2 + ---- + C_n^2 = \frac{(2n)!}{(n!)^2}$$

Proof: We know that

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 - - - - + C_n x^n$$

$$(x+1)^n = C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} - - - - - - - - + C_n$$

Multiplying the corresponding sides, we get

$$(1+x)^n (x+1)^n = \left(C_0 + C_1 x + C_2 x^2 + - - - C_n x^n\right) \times \left(C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + - - - - + C_n\right)$$

$$\Rightarrow (1+x)^{2n} = \left(C_0 + C_1 x + C_2 x^2 - - - - + C_n x^n\right)\left(C_0 x^n + C_1 x^{n-1} + - - - - + C_n\right) \quad \text{--- (1)}$$

Which is a an identity.

Hence co-efficients of x^n of both the sides are equal.

Co-efficients of x^n in R.H.S of (1)

$$= C_0^2 + C_1^2 + C_2^2 + - - - + C_n^2$$

Co-efficient of x^n in L.H.S of (1)

$$= C(2n, n) = \frac{\lfloor 2n}{\lfloor n \cdot \lfloor n} = \frac{\lfloor 2n}{(\lfloor n)^2}$$

$$\therefore C_0^2 + C_1^2 + C_2^2 + - - - - + C_n^2 = \frac{\lfloor 2n}{(\lfloor n)^2}$$

(proved)

Q -> Prove that

$$C_0 C_n + C_1 C_{n-1} + C_2 C_{n-2} - - - - - + C_n C_0 = \frac{(2n)!}{(n!)^2}$$

Proof, we know

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + - - - - + C_n x^n$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + - - - C_n x^n$

Multiplying corresponding sides, we get

$(1+x)^n (1+x)^n = (C_0 + C_1 x + C_2 x^2 + - - - - - + C_n x_n) \cdot$
$(C_0 + C_1 x + C_2 x^2 + - - - - + C_n x_n)$

$\Rightarrow (1+x)^{2n} = (C_0 + C_1 x + - - - + C_n x^n)(C_0 + C_1 x + - - - + C_n x^n)$ — (i)

Which is a identity

Hence the coefficiently of x^n of both the sides are equal.

Coefficient of x^n in R.H.S of (1)

$= C_0 C_n + C_1 C_{n-1} + - - - - - + C_n C_0$

Coefficient of x^n in L.H.S of (1)

$= C(2n, n) = \frac{\lfloor 2n}{\lfloor n \lfloor n} = \frac{\lfloor 2n}{(\lfloor n)^2}$

$\therefore C_0 C_n + C_1 C_{n-1} + - - - - + C_n C_0 = \frac{\lfloor 2n}{(\lfloor n)^2}$ []

Q → Prove that,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \cdots\cdots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

Proof

L.H.S $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \cdots\cdots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

$$= \frac{n}{1} + \frac{2n(n-1)}{2n} + \frac{3n(n-1)(n-2)}{6 \times \frac{n(n-1)}{2}} + \cdots + n \times \frac{1}{n}$$

$$= n + (n-1) + (n-2) + \cdots\cdots + 1$$

$$= 1+2+3+\cdots+n = \frac{n(n+1)}{2} = \text{R.H.S (Proved)}$$

Q → Prove that

$C_1 + 2C_2 + 3C_3 + \cdots\cdots + n\,C_n = n\,2^{n-1}$

(Try in Combination method)

Proof → We know,

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots\cdots + C_n x^n$

Differentiate both sides w.r.to x, we get

$n(1+x)^{n-1} = 0 + C_1 + 2C_2 x + \cdots\cdots + n\,C_n x^{n-1}$

which is a identity

Putting $x=1$, we get

$n.2^{n-1} = C_1 + 2C_2 + \cdots\cdots + n\,C_n$

(Proved)

Binomial Theorem for -ve integral index or fractional index

If n is a fraction or -ve integer and $|x| < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Examples →

$$\frac{1}{1+x} = (1+x)^{-1}$$

$$= 1 - x + \frac{(-1)(-2)}{2!}x^2 + \frac{(-1)(-2)(-3)}{3!}x^3 + \cdots$$

$$= 1 - x + x^2 - x^3 + \cdots$$

Provided $|x| < 1$

$$\frac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

Provided $|x| < 1$

$$\frac{1}{(1+x)^2} = (1+x)^{-2}$$

$$= 1 - 2x + 3x^2 - 4x^3 + \cdots + (-1)^r (r+1)x^r + \cdots$$

Provided $|x| < 1$

$$\frac{1}{(1-x)^2} = (1-x)^{-2}$$

$$= 1 + 2x + 3x^2 + 4x^3 + \cdots + (r+1)x^r + \cdots$$

Provided $|x| < 1$

Note :- General term in $(1+x)^n$

$= (r+1)$th term

$$= \frac{n(n-1)(n-2) \cdots (n-r+1)}{\lfloor r} x^r$$

Q-) Expand $\frac{x}{1+x^2}$ up to 4 terms

Solⁿ: $\frac{x}{1+x^2} = x(1+x^2)^{-1}$

$= x\{1 - x^2 + x^4 - x^6 + \cdots\}$

$= x - x^3 + x^5 - x^7 + \cdots$

Q-) Calculate $\sqrt{102}$ up to 4 decimal places using Binomial theorem

Solⁿ: $\sqrt{102} = (102)^{\frac{1}{2}} = (100+2)^{\frac{1}{2}}$

$= (100)^{\frac{1}{2}} \left\{1 + \frac{2}{100}\right\}^{\frac{1}{2}}$

$= 10 \{1 + 0.02)^{\frac{1}{2}}$

$$= 10 \left\{1 + \frac{1}{2}(0.02) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{\lfloor 2}(0.02)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\lfloor 3}(.02)^3 + \cdots\right\}$$

