'Jay Jagannath'

Determinant

A determinant of 2nd order is written as in the form $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ and its Value is ad-bc.

It has two rows and two columns. Here a, b, c, d are called elements. There are $2^2 = 4$ elements and $\lfloor 2$ terms. A determinant of 2nd order is also Called 2x2 (2 by 2) determinant.

A determinant of 3rd order is written in the form

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

It is also Called 3x3 determinant.

It has 3 rows and 3 ~~cows~~ Columns

The 1st row is $a_1\ b_1\ c_1$

The 1st Column is $a_1\ a_2\ a_3$.

It has $3^2 = 9$ elements

It has $\lfloor 3 = 6$ terms.

The above determinant Can be expan

either row wise or column wise.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$$

$$+ c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

(Expanding about 1st row)

$$= a_1 (b_2 c_3 - c_2 b_3) - b_1 (a_2 c_3 - c_2 a_3)$$
$$+ c_1 (a_2 b_3 - b_2 a_3)$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

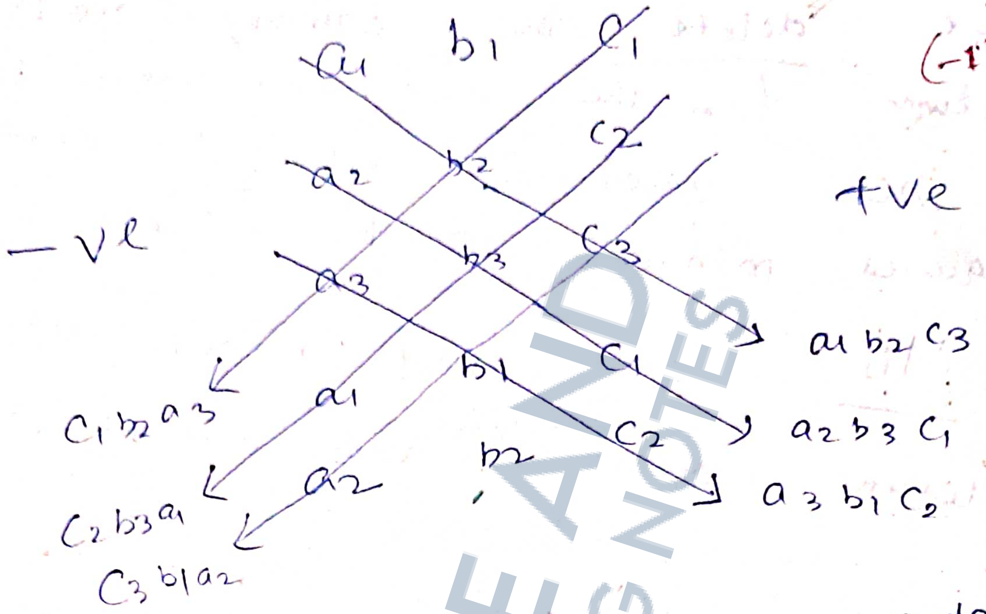
(Expanding about 1st column)

$$= a_1 (b_2 c_3 - c_2 b_3) - a_2 (b_1 c_3 - c_1 b_3)$$
$$+ a_3 (b_1 c_2 - c_1 b_2)$$

The sign ~~convention~~ concept in a determinant is as follows

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

The 3rd order determinant can be expanded by a method known as <u>method of sarrus</u>.



A determinant of n^{th} order has n rows and n columns. It is called $\underline{n \times n}$ determinant. It is of the form,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

Here, a_{ij} is the element which is situated at the intersection of <u>ith</u> row and <u>j th</u> column.

SCIENCE AND ENGINEERING NOTES

Minor of an element in a determinant

To find the minor of the element situated in the 'i th' row and 'j th' column, we delete the i th row and j th column from the determinant and we get a new determinant and it is the required minor of the element.

M_{ij} is called minors of the element in i th row and j th column.

Ex:
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Minor of 5 $= M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9-21 = -12$

Minor of 8 $= M_{32} = \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 6-12 = -6$

Co-factor of an element in a determinant

Co-factor of the element in ith row and jth column is denoted by C_{ij} or A_{ij}

Here $C_{ij} = (-1)^{i+j} M_{ij}$

i.e. co-factor of an element in the 'ith' row and 'jth' column.

$= (-1)^{i+j}$. Minor of the element a_{ij}

Minor with proper sign prefix is Co-factor.

Ex :
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Co-factor of 5 $= (-1)^{2+2}$. minor

$= (-1)^4 (-12)$

$= 1 \times (-12)$

$= -12$

Co-factor of 8

$= (-1)^{3+2} . (-6)$

$= (-1)^5 . (-6)$

$= 6$

Imp

Solution of ~~Simulat~~ simultaneous equations by ~~det~~ determinant method ~~on~~ on CRAMER'S RULE.

Suppose we have to solve

$a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$

we take, $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

$D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

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$x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$. Provided $D \neq 0$

Suppose, we have to solve

$a_1x + b_1y + c_1z = k_1$

$a_2x + b_2y + c_2z = k_2$

$a_3x + b_3y + c_3z = k_3$

Take $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

$$D_1 = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$$

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

Notes :

(i) Cramer's rule is applied if $D \neq 0$

SCIENCE AND ENGINEERING NOTES

(2) If $D = D_1 = D_2 = D_3 = 0$, the system of equation has infinite ~~no~~ number of solution.

(3). If $D = 0$, but atleast one of D_1, D_2, D_3 is not zero, then the system has no solution. Here, the system is called 'Inconsistent system'.

Unique sol^n → determinate, infinite sol^n → indeterminate.

4) The system of linear equations of the form

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

is called Homogeneous system.

The system given by

$$a_1x + b_1y + c_1z = k_1$$
$$a_2x + b_2y + c_2z = k_2$$
$$a_3x + b_3y + c_3z = k_3$$

is called non-homogenous system where at least one of k_1, k_2, k_3 is not zero.

5) For homogeneous system, $x = y = z = 0$ is a solution and it is called ~~trivial~~ trivial solution.

Here, if $D \neq 0$, then only trivial solution exists.

SCIENCE AND ENGINEERING NOTES

6) A homogeneous system has a non-trivial solution if $D=0$ and in this case, there are infinite number of solutions.

7) Homogeneous system is always consistent.

