

# Magneto statics

In nature some stones called lodestones behave as magnets. i.e. they possess north pole and south pole which indicate the geographical north and south of the earth. Later on, artificial magnets can be prepared by passing current in a coil that surrounds a piece of iron or steel. Only iron, Cobalt, nickel and some of their alloys possess molecular magnetic behavior spontaneously.

North pole and south pole always occur on the opposite sides of a magnet and they cannot be separated.

Earth is a huge magnet due to various reasons. and its south pole is located near the geographical north pole. The north pole of the earth's magnet is located near the geographical south pole.

## Coulomb's law in magnetism

Like poles repel and unlike poles attract. The magnitude of the force of attraction or repulsion is found to be directly proportional to the product of the poles.

Strengths when distance between the poles is kept constant.

$$\therefore F \propto m_1 m_2 \quad \text{when } d \text{ is kept constant.}$$

The magnitude of the force of attraction or repulsion is found to be inversely proportional to the square of the distance between the two poles when the pole strengths are kept constants.

$$\therefore F \propto \frac{1}{d^2} \quad \text{when } m_1, m_2 \text{ are kept constants}$$

This is called inverse square law in magnetism.

~~Combining~~ Combining these two variations,

$$\text{we have } F \propto \frac{m_1 m_2}{d^2} \quad \text{when all the quantities vary}$$

$$\Rightarrow F = K \frac{m_1 m_2}{d^2} \quad \text{where } K \text{ is a constant}$$

The reciprocal of  $K$  is called relative permeability ( $\mu$ )

$$\therefore F = \frac{1}{\mu} \frac{m_1 m_2}{d^2}$$



Naturally  $\mu$  must be depending on the nature of the medium in which the poles are kept. For air  $\mu = 1$ .

## Unit pole

The strength of a pole is measured by this unit in C.G.S. System.

## Defn of unit pole

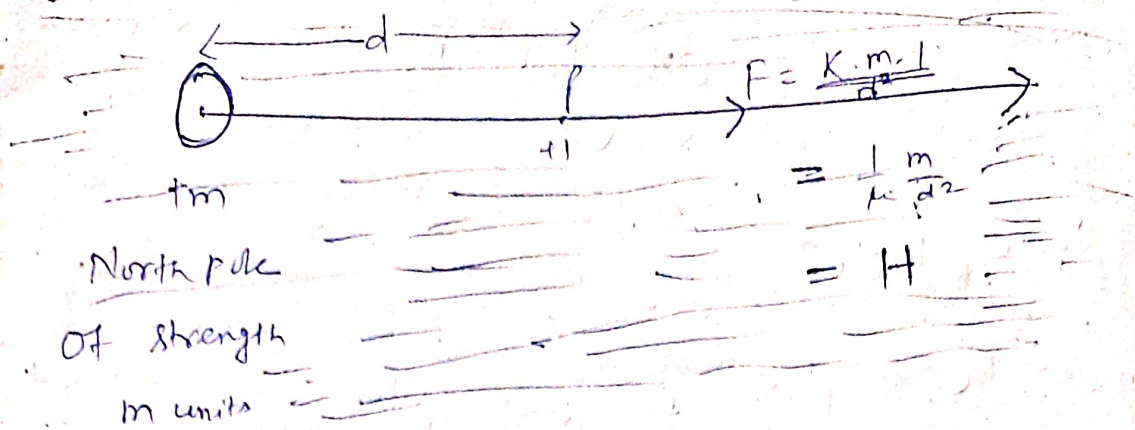
It is defined as that amount of pole strength which when kept at a distance of 1 cm away from an equal and similar pole repel each other with a force of 1 dyne when placed in air.

## Magnetic field

The space surrounding a pole where its influence is felt is called its magnetic field.

## Magnetic field strength or Magnetic field intensity

Magnetic field intensity at a point in a magnetic field is defined as the force experienced by a unit north pole placed at the point concerned.



If the <sup>'m' pole</sup> ~~test pole~~ be comparable to the <sup>unit</sup> pole then has created the field, then we have to use a smaller pole (whose strength is less than +1) If this pole has the strength m' units, then the force of repulsion experienced by this pole is

$$F = \frac{1}{4\pi} \frac{m \cdot m'}{d^2} = m' \cdot H$$

where  $\vec{H}$  = Magnetic field intensity at the point P

$$= \lim_{m' \rightarrow 0} \frac{\vec{F}}{m'}$$

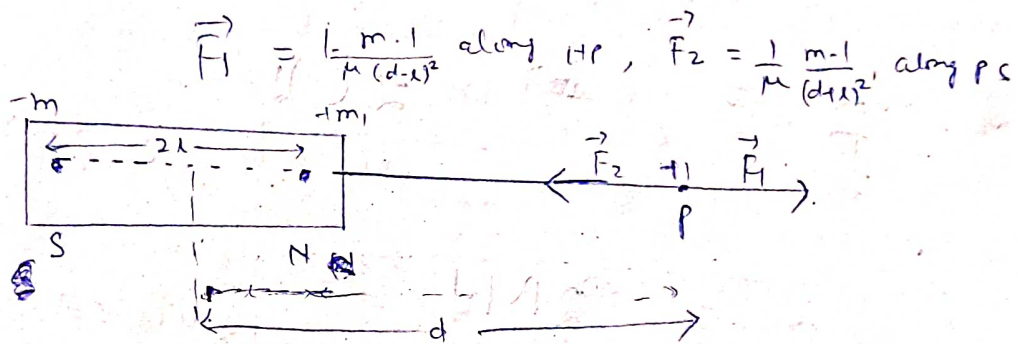
Magnetic field intensity due to a bar magnet at the end-on position

The bar magnet possess two poles placed at the separation  $2l$  which is called magnetic length and it is 85% of the geometric length.



The point P is at a distance d from the mid point of the bar magnet:

The point is situated on the axis of the bar magnet. Such a position is called end-on position. If a unit north pole be placed at the point P, then it will experience a repulsion due to the north pole and an attraction due to the south pole. In our diagram, the repulsive force is



greater than the attractive force.

i.e.  $F_1 > F_2$

Net force =  $F_1 - F_2$

$$= \frac{m}{\mu (d-l)^2} - \frac{m}{\mu (d+l)^2}$$

$$= \frac{m}{\mu} \left( \frac{1}{(d-l)^2} - \frac{1}{(d+l)^2} \right)$$

$$= \frac{m}{\mu} \left\{ \frac{(d+l)^2 - (d-l)^2}{(d-l)^2 (d+l)^2} \right\}$$

$$= \frac{m}{\mu} \left[ \frac{4dl}{\{(d-l)(d+l)\}^2} \right]$$

$$\vec{F} = \frac{m}{\mu} \left[ \frac{4dl}{(d^2 - l^2)^2} \right]$$

along magnetic moment of the bar magnet

as the product of pole strength and vector joining the south pole towards north pole

ie  $M = m \cdot 2l$   
~~of the~~

Thus  $F = \frac{2 \cancel{m} (m \cdot 2l) \cancel{2l}}{\mu (d^2 - l^2)^2}$

$$F = \frac{2Md}{\mu (d^2 - l^2)^2} \quad \text{directed along NP}$$

Since  $l^2 \ll d^2$  we can neglect

$l^2$  compared to  $d^2$  and the above

expression becomes  $F \approx \frac{2Md}{\mu d^3}$

$$\therefore F = \frac{2M}{\mu d^2}$$



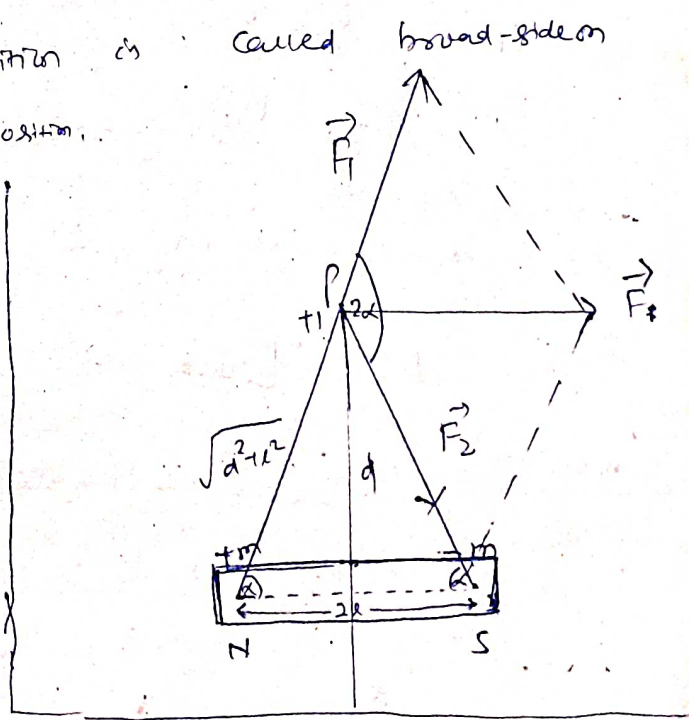
Electric field intensity at the broad-side-on position of an electric dipole

The bar magnet possess two poles placed at the separation  $2l$  which is ~~called~~ called magnetic length and it is 85% of the geometric length.

The point P is at a distance  $d$  from the mid point of the bar magnet. The point is situated on the  $\perp$  bisector of the magnetic length or (the line joining the two poles of bar magnet)

Such a position is called broad-side-on or equifocal position.

If a unit north-pole is placed at a point P then it will experience a repulsion ( $F_1$ ) due to the north pole and an attraction ( $F_2$ ) due to the south pole. ~~But~~ the ~~two~~ forces are equal.



The point P is ~~at~~ at a distance  $d$  from

The midpoint of the bar magnet.  $NP = SP = \sqrt{d^2 + l^2}$

$$\vec{F}_1 \text{ along } \vec{NP}$$

$$= \frac{1 \cdot m \cdot 1}{\mu \sqrt{d^2 + l^2} (NP)^2}, \text{ along } \vec{NP}$$

$$= \frac{m}{\mu (\sqrt{d^2 + l^2})^2}, \text{ along } \vec{NP}$$

$$= \frac{m}{\mu (d^2 + l^2)}, \text{ along } \vec{NP}$$

Magnetic field intensity at P due to a south pole of strength (-m) at S is

$$\frac{m}{\mu SP^2} = \frac{m}{\mu (d^2 + l^2)} = F_2, \text{ along } PS$$

$$\text{Thus } F_1 = F_2$$

Resultant intensity at P is obtained from the law of parallelogram of vectors.

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$$= \sqrt{2F_1^2 + 2F_1^2 \cos \alpha}$$

$$= \sqrt{2F_1^2 (1 + \cos \alpha)}$$

$$= \sqrt{2F_1^2 \cdot 2 \cos^2 \frac{\alpha}{2}}$$

$$= 2F_1 \cos \frac{\alpha}{2}$$



$$= 2 \cdot \frac{m}{\mu (d^2 - l^2)} \cdot \frac{l}{\sqrt{d^2 - l^2}}$$

$$F = \frac{M}{\mu (d^2 - l^2)^{\frac{3}{2}}}$$

where  $M = 2ml$

= magnetic moment of the bar magnet

The direction of  $\vec{F}$  is  $\parallel$  to NS.

When the magnet is short and the point of observation is at a long distance i.e.  $d^2 \gg l^2$ , we

have

$$F \approx \frac{M}{\mu d^3} = \frac{M}{\mu d^3}$$

## Problem

24.06.20

1. The strength of magnetic field at two points on the axis of a bar magnet at distances of 10 cm and 20 cm respectively from its centre are in the ratio 25 : 2. Calculate the magnetic length of the magnet. (Ans: 10 cm.)

Ans:

Magnetic field intensity at the end-on position

is given by  $F = \frac{2Md}{(d^2 - l^2)^2}$

$$\frac{F_1}{F_2} = \frac{2Md_1}{(d_1^2 - l^2)^2} \bigg/ \frac{2Md_2}{(d_2^2 - l^2)^2}$$

$$= \frac{2Md_1}{d_1^2 - l^2} \times \frac{d_2^2 - l^2}{2Md_2}$$

$$\frac{25}{2} = \frac{d_1}{d_2} \times \frac{(d_2^2 - l^2)^2}{(d_1^2 - l^2)^2}$$

$$\frac{25}{2} = \frac{10}{20} \times \frac{(d_2^2 - l^2)^2}{(d_1^2 - l^2)^2}$$

$$\Rightarrow 25d_1^2 + 25l^2 = d_2^2 - l^2$$

$$\Rightarrow 25d_1^2 - d_2^2 = 24l^2$$



Taking same divt of both the sides  
we get

$$5 = \frac{400 - l^2}{10 - l^2}$$

$$\Rightarrow 50 - 5l^2 = 400 - l^2$$

$$\Rightarrow 100 = 4l^2$$

$$\Rightarrow l^2 = 25$$

$$\Rightarrow l = 5 \text{ cm}$$

$$\therefore \text{Magnetic length} = 2l = 2 \times 5 = 10 \text{ cm}$$

2. Find the force between two like magnetic poles each of strength 20 units and placed 5 cm apart. What is the intensity of the magnetic field due to these poles at a point 5 cm from each.

