Logic

Axioms: Axioms are basic assumptions which are not proved and it is only necessary that they be consistent.

Theorem: The conclusions obtained from the axioms by logical reasoning are called theorem.

Propositions: Propositions are simple theorems.

Lemmas: Lemmas are helping theorem. (i.e conclusion obtained from lemmas help to proof a main theorem)

Corollaries: Conclusion obtained from theorems or propositions or lema's by simple logical reasoning are called corollaries

Proposition or Statement

Defn: A statement is a declarative sentence which is either true or false but not both.

If a statement is true then we say its truth value is and if it is false, we say its truth value is (F)

Statements are denoted by small letters like p, q, r, etc

Ex : ~~p~~ p : 2+3 = 5

p is the statement 2+3=5

Its truth value (T).

Negation of a statement

If p is a statement then the negation of p is denoted by ~p and is obtained by ~~sign~~ denying p.

Ex: p : 2+3 = 5

~p : 2+3 ≠ 5

q : 2 ≯ 5

~q : 2 > 5

Axiom of negation :

If p is true then ~p is false and if p is false the ~p is true.

Truth table for negation :

p	$\sim p$
T	F
F	T

Connectives :

Connectives are operations which can be performed on a pair of statements to produce a new statement.

There are 4 types of connectives

(i) conjuction

(ii) Disjunction (or Alternation)

(iii) Implication

(iv) Double implication (OR Equivalence)

Conjuction : {(And) $\wedge$}

Let p and q be two statements. The sentence p and q denoted by $p \wedge q$ becomes a statement if we accept its truth value from the following axioms.

Axiom for conjuction :

$p \wedge q$ is true only when both p and q are true and otherwise false

Truth table :

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Ex : $2+3=5 \wedge 2\times3=6$

Disjunction or Alternation {OR, ∨}

Let p and q be two statements. The sentence p or q denoted by p∨q becomes a statement we accept its truth value from the following axiom.

Axiom for disjunction

p∨q is true when atleast one of them is true.

Truth table

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Ex: $2+3=5 \lor 2\times 3=6$

(iii) Implication ($\Rightarrow$) or ($\rightarrow$) [Implise]

Let p and q be two statements.

The sentence ~~$p \Rightarrow q$~~ p implies q

~~denoted by~~ or if p then q

denoted by $p \Rightarrow q$ becomes a statement if we accept its truth value from the following axiom.

Here p is called hypothesis or antecedent, q is called Conclusion or Consequent.

Axiom for Implication:

An implication ~~p imp~~ $p \Rightarrow q$ is false when the hypothesis p is true and conclusion q is false

Truth table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex: ABC is a $\triangle$ $\Rightarrow AB^2 + BC^2 = AC^2$

Double implication or Bi Conditional

$(\Leftrightarrow, \leftrightarrow)$

Let p and q be two statements. The statement $p \Rightarrow q$ and $q \Rightarrow p$ is called double implication and is denoted by

$$p \Leftrightarrow q$$

Axiom

The double implication $p \Leftrightarrow q$ is true if either both ~~p∧q~~ p and q are true or both false

Truth table

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Note:

$p \Leftrightarrow q$ can be written as

p iff q (iff is called if and only if)

It can also be written as the "necessary and sufficient" condition for p is q.

Ex: ABC is a $\triangle \iff AB^2 + BC^2 = AC^2$

OR ABC is a $\triangle$ iff $AB^2 + BC^2 = AC^2$

OR The necessary and sufficient condition for ABC is a $\triangle$ is $AB^2 + BC^2 = AC^2$

Note: Necessary, implies, only if (Same)

is implied by, if, sufficient (Same)

Converse of an implication

The converse of $p \Rightarrow q$ is $q \Rightarrow p$

Inverse of an implication

The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

Contra position of an implication

The Contraposition of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

It is converse of Inverse

or Inverse of Converse.

Axiom

The Contraposition is true if the implication is true, The contraposition is false if the implication is false.

$p \Rightarrow q$	$\sim q \Rightarrow \sim p$
T	T
F	F

Ex:

If a^2 is even then a is even.

Its contraposition can be stated as

If a is odd then a^2 is odd

Equivalent Statement ($\Leftrightarrow$)

Two statements are equivalent if they have the same truth value

Ex: Implication and its contraposition are equivalent statement.

Tautology: If a composite statement is always true irrespective of the truth value of the individual statements then it is called a tautology.

~~The follo~~ Examples

1. Law of excluded middle: $p \vee \sim p$
2. Law of contradiction $\sim(p \wedge \sim p)$
3. Law of double negation $p \Leftrightarrow \sim(\sim p)$
4. Principle of syllogism

$$[(p \to q) \wedge (q \to r)] \to [p \to r]$$

5. Law of contra positive

$[\sim q \to \sim p] \Leftrightarrow [p \to q]$

Problem

1. Write the truth tables of the following statements and indicate which one is a tautology.

(i) $p \vee q \Rightarrow q \vee p$

Ans:

p	q	$p \vee q$	$q \vee p$	$p \vee q \Rightarrow q \vee p$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Since $p \vee q \Rightarrow q \vee p$ is always true ∴ This is a tautology.

2. Prove that $\sim(p \vee q) = \sim p \wedge \sim q$

OR Prove that $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

Proof

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

It follows from the above truth table that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ have the same truth value and so $\sim(p \vee q) = \sim p \wedge \sim q$

(proved)

Methods of proving theorems

There are two methods for proving theorems (i) direct method

(ii) Indirect method.

(i) Direct method

Suppose to proof $p \Rightarrow q$ then assume that p is true and prove that q is true.

(ii) Indirect method

There are 3 types

(a) Method of Contradiction.

(b) Method of Contra position.

(c) Method of Induction.

(a) Method of contradiction

Suppose to prove $p \Rightarrow q$. Assume p is true and $\sim q$ is true. If contradiction arises then assumption "$\sim q$ is true" is false i.e. q is true

$\therefore$ $p \Rightarrow q$ is established.