Higher secondary

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12. $$x+y+2z=4$$
$$2x-y-z=1$$
$$3x-2y-z=3$$

$$D=\begin{vmatrix}1 & 1 & 2\\ 2 & -1 & -1\\ 3 & -2 & -1\end{vmatrix} = 1(1-2)-1(-2+3)+2(-4+3)$$
$$= -1-1-2 = -4 \neq 0$$

$$D_1=\begin{vmatrix}4 & 1 & 2\\ 1 & -1 & -1\\ 3 & -2 & -1\end{vmatrix} = 4(1-2)-1(-1+3)+2(-2+3)$$
$$= -4-2+2 = -4$$

$$D_2=\begin{vmatrix}1 & 4 & 2\\ 2 & 1 & -1\\ 3 & 3 & -1\end{vmatrix} = 4$$

$$D_3=\begin{vmatrix}1 & 1 & 4\\ 2 & -1 & 1\\ 3 & -2 & 3\end{vmatrix} = -8$$

$$x=\frac{D_1}{D}=\frac{-4}{-4}=1$$

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$$y = \frac{D_2}{D} = \frac{4}{-4} = -1$$

$$z = \frac{D_3}{D} = \frac{-8}{-4} = 2$$

13 (ii) $x + 2y + 3z = 1$ —— (i)

$2x + 3y + 5z = 1$ —— (ii)

$3x + 4y + 7z = 1$ —— (iii)

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 5 \\ 3 & 1 & 7 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

Hence the system has infinite no. of solution

From (i) and (ii)

$$2x + 4y + 6z = 2$$
$$2x + 3y + 5z = 1$$
$$\overline{\quad y + z = 1} \Rightarrow y = 1 - z$$

SCIENCE AND ENGINEERING NOTES

From (i) and (iii)

$$2x + 4y + 6z = 2$$

$$3x + 4y + 7z = 1$$

$$\underline{-\quad -\quad -\quad -}$$

$$-x - z = 1 \Rightarrow x + z = -1$$

$$\Rightarrow x = -1 - z$$

Taking $z = \alpha$, $x = -1 - \alpha$, $y = 1 - \alpha$

Assuming different value of α, we get different solutions. Hence ~~there~~ there are infinite number of soln.

Q) Solve

$$2x + 3y + z = 3$$

$$x + 4y + z = 1$$

$$3x + 7y + 2z = 3$$

Soln:

$$D = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 4 & 1 \\ 3 & 7 & 2 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & 3 & 1 \\ 1 & 4 & 1 \\ 3 & 7 & 2 \end{vmatrix} \neq 0$$

The system has no solution and the system is thus inconsistent.

Q) For what value of 'k', do the following system of equation has a non-trivial solution over the set of rationals.

$$x + Ky + 3z = 0$$
$$3x + Ky - 2z = 0$$
$$2x + 3y - 4z = 0$$

For that value of K, find all the solution of the system.

Soln:- $D = \begin{vmatrix} 1 & K & 3 \\ 3 & K & -2 \\ 2 & 3 & -4 \end{vmatrix} = 33 - 2K$

The given system will have a non-trivial soln if $D = 0$

i.e $33 - 2K = 0 \Rightarrow K = \frac{33}{2}$

For this value of K, the given system ~~will~~ becomes

$$2x + 33y + 6z = 0 \quad \text{—(i)}$$
$$6x + 33y - 4z = 0 \quad \text{—(ii)}$$
$$2x + 3y - 4z = 0 \quad \text{—(iii)}$$

Solving (ii) and (iii), we get

$$\frac{x}{-132+12} = \frac{y}{-8+24} = \frac{z}{18-66}$$

$$\Rightarrow \frac{x}{-120} = \frac{y}{16} = \frac{z}{-48}$$

$$\Rightarrow \frac{x}{15} = \frac{y}{-2} = \frac{z}{6} = \lambda$$

$$\Rightarrow x = 15\lambda, \quad y = -2\lambda, \quad z = 6\lambda$$

For these values eqn (i) is satisfied.

∴ The given equation has infinite no. of solutions given by $(15\lambda, -2\lambda, 6\lambda)$ where $\lambda \in \mathbb{R}$

Do the problem of higher secondary.

Properties of determinant

1) The value of determinant remains unchanged by changing its rows (columns) into corresponding columns (rows)

Proof:

$a_1 \; a_2 \; a_3$
$b_1 \; b_2 \; b_3$
$c_1 \; c_2 \; c_3$

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Let $D_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

be the determinant obtained by changing the rows of D to corresponding columns.

$D = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3)$
$+ c_1(a_2b_3 - b_2a_3)$ (expanding about 1st row.)

$= a_1b_2c_3 - a_1c_2b_3 - b_1a_2c_3 - b_1c_2a_3$
$+ c_1a_2b_3 - c_1b_2a_3$

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$\mathcal{D}_1 = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - c_2a_3)$

$+ c_1(a_2b_3 - b_2a_3)$ (Expanding about 1st column)

$= D$ (proved)

② If any two rows (columns) of a determinant are interchanged, then the sign of determinant is changed.

Proof. Let

$$\mathcal{D} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Let $\mathcal{D}_1 = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$ be the determinant.

Obtained by interchanging 1st and 3rd row of $\mathcal{D}$.

$\mathcal{D} = a_1b_2c_3 - a_1c_2b_3 - b_1a_2c_3 + b_1c_2a_3$

$+ c_1a_2b_3 - c_1b_2a_3$

$\mathcal{D}_1 = a_3(b_2c_1 - c_2b_1) - b_3(a_2c_1 - c_2a_1)$

$+ c_3(a_2b_1 - a_1b_2)$

$= a_3b_2c_1 - a_3c_2b_1 - b_3a_2c_1 + b_3c_2a_1$

$c_3a_2b_1 - c_3a_1b_2$

$$\Delta_1 = +(a_1 b_2 c_3 - a_1 c_2 b_3 - b_1 a_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - c_1 b_2 a_3)$$

$$= -\Delta$$

Hence the sign has interchanged □

(3) If two rows (columns) of a determinant are identical the determinant vanishes.

Proof : Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

be the determinant whose 1st and 2nd rows are identical.

$$\therefore \Delta = a_1(b_1 c_3 - c_1 b_3) - b_1(a_1 c_3 - c_1 a_3) + c_1(a_1 b_3 - b_1 a_3)$$

$$= 0$$

Alternative method

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

be the determinant whose 1st and 2nd rows are identical.

SCIENCE AND ENGINEERING NOTES

Now
$$D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
be the determinant obtained by interchanging 1st and 2nd row of D.

Now $D_1 = -D$ (by property - 2)

But $D = D_1$

$\Rightarrow D = -D$

$\Rightarrow 2D = 0$

$\Rightarrow D = 0$ $\square$

4)) If each element of row (column) of a determinant be multiplied by the same quantity then the determinant is multiplied by that quantity.

Proof : Let

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad D_1 = \begin{vmatrix} Ka_1 & Kb_1 & Kb_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

be the determinant obtained by multiplying K with each element of 1st row of D

$$D = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

$$\Delta_1 = Ka_1(b_2c_3 - c_2b_3) - Kb_1(a_2c_3 - c_2a_3) + Kc_1(a_2b_3 - b_2a_3)$$

$$= K\left[a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\right]$$

$$= K.D \qquad \square$$

(5). If each element of any one row (column) of a determinant is the sum of two terms then the determinant can be expressed as the sum of two determinants.