$= 10\{1 + 0.01 - 0.000005\}$ approx

$= 10 \times 1.00995$ Approx

$= 10.0995$ approx (Ans

Greatest Term in the expansion of $(x+a)^n$

$(r+1)^{th}$ term $= T_{r+1} = C(n,r)\, x^{n-r} \cdot a^r$

r^{th} term $= T_r = C(n, r-1)\, x^{n-r+1} \cdot a^{r-1}$

$$\frac{T_{r+1}}{T_r} = \frac{C(n,r) \cdot x^{n-r} \cdot a^r}{C(n, r-1) \cdot x^{n-r+1} \cdot a^{r-1}}$$

$$= \frac{n-r+1}{r} \cdot \frac{a}{x}$$

Now $T_{r+1} > T_r$

$$\Rightarrow \frac{T_{r+1}}{T_r} > 1$$

$$\Rightarrow \frac{n-r+1}{r} \times \frac{a}{x} > 1 \Rightarrow (n-r+1)a > rx$$

$$\Rightarrow (n+1)a > rx + ra = r(x+a)$$

$$\Rightarrow r < \frac{(n+1)a}{x+a}$$

Hence the terms continue to increase as long as $r < \frac{(n+1)a}{x+a}$

Suppose $\frac{(n+1)a}{x+a}$ = an integer say K

$\therefore$ Terms continue to increase as long as $r < K$

when $r = K$, we have $T_{K+1} = T_K$ and

These are the two greatest terms in the expansion of $(x+a)^n$

In case $\frac{(n+1)a}{x+a}$ is a fraction, let its integral part be denoted by m. The terms continue to increase if $r=m$.

Thus T_{m+1} is the greatest term in the expansion of $(x+a)^n$

Q → Find the numerically greatest term in the expansion of $(3-x)^9$, when $x=2$.

Soln → $T_{r+1} = C(9,r)\, 3^{9-r} \times x^r$

(Numerically neglecting the −ve sign.)

$T_r = C(9, r-1)\, 3^{9-r+1}\, x^{r-1}$ (Numerically)

$$\frac{T_{r+1}}{T_r} = \frac{9-r+1}{r} \cdot \frac{x}{3} \quad \text{(numerically)}$$

$$= \frac{10-r}{r} \cdot \frac{x}{3}$$

$$= \frac{10-r}{r} \times \frac{2}{3} \quad \text{for } x=2$$

$$= \frac{20-2r}{3r}$$

Alternative method:-

$$\frac{(n+1)a}{x+a} = \frac{(9+1)2}{3+2}$$

$$= \frac{20}{5} = 4$$

$T_4 = T_5$ is the greatest term

$T_5 = T_4 = C(9,3) \cdot 3^6 \cdot x^3$
$= 84 \cdot 3^6 \cdot 2^3 = 489888$

If T_{r+1} is the numerically greatest term $\frac{T_{r+1}}{T_r} \geq 1$ i.e. $\frac{20-2r}{3r} \geq 1$ i.e. $20 \geq 5r$ i.e. $r \leq 4$

$\therefore$ T_5 and T_4 are numerically equal to each other and the greatest terms.

$\therefore T_5 = T_4 = C(9,3)\ 3^6 x^3$

$$= \frac{\lfloor 9}{\lfloor 6 \cdot \lfloor 3} \times 3^6 \times 2^3$$

$$= 489888$$

Q→ Find the greatest term in the ~~example~~ expansion of $(2+3x)^9$ when $x = \frac{3}{2}$.

Soln:

$$\frac{\cancel{7 \times 3^{13}}}{\cancel{2}}$$

$$\frac{(n+1)a}{x+a} = \frac{10 \times 3 \times \frac{3}{2}}{2+3 \cdot \frac{3}{2}}$$

$$= \frac{45 \times 2}{13}$$

$$= \frac{90}{13} = 6.9$$

Integral part

Greatest term

$$T_7 = C(9,6)\ 2^{9-6} \left(\frac{3}{2}\right)^6$$

~~Greatest term~~

$$= \frac{7 \times 3^{13}}{2}$$

Q→ Find the term independent of x in $(1-x)^3 \left(x-\frac{1}{x}\right)^7$

Soln: → $(1-x)^3 \left(x-\frac{1}{x}\right)^7$

$$= (1-3x+3x^2-x^3)\left(x-\frac{1}{x}\right)^7$$

$$= \left(x-\frac{1}{x}\right)^7 - 3x\left(x-\frac{1}{x}\right)^7 + 3x^2\left(x-\frac{1}{x}\right)^7 - x^3\left(x-\frac{1}{x}\right)^7 \quad \text{(i)}$$