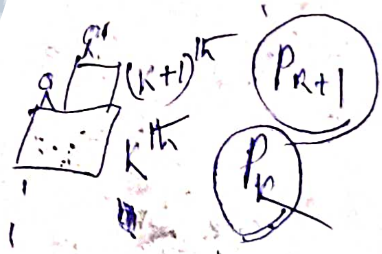
(ii) Method of contraposition

Suppose to prove $p \Rightarrow q$. then we prove $\sim q \Rightarrow -p$

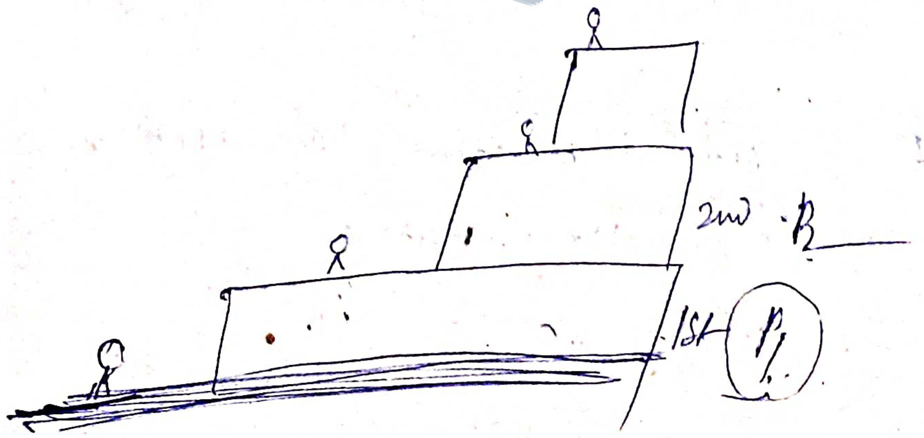
It is method of contra position.

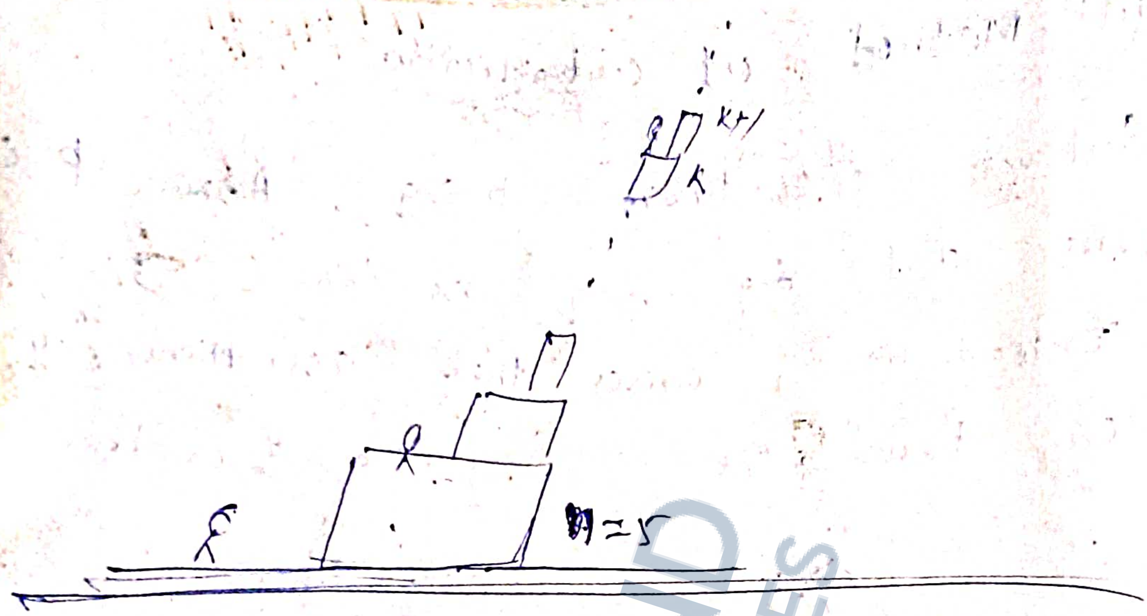
(iii) Method of induction

([illegible])



$n = 1, 2, \ldots$





Let p_1, p_2, ——— p_n

be statements. we have to prove p_n is true. ~~for all~~ $\forall n$ belongs ($\in$) to N

(i) Firstly we will prove p_1 is true

(ii) Secondly we assume that p_k is true. It is called induction hypothesis.

(iii) Thirdly we have to prove that p_{k+1} is true.

Thus it is established that p_n is true ~~for all~~ $\forall n \in N$

Note

1. Method of induction is applied when n is +ve integer.

Q.) Prove that $1+r+r^2+\cdots+\cdots+r^n$

$$= \frac{1-r^{n+1}}{1-r}, \text{ where } r \neq 1$$

by method of induction.

<u>Proof</u> :-

Let P_n be the statement

$$1+r+r^2 \text{-----} \text{---} + r^n = \frac{1-r^{n+1}}{1-r}$$

where $r \neq 1$ $\frac{1}{2}$

Now to ~~Proof~~ prove P_1 is true

Now P_1 is $1+r = \dfrac{1-r^2}{1-r}$

Now R.H.S of $P_1 = \dfrac{1-r^2}{1-r} = \dfrac{(1+r)(1-r)}{(1-r)}$

$$= 1+r = \text{L.H.S of } P_1$$

$\therefore$ P_1 is true.

1

Let's assume that P_K be true

i.e $1+r+r^2 \text{------} + r^K = \dfrac{1-r^{K+1}}{1-r}$

is true

1

<u>To prove that P_{K+1} is true</u>

Now P_{K+1} is $1+r+r^2 \text{ ---- } +r^K+r^{K+1}$

$$= \frac{1-r^{K+2}}{1-r}$$

LHS of P_{K+1}

$$= (1+r+r^2 \text{ ---- } r^K) + r^{K+1}$$

$$= \frac{1-r^{K+1}}{1-r} + r^{K+1}$$

(by induction hypothesis)

$$= \frac{1-\cancel{r^{K+1}}+\cancel{r^{K+1}}-r^{K+2}}{1-r}$$

$$= \frac{1-r^{K+2}}{1-r}$$

$$= \text{R.H.S of } P_{K+1}$$

$\therefore$ P_{K+1} is true [3]

$\therefore$ P_n is true $\forall n \in N$ by method of induction. [$\frac{1}{2}$]

2. Prove that $1+2+3+ \text{ ---- } n = \frac{n(n+1)}{2}$ by method of induction.

Ans.

Let P_n be the statement

$$1+2+3+4 \text{ ---- } +n = \frac{n(n+1)}{2}$$

Now to prove p_1 is true.

~~Now p_1 is $\frac{1(1+1)}{2}$~~

Now p_1 is $1 = \frac{1(1+1)}{2}$

Now R.H.S of p_1 : $\frac{1(1+1)}{2} = \frac{2}{2} = 1 = $ L.H.S of p_1

$\therefore p_1$ is true.

Lets assume that p_K be true.

i.e $1+2+3+ \text{———} +K = \frac{K(K+1)}{2}$ is true.