Proof:

Let $\Delta =$
$$\begin{vmatrix} a_1+\alpha_1 & b_1+\alpha_2 & c_1+\alpha_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

be the determinant in which each element of the 1st row is the sum of two terms.

$$\therefore \Delta = (a_1+\alpha)(b_2c_3 - c_2b_3) - (b_1+\alpha_2)(a_2c_3 - c_2a_3) + (c_1+\alpha_3)(a_2b_3 - b_2a_3)$$

(Expanding about 1st row)

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$$= \left[a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\right] + \left[\alpha_1(b_2c_3 - c_2b_3) - \alpha_2(a_2c_3 - c_2a_3) + \alpha_3(a_2b_3 - b_2a_3)\right]$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(6) If each element of any row (column) of a determinant be increased or decreased by a constant multiple of the corresponding elements of another row then value of determinant remains unchanged.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Let, $\Delta_1 = \begin{vmatrix} a_1 + Ka_2 & b_1 + Kb_2 & c_1 + Kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

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be the determinant obtained by replacing 1st row by the sum of the 1st row with K times the second row.

$$\therefore \Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} Ka_2 & Kb_2 & Kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(by Prop. 5)

$$= \Delta + K \begin{vmatrix} a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$ (by Prop. 4)

$= \Delta + K \times 0$ (By Prop. 3 and since 1st row and 2nd row are identical)

$= \Delta + 0$

$= \Delta$ $\square$.

Notes :

1. R_1, R_2, R_3 stand for 1st row, 2nd row and 3rd row respectively.

2. C_1, C_2, C_3 for 1st, 2nd and 3rd column respectively.

3. $R_1 + 3R_2 + 5R_3$ means replacing 1st row by 1st row + 3×2nd row + 5×3rd row.

Problems:

9. (i) Imp

$$\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} x & p & a \\ y & q & b \\ z & r & c \end{vmatrix}$$ (Interchanging rows and columns)

$$= - \begin{vmatrix} y & q & b \\ x & p & a \\ z & r & c \end{vmatrix}$$ (Interchanging R_1 and R_2)

$$= + \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$ Interchanging C_2 and C_3.

Again $$\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = - \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$ (Interchanging R_1 and R_3)

$$= + \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$$ Interchanging R_2 and R_3

□.

3. (ii) $$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

SCIENCE AND ENGINEERING NOTES

Replacing R_1 by $R_1+R_2+R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$$

9.(vii) $$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ (Taking a, b, c common from R_1, R_2 and R_3 respectively)

$$= abc \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$ (Replacing R_1 by R_1-R_2 and R_2 by R_2-R_3)

$$= abc(a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Taking common $(a-b)$ from R_1 and $(b-c)$ from R_2.

$$= abc(a-b)(b-c)\{1\cdot(b+c-a-b)\}$$

(Expanding about 1st ~~row~~ column)

$$= abc(a-b)(b-c)(c-a) \quad \square$$

SCIENCE AND ENGINEERING NOTES

Notes :-

While applying the six ~~group~~ properties attemp should be made.

(1) To reduce the size of the elements as far as practicable

(2) To reduce at least one of the elements to unity.

(3) To bring the max no of zeros in a certain rows or column and get this an element of unit value will be of immense help.

9. (viii)

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ Replacing R_1 by $R_1 - R_2 - R_3$

$$= -2 \begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ Taking -2 common from R_1

$$= -2\left[0 - c(ab + b^2 - bc) + b(bc - c^2 - ca)\right]$$

$= -2\left[-abc - cb^2 + bc^2 + b^2c - bc^2 - bca\right]$

$= -2\left[-2abc\right]$

$= 4abc \quad \square.$

9. (ii) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$= abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$ Taking a, b, c Common from ~~from~~ R_1, R_2 and R_3 respectively.

$= abc \begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Replacing by $R_1+R_2+R_3$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ Common from R_1

$= abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & \frac{1}{b} \\ -1 & -1 & \frac{1}{c}+1 \end{vmatrix}$

SCIENCE AND ENGINEERING NOTES

Replacing C_1 by $C_1 - C_3$ and C_2 by $C_2 - C_3$

$$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\left[1 + (0+1)\right]$$

(Expanding ~~as~~ about 1st row)

$$= abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1\right) \quad \text{(Proved)}$$

Q. (xii) Show that

$$\begin{vmatrix} (v+w)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix} = 2uvw(u+v+w)^3$$

Proof:

$$\begin{vmatrix} (w+v)^2 & u^2 & u^2 \\ v^2 & (w+u)^2 & v^2 \\ w^2 & w^2 & (u+v)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (v+w)^2 - u^2 & 0 & u^2 \\ 0 & (w+u)^2 - v^2 & v^2 \\ w^2 - (u+v)^2 & w^2 - (u+v)^2 & (u+v)^2 \end{vmatrix}$$

$C_1 - C_3$ and $C_2 - C_3$

SCIENCE AND ENGINEERING NOTES

$$= (u+v+w)^2 \begin{vmatrix} v+w-u & 0 & u^2 \\ 0 & w+u-v & v^2 \\ w-u-v & w-u-v & (u+v)^2 \end{vmatrix}$$

Taking $(u+v+w)$ common from C_1 and C_2

$$= (u+v+w)^2 \begin{vmatrix} v+w-u & 0 & u^2 \\ 0 & w+u-v & v^2 \\ -2v & -2u & 2uv \end{vmatrix}$$ R_3 by $R_3 - R_2 - R_1$

$$= (u+v+w)^2 \begin{vmatrix} v+w & u^2/v & u^2 \\ \frac{v^2}{u} & w+u & v^2 \\ 0 & 0 & 2uv \end{vmatrix}$$

By C_1 by $C_1 + \frac{C_3}{u}$ and C_2 by $C_2 + \frac{C_3}{v}$

$$= (u+v+w)^2 \left[2uv \left\{ (v+w)(w+u) - \frac{u^2}{v} \cdot \frac{v^2}{u} \right\} \right]$$

(Expanding about 3rd row)

$$= (u+v+w)^2 \, 2uv \{ vw + \cancel{vu} + w^2 + wu - \cancel{uv} \}$$

$$= (u+v+w)^2 \, 2uvw \, (u+v+w)$$

$$= 2uvw \, (u+v+w)^3 \quad \square$$

Page - 197 → Elements Q No.: 24

If $2s = a+b+c$, prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$$

SCIENCE AND ENGINEERING NOTES

Proof: Put $s-a = A$, $s-b = B$, $s-c = C$

Then $B+C = a$

$C+A = b$

$A+B = c$

$$\left\{\begin{aligned} B+C &= 2s-(b+c) = a \\ C+A &= 2s-(a+c) = b \\ A+B &= 2s-(a+b) = c \end{aligned}\right\}$$

$$\therefore D = \begin{vmatrix} (B+C)^2 & A^2 & A^2 \\ B^2 & (C+A)^2 & B^2 \\ C^2 & C^2 & (A+B)^2 \end{vmatrix}$$

(by 7(a) of Ex...)

$$= 2ABC(A+B+C)^3$$

$$= 2(s-a)(s-b)(s-c)s^3$$

$$= 2s^3(s-a)(s-b)(s-c)$$

(Proved)

Q:
$$\begin{vmatrix} (a+b)^2 & ca & bc \\ ca & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix}$$

$$= 2abc(a+b+c)^3$$

Hints: Multiply a, b, c with R_2, R_3 and R_1 respectively.