Consider $\left(x-\frac{1}{x}\right)^7$

General term $= (r+1)^{th}$ term

$$= C(7,r)\ x^{7-r} (-1)^r x^{-r}$$

$= (-1)^r \; C(7,r) \; x^{7-2r}$

Now there is no term independent of x in $(x-\frac{1}{x})^7$ & $3x^2(x-\frac{1}{x})^7$

To find term independent of x in $-3x(x-\frac{1}{x})^7$

Here the general term

$= (-1)^r \; C(7,r) \; x^{7-2r} . (-3x)$

$= (-1)^{r+1} . 3C(7,r) \; x^{8-2r}$

According to question $8-2r=0$

$\Rightarrow r=4$

i.e 5th term $= (-1)^5 \; C(7,4) . 3$

$= -3 . \frac{\cancel{\lfloor 7} \; 5\times 6\times 7}{\cancel{\lfloor 4} . \cancel{\lfloor 3}}$

$= -105$

To find term independent of x in $-x^3(x-\frac{1}{x})^7$

Here General term $= (-1)^r \; C(7,r) \; x^{7-2r} (-x^3)$

$= (-1)^{r+1} \; C(7,r) \; x^{10-2r}$

According to question $10-2r=0$

$\Rightarrow r=5$

$\therefore$ 6th term $= (-1)^6 \; C(7,5)$

$= \frac{\cancel{\lfloor 7} \; 6\times 7}{\lfloor 5 \times \cancel{\lfloor 2}} = 21$

$\therefore$ Term independent of x in (1)

$= -105 + 21 = -84$

Q → Find Co-efficient of x^n in $\frac{(1+x)^2}{(1-x)^2}$

Soln: $\frac{(1+x)^2}{(1-x)^2} = (1+x)^2 (1-x)^{-2}$

$= \{\cancel{1+x^2} + 2x + x^2\} \{1 + 2x + 3x^2 + 4x^3 + \cdots + (r+1) x^r + \cdots\}$

$= \{1 + 2x + 3x^2 + \cdots\} + 2x\{1 + 2x + 3x^2 \cdots\}$

$+ x^2 \{1 + 2x + 3x^2 + \cdots\}$ ——(1)

In the 1st bracket of (1) (because x^2 co-eff $= 3$, x^n --- $n+1$)

Co-efficient of $x^n = n+1$

In $2x\{1 + 2x + 3x^2 + \cdots\}$ Co-efficient of x^n

$= 2 \times$ Co-efficient of term contains x^{n-1}

$= 2 \times n$

In $x^2 \{1 + 2x + 3x^2 + \cdots\}$ ~~$2 \times n$~~ ~~$2 \times n$~~

Co-efficient of x^n

= Co-efficient of term containing x^{n-2}

~~= Co-efficient " " " x^{n-2}~~

$= n-1$

$\therefore$ The Coefficient of x^n in (1)

$= n+1 + 2n + n - 1 = 4n$ (Proved)

Topics
2mx

Q→ What is the sum of all 5 digit numbers formed using 1, 3, 5, 7, 9 without repetation? □ □ □ □ □

Soln → In unit place if 1 is placed, there are $4! = 24$ numbers.

If 3 is placed in unit place, there are $4! = 24$ numbers

Similarly if 5, 7, 9 are placed then there are $4! = 24$ number in each case.

∴ Sum due to unit place

$= 24(1+3+5+7+9) = 24 \times 25 = 600$

Due to ten place sum $= 24(1+3+5+7+9) \times 10$

$= 6000$

Due to ~~hund~~ hundred place sum $= 60,000$

" " Thousand " " $= 600\,000$

" " 10 " " " $= 6000\,000$

∴ The reaurred sum

$= 6000\,000 + 600\,000 + 60\,000 + 6000 + 600$

$= 6666600$ (Ans)

No- 34 Topic | a b c — —
Pg - 400

There are 2 empty spaces. We have to place at least 3 Consonants. Already 2 Consonants are present.

Hence we have to place at least 1 Consonant.

1 Consonant & 1 vowel → $C(13,1)\ C(4,1)\ 5!$

2 Consonant → $C(13,2) \times 5!$

Add 15600

If a is in 1st place then no. of words

$= C(13,1)\ C(4,1)\ 4! + C(13,2) \times 4!$

$= 3120$

If b and c are together then

no. of words $= \underbrace{C(13,1)}_{\to \text{ one consonant}} \times \underbrace{C(4,1)}_{\to \text{ one vowel}} 4! \times 2$

$+ C(13,2) \times 4! \times 2!$

$= 6240$

~~If b and c are together then no. of words~~

$\left\{ \begin{array}{l} \because\ bc - - - \Rightarrow 4! \\ b\ \&\ c \to 2! \end{array} \right\}$

41 Topics $\quad C(11,8) \times 4! \times \frac{18}{\lfloor 2 . \lfloor 2 \lfloor 2 . \lfloor 2} = 9979200$

14 (elements) P-66 $\quad$ How many integers between 100 and 1000 (both inclusive)

consists of ~~distinct~~ distinct odd integers

odd digits 1, 3, 5, 7, 9

Soln → Go $\quad$ □ □ □

$5 \times 4 \times 3$

Q → Find the no. of ways in which 12 identical coins can be put into 5 different purses, if more of the purposes remains empty.

