# Unit-I, Paper-II Electrostatics

By means of friction, charges (+ve and -ve) can be separated.

Ex:- Silk (-ve) and glass rod (+ve), Ebonite rod (-ve) Cat's skin (+ve)

The nature of charge on a body can be tested by means of an instrument called Gold-leaf electroscope. A body can be charged by the process of.

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(i) Conduction
(ii) Induction.

## Charging a body by the process of induction

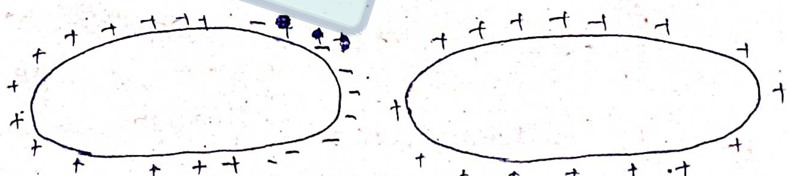
**Case I**

If we want to charge a body -vely, the following processes are to be adopted.

Fig-I

neutral body

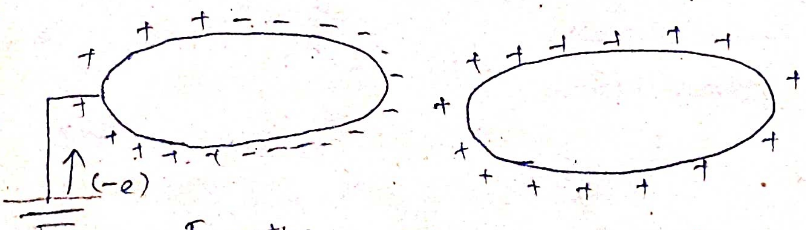
Fig-II



neutral ~~neutral~~ body

Positively Charged body.

Fig-III

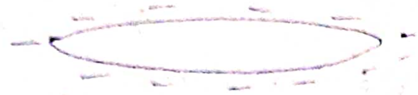



Earthed

Fig-4

Earth connection
cut-off

Fig 5



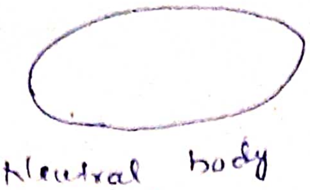
A body already charged +vely is brought near the neutral body. Due to its influence, there will be separation of charges in the neutral body as shown in fig(ii).

The ~~the~~ free end / extreme end of the neutral body is connected to the earth so that electrons will flow from the earth towards the neutral body and cancels the +ve charges present at that end. This has been shown in fig (iv).

The earth connection be cut off and the +vely charged body be withdrawn. The -ve charges present on the neutral body will be distributed through out and the neutral body now becomes -vely charged.

**Case II** If we want to charge a body +vely, the following ~~the~~ processes are to be adopted.

Fig (a)




(Fig-b)

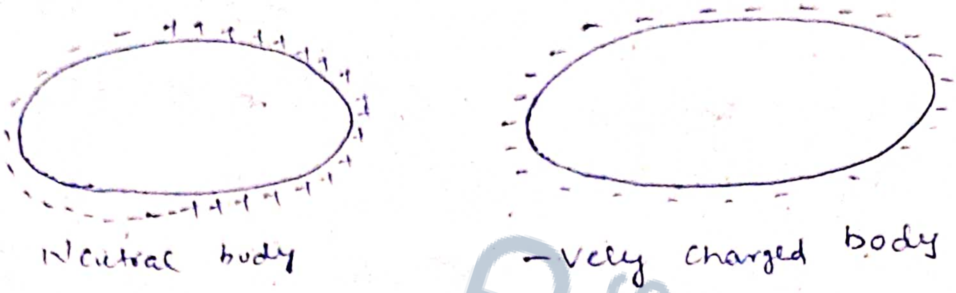
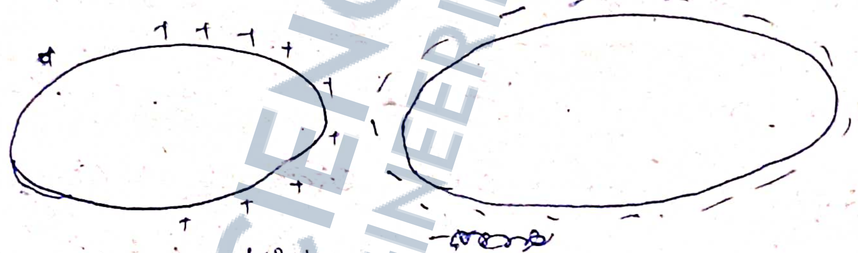



Fig-c


Fig-d



Earth connection is cut off

(Fig-e)

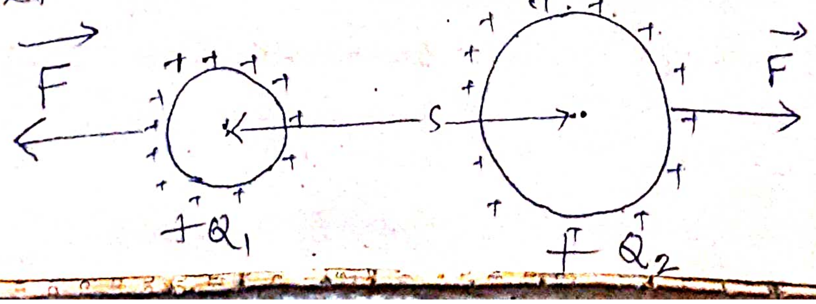
A body already charged -vely is brought near the neutral body. Due to its influence, there will be separation of charges in the neutral body as shown in fig (b)

The free end of the neutral body is connected to the earth so that all the electrons will flow towards earth from the neutral body. This has been shown in fig (d)

The earth connection be cut off and the ~~-vely +vely charge~~ -vely charged body be withdrawn. Then the +vely charges present in the body will be distributed through out and now the neutral body becomes +vely charged.

## Coulomb's law in scalar form.

Experimentally it was found that like charges repel and ~~oppo~~ unlike charges attract. The magnitude of the force between two point charges is directly proportional to the product of the charges when distance between them is kept constant and inversely proportional to the square of the distance between them when magnitude of the charges are kept constants.



$F \propto Q_1 Q_2$, when $S$ is kept constant

$F \propto \frac{1}{S^2}$, when $Q_1, Q_2$ are kept constants.

Combining these two variations, we get

$$F \propto \frac{Q_1 Q_2}{S^2}$$, when all the quantities vary.

$$\Rightarrow \boxed{F = K \cdot \frac{Q_1 \cdot Q_2}{S^2}}$$

where K is a constant whose value depends on the choice of the system of units used to express the charges and other quantities.

In C.G.S System of units,

the charge is expressed in ~~st~~ Stat coulomb or e.s.u (electro static unit) so that K becomes equal to unity.

1 Stat coulomb is defined as that amount of charge which when kept at a distance of 1 cm away from an equal and similar charge is repelled with a force of 1 Dyne.

Using Columbs law, we get

$$1\ \text{dyne} = K \cdot \frac{1\ \text{Stat C} \cdot 1\ \text{Stat C}}{(1\ \text{cm})^2}$$

$$\Rightarrow K = \frac{1\ \text{dyne} \cdot \text{cm}^2}{(\text{Stat C})^2}$$

In M.K.S system of units, the unit of change is ~~Colum~~ Coulomb, which is defined from the defn of current.

$$I = \frac{Q}{t}$$

i.e $1\ \text{Ampere} = \frac{1\ \text{Columb}}{1\ \text{sec}}$.

One coulamb is that amount of charge which when ~~when~~ flows through a conductor for 1 sec produces a current of 1 ampere in it. ~~when~~

when two similar bodies, each having a charge of 1 Culamb are kept at a separation of 1 metre in air repel each other with a force of $9 \times 10^9$ Newton.

Using Culambs law, we have

$$9 \times 10^9\ \text{Newton} = K \cdot \frac{1\ \cancel{\text{Colum}}\ \text{coul} \cdot 1\ \text{coul}}{(1\ \text{metre})^2}$$

$$\Rightarrow K = \frac{9 \times 10^9\ \text{Newton} \cdot (\text{metre})^2}{(\text{Coulomb})^2}$$

For future simplicity, K is written as $\frac{1}{4\pi\epsilon_0}$ where $\epsilon_0$ is called permittivity of air or Vacuum.

$$\therefore \frac{9\times10^9 \text{ Newton}\cdot\text{metre}^2}{(\text{Columb})^2} = \frac{1}{4\pi\epsilon_0}$$

$$\Rightarrow \epsilon_0 = \frac{1}{4\pi \; 9\times10^9 \text{ N.metre}^2/\text{Columb}^2}$$

$$= 8.85\times10^{-12} \frac{\text{Columb}^2}{\text{Newton metre}^2}$$

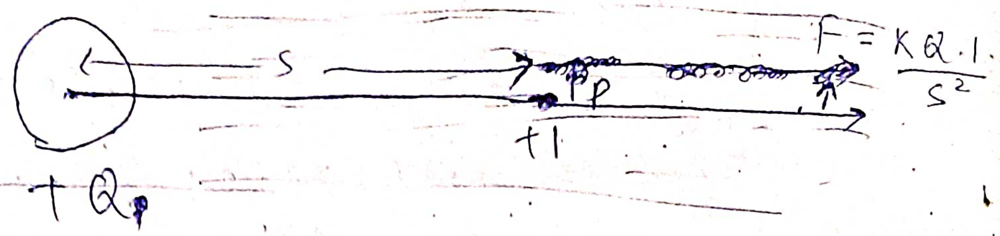
## Electric field

The space surrounding a charge where its influence is felt is called its electric field.

## Electric field intensity

The strength of the electric field at a point is expressed by a vector quantity called electric field intensity.

Electric field intensity at a point in an electric field is defined as the force experienced by a unit +ve charge placed at that point




Thus $\vec{E} = \vec{F} = \frac{KQ}{s^2}$, directed away from the +Q charge

If +Q charge is comparable to the unit +ve charge, then we have to use a much weaker test charge at the point P. If the test charge be +q, then the force experienced by it will be

$$F = K\frac{Q\,q}{s^2}$$

$$\Rightarrow \frac{F}{q} = \frac{KQ}{s^2} = E$$

$$\therefore \boxed{\vec{E} = \frac{\vec{F}}{q}} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

My hints of derivation of $F = qE$

According definition $E \cdot 1$ = Force

For unit +ve charge $F = E \cdot 1$

For q +ve " $F = qE$

Correct

<u>Units of $\vec{E}$</u> C.g.S system,

$$E = \frac{\text{Dyne}}{\text{Stat C}} \text{ or } \frac{\text{Dyne}}{\text{e.s.u}}$$

In M.K.S System,

$$E = \frac{\text{Newton}}{\text{Coulomb}}$$

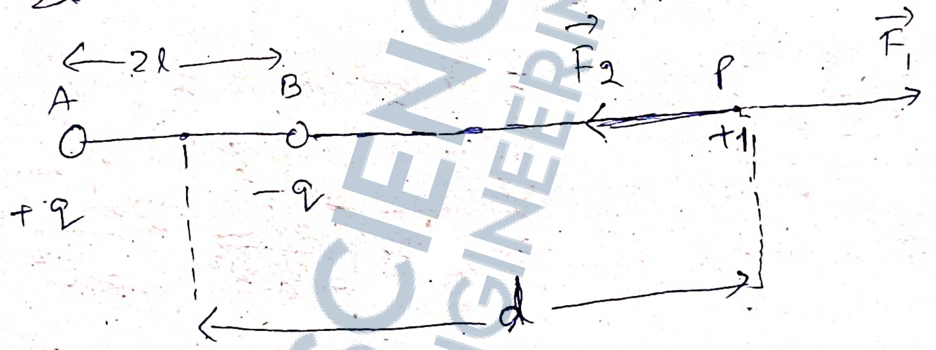
## Relation between the two units

Since 1 coulomb = $3\times10^9$ Stat C, we have

$$\frac{1\ \text{Newton}}{1\ \text{Coulomb}} = \frac{10^5\ \text{Dyne}}{3\times10^9\ \text{Stat C}} = \frac{1}{3\times10^4}\ \frac{\text{Dyne}}{\text{Stat C}}$$

## Electric field intensity at the end-on of an electric dipole or Axial position

An electric dipole consists of two charges, equal in magnitude, but opposite in sign. In our diagram AB represents the electric dipole whose charges are at a distance of $2l$ units.



P is a point situated on the extended part of the axis of the dipole. Such a position is called end-on position and at a distance d from the midpoint of electric dipole. The force of repulsion experienced by +1 charge present at P due to the charge +q at A be $\vec{F_1}$.

$$\therefore \vec{F_1} = \frac{Kq.1}{(AP)^2} = \frac{Kq}{(d+l)^2},\ \text{along}\ \vec{AP}$$

The force of attraction experienced by +1 Change present at P due to the Charge $-q$ at B be $\vec{F_2}$.

$$\therefore \vec{F_2} = \frac{Kq}{(BP)^2} = \frac{Kq}{(d-l)^2}, \text{ along } \vec{PB}$$

Net force experienced by the +1 Charge at P due to the Charges of the electric dipole. $= F_2 - F_1$, directed along $\vec{PB}$

$$\therefore F = \cancel{q} E = F_2 - F_1$$

$$= \frac{Kq}{(d-l)^2} - \frac{Kq}{(d+l)^2}$$

$$= Kq\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$

$$= Kq\left[\frac{(d+l)^2 - (d-l)^2}{(d-l)^2 (d+l)^2}\right]$$

$$= Kq\left[\frac{4d \cdot l}{(d-l)^2 \cdot (d+l)^2}\right]$$

<u>Defining</u> electric dipole moment as the product of ~~the~~ one of the charges of the dipole and the vector joining the $-ve$ Charge towards the $+ve$ Charge.

i.e $\vec{P} = q \cdot \vec{BA} = q \cdot 2l \cdot \hat{BA}$

Thus $p = 2ql$

$$\boxed{E = \frac{2kqPd}{(d^2-l^2)^2}}$$

Since $l << d$, we can neglect $l^2$ compared to $d^2$ and the above expression becomes

$$\boxed{E = \frac{2kP}{d^3}}$$



25.05.2K1

## Problems

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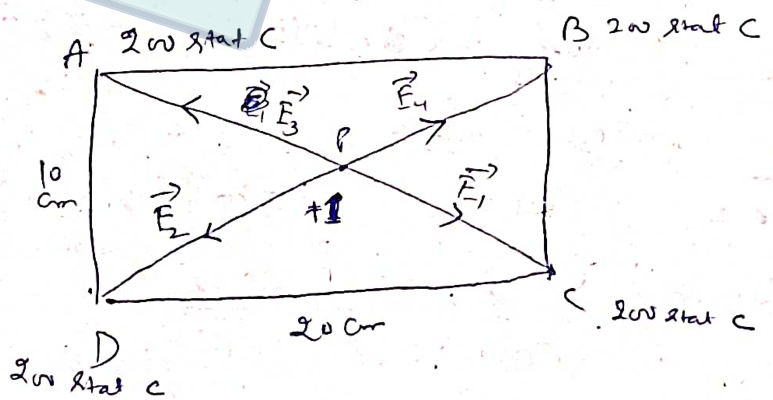
24. A, rectangle, 10 cm high and 20 cm wide, has +200 stat C charges placed on each ~~other~~ corner.




42000
6400
35600
$3.56 \times 10^4$

$BD = \sqrt{10^2+20^2} = \sqrt{100+400} = \sqrt{500} = 10\sqrt{5}$

Electric field intensity at P due to Charg A

$$E_1 = \frac{KQ}{AP^2} = \frac{1.200 \text{ stat } C}{(5\sqrt{5})^2 \text{ cm}^2} = \frac{\cancel{200}\ 40\cancel{8}}{\cancel{125}\ \cancel{25}\ 5} = \frac{8}{5} \text{ Dyne/statC}$$

Electric field intensity at P due to B

$$E_2 = \frac{KQ}{(BP)^2} = \frac{1 \times 200 \text{ stat } C}{(5\sqrt{5})\text{ cm}} = \frac{8}{5} \text{ Dyne/stat C}$$

Electric field intensity at P due to C

$$E_3 = \frac{KQ}{(CP)^2} = \frac{1 \times 200}{(5\sqrt{5})^2} = \frac{8}{5} \text{ dyne/stat C}$$

Electric field intensity at P due to D

$$E_4 = \frac{KQ}{(DP)^2} = \frac{1 \times 200}{(5\sqrt{5})^2} = \frac{8}{5} \text{ dyne/stat C}$$

Since $E_1$ and $E_3$ are equal and opposite they cancels each other.

Since $E_2$ and $E_4$ are equa and opposite they cancels each other.

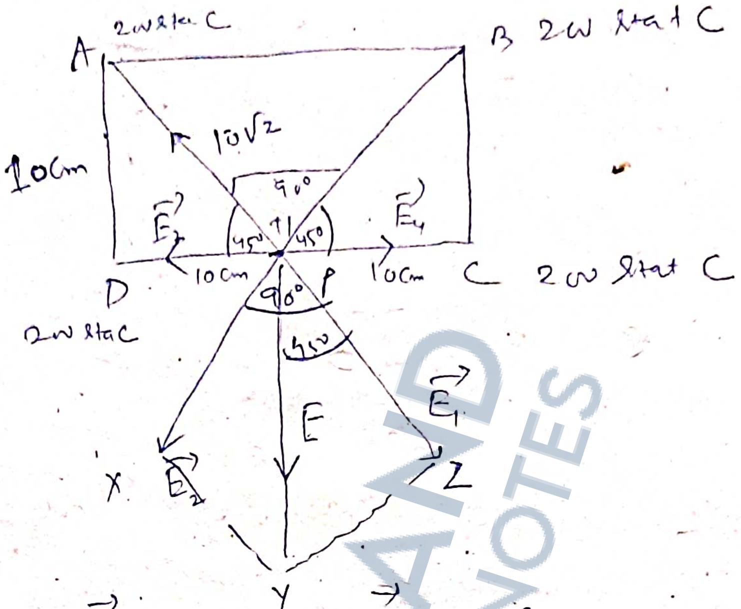
In second case

Electric field intensity due to

$$C = \frac{K \cdot Q_1}{(CP)^2} = \frac{1 \cdot 200}{(10)^2} = 2 \text{ dyne/stat C}$$

Electric field intensity due to D

$$= \frac{KQ}{(DB)^2} = \frac{1 \times 200}{10^2} = 2 \text{ dyne/stat C}$$




Since $\vec{E}_3$ and $\vec{E}_4$ are equal and opposite they cancel each other out.

~~The resultan~~

By parallelogram law, the resultant of $E_1$ and $E_2$ is $E$.

Electric field intensity due to A

$$= \frac{1.2\omega}{(10\sqrt{2})^2} = \frac{2\omega}{2\omega} = 1 \text{ dyne/stat c}$$

Electric field intensity due to B

$$= \frac{1 \times 2\omega}{(10\sqrt{2})^2} = \frac{2\omega}{2\omega} = 1 \text{ dyne / stat c}$$

The resultant is $\sqrt{1^2+1^2} = \sqrt{2} = 1.414$ dyne/statc.

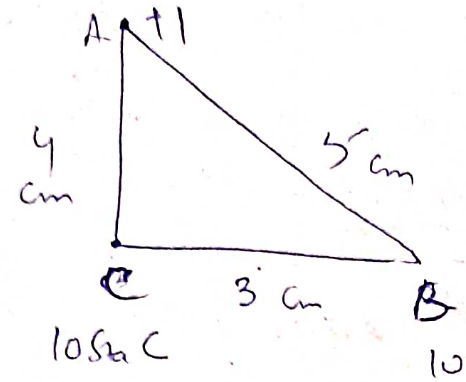
Since the diagonal bisect the $\angle 90°$,

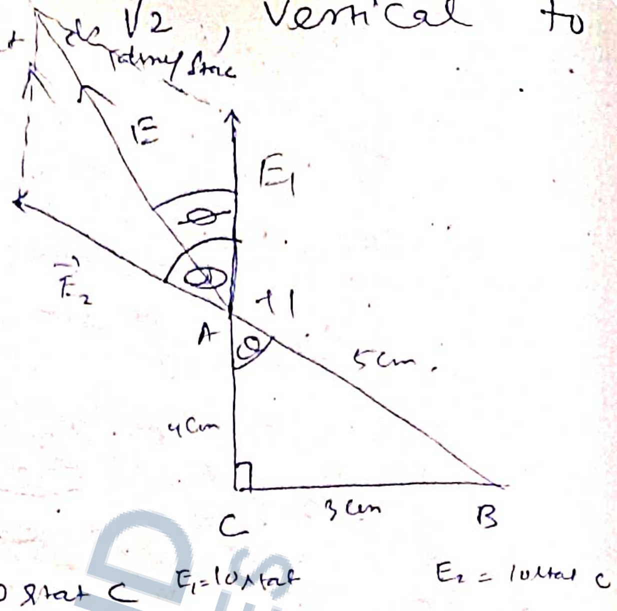
$\angle YPZ = 45°$, $\angle CPZ = 45°$

$\therefore \angle CPY = 90°$

∴ The resultant is $\sqrt{2}$, vertical to CD.

22.




Electric field intensity due to

$$C, E_1 = 1 \cdot \frac{10 \text{ Stat C}}{(4\text{ cm})^2} = \frac{10}{16} \frac{\text{Stat C}}{\text{cm}^2} = \frac{5}{8} \frac{\text{Stat C}}{\text{cm}^2}$$

Electric field intensity due to

$$D, E_2 = 1 \times \frac{10 \text{ Stat C}}{(5\text{ cm})^2} = \frac{10}{25} = \frac{2}{5} \frac{\text{Stat C}}{\text{cm}^2}$$

$$\cos\theta = \frac{4}{5}, \quad \sin\theta = \frac{3}{5}$$

The resultant of $E_1$ and $E_2$

$$\text{is } E = \sqrt{E_1^2 + E_2^2 + 2 E_1 E_2 \cos\theta}$$

$$= \sqrt{\frac{25}{64} + \frac{4}{25} + 2 \cdot \frac{5}{8} \cdot \frac{2}{5} \cdot \frac{4}{5}}$$

$$= \sqrt{\frac{625 + 256 + 64 \cdot 5 \cdot 2}{64 \cdot 25}}$$

$$= \sqrt{\frac{2494\ 840}{1600}} = \sqrt{\frac{1521}{40}}$$

$= \frac{39}{40}$ dyne / stat C.

$$\tan\phi = \cancel{\frac{E_1 \cos\theta}{E_1 + E_2 \sin\theta} = \frac{\frac{5}{8}\cdot\frac{4}{5}\cdot\frac{1}{2}}{\frac{5}{8}+\frac{4}{5}\cdot\frac{3}{5}}}$$

$$= \frac{E_2 \sin\theta}{E_1 + E_2 \cos\theta}$$

$$= \frac{\frac{2}{3}\cdot\frac{3}{5}}{\frac{5}{8}+\frac{2}{3}\cdot\frac{4}{5}} = \frac{\frac{6}{\cancel{25}}}{\frac{125+64}{\cancel{200}\ 8}} = \frac{48}{189}$$

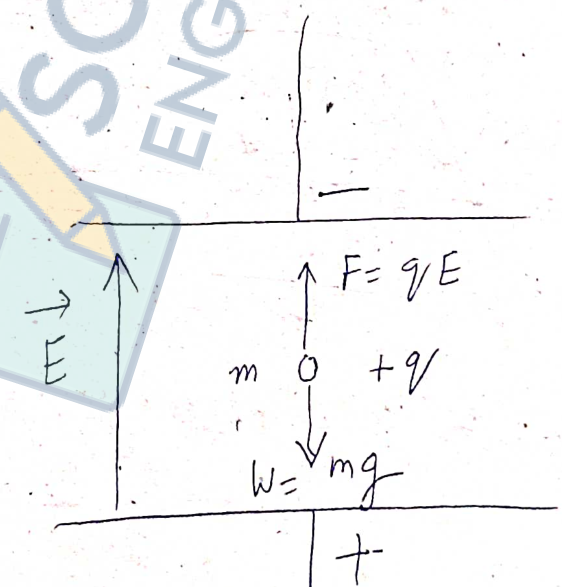
$$\Rightarrow \phi = \tan^{-1}\left(\frac{48}{189}\right) = \tan^{-1}(.253)$$

$= 14.25^\circ$ with the line AC

Q.19.

For equilibrium of the water drop

Upward force = downward force.



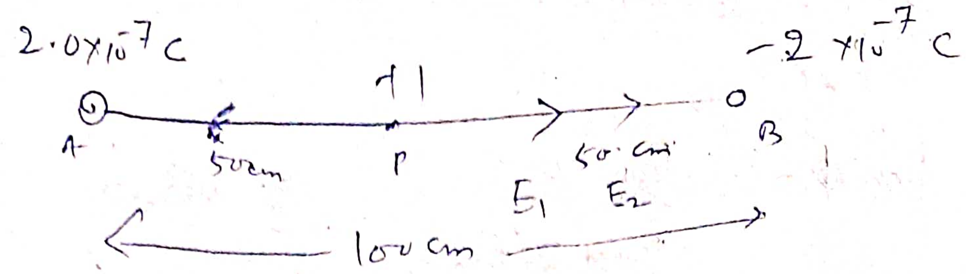

$\Rightarrow qE = mg$

$\Rightarrow 1\times10^{-7}\,\mu C \times E = 10\,\mu gram \times 9.8\ m/sec^2$

$\Rightarrow \frac{10^{-7}}{10^{6}} C \times E = \frac{10\times10^{-6}\,Kg}{10^{3}} \times 9.8$

$\Rightarrow 10^{-13} C \times E = 10^{-8} \times 9.8$ $\Rightarrow E = \frac{9.8\times10^{-8}}{10^{-13}} = 9.8\times10^{5}$ N/coloumb

14.




B is -vely charge it will attract the +1 charge

Electric field Intensity due to B = $\frac{2\times10^{-7}\,C \cdot 9\times10^{9}}{(.5)^2}$

$= \frac{2\times10^{-7}\times\cancel{100}}{.25}\times9\times10^{9}$

$E_2 = 9\times10^{9}\,\frac{2}{25}\times10^{-5}$ ~~ohms~~ Newton/ Stat c

Electric field intensity due to A

$E_1 = \frac{2\times10^{-7}\times1}{(.50)^2} = 9\times10^{9}\,\frac{2}{25}\times10^{-5}$ Newton/ Stat c

~~Resultant~~ electric field intensity $= E_1 + E_2$

$=$

Since $E_1$ and $E_2$ are towards B then they will be added for resultant

$E_1 + E_2$

$= 2\times\left(9\times10^{9}\,\frac{2}{25}\times10^{-5}\right) = \frac{4}{25}\times10^{-5}$

$$= \frac{18 \times 2}{25} \times 10^{6}$$

$$= \frac{36}{25} \times 10^{6}$$

Again

$$E_1 = \frac{9\times10^{9} \times 2\times10^{-7}}{.25 \; m^2} = \frac{18 \times 10^{2} \times 10^{2}}{25}$$

$$= \frac{18}{25} \times 10^{4}$$

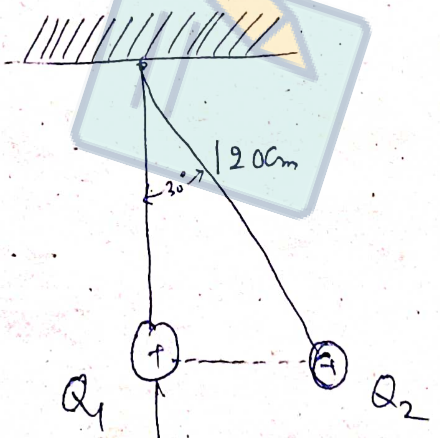
$$E_2 = \frac{18}{25} \times 10^{4}$$

$$\text{Resultant} = 2\left(\frac{18}{25} \times 10^{4}\right)$$

$$= \frac{36}{25} \times 10^{4}$$

$$= 1.44 \times 10^{4} \; N/C.$$

6.




When -ve 500 stat C charge sphere is touched with +ve 800 stat c the resultant charge is 300 statC. Since the spheres have same radius, so they have same surface area. So the 300 stat C charge is distributed between then two each having charge 150 stat C.

~~Force~~ between them $= \dfrac{K\, Q_1 \cdot Q_2}{r^2}$

$= 1 \cdot \dfrac{150 \times 150}{\cancel{\quad}\,(20)^2}$

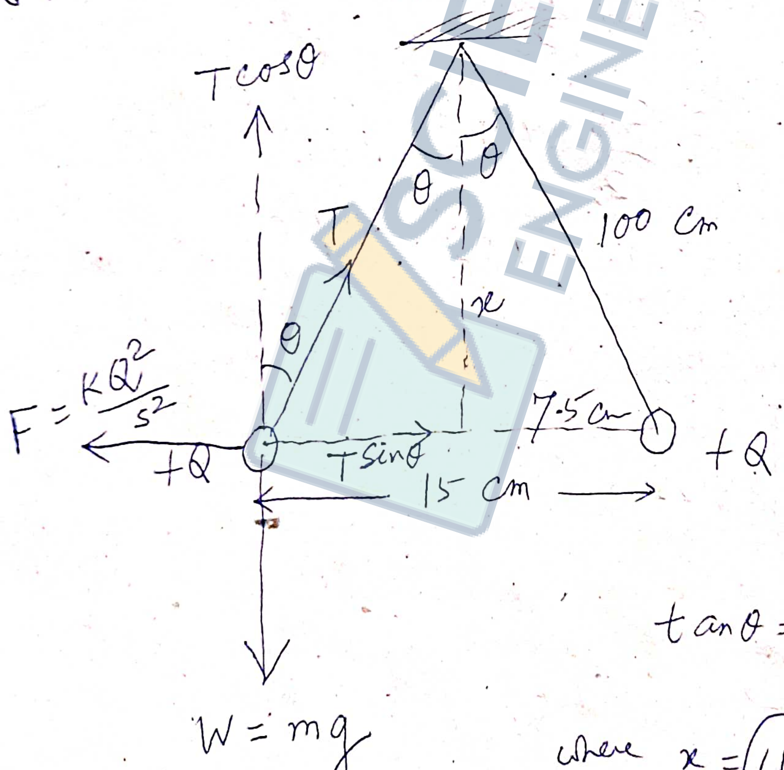
$= \dfrac{225\cancel{00}}{4\cancel{00}}$

~~= 1125 dyne / stat-c~~

$= 56.25$ dyne

$\vec{F} = q\vec{E}$

16.




$\tan\theta = \dfrac{7.5}{x}$ —— ①

where $x = \sqrt{(100)^2 - (7.5)^2}$

$T\sin\theta = F$

$T\cos\theta = mg$

dividing, $\tan\theta = F/mg$ —— ②

$$x = \sqrt{10000 - \cancel{\text{56.25}}\ 56.25}$$

$$= \sqrt{9943.75}$$

$$= 99.718 \text{ cm}.$$

$$\tan\theta = \frac{7.5}{99.718} = .0752$$

$$\Rightarrow \theta = \tan^{-1}(.0752)$$

$$= 4.301^\circ$$

$$F = W \cdot \tan\theta = (300 \text{ milligram})(980 \text{ cm/sec}^2) \times .0752$$

$$= 300 \times 10^{-3} \times 980 \times .0752$$

$$= .3 \times 980 \times .0752$$

$$= 22.1088 \text{ dyne}$$

$$F = \frac{KQ^2}{s^2}$$

$$\Rightarrow Q^2 = \frac{Fs^2}{K} = F \times (15)^2$$

$$= 22.1088 \times 225$$

$$= 4974.48$$

$$\Rightarrow Q = 70.52 \text{ Stat C} \quad \text{(Ans)}$$

No:- ~~6~~, ~~12~~, ~~8~~, ~~16~~, ~~14~~, ~~22~~, 29, ~~24~~, ~~9~~

Problem

1. Two Charges A of $2.00 \times 10^{-9}$ C and B of $-3 \times 10^{-9}$ C are 50 mm apart. Find the field intensity at a point C that is 30 mm from A and 40 mm from B.