SCIENCE AND ENGINEERING NOTES

Matrices

A matrix is an arrangement of elements (which may be numbers, functions etc) in certain rows and columns i.e

A matrix is an arrangement of elements.

If in a matrix there are m rows and n ~~coul~~ columns then the matrix is said to be $m \times n$ (m by n) matrix. Its order is $m \times n$. The elements of a matrix are also known as entries.

The elements of a matrix are enclosed within a square bracket or double lines ~~as~~ or parenthesis.

[ ] or ( ) or || ||

A matrix is denoted by capital letters like A, B, C etc

If in a matrix the no of rows and the no. of Columns are not equal i.e. $m \neq n$, then the matrix is Called Rectangular matrix of order $m \times n$.

If the no of rows = the no of the Columns i.e. $m = n$, then the matrix is Called a square matrix of order m.

A matrix having only one Column is Called a column matrix or a Column vector

A matrix having only one row is Called a row matrix or a row vector.

Ex:

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a square matrix of order 2

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ is a rectangular matrix of order 2×3

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ is a rectangular matrix of order 3×2

$(1, 2, 3, 4)$ is a row matrix of order 1×4

$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ is a column matrix of order 4×1

(1) is a square matrix of order 1×1

<u>Note</u> :-

The number of elements in <u>$m \times n$</u> order matrix is mn.

Consider an '$m \times n$' matrix as follows.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

Here the general element is a_{ij} which is situated at the intersection of <u>'ith' row</u> and <u>jth column</u>. The above matrix can be written as $(a_{ij})_{m \times n}$

SCIENCE AND ENGINEERING NOTES

Equality of two Matrices

Two matrices A and B are said to be equal if they are of the same order and their corresponding elements are equal.

Ex: $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\Rightarrow a = 1, \quad b = 2, \quad c = 3, \quad d = 4$

Sum of two matrices

We can find the sum of two matrices if they are of the same order.

Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}$

$A + B = \begin{pmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}$

Suppose $A = (a_{ij})_{m \times n}$

$B = (b_{ij})_{m \times n}$

$A + B = (a_{ij} + b_{ij})_{m \times n}$

SCIENCE AND ENGINEERING NOTES

Difference of two matrices

We can find the difference of two matrices if they are of the same order.

Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}$

$$A - B = \begin{pmatrix} -6 & -6 & -6 \\ -6 & -6 & -6 \end{pmatrix}$$

Suppose $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$

$$A - B = (a_{ij} - b_{ij})_{m \times n}$$

Q → Prove that matrix addition is commutative.

Let $A = (a_{ij})_{m \times n}$

$B = (b_{ij})_{m \times n}$

$$A + B = (a_{ij} + b_{ij})_{m \times n}$$

$$= (b_{ij} + a_{ij})_{m \times n}$$

$$= B + A \quad \text{(Proved)}$$

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Q → Prove that matrix addition is associative

Proof →

Let $A = (a_{ij})_{m \times n}$

$B = (b_{ij})_{m \times n}$

$C = (c_{ij})_{m \times n}$

Now

$A + (B + C) = (a_{ij})_{m \times n} + (b_{ij} + c_{ij})_{m \times n}$

$= \{(a_{ij})_{m \times n} + (b_{ij})_{m \times n}\} + (c_{ij})_{m \times n}$

$\{a_{ij} + b_{ij}\}_{m \times n} + (c_{ij})_{m \times n}$

$= (A + B) + C$

(Hence ... proved)

see last + ~~for~~ theory page for proof

Scalar multiplication

Let K be a scalar and $A = (a_{ij})_{m \times n}$

then we define $KA = (Ka_{ij})_{m \times n}$

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It is called Scalar multiplication.

Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$$7A = \begin{pmatrix} 7 & 14 & 21 \\ 28 & 35 & 42 \end{pmatrix}$$

Additive inverse of a matrix

The additive inverse of A is denoted by $-A$

Ex: Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

$$-A = \begin{pmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{pmatrix}$$

Zero matrix or Null matrix

If in a matrix all the elements are zero, then it is called a zero matrix or null matrix. It is denoted by O, (O not zero)

Ex: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times2}$ $(0)_{1\times1}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{2\times3}$

are null ~~matrices~~ matrices of different orders.

Notes :

1. The Zero matrices are of different orders so it is not unique. But zero matrix of a fixed order is unique.

2. The zero matrix is called additive identity.

3. $A + O = A = O + A$

4. $A + (-A) = O = (-A) + A$

Transpose of a Matrix

If the rows and columns of a matrix are interchanged then the new matrix obtained is called transpose of the given matrix. The transpose of A is denoted by A' or A^T or A^t.

Suppose A is of order $m \times n$ then A' is of order $n \times m$

Ex : $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3}$

$A' = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

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Unit ~~order~~ matrix or Identity matrix

A square matrix whose entries along the principal diagonal or leading diagonal (left top to right bottom) are all 1 and all other entries are zero is called unit matrix or identity matrix. It is denoted by 'I'.

Unit matrices are of different orders. So unit matrix is not unique. But for fixed order matrices the unit matrix is unique.

$\boxed{\text{(Matrix)} \times \text{(Unit matrix)} = \text{Matrix itself}}$ (into)

Ex:-

$(1)_{1\times 1}$, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times 3}$ are unit matrices of order 1, 2, 3 respectively.