Soln → No. of ways

= Coefficient of x^{12} in $(x+x^2+x^3+\cdots+x^{12})$

$= x^5(1+x+x^2+\cdots+x^{11})$

(∵ at least one coin in each purpose)

= Coefficient of x^{12} in $(1-x^{12})^5(1-x)^{-5}$

= Coefficient of x^7 in $(1-x)^{-5}$ $\left(\because \frac{1-x^{12}}{1-x} = x+x^2+\cdots+x^{12} \right)$

$= \frac{5 \times 6 \times 7 \cdots \times 11}{7!} = 330$

Ex — 3 (c)

Elements, pg - 70

14. How many factors does 1155 have that are divisible by 3.

Soln $1155 = 3 \times 5 \times 7 \times 11$

we can take 3 in 1 possible ways.

" " 5 in 2 " ways

" " 7 in 2 " " .

" " 11 " 2 " " .

∴ The total no of ways $= 1 \times 2 \times 2 \times 2 = 8$

19) $10,000 = 5^4 \times 2^4$

There are ~~posstible~~ possibilities for taking $5 = 5$.

There are possibilities for taking $2 : 1$ (0 time)

Total no of ways $= 5 \times 1 = 5$

But 1 does not end with 5

$\therefore$ Total no. of factors end with 5

$= 5 - 1 = 4$ (Ans)

4) A polygon having n sides has n vertices.

Two vertices are taken for 1 straight line.

$\therefore$ The no. of st. lines formed

$= \cancel{nC_2} = {}^nC_2$

But there are n sides.

$\therefore$ No of diagonals $= {}^nC_2 - n$

5) $|A| = n$, $|B| = m$

$\therefore$ $|A \times B| = nm$

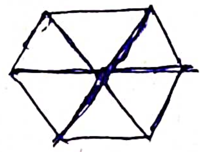
But subsets of $A \times B$ are relations.

$\therefore$ No of relations = No of subsets of $A \times B$

$= 2^{nm}$ (Including the relation ϕ & $A \times B$)

10) 7 points, we have to take 3 at a time in ${}^7C_3 = 35$ ways.

But 3 diagonals passes through centre.



∴ 3 triangles can not be formed.

∴ The no. of triangles formed $= 35 - 3$
$= 32$

22) There are 9 courses.
2 are compulsory.

we have to take 5.

Hence we select 3 courses out of 7 courses in $C(7,3)$ way $= 35$

If we take C_6 & C_8 together then we select only 1 course from 5 courses in $C(5,1) = 5$ ways.

Hence if we can not take C_6, C_8 together, then we get $35 - 5 = 30$ ways.

24) There are $(n+r)$ points & for a straightline we take 2 at a time in $C(n+r, 2)$ ways.

∴ Total no of st lines if no 3 are in one st. lines
$= C(n+r, 2)$

Given that n points are in one line.

Hence from these n points we may form C(n, 2) st. lines if they are not in one line.

Hence instead of C(n, 2) st. lines we take one st. lines only.

Hence total no of lines

$$= C(n+r, 2) - C(n, 2) + 1$$

$$= \frac{\lfloor n+r}{\lfloor n+r-2 . \lfloor 2} - \frac{\lfloor n}{\lfloor n-2 . \lfloor 2} + 1$$

$$= \frac{(n+r-1)(n+r)}{2} - \frac{(n-1)n}{2} + 1$$

$$= \frac{(n+r)^2 - n - r - (n^2 - n)}{2} + 1$$

$$= \frac{r^2 + 2rn - r}{2} + 1.$$

$$= \frac{r(r+2n-1)}{2} + 1$$

Exercise - 3(b)

22) (P.g 67) Keep one woman fixed. So 2 women arranged in 2! ways. Now in between 3 women there are 3 places around a round table

So, 3 men can be placed in 3 places in $P(3,3)$ ways

$\therefore$ No of ways = $P(3\times3) \times 2! = 12$

3(a) - Exercise (Page-62)

1. $\{1,2\} \rightarrow \{1,2,3\}$

For forming a function for 1 in the 1 set has 3 possibilites. It lies for corresponding ways.

For forming a function for 2 in the 1st set has 3 possibilities for corresponding ways.