To prove p_{K+1} is true

Now p_{K+1} is $1+2+3 \text{———} +K+K+1$

$$= \frac{(K+1)(K+2)}{2}$$

L.H.S of p_{K+1}

$$= 1+2+3+ \text{———} K+K+1$$

$$= \cancel{(\;K+K+1}$$

$$= (1+2+3+\dots K) + K+1$$

$$= \frac{K(K+1)}{2} + K+1 \quad \text{(by induction hypothesis)}$$

$$= \frac{K(K+1)+2K+2}{2}$$

$$= \frac{K^2+K+2K+2}{2} = \frac{K^2+3K+2}{2}$$

$\Rightarrow \frac{(K+1)(K+2)}{2} = R.H.S$ of P_{K+1}

$\therefore P_{K+1}$ is true

$\therefore P_n$ is true $\forall n \in N$ by method of induction.

3. Prove that $1 \times 3 \times 5 \times \text{———} (2n-1) \leq n^n$ by method of induction.

<u>Proof</u>

Let P_n be the statement

$1 \times 3 \times 5 \times \text{———} (2n-1) \leq n^n$

To prove that P_1 is true

Now P_1 is $1 \leq 1^1$ i.e $1 \leq 1$

which is true.

Lets assume that P_K be true.

i.e

$1 \times 3 \times 5 \text{———} (2K-1) \leq K^K$

is true

<u>To prove that P_{K+1} is true</u>

i.e to prove that

$$1\times 3\times 5 \cdots \times(2k-1)\ 2(k+1)-1 \le (k+1)^{(k+1)}$$

i.e to prove that

$$1\times 3\times 5 \cdots (2k-1)(2k+1) \le (k+1)^{k+1}$$

Now $1\times 3\times 5 \cdots (2k-1)(2k+1) \le k^k.(2k+1)$ (by induction hypothesis)

$\therefore$ we have to prove that

$$k^k.(2k+1) \le (k+1)^{k+1}$$

i.e to prove

$$k^{k+1}\left(2+\frac{1}{k}\right) \le (k+1)^{k+1}$$

i.e to prove

$$2+\frac{1}{k} \le \left(\frac{k+1}{k}\right)^{k+1} = \left(1+\frac{1}{k}\right)^{k+1}$$

i.e to prove

$$2+\frac{1}{k} \le 1+(k+1)\times\frac{1}{k}+\frac{(k+1)\times k}{\lfloor 2}\times\frac{1}{k^2}+\cdots \left(\frac{1}{k}\right)^{k+1}$$

i.e to prove

$$2+\frac{1}{k} \le 1+1+\frac{1}{k}+\frac{(k+1)\times k}{\lfloor 2}\times\frac{1}{k^2}+\cdots\left(\frac{1}{k}\right)^{k+1}$$

which is true

$\therefore$ P_{k+1} is true

$\therefore$ P_n is true $\forall n \in \mathbb{N}$

by induction method.

Note:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \cdots + x^n$$

Set Theory

Set is undefined term in mathematics. Rather we can give the concept of set as follows.

A set is a collection of well defined and distinct objects. The objects are called elements or members of the set.

Notation:

Usually capital letters such as A, B, C, D, ———— etc denote sets and small letter such as x, y, z denotes elements of a set.

$x \in A$ is written for x is an element of A. If x is not an element of A then $x \notin A$.

Russell's Paradox

A barber in a certain town ~~is~~ shaved all those who donot save themselves and only those.

Let S be the set of all people whome barber ~~sh~~ shaved.

Does the barber belong to the set?

Here there is a paradox to this question.

Ex:- Collection of all big cities of India is not a set. Collection of all cities in India more than 10 lakh ~~pupeop~~ ~~people~~ population is a set.

Note: The set theory is discovered by ~~Jorg~~ George Cantor in 1874

Description of Sets

Sets can be described in two ways given below.

1. Tabular form or Roster method
   ~~on~~ or method of ~~ex~~ extension

Here we describe the set by listing all the elements in a row within {curly} bracket.
or (braces)

Ex

(i) Set of all vowels in English alphabets

$\Rightarrow$ $A = \{a, e, i, o, u\}$

(ii) Set builder or set selector form or Method of Specification. or method or intension ([illegible])

In this method we describe the set by defining property which is common to all the elements

Ex

Set of all vowels in English alhabets

$A = \{x \mid x \text{ is a vowel in English alhabet}\}$

Equality of Sets

Two sets A and B are equal A is iff they have the same elements.

Here we write

$$A = B$$

Notes

(1) Order of writing elements does not affect the set.

(2) Repetation of small elements does not affect the set.

Empty Set or Null set or Void set

A set having no element is called an empty set. It is denoted by (ϕ).

Question: Write the null set in two methods.

In roaster method

$$\phi = \{ \quad \}$$

In set builder method $\phi = \{x \mid x \neq x\}$

Finite Set

A set having finite number of elements is called finite set.

Infinite set

A set which is not finite is called infinite set.

Ex : Set of all sand particles in Puri sea shore is finite set.

Set of all natural number is infinite set.

Single Ton ~~Tone~~ Set

A set having only one element is called singleton set.

Ex : $\{0\}$

Double Ton Set

A set having 2 elements is called doubleton ~~tone~~ set.

Ex : $A = \{1, 2\}$

CARDINALITY OF A SET

Let S be any set, The cardinality

of S is the number of elements of S. It is denoted by $|S|$ or $O(S)$ or $n(x)$ or $Card(S)$

It is also called order of the set S

$$\boxed{|\phi| = 0}$$

Sub Sets

A set 'A' is called a subset of 'B' iff every element of 'A' is in 'B'. Here we denote $A \subset B$.

Now $A \subset B \Leftrightarrow x \in A \Rightarrow x \in B$

A is called proper subset of B (Denoted by $A \subsetneq B$) iff $y \in A \Rightarrow y \in B$ and $\exists$ at least one element $x \in B$ such that $x \notin A$.

Notes 1. Every set is a subset of itself. It is called improper subset.

2. Null set (ϕ) is a subset of

B every set.

3. $A \not\subset B$ means A is not a subset of B.

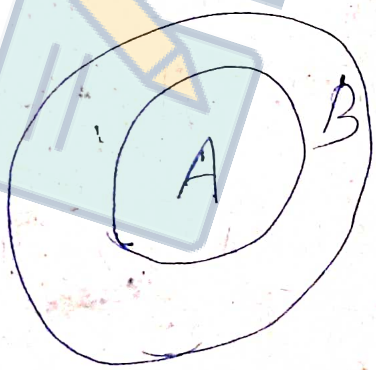
Super set

If A is a subset of B then B is called super set of A and we write $B \supset A$

Venndiagram

Sets can be illustrated by the help of diagrams known as venndiagrams which is named after English logicia John venn.

Venndiagram of a subset



$A \subset B$

Sets can be illustrated as a closed figure in venn diagram.

Set of sets

When the elements of a set are some sets then the set is called

Set of sets.

Power Set

Suppose 'S' is a set. The power set of S is the set of all subsets of S and is denoted by P(S). Power set is a set of sets.