Multiplication of Matrices

Suppose A is a matrix of order $(m \times n)$ and $A = (a_{ij})_{m\times n}$, B is a matrix of order $n \times p$ and

$B = (b_{jk})_{n \times p}$,

Then the product of matrix $AB = C$ is a matrix of order $m \times p$ and

$C = (c_{ik})_{m \times p}$

$A = (a_{ij})_{m \times n}$
$i = 1, 2, \cdots m$
$j = 1, 2, \cdots n$
$B = (b_{jk})_{n \times p}$
$k = 1, 2, \cdots p$

Also
$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

$$= a_{i1} b_{1k} + a_{i2} b_{2k} + a_{i3} b_{3k} + \cdots + a_{in} b_{nk}$$

$$= \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix} \begin{pmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{pmatrix}$$

Hence element in ith row and kth column of AB

$$= \begin{pmatrix} i\text{th row of } A \end{pmatrix} \begin{pmatrix} k\text{th column of } B \end{pmatrix}$$

$$\boxed{i\text{th row of } A} \quad \boxed{\begin{matrix} k\text{th} \\ \text{column} \\ \text{of} \\ B \end{matrix}}$$

8) (iii) $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}_{2 \times 2}$ $\begin{pmatrix} 4 & 2 \\ -1 & -2 \end{pmatrix}$

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$$B = \begin{pmatrix} 4 & 2 \\ -1 & -2 \end{pmatrix}_{2\times 2}$$

AB is a matrix of order 2×2

$$AB = \begin{pmatrix} 1\times 4 + (-1)\times(-1) & 1\times 2 + (-1)\times -2 \\ 2\times 4 + 3\times(-1) & (2\times 2) + 3\times(-2) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 \\ 5 & -2 \end{pmatrix}$$

BA is a matrix of order 2×2

$$BA = \begin{pmatrix} 4\times 1 + 2\times 2 & 4\times(-1) + 2\times 3 \\ -1\times 1 + (-2)\times 2 & (-1)(-1) + (-2)3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 2 \\ -5 & -5 \end{pmatrix}$$

$AB \neq BA$

∴ Commutative law for multiplication of matrices is not satisfied.

<u>Ex:</u>

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 1 & -1 \end{pmatrix}_{2\times 3}$$

$$B = \begin{pmatrix} 2 & 4 \\ 1 & 2 \\ 3 & -1 \end{pmatrix}_{3\times 2}$$

$$AB = \begin{pmatrix} 1\times 2 + (-2)\times 1 + 2\times 3 & 1\times 4 + (-2)\times 2 + 2(-1) \\ 3\times 2 + 1\times 1 + (-1)\times 3 & 3\times 4 + 1\times 2 + (-1)(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -2 \\ 4 & 15 \end{pmatrix}$$

Ex: Calculate 'BA'

$$A = \begin{pmatrix} 3 & 2 & 4 \end{pmatrix}_{1\times 3}$$

$$B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}_{3\times 1}$$

AB is a matrix of order 1×1

$$AB = (3\times 1 + 2\times 0 + 4\times 2) = (11)_{1\times 1}$$

Difference between Matrix and Determinant

① Determinant is a quantity where as matrix is an arrangement in rows and Columns

② In a determinant the no of rows is equal to the number of Columns But in a matrix the no of rows may not be equal to the no of Columns. A square matrix has the

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determinant

(3) If a scalar is multiplied with matrix, then it is multiplied with all the elements of the matrix, with a determinant if a scalar is multiplied than it is multiplied with the elements of a row or of a column.

Adjoint of a square matrix

The adjoint of a square matrix A is denoted by adj A. It can be found as follows.

First find the co-factors of the elements of A and arrange them according to their order, the matrix obtained is called matrix of co-factors. The transpose of matrix of co-factor is called adj-A

Singular and non-singular matrices

A square matrix A is called singular if $|A| = 0$ otherwise it is non-singular.

Inverse of a square matrix (6.2)

Suppose A is a square matrix and it is non-singular, then its inverse exists and is denoted by A^{-1} and is defined by

$$A \cdot A^{-1} = I = A^{-1} \cdot A = \frac{Adj A}{|A|} \times A$$

Also $A^{-1} = \dfrac{adj\, A}{|A|}$ & $A\, Adj.A = |A| I$

Q → Inverse of square matrix if exists is unique

Proof : Let A be a square matrix and is non-singular. Then inverse of A exists.

To prove that inverse of A is unique

If not, let B and C be two inverses of A, and $B \neq C$

$$AB = I = BA$$

and $$AC = I = CA$$

Now $$(CA)B = C(AB)$$

$$\Rightarrow IB = IC$$

$$\Rightarrow B = C$$ which is a contradiction.

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$\therefore$ Inverse of A is unique proved.

Q) Find the inverse of the following matrix

Imp

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 2 \\ 1 & -1 & -1 \end{pmatrix}$$

Ans): Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 2 \\ 1 & -1 & -1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -3 & 2 \\ 1 & -1 & -1 \end{vmatrix} = 10 \neq 0$$

$\therefore$ A is non-singular.

$\therefore A^{-1}$ exists.

$$C_{11} = (-1)^{1+1} \begin{vmatrix} -3 & 2 \\ -1 & -1 \end{vmatrix} = 5$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = 3$$

$C_{13} = 2$

$C_{21} = 0$, $C_{22} = -2$, $C_{23} = 2$

$C_{31} = 5$ $C_{32} = -1$, $C_{33} = -4$

Matrix of co-factors $= \begin{pmatrix} 5 & 3 & 2 \\ 0 & -2 & 2 \\ 5 & -1 & -4 \end{pmatrix}$

$$\text{Adj } A = \begin{pmatrix} 5 & 0 & 5 \\ 3 & -2 & -1 \\ 2 & 2 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{10}\begin{pmatrix} 5 & 0 & 5 \\ 3 & -2 & -1 \\ 2 & 2 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{10} & -\frac{1}{5} & -\frac{1}{10} \\ \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \end{pmatrix} \quad \text{(Ans)}$$

Solution of simultaneous eqns by matrix method

Suppose we have

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The given system of eqns can be written in matrix form as

$$AX = B$$

where $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}_{3\times 3}$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{3\times 1}$$

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$$B = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{3\times 1}$$

Now $AX = B \Rightarrow A^{-1}(AX) = A^{-1}B$

$$\Rightarrow (A^{-1}A)X = A^{-1}B$$

$$\Rightarrow I X = A^{-1}B \Rightarrow X = A^{-1}B$$

Q → <u>Solve by matrix method</u>

$$x + y + z = 2$$
$$x - 3y + 2z = 0$$
$$x - y - z = -4$$

<u>Soln</u>) The system of eqn

$$x + y + z = 2$$
$$x - 3y + 2z = 0$$
$$x - y - z = -4$$

The given system can be written in matrix form as $AX = B$

where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 2 \\ 1 & -1 & -1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$B = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

Now $AX = B \Rightarrow X = A^{-1}B$

Now $A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{10} & -\frac{1}{5} & -\frac{1}{10} \\ \frac{1}{2} & \frac{1}{2} & -\frac{2}{5} \end{pmatrix}$

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$$\therefore X = A^{-1}B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{10} & -\frac{1}{5} & -\frac{1}{10} \\ \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 0 - 2 \\ \frac{3}{5} + 0 + \frac{2}{5} \\ \frac{2}{5} + 0 + \frac{8}{5} \end{pmatrix} = \begin{pmatrix} -1 \\ +1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \Rightarrow x = -1, y = 1, z = 2$$

9)
$$\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$$

$$= \begin{vmatrix} b+c & a+b & c+a \\ a+c & -2b & b+c \\ a+b & b+c & -2c \end{vmatrix}$$ Replacing C_1 by $C_1 + C_2 + C_3$

Expanding about 1st row

(Done - Later in the notes or in third page from here)

<u>Alt method</u>

Let $\Delta = \begin{vmatrix} -2a & a+b & c+a \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$

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Putting $b=-a$ in Δ we see that $\Delta=0$

$\therefore$ $a+b$ is a factor of Δ

Similarly, putting $b=-c$ and $c=-a$

we see that $\Delta=0$

$\therefore$ $b+c$ and $c+a$ are factors of Δ

$\therefore$ $(a+b)(b+c)(c+a)$ is a factor of Δ.