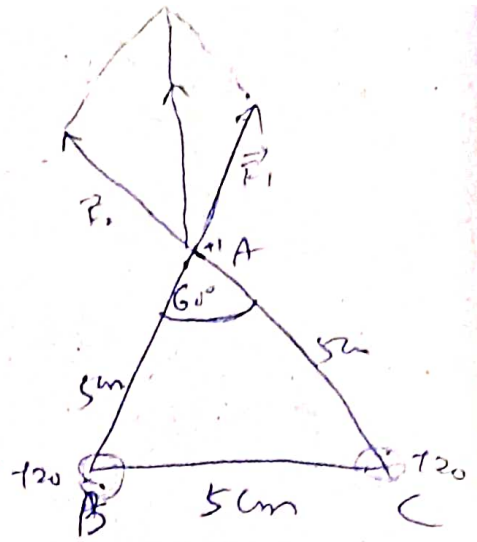
(Ans: 16 dyne, 1.39 oersted)

According to  
Coulomb's law

Force between  
two magnetic poles

$$= \frac{1}{\mu} \frac{m_1 m_2}{r^2}$$

$$= \frac{1}{1} \cdot \frac{20 \times 20}{(5)^2} = \frac{400}{25} = 16 \text{ dyne.}$$



$$F_1 = F_2 = \frac{20 \times 1}{5^2} = \frac{20}{25} = \frac{4}{5} \text{ dyne}$$

~~F =~~

Net field intensity

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^2 + 2 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \cos 120^\circ}$$

$$= \sqrt{\frac{16}{25} + \frac{16}{25} + \frac{32 \times 16}{25} \times \frac{1}{2}}$$

$$= \sqrt{\frac{3 \times 16}{25}}$$

$$= \sqrt{\frac{48}{25}}$$

$$= \frac{\sqrt{48}}{25} = \frac{2\sqrt{3} \cdot 2}{25} = \frac{4\sqrt{3}}{5} = \frac{4 \times 1.732}{5}$$

~~4.24~~



$$= \frac{6.928}{5} = 1.385$$

$\approx 1.39$  Oersted.

$$\boxed{\frac{\text{Oersted-dyne}}{\text{Gauss}^2}}$$

3) Magnetic North poles of strengths 50 and 90 units are placed at the corners B and C of an equilateral triangle ABC of side length 10 cm. If a South pole of strength 80 units be placed at A, find the resultant force on A.

(Ans: 98.3 dyne,  $20.63^\circ$  with AC)

4) Two poles, one of which 5 times as strong as the other exert on each other a force equal to the weight of 800 mg when they are placed 10 cm apart. Find the strength of each pole. (Ans:  $4\sqrt{8}$ ,  $20\sqrt{9}$ )

5) Two north poles repel one another with a force of 2.4 dyne when their distance apart is 2 cm. What will be their distance apart when the force is 3.6 dyne. (Ans: 1.63 cm)

Q Find the repulsive force when their distance apart is 3 cm (Ans: 1.07 dyne)  
(1.066)

Ans: 3.

Force between A and B

$$= \frac{1 \times 50 \times 80}{100}$$

$$= 40 \text{ dyne}$$

$$\therefore \vec{AB} = 40 \text{ dyne}$$

Force between A and C =  $\frac{1 \times 80 \times 90}{100}$   
= 72 dyne

$$\therefore \vec{AC} = 72$$

Using parallelogram law, Net force on A

$$= \sqrt{(40)^2 + (72)^2 + 2 \cdot 40 \cdot 72 \cdot \cos 60^\circ}$$

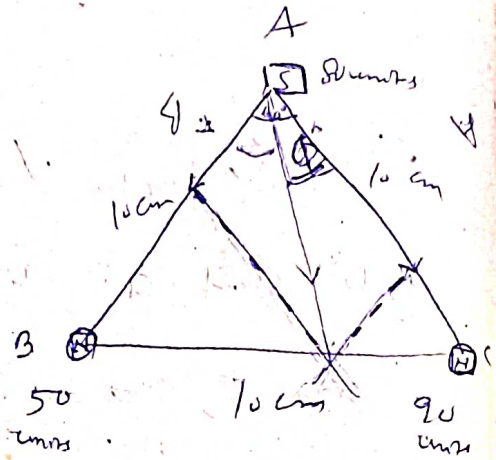
$$= \sqrt{1600 + 5184 + 2880}$$

$$= \sqrt{9664}$$

$$= 98.3 \text{ dyne}$$

$$f_{\text{amp}} = \frac{B \sin \alpha}{A + B \cos \alpha} = \frac{AB \sin 60^\circ}{A + AB \cos \alpha}$$

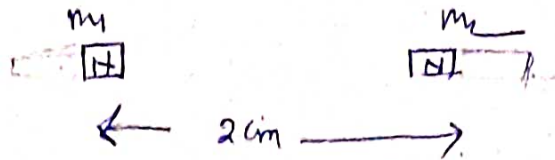
$$= \frac{40 \times \frac{\sqrt{3}}{2}}{72 + 40 \times \frac{1}{2}} = \frac{5 \times 20\sqrt{3}}{72 + 20} = \frac{5 \times 17.32}{92} = 23$$







5.



$$\frac{m_1 m_2}{4} = 2 \cdot 4 \quad \text{--- (i)} \quad \text{and} \quad \frac{m_1 m_2}{d^2} = 3 \cdot 6 \text{ dyne (ii)}$$

Dividing eqn (ii) by eqn (i)

$$\frac{\frac{m_1 m_2}{d^2}}{\frac{m_1 m_2}{4}} = \frac{3 \cdot 6}{2 \cdot 4}$$

$$\Rightarrow \frac{m_1 m_2}{d^2} \cdot \frac{4}{m_1 m_2} = \frac{36}{24} = \frac{3}{2}$$

$$\Rightarrow \frac{4}{d^2} = \frac{3}{2}$$

$$\Rightarrow d^2 = \frac{8}{3} \Rightarrow d = \sqrt{\frac{8}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$= \frac{2 \times 1.414}{1.732} = \frac{2.828}{1.732}$$

$$\therefore d = \frac{2.828}{1.732} = 1.63 \text{ cm}$$

Repulsive force between them

$$\frac{m_1 m_2}{9}$$

But from previous question  $\frac{m_1 m_2}{4} = 2 \cdot 4$

$$= \frac{9 \cdot 6}{7}$$

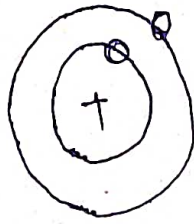
$$\Rightarrow m_1 m_2 = 9 \cdot 6$$

$$= \frac{96}{90} = 1.066$$

$$\approx 1.07 \text{ dyne}$$



18.



H atom

H atom hai

one electron  
and one proton.

According to ~~the~~ Coulomb's law

attractive force between them

$$= \frac{k Q_1 Q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{\left(\frac{6 \times 10^{-7}}{10^2}\right)^2}$$

$$= \frac{(9 \times 1.6 \times 1.6) \times 10^{-29} \times 10^4}{36 \times 10^{-14}}$$

$$= \left(\frac{9 \times 1.6 \times 1.6}{36 \times 10^4}\right) \times 10^{-25} \times 10^{14}$$

$$= (0.4 \times 1.6) \times 10^{-11}$$

$$= 0.4 \times 10^{-11}$$

$$= 6.4 \times 10^{-12} \text{ N}$$

(ii) Part here Centrifugal force = ~~Attractive force~~  
Attractive force

$$\Rightarrow \frac{mv^2}{r} = 6.4 \times 10^{-8} \text{ N}$$

$$\Rightarrow \frac{9.11 \times 10^{-31} \times v^2}{6.07 \times 10^{-9}} = 6.4 \times 10^{-8}$$



$$2) \quad V^2 = \frac{6.4 \times 10^8 \times 6 \times 10^9}{9.11 \times 10^{31}}$$

$$= \left( \frac{6.4 \times 6}{9.11} \right) \times 10^{(8+9-31)}$$

$$= (4.2) \times 10^4$$

$$2) \quad V = 2.1 \times 10^7 \text{ m/s}$$

$$V = r \omega$$

$$\Rightarrow \omega = \frac{V}{r} = \frac{2.1 \times 10^7}{6 \times 10^9} = \frac{21}{6} \times 10^6 \times 10^9$$

$$= 3.5 \times 10^{15} \text{ rad/sec}$$

$$= \frac{3.5}{2\pi} \times 10^{15} \text{ rev/sec}$$

$$= \frac{3.5}{2 \times 3.14} \times 10^{15}$$

$$= \frac{3.5}{6.28} \times 10^{15}$$

$$= \frac{35}{6.28} \times 10^{14} \text{ rev/sec}$$

$$= 5.5 \times 10^{14} \text{ rev/sec}$$

Q.

28.  $F = 40 \text{ N}$

To hold a charge of  $20 \mu\text{C}$   
 $= 200 \times 10^{-6} \text{ C}$

$$F = E \times q$$

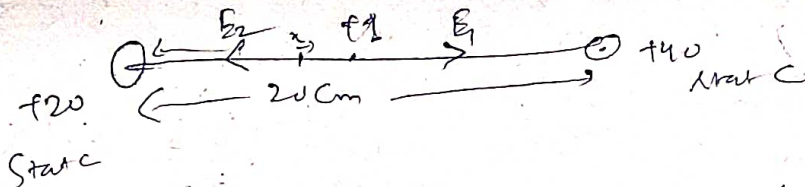
$$\Rightarrow 40 = E \times 200 \times 10^{-6}$$

$$\Rightarrow E = \frac{40}{200 \times 10^{-6}} = \frac{40 \times 10^6}{200}$$

$$= \frac{400 \times 10^5}{200}$$

$$= 2 \times 10^5 \text{ N/C} \text{ - direction}$$

26.



(i) Force acting on them =  $\frac{20 \times 40}{400} = 2 \text{ dyn}$

(ii) Electric field intensity =  $E_2 - E_1 = \frac{20 \times 1}{(10)^2} + \frac{40 \times 1}{(10)^2}$

$$= \frac{20}{100} + \frac{40}{100}$$

$$= 0.2 + 0.4$$

$$= 0.6 \text{ dyn/cm}$$

(iii) Field intensity = 0 when

$$E_1 = E_2$$

$$\Rightarrow \frac{20 \times 1}{(2-x)^2} = \frac{40}{(2+x)^2}$$

3)