$|P(S)| = (2^n)$

Ex. Let $A = \{a, b, c\}$

~~$P(A) = \{\phi \{a\}, \{b\}, c$~~

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

Question: Write the power set of ϕ

Ans: $P(\phi) = \{\phi\}$

One - to - One Correspondence

(1 - 1 Correspondence)

1 - 1 correspondence between two sets A and B (finite or infinite) is established if for every element of A we associate exactly one element of B

and to each element of B we associate exactly one element of A.

Ex: There is a 1-1 correspondence between the set of natural number (N) and the set of +ve ~~Part~~ even integers. ~~Are in Equal one-one correspondence.~~

$$\begin{array}{lcccccc} N: & 1 & 2 & 3 & 4 & 5 & - - - - \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ A: & 2 & 4 & 6 & 8 & 10 & - - - - \end{array}$$

Equivalant sets OR Similar Sets

Two sets A and B are called similar or equivalant if $\exists$ a 1-1 ~~Corre~~ Correspondence between A and B. Here we write $A \sim B$

Ex: Set of all natural number and Set of all +ve even integers are equivalant.

Note: Equivalant sets have the

Same Cardinality

Universal Sets

Universal set is that set which contains all elements under discussion.

It varies from discussion to discussion but is fixed in one discussion.

It is denoted by E or U or X.

It can be shown as the interior of a rectangle in venn diagram.

Note:

1. Two sets A and B are equal iff $A \subset B$ and $B \subset A$

i.e

$$A = B \iff A \subset B \text{ and } B \subset A$$

Question

16. (i) $P(\phi) = \{\phi\}$

$|P(\phi)| = 1 = 2^0$ (verified)

Sat - 7.5

~~Sets~~ Aim

(1-b)

18. Proof

We have to prove that $|A| = n$

$\Rightarrow |P(A)| = 2^n$

where $n = 0, 1, 2 \ldots\ldots$

Case-I

Suppose $n = 0$ i.e $|A| = 0$

$A = \phi$ then $P(A) = \{\phi\}$

$|P(A)| = 1 = 2^0$

$\therefore |A| = 0 \Rightarrow |P(A)| = 2^0$

Case-II

Suppose $n = 1, 2, 3, \ldots\ldots$

we have to prove $|A| = n \Rightarrow |P(A)| = 2^n$ by method of induction.

Let P_n be the statement

$|A| = n \Rightarrow |P(A)| = 2^n$

Now to prove P_1 is true

Now P_1 is $|A| = 1 \Rightarrow |P(A)| = 2^1$

Let $|A| = 1$ i.e A is a singleton set

Let $A = \{a\}$ $\therefore P(A) = \{\phi, \{a\}\}$.

$\therefore \quad |P(A)| = 2 = 2^1$

$\therefore \quad P_1$ is true

Let's assume that P_K be true.

i.e $|A| = K \Rightarrow |P(A)| = 2^K$ is true.

Now to prove that

P_{K+1} is true

Now P_{K+1} is $|A| = K+1 \Rightarrow |P(A)| = 2^{K+1}$

Let $|A| = K+1$ i.e A is a set having $K+1$ number of elements.

Let ~~B~~ $A = \{a_1, a_2, \ldots\ldots a_K, a_{K+1}\}$.

Let $B = \{a_1, a_2 \ldots\ldots a_K\}$.

Here all subsets of B are subsets of A but $|B| = K$

$\therefore |P(B)| = 2^K$

by induction hypothesis.

i.e the total number of subsets of B is 2^K. More over A contains all elements of B and one element ~~a_{K+1}~~ a_{K+1} more than B

Now inserting this element a_{k+1} into all 2^k subset of B, another 2^k number of subset of A are obtained

(For ex $\{a_1, a_2\} \subset B$

$\therefore \{a_1, a_2, a_{k+1}\} \subset A$)

$\therefore$ Total number of subsets of A

$= 2^k + 2^k$

$= 2 \cdot 2^k$

$= 2^1 \cdot 2^k$

$= 2^{k+1}$

$\therefore |P(A)| = 2^{k+1}$

$\therefore P_{k+1}$ is true

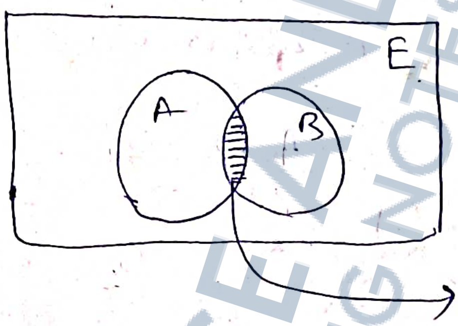
$\therefore P_n$ is true $\forall n = 1, 2, 3 ----$

by method of induction.

$\therefore |A| = n \Rightarrow |P(A)| = 2^n \; \forall n = 0, 1, 2 \cdot$

Operation of Sets

Intersection :- The intersection of A and B is denoted by $A \cap B$, and is the set of all elements common to both A and B.



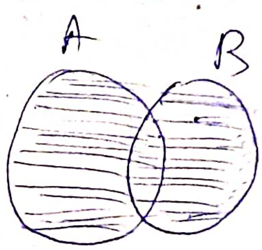
$A \cap B$

The ~~shaded~~ lined portion denotes $A \cap B$.

$$x \in A \cap B \iff x \in A \wedge x \in B$$

Union

The union of A and B is denoted by $A \cup B$ and is the set of all elements of A and B taken together.



The lined portion denotes $A \cup B$.

$$x \in A \cup B \iff x \in A \vee x \in B$$

'OR' 2 types (i) Inclusive OR
(ii) Exclusive OR.

In the union the 'OR' used is inclusive.

Disjoint set or Mutual exclusive set

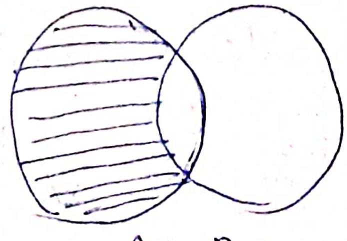
Two sets A and B are called disjoint if $A \cap B = \phi$.

Difference of two sets

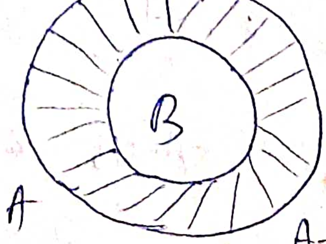
The difference of two sets A and B is denoted by $A \setminus B$ or $A-B$ and is defined.