Taking product of diagonal elements of Δ, it is seen that Δ is of 3rd degree in a, b, c (3)

Hence we can write

$$\Delta = K(a+b)(b+c)(c+a)$$

where K is a constant.

Put $a=0$, $b=1$, $c=1$ we get

$$8 = K(1)(2)(1)$$

$\Rightarrow K = 4$

$\therefore$ $\Delta = 8$ for $a=0$, $b=1$, $c=1$

$\therefore$ $\Delta = 4(a+b)(b+c)(c+a)$ $\square$.

Diagonal Matrix :

If in a square matrix the elements which are not in principal diagonal are zero then it is called Diagonal Matrix. It is of the form -:

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$$= \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}$$

Scalar Matrix

It is a diagonal matrix, all the elements in the principal diagonal are same then it is called scalar matrix. It is of the form

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$$

Symmetric Matrix

A square matrix A is called symmetric if $A = A^T$

Ex $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Skew - Symmetric Matrix

A square matrix A is called skew symmetric if $\boxed{A^t = -A}$ $\Rightarrow 2A = 0$ $\Rightarrow A = 0$

Ex: $\begin{pmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & -h & g \\ h & 0 & -f \\ -g & f & 0 \end{pmatrix}$

$2 \begin{pmatrix} 0 & h & -g \\ -h & 0 & d \\ g & -d & 0 \end{pmatrix}$

Problems :

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Q1.
$$\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$$

$$= \begin{vmatrix} b+c & a+b & a+c \\ c+a & -2b & b+c \\ a+b & b+c & -2c \end{vmatrix}$$

Replace C_1 by $C_1+C_2+C_3$. Expanding about 1st row.

$$=(b+c)\left[4bc-(b+c)^2\right]-(a+b)\left[-2c(a+c)-(a+b)(b+c)\right]+(a+c)\left[(c+a)(c+b)+2b(a+b)\right]$$

$$=(b+c)\left[-(b-c)^2\right]+\left[(a+b)^2+(a+c)^2(b+c)+2(a+b)(a+c)(b+c)\right]$$

$$=(b+c)\left[-(b-c)^2+(a+b)^2+(a+c)^2+2(a+b)(a+c)\right]$$

$$=(b+c)\left[(a+b+b-c)(a+b-b+c)+(a+c)(a+c+2a+2b)\right]$$

$$=(b+c)(a+c)(a+2b-c+3a+2b+c)$$

$$=(b+c)(a+c)(4a+4b)=4(a+b)(b+c)(c+a)$$

(Proved).

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23.(i) Elements

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^3+a & ab^2 & ac^2 \\ a^2b & b^3+b & bc^2 \\ a^2c & b^2c & c^3+c \end{vmatrix}$$ $C_1.a, C_2.b, C_3.c$

$$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$ Taking a, b, c common from R_1, R_2, R_3

$$= \begin{vmatrix} a^2+b^2+c^2+1 & b^2 & c^2 \\ a^2+b^2+c^2+1 & b^2+1 & c^2 \\ a^2+b^2+c^2+1 & b^2 & c^2+1 \end{vmatrix}$$ Replacing C_1 by $C_1+C_2+C_3$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$ R_2 by R_2-R_1, R_3 by R_3-R_1

$$= (1+a^2+b^2+c^2) \cdot 1$$ Expanding about 1st Column

$$= (1+a^2+b^2+c^2)$$

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$$\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B & 0 \\ \sin^2 B - \sin^2 C & \cot B - \cot C & 0 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$(R_1 - R_2,\ R_2 - R_3)$

$$= \begin{vmatrix} 0 & \sin^2 A - \sin^2 B & \cot A - \cot B \\ 0 & \sin^2 B - \sin^2 C & \cot B - \cot C \\ 1 & \sin^2 C & \cot C \end{vmatrix}$$

$= (\sin^2 A - \sin^2 B)(\cot B - \cot C) - (\sin^2 B - \sin^2 C)(\cot A - \cot B)$

But $\sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$

$= \sin C \ \sin(A-B)$ $\quad (\because A+B = \pi - C)$

$\therefore \sin^2 B - \sin^2 C$

$= \sin A \cdot \sin(B-C)$

$\cot B - \cot C = \frac{\cos B}{\sin B} - \frac{\cos C}{\sin C} = \frac{\sin(C-B)}{\sin B \sin C}$

$\cot A - \cot B = \frac{\sin(B-A)}{\sin A \cdot \sin B}$

$\therefore \Delta = \sin C \cdot \sin(A-B) \cdot \frac{\sin(C-B)}{\sin B \cdot \sin C} - \sin A \cdot \sin(B-C) \times$

$$\frac{\sin(B-A)}{\sin A \cdot \sin B}$$

$$\Delta = \frac{1}{\sin B}\left[\sin(A-B)\,\sin(C-B) - \sin(C-B).\sin(A-B)\right]$$

$$= 0.$$

Q28 - elements

$$\sum_{j=1}^{n} \Delta_j = \begin{vmatrix} \sum_{j=1}^{n} j & a & \frac{n(n+1)}{2} \\ \sum_{j=1}^{n} j^2 & b & \frac{n(n+1)(2n+1)}{6} \\ \sum_{j=1}^{n} j^3 & c & \frac{n^2(n+1)^2}{4} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & a & \frac{n(n+1)}{2} \\ \frac{n(n+1)(2n+1)}{6} & b & \frac{(2n+1)(n+1)n}{6} \\ \frac{n^2(n+1)^2}{4} & c & \frac{n^2(n+1)^2}{4} \end{vmatrix}$$

$= 0$ ($\because$ Two columns are identical)

Q3.(ix)

$$\begin{vmatrix} ax-by-cz & ay+bx & az+cx \\ bx+ay & by-cz-ax & bz+cy \\ cx+az & cy+bz & cz-ax-by \end{vmatrix}$$

SCIENCE AND ENGINEERING NOTES

$$=\frac{1}{x}\begin{vmatrix} ax^2-bxy-czx & ay+bx & az+cx \\ bx^2+ayx & by-cz-ax & bz+cy \\ cx^2+azx & cy+bz & cz-ax-by \end{vmatrix}$$