$\therefore$ No of ways $= 3\times3 = 9$

3) $^5C_2$ { Out of 5 we take 2 to find a quad order is not taken, so A→B is same as B→A }

$= \frac{\lfloor 5}{\lfloor 2 \times \lfloor 3} = 10$ (4×5)

4) ~~Percentage~~ Pentagon has 5 sides & 5 vertices

$\therefore$ No of St. lines $= {}^5C_2 = 10$ (2 points are taken for st line)

Diagonals = Total no. of st. lines ~~sides~~ sides

$= 10 - 5 = 5$

5) $2 \times 3 = 6$

6) $4 \times 4 \times 4 \times 4 = 256$

For a, For b, For c, For d

7) Sum in unit place

$= 21 \; (1+2+3)$

$= 2 \times 6 = 12$

Sum in the ten place $= 120$

" hundred place $= 1200$

Sum $= 1200 + 120 + 12 = 1332$.

8. $\boxed{\text{vowel}}$ $\boxed{\text{Consonant}}$ $C(2,1)\,C(4,1) \times 2!$

$2 \times 4 = 8 \rightarrow$ Starting with a vowel.

Vowel & Consonant can be arranged in 2 ways.

$\boxed{\text{Consonant}} \times \boxed{\text{Vowel}}$

$4 \times 2 = 8$. ($\therefore$ Total $= 2 \times 8 = 16$ ways)

9) $4 \times 4 = 16$ (For enter – 4 possibilities
" exit – 4 ")

3(b) Pg-67

21)(i) If $m = n$. A has n elements
B has functions from $A \rightarrow B$ are 1-1

Let $A = \{a_1, a_2, \ldots\ldots a_n\}$

a_1 Corresponding to any one of n elements of B in n ways.

a_2 Corresponds to any one rest (n-1) elements of B in (n-1) ways.

----

a_n Corresponds to any rest ____________

____

$\therefore$ No of ways $= n(n-1) \cdots 1 = \lfloor n$

(ii) If $m < n$, then A has more no of elements than B so no 1-1 functions is formed. (Ans = 0 ($\because$ ${}^{m}P_r$ cannot formed)

(iii) If $m \geq n$

a_1 Corresponds in m ways

a_2 " " $(m-1)$ ways

----

a_n " $m-(n-1) = m-n+1$ ways

$\therefore$ Total no of ways $= m(m-1) \cdots (m-n+1)$

$$= {}^{m}P_n = \frac{\lfloor m}{\lfloor m-n}$$

18. BOOKLET (To form 4 letter words)

Taking one "O", we have to take 4 letters out of 6 letters in $P(6,4) = \frac{6!}{2!}$

$= 360$ ways

Taking two O's 2 other letters are chosen out of 5 letters in $P(5,2)$

$= \frac{5!}{3!} = 20$ ways and each time

Of these are 20 ways two O's can be chosen as follows.

Consider 2 O's & 2 other letters as one kind, then no of ways $= \frac{4!}{2!\,2!} = 6$

∴ If two O's are taken no of ways $= 20 \times 6 = 120$.

∴ Total number of ways $360 + 120 = 480$

12. One digit no → 6

Two digit no:

$\square \; \square$

$6 \times 6 = 36$

3 digit

1 or 2 or 3 ← $\square$ → 3 $\quad \square \; \square$

$3 \times 6 \times 6 = 108$

150

11. $\square \; \square \; \square \; \square \; \square$ 1 or 3

$3 \times 3 \times 2 \times 1 \times 2 = 36$

11. $4 \times 2 = 8$

12. 13. 0 → not placing

0 Cannot placed

$6 \times 5 \times 4 \times 1 = 120$

2 or 4 or 6

$5 \times 5 \times 4 \times 3 = 300$

Total = 420

14. $5 \times 4 \times 3 = 60$

16. 5   6 or 8

$1 \times 2 \times 5 \times 4 = 40$

6 or 8

$2 \times 6 \times 5 \times 4 = 240$

$7 \; 6 \; 5 \; 4 \; 3 = 2520$

$7 \; 6 \; 5 \; 4 \; 3 \; 2 = 5040$

$7 \; 6 \; 5 \; 4 \; 3 \; 2 \; 1 = 5040$

12880

Total: 12880

P. 80

3. 4(v), 6(i) 9 11

P-76 - 7(b), 7(a)

P-77: 12, ~~Page 80 4(v)~~

Page - 802 4(v)

Binomial theorem

Elements - 4 (v)

$$C_0 - 2C_1 + 3C_2 + \cdots + (-1)^n (n+1) C_n$$

$$= (C_0 - C_1 + C_2 - C_3 \cdots + (-1)^n C_n)$$

$$- (C_1 - 2C_2 + 3C_3 \cdots (-1)^{n-1}_n C_n)$$

Take $x(1+x)^n = \cdots$ Differentiate & put $x=-1$

Calculus method

$$= 0 - \left[n - \frac{2 \times n(n-1)}{\lfloor 2} + \frac{3 \times n(n-1)(n-2)}{\lfloor 3} \cdots\right]$$

$$= -n \left[1 - (n-1) + \frac{(n-1)(n-2)}{\lfloor 2} \cdots\right]$$

$$= -n \left[(1-1)^{n-1}\right] = (-n).\, 0 = 0 \quad \text{(Proved)}$$

4. (vi)