2. Calculate the ratio of the electrostatic and gravitational forces between two electrons that are 1.0 m apart.

m air Ans $(4.1713 \cdot 2 \times 10^{38})$

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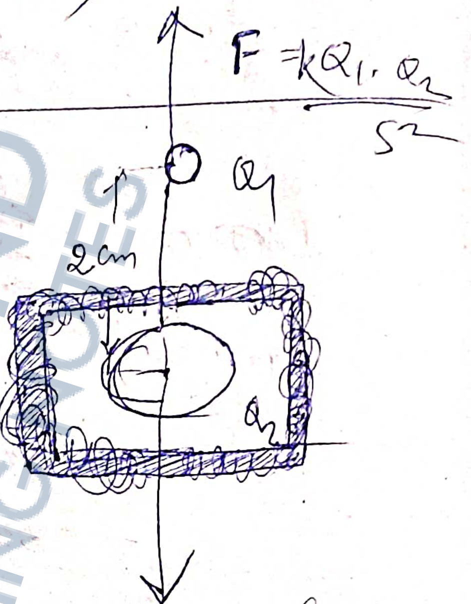
<u>584 page</u> 2.




From figure, for equilibrium

$$mg = F$$

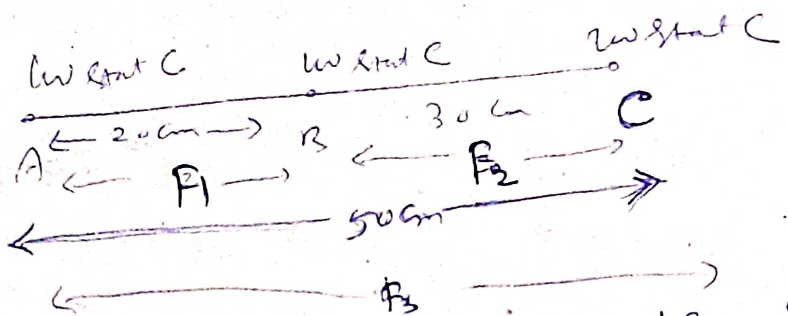
$$\Rightarrow .25 \times 980 = K. \frac{Q_1 \cdot Q_2}{S^2} \qquad mg = (.25 \times 980)$$

$$\Rightarrow 2.5 \times 98 = \frac{1 \times Q_1 \times 20}{4}$$

$$\Rightarrow Q_1 = \frac{\cancel{25} \times \cancel{4} \times \cancel{98}^{49}}{\cancel{20}_{2}} = 49 \text{ Stat C}$$

Since $Q_2$ is $+20$ Stat C so $Q_1$ must be $-$ve ie $-49$ stat C

8. Three 100 stat C charges are arranged in a straight line. B is 20 cm to the right of the first




Charge exerted by A and B on C

$= F_3 + F_2$

$= 1 \cdot \frac{100 \times 100}{(50)^2} + \frac{100 \times 100}{(30)^2}$

$= \frac{10000}{2500} + \frac{10000}{900}$

$= \frac{900 \times 10^4 + 25 \times 10^2 \times 10^4}{2500 \times 900}$

$= \frac{10^2 (9 + 25)}{(25 \times 9) \times 10^4} = \frac{3400}{225}$

$= 15.1$ dyne

The force exerted by A and C on B

$E_1 - E_2$

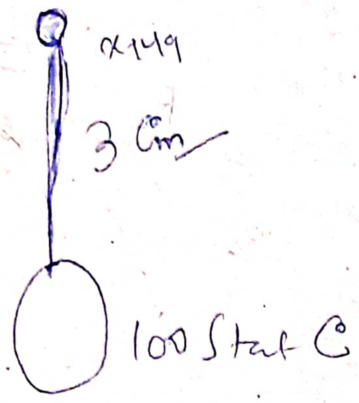
$= \frac{100 \times 100}{400} - \frac{100 \times 100}{(30)^2}$

$= 25 - \frac{100}{9}$

$= 25 - 11.11$

$= 13.89$ dyne

Q. 9.




$$F = \frac{Q_1 \cdot Q_2}{r^2}$$

$$\Rightarrow 49 = \frac{Q_1 \cdot 100}{9}$$

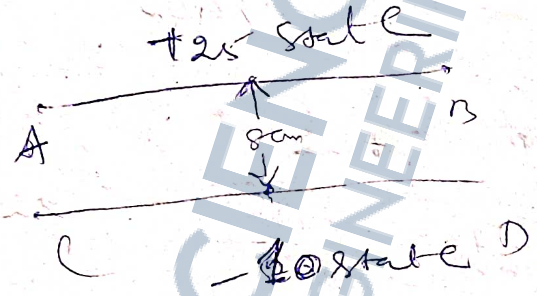
$$\Rightarrow Q = \frac{49 \times 9}{100}$$

$$= \frac{441}{100}$$

$$= 4.41$$

Charge on the bead is ~~ae~~ −4.41 Stat C

20.




Since two conductors ~~ar~~ have different charges they will attract

the net charge between them is 15 Stat C.

Since the conductors are similar it will distribute equally each having 7.5 Stat C

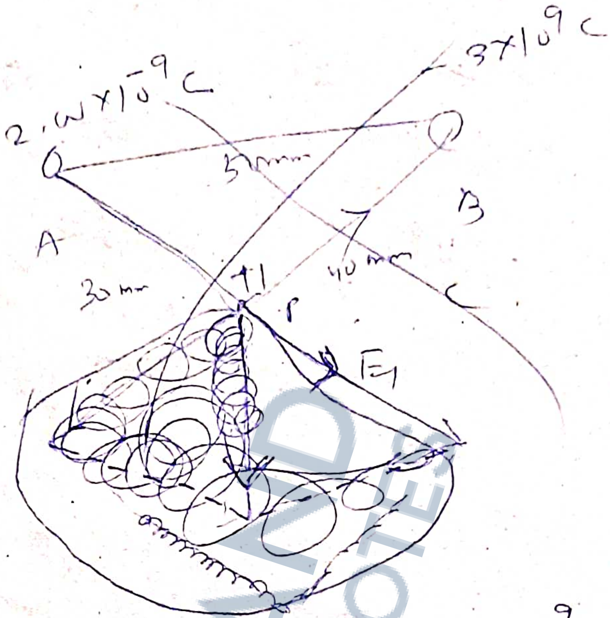
when they are 8 Cm apart

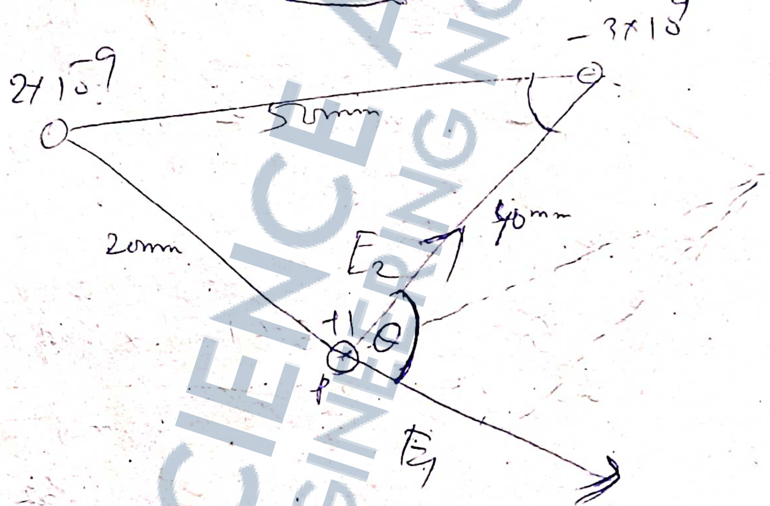
$$\text{net force} = \frac{7.5 \times 7.5}{64} = \frac{56.25}{64}$$

$$= .87 \text{ dyne.}$$

29.

Extra (1.2)




$$E_1 = \frac{1 \times \cancel{2\times 10^{-9}} \times 2\times10^{-9} \times 1}{(20)^2} = \frac{2\times10^{-9}}{2\cdot4\times100}$$

$$= .5 \times 10^{-11}$$

$$= 5 \times 10^{-12}$$

$$E_2 = 1 \times \frac{3\times10^{-9}\times 1}{(40)^2} = \frac{3\times10^{-9}}{16\times10^{2}} = \frac{3}{16}\times10^{-7}$$

Net force E

$$= \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

Q.

$1.6\times10^{-19}$ C          $1.6\times10^{-19}$ C

e      1m      e

$9.1\times10^{-31}$ kg          $9.1\times10^{-31}$ kg

Gravitational force between two electrons

is $F = \frac{GMM}{r^2} = \frac{6.67\times10^{-11}\times(9.1\times10^{-31})^2}{(1m)^2}$

$= 6.67\times(9.1)^2\times10^{-11}\times10^{-62}$

$= 552.3427\times10^{-73}$

electrostatic force between them

$\frac{KE_1E_2}{r^2} = F = \frac{9\times10^9\times(1.6\times10^{-19})^2}{(1m)^2}$

$F = 9\times2.56\times10^9\times10^{-38}$

$= 23.04\times10^{-29}$

$$\frac{E}{F} = \frac{23.04 \times 10^{-29}}{552.3427 \times 10^{-73}}$$

$$= \frac{2304}{552.3427} \times 10^{-31} \times 10^{73}$$

$$= 4.171323 \times 10^{42}$$

$$= 41713.23 \times 10^{38}$$

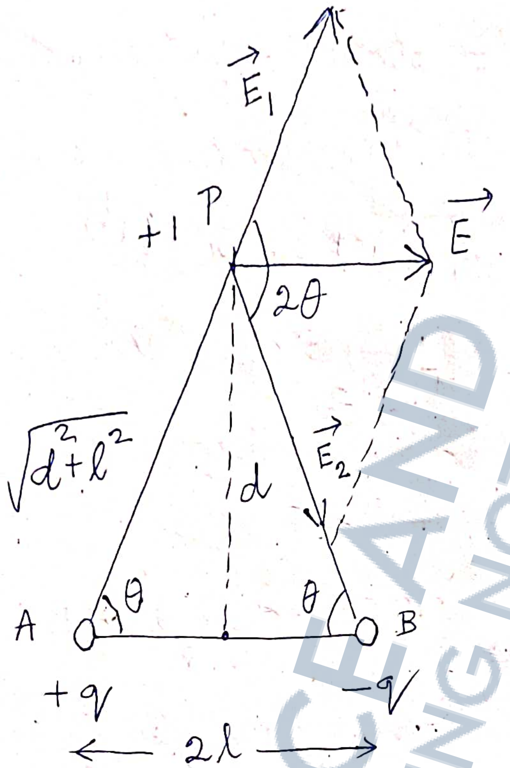
— o —

## Electric Field Intensity at the Broad-side-on position of an electric dipole

An electric dipole consists of two charges equal in magnitude, but opposite in sign. In our diagram AB represents the electric dipole whose charges are at a distance of $2l$ units.

P is a point situated on the $\perp$ bisector

Of the line ~~joing~~ joining the two charges of the electric dipole.




Such position is called broad-side-on or equitorial position.

The force of repulsion experienced by +1 charge present at P due to the charge +q at A be $\vec{F}_1 = \vec{E}_1$

~~distance d is between distance~~ P is at a distance d from the midpoint of dipole

$$= \frac{K \cdot q \cdot 1}{AP^2}, \text{ along } \vec{AP}$$

$$= \frac{Kq}{(\sqrt{d^2+l^2})^2}, \text{ along } \vec{AP}$$

$$= \frac{Kq}{d^2+l^2}, \text{ along } \vec{AP}$$

The force of attraction experienced by +1 charge present at P due to -q charge at B be

$$\vec{F_2} = \vec{E_2}$$

$$= \frac{K.q.1}{(PB)^2} \text{, along } \vec{PB}$$

$$= \frac{Kq}{\left(\sqrt{d^2+l^2}\right)^2} \text{, along } \vec{PB}$$

$$= \frac{Kq}{d^2+l^2} \text{, along } \vec{PB}$$

Thus $E_1 = E_2$

The net force experienced by the +1 charge at P due to the two charges of the electric dipole is obtained by the law of parallelogram of vectors.

$$\therefore F = E = \sqrt{E_1^2 + E_2^2 + 2E_1 . E_2 \cos 2\theta}$$

$$= \sqrt{E_1^2 + E_1^2 + 2E_1 . E_1 \cos 2\theta}$$

$$= \sqrt{2E_1^2 + 2E_1^2 \cos 2\theta}$$

$$= \sqrt{2E_1^2 (1 + \cos 2\theta)}$$

$$= \sqrt{2E_1^2 \, 2\cos^2\theta}$$

$$= 2E_1 \cos\theta$$

$$= 2 \frac{Kq}{d^2+l^2} \cdot \frac{l}{\sqrt{d^2+l^2}}$$

$$\boxed{E = \frac{2Kql}{(d^2+l^2)^{\frac{3}{2}}}}$$

Defining electric dipole moment as the product of one of the charges of the dipole and the vector joining -ve charge towards +ve charge

i.e $\vec{P} = q.\vec{BA} = q.2l\hat{BA}$

Thus $P = 2ql$

$$E = \frac{KP}{(d^2+l^2)^{\frac{3}{2}}}$$

Since $l << d$ we can neglect $l^2$ compared to $d^2$ and above expression becomes

$$\boxed{E = \frac{KP}{d^3}}$$

N.B Electric field intensity in axial/end on position is almost 2 times than in broad side on or equitorial position

## Electric lines of force

These are imaginary lines drawn in space such that the tangent at any point on it will show the direction of the field intensity at that point.

A few properties of the electric -

lines of force are given below.

1. They start from +vely Charged Objects and terminate on the negatively Charged Objects.

2. They start ⊥larly from the +vely Charged objects and terminate ⊥larly on negatively Charged objects.

Imp

3. Two electric lines of force never intersect.


<u>Proof</u> : AB and CD be two electric lines Of force which intersect at the point P. At the point P, two tangents Can be drawn for the two curves indicating two electric field intensities $\vec{E_1}$ and $\vec{E_2}$ at one point P.

According to the defn of electric field intensity, there can be only one specific magnitude and direction at one point in an electric field.

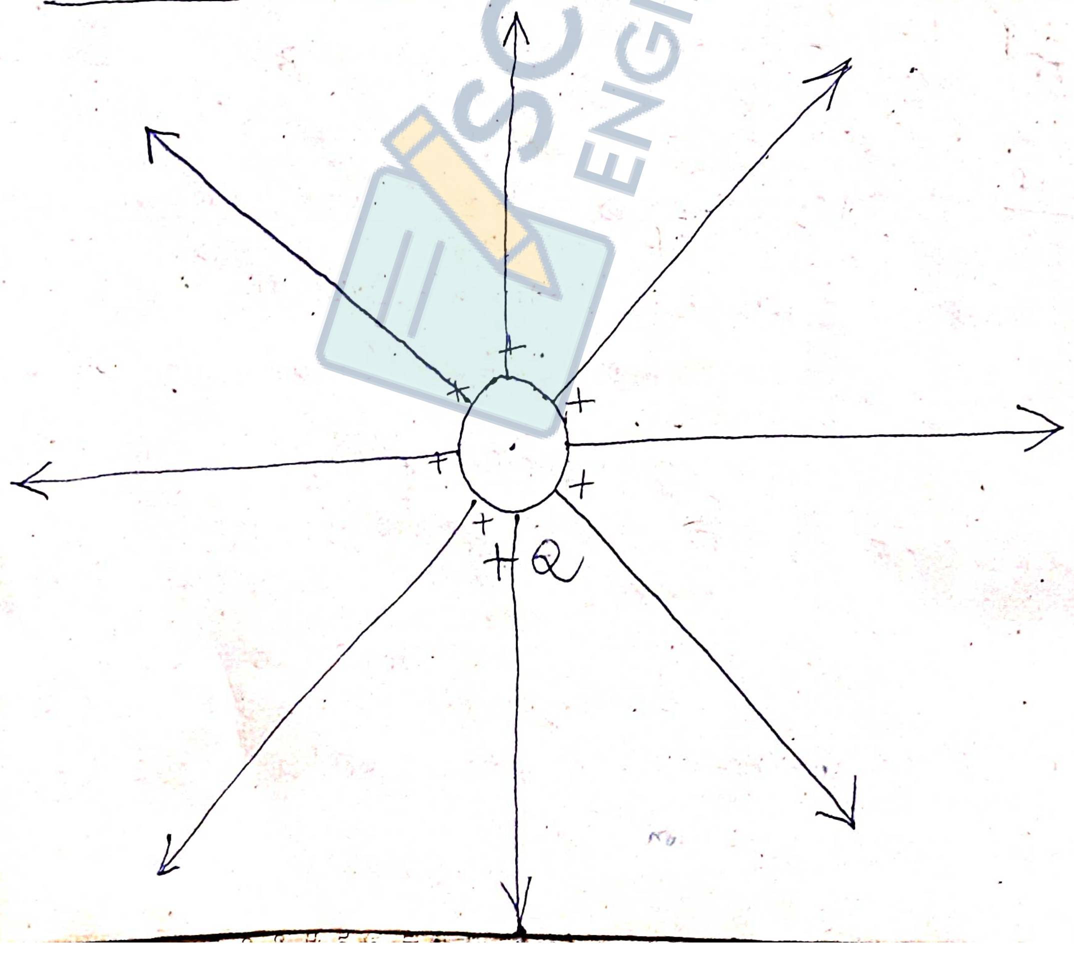
Therefore, two electric lines or force can not intersect.

4. Two electric lines of force repels each other.

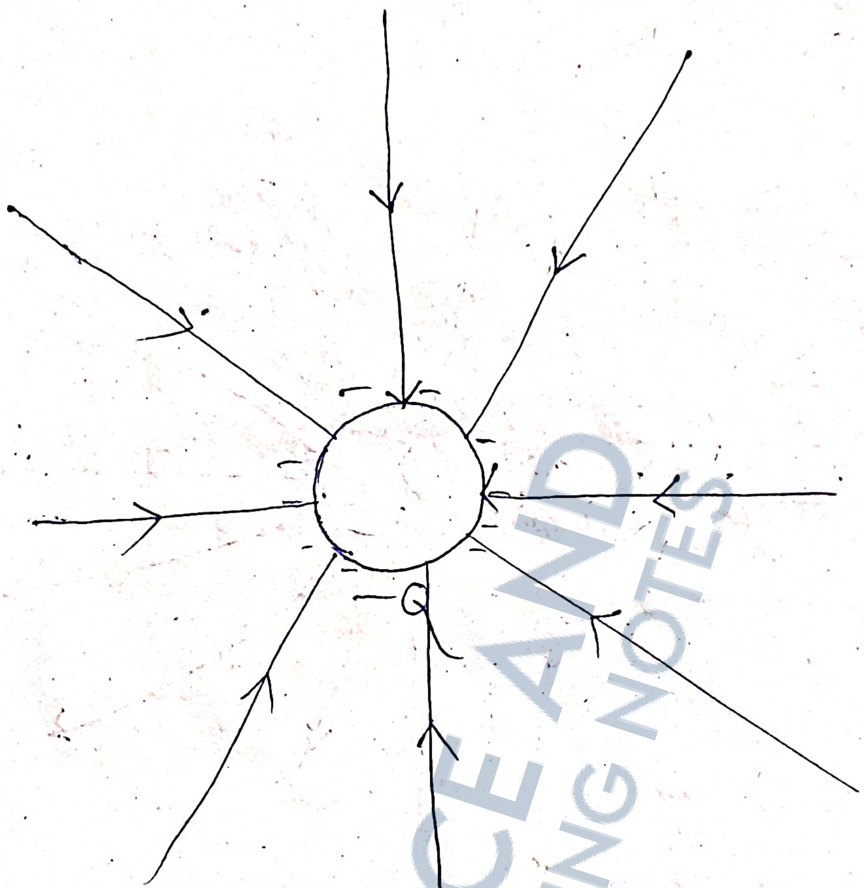
5. The electric lines of force terminate on charged objects and do not continue inside the objects. This property distinguishes them from magnetic lines or forces which continue inside the magnetic objects.

## Electric lines of force in some specific cases

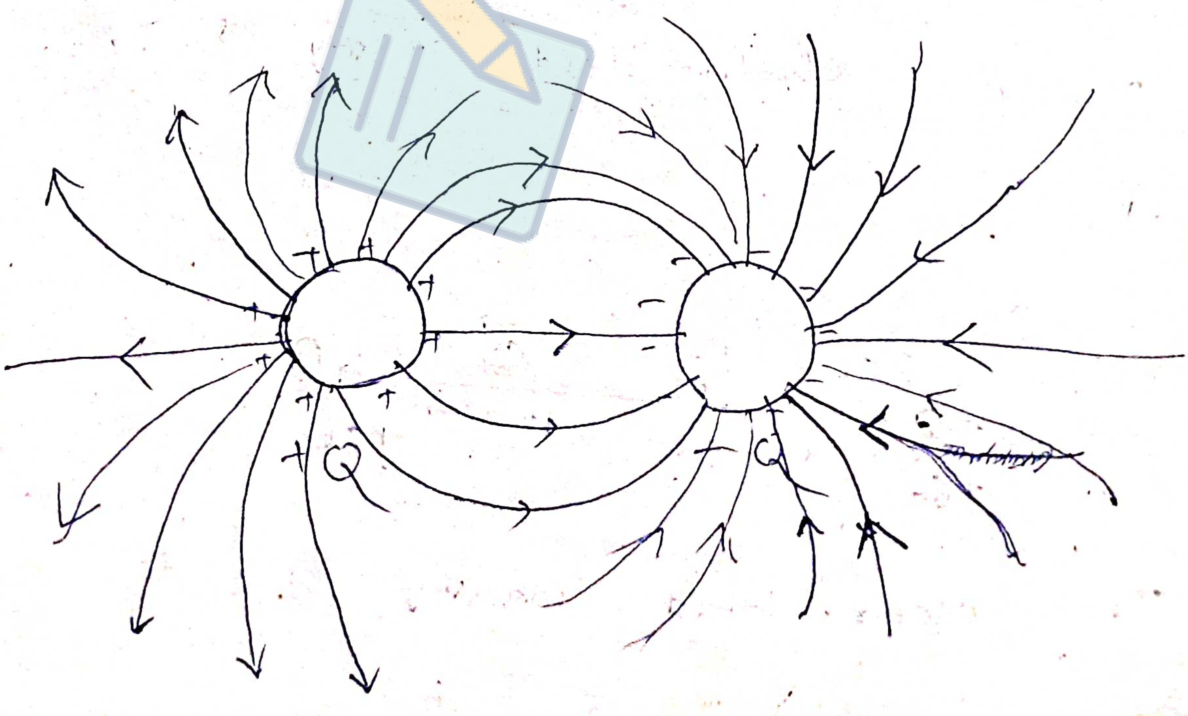
1. Due to an isolated +vely charged sphere



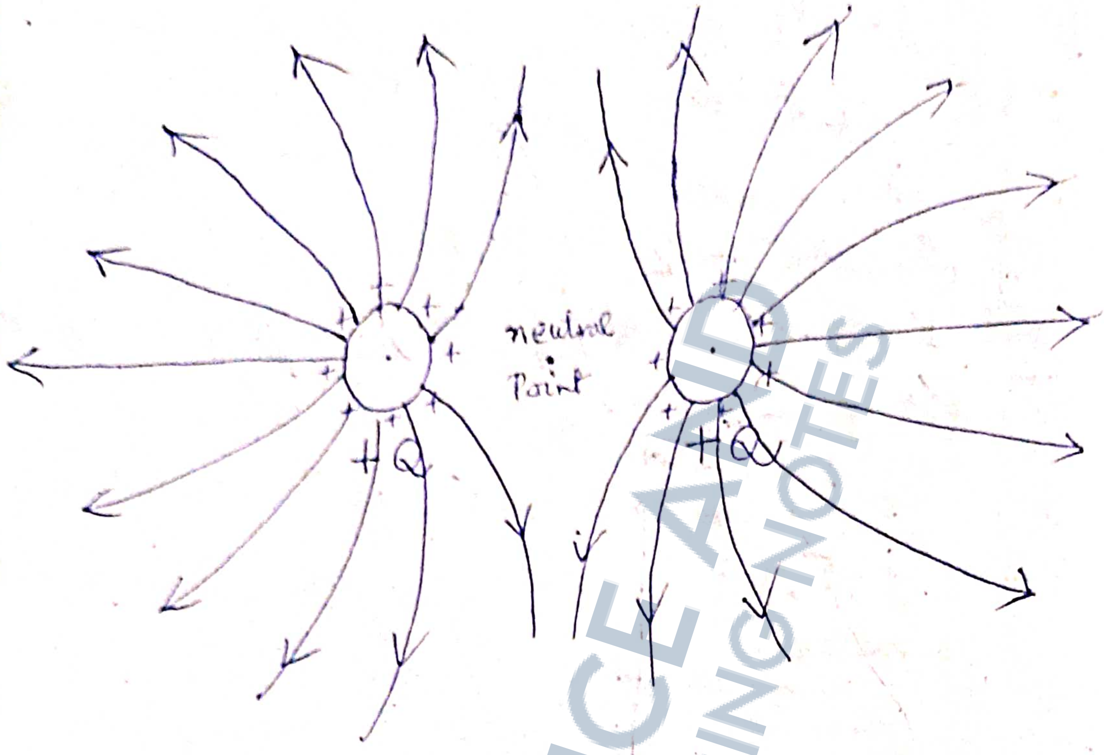
2. Due to an isolated -vely charged sphere




3. Due to a +vely charged sphere and -vely charged sphere kept near each other

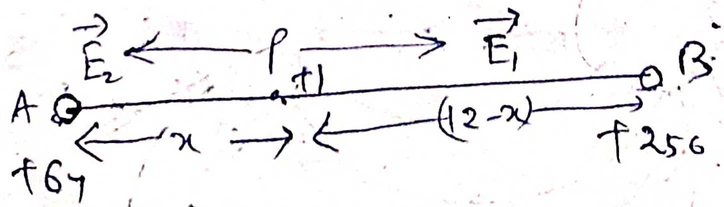


4. Due to two +vely charged spheres placed near each other.




Problem

1. Two charges +64 e.s.u and +256 e.s.u are kept at a separation of 12 metre in air. Find the neutral point. (where the net electric field intensity is zero.)




The electric field intensity due to A on P is $E_1$

i.e $E_1 = \frac{K \cdot 64}{x^2} = \frac{1 \cdot 64}{x^2}$

Electric field intensity due to B on P is $E_2$

i.e $E_2 = \frac{1 \cdot 256}{(12-x)^2}$

At neutral point $E_1 = E_2$

$$\Rightarrow \frac{64}{x^2} = \frac{256}{(12-x)^2}$$

$$\Rightarrow \frac{\cancel{64}}{x^2} = \frac{\cancel{256}\ 4}{144+x^2-24x}$$

$$\Rightarrow 144+x^2-24x = 4x^2$$

$$\Rightarrow 3x^2+24x-144=0$$

$$\Rightarrow x^2+8x-48=0$$

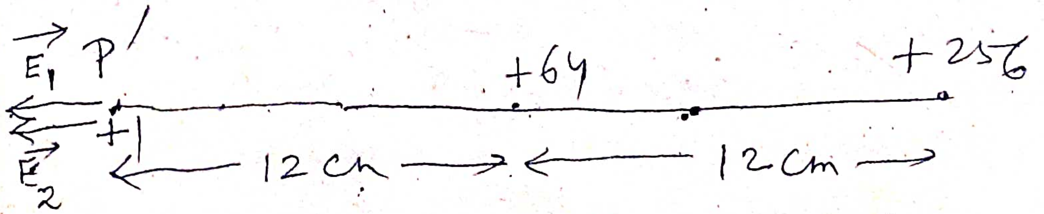
$$\Rightarrow x^2+12x-4x-48=0$$

$$\Rightarrow x(x+12)-4(x+12)=0$$

$$\Rightarrow (x+12)(x-4)=0$$

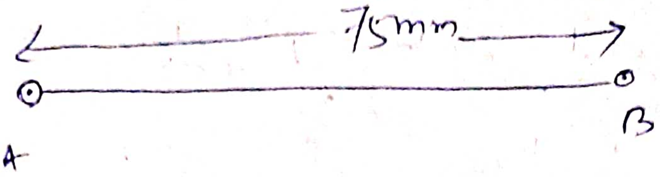
Either $x = 12$ or $x = 4$ cm

If $x = -12$, then $\vec{E_1}$ and $\vec{E_2}$ will not cancel as shown in the figure. Hence $x = 4$ cm is the correct ans.




∴ The neutral point is at 4 cm from +64 charge.

(13.)




$\cdot 0250\,\mu C$ $\qquad\qquad$ $-\cdot 0075\,\mu C$

~~The force bet~~

The Charge on A $= +\cdot 025\,\mu C$

$$= +\frac{.025}{10^{6}}\ C$$

$$= +\frac{25}{10^{9}}\ C$$

The Charge on B $= \frac{75}{10^{10}}$ C

(i) Force between the two Charges

$$= \frac{9\times10^{9}\ \frac{25}{10^{9}}\cdot\frac{75}{10^{10}}}{(.075)^2}$$

.075

$\frac{.005625}{10^{6}}$

$$= \cancel{\frac{9\times10^{9}\ 25\times75}{10^{19}\ 10^{7}}\times\frac{10^{2}}{5625}}$$

$$= \cancel{\frac{1}{3}\times10^{-7}\times9\times10^{9}}$$

$$= \cancel{.33\times10^{-7}}$$

$$= \cancel{3.3\times10^{-8}}\ \text{Co}$$

$$= \frac{\cancel{9}^{3} \times 10^{9} \times \cancel{25} \times \cancel{75}}{\cancel{10^{19}}\ 10^{13}} \times \frac{\cancel{10^{6}}}{\cancel{5625}\ \cancel{75}\ 3}$$

$$= 3 \times 10^{-4}\ \text{~~Coulomb~~. Newton}$$

(ii) when the spheres brought into contact the charge remains

$$\frac{25}{10^9} - \frac{75}{10^{10}}$$

$$= \frac{250 - 75}{10^{10}} = \frac{175}{10^{10}} = 175 \times 10^{-10} \text{ coulombs.}$$

It will be distribute between them each

$87.5 \times 10^{-10}$ Coulombs.

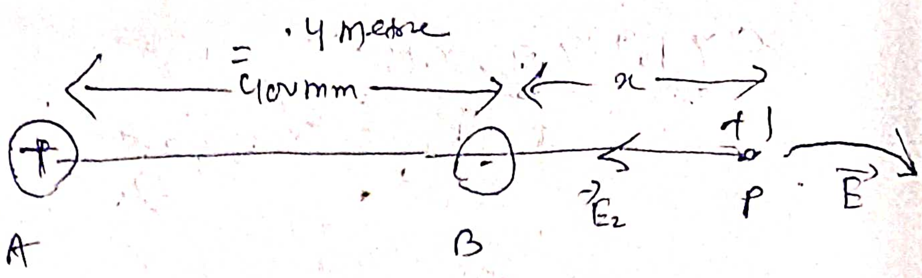
Force between ~~each~~ them ~~one~~

$$\frac{87.5 \times 10^{-10} \times 87.5 \times 10^{-10} \times 9 \times 10^{9}}{(.75)^2}$$

$$= 122500 \times 10^{-11} \text{ Newton.}$$

$$= 1225 \times 10^{-9}$$

11.




A        B

$1.67\,\mu C$        $-.600\,\mu C$

$= \frac{1.67}{10^6}\,C$        $= -\frac{.6}{10^6}\,\mu C$

The electric field intensity due to A on P is

$$E_1 = \frac{\frac{1.67}{10^6} \cdot 9\times10^9 \times 1}{(.4+x)^2}$$

$$= \frac{1.67\times9\times10^3\times10^2}{.16+x^2+.8x}$$

$$= \frac{16.7\times9\times10^4}{.16+x^2+.8x}$$

$$= \cancel{09.37}$$

Electric field intensity due to B on P is

$$E_2 = \frac{\frac{.6}{10^6}\times 9\times10^9\times1}{x^2}$$

$$= \frac{.6\times9\times10^3}{x^2}$$

$$= \frac{5.4\times10^3}{x^2}$$

$$= \frac{5400}{x^2}$$

when third charge experience no force,

there $E_1 = E_2$

$$\Rightarrow \frac{16.7 \times \cancel{9} \times \cancel{10^9}/10^2}{.16 + x^2 + 1.8x} = \frac{\cancel{5400}\ \cancel{600}\ 6}{x^2}$$

$$\Rightarrow \cancel{1670 x^2 = 9.6 + 6x^2 + 4.8x}$$

$$E_1 = E_2$$

$$\Rightarrow \frac{\frac{1.67}{\cancel{10^6}} \times \cancel{9} \times \cancel{10^9}}{(.4+x)^2} = \frac{\frac{.6}{\cancel{10^6}} \times \cancel{9} \times \cancel{10^9}}{x^2}$$

$$\Rightarrow \frac{1.67}{(.4+x)^2} = \frac{.6}{x^2}$$

$$\Rightarrow 1.67x^2 = .6(.16 + x^2 + .8x)$$

$$\Rightarrow 1.67x^2 = .096 + .6x^2 + .48x$$

$$\Rightarrow 167x^2 = 9.6 + 60x^2 + 48x$$

$$\Rightarrow 107x^2 - 48x - 9.6 = 0$$

$$\Rightarrow x = \frac{48 \pm \sqrt{(-48)^2 - 4.(107).(-9.6)}}{2.107}$$

$$= \frac{48 \pm \sqrt{2304 + 4108.8}}{214}$$

$$= \frac{48 \pm \sqrt{6412.8}}{214}$$

$$= \frac{48 \pm 80.07}{214}$$

$$= \frac{48 + 80.07}{214} \quad \text{or} \quad \frac{48 - 80.07}{214}$$

$$= \frac{128.07}{214} \quad \text{or} \quad \frac{-32.07}{214}$$

$= .59$ m or $-.149$ metre.