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

i.e $x \in A-B \iff x \in A$ and $x \notin B$



$A \setminus B$

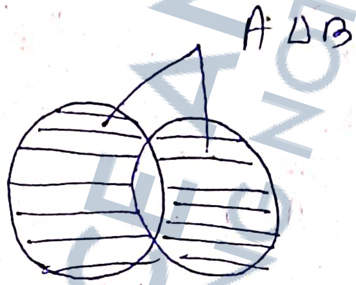


$A-B$

Symmetric difference

The symmetric difference of two sets A and B is denoted by $A \Delta B$ and is defined by

$$A \Delta B = (A-B) \cup (B-A)$$

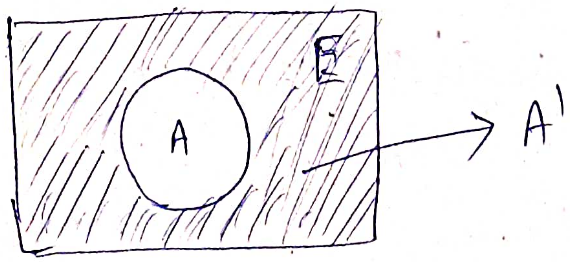


Complement of a set

Let E be the Universal set.

Let A be any set in E

The complement of A is denoted by A' or A^c and is defined as $A' = \{x \in E \mid x \notin A\}$



$\therefore$ $x \in A' \Leftrightarrow x \notin A$ also

$A' = E - A$

Notes

(1) $x \notin A \cup B \iff \sim (x \in A \cup B) \iff$

$\sim (x \in A \vee x \in B) \iff$

$\sim (x \in A)$ and $\sim (x \in B)$

$(\because \sim (p \vee q) = \sim p \wedge \sim q)$

$\iff$ $x \notin A$ and $x \notin B$

(2) $x \notin A \cap B$

$\iff$ $x \notin A$ OR $x \notin B$

$(\because \sim (p \wedge q) = \sim p \vee \sim q)$

2 prove
2
for ex 2
$A = \{a_1, a_2, a_{k+1}\}$
$(a_1, a_2 \ldots a_{k+1})$

3. $x \in \bigcup_{i=1}^{n} A_i$

$\iff x \in A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$

$\iff x \in A_i$ for some (at least) $i = 1, 2, \ldots n$

4. $x \in \bigcap_{i=1}^{n} A_i$

$\iff x \in A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$

$\iff x \in A_i \ \forall i = 1, 2, 3 \ldots n$

5. $x \notin \bigcup_{i=1}^{n} A_i$

$\iff x \notin A_i \ \forall i = 1, 2, 3 \ldots n$

6. $x \notin \bigcap_{i=1}^{n} A_i$

$\Leftrightarrow x \notin A_i$ for some $i = 1, 2, 3, \ldots, n$.

Idempotent law for union.

$A \cup A = A$

Proof:

Let $x \in A \cup A \Rightarrow x \in A$ OR $x \in A$

$\Rightarrow x \in A$

$\therefore A \cup A \subset A$ ——— (i)

Let $x \in A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A \cup A$

$\therefore A \subset A \cup A$ ——— (ii)

From (i) and (ii) we get

$A \cup A = A$ (Proved)

Alternative method

Let $x \in A \cup A$

$\Leftrightarrow x \in A$ OR $x \in A$

$\Leftrightarrow x \in A$

$\therefore A \cup A \subset A$ —(i) and $A \subset A \cup A$ —(ii)

From (i) and (ii) we get $A \cup A = A$ (Proved)

Idempotent law for intersection

$A \cap A = A$

Proof

Let $x \in A \cap A$

$\Leftrightarrow x \in A$ and $x \in A$

$\Leftrightarrow x \in A$

$\therefore A \cap A \subset A$ —— (i)

and $A \subset A \cap A$ —— (ii)

From (i) and (ii) $A \cap A = A$ (Proved)

Commutative law for union

$A \cup B = B \cup A$

Proof

Let $x \in A \cup B$

$\Leftrightarrow x \in A$ OR $x \in B$

$\Leftrightarrow x \in B$ OR $x \in A$ $\left(\because p \vee q = q \vee p\right)$

$\Leftrightarrow x \in B \cup A$

$\therefore A \cup B \subset B \cup A$ —— (i)

and $B \cup A \subset A \cup B$ —— (ii)

from (i) and (ii) $A \cup B = B \cup A$ (Proved)

Commutative law for intersection

$A \cap B = B \cap A$

Proof

Let $x \in A \cap B$

$\Leftrightarrow x \in A$ and $x \in B$

$\Leftrightarrow x \in B$ and $x \in A$ $\quad (\because P \wedge Q = Q \wedge P)$

$\Leftrightarrow x \in B \cap A$

$\therefore \quad A \cap B \subset B \cap A$ —— (i)

$B \cap A \subset A \cap B$ —— (ii)

From (i) and (ii) $A \cap B = B \cap A$ (Proved)

Associative law for Union

$A \cup (B \cup C) = (A \cup B) \cup C$

Proof Let $x \in A \cup (B \cup C)$

$\Leftrightarrow x \in A$ or $x \in (B \cup C)$

$\Leftrightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Leftrightarrow (x \in A$ or $x \in B)$ or $x \in C$

$(\because P \vee (Q \vee r) = (P \vee Q) \vee r$

$\Leftrightarrow x \in A \cup B$ or $x \in C$

$\Leftrightarrow x \in (A \cup B) \cup C$

$\therefore$ $A \cup (B \cup C) \subset (A \cup B) \cup C$ —(i)

and $(A \cup B) \cup C \subset A \cup (B \cup C)$ —(ii)

From (i) and (ii)

$$A \cup (B \cup C) = (A \cup B) \cup C$$

(Proved)

Associate law for intersection

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Proof

Let $x \in A \cap (B \cap C)$

$\Leftrightarrow x \in A$ and $x \in (B \cap C)$

$\Leftrightarrow x \in A$ and ($x \in B$ and $x \in C$)

$\Leftrightarrow$ ($x \in A$ and $x \in B$) and $x \in C$ $\quad (\because p \wedge (q \wedge r) = (p \wedge q) \wedge r)$

$\Leftrightarrow x \in A \cap B$ and $x \in C$

$\Leftrightarrow x \in (A \cap B) \cap C$

$\therefore$ $A \cap (B \cap C) \subset (A \cap B) \cap C$ —(i)

$(A \cap B) \cap C \subset A \cap (B \cap C)$ —(ii)

From (i) and (ii)

$$A \cap (B \cap C) = (A \cap B) \cap C$$

(Proved)

Notes

1. $A \cap B \subset A$ and $A \cap B \subset B$

2. $\phi \cap A = \phi$

Proof

Also $\phi \cap A \subset \phi$ —(i) ($\because A \cap B \subset A$)

but $\phi \subset \phi \cap A$ —(ii) (ϕ is a subset of every set)

$\therefore$ From (i) and (ii)

$\phi \cap A = \phi$ (Proved)

3. $A \subset A \cup B$ and $B \subset A \cup B$

4. $\phi \cup A = A$

Proof We know $A \subset \phi \cup A$ —(i)