$$C_0 + 3C_1 + 5C_2 + \cdots + (2n+1) C_n$$

$$= (C_0 + C_1 + C_2 + \cdots + C_n) + 2(C_1 + 2C_2 + \cdots + nC_n)$$

$$= 2^n + 2 \times n \times 2^{n-1} = 2^n + n\, 2^n = 2^n (n+1)$$

3. $$\frac{(1+k)\left(1+\frac{k}{2}\right) \cdots \left(1+\frac{k}{m}\right)}{(1+m)\left(1+\frac{n}{2}\right) \cdots \left(1+\frac{n}{k}\right)}$$

$$= \frac{\dfrac{(1+k)(2+k) \cdots (n+k)}{1 \times 2 \times 3 \cdots \times n}}{\dfrac{(1+n)(2+n) \cdots (k+n)}{1 \times 2 \times 3 \cdots \times k}}$$

$$= \frac{\dfrac{(k+1)(k+2)\ldots\ldots(k+n)}{\lfloor n}}{\dfrac{(n+1)(n+2)\ldots\ldots(n+k)}{\lfloor k}}$$

$$= \frac{\dfrac{(k+n)(k+n-1)\ldots\ldots\{k+n-(n-1)\}}{\lfloor n}}{\dfrac{(n+k)(n+k-1)\ldots\ldots\{(n+k)-(k-1)\}}{\lfloor k}}$$

$$= \frac{{}^{k+n}C_n}{{}^{n+k}C_k} = \frac{{}^{k+n}C_{k+n-n}}{{}^{n+k}C_k} = \frac{{}^{n+k}C_k}{{}^{n+k}C_k}$$

$= 1$ (proved)

9) $\frac{1}{1!9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!1!}$

$$= \frac{1}{10!}\left[\frac{10!}{1!9!} + \frac{10!}{3!7!} + \frac{10!}{5!5!} + \frac{10!}{7!3!} + \frac{10!}{9!1!}\right]$$

$$= \frac{1}{10!}\left[{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right]$$

$$= \frac{1}{10!}\, 2^{10-1} = \frac{2^9}{10!} = \frac{2^a}{b!}$$

$\therefore\ a = 9,\ b = 10$

10. (a) Suppose $x+y=5$ then

$$x^{99}+y^{99} = x^{99} + (5-x)^{99}$$

$$= \cancel{x^{99}} + \left\{ 5^{99} - 99x\,5^{98} + \cdots + 99x^{98}\cdot 5 - \cancel{x^{99}} \right\}$$

$$= 5\left\{ 5^{98} - 99x\,5^{97} + \cdots + 99x^{98} \right]$$

Now taking $x=1$, $y=4$ we have $1^{99}+4^{99}$ is divisible by 5.

Taking $x=2$, $y=3$ we have $2^{99}+3^{99}$ is divisible by 5.

Also 5^{99} is divisible by 5.

$\therefore$ $1^{99}+2^{99}+3^{99}+4^{99}+5^{99}$ is divisible by 5

(b) If $x+y=3$ then $x^{99}+y^{99}$ is divisible by 3

Taking $x=1$, $y=2$ we get $1^{99}+2^{99}$ is divisible by 3

Also 3^{99} is " "

Again if $x+y=9$ then $x^{99}+y^{99}$ is divisible by 9.

Taking $x=4$, $y=5$ we get $4^{99}+5^{99}$ " " " "

i.e " " " by 3

$\therefore\ 1^{99}+2^{99}+ \cdots - +5^{99}$ is divisible by 3 & hence divisible by 15.

page 76-77 (Elements)

7. (d) General term $= (r+1)^{th}$ term in $\left(\frac{3}{a}+\frac{a}{3}\right)^{10}$

$$= {}^{10}C_r \left(\frac{3}{a}\right)^{10-r}\left(\frac{a}{3}\right)^r = {}^{10}C_r\, 3^{10-r}\, a^{2r-10}\, 3^{-r}$$

For -ve powers of a, $2r-10<0$

i.e $r<5$

i.e $r=0,1,2,3,4$ i.e 5 terms

For +ve powers $2r-10>0$ i.e

$r>5$ i.e $r=6,7,8,9,10$ i.e 5 terms.

(a) $(r+1)^{th}$ term in $(x+*)^n$ is

$${}^nC_r\, x^{n-r}(*)^r$$

6^{th} term $= {}^nC_5\, x^{n-5}(*)^5 = {}^nC_5\, x^{n-10}$ (given)

$$\Rightarrow (*)^5 = x^{-5} = (x^{-1})^5$$

$$\Rightarrow * = x^{-1} = \frac{1}{x}$$

12. (a) $(1+3x+10x^2)\left(x+\frac{1}{x}\right)^{10}$

$$= \frac{1+3x+10x^2}{x^{10}}(1+x^2)^{10}$$

$= \frac{1}{x^{10}}(1+x^2)^{10} + \frac{3}{x^9}(1+x^2)^{10} + \frac{10}{x^8}(1+x^2)^{10}$