$= 590$ m.m or $-150$ mm

Although at $x = -150$ mm $E_1 = E_2$ but they don't cancel each other.

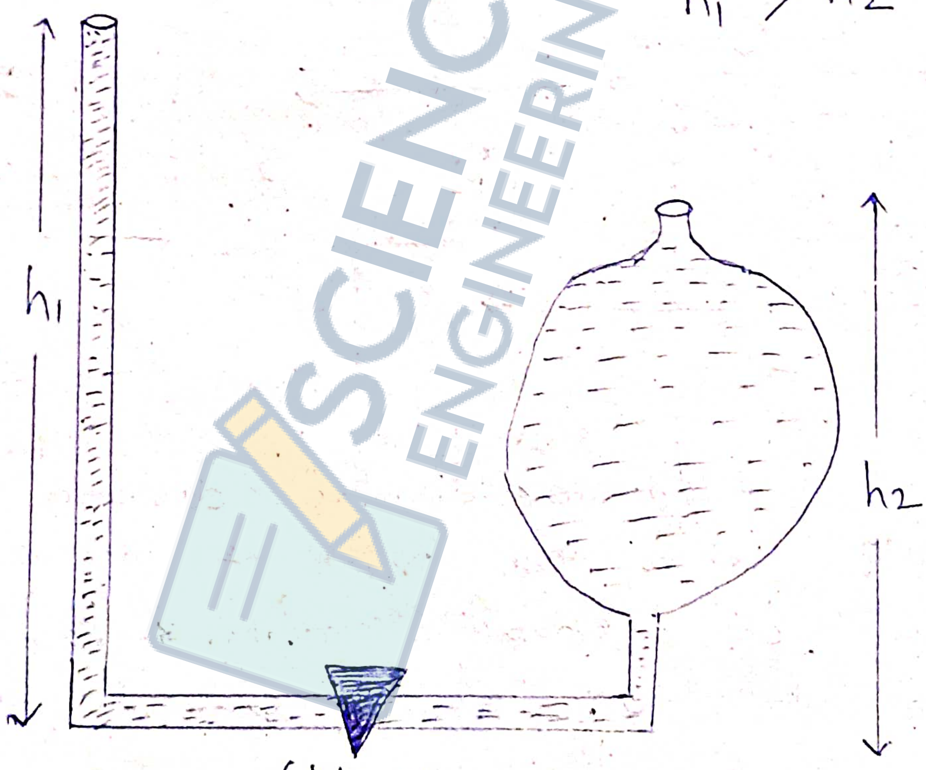
$\therefore$ The neutral point is 590 mm from $-60\mu C$

# Electric Potential

It is the electrical condition of a body which decides whether a body will give charge or receive charge when brought into contact with another body.

In this regard, electric potential can be compared with temperature in ~~heat theory~~ heat phenomena or height of a liquid column in hydrostatics.

$h_1 > h_2$



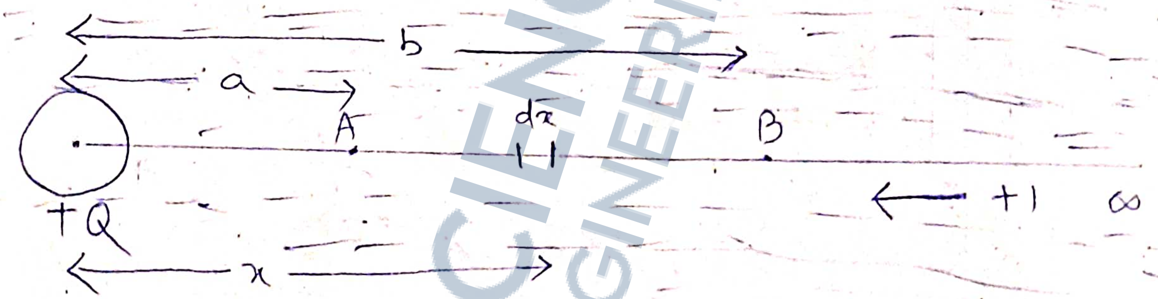

Stopcock

Although amount of water present in the cylinder is much less compared to that of the sphere, yet water flows from the cylinder towards the sphere till the levels become the same when the ~~sto~~ stopcock is opened. Similarly charges will flow from the body with high potential towards the body with lower potential till their potentials become equal.

Consequences

## Expression for electric potential at a point near an isolated charge

Electric potential at a point in an electric field due to an isolated charge can be measured by calculating the amount of workdone to bring a unit +ve charge from infinity up to the point concerned.




Let's devide the distance from infinity up to the point A into large number of small segments, each of width dx. One such segment has been shown in the figure.

dW = Small amount of work done to displace the +1 charge through a distance dx.

= Force × displacement

$= \frac{KQ.1}{x^2} . dx$

Total amount of workdone can be obtained by integrating both the sides with proper limits.

$$\int_0^W dW = KQ\int_a^{\infty} x^{-2}.\,dx$$

$$\Rightarrow (W)\Big|_0^W = KQ\left(\frac{x^{-2+1}}{-2+1}\right)\Bigg|_a^{\infty}$$

$$\Rightarrow W - 0 = KQ\left(-x^{-1}\right)\Big|_a^{\infty}$$

$$\Rightarrow W = KQ\left(-\frac{1}{x}\right)\Bigg|_a^{\infty}$$

$$= KQ\left(-\frac{1}{\infty} + \frac{1}{a}\right)$$

$$= \frac{KQ}{a}$$

$$\Rightarrow W = \frac{KQ}{a} = V_A \quad \text{——— (i)}$$

= Electric potential at A

In the same manner, the distance from infinity upto B can be devided into large number of small segments and limits of integration will be from b to $\infty$. This will give the expression for the electric

Potential at B as

$$V_B = \frac{KQ}{b} \quad \text{------------} (ii)$$

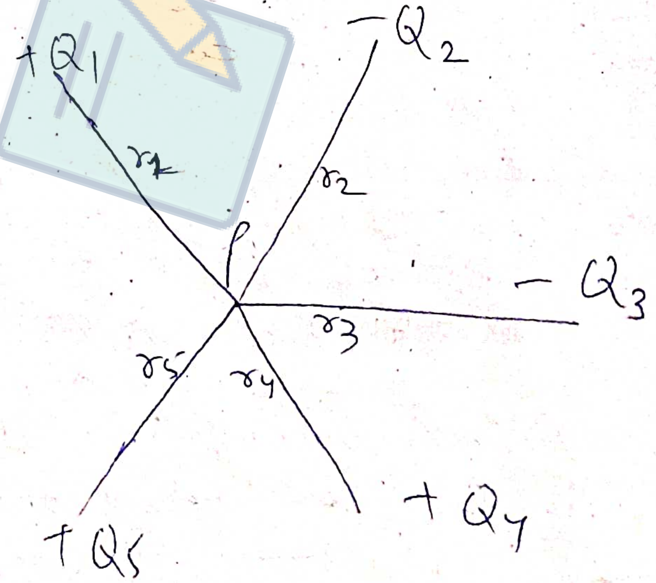
The amount of workdone to bring the +1 Charge from the point B to the point A will be

$$\Delta W = V_A - V_B \quad \text{------------} (iii)$$

Instead of a +1 charge suppose a Charge +q be brought from the point B to the point A, then the amount of workdone will be given by

$$\Delta W = q(V_A - V_B) \quad \text{------------} (iv)$$

<u>Electric potential at a point due to several Changes</u>



Since, electric potential is a scalar quantity, it can be added with proper signs.

Electric potential at P due to the 5 charges is given by

$$V_P = \frac{KQ_1}{r_1} + \frac{K(-Q_2)}{r_2} + \frac{K(-Q_3)}{r_3} + \frac{KQ_4}{r_4} + \frac{KQ_5}{r_5}$$

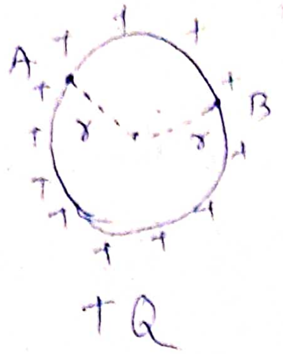
## Equipotential surface

If the electric potential at all points will be the same, then we call such a surface as equipotential surface.

The amount of workdone to shift any charge from one point to another point on the equipotential surface is zero.

$$\therefore \Delta W = q(V_A - V_B)$$
$$= q \cdot 0$$
$$= 0$$

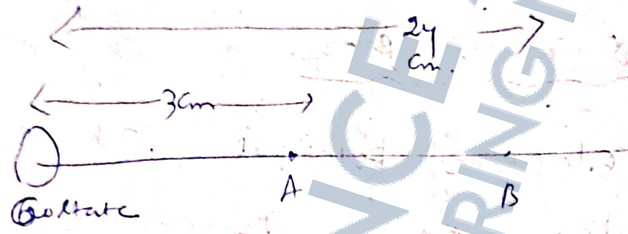
Example : When a metallic sphere (hollow or solid) is given some charge, then the charge is uniformly distributed on its surface. From Gauss's theorem, it has been proved that these charges behave as if they are concentrated at the same centre.




$$V_A = \frac{KQ}{r} = V_B$$

---

14.




$$V_A = \frac{KQ}{a} = \frac{1.60}{3} = 20$$

$$V_B = \frac{KQ}{b} = \frac{1.60}{24} = \frac{60}{24}$$

Work done $= q \cdot (V_A - V_B)$

$$= 5\left(20 - \frac{60}{24}\right)$$

$$= 5\left(\frac{480 - 60}{24}\right)$$

$$= \frac{5 \times \cancel{420}\,\cancel{210}\,70}{\cancel{24}\,\cancel{12}\,\cancel{8}\,4} = \cancel{420}\,\cancel{35}\,\frac{350}{4}$$

$$= 87.5 \text{ erg}$$

8. Force $= 4.8\times10^{-2}$ N

$\cancel{\Rightarrow \frac{KQ}{d^2}} = 4.8\times10^{-2}$ N

K in M.K.S unit $= 9\times10^9 \frac{\text{Newton metre}}{C^2}$

$q = 40\mu C = 40\times10^{-6}$ C

Workdone = Force × displacement

$= 4.8\times10^{-2}\times .2$

Workdone $= q(V_A - V_B)$

$\Rightarrow V_A - V_B = \frac{4.8\times10^{-2}\times .2}{40\times10^{-6}}$

$= \frac{4.8\times\cancel{2}\times10^{-2}\times\cancel{10^{6}}^{10^5}\times10^{-1}}{\cancel{20}\cancel{40}}$

$= 2.4\times10^2$ V

$= 240$ Volt

30. Potential difference $\Delta V = 40$ mV $= 40\times10^{-6}$ V.

Charge of electron $q = 1.6\times10^{-19}$ C

~~This much work~~

$\Delta W = q.\Delta V$

$\Rightarrow \Delta W = 1.6\times10^{-19}\times40\times10^{-6}$

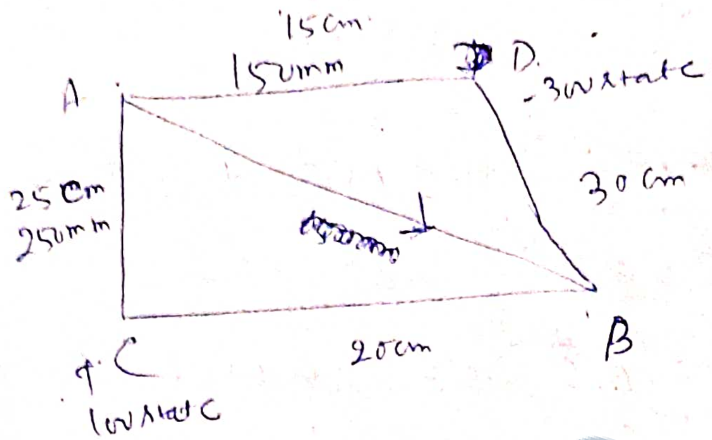
$= 64\times10^{-13}$ J

$= 6.4\times10^{-12}$ J

This much of workdone will be stored as the

Potential energy.

11.




$V_A = \frac{K.Q}{25} = \frac{K.100.4}{25} = 4$ volt

$V = \frac{K.Q}{15} = \frac{1.(-300)}{15} = -20$ volt

Electric potential at A due to C and D

is $V_A = \frac{KQ}{25} + \frac{KQ}{15}$

$= 4 - 20$

$= -16$ volt

Electric potential at B due to C and D

$V_B = \frac{KQ}{20} + \frac{KQ}{30}$

$= \frac{1.100}{20} + \frac{1.(-300)}{30}$

$= 5 - 10$

$= -5$

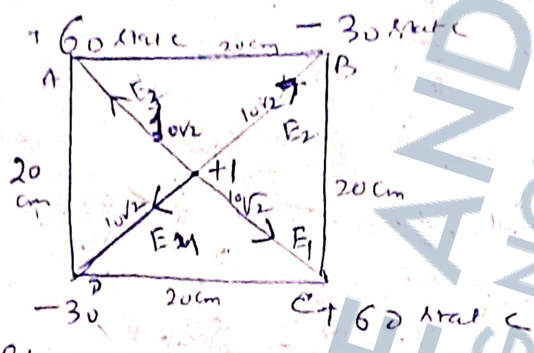
$$\Delta W = Q(V_B - V_A)$$

$$= 50 \text{ statc} (-5 + 16)$$

$$= 50 \times 11$$

$$= 550 \text{ erg}$$

24.




$$E_1 = \frac{K \cdot 60}{(10\sqrt{2})^2} = \frac{\cancel{60}^{3}}{\cancel{200}_{10}} = \frac{3}{10} \text{ dyne/statC}$$

$$E_3 = \frac{1 \cdot 60}{(10\sqrt{2})^2} = \frac{3}{10} \text{ dyne/stat C}$$

So $E_1$ and $E_3$ Cancel each other, because they are equal in magnitude and opposite in direction

$$E_2 = \frac{1 \cdot 30}{(10\sqrt{2})^2} = \frac{30}{200} = \frac{3}{20} \text{ dyne/stat C}$$

$$E_4 = \frac{1 \cdot 30}{(10\sqrt{2})^2} = \frac{30}{200} = \frac{3}{20} \text{ dyne/stat C}$$

$E_2$ and $E_4$ Cancel each other because they are equal in magnitude and opposite in direction.

$\therefore$ Net electric intensity is Zero.

Electric potential at P

$$V_P = \frac{K \cdot 60}{10\sqrt{2}} + \frac{K(60)}{10\sqrt{2}} + \frac{K(-30)}{10\sqrt{2}} + \frac{K(-30)}{10\sqrt{2}}$$

$$= \frac{60\ 6}{10\sqrt{2}} + \frac{60}{10\sqrt{2}} - \frac{60}{10\sqrt{2}}$$

$$= \frac{36\sqrt{2}}{2 \cdot 2}$$

$$= 3\sqrt{2}$$

$$= \cancel{3 \cdot (1.414)} \quad = 3\,(1.414)$$

$$= 4.242 \text{ statV}$$

599

(2)

$\Delta V = 8.0\,V$

$q = 400\,\mu C = 400 \times 10^{-6}\,C$

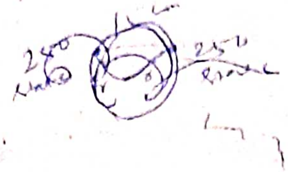
$K = 9 \times 10^{9}\ \frac{\text{Newton}\cdot\text{meter}^2}{C^2}$

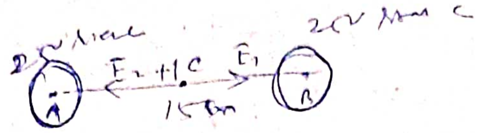
$\Delta W = q \cdot \Delta V$

$= 400 \times 10^{-6}\,C \times 8\,V$

$= 32 \times 10^{-4}$ Joule

4.





Electrical field intensity at C

due to A, $E_1 = \frac{1 \cdot 250 \cdot 1}{(7.5)^2} = \frac{250}{(7.5)^2}$ dyne/statc

Electrical field intensity at C

due to B, $E_2 = \frac{1 \cdot 250 \cdot 1}{(7.5)^2} = \frac{250}{(7.5)^2}$ dyne/statc

Since $E_1$ and $E_2$ are equal in magnitude and opposite in direction, they cancel each other.

∴ Electric field intensity at C is zero.

Electric ~~field intensity~~ potential at C due to A

$= \frac{KQ}{a} = \frac{250}{7.5}$ V

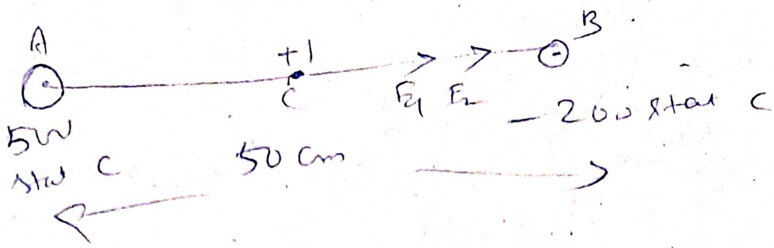
Electric potential at C due to B

$= \frac{1 \cdot 250}{7.5}$ V

At C total electrical potential

$= \frac{250}{7.5} + \frac{250}{7.5} = \frac{500}{7.5} = \frac{\cancel{5000}\,\cancel{1000}\,200}{\cancel{75}\,\cancel{15}\,3} = 66.66$ Stat V

6.




Electric field intensity due to A

$$= \frac{500 \times 1}{(25)^2} = \frac{500}{625}$$

Electric ~~fitot~~ field intensity due to B

$$= \frac{200 \times 1}{(25)^2} = \frac{200}{625}$$

~~∴ Electrical~~
Net
∴ Electric field intensity at C

$$= \frac{500}{625} + \frac{200}{625} = \frac{700}{625} = 1.12 \text{ dyne/statC}$$

Electric potential at C due to A and B

$$= \frac{KQ}{a} + \frac{KQ}{b}$$

$$= \frac{1 \times 500}{25} + \frac{1 \times (-200)}{25}$$

$$= 20 - 8$$

$$= 12 \text{ StatV}$$

work done to bring a charge +23.5 statC

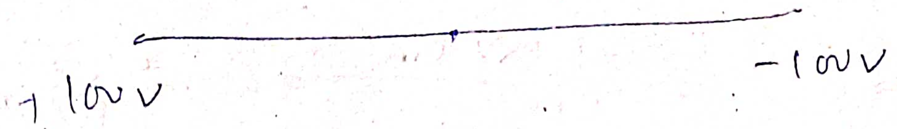
to that point $= q(V_C - V_\infty)$ $\quad V_\infty$

$$= q \times (12 - 0) \quad q^{\,0}$$

$$= 23.5 \times 12$$

$$= 282 \text{ erg.}$$

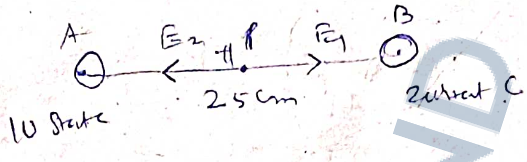
8. 10.




Potential difference $= 100V - (-100V)$

$= 200V$

12.




Electric field intensity at P Since $E_2 > E_1$
Net " " will be $E_2 - E_1$

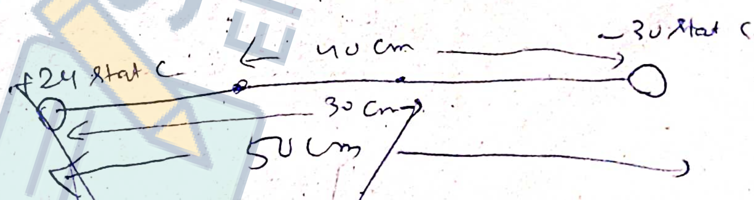
$= -\frac{1 \times 10 \times 1}{(12.5)^2} + \frac{20}{(12.5)^2}$

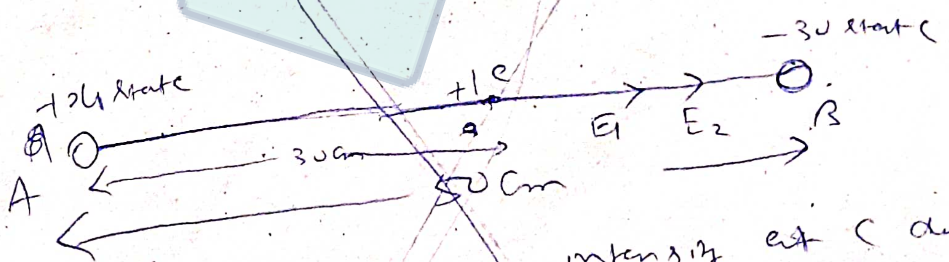
$= -\frac{10}{156.25} + \frac{20}{156.25}$

$= \frac{10}{156.25}$

$= 0.064$ dyn/statC

20.



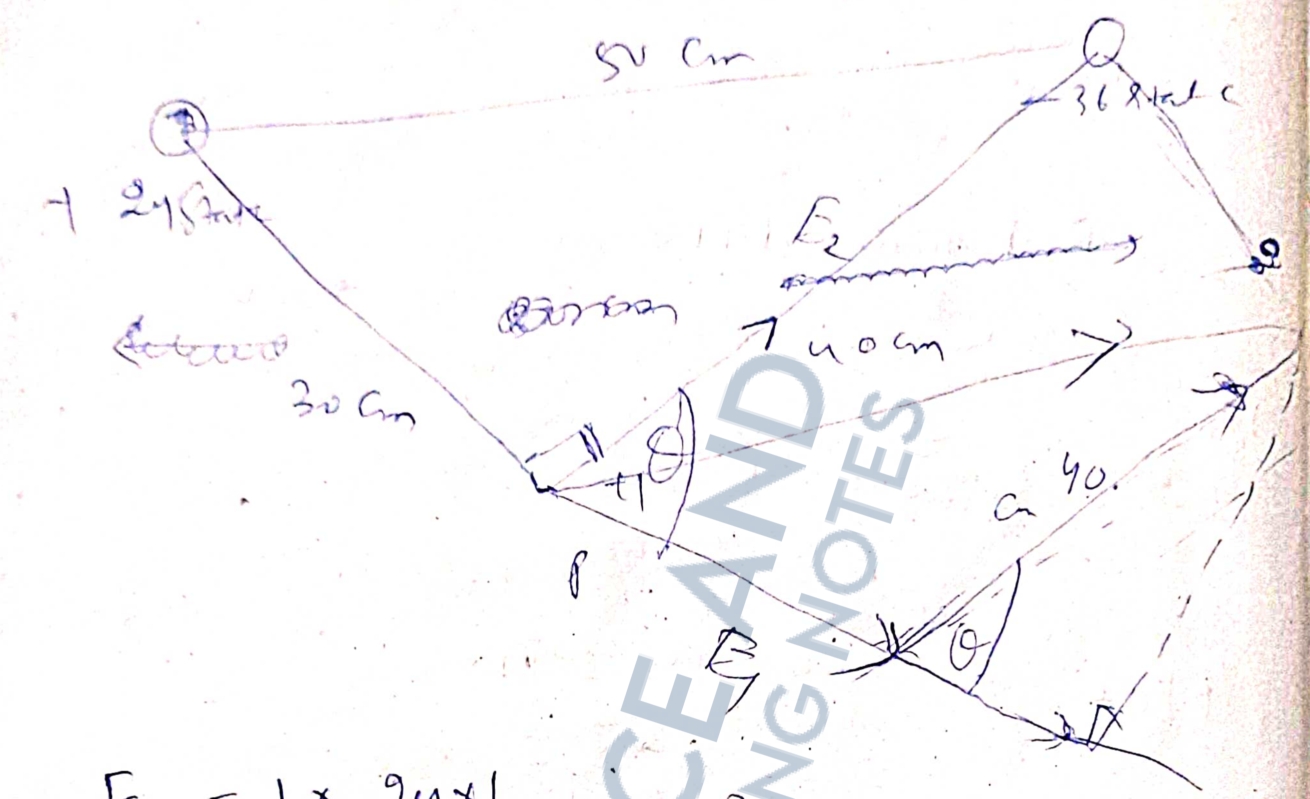




Fig(i) Electrical field intensity at C due to A and B is

$= \frac{24}{(30)^2} + \frac{30}{(20)^2} = \frac{24}{900} + \frac{30}{400}$

$= \frac{96 + 270}{3600} = \frac{366}{3600} =$

20.




$$E_1 = 1 \times \frac{24 \times 1}{(30)^2} = \frac{24}{900}$$

$$E_2 = \frac{1 \times 36 \times 1}{(40)^2} = \frac{36}{1600}$$

Resultant $E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\alpha}$

$$= \sqrt{\frac{576}{810000} + \frac{1296}{2560000} + 2 \cdot \frac{24}{900} \times \frac{36}{1600} \times}$$

22.

$q = 4\,\mu C = 4\times10^{-6}\,C$

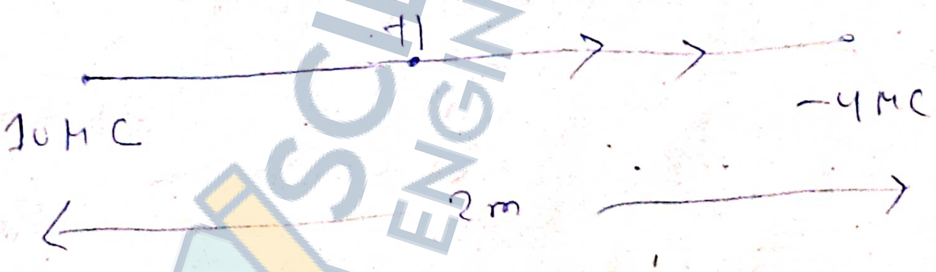
$V = 2.4\times10^{5}$

$W = qV$

$F\times d = 4\times10^{-6}\times 2.4\times10^{5} = 9.6\times10^{-1} = .96$

$\Rightarrow d = \dfrac{.96}{F} =$

26.




Potential at the mid point

$= \left\{ \dfrac{10\times10^{-6}}{1\,m} + \dfrac{-4\times10^{-6}}{1} \right\} 9\times10^{9}$

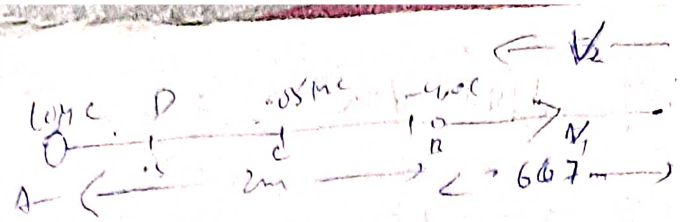
$= \left\{ 10^{-5} + (-4\times10^{-6}) \right\} 9\times10^{9}$

$= 10^{-5}\,(1 - 4\times10^{-1})$

$= 10^{-5}\times 6 = 6\times10^{-6}\times 9\times10^{9}$

$= 5.4\times10^{4}\,V$

(b)




$V_1 = \frac{9\times10^{9}\times10^{-6}\times10}{2.667} = \frac{9}{2.667}\times10^{4} = 3.37\times10^{4}$

$V_2 = \frac{9\times10^{9}\times(-4)\times10^{-6}}{.667} = -\frac{9\times4}{.667}\times10^{3}$

$= -\frac{36}{.667}\times10^{3}$

Work require = $q(V_D - V_C)$

$V_D = \frac{9\times10^{9}\times10\times10^{-6}\times .05\times10^{-6}}{(.5)^2}$

$= \frac{(9\times .05)\times10^{-2}}{.25} = \frac{9\times\frac{5}{100}\times\cancel{10^{-2}}\times\cancel{10^{2}}}{25\times10^{-2}}$

$= \frac{9\times5\times25}{100}$

$V_D = \frac{9\times10^{9}\times10\times10^{-6}\times .05\times10^{-6}}{(1)^2}$

$= \frac{9\times5}{100}\times10^{-2}$

$= \frac{.45}{100}$

Work require = $.05\times10^{-6}\left(\frac{1125}{100} - \frac{.45}{100}\right)$

$$= \frac{5}{100} \times \frac{1}{10^6} \times \left( \frac{1127.55}{100} \right)$$

$$= \frac{5622.75}{10^{10}}$$

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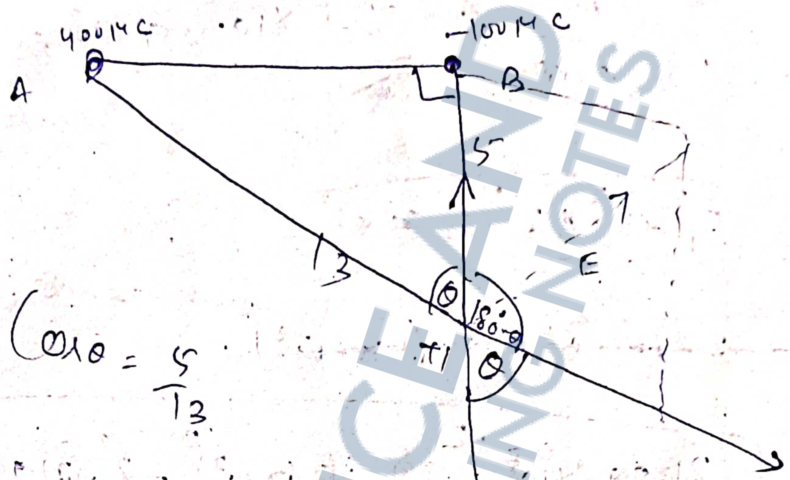
**23.**




$\cos\theta = \frac{5}{13}$

$$E_1 = \frac{400\times10^{-6}}{(13)^2} = \frac{400\times10^{-6}}{169} = 2.366 \text{ dyne/statc}$$

$$E_2 = \frac{100\times1}{25} = 4 \text{ dyne/statc.}$$

$$E = \sqrt{(E_1)^2+(E_2)^2+2E_1E_2\cos(180-\theta)}$$

$$= \sqrt{5.6+16-2.(2.366).(4).\frac{5}{13}} \quad \left(\because \cos(180°-\theta) = -\cos\theta\right)$$

$$= \sqrt{21.60-7.28}$$

$$= \sqrt{14.32}$$

$= 3.784$ dyne/statc

$$E_1 = 9\times10^9 \frac{400\times10^{-6}}{(.13)^2} = \frac{4\times10^{-4}}{.0169} = \frac{4\times10^{-4}}{1.69\times10^{-2}} = 2.366\times10^{-2}\times9\times10^9 = 21.294\times10^{7}$$

$$E_2 = 9\times10^9 \frac{100\times10^{-6}}{(.05)^2} = \frac{100\times10^{-6}}{.0025} = \frac{100\times10^{-6}}{25\times10^{-4}} = 4\times10^{-2}\times9\times10^9 = 36\times10^{7}$$

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\alpha}$$

$$= \cancel{(2.366\times10^{-2})^2 + (4\times10^{-2})^2}$$

$$= \sqrt{(21.294\times10^7)^2 + (36\times10^7)^2 - 2\cdot(21.294\times10^7)(36\times10^7)\times\frac{5}{13}}$$

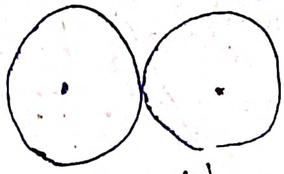
$$= \sqrt{(21.294)^2\times10^{14} + (36)^2\times10^{14} - \left(2\times21.294\times36\times\frac{5}{13}\right)\times10^{14}}$$

$$= 10^7\sqrt{453.43 + 1296 - (589.68)}$$

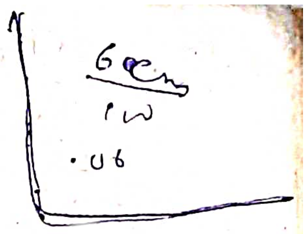
$$= 10^7 \times 34.055 \text{ N/C}$$

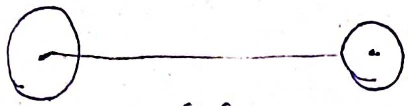
(5) When a neutral body is comes towards the copper sphere frome, Here induction precedes attraction. So these neutral sphere is -vely charged. So there is a force of attraction

(b)



$700\times10^{-6}$ C    Neutral body.