$$\Rightarrow \frac{20}{n^2} = \frac{40 - 2}{400 + n^2 - 40n}$$

~~$$\Rightarrow \frac{40n^2}{n^2} = \frac{800n - 2n^2}{400 + n^2 - 40n}$$~~

$$\Rightarrow 2n^2 = 400 + n^2 - 40n$$

$$\Rightarrow n^2 + 40n - 400 = 0$$

$$\Rightarrow x = \frac{-40 \pm \sqrt{1600 - 4 \cdot (1) \cdot (-400)}}{2}$$

$$= \frac{-40 \pm \sqrt{1600 + 1600}}{2}$$

$$= \frac{-40 \pm \sqrt{3200}}{2}$$

$$= \frac{-40 \pm (4 \times 1.414) \times 10}{2}$$

$$= \frac{-40 \pm 56.56}{2}$$

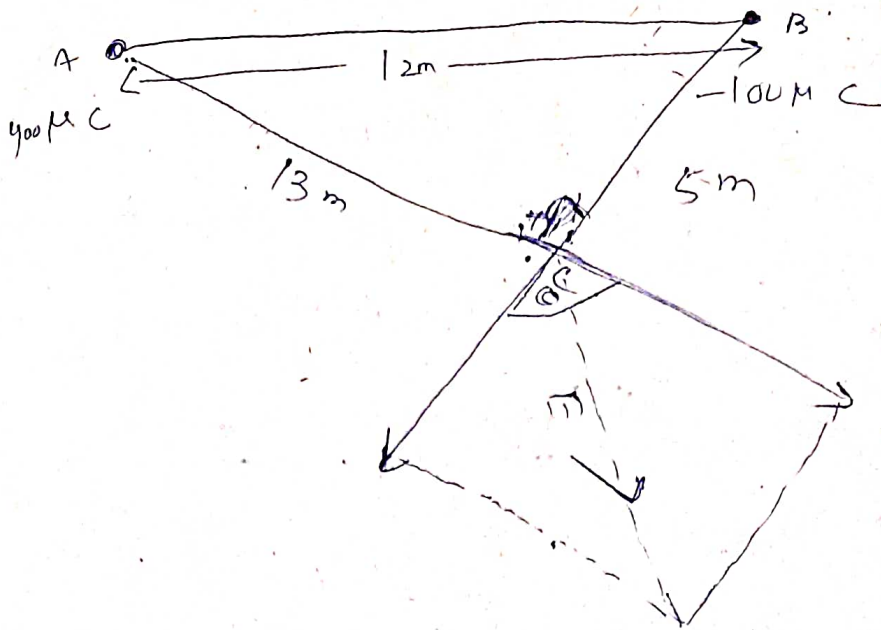
$$= \frac{-40 + 56.56}{2}$$

$$= \frac{56.56 - 40}{2}$$

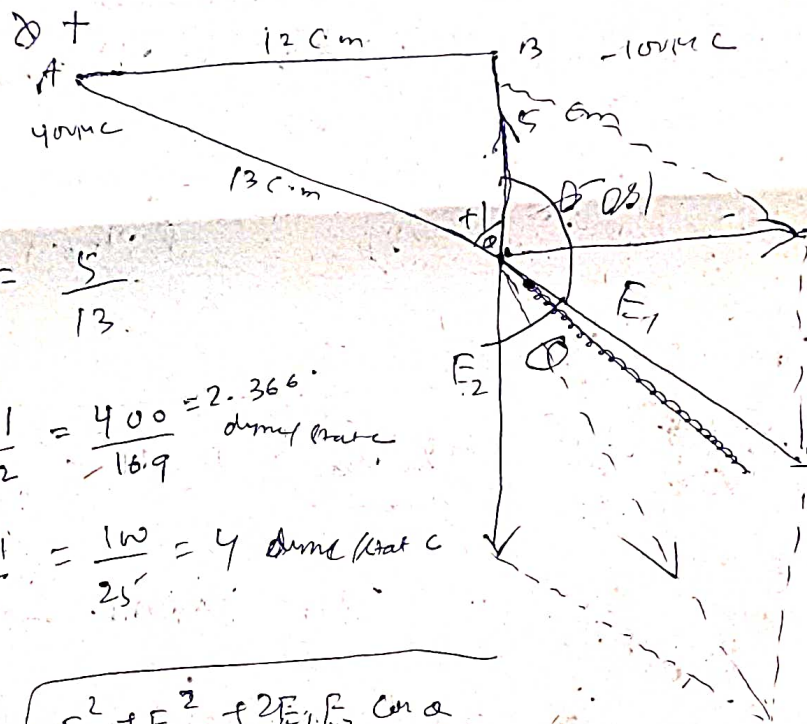
$$= \frac{16.56}{2} = 8.28 \text{ form } 20000$$

which is not possible





Q 23.



$$\cos \alpha = \frac{5}{13}$$

$$E_1 = \frac{400 \times 1}{(13)^2} = \frac{400}{169} = 2.366 \text{ down (Nat C)}$$

$$E_2 = \frac{100 \times 1}{(5)^2} = \frac{100}{25} = 4 \text{ down (Nat C)}$$

$$\text{Resultant } E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos \alpha}$$

$$= \sqrt{\frac{160000}{28561} + 16 + 2 \cdot \frac{400}{169} \times 4 \times \frac{5}{13}}$$

$$= \sqrt{28561 \times 2197}$$

$$= \sqrt{5.6 + 16 + 2 \cdot (2.366) \times 4 \times \frac{5}{13}}$$

$$= \sqrt{21.6 + 7.28} = 5.37$$

## Lines of force due to a bar magnet

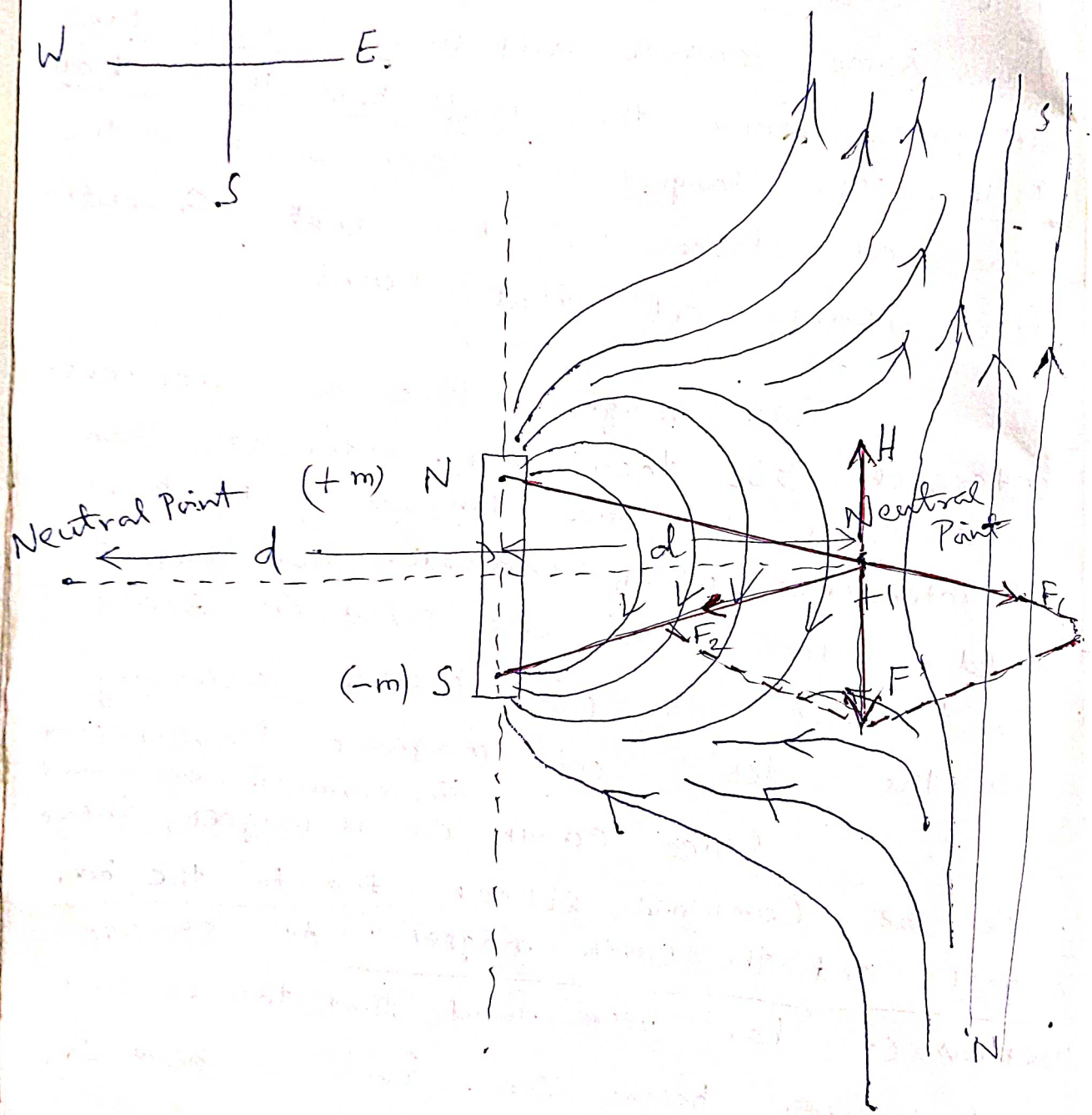
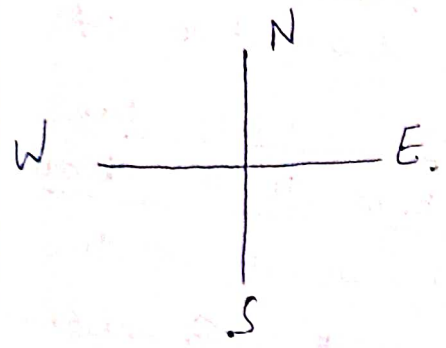
### Neutral point

Lines of force always starts from North Pole and terminate at the South Pole of the same magnet and move inside the magnet from the South Pole to the North Pole. The tangent at any point on the line of force indicates the resultant field intensity at that point.

Two magnetic lines of force never intersect. If they will intersect, then there will be two tangents at the point of intersection which indicates two magnetic field intensities at one point in space. But, this is not possible according to the def<sup>n</sup> of magnetic field intensity.

Since earth is a magnet, there will be combined effect due to the bar magnet and the earth magnet. At specific positions, it has been found that the resultant field intensity become zero. Such a point is called neutral point, where the resultant field intensity due to the bar magnet is exactly cancelled by ~~bars~~ the horizontal component of the earth's field intensity.

Case - 1 North pole of the bar magnet  
Pointing towards the geographical North  
direction



In this case, the neutral point is situated on the broad-side - on either side of the magnet.



$$\frac{M}{(d^2 - l^2)^{\frac{3}{2}}} = H$$

$$\Rightarrow M = H (d^2 - l^2)^{\frac{3}{2}}$$

$$\text{and } m = \frac{M}{2l}$$

Thus, location of the neutral point ~~and~~ enables us to find the magnetic moment and pole strength of the bar magnet.

### Case-II

North pole of the bar magnet point  
towards the geographical South pole

In this case, the neutral point is situated on the axis of the bar magnet (End-on-position)

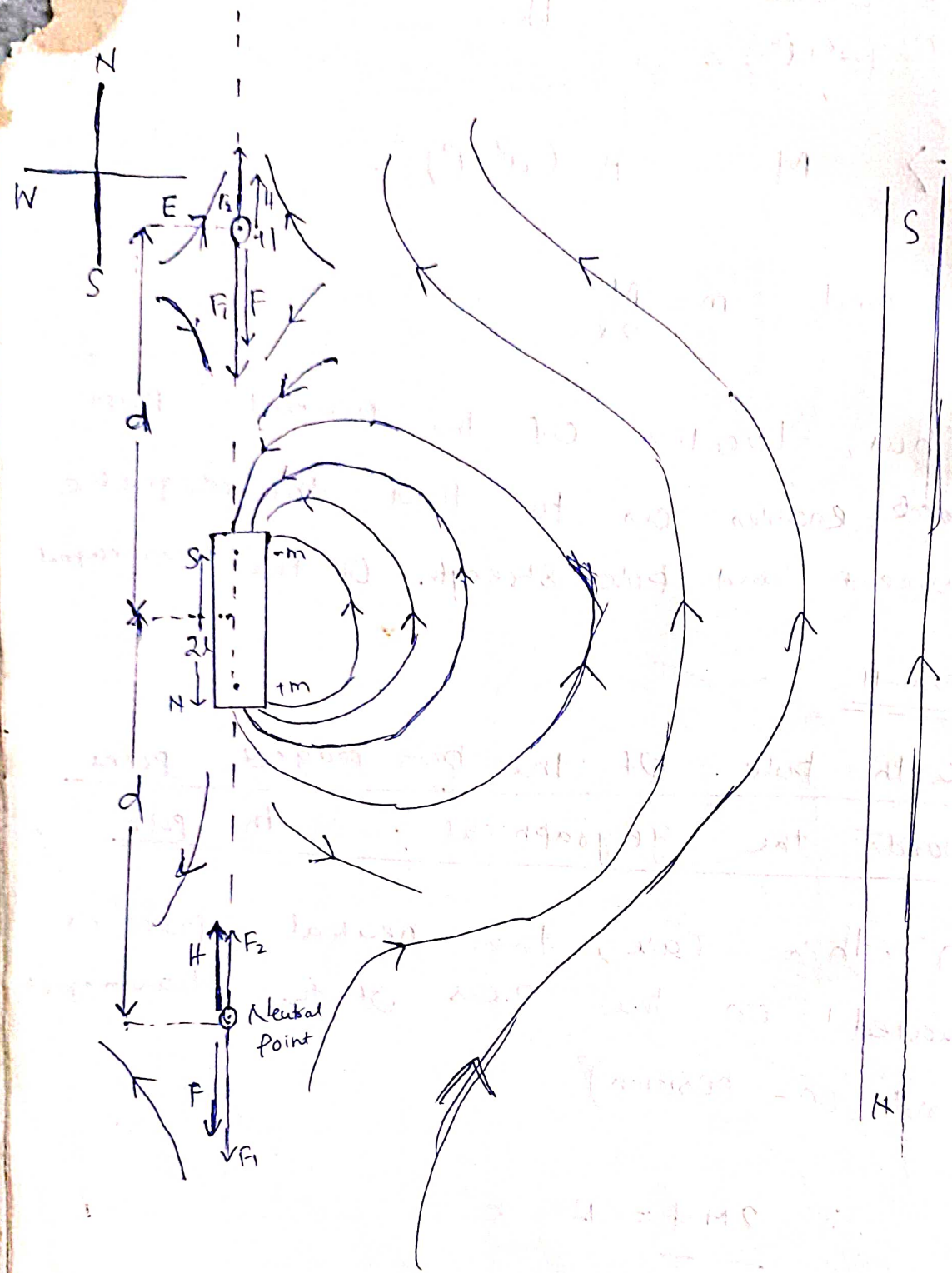
$$\therefore \frac{2Md}{(d^2 - l^2)^2} = H$$

$$\Rightarrow M = \frac{H (d^2 - l^2)^2}{2d}$$

$$\text{and } m = \frac{M}{2l}$$

Thus, the location of the neutral point helps us to find the magnetic moment and

length.