Multiply x in & C_1 and dividing by x

$$=\frac{1}{x}\begin{vmatrix} a(x^2+y^2+z^2) & ay+bx & cx+az \\ b(x^2+y^2+z^2) & by-cz-ax & bz+cy \\ c(x^2+y^2+z^2) & cy+bz & cz-ax-by \end{vmatrix}$$

Rept C_1 by $C_1+yC_2+zC_3$

$$=\frac{x^2+y^2+z^2}{x}\begin{vmatrix} a & ay+bx & cx+az \\ b & by-cz-ax & bz+cy \\ c & cy+bz & cz-ax-by \end{vmatrix}$$

$$=\frac{x^2+y^2+z^2}{xa}\begin{vmatrix} a^2 & a^2y+abx & acx+a^2z \\ b & by-cz-ax & bz+cy \\ c & cy+bz & cz-ax-by \end{vmatrix}$$

Multiplying a in R_1 and dividing by a

$$=\frac{x^2+y^2+z^2}{xa}\begin{vmatrix} a^2+b^2+c^2 & y(a^2+b^2+c^2) & z(a^2+b^2+c^2) \\ b & by-cz-ax & bz+cy \\ c & bz+ay & cz-ax-by \end{vmatrix}$$

Replacing R_1 by $R_1+bR_2+cR_3$

$$=\frac{(x^2+y^2+z^2)(a^2+b^2+c^2)}{ax}\begin{vmatrix} 1 & y & z \\ b & by-cz-ax & bz+yc \\ c & bz+cy & cz-ax-by \end{vmatrix}$$

$$= \frac{(x^2+y^2+z^2)(a^2+b^2+c^2)}{ax} \begin{vmatrix} 1 & 0 & 0 \\ b & -(cz+ax) & -cy \\ c & bz & -ax-by \end{vmatrix}$$

Repl C_2 by $C_2 - yC_1$ and C_3 by $C_3 - zC_1$

$$= \frac{(x^2+y^2+z^2)(a^2+b^2+c^2)}{ax} \left\{ (ax+by)(cz+ax) - bcyz \right\}$$

$$= \frac{(x^2+y^2+z^2)(a^2+b^2+c^2)}{ax} \left\{ acxz + a^2x^2 + bcyz + abxy - bcyz \right\}$$

$$= \frac{(x^2+y^2+z^2)(a^2+b^2+c^2)(ax+by+cz)\,ax}{ax}$$

$$= (x^2+y^2+z^2)(a^2+b^2+c^2)(ax+by+cz)$$

□

Q-1

1. Prove that Matrix multiplication is distributive w.r.t to addition i.e

$$A(B+C) = AB + AC$$

2. $(AB)^T = B^T A^T$ (Reversal law).

3. Prove that $A^{-1} = \frac{\text{Adj}\,A}{|A|}$ provided A is non-singular.

SCIENCE AND ENGINEERING NOTES

1) Let $A = (a_{ij})_{m \times n}$ $B = (b_{jk})_{n \times p}$

$C = (c_{jk})_{n \times p}$

$B + C = (b_{jk} + c_{jk})_{n \times p}$

Now $AB = \left(\sum_{j=1}^{n} a_{ij} \, b_{jk} \right)_{m \times p}$

$AC = \left(\sum_{j=1}^{n} a_{ij} \, c_{jk} \right)_{m \times p}$

$AB + AC = \left(\sum_{j=1}^{n} a_{ij} \, b_{jk} + \sum_{j=1}^{n} a_{ij} \, c_{jk} \right)_{m \times p}$

$= \left(\sum_{j=1}^{n} a_{ij} \left(b_{jk} + c_{jk} \right) \right)$

$= A(B + C)$ (Proved)

2) Let $A = (a_{ij})_{m \times n}$

$B = (b_{jk})_{n \times p}$

$A^T = (a'_{ji})_{n \times m}$ where $a'_{ji} = a_{ij}$

$B^T = (b'_{kj})_{p \times n}$ where $b'_{kj} = b_{jk}$

$B^T A^T = \left(\sum_{j=1}^{n} b'_{kj} \, a'_{ji} \right)$

$= \left(\sum_{j=1}^{n} b_{jk} \, a_{ij} \right)$

$$= \left(\sum_{j=1}^{n} a_{ij} b_{jk} \right)$$

Let $C = AB$

but $C = (c_{ik})_{m \times p}$ where

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

$$(AB)^T = C^T = (c'_{ki})_{p \times m}$$

where $c'_{ki} = c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

$$= \sum_{j=1}^{n} a_{ij} b_{jk}$$

$$= B^T A^T \quad \square$$

Q.(ix)

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & cb & a^2+b^2 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} (b^2+c^2)a & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & cb^2 & c(a^2+b^2) \end{vmatrix}$$

multiplying C_1, C_2, C_3 by

a, b, c respectively.

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$ Taking a, b, c common from R_1, R_2 and R_3 respectively

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2+c^2 & c^2 \\ b^2 & b^2 & a^2+b^2 \end{vmatrix}$$ $C_1 = C_2 - C_3$ and taking -2 common

$$= -2\left[-b^2(c^2a^2) + c^2(-a^2b^2)\right] = 4a^2b^2c^2$$

D.

23 (vi)
$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$ by $C_1 - 2C_3$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$ $C_1 + C_2$

$$= (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= (a^2+b^2+c^2)\begin{vmatrix} 0 & c^2-b^2 & c(b-a) \\ 0 & b^2-c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix} \quad R_1-R_3 ; R_2-R_3$$

$$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$$

$$= (a^2+b^2+c^2)(a-b)(b-c)\left\{(a+b)(-a)+c(b+c)\right\}$$

$$= (a^2+b^2+c^2)(a-b)(b-c)\left\{-a^2-ab+bc+c^2\right\}$$

$$= (a^2+b^2+c^2)(a-b)(b-c)\left\{c^2-a^2+b(c-a)\right\}$$

$$= (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$$

Q7: Let A be the 1st term of G.P. R be the c.r. of G.P.