$350\times10^{-6}$ C    $350\times10^{-6}$ C

Force between them $= \dfrac{350\times10^{-6}\,C \times 350\times10^{-6}\,C}{(.06)^2}$

$= \dfrac{35\times35}{36} \times \dfrac{10^{-12}}{10^{-4}}$

$= 34.02 \times 10^{-8}$ Newton

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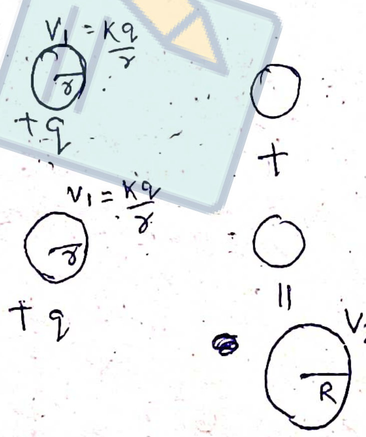
18.




$V_2 = \dfrac{KQ}{R} = \dfrac{K.2q}{R}$

$+Q = 2q$

The relation between r and R is obtained from the eqn

$$\cancel{\frac{4}{3}\pi} R^3 = 2\cdot\cancel{\frac{4}{3}\pi} r^3$$

$$\Rightarrow R^3 = 2r^3$$

$$\Rightarrow R = 2^{\frac{1}{3}} r$$

Now $$\frac{V_2}{V_1} = \frac{\frac{\cancel{K}2\cancel{q}}{R}}{\frac{\cancel{K}\cancel{q}}{r}} = \frac{2}{\frac{2^{\frac{1}{3}}\cancel{r}}{\cancel{r}}}$$

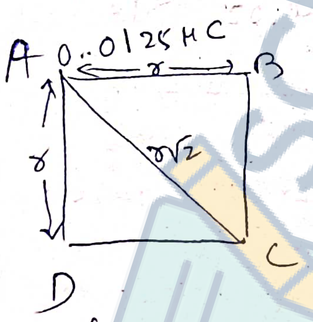
$$= 2^{1-\frac{1}{3}} = 2^{\frac{2}{3}}$$

$$= (2^2)^{\frac{1}{3}}$$

$$= 4^{\frac{1}{3}}$$

$$\boxed{\Rightarrow V_2 = 4^{\frac{1}{3}} V_1}$$

13.




The electrostatic potential energy of the system of four charges present at the 4 corners of the square ABCD is equal to the amount of workdone to bring the 4 charges one by one from infinity up to the points A, B, C, and D.

Let $\Delta W_1$ = Amount of workdone to bring $+q$ Charge from infinity up to the point A

$$= q\,(V_A - V_{infinity})$$

$$\left( \because V_A = \frac{Kq}{r} - \frac{Kq}{r} = 0 \quad \rightarrow V_\infty = 0 \right)$$

$$= q\,(V_A - V_\infty)$$

$$= q\,(0 - 0)$$

$$= 0$$

$\Delta W_2$ = Amount of workdone to bring $+q$ Charge from infinity up to the point B

$$= q\,(V_B - V_\infty)$$

$$= q\left(\frac{Kq}{r} - 0\right)$$

$$= \frac{Kq^2}{r}$$

$\Delta W_3$ = Amount of workdone to bring $+q$ charge from infinity up to the C

$$= q\,(V_C - V_\infty)$$

$$= q\left(\frac{Kq}{r\sqrt{2}} + \frac{Kq}{r} - 0\right)$$

$$= \frac{Kq^2}{r\sqrt{2}} + \frac{Kq^2}{r}$$

$\Delta W_4$ = Amount of workdone to bring $+q$ charge from infinity up to the point D

$$= q\,(V_D - V_\infty)$$

$$= q\left(\frac{Kq}{r} + \frac{Kq}{r\sqrt{2}} + \frac{Kq}{r} - 0\right) = \frac{2Kq^2}{r} + \frac{Kq^2}{r\sqrt{2}}$$

Total amount of workdone to bring the 4 Charges

$= \Delta W_1 + \Delta W_2 + \Delta W_3 + \Delta W_4$

$= 0 + \frac{Kq^2}{r} + \frac{Kq^2}{r\sqrt{2}} + \frac{Kq^2}{r} + \frac{2Kq^2}{r} + \frac{Kq^2}{r\sqrt{2}}$

$= \frac{Kq^2}{r}\left(1 + \frac{1}{\sqrt{2}} + 1 + 2 + \frac{1}{\sqrt{2}}\right)$

$= \frac{Kq^2}{r}(4 + \sqrt{2})$

Here $K = 9\times10^{9} \frac{C^2}{Nm^2}$

$q = .0125\times10^{-6}\ C$

$r = 1m$

$\therefore W = \frac{9\times10^{9} \times (.0125)^2 \times 10^{-12}}{1} \times (4+\sqrt{2})$

$= 9\times10^{-3} \times 1.5625\times10^{-4}\,(4+\sqrt{2})$

$= (9\times1.5625)(4+1.414)\times10^{-7}$

$= 9\times1.5625\times5.414\times10^{-7}$

$= 76.13\times10^{-7}\ J$

$= 7.613\times10^{-6}\ J$ This is the potential energy

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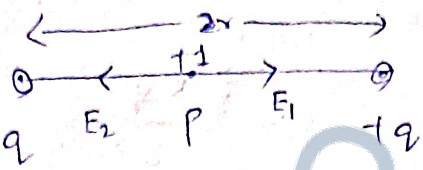
(16)

potential = same, field = 0

potential ≠ 0, ... even.

(a) Lets consider two charges +q each kept at a separation of $2r$



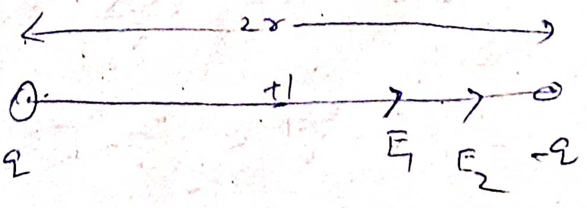

$$E_1 = \frac{Kq}{r^2}, \quad E_2 = \frac{Kq}{r^2}$$

At the mid point net electric field intensity is zero, because $E_1 = E_2$, equal in magnitude, opposite in direction.

$$V_P = \frac{Kq}{r} + \frac{Kq}{r} = 2\frac{Kq}{r} \neq 0$$

This is an example where electric field intensity is zero but electric potential is not zero.

(b) Let's consider two charges +q, -q kept at a separation of 2r.




$$E_1 = \frac{Kq}{r}, \quad E_2 = \frac{Kq}{r}$$

At the mid point of net electric field intensity $= E_1 + E_2$

$$= \frac{Kq}{r^2} + \frac{Kq}{r^2}$$

$$= \frac{2Kq}{r^2}$$

$$\neq 0$$

$$V_P = \frac{Kq}{r} + \frac{K(-q)}{r} = 0$$

This is an example where electric field intensity is not zero but electric field potential is zero

(25) For outside points and points on the surface of charged sphere, the charges behave as if they are concentrated at the centre (From Gauss theorem)

At the outside point

(a) $V = \cancel{\frac{KQ(r)}{Qr}} = \frac{KQ}{r}$

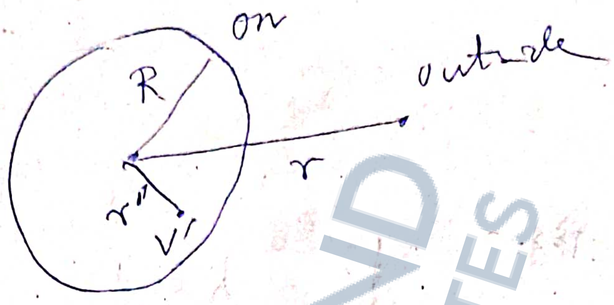
$$= \frac{9\times10^9 \times \cdot 3\times10^{-6}}{\cdot 27}$$

$$= \frac{\cancel{9}\times10^9\times\cancel{3}\times10^{-7}\times10^2}{\cancel{27}} = 10^4 \text{ Volt}$$

(b) On the surface of the sphere

$$V = \frac{KqQ}{\gamma R}$$

$$= \frac{9\times10^{9} \times 3\times10^{-7} \times 10^{2}}{1.8}$$ $= 1.5 \times 10^{4}$ volt




(c) From an application of Gauss theorem, it has been proved that $E = 0$ at all inside points of the hollow charged sphere.

But $E = -\frac{dV}{dR}$

= Potential gradient

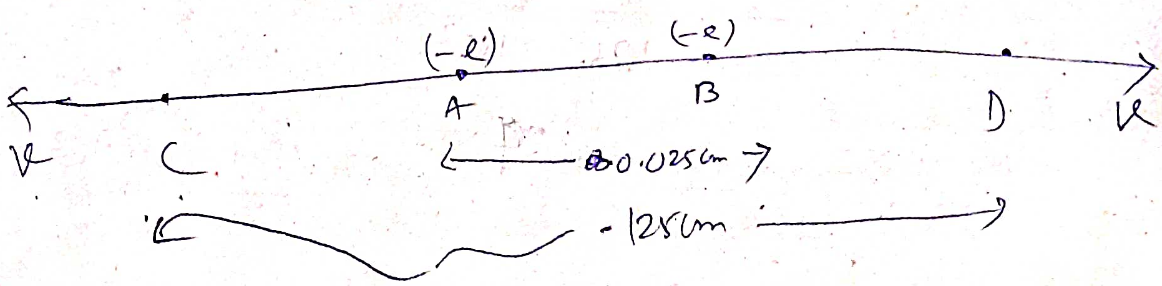
$\Rightarrow 0 = dV$

$\Rightarrow V$ is constant = Potential on the surface

$= 1.5\times10^{4}$ volt

Q. electric field intensities $\rightarrow \frac{KQ}{r^2} = \frac{Ke}{(27)^2}, \frac{Ke}{(.18)^2}, \frac{KQ}{(.9)^2}$

27.



We know that $\Delta W = \Delta E_K$

$$\Rightarrow q.\Delta V = \frac{1}{2} m v^2 - \frac{1}{2} m (0)^2$$

$$\Rightarrow (-e).\Delta V = \frac{1}{2} m v^2 \quad \text{——— (i)}$$

From the figure, we see that

$$V_1 = \frac{K(-e)}{.025 \times 10^{-2}} = $$ Potential on electron at B due to another electron at A.

$$V_2 = \frac{K(-e)}{0.125 \times 10^{-2}} = $$ Potential on the electron at D due another electron at C

Thus $$(-e).\left[\frac{K(-e)}{.025 \times 10^{-2}} - \frac{K(-e)}{.125 \times 10^{-2}}\right] = \frac{1}{2}(9.1 \times 10^{-31}) v^2$$

$$\Rightarrow Ke^2 \left[\frac{1}{.025 \times 10^{-2}} - \frac{1}{.125 \times 10^{-2}}\right] = \frac{9.1}{2} \times 10^{-31} v^2$$

$$\Rightarrow Ke^2 \left[\frac{5 - 1}{.125 \times 10^{-2}}\right] = \frac{9.1}{2} \times 10^{-31} v^2$$

$$\Rightarrow \frac{9 \times 10^{9} \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 4}{.125 \times 10^{-2}} = \frac{9.1}{2} \times 10^{-31} v^2$$

$$\Rightarrow \frac{9 \times 2.56 \times 4}{.125} \times 10^{-29} \times 10^{2} = \frac{9.1 \times 10^{-31} v^2}{2}$$

$$\Rightarrow V^2 = \frac{72 \times 2.56 \times 10^2}{9.1 \times .125 \times 10^{-5}}$$

$$= \frac{72 \times 2.56}{9.1 \times .125} \times 10^7$$

$$\Rightarrow V = 12.72 \times 10^2$$

$$= 1.272 \times 10^3 \text{ m/sec}$$

Relative velocity of electrons because they are moving in opposite direction

$$= 2V$$

$$= 2 \times 1.272 \times 10^3$$

$$= 2.544 \times 10^3 \text{ m/sec.}$$

Q→ Derive expressions for electric potential at the end on and broad-side on positions of an electric dipole

An electric dipole consists of two charges equal in magnitude, but opposite in sign. In our


diagram AB represents electric dipole whose charges are at a distance 2l units.

When the point P is situated on the extension of the axis of dipole, we call it end-on position.

$V_P$ = Electric potential at P due to Charges at A and B

$$= \frac{Kq}{AP} + \frac{K(-q)}{BP}$$

$$= \frac{Kq}{d+l} - \frac{Kq}{d-l}$$

$$= Kq\left[\frac{\cancel{d}-l-\cancel{d}-l}{(d+l)(d-l)}\right]$$

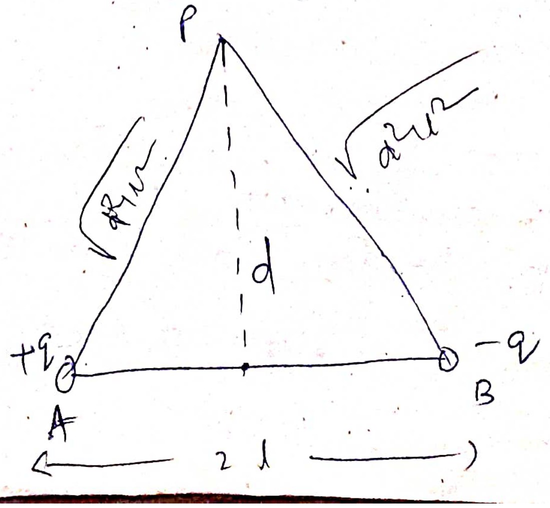
$$= -\frac{2Kql}{d^2-l^2}$$

$$= -\frac{Kp}{d^2-l^2}$$ where $p$ = Magnitude of the electric dipole moment $= 2ql$

For an electric dipole $l^2$ can be neglected compared to $d^2$.

$$\therefore \boxed{V_P = -\frac{Kp}{d^2}}$$

(ii)



When the point is situated on the perpendicular bisector of the line joining the two charges of the dipole then this position is called broad side on position.

$V_P$ = Electric potential at P due to charges at A and B

$$= \frac{Kq}{AP} + \frac{K(-q)}{BP}$$

$$= \frac{Kq}{\sqrt{d^2+r^2}} - \frac{Kq}{\sqrt{d^2+r^2}}$$

$$= 0$$

Q-) Establish the relation between volt and stat volt

(Hint: use $\Delta W = q \cdot \Delta V$)

Ans:

We know that $\Delta W = q \cdot \Delta V$

In M.K.S unit $1\,\text{Joule} = 1\,C \cdot 1\,V$

In C.G.S unit $1\,\text{erg} = 1\,\text{statC} \cdot 1\,\text{statV}$

Dividing both the sides, $\frac{1\,\text{Joule}}{1\,\text{erg}} = \frac{1\,C}{1\,\text{statC}} \cdot \frac{1\,V}{1\,\text{statV}}$

$\Rightarrow \cancel{10^7} = 3 \times \cancel{10^9}\,{}^2 \times \frac{1\,V}{1\,\text{statV}}$

$\Rightarrow 1\,\text{statV} = 300\,V$

Formula :

$$C = \frac{Q}{V}$$

where $C$ = Capacitance in Farads

$Q$ = Charge in Coulomb.

$V$ = Electric potential in Volt

$\therefore$ $1 \text{ Farad} = \frac{1 \text{ Colomb}}{1 \text{ Volt}}$ —— (1)

In C.G.S System of units

$1 \text{ Stat Farad} = \frac{1 \text{ Stat C}}{1 \text{ Stat V}}$ —— (2)

A relation between Farads and Stat Farads can be obtained

Dividing both the sides of (1) and (2), we get

$$\frac{1 \text{ Farad}}{1 \text{ Stat Farad}} = \frac{1\,C}{1\,V} \times \frac{1 \text{ Stat V}}{1 \text{ Stat C}}$$

$$= \frac{1\,C}{1 \text{ Stat C}} \times \frac{1 \text{ Stat V}}{1\,V}$$

$$= 3 \times 10^9 \times 300$$

$$= 9 \times 10^{11}$$

$\therefore$ 1 Farad = $9\times10^{11}$ stat farad.

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16. We know that $C = \dfrac{Q_1}{V_1}$

Let $C = \dfrac{Q}{V_1} = \dfrac{Q_1}{150}$

$\Rightarrow 3\times10^{-6}\times150 = Q_1$

$\Rightarrow Q_1 = 45\times10^{-5}$ columb.

$Q_2 = CV_2 = 3\times10^{-6}\times500$

$= 15\times10^{-4}$ columb.

$\Delta Q = Q_2 - Q_1 = 15\times10^{-4} - 45\times10^{-5}$

$= 10^{-4}(15 - 4.5)$

$= 10^{-4}\times10.5$ columbs.

$\Delta t = 5.25\times10^{-3}$ sec.

$I = \dfrac{\Delta Q}{\Delta t} = \dfrac{10^{-4}\times\cancel{10.5}^{\,2}}{\cancel{5.25}\times10^{-3}} = 2\times10^{-1}$

$= 0.2$ A

Problem 1. When capacitors are connected are series the effective capacitance ($C_s$) is given by $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots$

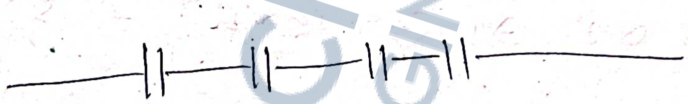
When Capacitors are connected in parallel the effective Capacitance ($C_p$) is given by

$$C_p = C_1 + C_2 + C_3 + \text{-----}$$

1. Several 1 MF capacitors are provided. How you will connect them to get capacitances like 0.25, 0.5, 0.75, 1.5, 2, 2.5 MF.

Ans:

(a) To get 0.25 MF,

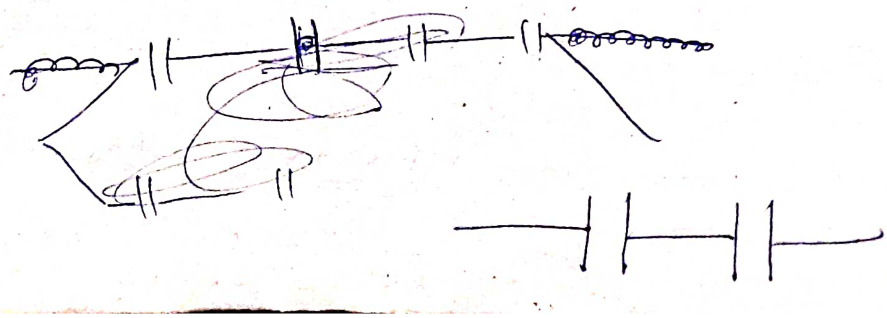


$$\frac{1}{C_s} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$$

$$\Rightarrow \frac{1}{C_s} = 4$$

$$\Rightarrow C_s = \frac{1}{4} = 0.25 \text{ MF}$$

(b) To get 0.5 MF,



$$\frac{1}{C_s} = \frac{1}{1} + \frac{1}{1}$$

$$\Rightarrow \frac{1}{C_s} = 2$$

$$\Rightarrow C_s = \frac{1}{2} = 0.5 \mu F$$

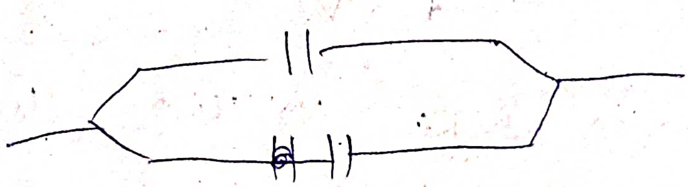
(iii) To get $.75 \mu F$

$$C_p = C_1 + C_2$$
$$= .25 + .5$$
$$= 0.75 \mu F$$

(iv)

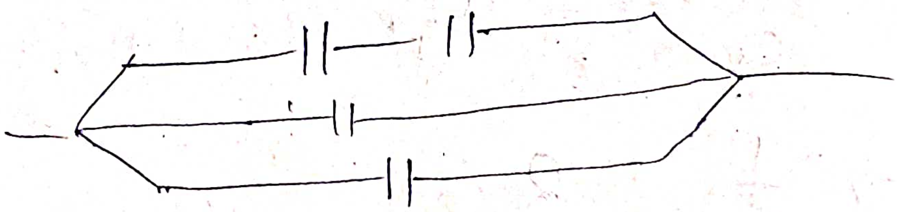
$$C_p = C_1 + C_2$$
$$= 1 + .5$$
$$= 1.5 \mu F$$

(v)



$$C_p = C_1 + C_2 = 1 + 1 = 2 \mu F$$

Q.7



$$C_p = C_1 + C_2 + C_3$$
$$= 0.5 + 1 + 1$$
$$= 2.5\ \mu F$$

---

## Electron volt (eV)

It is a unit of work or energy which is used in atomic and nuclear physics.

It is defined as the amount of workdone to displace an electron between two points at a potential difference of 1 volt.

$$\Delta W = q \cdot \Delta V$$
$$= 1.6 \times 10^{-19}\ \text{coul} \times 1V$$
$$= 1.6 \times 10^{-19}\ \text{Joule}$$
$$= 1\ eV$$

Bigger units like million electron volt, Giga electron volt are used.

$$1\,MeV = 10^6\ eV$$
$$= 10^6 \times 1.6 \times 10^{-19}\ \text{Joule}$$
$$= 1.6 \times 10^{-13}\ \text{Joule}$$

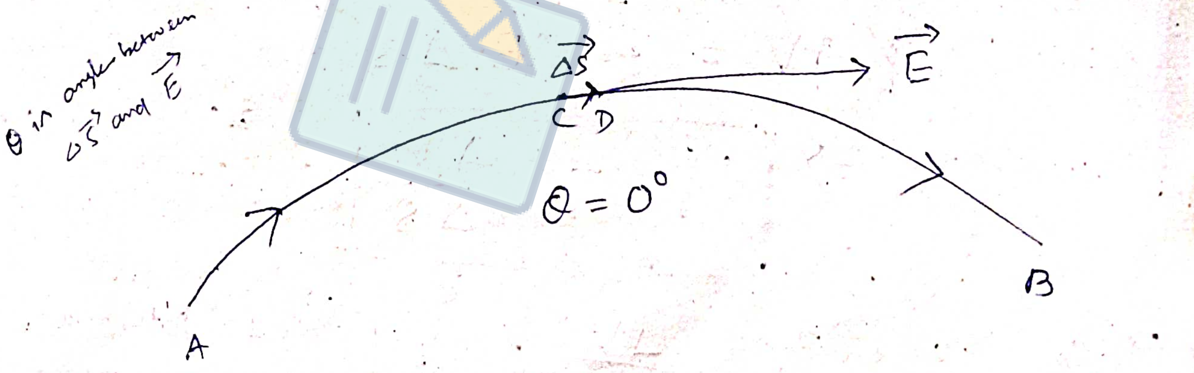
$$1\,GeV = 10^9\ eV$$
$$= 10^9 \times 1.6 \times 10^{-19}\ \text{Joule}$$
$$= 1.6 \times 10^{-10}\ \text{Joule}.$$

## Relation between electric field intensity and electric potential

**Case - I**

Two points C and D on the electric line of force, are very close.

Lets consider an electric line of force AB on which C and D are two points - very close to one another.




The tangent at C represents the electric field intensity and $\vec{CD} = \Delta\vec{S}$ is the displacement vector.

Workdone to shift a charge $+q$ from the point C to the point D

$= \Delta W$ (say)

$= q . \Delta V$ ——— (i)

But $\Delta W =$ Force $\times$ displacement

$= q\vec{E} \cdot \vec{\Delta S}$

$= qE\Delta S \cos 0^\circ$

$= qE\Delta S$

A $-$ve sign can be given to indicate that the workdone is $-$ve

$\therefore \Delta W = -qE\Delta S$ ——— (ii)

Equating these two expressions (i) and (ii), we get

$\cancel{q}\Delta V = -\cancel{q}E\Delta S$

$\Rightarrow \boxed{E = -\dfrac{\Delta V}{\Delta S}}$

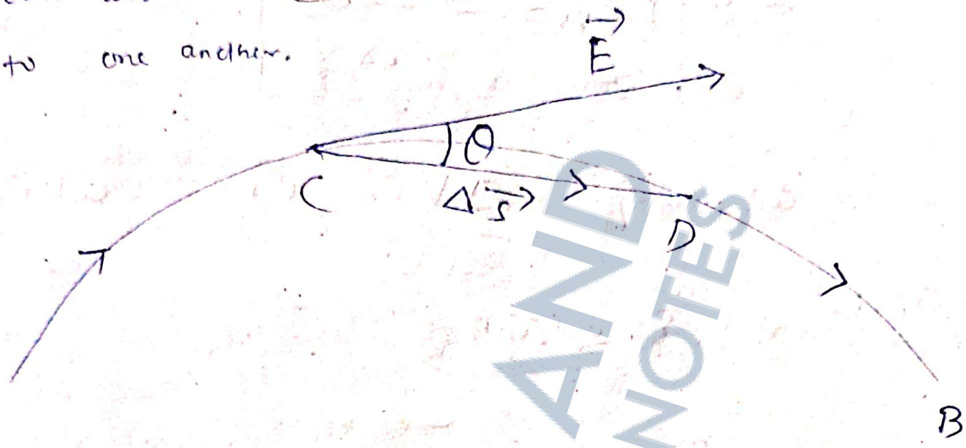
This quantity $\dfrac{\Delta V}{\Delta S}$ is called potential gradient.

Thus electric field intensity $= -$ Potential gradient.

Case - II

Two points C and D on the electric line of force, are not very close.

Lets consider an electric lines or force AB on which C and D are two points not close to one another.




The tangent at C represents the electric field intensity and $\vec{CD} = \Delta\vec{s}$ is the displacement vector.

Work done to shift a charge +q from the point C to the point D

$= \Delta W$ (say)

$= q \cdot \Delta V$ ——— (iii)

But $\Delta W =$ Force × displacement

$= q\vec{E} \cdot \Delta\vec{s}$

$= qE\,\Delta s \cdot \cos\theta$

A -ve sign can be given to indicate that the workdone is -ve

$\therefore \Delta W = -qE\Delta s\cos\theta$ ——— (iv)

Equating these two expressions (iii) and (iv), we get

$$q\,\Delta V = -q E \Delta S \cos\alpha$$

$$\Rightarrow \boxed{E\cos\theta = -\frac{\Delta V}{\Delta S}}$$

The quantity $\frac{\Delta V}{\Delta S}$ is called potential gradient

Thus component of electric field intensity along displacement direction = − Potential gradient.

Problem → 1.

Charges of +10 statC are placed at the corner points A, B and D of a square of side length 5 cm. Calculate the amount of workdone to shift a charge of 10 statC from the point O (Point of intersection of diagonals) upto the point C

Ans: − 30.7 erg

# Capacitor or Condenser

[See for dimension in 1st yr Kinem...]

It is a device to store charges. The Capacity or Capacitance is defined as the amount of charge required to raise the potential by 1 unit.

i.e $\boxed{C = \frac{Q}{V}}$

[Dimension of C = $[M^{-1} L^{-2} T^{4} A^{2}]$]

In C.G.S system of units, the charge is expressed in e.s.u or Stat C, and potential in e.s.u or stat V

Capacity in e.s.u or Stat Farad.

$\therefore$ 1 Stat Farad = $\frac{1 \text{ Stat C}}{1 \text{ Stat V}}$

In M.K.S system or S.I system or practical system of measurement, the electric charge is measured in Coulomb, potential in Volt and Capacity in Farad.

$\therefore$ 1 Farad = $\frac{1 \text{ Coulomb}}{1 \text{ Volt}}$ or $\frac{1 \text{ Coul}}{1 \text{ volt}}$

## Defn of farad

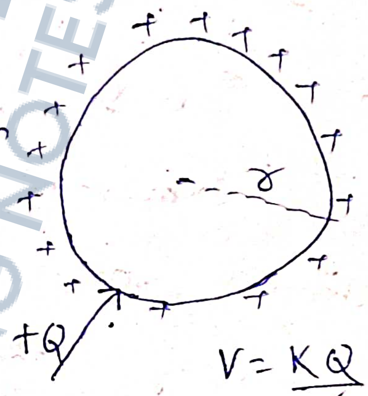
A body is said to possess a capacity of 1 Farad when its potential is raised by 1 volt due to supply of charge

Of amount 1 Coulomb.

Smaller units like micro Farad ($\mu F$), micro micro Farad or Pico farad ($\mu\mu F$ or $PF$) are also used.

## Capacity of a metalic Sphere

The Charge supplied to a metalic sphere is uniformly distributed on its surface. From an application of Gauss theorem, it has been proved that all the charges behave as if they are concentrated at the centre.




$V = \frac{KQ}{r}$

Electric potential at any point on the surface is $V = \frac{KQ}{r}$

Capacity of the Sphere $= C = \frac{Q}{V} = \frac{Q}{\frac{KQ}{r}}$

$$= \frac{r}{K}$$

In C.G.S system of units, $K = 1$ so that $C = r$

i.e Capacity of a metalic sphere in Stat F is numerically equal to the radius or the sphere expressed in centimeter.

In M.K.S ~~centers~~ system of units

$$K = 9\times10^9$$
$$= \frac{1}{4\pi\epsilon_0}$$

$$\therefore C \text{ in Farad} = \frac{\text{Radius of the sphere expressed in meter}}{9\times10^9}$$

$$= \frac{r}{\frac{1}{4\pi\epsilon_0}}$$

$$= 4\pi\epsilon_0 r$$

## Energy of a Charged Capacitor

The amount of workdone to bring the charges from infinity up to the capacitor is stored in c.t. as electrostatic potential energy.

During the charging process, gradually more and more work has to be done because the charges already present on the capacitor repel the fresh charge that is being added.

At any instant of time, let the charge on the capacitor be $+q$. Electric potential developed be $V$ units.

$$\therefore \quad C = \frac{q}{V}$$

$$\Rightarrow V = \frac{q}{C}$$

The amount of work done to bring an additional charge $+dq$ is given by

$$dW = dq.(V - V_\infty)$$

$$= dq\left(\frac{q}{C} - 0\right)$$

$$= \frac{1}{C}(q.dq)$$

Integrating both the sides of the above eqn with proper limits, we get

$$\int_0^W dw = \frac{1}{C}\int_0^Q q.dq$$

$$\Rightarrow (W)\Big|_0^W = \frac{1}{C}\left(\frac{q^2}{2}\right)\Big|_0^Q$$

$$\Rightarrow W - 0 = \frac{1}{C}\left(\frac{Q^2}{2} - 0\right)$$

$$\Rightarrow W = \frac{Q^2}{2C}$$

If the final potential developed in the capacitor be $V_f$, then $C = \frac{Q}{V_f}$

$$\Rightarrow Q = C V_f.$$

$$\therefore \boxed{W = \frac{Q^2}{2C} = \frac{(C \cdot V_f)^2}{2C} = \frac{1}{2} C V_f^2}$$

$\therefore$ Energy stored in the capacitor is

$$\frac{1}{2} C V_f^2$$

## Sharing of charge between two capacitors initially at different potentials, but connected by a fine metallic wire

If these two spheres be connected by a metalic wire, then charges will flow from the sphere with higher potential to the sphere having lower potential till their potentials become ~~same~~ same.