Problems

Ex. A bar magnet is placed with its north pole pointing south. If the neutral point is at a distance of 20cm from a nearest pole of the magnet. Calculate

The magnetic moment of the magnet ..

$$H = 0.36$$
$$2l = 10 \text{ cm.}$$

(Ans: 2592 C.G.S unit)

② A magnet whose poles are 12 cm apart is placed in the magnetic meridian. The field due to this magnet counter balances the earth's horizontal field (0.35 C.G.S unit) at a point 10 cm from each pole.

Find the pole strength of the magnet  
(Ans: 29.17 unit pole)

③ A short bar magnet is placed in the magnetic meridian with its north pole pointing south. The neutral point is 24 cm from the south pole of the magnet and upon the prolongation of its axis.

Find the intensity of the field at a point on the axis 20 cm from the south pole and north pole of it.

Given  $H = 0.18$  Oersted (Ans: 0.13 C.G.S unit)

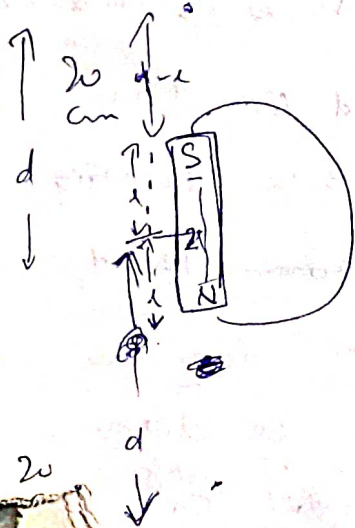
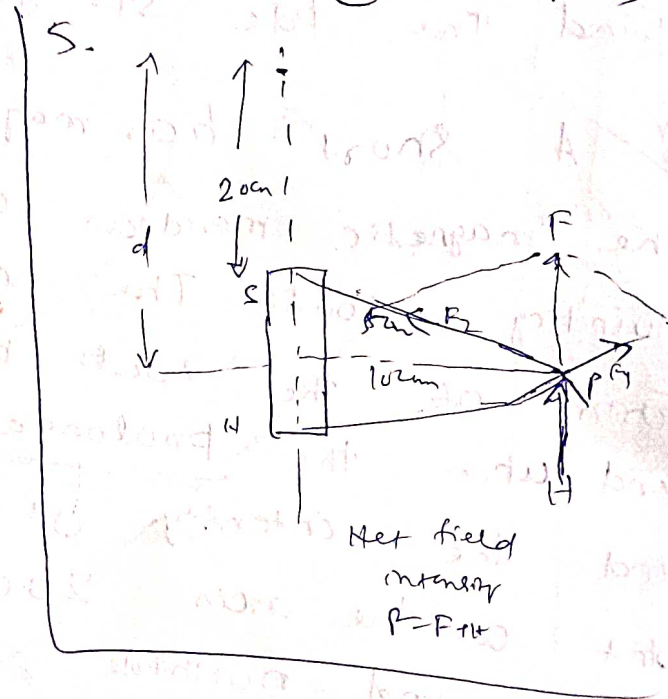
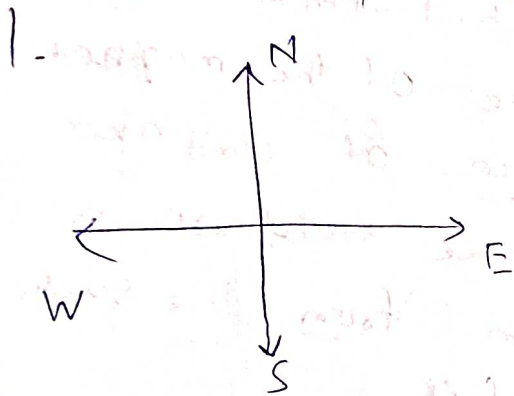
④ A short magnet is placed on the drawing board with its north pole pointing north. The neutral points are ~~found~~ found to be at a distance of 20 cm from the centre of the magnet. What will be the distance of the neutral point when the magnet is reversed? (Ans: 25.2 cm)



A bar magnet having poles 10 cm apart placed on the magnetic meridian with its north pole pointing south. The neutral point is at a distance 20 cm from the nearer pole. Find the intensity of the resultant field at a point on the  $\perp$  bisector of the axis of a magnet and at a distance of 10 cm from the centre of the magnet.

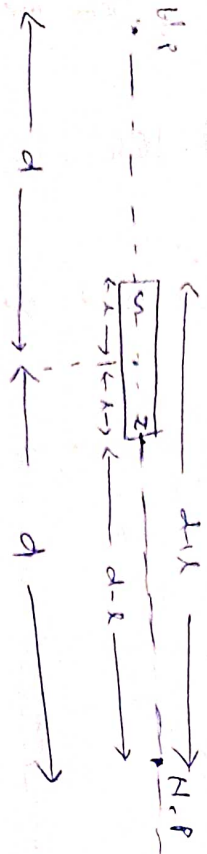
$H = 0.4$  gauss [Ans 2.46 gauss]

Ans.



$d = 20$  cm.

Since the magnet is placed with north pole pointing south, then the neutral point will be at the end on position at a distance  $d$  from the mid point.



Here  $d+l = 20 \text{ cm}$ .  
 But  $2l = 10 \text{ cm}$   
 $l = 5 \text{ cm}$

$$\Rightarrow d = 20 - 5 = 15 \text{ cm}$$

$$H = 0.36$$

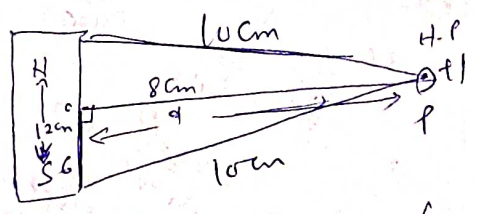
$$M = \frac{H (d^2 - l^2)^2}{2dl}$$

$$= \frac{0.36 \times (225 - 25)}{50}$$

$$= \frac{0.36 \times 200}{50} = 1.44 \text{ G.O.S. unit}$$

of the magnetic length  $2l$ .

2. Length between the two poles =  $2l = 12 \text{ cm}$   
 $\Rightarrow l = 6 \text{ cm}$ .



Earth's horizontal field  $H = 0.35$

$$d = \sqrt{(SP)^2 - (OP)^2} = \sqrt{10^2 - (6)^2} = 8 \text{ cm}$$

Strength of the magnet (m)

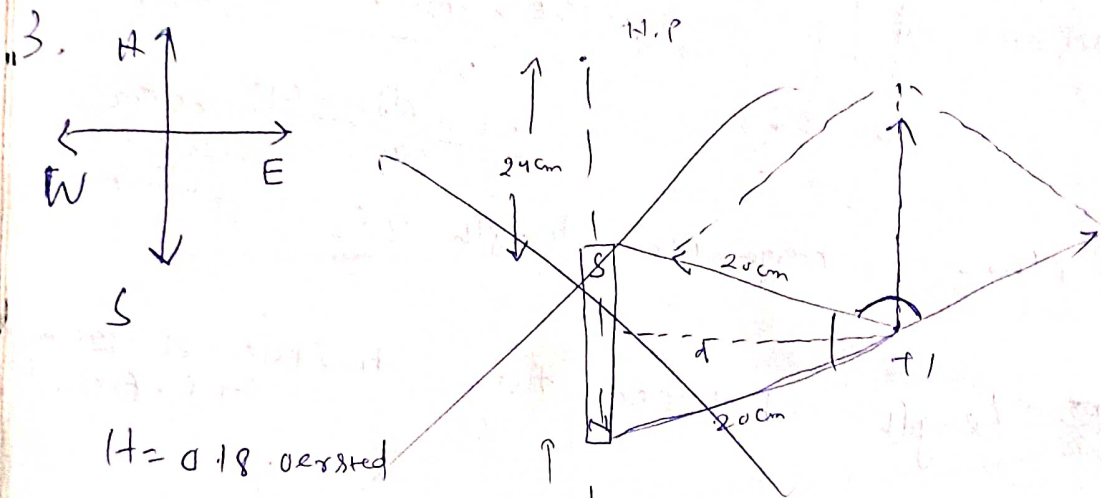
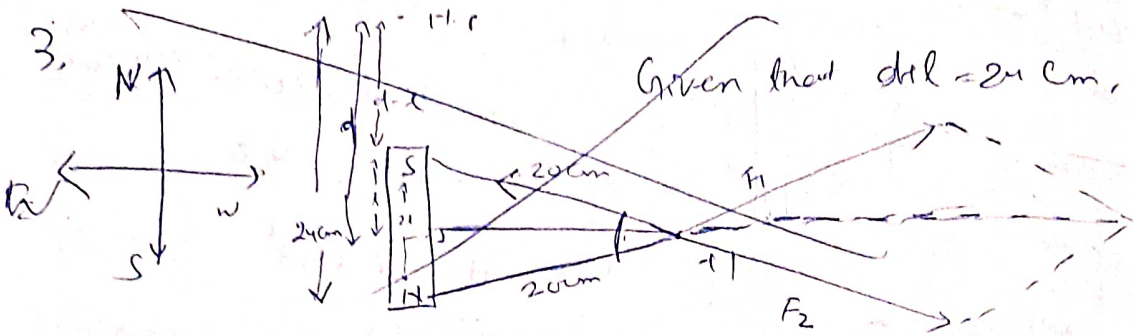
$$= \frac{M}{2d}$$

$$\text{But } M = H (d^2 + l^2)^{\frac{3}{2}} = 0.35 (64 + 36)^{\frac{3}{2}}$$

$$= 0.35 (100)^{\frac{3}{2}} = 0.35 \times (10)^2 \times \frac{3}{2} = 52.5$$

Pole strength of the bar magnet

$$= \frac{M}{2l} = \frac{350}{12} = 29.166 \approx 29.17 \text{ C.G.S units}$$



3.

5.

$$H = \frac{M}{(d^2 - l^2)^{3/2}} = \frac{2850}{(25^3 - 12^3)^{3/2}}$$

$$= \frac{2850}{12177} = 2.091$$

$$H = \frac{2M}{(d^2 - l^2)^{3/2}} = \frac{2M}{(625 - 144)^{3/2}}$$

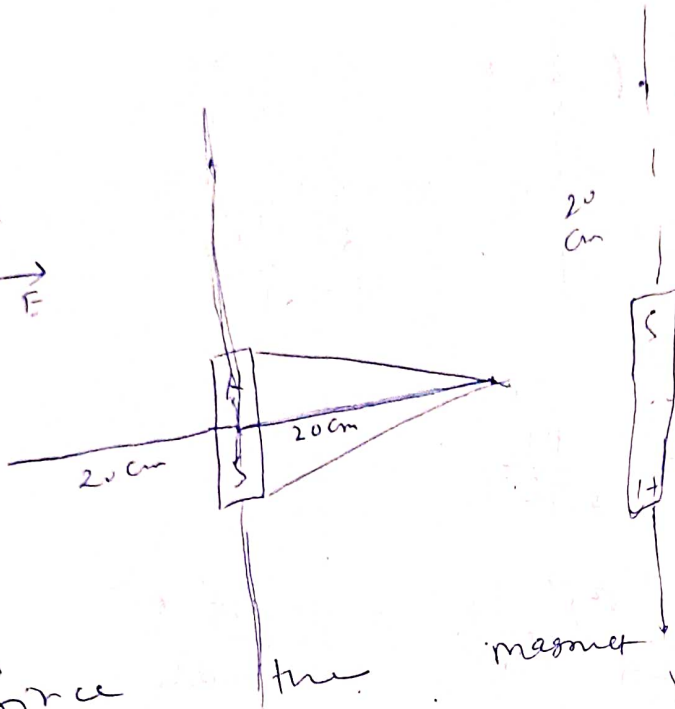
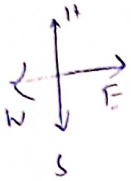
$$= \frac{2M}{225} = \frac{2M}{15^3}$$

$$M = \frac{H (d^2 - l^2)^{3/2}}{2}$$

$$= \frac{0.18 \times (625 - 144)^{3/2}}{2} = \frac{0.18 \times 600 \times 1260}{2} = 13608 \text{ CGS}$$



4.