($\because A \subset A \cup B$)

Let $x \in \phi \cup A$

$\Rightarrow x \in \phi$ or $x \in A$

$\Rightarrow x \in A$ ($\because x \notin \phi$)

$\therefore \phi \cup A \subset A$ —(ii)

From (i) and (ii) $\phi \cup A = A$

Distributive law of Union over intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proof

Let $x \in A \cup (B \cap C)$

$\Leftrightarrow x \in A$ or $x \in (B \cap C)$

$\Leftrightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Leftrightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$(\because p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r))$

$\Leftrightarrow (x \in A \cup B)$ and $(x \in A \cup C)$

$\Leftrightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ — (i)

and $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ — (ii)

From (i) and (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(proved)

Distributive law of intersection over union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Proof

Let $x \in A \cap (B \cup C)$

$\Leftrightarrow x \in A$ and $x \in (B \cup C)$

$\Leftrightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Leftrightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$(\because p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

$\Leftrightarrow (x \in A \cap B)$ or $(x \in A \cap C)$

$\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ —(i)

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ —(ii)

From (i) and (ii) $A \cap (B \cup C)$
$= (A \cap B) \cup (A \cap C)$ (Proved)

Prove that $A \Delta A = \phi$

Proof

$A \Delta A = (A - A) \cup (A - A)$

$= \cancel{A \cup A}$

$= \phi \cup \phi$

$= \phi$

Prove that $A \Delta B = B \Delta A$

Proof

$A \Delta B = (A - B) \cup (B - A)$

$= (B - A) \cup (A - B)$

(by Commutative law for union)

$= B \Delta A$ (proved)

p, q, r condition

Question

11. Prove that $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Proof

Let $x \in A \cap (B \Delta C)$

$\Leftrightarrow x \in A$ and $x \in (B \Delta C)$

$\Leftrightarrow x \in A$ and $(x \in B - C \cup C - B)$

$\Leftrightarrow x \in A$ and $(x \in B - C$ or $x \in C - B)$

$\Leftrightarrow (x \in A$ and $x \in (B - C))$ or $(x \in A$ and $x \in C - B)$

$(\because p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r))$

$\Leftrightarrow (x \in A$ and $(x \in B$ and $x \notin C))$ or
$(x \in A$ and $(x \in C$ and $x \notin B))$

$\Leftrightarrow ((x \in A$ and $x \in B)$ and $x \notin C)$ or
$((x \in A$ and $x \in C)$ and $x \notin B)$

$(\because p \wedge (q \wedge r) = (p \wedge q) \wedge r)$

$\Leftrightarrow (x \in A \cap B$ and $x \notin C)$ or
$(x \in A \cap C$ and $x \notin B)$ $\rightarrow$ Turning point

$\Leftrightarrow (x \in A \cap B$ and $x \notin A \cap C)$ or $(x \in A \cap C$ and $x \notin A \cap B)$

$(\because\ x \notin C$ and already $x \in A$
$\Leftrightarrow x \notin A \cap C$

Similarly $x \notin B$ and already $x \in A$
$\Leftrightarrow x \notin A \cap B$).

$\Leftrightarrow (x \in A \cap B - A \cap C)$ or $(x \in A \cap C - A \cap B)$

$\Leftrightarrow x \in (A \cap B - A \cap C) \cup (A \cap C - A \cap B)$

$\Leftrightarrow x \in (A \cap B) \Delta (A \cap C)$

$\therefore A \cap (B \Delta C) \subset (A \cap B) \Delta (A \cap C)$ —(i)

and $(A \cap B) \Delta (A \cap C) \subset A \cap (B \Delta C)$ ——(ii)

$\therefore$ From (i) and (ii)
$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

(Proved)

4. Prove that $A \Delta B = (A \cup B) - (A \cap B)$

Proof Let $x \in (A \cup B) - (A \cap B)$

$\Leftrightarrow x \in A \cup B$ and $x \notin A \cap B$

~~$\Leftrightarrow x \in (x \in A$~~

$\Leftrightarrow (x \in A$ or $x \in B)$ and $(x \notin A$ or $x \notin B)$

$(\because\ \sim(p \wedge q) = \sim p \vee \sim q)$

$\Leftrightarrow (x \in A$ and $(x \notin A$ or $x \notin B))$

or $(x \in B$ and $(x \notin A$ or $x \notin B))$

$\Leftrightarrow ((x \in A$ and $x \notin A)$ or $(x \in A$ and $x \notin B))$ or

$((x \in B$ and $x \notin A)$ or $(x \in B$ and $x \notin B))$

$\Leftrightarrow$ $(x \in (A - A)$ or $x \in (A - B))$ or
$(x \in (B - A)$ or $x \in (B - B))$

$\Leftrightarrow$ $(x \in \phi$ or $x \in A - B)$ or $(x \in B - A$ or $x \in \phi)$

$\Leftrightarrow$ $(x \in (\phi \cup A - B))$ or $(x \in (B - A \cup \phi))$

$\Leftrightarrow$ $(x \in A - B)$ or $(x \in B - A)$

$\Leftrightarrow$ $x \in A - B \cup B - A$

$\Leftrightarrow$ $x \in A \Delta B$

$\therefore$ $(A \cup B) - (A \cap B) \subset A \Delta B$ ---(i)

$A \Delta B \subset (A \cup B) - (A \cap B)$ ---(ii)

From (i) and (ii)

$$A \Delta B = (A \cup B) - (A \cap B)$$

(Proved)

5. Prove $A - (B \cup C) = (A - B) \cap (A - C)$

Proof

Let $x \in A - (B \cup C)$

$\Leftrightarrow$ $x \in A$ and $x \notin B \cup C$

$\Leftrightarrow$ $x \in A$ and $(x \notin B$ and $x \notin C)$

$(\because \sim (p \vee q) = \sim p \wedge \sim q)$

$\Leftrightarrow (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)$

$\Leftrightarrow (x \in A - B) \text{ and } (x \in A - C)$

$\Leftrightarrow x \in A - B \cap A - C$

$\therefore A - (B \cup C) \subseteq (A - B) \cap (A - C)$ — (i)

and $(A - B) \cap (A - C) \subseteq A - (B \cup C)$

From (i) and (ii)

$$A - (B \cup C) = (A - B) \cap (A - C)$$

6. imp Prove that $A - (B \cap C) = (A - B) \cup (A - C)$

Let $x \in A - (B \cap C)$

$\Leftrightarrow x \in A$ and $x \notin B \cap C$

$\Leftrightarrow x \in A$ and $(x \notin B$ or $x \notin C)$

~~$\Leftrightarrow x \in A$~~ $(\because \sim (p \wedge q) = \sim p \vee \sim q)$

$\Leftrightarrow (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$

$\Leftrightarrow x \in A - B$ or $x \in A - C$

$\Leftrightarrow x \in (A - B) \cup (A - C)$

$\therefore A - (B \cap C) = (A - B) \cup (A - C)$ — (i)

and $(A - B) \cup (A - C) = A - (B \cap C)$ — (ii)

From (i) and (ii)

$A - (B \cap C) = (A - B) \cup (A - C)$

Notes :-

1. $\phi' = E$ where E is the universal set.