Let $V_1 > V_2$


$C_1 = r_1 = \frac{Q_1}{V_1}$

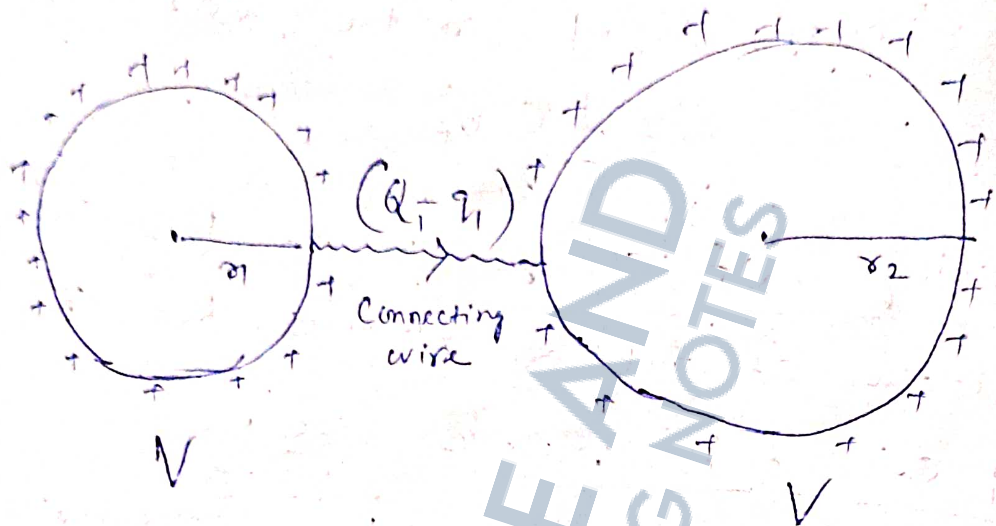
(In C.G.S units)

$C_2 = r_2 = \frac{Q_2}{V_2}$

$\Rightarrow V_1 = \frac{Q_1}{C_1}$

$\Rightarrow V_2 = \frac{Q_2}{C_2}$.

Let this common potential be V units and the charges present in the spheres be $+q_1$ and $+q_2$




$C_1 = r_1 = \frac{q_1}{V}$

$C_2 = r_2 = \frac{q_2}{V}$

$\Rightarrow q_1 = C_1 V$

$\Rightarrow q_2 = C_2 V$

But charge is always conserved.

i.e Total charge of two spheres before contact = Total charge of the two spheres after contact

$\Rightarrow Q_1 + Q_2 = q_1 + q_2$

$\Rightarrow C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$
$= (C_1 + C_2) V$

$\Rightarrow \boxed{V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}}$ = Common potential of the two capacitors after contact

Total electrostatic energy present in the two capacitors before contact

$$= E_i$$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

Total electrostatic energy present in the two ~~[illegible]~~ capacitors after energy

$$= E_f$$

$$= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

It is experimentally found that $E_i > E_f$

$\Delta E = E_i - E_f$ = Electrostatic energy converted into heat energy in the connecting wire which is radiated into the atmosphere

Problems:

1. Two metallic spheres of radii 3 cm and 5 cm are charged to potential 10 and 15 ~~[illegible]~~ e.s.u. respectively. They are then connected by a thin metallic wire, calculate the loss of electric energy in this process. What happens to the energy?

Ans: 23.4375 erg.

3. Two spheres of 2 and 6 cm radii are charged respectively with 80 and 30 units of electricity. Compare their potentials. If they are connected by a fine wire, how much electricity will pass along it.

Ans: → 8 : 1 , 52.5 e.s.u

4. When a charge of 50 units is given to a sphere, it is found to have a potential 20. After being connected to a second sphere, the potential falls to 8. Find the radius of second sphere. ~~Find the radius of second sphere~~ (uncharged)

Ans: 3.75 cm

5. 4 metalic spheres of diameters 4 cm, 5 cm, 8 cm and 10 cm are joined together by a fine metalic wire and a charge of 810 e.s.u is imparted to the system. Find the charge on each sphere and their common potential.

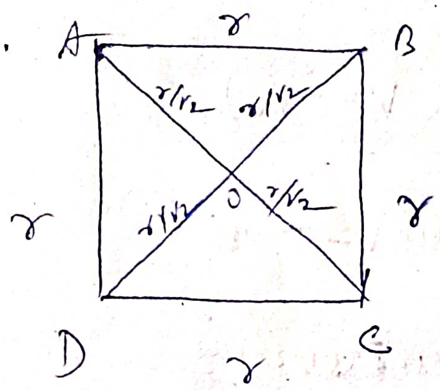
Ans → 120, 150, 240, 300 stat C, 60 stat V

— u —

$Q_1 = \frac{50 \text{ units}}{}$ , $\frac{V_1 = 20}{V}$ $Q_1 + Q_2 = q_1 + q_2$

$\Rightarrow 50 =$ $C_1 V + C_2 V$

## Problems:

1. 


The potential developed at C due to charges at A, B, D

is $V_C = \frac{Kq}{r\sqrt2} + \frac{Kq}{r} + \frac{Kq}{r}$

$= \cancel{\frac{1\cdot10^{-2}}{5\sqrt2} + \frac{1\cdot10^{-2}}{5} + \frac{1\cdot10^{-2}}{5}}$

$\cancel{= 3\sqrt2}$

$= \frac{Kq}{r}\left(\frac{1}{\sqrt2} + 1 + 1\right)$

$= \frac{1\cdot10^{-2}\sqrt2}{\cancel{5}}\left(\frac{1+\sqrt2+\sqrt2}{\sqrt2}\right)$

$= \sqrt2\,(1+2\sqrt2)$

$= 4+\sqrt2$

Potential developed at O due to charges at A, B, D is $V_O =$

$\frac{Kq}{r/\sqrt2} + \frac{Kq}{r/\sqrt2} + \frac{Kq}{r/\sqrt2}$

$= \sqrt2\frac{Kq}{r} + \sqrt2\frac{Kq}{r} + \sqrt2\frac{Kq}{r}$

$= 3\times\sqrt2\frac{Kq}{r} = \frac{3\times\sqrt2\times1\cdot10^{2}}{5} = 6\sqrt2$

When a 10 statc charge is shifted from O to C, then amount of workdone is $q\,(V_c - V_o)$

$$= 10\,(4 + \sqrt{2} - 6\sqrt{2})$$

$$= 10\,(4 - 5\sqrt{2})$$

$$= 10\,\{4 - 5(1.414)\}$$

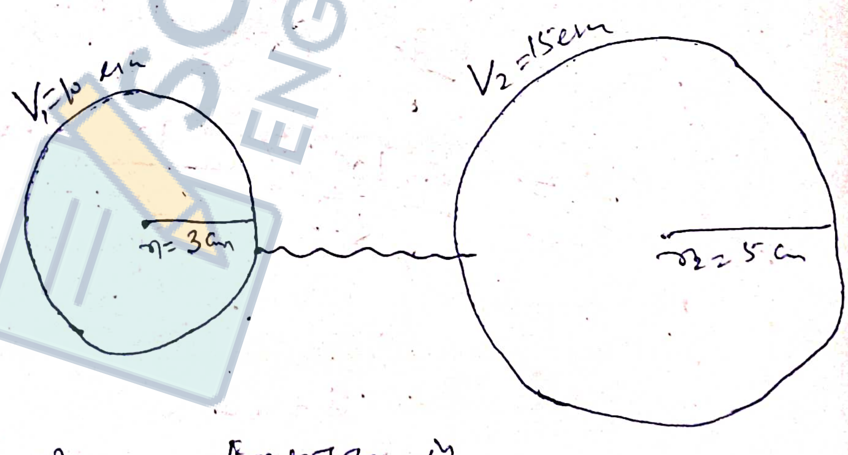
$$= 10\,\{4 - 7.070\}$$

$$= 10 \times (-3.07)$$

$$= -30.7 \text{ erg}$$

∴ The Amount of workdone $-30.7$ erg.

2.




Loss of Energy is

$$\Delta E = E_i - E_f$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}C_1\left(V_1^2 - V^2\right) + \frac{1}{2}C_2\left(V_2^2 - V^2\right)$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$C = ?$ stat farad

$C_1 = 3$ stat farad

$C_2 = 5$ stat farad

$$= \frac{3 \cdot 10 + 5 \cdot 15}{3+5}$$

$$= \frac{30+75}{8}$$

$$= \frac{105}{8} \text{ Stat V}$$

$$UE = \frac{1}{2} \cdot 3\left(100 - \frac{(105)^2}{8^2}\right) + \frac{1}{2} 5 \cdot \left(225 - \frac{(105)^2}{8^2}\right)$$

$$= \frac{3}{2}\left(\frac{6400 - 11{,}025}{64}\right) + \frac{5}{2}\left(\frac{14400 - 11{,}025}{64}\right)$$

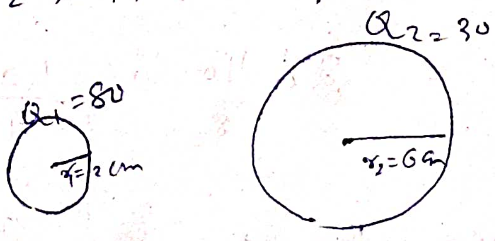
$$= \frac{3}{2} \times \frac{-4625}{64} + \frac{5}{2} \times \frac{3{,}375}{64}$$

$$= \frac{-13875 + 16875}{64 \times 2}$$

$$= \frac{3000}{128}$$

$$= 23.4375 \text{ erg}$$

3.




(i) $$\frac{V_1}{V_2} = \frac{\frac{q_1}{c_1}}{\frac{q_2}{c_2}} = \frac{\frac{80}{2}}{\frac{30}{6}} = \frac{40}{5} = 8:1$$

11)

$C = \frac{Q}{V}$, $V = \frac{Q}{C}$

$V_1 = \frac{Q_1}{C_1} = \frac{80}{2} = 40$ volt

$V_2 = \frac{Q_2}{C_2} = \frac{30}{6} = 5$ volt.

~~$V = \frac{V_1 + V_2}{2} = \frac{40+5}{2} = \frac{45}{2} = 22.5$ Volt~~ Start

~~Let the remaing charge after equilibrium be $q_1$ and $q_2$~~

~~$q_1 = C_1 V = 2 \times 22.5 = 45$~~ start calm.

~~Amount of electricity flows $Q_1 - q_1$~~

~~$= 80 - 45$~~

~~$= 35$~~

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{2 \cdot 40 + 6 \cdot 5}{2+6} = \frac{80+30}{8} = \frac{110}{8}$$

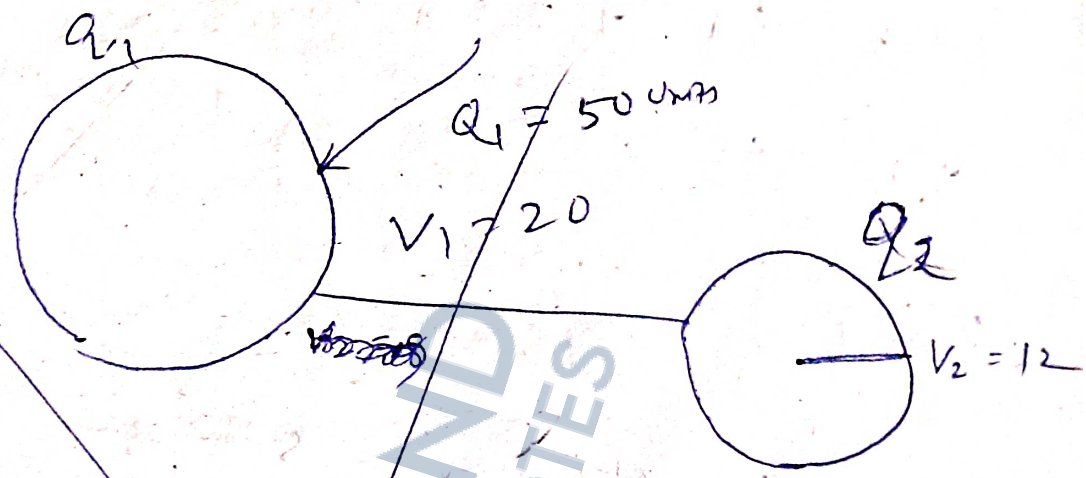
Let the remaining charge at the first sphere after equilibrium be $q_1$.

$$q_1 = C_1 V = \cancel{2} \times \frac{110}{\cancel{8}\,4} = \frac{110}{4}$$

Amount of electricity pass in the wire $= Q_1 - q_1$

$$= 80 - \frac{110}{4} = \frac{320-110}{4} = \frac{210}{4} = 52.5$$ esu

4.




Charge is always conserved

$\Rightarrow Q_1 + 0 = q_1 + q_2$

$= C_1V + C_2V$ ——— (i)

$\Rightarrow 50 =$

$Q_1 = 50$ units, $V_1 = 20V$, $C_1 = \frac{Q_1}{V_1} = \frac{50}{20} = \frac{5}{2}$

$\therefore 50 = \frac{5}{2} \cdot V + C_2V$

$50 = \frac{5V + 2C_2V}{2}$

$\Rightarrow 100 = 5V + 2C_2V$ ——— (ii)

But $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$\Rightarrow V = \dfrac{\frac{5}{2}\cdot 20 + C_2\cdot 12}{\frac{5}{2} + C_2}$

$\Rightarrow V = \dfrac{50 + 12C_2}{\frac{5+2C_2}{2}} = \dfrac{2(50+12C_2)}{5+2C_2}$

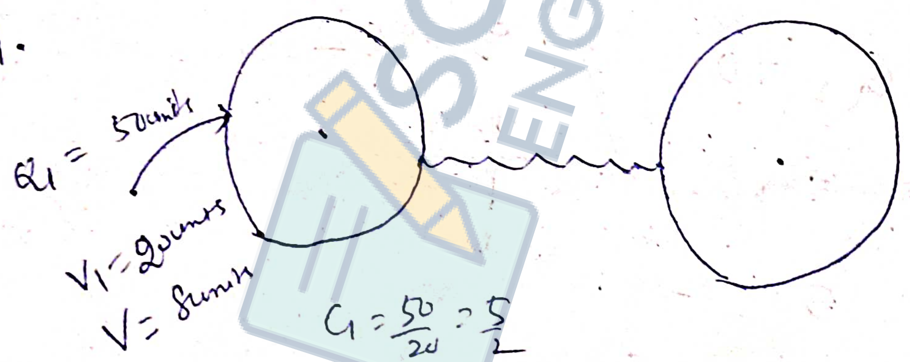
$\Rightarrow 5V + 2C_2V = 100 + 24C_2$

$\Rightarrow 5V - 22C_2V = 100$ ——— (iv)

Equating eqn (iii) and (iv) we get

$5V + 2C_2V = 5V - 22C_2V$.

4.




$C_1 = \frac{50}{20} = \frac{5}{2}$

Charge is always conserved

$Q_1 + Q_2 = q_1 + q_2$

$\Rightarrow Q_1 + 0 = C_1V + C_2V$

$\Rightarrow 50 = \frac{5}{2}\cdot 8 + C_2\cdot 8$

$\Rightarrow 50 - 20 = C_2 8$

$\Rightarrow C_? = \frac{30}{8} = \frac{15}{4}$ Stat farad.

Radius $= \frac{15}{4}$ i.e 3.75 cm.

5. 4 metalic spheres have diameter 4, 5, 8, 10 cm

radius are 2, 2.5, 4, 5 cm.

Capacitance are 2, 2.5, 4, 5 stat farad.

The charge given to the four spheres is ~~oo~~ distributed according to their capacitance.

After distribution they have a common potential V.

810 e.s.u charge is given to 4 sphere

i.e $q_1 + q_2 + q_3 + q_4 = 810$

$\Rightarrow C_1V + C_2V + C_3V + C_4V = 810$

$\Rightarrow 2V + 2.5V + 4V + 5V = 810$

$\Rightarrow 13.5V = 810$

$\Rightarrow V = \frac{810}{13.5} = 60$ stat V.

Charge on 1st sphere $= C_1V = 2 \times 60 = 120$ stat c

2nd " $= C_2V = 2.5 \times 60 = 150$ "

3rd " $= C_3V = 4 \times 60 = 240$ "

4th " $= C_4V = 5 \times 60 = 300$ "

Common ~~...~~ potential is $= 60$ stat V

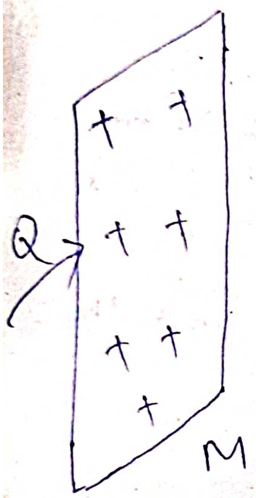
Ans → the doubts [box] before 21-ap

## Parallel plate capacitor

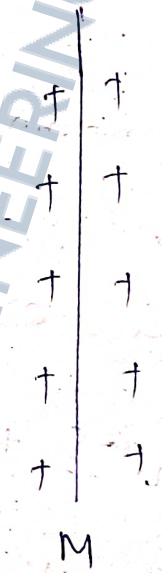
It is a device to increase the capacitance of a metalic plate by bringing another uncharged metalic plate near it.

**Step-1**

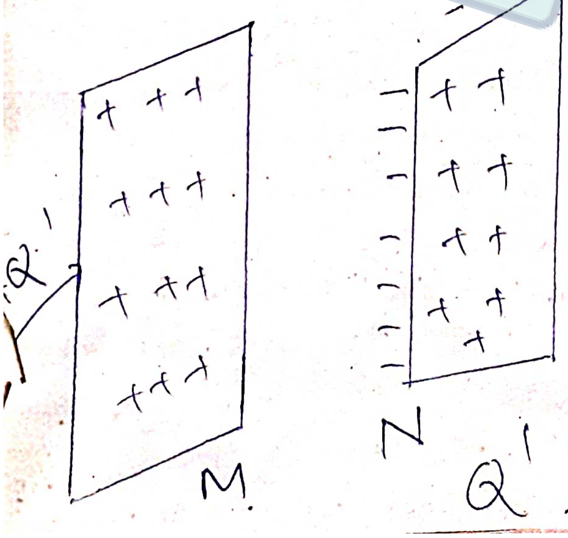
Let the max$^m$ charge that can be stored in a metalic plate M be +Q units.



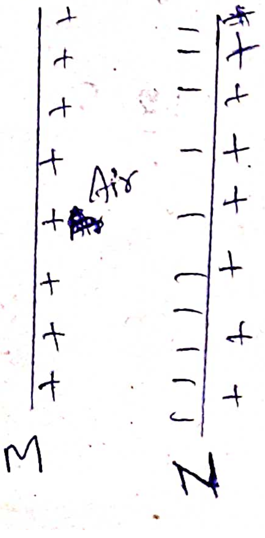

Or




**Step-II**




or



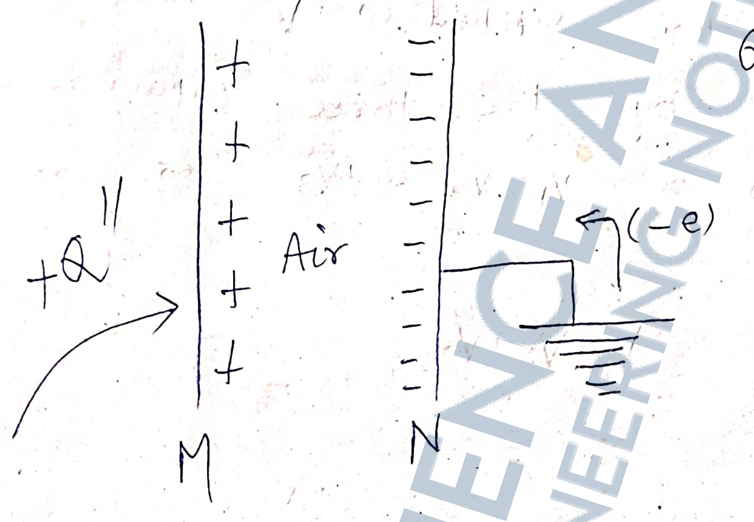

$Q' > Q$

when another uncharged, similar metalic plate N is brought near it, the capacity of the plate M increases.

## Step-3

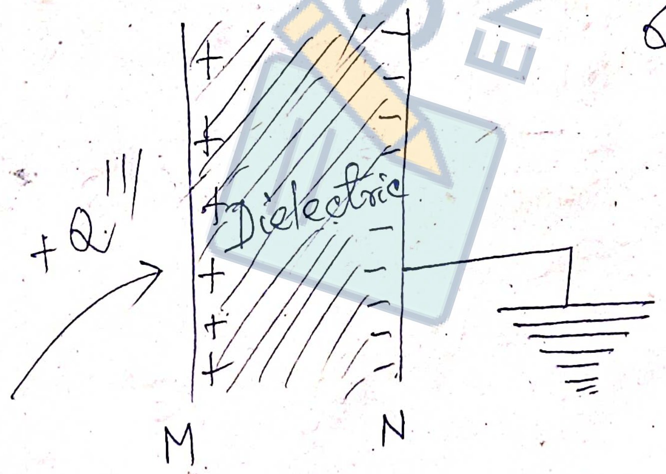
When the plate N is earthed, the capacity of the plate M is further increased.

$Q'' > Q'$




## step-4

$Q''' > Q''$



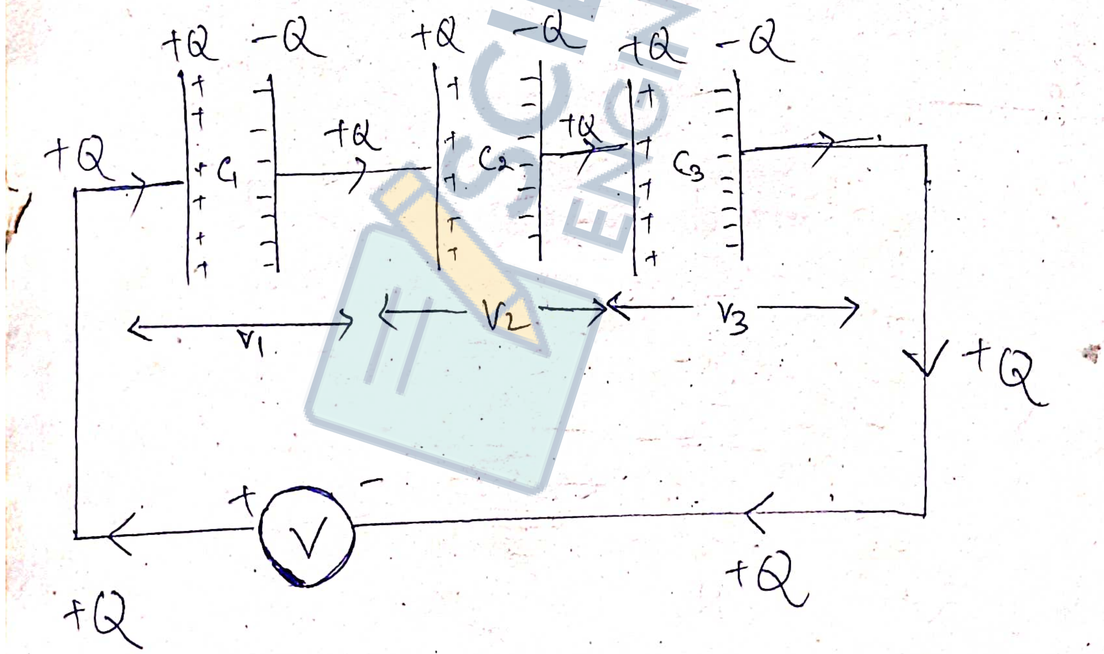

When a dielectric material is introduced in between the two plates M and N, then the capacitance of M is further increased.

# Grouping of Capacitors

1. Capacitors are said to be connected in series when the auxillary plate of one capacitor is connected to the main plate of another capacitor and the free ends are connected to the terminals of a battary

The voltage supplied is $V$ units which is shared by the three capacitors. If these voltages be $V_1$, $V_2$ and $V_3$, then

$$V = V_1 + V_2 + V_3 \quad \text{——— (1)}$$

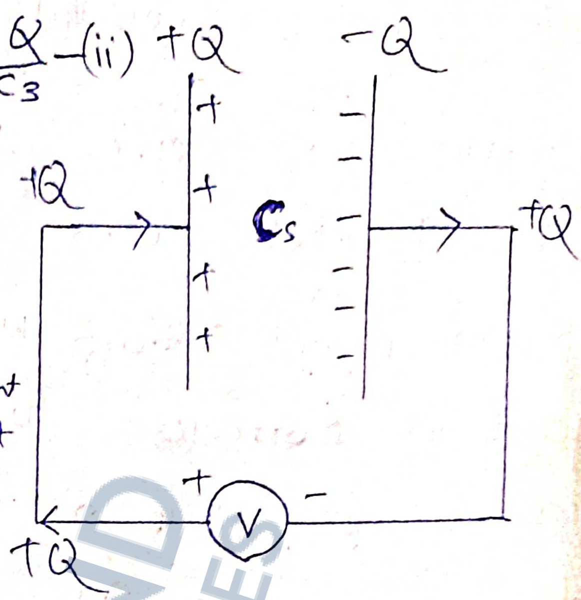



The same charge appears in each capacitor

$$\therefore \quad C_1 = \frac{Q}{V_1}, \quad C_2 = \frac{Q}{V_2}, \quad C_3 = \frac{Q}{V_3}$$

$\Rightarrow V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$ ——(ii)

If the equivalant capacitance of these threea capacitors be $C_s$, then




Equivalant circuit

$$C_s = \frac{Q}{V}$$

$$\Rightarrow V = \frac{Q}{C_s} \text{ ———— (iii)}$$

Using eqn s (ii) and (iii) in eqn (i), we get

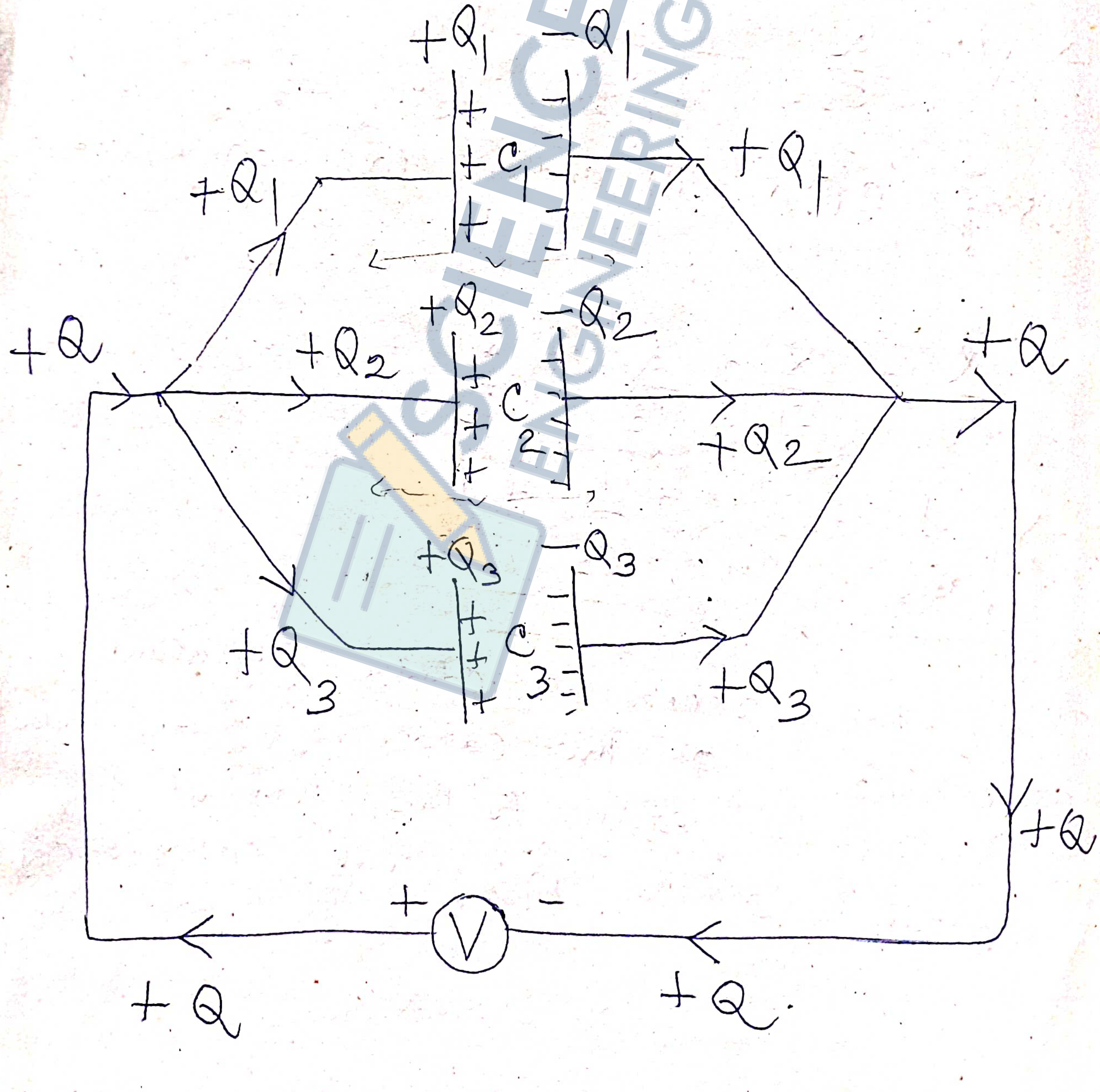
$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Thus, the reciprocal of the equivalant capacitance of the capacitors when connected in series is equal to the sum of the reciprocal of the individual capacitances

# Capacitors connected in parallel → See for dimensions

Capacitors are said to be connected in parallel when ~~the~~ the +ve plate plates or main plates of all the capacitors are connected to one point and the auxiliary plate or negative plate or all the capacitors are connected to another points and these two points are connected to the terminals of a battery.



The Voltage supplied is V units which is experienced by ~~each~~ three capacitors

The charge coming from battery is distributed into three parts such that

$$Q = Q_1 + Q_2 + Q_3 \quad \text{—— (i)}$$

Same Voltage appears in each capacitor

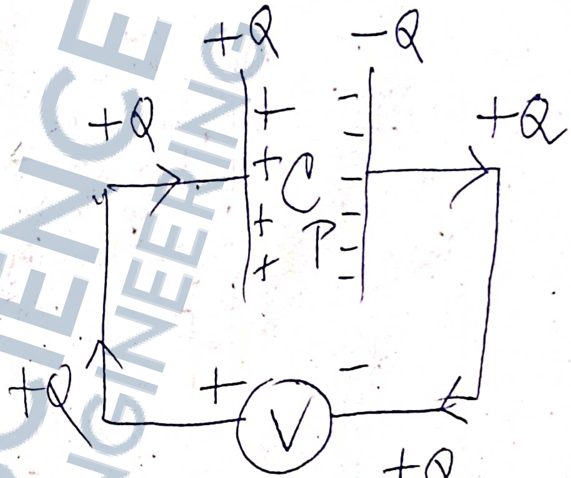
$$\therefore \quad C_1 = \frac{Q_1}{V}, \quad C_2 = \frac{Q_2}{V}, \quad C_3 = \frac{Q_3}{V}.$$

$$\Rightarrow \cancel{V = \frac{Q_1}{C_1}, \quad V = 0}$$

$$\Rightarrow Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

$$Q_3 = C_3 V \quad \text{—(ii)}$$




Equivalent circuit

If the equivalent capacitance of these three capacitors be $C_p$, then

$$C_p = \frac{Q}{V}$$

$$\Rightarrow Q = C_p V \quad \text{—— (iii)}$$

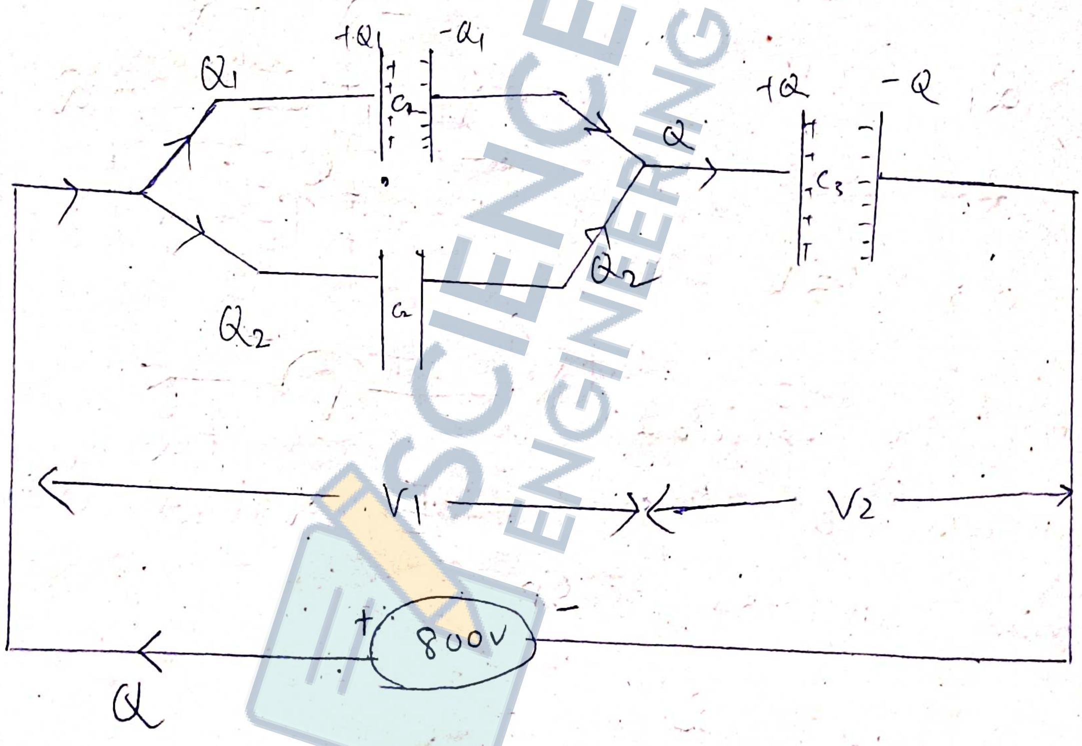
Using Eqn (ii) and (iii) in eqn (i), we get

$$C_p V = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow \boxed{C_p = C_1 + C_2 + C_3}$$

Thus, the equivalant capacitance of the capacitors when connected in parallel is equal to the sum of the individual capacitances.