Since the magnet is short  
 Since north pole point the north.

$$\therefore H = \frac{2M}{d_1^3}$$

When it is reversed

$$H = \frac{2M}{d_2^3}$$

Dividing

$$1 = \frac{M}{d_1^3} \times \frac{d_2^3}{2M} = \frac{d_2^3}{2d_1^3} = \frac{d_2^3}{2 \times (20)^3}$$

~~$$\Rightarrow d_2^3 = 8000$$~~
~~$$\Rightarrow \frac{d_2^3}{2} = 8000$$~~

$$\Rightarrow d_2^3 = 8000 \times 2$$

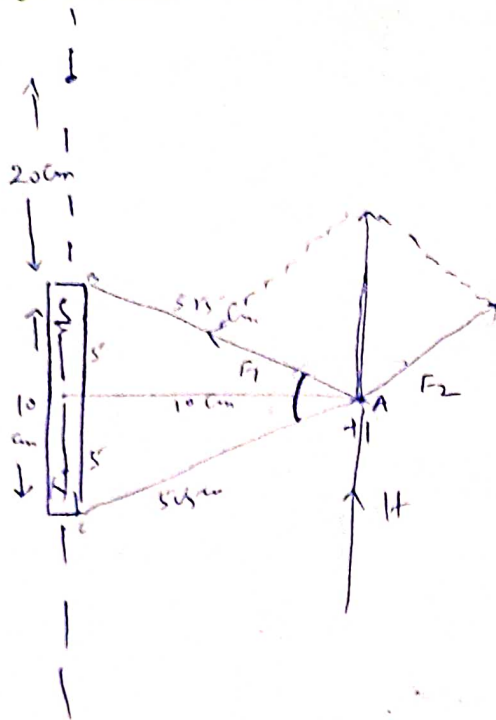
$$\Rightarrow d_2 = 2^{\frac{1}{3}} \times 20 \text{ cm.}$$

Let  $x = 2^{\frac{1}{3}} \Rightarrow \ln x = \frac{1}{3} \ln 2 = \frac{1}{3} \times 0.693 = 0.231$

2)  $x = \text{antilog}(0.231) = 1.26$

$$d_2 = 1.26 \times 20 = 25.2 \text{ cm}$$

5.



$$\frac{2Md}{(d^2 - l^2)^2} = H$$

$$M = ?$$

Pile strength =  $\frac{M}{2l}$

$$= \frac{H(d^2 - l^2)^2 \times 1}{2d \times 2l} = \frac{H(25^2 - 10^2)^2}{4dl}$$

$$= \frac{0.4 \times (25^2 - 10^2)^2}{4 \times (20) \times (5)} = \frac{0.4 \times 600^2}{4 \times 100}$$

~~$$= \frac{0.4 \times 375 \times 75 \times 15 \times 3}{4 \times 20 \times 4} = \frac{3}{5}$$~~

$$= \frac{3}{8}$$

Net force

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$$F = \frac{M}{[(10)^2 + (5)^2]^{3/2}}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(5\sqrt{5})^2 + (5\sqrt{5})^2 - (10)^2}{2 \cdot (5\sqrt{5})(5\sqrt{5})}$$

$$= \frac{125 - 125 - 100}{250}$$

$$= \frac{250 - 100}{250}$$

$$= \frac{150}{250}$$

$$= \frac{3}{5}$$

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos(180^\circ - A)}$$

$$= \sqrt{2F_1^2 - 2F_1^2 \cos A}$$

$$= \sqrt{2F_1^2 \left(1 - \frac{3}{5}\right)}$$

$$= \sqrt{2} \times F_1 \times \sqrt{\frac{2}{5}}$$

$$= \sqrt{2} \times \frac{m_1 m_2}{r^2} \times \frac{\sqrt{2}}{\sqrt{5}}$$

$$= \frac{2 \times \frac{3}{5} \times \frac{3}{5}}{(5\sqrt{5})^2 \times \sqrt{5}}$$

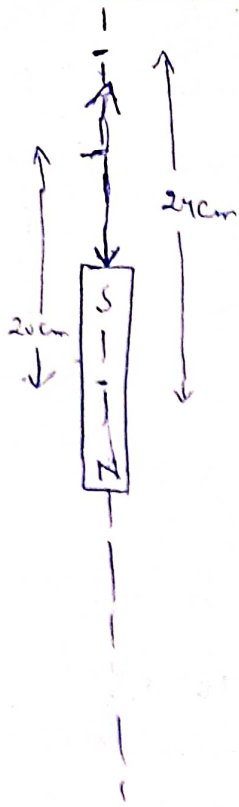
$$= \frac{18}{125 \times \sqrt{5}} = \frac{18}{32 \times 5 \times \sqrt{5}} = \frac{18}{25 \times 125 \times \sqrt{5}}$$

$$\text{Net force} = F + H$$

$$= \frac{9}{32 \times 125 \times \sqrt{5}} + 0.4$$



3.



Since it is a short magnet the magnetic length can be neglected,  $l \ll d$ .  
 Actual dist = 24 cm but  $l \ll d$ ,  $\therefore$  down

$$\therefore \frac{2M}{d^3} = H$$

$$\Rightarrow \frac{2M}{(24)^3} = 0.18$$

$$\Rightarrow M = \frac{(24)^3 \times (0.18)}{2}$$

$$= 1244.16$$

At 20 cm from the centre of

field intensity  $\frac{2M}{d^3}$

$$= \frac{2 \times 1244.16}{(20)^3} = \frac{2 \times 1244.16}{8000} = \frac{2488.32}{8000} = 0.31104$$

~~3.215~~ = 0.31104

Net field intensity = ~~3.215~~

Since at ~~field~~ At 24 cm

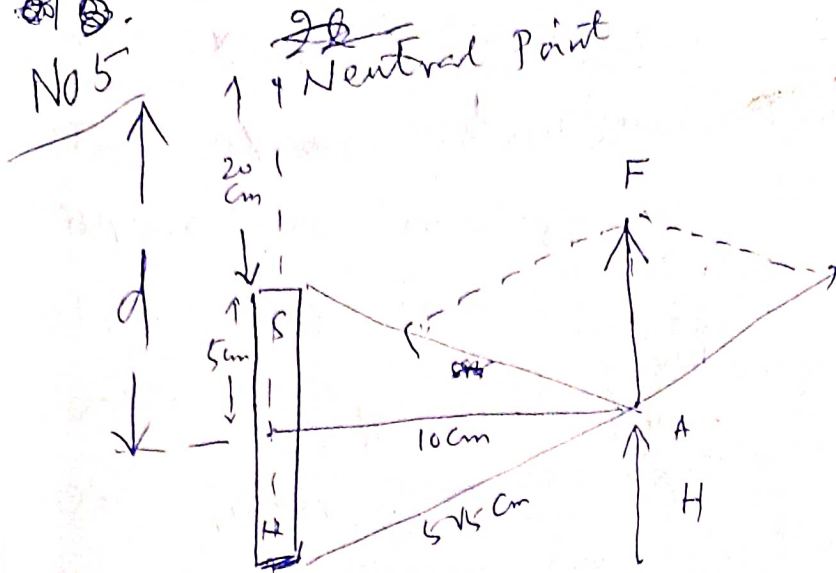
0.31104
- 0.18000
-----
0.13104
(Am)
C. G. S
Unit -

The earth field intensity = Magnet field intensity  
 but 20 cm is nearest magnet

$$\therefore \text{Net field intensity} = \text{Magnet F.I} - \text{earth F.I}$$

$$= 0.31104 - 0.18 = 0.13104 \text{ (Am)}$$

No 5



Here, the neutral Point is situated at the end-on position  
Magnetic moment ~~at~~  $A$  be  $M$ .

$$\therefore F = H$$

$$\frac{2Md}{(d^2 - l^2)^2} = H$$

$$\Rightarrow \frac{2 \times M \times 25}{[(25)^2 - (5)^2]^2} = H = 0.4$$

$$\Rightarrow M = 0.4 \times \frac{600 \times 600 \times 12}{50} = 7200 \times \frac{4}{10} = 2880 \text{ CGS units}$$

At a point 10 cm away on the broad-side-on position,

$$F = \frac{M}{(d^2 + l^2)^{3/2}} = \frac{2880}{[(10)^2 + (5)^2]^{3/2}}$$

$$F = \frac{2880}{(125)^{\frac{3}{2}}} = \frac{2880}{125 \cdot 125^{\frac{1}{2}}} = \frac{572}{\cancel{2880} \sqrt{5}}$$

~~$$= \frac{572}{\cancel{125} \sqrt{5}}$$

$$= \frac{.9152}{\sqrt{5}} = \frac{.9152}{2.2360679}$$

$$= \cancel{.409}$$~~

$$= \frac{572 \sqrt{5}}{125 \times 5} = \frac{572 \times (2.236)}{\cancel{625}}$$

$$= 2.046 \text{ gauss}$$

But Net field intensity =  $2.046 + 0.4$

$$2.046 + .4$$

$$= 2.446 \text{ gauss}$$



# Terrestrial Magnetism

The study of magnetic field of the earth involves the knowledge of three quantities which vary from place to place on the earth's surface. The following 3 quantities are called magnetic elements of the earth and they specify completely the magnetic field of the earth at any place.

1. Declination or Variation
2. The Dip or Inclination
3. The horizontal Intensity.

## (1) Declination or Variation ( $\theta$ )

It is the angle which the magnetic meridian at a given place makes with the geographical meridian.

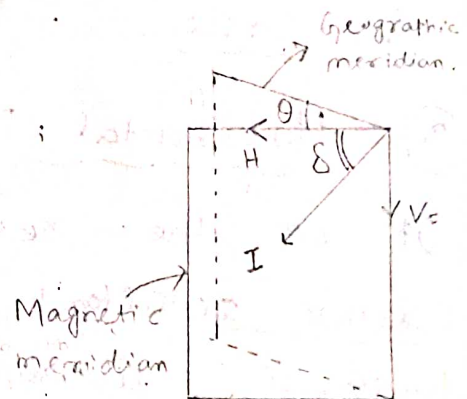
It is expressed as

$\theta^\circ E$  or  $\theta^\circ W$ .

Ex: Declination at Delhi

is  $2^\circ E$  which means that

the north pole of a compass needle will point  $2^\circ$  East of the geographical north-south direction.



where

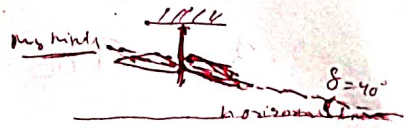
$H$  = Horizontal intensity,

$I$  = Earth's resultant field intensity.

## ② Dip or Inclination ( $\delta$ )

The dip or inclination at a place is the angle which the earth's resultant magnetic field intensity at that place makes with the horizontal ~~direction~~ line in the magnetic meridian at that place.

It will be  $\delta^{\circ}N$  or  $\delta^{\circ}S$  according as the point lies in the northern or southern hemisphere. For example, at Delhi, the dip is  $40^{\circ}N$  which means that the dip needle will be inclined to the horizontal at  $40^{\circ}$  with its north pole dipping downwards at Delhi.



The angle of dip is measured in the laboratory by an instrument called dip-circle.