2. $E' = E - E = \phi$

3. $A' \cup A = E$

Proof we know $A' \cup A \subset E$ —(i)

Let $x \in E$

$\Rightarrow x \in A$ or $x \notin A$

$\Rightarrow x \in A$ or $x \in A'$

$\Rightarrow x \in A \cup A'$

$\Rightarrow x \in A' \cup A$

$\therefore E \subset A' \cup A$ —(ii)

~~$A' \cup A \subset$~~

From (i) and (ii) we have

$A' \cup A = E$ (Proved)

4. $A \cap A' = \phi$

Proof :- we know $\phi \subset A \cap A'$ —(i)

Let $x \in A \cap A'$

$\Rightarrow x \in A$ and $x \in A'$

$\Rightarrow x \in A$ and $x \notin A$

$\Rightarrow x \in A - A$

$\Rightarrow x \in \phi$

$\therefore A \cap A' \subset \phi$ —— (ii)

From (i) and (ii) $A \cap A' = \phi$ (proved)

5. $(A')' = A$

Proof : Let $x \in (A')'$

$\Leftrightarrow x \notin A'$

$\Leftrightarrow x \in A$

$\therefore (A')' \subset A$ —— (i)

and $A \subset (A')'$ —— (ii)

From (i) and (ii) $(A')' = A$ (proved)

6. Prove that $A \subset B \Leftrightarrow B' \subset A'$

Proof Let $A \subset B$ to prove $B' \subset A'$

Let $x \in B'$

$\Rightarrow x \notin B$

$\Rightarrow x \notin A$ [$\because A \subset B$ i.e $x \in A \Rightarrow x \in B$ i.e $x \notin B \Rightarrow x \notin A$ contraposition)

$\Rightarrow x \in A'$

$\therefore B' \subset A'$

Conversely let ~~$x \in B$~~

$B' \subset A'$ to prove

$A \subset B$

Let $x \in A$

$\Rightarrow x \notin A'$

$\Rightarrow x \notin B'$ ($\because B' \subset A'$ i.e $x \in B' \Rightarrow x \in A'$ i.e $x \notin A' \Rightarrow x \notin B'$ contraposition)

$\Rightarrow x \in B$

$\therefore$ ~~xde~~ $A \subset B$ (Proved)

Prove that $A - B = A \cap B'$

Let $x \in A - B$

$\Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B'$

$\Leftrightarrow x \in A \cap B'$

$\therefore A - B \subset A \cap B'$ —(i)

and $A \cap B' \subset A - B$ —(ii)

From (i) and (ii) $A - B = A \cap B'$ (Proved)

De Morgan's laws

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof

(i) Let $x \in (A \cup B)'$

$\Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$(\because \sim(p \vee q) = \sim p \wedge \sim q)$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$

$\therefore (A \cup B)' \subset A' \cap B'$ — (i)

$A' \cap B' \subset (A \cup B)'$ — (ii)

From (i) and (ii) $(A \cup B)' = A' \cap B'$

(Proved)

(ii) Let $x \in (A \cap B)'$

$\Leftrightarrow x \notin A \cap B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$(\because \sim(p \wedge q) = \sim p \vee \sim q)$

$\Leftrightarrow$ $x \in A'$ or $x \in B'$

$\Leftrightarrow$ $x \in A' \cup B'$

$\therefore$ $(A \cap B)' \subset A' \cup B'$ —— (i)

$A' \cup B' \subset (A \cap B)'$ —— (ii)

From (i) and (ii) $(A \cap B)' = A' \cup B'$

(Proved)

(ii) Prove that

$$A - (B \cup C) = (A - B) - C$$

<u>Let</u> $x \in A - (B \cup C)$

$\Leftrightarrow$ $x \in A$ and $x \notin B \cup C$

$\Leftrightarrow$ $x \in A$ and ($x \notin B$ and $x \notin C$)

$(\because \sim(p \vee q) = \sim p \wedge \sim q)$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) & $x \notin C$

~~$(\because p \wedge (q \wedge r))$~~

~~$(\because (p \wedge q) \wedge r = p \wedge$~~

$\because$ $(p \wedge (q \wedge r) = (p \wedge q) \wedge r$

$\Leftrightarrow$ $x \in (A-B)$ and $x \notin C$

$\Leftrightarrow$ $x \in (A-B)-C$

$\therefore$ $A-(B\cup C) \subset (A-B)-C$ — (i)

$(A-B)-C \subset A-(B\cup C)$ — (ii)

From (i) and (ii)

$$A-(B\cup C) = (A-B)-C$$

(proved)

(iv) <u>Prove that</u>

$$(A\cap B)-C = A\cap(B-C)$$

<u>Let</u> $x \in (A\cap B)-C$

$\Leftrightarrow$ $(x \in A\cap B)$ and $x \notin C$

$\Leftrightarrow$ $(x \in A$ and $x \in B)$ and $x \notin C$

$\Leftrightarrow$ $x \in A$ and $(x \in B$ and $x \notin C)$

$(\because (p \wedge q)\wedge r = p\wedge(q\wedge r))$

$\Leftrightarrow$ $x \in A$ and $(x \in B-C)$

$\Leftrightarrow$ $x \in A\cap(B-C)$

$\therefore$ $(A\cap B)-C \subset A\cap(B-C)$ — (i)

and $A\cap(B-C) \subset (A\cap B)-C$ — (ii)

From (i) and (ii),

$$(A \cap B) - C = A \cap (B - C)$$

(IV) Imp

Prove that

$$B - \bigcup_{i=1}^{n} A_i = \bigcap_{i=1}^{n} (B - A_i)$$

Proof

Let $x \in B - \bigcup_{i=1}^{n} A_i$

$\Leftrightarrow x \in B$ and $x \notin \bigcup_{i=1}^{n} A_i$

$\Leftrightarrow x \in B$ and $x \notin A_i \ \forall\ i = 1, 2, 3 \ldots n$

$\Leftrightarrow x \in (B - A_i)$ for $\forall\ i = 1, 2, 3 \ldots n$

$\Leftrightarrow x \in \bigcap_{i=1}^{n} (B - A_i)$

$\therefore B - \bigcup_{i=1}^{n} A_i \subset \bigcap_{i=1}^{n} (B - A_i)$ —(i)

and $\bigcap_{i=1}^{n} (B - A_i) \subset B - \bigcup_{i=1}^{n} A_i$ (ii)

From (i) and (ii)

$$B - \bigcup_{i=1}^{n} A_i = \bigcap_{i=1}^{n} (B - A_i)$$

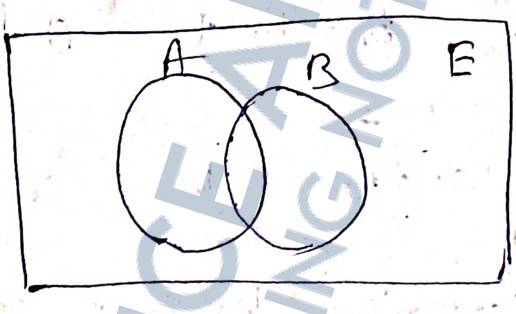
Theorem

Let A and B be finite sets

Then $|A \cup B| = |A| + |B| - |A \cap B|$.