Problem → 6.11 → 13






$C_1 = 2\mu F$ , $C_2 = 4\mu F$ , $C_3 = 3\mu F$

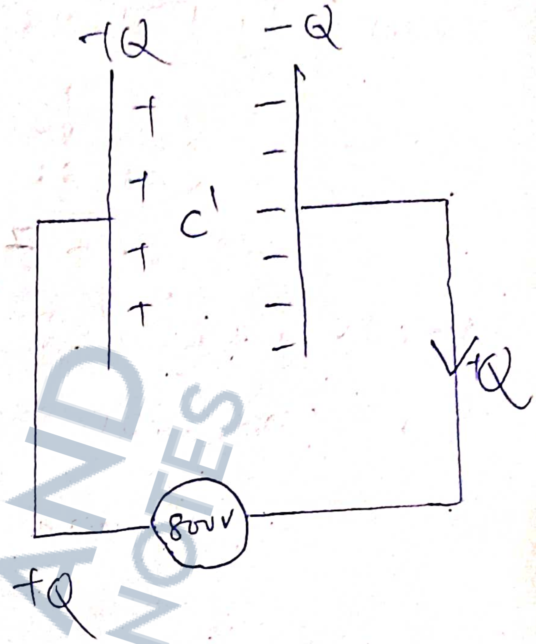
$C_1$ and $C_2$ are in ~~parallel connection~~ connect in parallel.

$\therefore$ $C_P = C_1 + C_2 = 2\mu F + 4\mu F = 6\mu F$

and $C_3$ and $C_P$ are connected in

Series .

$$\frac{1}{C_s} = \frac{1}{C_P} + \frac{1}{C_3}$$




$$\Rightarrow \frac{1}{C_s} = \frac{1}{6} + \frac{1}{3}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1+2}{6} = \frac{3}{6}$$

$$\Rightarrow C_s = \frac{6}{3} = 2\ \mu F$$

$= C'$

= Equivalant capacitance .

Now $C' = \frac{Q}{V} = \frac{Q}{800}$

$$\Rightarrow 2 \times 10^{-6} = \frac{Q}{800}$$

$$\Rightarrow Q = 1600 \times 10^{-6} \text{ Farad}$$

$= 1600\ \mu F$

This charge will appear in third capacitor

$$\cancel{\Rightarrow Q = \frac{C_3}{V}}$$

$$\cancel{\Rightarrow 1600\ \mu F = \frac{3\ \mu F}{V}}$$

$$\cancel{\Rightarrow V = \frac{3\ \mu F}{1600\ \mu F}}$$

$$C_3 = \frac{Q}{V_2}$$

$$\Rightarrow 3\ \mu F = \frac{1600\ \mu F}{V_2} \Rightarrow V_2 = \frac{1600}{3} = 533.33 \text{ volt}$$

$V = V_1 + V_2$

$\Rightarrow 800 = V_1 + 533.33$

$\Rightarrow V_1 = \begin{array}{r} 800.00 \\ -533.33 \\ \hline 266.67 \end{array}$

~~Also~~ So, $C_1, C_2$ has equal potential difference 266.67

Thus potential difference ~~between~~ across the capacitors are 266.67, 266.67 and 533.33 volt.

16 → 610 page

16. Three capacitors has capacitance 1 μF, 2 μF, 3 μF

Let $C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 3\mu F$

when they are connected in series, then

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$$

$$\Rightarrow C_s = \frac{6}{11}\mu F$$

In an equivalant circuit of $C_1, C_2, C_3$

$C_s = \frac{Q}{V} \Rightarrow \frac{6}{11} = \frac{Q}{300V}$

$$\Rightarrow Q = \frac{6 \times 300}{11}$$

$$= \frac{1800}{11} \, \mu C$$

This charge appears in each capacitor, because they are in series.

$$V_1 = \frac{Q}{C_1}$$

$$= \frac{\frac{1800}{11}}{1} = \frac{1800}{11} = 163.6 \text{ Volt}$$

$$V_2 = \frac{Q}{C_2} = \frac{\frac{1800}{11}}{2} = \frac{1800}{22} = 81.6 \text{ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{\frac{1800}{11}}{\frac{3}{1}} = \frac{1800}{33} = 54.4 \text{ V}$$

∴ The first capacitor or capacitance $1 \, \mu F$ will puncture.

(b) When the capacitors are connected in parallel then the voltage is equally experienced i.e. each capacitor will accure 300V.

So 3 capacitors will be punctured at the same time.

Q→ ~~4~~, ~~6~~, ~~7~~, ~~8~~, ~~9~~, ~~10~~, ~~11~~, ~~12~~, ~~14~~, ~~17~~, ~~18~~, 19, 2?

$\frac{1}{2} \varepsilon v^2$ (Ans – 400V) (Ans – 200V)

# Problems

4. Capacitor has capacitance 2.00 MF

$\therefore C = 2.00$ MF

Potential difference $V = 100V$

Charge on the capacitor $Q = C \times V$

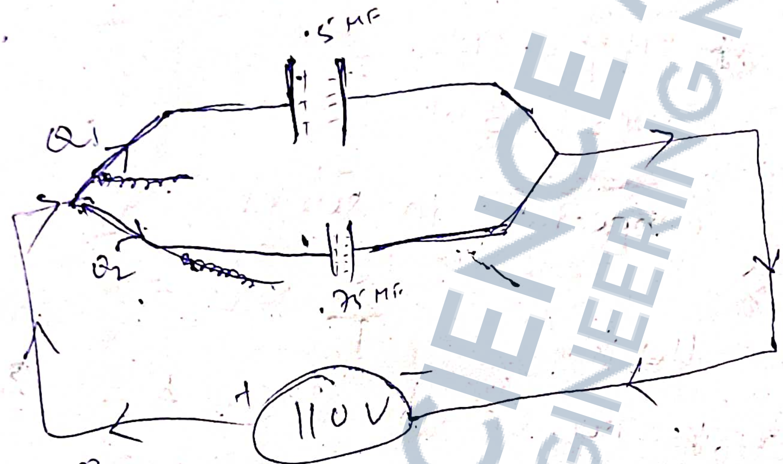
$= 2 \times 10^{-6} \times 100$

$= 2 \times 10^{-4}$ Coulomb

$= 200 \mu C$

(Ans)

6.




Q

$C_p = .5 \text{ MF} + .75 = 1.25 \text{ MF}$

$V = 110V$

$C_1 = .5$ MF, $C_2 = .75$ MF

~~Q~~ $C_p = \frac{Q}{V} = \frac{Q}{110V}$

$\Rightarrow 1.25 \text{ MF} = \frac{Q}{110V}$

$\Rightarrow Q = 137.5$ MF

$Q_1 = C_1 V = .5 \times 110 = 55\ \mu C.$

$Q_2 = C_2 V = .75 \times 110 = 82.5\ \mu C$

$\therefore$ The charge taken from the source is 137.5, each have charges $55\ \mu C$ and $82.5\ \mu C$

7. Two capacitors have capacitances

(ii) $C_1 = 3\ \mu F$

$C_2 = 5\ \mu F$

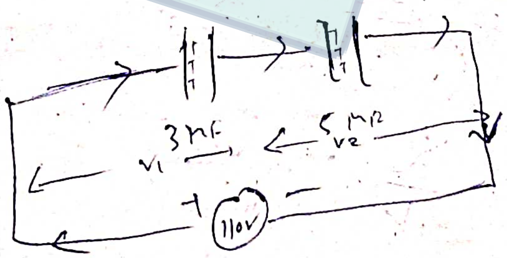
They are connected in series.

Potential $V = 110\ V$.

Potential difference across $3\ \mu F$ be $V_1$ ?

and $V_1 = ?$

Energy across $5\ \mu F$ be $\frac{1}{2} C_2 V_2^2$.




$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$$

$$\Rightarrow C_s = \frac{15}{8}\ \mu F$$

16/7/01

$C_S = \frac{Q}{V} \Rightarrow Q = C_S \cdot V$

$= \frac{15}{8} \mu F \times 110 V$

$= \frac{1650}{8} \mu C$

B1

Potential across $3 \mu F = V_1 = \frac{Q}{C_1} = \frac{\frac{1650}{8}}{3}$

$= \frac{\cancel{1650}^{550}}{8 \times \cancel{3}}$

$= 68.6$ Volt

Potential across $5 \mu F = V_2 = \frac{Q}{C_2} = \frac{\frac{1650}{8}}{5}$

$= \frac{165\cancel{0}}{4\cancel{0}}$

$= 41.25$ Volt

Energy in the $5 \mu F$ Capacitor

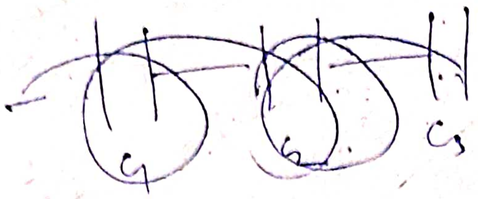
$= \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 5 \mu F \times (41.25)^2 \text{ volt}^2$

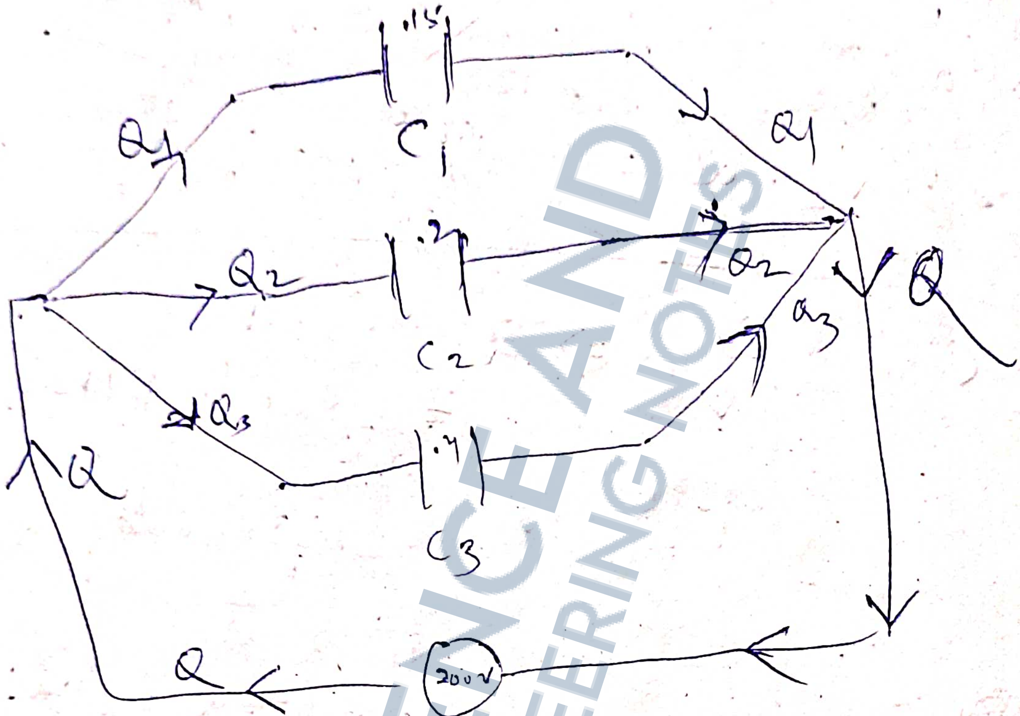
$= 2.5 \times 1701.5625 \times 10^{-6}$

$= .004253.90625 \mu$ Joule

8.

$C_1 = 0.15\,\mu F,\ C_2 = .2\,\mu F,\ C_3 = .4\,\mu F$






In parallel connection V is constant

$$Q = Q_1 + Q_2 + Q_3$$

$$C_1 = \frac{Q_1}{V} \Rightarrow V = \frac{Q_1}{C_1}$$

$$V = \frac{Q_2}{C_2}$$

$$V = \frac{Q_3}{C_3}$$

$C_1, C_2, C_3$ when connected in parallel then

$$C_P = C_1 + C_2 + C_3 = 0.15 + 0.2 + 0.4$$

$$= .75\ \mu F$$

(i) Charge on each capacitors,

$\cancel{C_1 Q = \frac{V}{Q_1}} =$

$C_1 = \frac{Q_1}{V} \Rightarrow Q_1 = C_1 V = (0.15\,\mu F) \times 200$

$= 30\,\mu\text{coulomb}.$

$Q_2 = C_2 V = (0.2) \times 200 = 40\,\mu C.$

$Q_3 = C_3 V = (0.4) \times 200 = 80\,\mu C.$

(ii) Total capacitance $C_P = 0.75\,\mu F$

(iii) Total charge $Q = Q_1 + Q_2 + Q_3$

$= (30 + 40 + 80)\,\mu C$

$= 150\,\mu C$

$= 1.5 \times 10^2\,\mu C$

or $Q = C_P V = (.75) \times (200) = 1.5 \times 10^2\,\mu C$

9. Potential difference 75.4 Volt is applied to a combination to the capacitors $1.25\,\mu F$, $0.572\,\mu F$ capacitor connected in series.

$C_1 = 1.25\,\mu F$, $C_2 = 0.572\,\mu F$

$V = 75.4$ volt

Charge is constant in series connection.

$C_S = \frac{Q}{V}$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{1.25} + \frac{1}{0.572} = \frac{0.572 + 1.25}{1.25 \times 0.572}$$

$$\Rightarrow C_s = \frac{1.25 \times 0.572}{1.822} = \frac{.71500}{1.822}$$

$$= \frac{71500}{182200}$$

$$= .39\ \mu F$$

The charge on each capacitors ~~$Q_1, Q_2, Q_3$~~ Q

$$C_s = \frac{Q}{V} \Rightarrow Q = C_s \times V$$

$$= (0.39\ \mu F) \times 75.4\ V$$

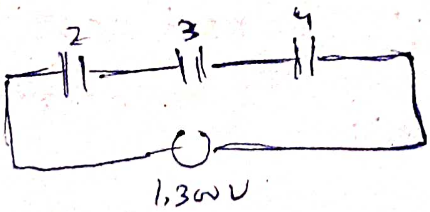
$$= 29.406\ \mu C$$

Potential difference ~~acros~~ across the 1.25 MF capacitor $= V_1 = \frac{Q}{C_1} = \frac{29.406}{1.25}$

$$= \frac{29.406}{.125}$$

$$= 23.5 \text{ volt}$$

10. $C_1 = 2$, $C_2 = 3$, $C_3 = 4$ MF



Since the Capacitors are connected in series

So charge (Q) is constant.

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = \frac{13}{12}$$

$$\Rightarrow C_s = \frac{12}{13}$$

$$C_s = \frac{Q}{V} \Rightarrow \frac{12}{13} = \frac{Q}{1300}$$

$$\Rightarrow Q = \frac{\cancel{1300}^{100} \times 12}{\cancel{13}} = 1200$$

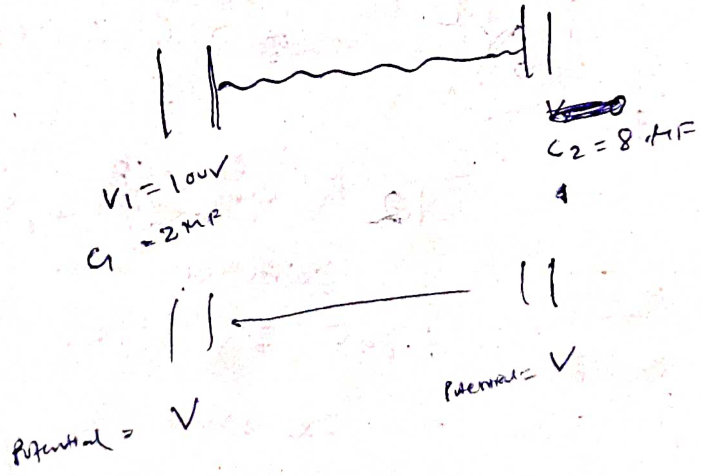
$$V_1 = \frac{Q}{C_1} = \frac{1200}{2} = 600\,V$$

$$V_2 = \frac{Q}{C_2} = \frac{1200}{3} = 400\,V$$

$$V_3 = \frac{Q}{C_3} = \frac{1200}{4} = 300\,V$$

11. $C_1 = 2\mu F$, $V_1 = 100\,V$

$C_2 = 8.0\mu F$, $V = ?$



Charge is always conserved

$$Q_1 + Q_2 = q_1 + q_2$$

$$\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow (2 \times 100V) + 0 = V(2+8) = 10V$$

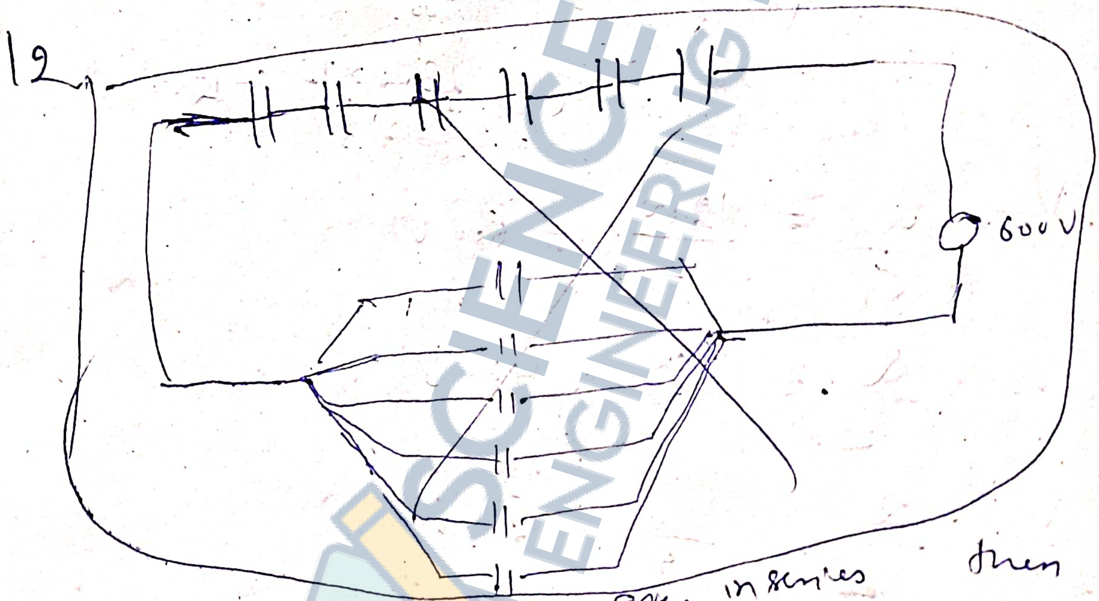
$$\Rightarrow 200\,\mu C \quad 10\,V$$

$$\Rightarrow V = 20 \text{ Volt}$$

~~Charge~~ potential of the second capacitor = 20 volt

Charge on 2nd capacitor $C_2V = 8 \times 20 = 160\,\mu C$.

12.




If 6, $\frac{1}{2}\,\mu F$ capacitors are in series then

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_6} = 6\left(\frac{1}{C}\right)$$

$$\Rightarrow \frac{1}{C_S} = \left\{6 \times \frac{1}{\frac{1}{2}}\right\} = 6 \times 2 = 12$$

$$\Rightarrow C_S = \frac{1}{12}\,\mu F = .083\,\mu F$$

$= 0.083 \times 10^{-6} F = 83 \times 10^{-3} \mu F$

When connected in parallel

$$C_P = 6 \times \left(\frac{1}{2}\,\mu F\right)$$

$$= 3\,\mu F$$

They ~~are assumed~~ connected in series.

$$\therefore \frac{1}{C_s} = \cancel{\frac{1}{\frac{1}{3}} + \frac{1}{6}}$$

$$\Rightarrow \frac{1}{C_s} = \cancel{3 + \frac{1}{6}}$$

When first group is connected to 600V

battery. $Q = CV$ ~~= .83 × 600~~

$= .083 \times 600$

$= 49.800$

$= 49.8 \mu C$

When 2nd group is connected to 600 V

$Q = CV = (3) \times 600 = 1800 \mu C$

14. Three capacitors capacitances

$C_1 = .1$, $C_2 = 0.2$, $C_3 = 0.5$

Connected in parallel

$C_p = C_1 + C_2 + C_3 = 0.1 + 0.2 + 0.5 = 0.8 \mu F$

This group connect with another group of $C_p = 0.8 \mu F$

~~are~~ connected in series

Then two groups

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = 2\left(\frac{1}{0.8}\right) = \frac{20}{8} = \cancel{8}$$

$\Rightarrow C_s = \frac{8}{20}$

Then another three capacitors connected in series.

$$\frac{1}{C_s} = \frac{1}{.1} + \frac{1}{.2} + \frac{1}{.5} = 10 + 5 + 2 = 17$$

$C_s = \frac{1}{17} \mu F$ , $C_P \cdot 8 \mu F$

then they are connected in series

$$\frac{1}{C_s} = \frac{1}{C_s} + \frac{1}{C_P} = \frac{1}{\frac{1}{17}} + \frac{1}{.8}$$

$$= \cancel{8 + \frac{10}{8} = \frac{64+10}{8} = \frac{74}{8}}$$

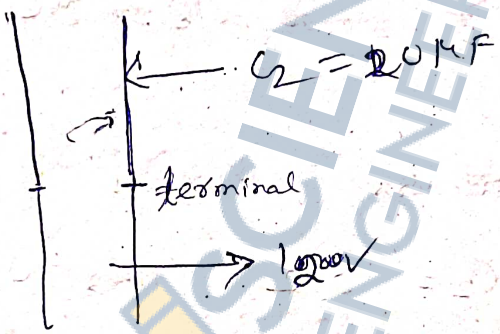
$$= 17 + \frac{10}{8} = \frac{136+10}{8}$$

$$\Rightarrow C_s = \frac{8}{146} = .0547 \mu F$$

$$= .0547 \times 10^6 pF$$

$$= 547 \times 10^2 pF$$

17.




$C_1 = 10 \mu F$

$$Q_1 = C_1 V_1 = 10 \mu F \times 1200 V = 12{,}000 \mu C$$

Charge is conserved.

$$Q_1 + Q_2 = q_1 + q_2$$

$$\Rightarrow 10^4 + 0 = C_1 V + C_2 V$$

$$\Rightarrow 12000 = 10\mu F \cdot V + 20\mu F \cdot V$$

$$= V(30\mu F)$$

$\Rightarrow V = \dfrac{12000}{30\,\mu F}$ ~~$= \frac{1200}{3}$ volt~~

$= 400$ volt

18. $C_1 = 5\,\mu F$

$V = 800V$

Energy discharged through conductance,

The given amount of energy during discharge,

= Energy stored in the capacitor

$= \frac{1}{2} \cdot 5 \times (800)^2$

$= \frac{1}{2} \times 5\,\mu F \times 64 \times 10^4$

$= \frac{1}{\cancel{2}} \times 5 \times 10^{-6} \times \cancel{64}^{32} \times 10^4$

$= 160 \times 10^{-2}$

$= 1.6$ Joule

19. $C_1 = 1\,\mu F$, $V_1 = 100\,V$

$C_2 = 1\,\mu F$, $V_2 = 200\,V$

$C_3 = 1\,\mu F$, $V_3 = 300\,V$

The capacitors are joined in the series

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = 1 + 1 + 1 = 3$

$\Rightarrow C_s = \frac{1}{3}\,\mu F$

20.

$C_1 = 2.0\mu F$, $C_2 = 3.0\mu F$, $C_3 = 6.\mu F$

$V = 60\ V$;

$$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6} = 1$$

$\Rightarrow C_s = 1 \times 10^{-6} F$

$C_p = C_1 + C_2 + C_3 = 2+3+6 = 11\mu F$

Energy stored in the capacitors, when connected in series

$$= \frac{1}{2} C_s V^2$$

$$= \frac{1}{2} \cdot 1 \cdot (60)^2 \times 10^{-6}$$

$$= \frac{3600}{2} \times 10^{-6}$$

$$= 1800 \times 10^{-6}$$

$= .0018$ Joule

Energy stored in the capacitors, when connected in parallel

$$= \frac{1}{2} C_p V^2$$

$$= \frac{1}{2} \cdot (11) \times 10^{-6} \times 3600$$

$$= 5.5 \times 36 \times 10^{-4}$$

$$= 198 \times 10^{-4}$$

$= .0198$ Joule $\approx 0.02$ Joule

21. Capacitor has Capacitance $C_1$ 10 μF

$V_1 = 1000\,V$

Uncharged sphere, $C_2 = 40\,\mu F$

Charge is conserved

$$Q_1 + Q_2 = q_1 + q_2$$

$$\Rightarrow C_1V_1 + 0 = C_1V + C_2V$$

$$\Rightarrow (10\,\mu F) \times (1000\,V) = V(10+40) = 50\,V$$

$$\Rightarrow \frac{10000\,\mu C}{50} = V$$

$$\Rightarrow 200 \text{ Volt} = V$$

∴ Net potential difference 200 volt

~~Air dielectric~~ ~~between beginning of fraction~~



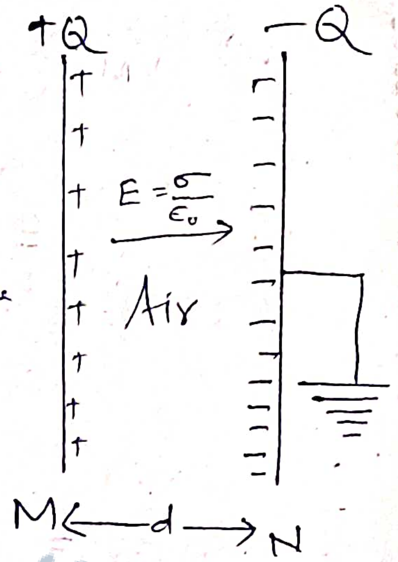
## Expressions for the capacitance of parallel plate capacitors

Case-1

Air as the dielectric present in between the two parallel plates.

From an application of Gauss theorem, it has been proved that the electric field

Intensity in between the two oppositely charged plate kept near each other is given by $\vec{E} = \dfrac{\sigma}{\epsilon_0}$, directed from the +ve plate towards the −ve plate.




where $\sigma$ = Surface charge density
= Charge per unit surface area.
$= \dfrac{Q}{A}$

where A = ~~Area~~ Total surface area of the plate M

$V_M - V_N$ = Potential difference between the plates M and N.
= Workdone to shift a +1 charge from the plate N to the plate M

My hints because work should be +ve, shifting +1 charge towards +ve is +ve, because of repulsion)

= Force × displacement
= ~~Q.E × d~~
$= q.E \times d$
$= 1 . \dfrac{\sigma}{\epsilon_0} . d$

where $\epsilon_0$ = Permitivity of free space or ~~of~~ ~~air~~. air

But $V_N = 0$ ( being connected to the earth which is at 0 potential.

$\therefore \quad V_M = \dfrac{\sigma d}{\epsilon_0}$

$\therefore \quad C$ = Capacity of the parallel plate capacitor

$$= \frac{Q}{V_M}$$

$$= \frac{\sigma A}{\frac{\sigma d}{\epsilon_0}}$$

$$= \frac{\epsilon_0 A}{d}$$

$$\therefore \quad \boxed{C = \frac{\epsilon_0 A}{d}}$$

Thus $C \propto A$, when $d$, is kept constant

$C \propto \dfrac{1}{d}$, when $A$ is kept constant.

## Case - II

A dielectric that completely fills the space in between the two plates of the capacitor

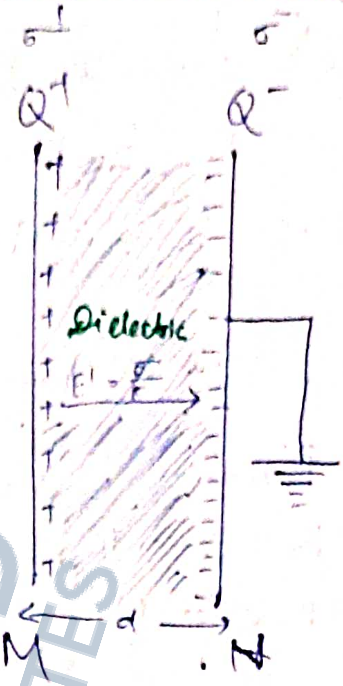
(write See the mark in case I < >)

The electric field intensity in between the two plates is

$$E' = \frac{\sigma}{\epsilon}, \text{ directed along } \overrightarrow{MN}$$

where $\epsilon$ = Permittivity of the dielectric medium.

Proceeding as before, it can be shown that the capacity of such a parallel plate capacitor is




$$C' = \frac{\epsilon A}{d}$$

Defining relative permitivity $\epsilon_r(\epsilon_r)$ or dielectric constant (K) as the ratio of $\epsilon$ and $\epsilon_0$;

i.e $\epsilon_r = K = \frac{\epsilon}{\epsilon_0}$

$$\Rightarrow \epsilon = K\epsilon_0$$

$$\therefore \boxed{C' = \frac{\epsilon A}{d} = \frac{K\epsilon_0 A}{d} = KC}$$

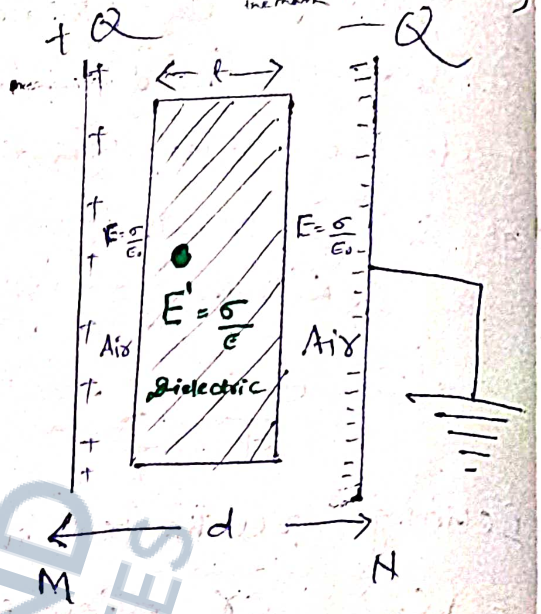
For air K=1 ~~and~~ and for other dielectric K > 1. Hence C' > C. This explains why the capacity of a ~~p~~ parallel plate capacitor increases due to the introduction of the dielectric.