Coefficient of x^4 in $\frac{1}{x^{10}}(1+x^2)^{10}$

= Coefficient of x^{14} in $(1+x^2)^{10}$

$(r+1)^{th}$ term in $(1+x^2)^{10} = {}^{10}C_r\, x^{2r}$

$\therefore 2r = 14 \Rightarrow r = 7$

$\therefore$ Coefficient $= {}^{10}C_7 = 120$

Co-efficient of x^4 in $\frac{3}{x^9}(1+x^2)^{10}$

= Coefficient of x^{13} in $3(1+x^2)^{10} = 0$

($\because x^{13}$ will not occure as $2r = 13$
$\Rightarrow r = \frac{13}{2}$
not possible)

Coefficient of x^4 in $\frac{10}{x^8}(1+x^2)^{10}$

= Coefficient of x^{12} in $10(1+x^2)^{10}$

$= 10 \times {}^{10}C_6$ ($\because 2r = 12 \Rightarrow r = 6$)

$= 10 \times 210$

$= 2100$

$\therefore$ Co-eff of x^4 in $(1+3x+10x^2)$

$\left(x+\frac{1}{x}\right)^{10}$

$= 120+2100 = 2220$

b) Co-efficient of x^0 in

$\frac{1}{x^{10}}(1+x^2)^{10}$ = Co-efficient of x^{10} in $(1+x^2)^{10}$

$= {}^{10}C_5 \quad (\because 2r=10 \Rightarrow r=5)$

$= 252$

Co-efficient of x^0 in $\frac{3}{x^9}(1+x^2)^{10}$

= Coeff of x^9 in $(1+x^2)^{10} = 0$

$(\because 2r=9$ not possible$)$

Co-efficient of x^0 in $\frac{10}{x^8}(1+x^2)^{10}$

= Co-efficient of x^8 in $10(1+x^2)^{10}$

$= 10 \times {}^{10}C_4 = 2100$

$\therefore$ Co-efficient of x^0 in given exp $= 2100+252$

13. General term $= C(m+n, r)a^r$

Taking $r=m$, Co-eff $= C(m+n, m)$

Taking $r=n$, Co-eff $= C(m+n, n)$

14. The expansion of $(a+b+c)^n$ is the product of n factors

each equal to $(a+b+c)$ and every term in the expansion is formed by taking one letter out of each of these n factors and therefore the no of ways in which any term $a^p b^q . c^r$ will appear in the final product is equal to the no. of ways of arranging n letters when p of them must be a, q, of them must be b, and r of them must be c.

in coeff of $a^p . b^q c^r = \frac{\lfloor n}{\lfloor p . \lfloor q . \lfloor r}$ (Proved)

where $p+q+r=n$.

page no 76 12 - 10 gives will be

arranged among themselves in 10! ways.

Between 10 girls there are 11 gaps & in this 11 gaps 10 boys will sit in $P(11, 10) = \frac{\lfloor 11}{\lfloor 1} = \lfloor 11$ ways.

$\therefore$ Total no. of ways $= \lfloor 10 \cdot \lfloor 11$

13. The 7 girls will sit together. So we take these 7 girls in one place & 6 men require 6 places. Here total places = 7. These can be arranged in 7! ways. Each times of these arrangement 7 girls can be arranged in 7! ways.

$\therefore$ Total no of ways $= 7!\ 7!$

16. n is the product of K distinct primes.
10-70

$n_1, n_2, \text{-----}\ n_K$.

ie $n = n_1 n_2 \text{---}\ n_K$

So we choose n_1 in 2 ways.

1 8826 " " n_2 " " "

" " n_K " 2 ways

∴ Total no of ways = $2 \times 2 \times 2 \cdots$ K times

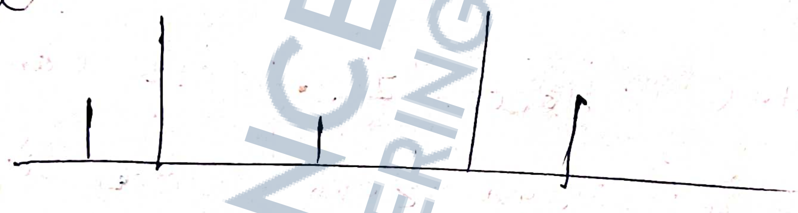
Total no factors $= 2K = 2^K$

Excluding 1 & the number n itself the total no of factors $= 2^K - 2$

18.

$\frac{4 \; 8 \; 5}{0} \quad \frac{10}{0} \quad \frac{4 \; 8 \; 15}{0} \quad \frac{20}{0} \rightarrow$ 4 terms.