In particular if A and B are disjoint then $|A \cup B| = |A| + |B|$

Proof:-



{1, 2, 3, 4}
{3, 4, 5, 6}

When we count the number of elements in $A \cup B$, elements of $A \cap B$ are counted twice, once in the counting of elements of A and second time in the counting of the elements of B.

So the number of element in $A \cup B$ would be the sum of the number in A and B taking separately less by the number of elements in $A \cap B$. Hence $|A \cup B| = |A| + |B| - |A \cap B|$

In particular A and B are disjoint Then $A \cap B = \phi$

$\Rightarrow |A \cap B| = |\phi| = 0$

$\therefore |A \cup B| = |A| + |B| - 0$

$= |A| + |B|$ (proved)

Cartesian product of sets

Let A and B be any two sets.

The Cartesian product of A and B is denoted by $(A \times B)$.

and is defined by

$$A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$$

Notes:

1. (x, y) is called an ordered pair or ordered 2-tuple

(x, y, z) is called an ordered triad or ordered 3-tuple.

$(x_1, x_2, \ldots x_n)$ is called ordered n-tuple

2. ~~2~~ $(x_1, x_2 ---- x_n, y_1, y_2 ---- y_n)$

is called ordered 2n tuple.

2. In ordered pair (x, y), x is called first constituent and y is called 2nd constituent

3. Two order pairs (x, y) and (a, b) are equal if $x = a$ and $y = b$

4. Cartesian products can be illustrated by diagrams known as Tree-diagrams.

5. $A \times B \times C = \{(x, y, z) \mid x \in A, y \in B \text{ and } z \in C\}$

Ex $A = \{1, 2, 3\}$, $B = \{x, y\}$

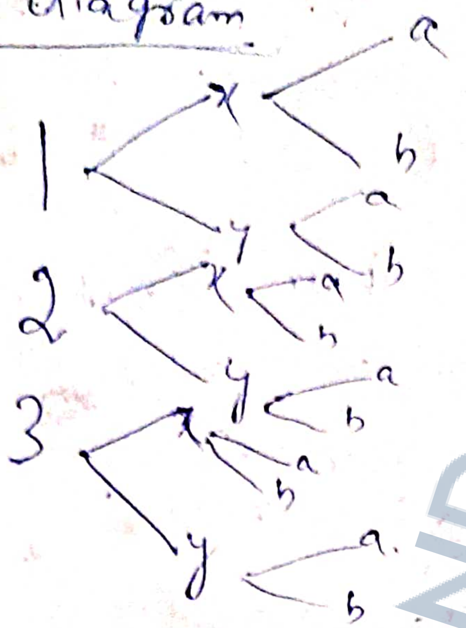
$C = \{a, b\}$

$A \times B = \{(1,x), (1,y), (2,x), (2,y), (3,x), (3,y)\}$

~~$A \times B \times C = \{(1,x,a), (1,x,b), (1,y,a, (1,y,b)$~~

$A \times B \times C = \{(1, x, a), (1, x, b), (1, y, a), (1, y, b)$
$(2, x, a)(2, x, b), (2, y, a), (2, y, b), (3, x, a)$
$(3, x, b), (3, y, a), (3, y, b)\}$

Tree diagram



Note

$$A \times B \neq B \times A$$

Theorem

Let A, B, C be sets then

V.V. imp (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

V.V. imp (ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Proof

(i) Let $(x, y) \in A \times (B \cap C)$

$\Leftrightarrow$ $x \in A$ and $y \in B \cap C$

$\Leftrightarrow$ $x \in A$ and $(y \in B$ and $y \in C)$

$\Leftrightarrow$ $(x \in A$ and $y \in B)$ and $\cancel{(y \in C)}$ $(x \in A$ and $y \in C)$

$\{ \because \; p \wedge (q \wedge r) = (p \wedge q) \wedge (p \wedge r) \}$

$\Leftrightarrow$ ~~$(x,y) \in A \cap B$~~ and

$\Leftrightarrow \; (x,y) \in A \times B$ and $(x,y) \in A \times C$

$\Leftrightarrow \; (x,y) \in (A \times B) \cap (A \times C)$

$\therefore \; A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ (i)

$(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ (ii)

From (i) and (ii)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(Proved)

(ii) Let $(x,y) \in A \times (B \cup C)$

$\Leftrightarrow \; x \in A$ and $y \in B \cup C$

$\Leftrightarrow \; x \in A$ and $(y \in B$ or $y \in C)$

$\Leftrightarrow \; (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$(\because \; p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r))$

$\Leftrightarrow \; \{(x,y) \in A \times B\}$ or $\{(x,y) \in A \times C\}$

$\Leftrightarrow \; (x,y) \in A \times B \cup A \times C$

$\therefore \; A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ (i)

$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$

From (i) and (ii)

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$ (proved)

Note.

1. $A \times A$ is written as A^2

Imp problem

1. Suppose X is universal set. Let A and B be subsets of X. ~~prove~~ ~~$A \subseteq B$~~ ~~Prove that~~ $\cancel{A' \cup B}$

$A \subset B$ iff $A' \cup B = X$ Prove it:

Proof Suppose X is universal set.

Let A and B be subsets of X.

Let $A \subset B$ i.e

$x \in A$ ~~and~~ $\Rightarrow x \in B$

To prove that

$A' \cup B = X$

$\because$ we know $A' \cup B \subset X$ ——— (i)

($\because$ Any set is subset of universal set)

$A \subset B \Rightarrow A \cup A' \subset B \cup A'$

$\Rightarrow X \subset A' \cup B$ (ii)

From (i) and (ii)

$A' \cup B = X$

Conversely let

$A' \cup B = X$, To prove that $A \subset B$

Since $A' \cup B = X$

$\therefore A' \cup B \subset X$

and $X \subset A' \cup B$

Now $X \subset A' \cup B$

$\Rightarrow A' \cup A \subset A' \cup B$

$\Rightarrow A \subset B$ (Proved)