## ③ Horizontal Intensity

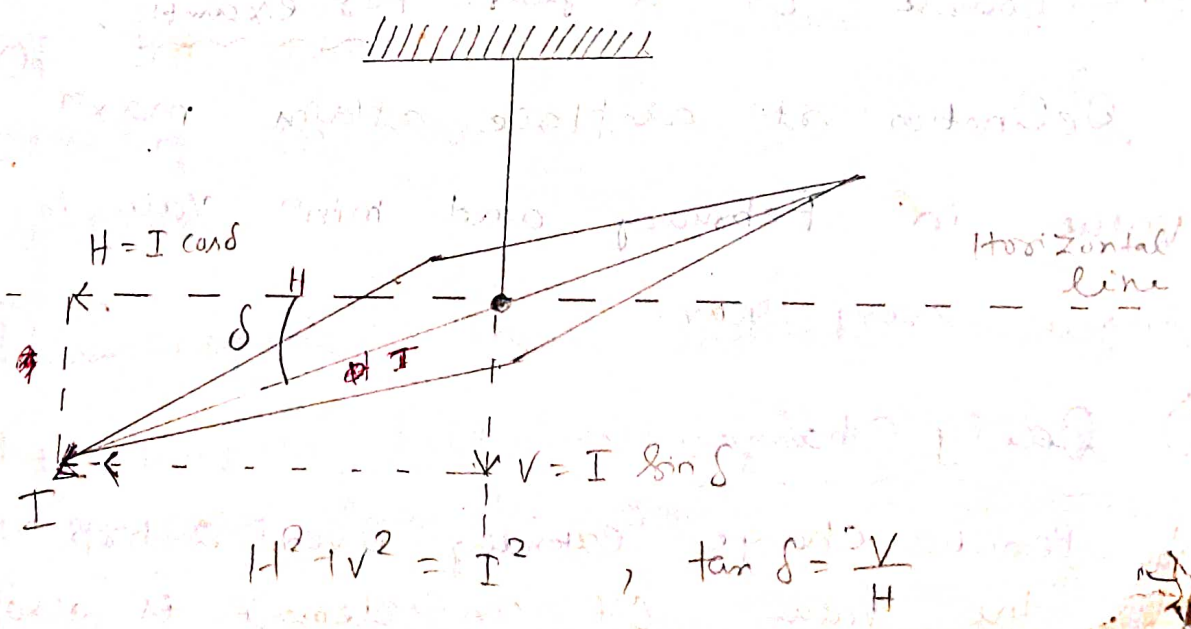
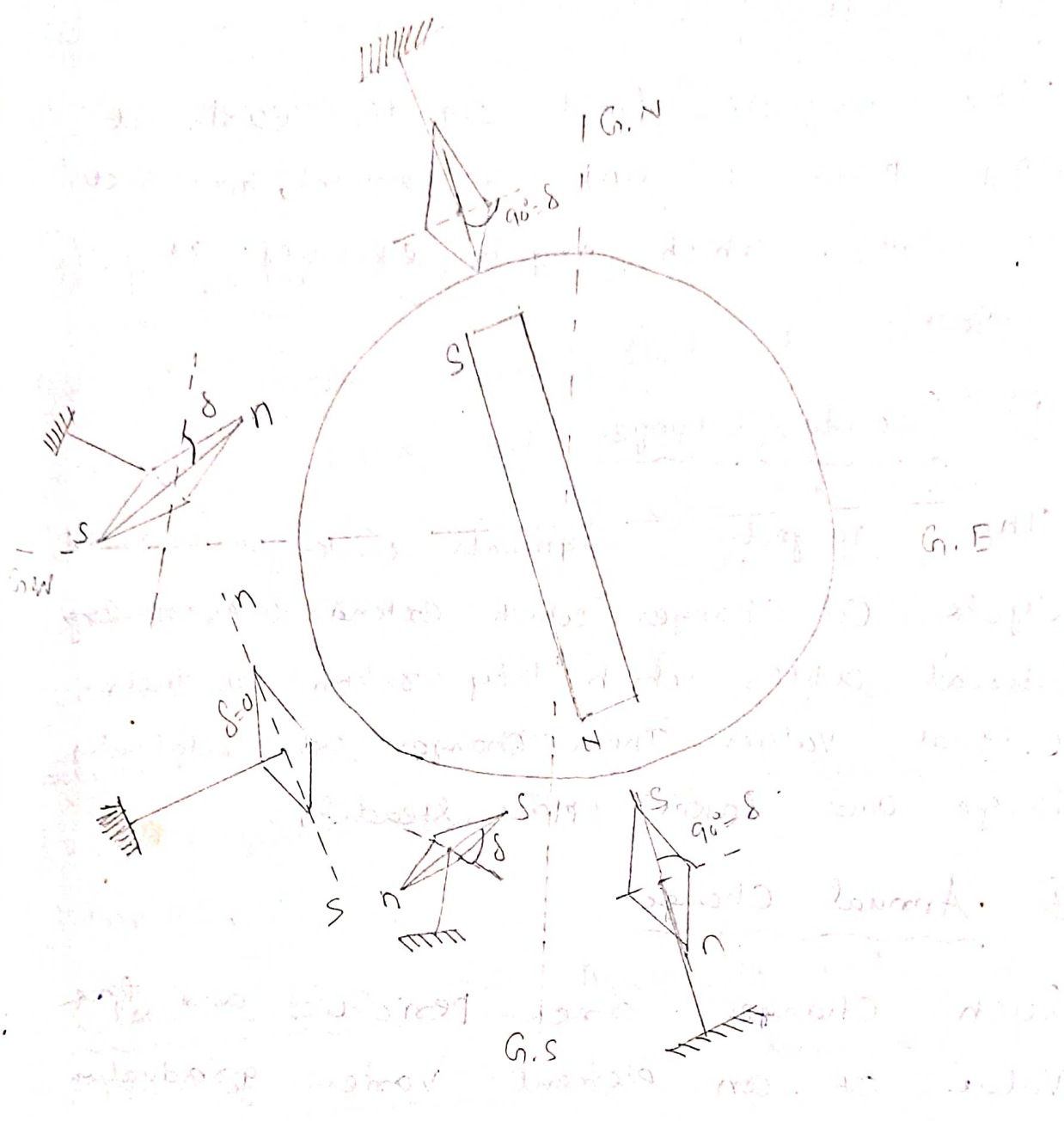
It is the resolved part of the earth's resultant intensity at a place in the horizontal direction in the magnetic meridian.

$$H = I \cos \delta$$

The vertical intensity ( $V$ ) is the resolved part of the earth's resultant intensity at a place in the vertical direction.

$$\therefore V = I \sin \delta$$

Fig of Inclination





## Changes in the values of magnetic elements at a place

The magnetic field of the earth at any place is not a constant, but subject to changes which may be classified as follows :-

### ① Secular Change

The magnetic elements undergo a gradual cycle of changes which extend over a long interval after which they return to their original values. These changes are relatively large and takes place steadily.

### ② Annual Change

Such changes are periodic and the value of an element varies gradually between a max<sup>m</sup> value and a min<sup>m</sup> value in course of a year. For example :-

Declination at a place attains max<sup>m</sup> value in February and min<sup>m</sup> value in August every year.

### ③ Daily Change

A periodic change extending over 24 hrs the value of an element is also

noticed. An element reaches the max<sup>m</sup> value at some hour of the day and the min<sup>m</sup> value at some other hour. Characteristic of the element.

#### ④ Magnetic storm

It has been found that during volcanic eruptions, display of Aurora Borealis, Appearance of Sun-spots etc, sudden and violent changes occur in the indications of recording instruments measuring the magnetic element. These changes are said to be due to magnetic storm. They are obviously non-periodic.

#### Classification of Materials into dia-para - and ferrimagnetic substances

All materials available on the surface of the earth can be classified into 3 types depending on the magnetic behaviour of substances.

① Diamagnetic : These substances are repelled by magnetic field. Therefore, they always try to go away from the region of stronger magnetic field to the region of weaker magnetic field.



## (2) Paramagnetic

These substances are attracted by the magnetic field with a weak force. When put in a magnetic field they try to go from the weaker field towards the stronger field.

## (3) Ferromagnetic :-

Iron, Cobalt and nickel and some of their alloys come under this category. They possess molecules or atoms which have spontaneous magnetic behaviour. They can be converted into permanent magnets and they are strongly attracted by the magnets.

## Distinction between Dia- and Paramagnetic Substances

### Diamagnetic

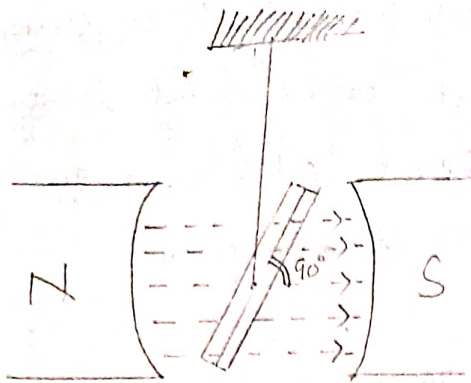
(1) A dia-magnetic solid in the form of a bar or rod when suspended in between the pole-pieces of a strong magnet is found to lie in the direction making  $90^\circ$  with the lines of force.

### Paramagnetic

(1) A para-magnetic solid in the form of a bar or rod when suspended in between the poles pieces of a strong magnet is found to lie in a direction parallel with the lines of force.

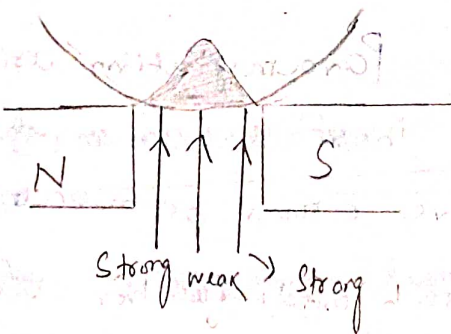


Fig:



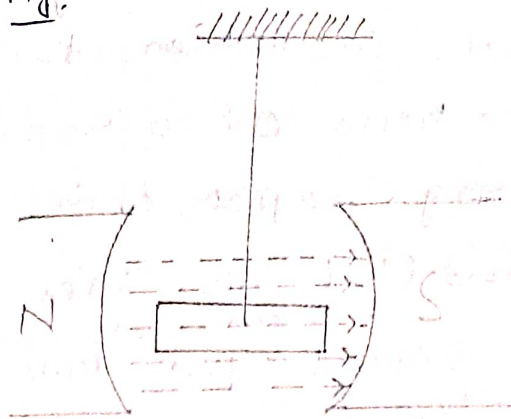
(2) A small amount of dia-magnetic liquid be kept on a watch glass. This watch glass be placed on the pole pieces of a strong magnet having a ~~small~~ <sup>10cm</sup> separation, then it is seen that the middle part of the liquid gets ~~depressed~~ <sup>raised</sup> where as the liquid ~~depressed~~ <sup>depressed</sup> on the sides.

Fig:



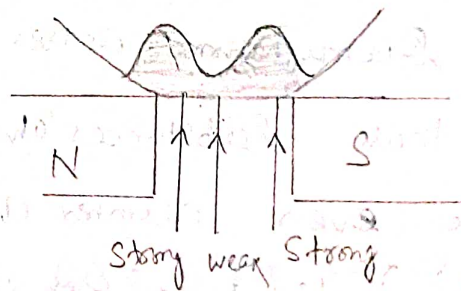
Because magnetic field is weaker than that of corner region.  
 (3) When a dia-magnetic liquid is taken in a U-tube with one of the arms

Fig:



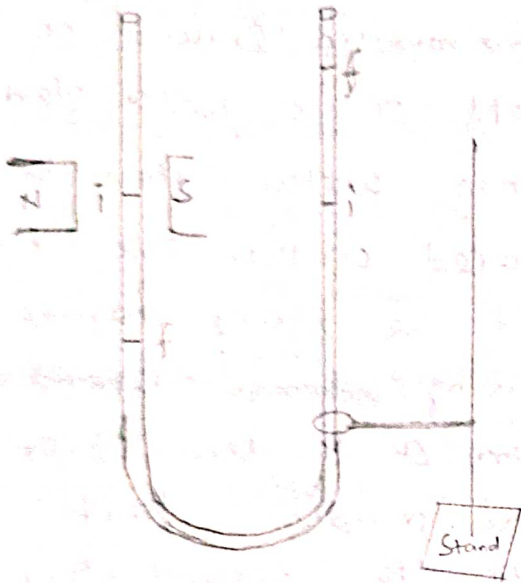
(2) A small amount of paramagnetic liquid be kept on a watch glass. This watch glass be placed on the pole pieces of a strong magnet having ~~small~~ <sup>10cm</sup> separation, then it is seen that the middle part of the liquid gets ~~depressed~~ <sup>(depressed)</sup> where as liquid on the sides get ~~depressed~~ <sup>raised</sup>.

Fig:

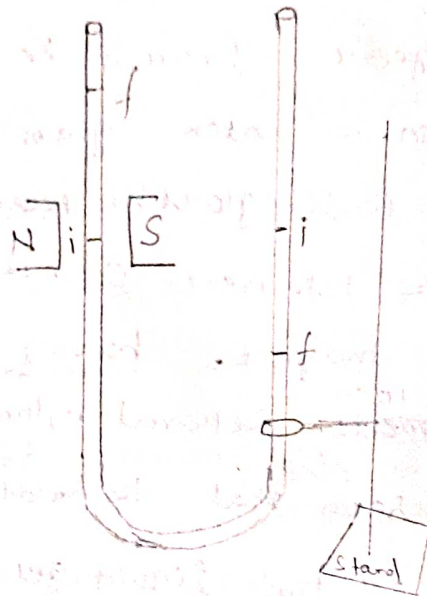


in the central region.  
 (3) When a para-magnetic liquid is taken in a U-tube with one of the arms of the U-t

Of the U-tube being kept in between the pole pieces of a strong magnet, it is found that the level of liquid goes down in that arm.



being kept in between the pole pieces of a strong magnet, it is found that the level of liquid goes up in that arm.



$i =$  Initial level of the liquid

$f =$  Final level of the liquid after magnetic field is applied.

3. Diamagnetism arises in those substances which have even number of electrons. As a result, the net spin magnetic moment will be zero. It arises due to the change in orbital frequency of

4. Paramagnetism arises in those substances which have atoms or molecules with odd number of electrons. Then the net spin will not be zero. When the external magnetic field



the electrons due to Lorentz force

$$\vec{F} = q(\vec{v} \times \vec{B})$$

⑤ This diamagnetism is independent of change of temp.

⑥ The examples of diamagnetic substances are Bismuth, antimony, gold, water, alcohol, quartz, hydrogen gas etc.

⑦  $\chi_m$  is negative but small.

$\chi_m$  is magnetic susceptibility.

## Ferromagnetism

Ferro-magnetic substances like iron, cobalt, and nickel are characterised by larger number of molecular magnets present inside it which are directed at random as shown in fig(i). This is called

is applied, these spin moments of different atoms or molecules get aligned along the field direction. This gives rise to paramagnetism.