## Case - III

A dielectric of thickness less than the separation between the plates is introduced in between the plates of the capacitor

< First write the introduction & exp. gauss (application of ... in first 'air' capacitor upto A = Area) (see the mark)

The thickness of dielectric is $t$ which is less from $d$. Now




$V_M - V_N$

= Potential difference between the plates M and N

= Workdone to shift a +1 charge from plate N to the plate M

= workdone in air + workdone in dielectric

= Force × displacement + Force × displacement

$$= q.E(d-t) + q.E't$$

$$= 1.\frac{\sigma}{\epsilon_0}(d-t) + 1.\frac{\sigma}{\epsilon}.t$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{\epsilon_0 K}.t$$

$$= \frac{\sigma}{\epsilon_0}\left(d-t+\frac{t}{K}\right)$$

$$= \frac{\sigma}{\epsilon_0}\left\{d-t\left(1-\frac{1}{K}\right)\right\}$$

$$= \frac{\sigma}{\epsilon_0}\left\{d-x\right\}$$ where $x = t\left(1-\frac{1}{K}\right)$

= +ve quantity.

But $V_N = 0$ ( because the plate N is connect to earth which is at

Zero potential)

$$\therefore \quad V_M = \frac{\sigma}{\epsilon_0}\left[d - t\left(1-\frac{1}{k}\right)\right]$$

$$= \frac{\sigma}{\epsilon_0}(d-x)$$

Capacity of the parallel plate capacitor is

$$C_1 = \frac{Q}{V_M} \qquad \left(\because \sigma = \frac{Q}{A} = \text{surface charge density} \quad \therefore Q = \sigma A\right)$$

$$= \frac{\sigma A}{\frac{\sigma}{\epsilon_0}\left[d - t\left(1-\frac{1}{k}\right)\right]}$$

$$\boxed{C_1 = \frac{A\epsilon_0}{\left\{d - t\left(1-\frac{1}{k}\right)\right\}}}$$

$$= \frac{\epsilon_0 A}{d-x}$$

The above expression for $C_1$ clearly shows that the capacity increases due to the introduction of the dielectric.

i.e. $C_1 > C$

## Special cases

### (1) No dielectric present

i.e $t = 0$

$$\therefore C_1 = \frac{\epsilon A}{d - 0} = C$$

### (2) Dielectric of thickness $t = d$

$$\therefore C_1' = \frac{A \cdot \epsilon_0}{d - d\left(1 - \frac{1}{K}\right)}$$

$$= \frac{A \cdot \epsilon_0}{\cancel{d} - \cancel{d} + \frac{d}{K}}$$

$$= \frac{K A \epsilon_0}{d}$$

$$= K C$$

$$\& = C'$$

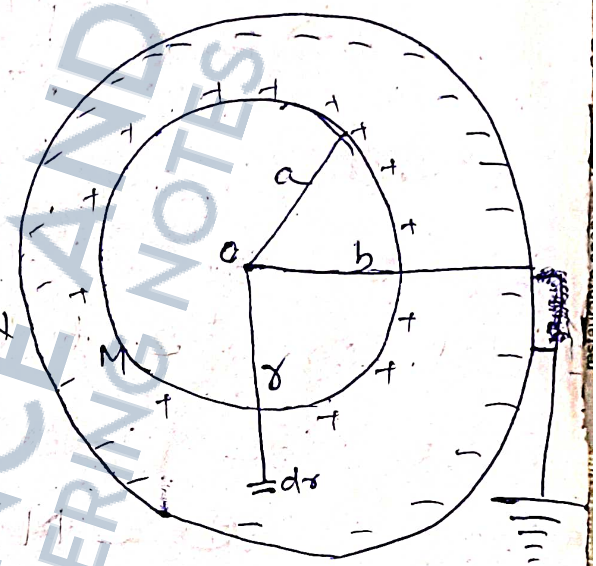
## Spherical Capacitor

It consists of two hollow, concentric, metalic spheres having radii a and b with $b > a$ (say). The following two cases arises.

1. Charge given to the inner sphere and the outer sphere is earthed.

2. Charge given to the outer sphere and the inner sphere is earthed.

<u>Case-1</u>




The inner metalic sphere is given some charge, +Q (say).

The outer sphere is uncharged and induction takes place. As a result -ve charges appear on its inner surface and +ve charges appear on its outer surface. If the outer side of the sphere N is earthed, then electrons from earth will flow ~~the~~ towards the outerside of the sphere N and ~~neut~~ the +ve charges get neutralised.

From an application of Gauss theorem, it has been proved that the ~~poss~~ concen- charges behave as if they are treated at the centre (for points on or out side ~~the surface~~ of the sphere)

Electric field intensity at a point distanant $r$ from the centre $(a<r<b)$ is given by

$$\vec{E} = \frac{KQ}{r^2}, \text{ directed along } \overrightarrow{MN}$$

$$\Rightarrow E \propto \frac{1}{r^2}$$

Therefore, we have to use the method of calculus to find the total workdone to shift a +1 charge from the outer sphere N to the inner sphere M

$\therefore dW$ = Small amount of work done to shift +1 charge through a small distance $dr$

= Force × displacement

$= q \cdot E \times dr$

$= 1 \cdot \frac{KQ}{r^2} dr$

Integrating both the sides with proper limits, we get

$$\int_0^W dW = KQ \int_a^b r^{-2} dr$$

$$\Rightarrow (W)\Big|_0^W = KQ \left(\frac{r^{-2+1}}{-2+1}\right)\Big|_a^b$$

$$\Rightarrow W - 0 = KQ \left(\frac{r^{-1}}{-1}\right)\Big|_a^b$$

$$= KQ \left(-\frac{1}{r}\right)\Big|_a^b$$

$$= -KQ \left(\frac{1}{b} - \frac{1}{a}\right)$$

$$= KQ \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\therefore V_M - V_N = KQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

But $V_N = 0$ because the outer sphere is connected to the earth

$$\therefore V_M = KQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{V_M} = \frac{Q}{KQ\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{K\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C = \frac{1}{K\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{K\left(\frac{b-a}{ab}\right)}$$

$$C = \frac{ab}{K(b-a)}$$

In C.G.S system $K = 1$

$$\therefore \boxed{C = \frac{ab}{b-a}}$$

In M.K.S system of units,

$$K = \frac{1}{4\pi\epsilon_0}$$

$$\therefore \boxed{C = \frac{4\pi\epsilon_0 \cdot ab}{b-a}}$$

If a dielectric be introduced in between the two hollow spheres, then the Capacity of the spherical capacitor becomes $C' = \cancel{4\pi\epsilon_0} \frac{4\pi\epsilon\, ab}{b-a}$

$$\therefore \frac{C'}{C} = \frac{\epsilon}{\epsilon_0} = K = \text{Dielectric Constant}$$

$$\Rightarrow C' = KC$$

Here also, we see that $C' > C$

i.e Capacity increases when a dielectric is introduced in between the two ~~hollow~~ spheres.

— o —

3-5, 5-7 → Saturday.

(19) Three capacitors, each have capacitance 1 μF.

$C_1 = 1\mu F$, $C_2 = 1\mu F$, $C_3 = 1\mu F$

$V_1 = 100\,V$, $V_2 = 200\,V$, $V_3 = 300\,V$.

~~Certain~~

Energy before connection

$= \frac{1}{2}\{C_1V_1^2 + C_2V_2^2 + C_3V_3^2)$

$= \frac{1}{2}\{1.(100)^2 + 1.(200)^2 + 1.(300)^2\}$

$= \frac{1}{2}\{10,000 + 40,000 + 90,000)$

$= \frac{1}{2} \times 140,000 \times 10^{-6}$

$= 70,000 \times 10^{-6}$

$= .07$ Joule

When the 3 capacitors are connected to 450 volt, then each capacitor acquires voltage of 150 volt.

Energy after connection

$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2$

$= \frac{1}{2}V^2 \quad (C_1 + C_2 + C_3)$

$= \frac{1}{2}.(150)^2 \times 3\,\mu F = \frac{1}{2} \times 22500 \times \overset{1.5}{\cancel{3}} \times 10^{-6}$

$= 33750 \times 10^{-6} = .03375$ Joule

$\Delta E = 0.070000 - 0.033750$

$$\begin{array}{r} 0.070000 \\ 0.033750 \\ \hline .036250 \end{array} \text{ Joule.}$$

This amount of energy is converted to heat energy and transfered to the atmosphere.

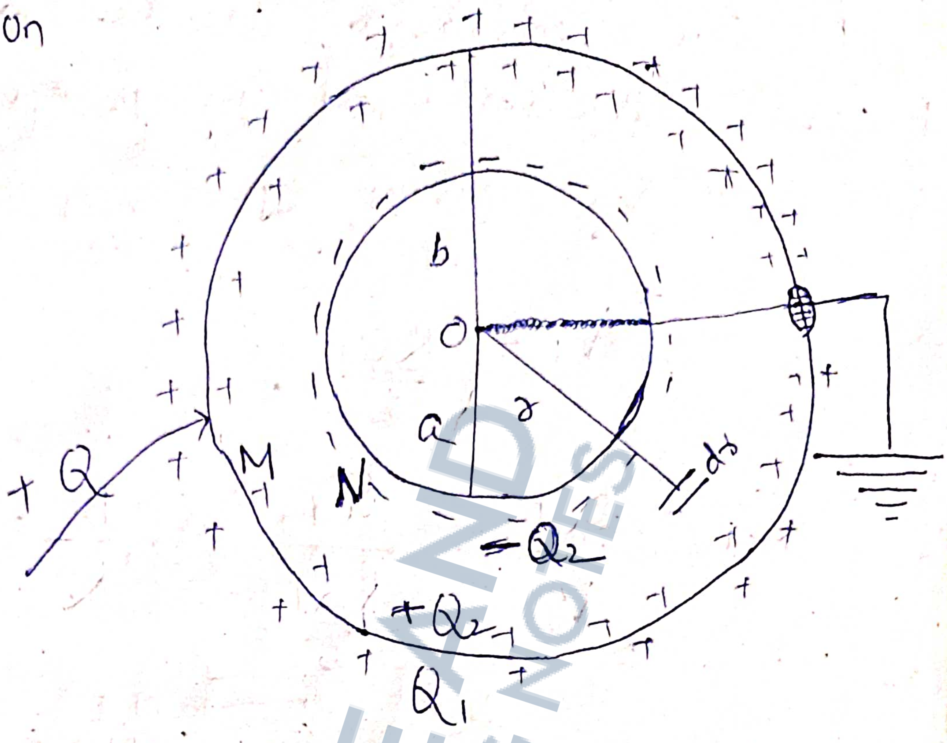
D > Think as —

## Case-II

**Charge given to the outer sphere and inner sphere is earthed.**

There are two hollow, concentric metalic spheres of radii a and b $(b > a)$. The inner sphere is earthed and the outer sphere is given some amount of charge $(+Q)$, (Say). Out of $+Q$ amount of charge, an amount $+Q_1$ remains on the outer surface and $+Q_2$ goes to the inner ~~oth~~ surface. Due to the process of induction, a charge of

$-Q_2$ is induced on the outer surface of the inner sphere and $+Q_2$ is induced




at the inner surface ~~at~~ of that sphere.

When the inner sphere is earthed electron flow from the earth towards the hollow sphere and $+Q_2$ charge will be neutralised. This situation has been shown in the figure.

This type of spherical capacitor can be regarded as it two capacitors are connected in parallel.

(i) A hollow charged sphere of radius b having a charge $+Q_1$

(ii) Two hollow metalic spheres of radii a and b having charges $-Q_2$ and $+Q_2$ respectively

The charges of $+Q_1$ behave as if they are concentrated at the centre. Electric potential at any point on its surface is given by

$$V_1 = \frac{KQ_1}{b}$$

$$C_1 = \frac{Q_1}{V_1} = \frac{Q_1}{\frac{KQ_1}{b}} = \frac{b}{K} = \frac{b}{\frac{1}{4\pi\epsilon_0}}$$

$$C_1 = b4\pi\epsilon_0$$ (when M.K.S system or units is used)

To find $C_2$, lets find $V_m - V_N$.

$V_M - V_N$ = Amount of work done to bring +1 charge from the sphere N towards the sphere M.

= Force $\times$ displacement

$$= \frac{KQ_2}{r^2} \cdot dr$$

Since $V_N = 0$, as the sphere N has been connected to the earth, we have

$$V_M = \int_a^b \frac{KQ_2}{r^2} \cdot dr$$

$$V_M = KQ_2 \int_a^b r^{-2} \cdot dr$$

$$= KQ_2 \left( \frac{r^{-2+1}}{-2+1} \right) \Bigg|_a^b$$

$$= KQ_2 \left( -\frac{1}{r} \right) \Bigg|_a^b$$

$$= -KQ_2 \left( \frac{1}{b} - \frac{1}{a} \right)$$

$$= KQ_2 \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$= KQ_2 \left( \frac{b-a}{ab} \right)$$

$$= \frac{Q_2 (b-a)}{4\pi \epsilon_0 \; ab}$$

$$C_2 = \frac{Q_2}{V_M} = \frac{Q_2}{\frac{Q_2 (b-a)}{4\pi\epsilon_0 ab}} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$\therefore C_p = C_1 + C_2$$

$$= b 4\pi\epsilon_0 + \frac{4\pi\epsilon_0 ab}{b-a}$$

$$= b 4\pi\epsilon_0 \left( 1 + \frac{a}{b-a} \right)$$

$$= 4\pi\epsilon_0 \cdot b \left( \frac{b - \cancel{a} + \cancel{a}}{b-a} \right)$$

$$\boxed{C_1 = \frac{b^2 4\pi\epsilon_0}{b-a}}$$

If the space in between the two spheres be filled with a dielectric medium of perimitivity $\epsilon$, then

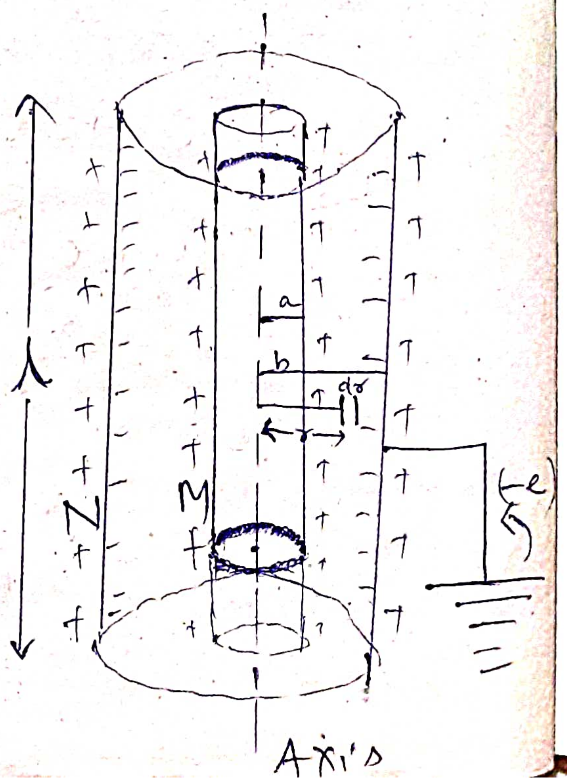
$$C' = \frac{4\pi\epsilon b^2}{b-a}$$

$$\therefore \frac{C'}{C} = \frac{\epsilon}{\epsilon_0} = \epsilon_r = K = \text{dielectric constant}$$

Since $K > 1$ for all the dielectrics except air, $C' > C$. This shows that capacitance increases with the presence of dielectric in between the two hollow spheres.

## * Cylindrical Capacitor

It consists of two hollow co-axial, metalic cylinders of radii $a, b$ ($b > a$).




The inner cylinder is given some amount of charge say $(+Q)$. The

Outer Cylinder is earthed :

From an application of Gauss theorem, an expression for electric field intensity at a point in between the two cylinders is given by

$$E = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{r}$$

where $\lambda$ = Linear charge density
= Charge per unit length
$= \frac{Q}{l}$

$\therefore E \propto \frac{1}{r}$

Therefore, we have to use the method of Calculus to find the total workdone to shift +1 charge from outer cylinder N to inner cylinder M.

$\therefore dW$ = Small amount of work done to shift +1 charge through a small distance dr
= Force x displacement
= $q.E \times dr$

$$\therefore dW = \cancel{\lambda \cdot \frac{KQ}{r^2}\,dr} = \frac{\lambda}{2\pi\epsilon_0 r} \cdot dr$$

Integrating both the sides with proper limits, we get

$$\int_0^W dw = \int_a^b \frac{\lambda}{2\pi\epsilon_0}\,\cancel{r}\cdot\frac{dr}{r}$$

$$\Rightarrow (W)\Big|_0^W = \frac{\lambda}{2\pi\epsilon_0}\int_a^b \frac{dr}{r}$$

$$\Rightarrow W - 0 = \frac{\lambda}{2\pi\epsilon_0}(\ln r)\Big|_a^b$$

$$\Rightarrow W = \frac{\lambda}{2\pi\epsilon_0}(\ln b - \ln a)$$

$$\Rightarrow W = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a}$$

$$= \frac{\frac{Q}{l}}{2\pi\epsilon_0}\ln\frac{b}{a}$$

$$= \frac{Q}{2\pi\epsilon_0 l}\ln\frac{b}{a}$$

$$\therefore V_M - V_N = \frac{Q}{2\pi\epsilon_0 l}\ln\frac{b}{a}$$

But $V_N = 0$ because the outer sphere is connected to earth

$$\Rightarrow V_M = \frac{Q}{2\pi\epsilon_0 l} \ln\frac{b}{a}$$

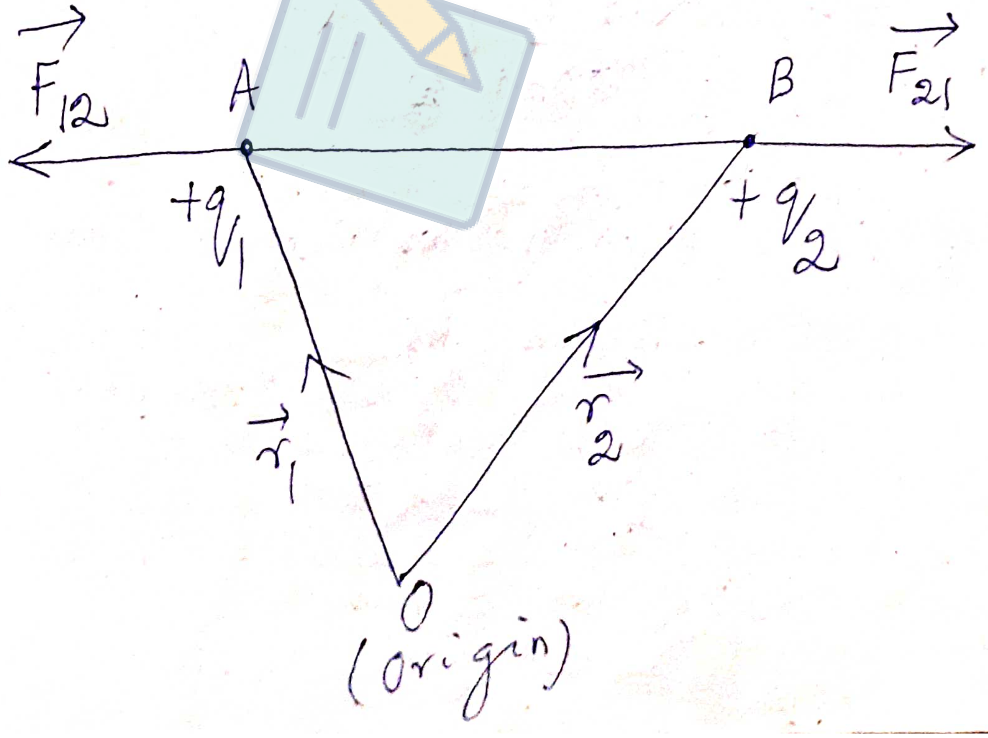
$$C = \frac{Q}{V_M} = \frac{Q}{\frac{Q \ln\frac{b}{a}}{2\pi\epsilon_0 l}} = \frac{\cancel{Q}\, l . 2\pi\epsilon_0}{\cancel{Q} \ln\frac{b}{a}}$$

$$C = \frac{l . 2\pi\epsilon_0}{\ln\frac{b}{a}}$$

$$\frac{l\, 2\pi\epsilon_0}{k_0 \ln(\frac{b}{a})}$$

## Coulomb's law in vector form

Let there be two charges $+Q_1$ and $+Q_2$ kept at the points A and B having position vectors $\vec{r_1}$ and $\vec{r_2}$ with respect to some origin O



From the triangle law of vectors, we can write.

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\Rightarrow \vec{r_1} + \vec{AB} = \vec{r_2}$$

$$\Rightarrow \vec{AB} = \vec{r_2} - \vec{r_1}$$

$$AB = |\vec{AB}| = |\vec{r_2} - \vec{r_1}|$$

Now $\vec{BA} = -\vec{AB} = -(\vec{r_2} - \vec{r_1}) = \vec{r_1} - \vec{r_2}$

$$\therefore BA = |\vec{BA}| = |\vec{r_1} - \vec{r_2}| = AB$$

$\vec{F_{21}}$ = Force of repulsion experienced by the charge $+Q_2$ due to the charge $+Q_1$

$$= \left( \frac{K q_1 q_2}{(AB)^2} \right) \hat{AB}$$

where $\hat{AB}$ = Unit vector along the direction AB

$$= \frac{\vec{AB}}{|\vec{AB}|}$$

$$= \frac{\vec{AB}}{AB}$$

$$\therefore \vec{F}_{21} = \frac{K q_1 q_2}{AB^2} \cdot \frac{\vec{AB}}{AB}$$

$$= \frac{K q_1 q_2}{AB^3} \cdot \vec{AB}$$

$$= \frac{K q_1 q_2 \cdot (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

$\vec{F}_{12}$ = Force of repulsion experience by the charge $q_1$ due to the charge $q_2$.

$$= \left(\frac{K q_1 q_2}{(AB)^2}\right) \cdot \hat{BA}$$

where $\hat{BA}$ = Unit vector along the direction BA

$$= \frac{\vec{BA}}{|\vec{BA}|}$$

$$= \frac{\vec{BA}}{BA}$$

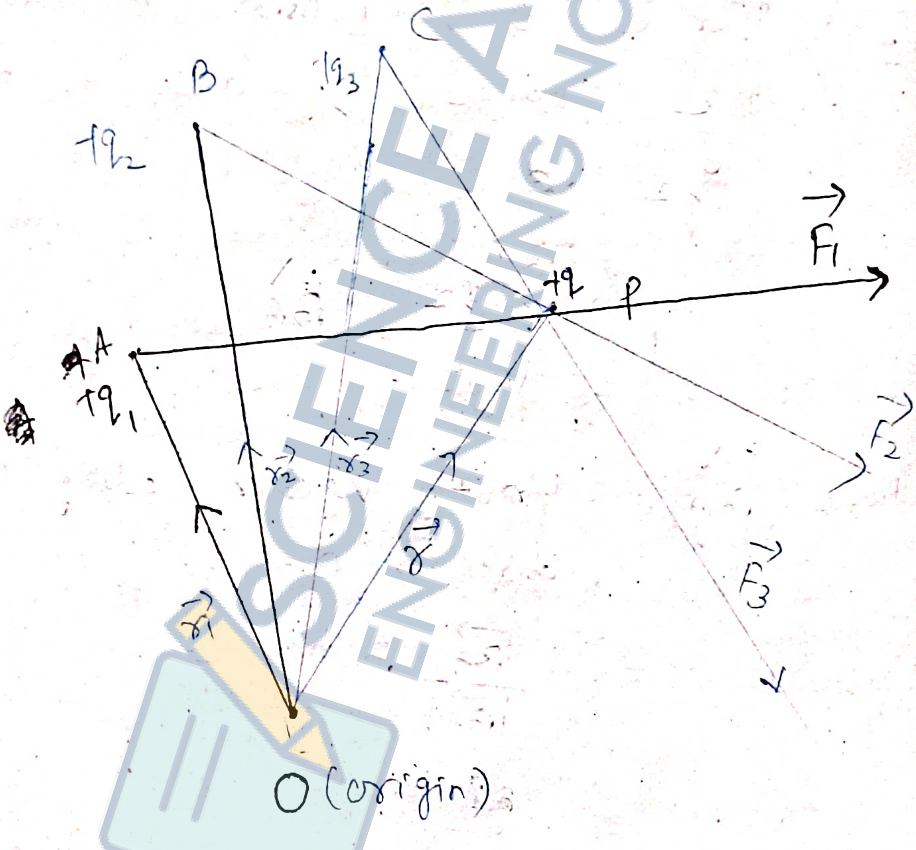
$$\therefore \vec{F}_{12} = \frac{K q_1 q_2}{(BA)^2} \cdot \frac{\vec{BA}}{BA}$$

$$= \frac{K q_1 q_2 \cdot \vec{BA}}{BA^3}$$

$$= \frac{K q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

From these two expressions it is easily proved that $\vec{F}_{12} = -\vec{F}_{21}$ (As expected from Newton's third law)

## Force on a point charge placed near many isolated charges




P is a point having position vector $(\vec{r})$ with respect to the origin O. There are charges $+q_1, +q_2, +q_3$, ------- located at points A, B, C, ------ having position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$, ------ respectively.

The repulsive forces experienced by the charge $+q$ at P are $\vec{F_1}, \vec{F_2}, \vec{F_3},$ - - — .

Net force on the charge $+q$ at P is

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \text{---}$$

$$= \frac{Kq_1 q}{AP^2}\,\hat{AP} + \frac{Kq_2 q}{BP^2}\,\hat{BP} + \frac{Kq_3 q}{CP^2}\,\hat{CP} + \text{---}$$

$$= Kq\left[\frac{q_1}{AP^2}\cdot\frac{\vec{AP}}{AP} + \frac{q_2}{BP^2}\cdot\frac{\vec{BP}}{BP} + \frac{q_3}{CP^2}\cdot\frac{\vec{CP}}{CP} + \text{---}\right]$$

$$= Kq\left[\frac{q_1(\vec{r}-\vec{r_1})}{|r-r_1|^3} + \frac{q_2(\vec{r}-\vec{r_2})}{|r-r_2|^3} + \frac{q_3(\vec{r}-\vec{r_3})}{|r-r_3|^3} + \text{---}\right]$$

$$= Kq\left[\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}\right]$$

Electric field intensity at a point in an electric field created by several isolated charges

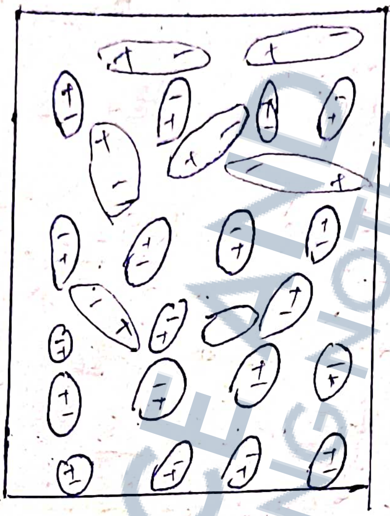
By defn, electric field intensity at a point is given by

$$\vec{E} = \lim_{q\to 0}\frac{\vec{F}}{q}$$

$$\therefore \vec{E} = K\left[\sum_{i=1}^{n} \frac{q_i\,(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}\right]$$

## ~~Ditec~~ Dielectric, an atomic view

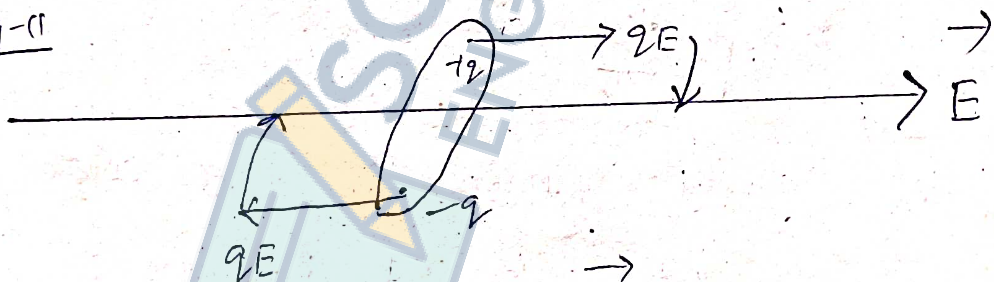
Fig-I

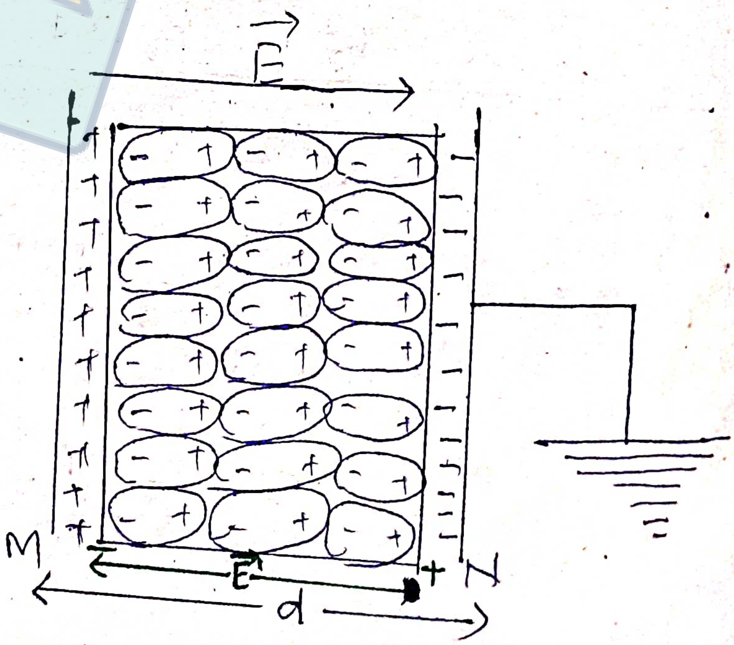


$\vec{E} = 0$

Field direction.

Fig-II



A dielectric is a non-conducting material having no free-charge. However, it contains large number of electric dipoles which are randomly oriented in the absence of an external electric field as shown in fig(i)

When such a material is introduced in between the plates of a parallel plate capacitor, each dipole will experience a couple. The torque due to the couple will try to align the dipole along the field direction.

If the electric field intensity be very strong, all the electric dipoles will be aligned along the field direction. This is shown in fig(iii)

Before the dielectric was introduced,

$V_M - V_N$ = Work done to shift a +1 charge from the plate N to the plate M

$= qEd$

$= 1 \cdot E \cdot d$

or $V_M = Ed$ ——— (1) $(\because V_N = 0)$

If there is a charge +Q on the plate M, then capacity is

$$\therefore C = \frac{Q}{V_M} = \frac{Q}{Ed} \qquad \text{———} (2)$$

When the dielectric of thickness d' be placed in between the two plates, +ve and -ve polarities develop as shown in the diagram. This produces an electric field intensity $\vec{E'}$ ($E' < E$, due to principle of causality).

$\therefore$ Net ~~elet~~ electric field intensity in the space between the two plates = $E - E'$

$$\therefore V_M' - V_N' = q(E - E')d$$
$$= 1 \cdot (E - E')d$$

$$\Rightarrow V_M' = (E - E')d \qquad \text{———} (3)$$

$$(\because V_N' = 0)$$

Then capacity will become

$$C' = \frac{Q}{V_M} = \frac{Q}{(E - E')d} \qquad \text{———} (4)$$

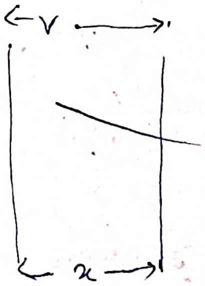
Looking at the eqn 1 and 3 we see that the potential of the plate M decreases due to the introduction of the dielectric.

Looking at the eqⁿs (2) and (4) we see that the Capacity increases due to the introduction of the dielectric.

## Problem-1

1. A parallel plate condenser is charged to a potential difference V e.s.u. A slab 3 mm thick is inserted between the plates and it becomes necessary to increase the distance between the plates by 2.4 mm to maintain the p.d. Find K)

Ans:

Fig(i)

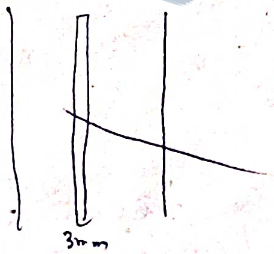
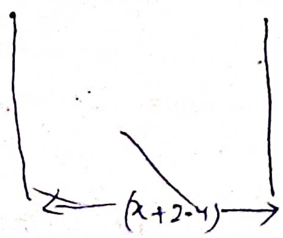


Fig (ii)



Fig(iii)

Before the slab was introduced

$$C = \frac{\epsilon_0 A}{d}$$ and after the slab was introduced the capacitance becomes $C_1 = \frac{\epsilon_0 A}{d-x}$.