19 ch: 3GM



The 2 boys can sit in $\underline{2!}$ ways. Before & 2 boys there are 3 places where 3 girls can sit in $P(3,3) = 3!$ ways

Hence the no. of ways of arrangement

$= 2!\,3!$

$= 12$

—o—

Q-) $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$

$$= \int_0^1 \frac{x^4(1-4x+6x^2-4x^3+x^4)}{1+x^2}\,dx$$

$$= \int_0^1 \frac{x^4-4x^5+6x^6-4x^7+x^8}{1+x^2}\,dx$$

Consider

$x^2+1 \,\big)\, x^8-4x^7+6x^6-4x^5+x^4 \,\big(\, x^6-4x^5+5x^4$

$\quad \cancel{x^8} + x^6$ $\;(-)\;(-)$

$\quad -4x^7+5x^6-4x^5+x^4$

$\quad -4x^7 - 4x^5$ $\;(+)\;(+)$

$\quad 5x^6 + x^4$

$\quad 5x^6 + 5x^4$ $\;(-)\;(-)$

$\quad -4x^4$

$$= \int_0^1 \left(x^6-4x^5+5x^4-\frac{4x^4}{x^2+1}\right)dx$$

$$= \int_0^1 (x^6 - 4x^5 + 5x^4)\,dx - \int_0^1 \frac{4x^4}{x^2+1}\,dx$$

$$= \left[\frac{x^7}{7} - \frac{\cancel{4}^2 x^6}{\cancel{6}_3} + \frac{\cancel{5}x^5}{\cancel{5}}\right]_0^1 - \int_0^1 \left(4x^2 - 4 + \frac{4}{x^2+1}\right)dx$$

$$= \left\{\left(\frac{1}{7} - \frac{2\cdot 1}{3} + 1\right) - (0 - 0 + 0)\right\} - \left[\frac{4\cdot x^3}{3} - 4x + 4\cdot \tan^{-1}x\right]_0^1$$

$$= \frac{10}{21} - \left\{\left(\frac{4}{3} - 4 + 4\cdot\frac{\pi}{4}\right) - (0 - 0 - 0)\right\}$$

$$= \frac{10}{21} - \frac{4}{3} + 4 - \pi$$

$$= \frac{10 - 28 + 84}{21} - \pi$$

$$= \frac{66}{21} - \pi$$

$$= \frac{22}{7} - \pi$$

Corollary → The no. of permutations of n different objects taken all at a time is $P(n,n) = n!$

Proof ÷ The number of permutations of n different objects taken n at a time is the same as the number of different arrangements of n objects in n places in a row.

1st	2nd	3rd	----	nth
$\square$	$\square$	$\square$	----	$\square$
n	$\times\, n-1$	$\times\, n-2$		$\times\, 1$

The 1st place can be filled by anyone of n objects in n different ways. Keeping one object in 1st place there are $(n-1)$ objects left for the 2nd place. So the 2nd place can be filled up in $(n-1)$ ways.

∴ The 1st two places can be filled up in $n(n-1)$ different ways.

After the 1st two places are filled up there are $(n-2)$ objects left for 3rd place. So the 3rd place can be filled up in $(n-3)$ different ways.

∴ The 1st three places can be filled up in $(n)(n-1)(n-2)$ ways (by F.P.C.)

Similarly continuing up to n^{th} place can be filled ~~up~~ up by 1 ~~place~~ way.

So all the n places can be filled up in $n.(n-1)(n-2)$ ———— 1 way.

(By F.P.C)

Hence total no of ways of filling all the n places

$= P(n,n) = n(n-1)(n-2) ---- 1$ (Answer)

$= \lfloor n$

Proof →

Let $a, x \in R$ and n is any +ve integer.

Let P_n be the statement.

$(a+x)^n = C(n,0)\,a^n + C(n,1)\,a^{n-1}x + C(n,2)\,a^{n-2}.x^2$

$+ C(n,3)\,a^{n-3}\,x^3 + ------ + C(n,r)\,a^{n-r}.x^r + ----- + C(n,n)\,x^n$

Porve that the no of permutation of n dibberent Objects taken all at a time is $P(n,n) = n!$

Porf : The number of permutation of n ~~obj~~ different objects taken all at a time is same as the number of dibberent arrangements of n objects in n places in a row.

1st . . . 2nd . . . 3rd . . . . . . $n-1$ th . . . nth

$n \times (n-1) \times (n-2) \times \ldots \ldots \times n-(n-2) = 2 \times n-(n-1) = 1$

The 1st place can be filledup by any one of ~~ool~~ n Objects in n dibberent ways. Keeping one object in the first place there $(n-1)$ objects lebt for the 2nd place. So the 2nd place can be ~~can~~ filled up in $(n-1)$ ways

The first two places can be filled up in $n(n-1)$ dibberent ways (by F.P.S)

After the 1st two places are filled up there are $(n-2)$ leuf for the 3rd place. So the 3rd place can be filled up in $(n-2)$ dibberent ways.

∴ The first three place can be filled up by $n(n-1)(n-2)$ different ways.

Similarly continuing up to n^{th} place

∴ The n^{th} place can be filled up in 1 way. So all the n places can be filled up by $n.(n-1).(n-2)$ - - ——— 1 different ways, (By F.P.C)

∴ Here the total no of ways of filling n places

$= n(n-1)(n-2)$ - - — — — ——— 1

$= \lfloor n$

i.e $P(n,n) = n(n-1).(n-2)$ - ——— $1 = \lfloor n$

(Proved)