$\therefore$ pth term $= x = AR^{p-1}$

$\therefore$ $\log x = \log A + (p-1)\log R$

$\log y = \log A + (q-1)\log R$

$\log z = \log A + (r-1)\log R$

$$\therefore \begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = \begin{vmatrix} \log A+(p-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log A+(r-1)\log R & r & 1 \end{vmatrix}$$

$$= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

$$= 0 + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$$

by $C_2 - C_3$

$$= 0 + 0 = 0$$

<u>3U</u>. Let A be the 1st term and R be the C.R of the ith term $= a_i = AR^{i-1}$

$$\log a_i = \log A + (i-1)\log R$$

$$= \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+1} & \log a_{n+2} & \log a_{n+3} \\ \log a_{n+2} & \log a_{n+3} & \log a_{n+4} \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (n-1)\log R & \log A + n\log R & \log A + (n+1)\log R \\ \log A + n\log R & \log A + (n+1)\log R & \log A + (n+2)\log R \\ \log A + (n+1)\log R & \log A + (n+2)\log R & \log A + (n+3)\log R \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (n-1)\log R & \log A + n\log R & \log A + (n+1)\log R \\ \log R & \log R & \log R \\ 2\log R & 2\log R & 2\log R \end{vmatrix}$$

$R_2 - R_1$ and $R_3 - R_1$:

31) $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

$= \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$ $C_1 \to C_1 + C_2$

$= \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$ $(R_2 - R_1)$ and $(R_3 - R_1)$

$= 2 + 4\sin 2x$

Min^m value of $\sin 2x = -1$

Min^m value of $f(x) = 2 - 4 = -2$

32) $f(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_3(x) & h_3(x) \end{vmatrix}$

$= f_1(x)\{g_2(x)\, h_3(x) - g_3(x)\, h_2(x)\}$

$- f_2(x)\{g_1(x) + h_3(x) - g_3(x)\, h_1(x)\}$

$+ f_3(x)\{g_1(x)\, h_2(x) - g_2(x)\, h_1(x)\}$

SCIENCE AND ENGINEERING NOTES

$$f'(x) = \Big[f_1'(x) \{ g_2(x)\, h_3(x) - g_3(x)\, h_2(x) \}$$

$$+ f_1(x) \{ g_2(x)\, h_3'(x) + g_2'(x)\, h_3(x) \} - g_3(x)\, h_2' x - g_3'(x)\, h_2(x) \Big]$$

$$= f_1'(x) \{ g_2(x)\, h_3(x) - g_3(x)\, h_2(x) \}$$

$$+ \big[f(x) (g_2'(x)\, h_3(x) - g_3'(x)\, h_2(x) \big]$$

$$+ \big[f_1(x) \{ g_2 x\, h_3' x - g_3(x)\, h_2'(x) \big]$$

$$= \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

$$f'(a) = \begin{vmatrix} f_1'(a) & f_2'(a) & f_3'(a) \\ g_1(a) & g_2(a) & g_3(a) \\ h_1(a) & h_2(a) & h_3(a) \end{vmatrix} +$$

SCIENCE AND ENGINEERING NOTES

$$\begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1'(a) & g_2'(a) & g_3'(a) \\ h_1'(a) & h_2(a) & h_3(a) \end{vmatrix} + \begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1(a) & g_2(a) & g_3(a) \\ h_1(a) & h_2'(a) & h_3'(a) \end{vmatrix}$$

$= 0 + 0 + 0 = 0 \quad \left(\because g_1 a = h_1(a) = f_1(a) \text{ etc} \right)$

Q9. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 9-4 & -12+4 \\ 3-1 & -4+1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 1+2\times2 & -4\times2 \\ 2 & -1-(2\times2) \end{bmatrix}$

$\therefore$ It is true for $k=2$

Let it be true for $k = r$

i.e $A^r = \begin{bmatrix} 1+2r & -4r \\ r & 1-2r \end{bmatrix}$

To prove that it is true for $k = r+1$

$A^{r+1} = A^r \cdot A$

$$= \begin{bmatrix} 1+2r & -4r \\ r & 1-2r \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+6r-4r & -4-8r+4r \\ 3r+1-2r & -4r-1+2r \end{bmatrix}$$

$$= \begin{bmatrix} 3+2r & -4-4r \\ r+1 & -1-2r \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(r+1) & -4(r+1) \\ r+1 & 1-2(r+1) \end{bmatrix}$$

$\therefore$ It is true $k = r+1$

$\therefore$ It is proved $\forall k \in N$ by method of induction.

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$ax + hy + g = 0, \quad hx + by + f = 0$

$gx + fy + c - \lambda = 0$

Eliminating x and y, we get

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} + \begin{vmatrix} a & h & 0 \\ h & b & 0 \\ g & f & -\lambda \end{vmatrix} = 0$$

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$$\Rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} + a(-b\lambda) - h(-h\lambda) = 0$$

$$\Rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \lambda(ab - h^2)$$

$$\Rightarrow \lambda = \frac{1}{ab-h^2} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(Proved)

$\boxed{\text{Matrix or matrice}}$

Let $A = (a_{ij})_{m\times n}$

$B = (b_{ij})_{m\times n}$

$C = (c_{ij})_{m\times n}$

$$(A+B)+C = \{(a_{ij})_{m\times n} + (b_{ij})_{m\times n}\} + (c_{ij})_{m\times n}$$

$$= \{(a_{ij} + b_{ij})_{m\times n}\} + (c_{ij})_{m\times n}$$

$$= (a_{ij} + b_{ij} + c_{ij})_{m\times n}$$

$$= (a_{ij})_{m\times n} + \{(b_{ij} + c_{ij})\}_{m\times n}$$

$$= A + B+C$$

Proof : To Prove

$$(A+B)+C = A+(B+C)$$

Let $A = (a_{ij})_{m\times n}$

$B = (b_{ij})_{m\times n}$

$C = (c_{ij})_{m\times n}$

$A+B = (a_{ij}+b_{ij})_{m\times n}$

$(A+B)+C = (a_{ij}+b_{ij})_{m\times n} + (c_{ij})_{m\times n}$

$= \left((a_{ij}+b_{ij})+c_{ij}\right)_{m\times n}$

$= \left(a_{ij}+(b_{ij}+c_{ij})\right)_{m\times n}$

$= (a_{ij})_{m\times n} + (b_{ij}+c_{ij})_{m\times n}$

$= A+(B+C)$ $\square$

SCIENCE AND ENGINEERING NOTES

(iii) Let $A = (a_{ij})_{m \times n}$

$$\operatorname{Adj} A = (A_{ji})_{n \times n}$$

$$A \times \operatorname{Adj} A = \left(\sum_{j=1}^{n} a_{ij} \times A_{ji} \right)$$

$$= \begin{pmatrix} |A| & 0 & \cdots & 0 \\ 0 & |A| & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & |A| \end{pmatrix}$$

$$= |A| \times I$$

$$\left(\because \sum_{j=1}^{n} a_{ij} A_{ji} = |A| \text{ for } i = j, \text{ and } 0 \text{ for } i \neq j \right)$$

Similarly $\operatorname{Adj} A \times A = |A| I$

$$\therefore A \cdot \operatorname{Adj} A = \operatorname{adj} A \cdot A = |A| I$$

$$\Rightarrow \frac{A \operatorname{Adj} A}{|A|} = \frac{\operatorname{adj} A}{|A|} \cdot A = I$$

$$\Rightarrow A^{-1} = \frac{\operatorname{Adj} A}{|A|}$$

SCIENCE AND ENGINEERING NOTES