⑤ The paramagnetism decreases with increase of temp. This has been theoretically established by Langevin.

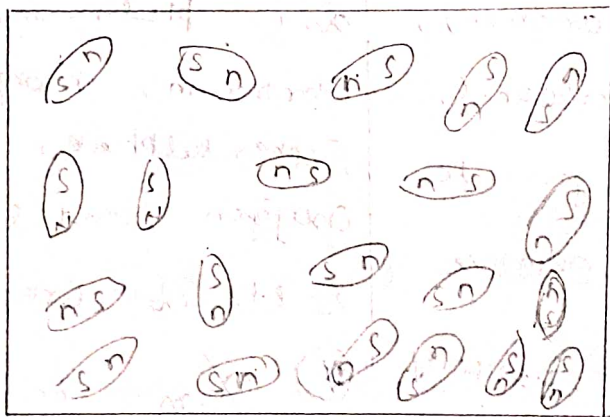
⑥ The examples of paramagnetic substances are platinum, aluminium, chromium, manganese, copper sulphate, liquid oxygen and  $\text{Lu}^{\text{III}}$  salts of iron and nickel

⑦  $\chi_m$  is +ve but small.



"Weiss molecular theory". When a strong magnetic field is applied, each dipole experiences a couple as shown in fig (i). Ultimately, all the molecular magnets will be aligned along the field direction. This is shown in fig (ii). Due to this alignment of molecular dipoles, one side exhibits North polarity and the other side shows South polarity i.e. a magnet is formed.

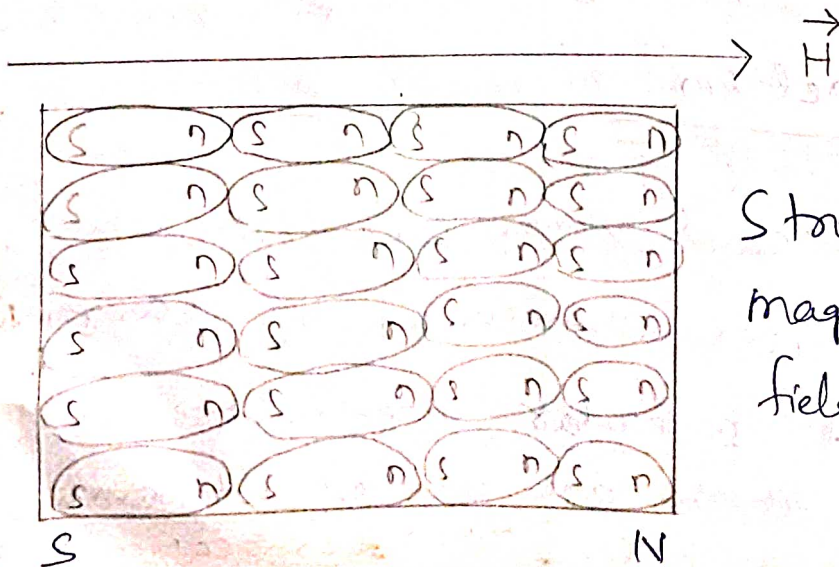
Fig = 1



No magnetic field applied.

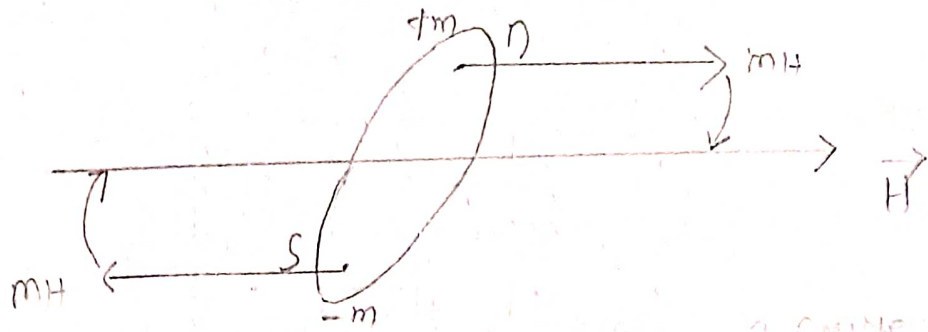
$$\vec{H} = 0$$

Fig = ii



Strong magnetic field is applied.

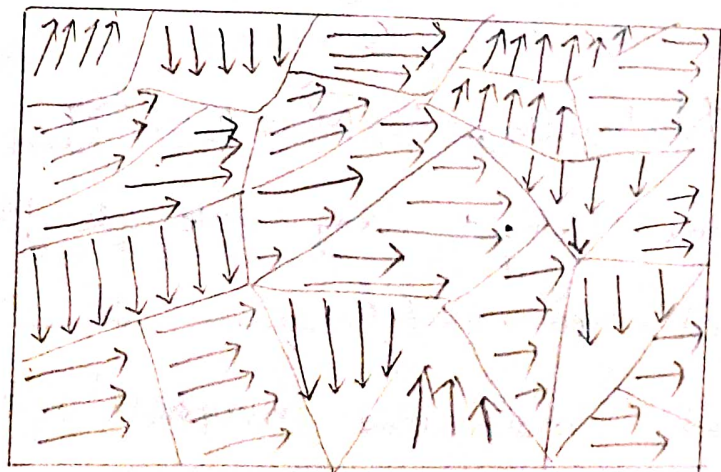
Fig-111



Magnetic dipole experiencing a couple

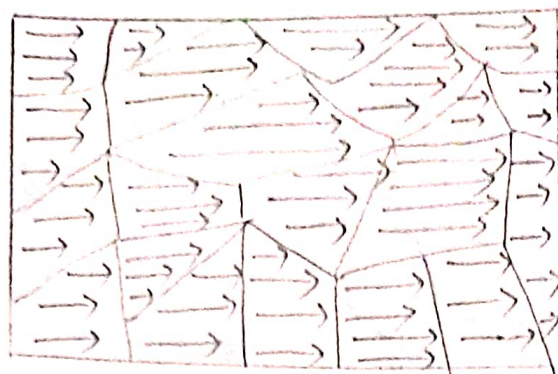
Modern theory of ferromagnetism says that there are domains present inside the ferromagnetic materials. All the molecular magnets present in one domain behave identically, i.e. all of them point in one direction. Due to the application of a strong field, these domains as a whole get oriented and magnet is formed as shown in Fig 4(b).

Fig-112



No field magnetic field.

Fig. 4(b)



$\rightarrow H$   
(Strong magnetic field)

## Magnetic ~~Field~~ Moment ( $\vec{M}$ )

It is defined as the vector sum of all the molecular magnetic moments present in a specimen.

$$\vec{M} = \sum_{i=1}^N m_i \vec{l}_i, \quad m_i = \text{pole strength of a molecular magnet.}$$

where  $\vec{l}_i$  = Vector directed from the south pole to the north pole of molecular magnet.

Naturally,  $\vec{M} = 0$  for a ferro-magnetic material without any external magnetic field applied to it. (Because of random direction of the molecular magnets). With increase of external field, more and more molecular dipoles will get aligned along



The field direction. Then  $\vec{M}$  will gradually increase.

Due to ~~to~~ very strong magnetic field all the molecular dipoles will get aligned along the field direction and  $\vec{M}$  will be  $\text{max}^m$ . This is called saturation stage. Any further increase of magnetic field is quite useless.

### Intensity of magnetisation (I)

It is defined as the magnetic moment per unit volume.

$$\therefore I = \frac{M}{V}$$

"I" can be regarded as a quantity by which the extent of magnetisation of a ferromagnetic material due to varying magnetic field can be known.

### Magnetic susceptibility ( $\chi_m$ )

It is defined as the intensity of magnetisation developed in a specimen by the application of a magnetic field.

Of strength unity.

$$\chi_m = \frac{I}{H}$$

For a diamagnetic material  $\chi_m$  is -ve but small. For a paramagnetic material it is +ve but small and for a ferromagnetic material it is +ve and large ( $\approx 2000$ ).

Magnetic Permeability

From Coulomb's law on magnetism we know that force between two magnetic poles kept at a separation  $d$  in vacuum or air is given by the expression.

$$F_1 = \frac{1}{\mu_0} \frac{m_1 m_2}{d^2} \quad \text{--- (i)}$$

If the same two poles be kept the same distance apart in a medium with permeability  $\mu$ , then the force becomes

$$F_2 = \frac{1}{\mu} \frac{m_1 m_2}{d^2} \quad \text{--- (ii)}$$

Dividing eq<sup>n</sup> (i) by eq<sup>n</sup> (ii), we get

$$\frac{F_1}{F_2} = \frac{M}{M_0} = \mu_r = \text{Relative Permeability}$$

Thus, relative permeability of a medium can be defined as the ratio of the force between two magnetic poles placed some distance apart in air to the force between the same two poles placed the same distance apart in the medium concerned.

### Cycle of Magnetisation, Hysteresis

When a ferro-magnetic material is subjected to a magnetic field that gradually increases from zero to certain max<sup>m</sup> value, then the intensity of magnetisation (I) is also found to increase. This refers to a part of the curve ~~at~~ in fig (6). For a certain magnetic field H<sub>max</sub>, I is max<sup>m</sup> and this refers to a saturation stage. Ewing, decreased the magnetic field gradually and found that OB amount



Of  $I$  is left in the specimen even after the field is removed. Thus AB part of the curve is described.

Fig-5

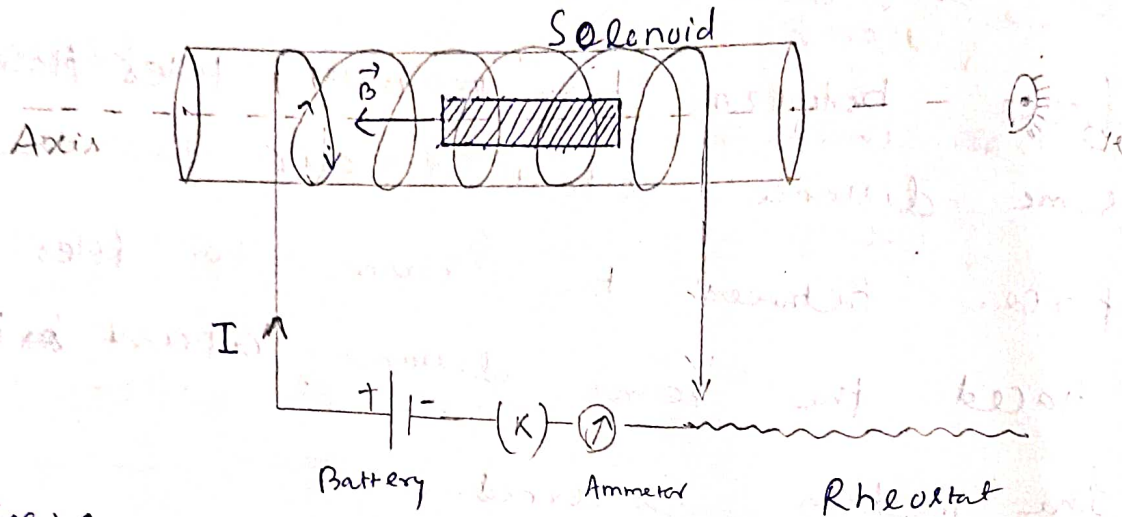
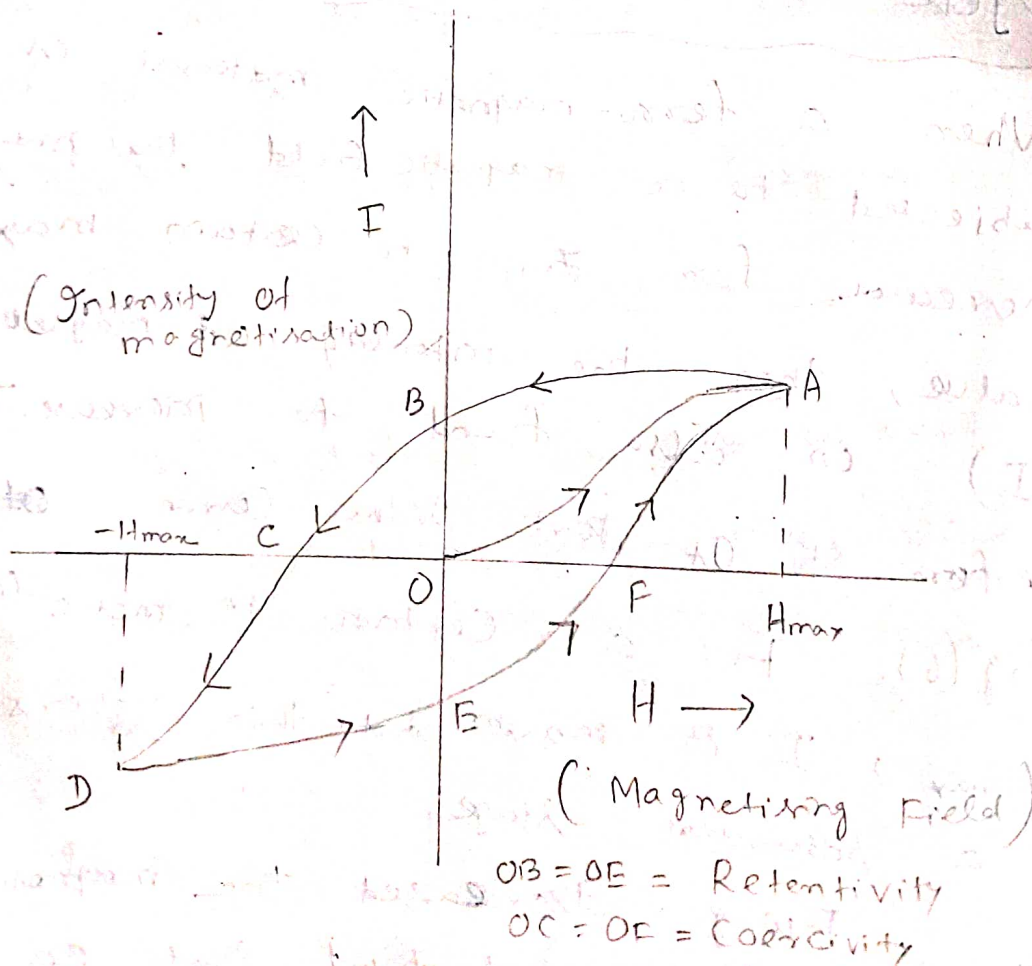


Fig-6



To destroy this residual magnetism left in the specimen, reverse magnetic field has to be applied. For OC amount of reverse magnetic field called Coercive force, the magnetism completely vanishes. This BC part of the curve is described.

This field be gradually decreased and DE part of curve is described. OE amount of retentivity is left in the specimen even after the reverse magnetic field be decreased to zero. To destroy the retentivity, the magnetic field be gradually increased in the original direction. Then EF part is described.

Now the specimen is again demagnetised. If the original field be increased further then FA part of the curve is described. This completes the cycle.

All these processes together are called cycle of magnetisation and the loop ABCDEFA is called Hysteresis loop.

The area of the loop in C.G.S

units when divided by  $4.2 \times 10^7$  gives  
the amount of heat produced in  
Calory per cycle.

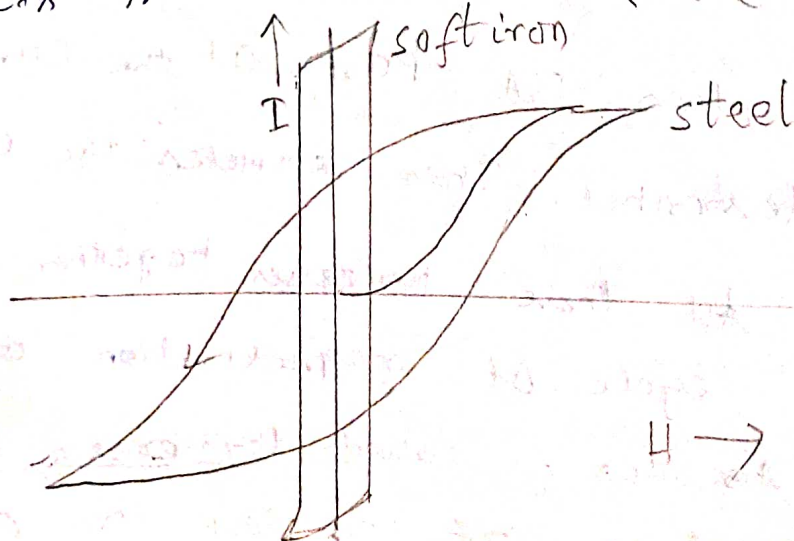
## Definition of hysteresis

The lagging of the intensity of  
magnetisation behind the applied magnetic  
field is called hysteresis.

## Applications

Study of the hysteresis loop of  
different materials helps us to select  
proper materials for specific purposes.

e.g → To make permanent magnets  
~~steel~~ steel is used due to its high  
coercivity where as for the core of  
transformers soft iron is chosen due  
to its small loop area.





Curie's law states that the magnetic susceptibility is inversely proportional to the absolute temp.

$$\chi_m \propto \frac{1}{T}$$

$$\Rightarrow \chi_m = \frac{C}{T}$$

where  $C$  is a constant called Curie's constant.

### Curie-Weiss' law

When a ferromagnetic material is heated, it is found that at a particular temperature (characteristic of material), the ferromagnetic material gets converted into a paramagnetic material. This temp is called Critical temperature. ( $T_c$ ).

Ex:  $T_c$  for Fe =  $770^\circ\text{C}$

$T_c$  for Cobalt =  $1150^\circ\text{C}$

$T_c$  " Ni =  $360^\circ\text{C}$

$T_c$  " Ni-Fe alloy =  $70^\circ\text{C}$

$T_c$  is also called Curie point or

Curie temperature.

Curie-Weiss law can be stated by the formula  $\chi_m = \frac{C}{T - T_c}$

For  $T > T_c$ , this formula is correct.

## Relation between Magnetic susceptibility and magnetic permeability

The relation between magnetic induction (B) and magnetic field intensity (H) in C.G.S units has been established as

$$B = H + 4\pi I$$

where I = Intensity of magnetization.

Dividing by 'H' through out, we get

$$\frac{B}{H} = 1 + 4\pi \frac{I}{H}$$

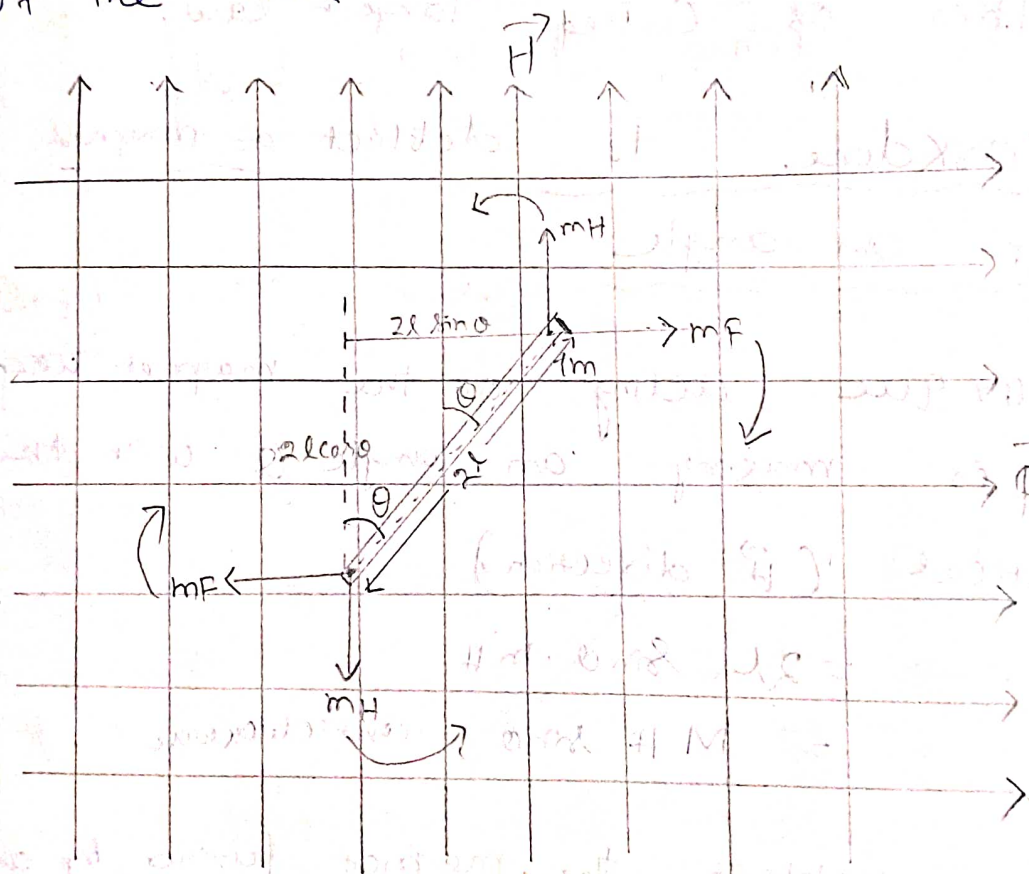
$$\Rightarrow \boxed{\mu_r = 1 + 4\pi \chi_m} \quad \text{where}$$

where  $\mu_r$  = Relative permeability

$\chi_m$  = Magnetic susceptibility

## Tangent law

When a suspended magnet is subjected to crossed magnetic field, then this law holds good. The two magnetic fields must be at right angles to one another and the plane of the magnet should be in the same plane as that of the magnetic fields.



The torque produced by the two couples must balance one another.

Torque due to the deflecting couple  
(Clockwise)

= Torque due to the anticlockwise couple  
(Called restoring couple by the earth's field  $\vec{H}$ )



$$\therefore 2l \cos \alpha \cdot mF = 2l \sin \alpha \cdot mH$$

$$\Rightarrow F = H \tan \alpha$$

If  $H$  be the earth's field which is a constant, at a particular place, then

$$F \propto \tan \alpha$$

This is called Tangent law.

Work done to deflect a magnet by an angle

Torque acting on the magnet when it is making an angle  $\alpha$  with the vertical ( $H$  direction)

$$= 2l \sin \alpha \cdot mH$$

$$= MH \sin \alpha, \text{ anticlockwise}$$

To deflect the magnet further by an angle  $d\alpha$ , the amount of work done is given by

$$dW = \tau d\alpha = MH \sin \alpha \cdot d\alpha$$

Integrating both the sides with proper limits, we get

$$\int_0^{\theta_0} dW = \int_0^{\theta_0} M l \sin \alpha \cdot d\alpha$$

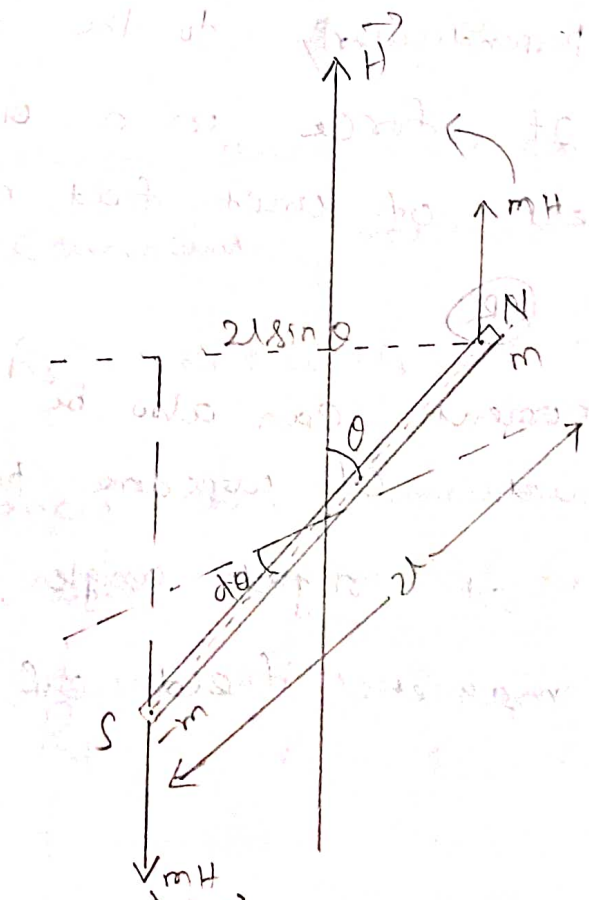
$$\Rightarrow W|_0^{\theta_0} = M l \int_0^{\theta_0} \sin \alpha \cdot d\alpha$$

$$\Rightarrow W - 0 = M l \left( -\cos \alpha \right) \Big|_0^{\theta_0}$$

$$\Rightarrow W = M l \left[ -\cos \theta_0 - (-\cos 0^\circ) \right]$$

$$\therefore W = M l (1 - \cos \theta_0)$$

Fig:



Some special cases

①  $\theta_0 = 90^\circ$

$$W = M l (1 - 0) = M l$$

(2)

$$\theta_0 = 180^\circ$$

$$W = MH (1 - \cos 180^\circ) = MH (1 - (-1)) \\ = 2MH$$

Another def<sup>n</sup> of magnetic moment  
can be obtained from this theory

Def<sup>n</sup> of 'M'

The magnetic moment of a magnet  
can be defined as the moment of  
couple or torque required to hold  
the magnet perpendicular to the direction  
of lines of force in a uniform  
magnetic field of unit field intensity

(OR)

Magnetic moment can also be defined  
as the amount of work done to keep  
the magnet at right angles to  
a uniform magnetic field of strength  
unity.