If the separation be increased from $d$ to $d+x$ then $C_1 = \frac{\epsilon_0 A}{d+\cancel{x}-\cancel{x}}$

$$\therefore C_1 = \frac{\epsilon_0 A}{d} = C$$

Thus, to keep the p.d and capacitance of the capacitors are constant, the plates to be separated further by an amount

$$x = t\left(1-\frac{1}{K}\right)$$

$$2.4\ mm = 3\ mm\left(1-\frac{1}{K}\right)$$

$$\Rightarrow \frac{2.4}{3} = \frac{K-1}{K}$$

$$\Rightarrow 2.4K = 3K - 3$$

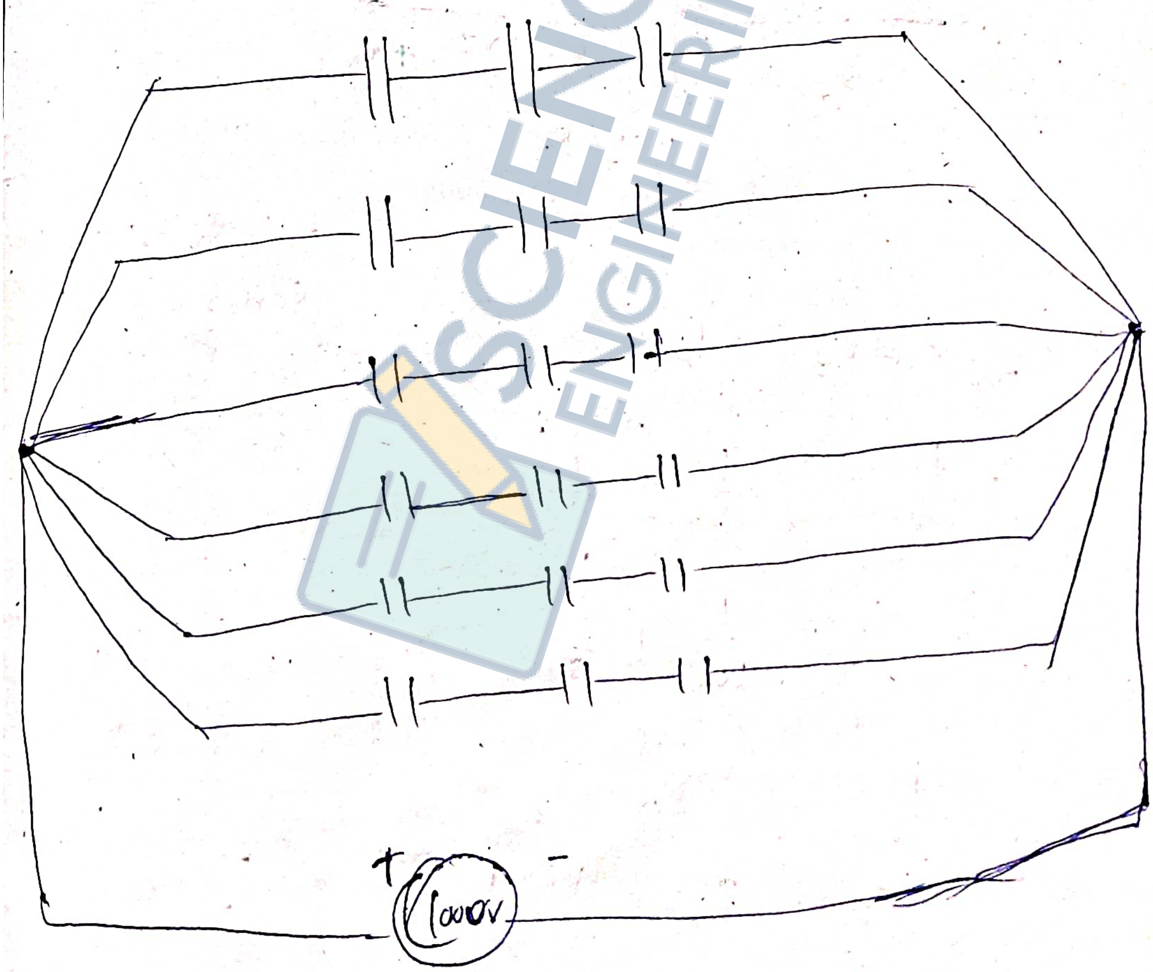
$$\Rightarrow .6K = 3$$

$$\Rightarrow K = \frac{3}{.6} = \frac{30}{6} = 5 \quad \text{(Ans)}$$

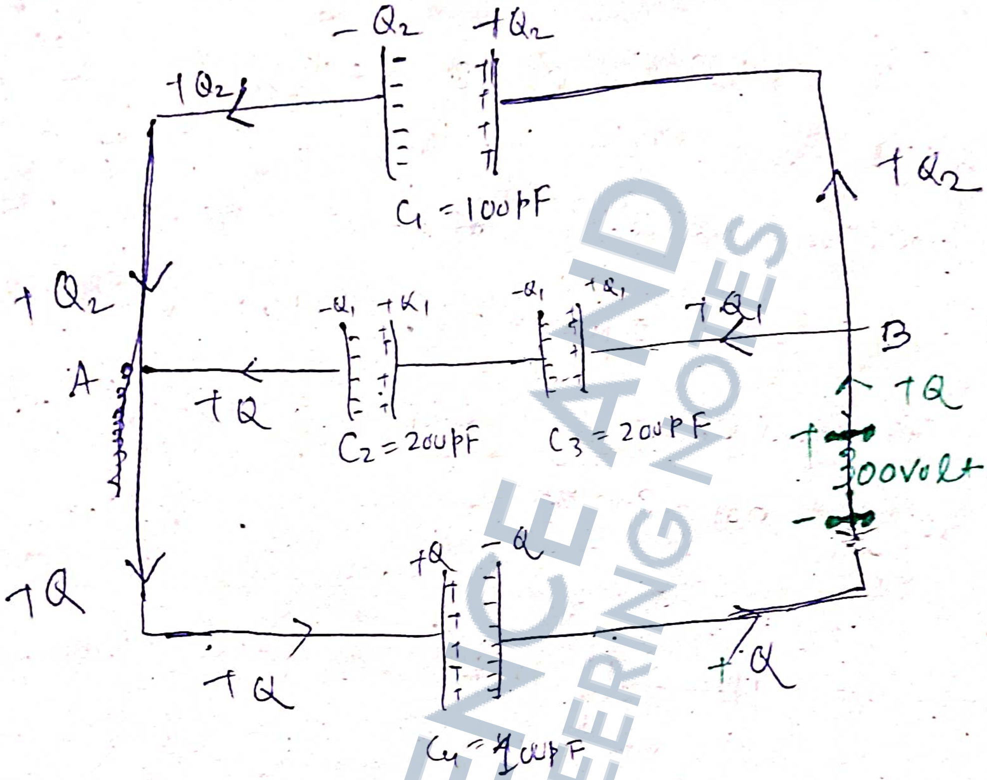
2. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 K volt. A large number of 1 μF capacitors

are available to him each of which can withstand a p.d of not more than 400 volt. Suggest a possible arrangement that requires a minm number of capacitors.

Ans :- A minm of 18 capacitors, each of capacitance 1MF with 6 branches and 3 capacitor in each branch has to be used to get 2MF with 1000 volt supplied. Each capacitors will experience 333.33 volt and will not punkture



3. Find the equivalent capacitance of the following network. Determine the charge and voltage across each capacitor.




$\therefore$ $C_2$ and $C_3$ are in series.

$$\frac{1}{C_s} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{200} + \frac{1}{200}$$

$$\frac{1}{C_s} = \frac{2}{200}$$

$$\Rightarrow C_s = \frac{200}{2} = 100\,pF$$

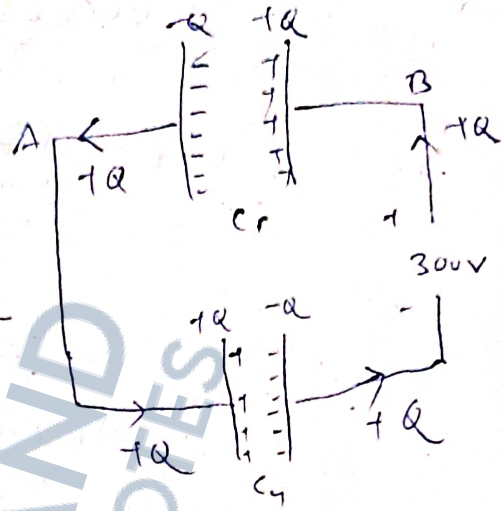
The equivalent capacitance between A and B is $C_p$

$$= C_1 + C_B$$

$$= 100\,pF + 100\,pF$$

$$= 200\,pF$$

Now $C_P$ and $C_4$ are in series




∴ Equivalant capacitance be $C_s'$

$$\frac{1}{C_s'} = \frac{1}{C_P} + \frac{1}{C_4}$$

$$\Rightarrow \frac{1}{C_s'} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200} = \frac{3}{200}$$

$$\Rightarrow C_s' = \frac{200}{3}\,pF$$

But $C_s' = \dfrac{Q}{300}$

$$\Rightarrow \frac{200}{3} = \frac{Q}{300}$$

$$\Rightarrow Q = \frac{200 \times \cancel{300}^{100}}{\cancel{3}} = 20{,}000 \times 10^{-12}$$

$$= 2 \times 10^{-8}\ \text{coul.}$$

$$C_4 = \frac{Q}{V_4} = \frac{2\times10^{-8}}{V_4}$$

$$\Rightarrow 100 \times 10^{-12} = \frac{2\times10^{-8}}{V_4}$$

$$\Rightarrow V_4 = \frac{2\times10^{-8}}{100\times10^{-12}} = \frac{2\times10^{4}}{100} = 200\ \text{volt}$$

∴ $V_A - V_B = 200$ volt which is experienced by $C_1$

$\cancel{V_1 = \frac{Q_1}{C_1}} =$

$\therefore \quad V_1 = 100$ volt

$Q_2 = C_1 V_1 = 100 \times 100 = 10^{4} \times 10^{-12}$ Coulomb.

$= 10^{-8}$ coulomb.

But. $Q = Q_1 + Q_2$

$\Rightarrow \quad 2 \times 10^{-8} = Q_1 + 10^{-8}$

$\Rightarrow \quad Q_1 = \cancel{6}\ 2 \times 10^{-8} - 10^{-8} = 10^{-8}$ Coulomb.

Now for the Capacitor $C_2$.

$C_2 = 200\,pF, \; Q_1 = 10^{-8}$.

$$V_2 = \frac{Q_1}{C_2} = \frac{10^{-8}}{200 \times 10^{-12}} = \frac{10^{4}}{200} = \frac{10000}{200}$$

$= 50$ Volt

$V_2 + V_3 = V_A - V_B = 100$ volt.

$\Rightarrow \quad 50$ volt $+ V_3 = 100$ volt

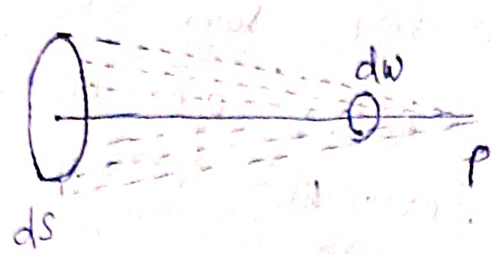
$\Rightarrow \quad V_3 = 100$ volt $- 50$ volt $= 50$ volt.

$\therefore \quad V_1 = 100$ volt, $V_2 = 50$ volt, $V_3 = 50$ volt

$Q_1 = 10^{-8}$, $Q_2 = 10^{-8}$, $Q = 2 \times 10^{-8}$

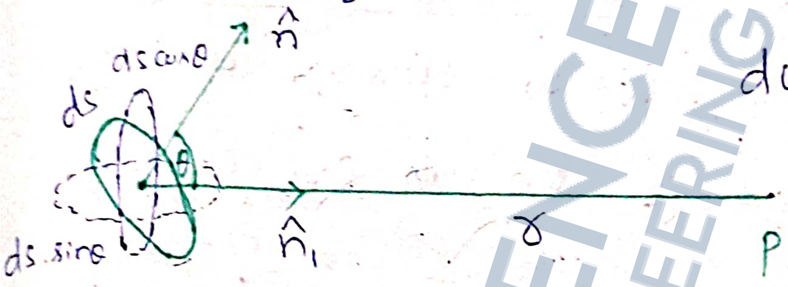
Coulomb.

# Solid angle




dω = Solid angle formed by P by the surface ds

$$= \frac{ds}{r^2}$$




$$d\omega = \frac{ds \cos\theta}{r^2}$$

Total solid angle formed by a sphere at its ~~distance~~ centre

$$= \frac{4\pi r^2}{r^2} = 4\pi \text{ steradian} \quad (\because \text{Surface area of sphere} = 4\pi r^2)$$

= Also the solid angle formed by a closed surface at any point inside it.

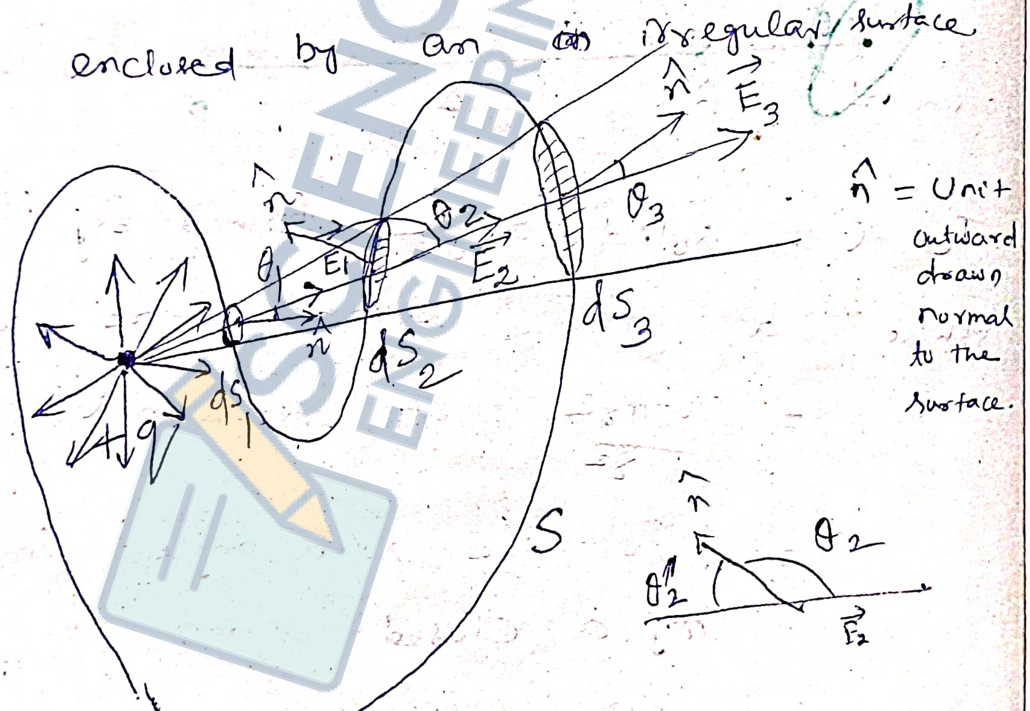
# Gauss theorem on electrostatics

**Statement:** The total electric flux over a closed surface enclosing some charge is equal to the ratio of the charge enclosed and the permittivity of air.

$$\oint_S \vec{E}\cdot\hat{n}\,ds = \frac{\Sigma q}{\epsilon_0} \quad (\text{In M.K.S system})$$

## Derivation of Gauss law from Coulomb's law

For simplicity, let us consider a single charge enclosed by an irregular surface




$\theta$ is angle between $\hat{n}$ & $\vec{E}$

Electric lines of force emerge from the isolated charge and they cut the surface at odd number of times. Considering the figure we see that

$$d\phi_e = \vec{E_1}\cdot\hat{n}\, ds_1 + \vec{E_2}\cdot\hat{n}\, ds_2 + \vec{E_3}\cdot\hat{n}\, ds_3$$

where $\vec{E_1}$, $\vec{E_2}$ and $\vec{E_3}$ are the electric field intensities at the sides of $ds_1, ds_2$, $ds_3$ respectively.

From Coulomb's law, one can write

$$E_1 = \frac{Kq}{r_1^2}, \quad E_2 = \frac{Kq}{r_2^2}, \quad E_3 = \frac{Kq}{r_3^2}$$

$$\therefore d\phi_e = \frac{Kq}{r_1^2}\cdot 1\cdot \cos\theta_1\, ds_1 + \frac{Kq}{r_2^2}\cdot 1\cdot \cos\theta_2\, ds_2 + \frac{Kq}{r_3^2}\cdot 1\cdot \cos\theta_3\, ds_3$$

$$d\phi_e = Kq\left[\frac{ds_1\cos\theta_1}{r_1^2} + \frac{ds_2\cos(180^\circ - \theta_2')}{r_2^2} + \frac{ds\cos\theta_3}{r_3^2}\right]$$

where $\theta_2'$ is the actual angle between $\hat{n}$ and $\vec{E_2}$

$$\therefore d\phi_e = Kq\left[dw - \frac{ds_2\cos\theta_2'}{r_2^2} + dw\right]$$

where $dw =$ Solid angle formed by the surfaces $ds_1, ds_2, ds_3$ at the site of the charge $+q$

$\therefore d\phi_e = Kq[d\omega - d\omega + d\omega]$

$= Kq\, d\omega$

Integrating both the sides over the closed surface, we get

$$\oint d\phi_e = Kq \oint d\omega$$

$$\Rightarrow \oint \vec{E}\cdot\hat{n}\, ds = Kq\, 4\pi = \frac{1}{4\pi\epsilon_0} q\, 4\pi = \frac{q}{\epsilon_0}$$

If more charges will be present inside the surface then the above expressions becomes

$$\oint_s \vec{E}\cdot\hat{n}\, ds = \frac{\Sigma q}{\epsilon_0} \quad \text{(proved)}$$

Corollary :-

If the charge will be present outside the closed surface then the electric lines of force will cut the surface even numbers of time and $\phi_e = 0$

20. 6. 21

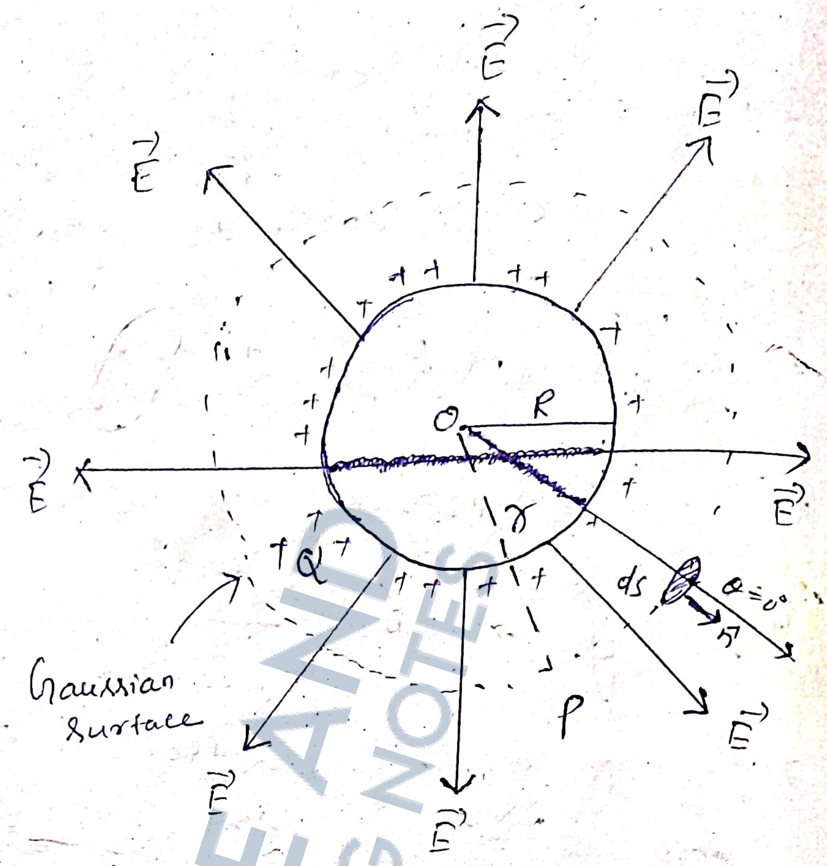
# Application of Gauss theorem

## 1. To find $\vec{E}$ at points inside, on and outside of a hollow charged sphere

### (a) Outside point

The Gaussian surface is an imaginary surface which is a sphere of radius $r$ (In this case). It encloses all the charges present on the surface of hollow sphere ($+Q$, say). The electric lines of force starting from the hollow sphere pierce (Penetrate) through the Gaussian surface so that $\hat{n}$

Of an elementary area ds of Gaussian surface and $\vec{E}$ make $0^\circ$ with each other.




$$d\phi_e = \vec{E} \cdot \hat{n}\, ds$$

$$= E \cdot 1 \cdot \cos 0 \cdot ds$$

$$= E\, ds$$

Integrating both the sides, we get

$$\oint d\phi_e = E \oint ds$$

{ Sphere ds than, area is $4\pi r^2$, because sphere is symmetrical about all the axis) so $\vec{E}$ is constant

$$\Rightarrow \phi_e = \text{Total electric flux} = E \cdot 4\pi r^2$$

$$\Rightarrow \frac{Q}{\epsilon_0} = E \cdot 4\pi r^2$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} = \frac{KQ}{r^2}}$$

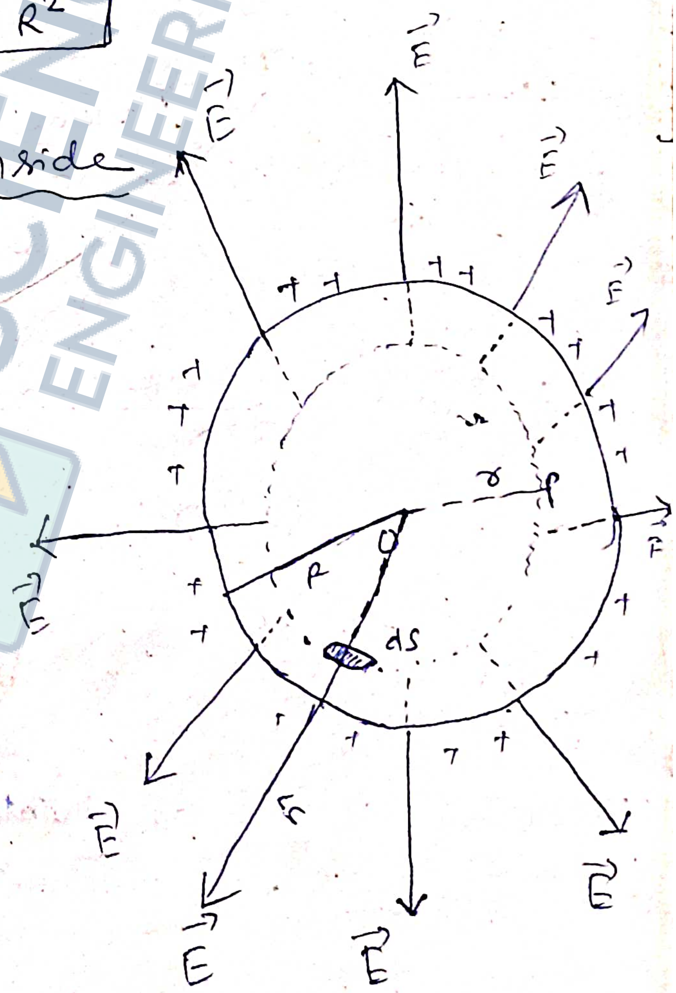
This expression shows that the charges present on the hollow sphere as if they are concentrated at the centre for outside points.

b) Point on the Surface

Here, the Gaussian surface coincides with the surface of the sphere and proceeding as in case (a), we can get

$$\boxed{E_{on} = \frac{KQ}{R^2}}$$

(c) Point is inside




The dotted circle represents the spherical Gaussian surface.

Imagining the electric lines to start from the Gaussian surface flux can be calculated as before.

(Charges are outside of the Gaussian surface)

(Charges are on the surface $\therefore q = 0$)

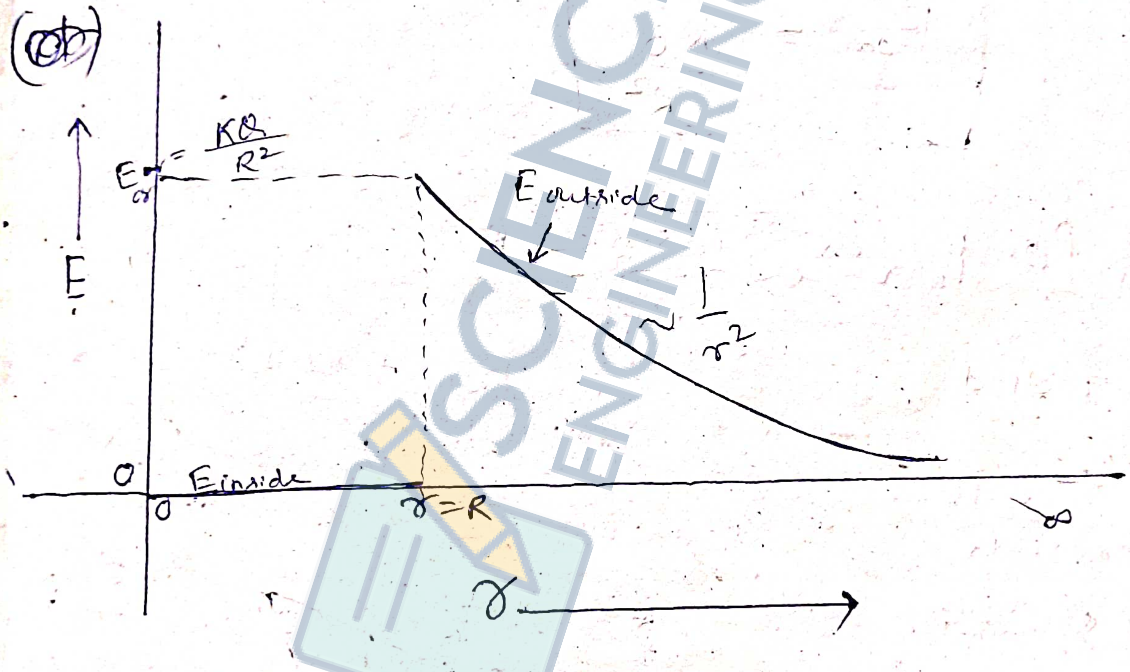
$$\therefore \quad \phi_e = E \cdot 4\pi r^2$$

$$\Rightarrow \frac{0}{\epsilon_0} = E \cdot 4\pi r^2$$

$$\Rightarrow \boxed{E_{inside} = 0}$$

Thus the electric field intensity at any point inside the hollow charged sphere is zero.

(d)




## 2nd application

To find $\vec{E}$ at points inside, on and outside of uniformly charge solid sphere

As in the first application, one can show that,

$\vec{E}_{outside} = \frac{KQ}{r^2}$, directed away from the sphere.

$\vec{E}_{on} = \frac{KQ}{R^2}$, directed away from the sphere.

If the point will be inside, then the Gaussian surface is a sphere of radius $r$ which encloses a charge $Q'$ given by $Q' = \rho \cdot \frac{4}{3}\pi r^3$

where $\rho$ = volume charge density

$= \frac{Q}{\frac{4}{3}\pi R^3}$

$\therefore Q' = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\frac{4}{3}\pi} r^3$

$= \frac{Q r^3}{R^3}$

$\phi_e$ = Total electric flux over the gaussian surface

$= E_{inside} \cdot 4\pi r^2$

$\Rightarrow \frac{Q'}{\epsilon_0} = E_{inside} \, 4\pi r^2$

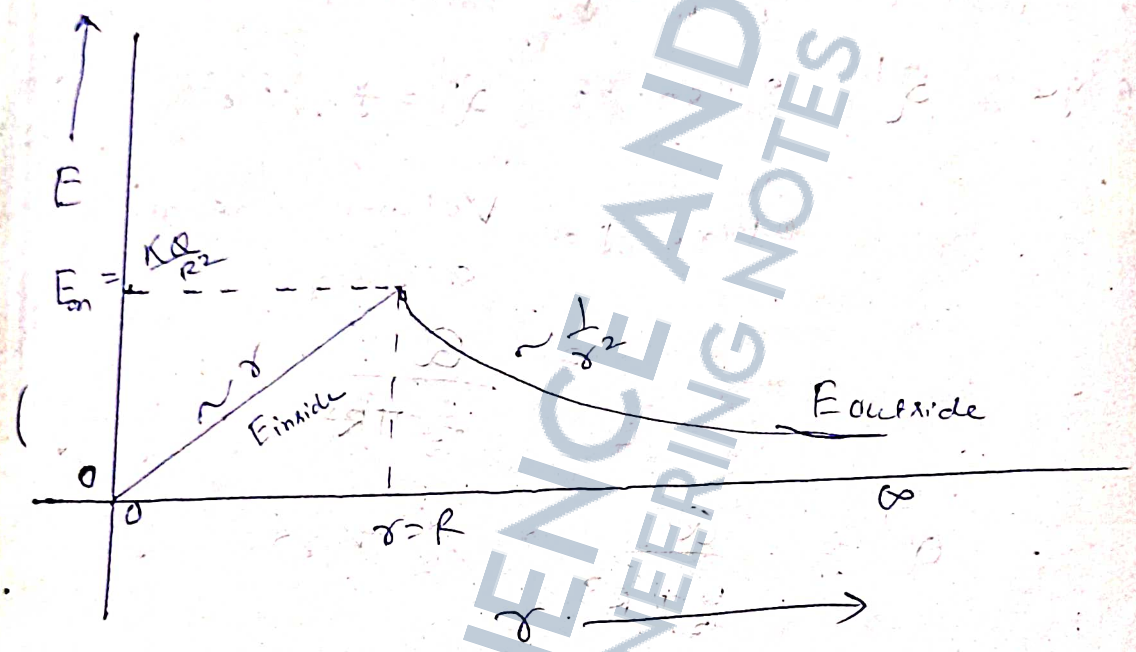
$\Rightarrow \frac{Q \cancel{r^3} r}{R^3 \epsilon_0} = E_{inside} \, 4\pi \cancel{r^2}$

$\Rightarrow$ ~~Ear~~ $E_{inside} = \frac{Qr}{R^3 \epsilon_0} \times \frac{1}{4\pi} = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{KQr}{R^3}$

Thus $E_{inside} \propto r$

If $r = R$, then

$$E_{on} = \frac{KQ \cdot \cancel{R}}{\cancel{R^3} R^2} = \frac{KQ}{R^2} \text{ (verified)}$$



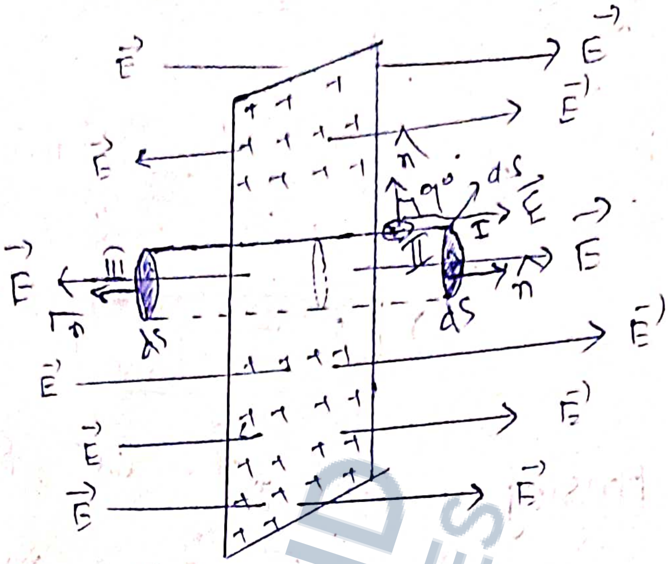

~~3rd~~ application

**To find $\vec{E}$ near a uniformly charged plane sheet**

Lets imagine a pill box as the Gaussian surface which lies on both sides of the plane sheet of charge.

It encloses a charge $\sigma \cdot 2\pi r^2$ (where $\sigma = \frac{Q}{2A}$, A = Surface area of one side of the metalic surface)

Total electric flux over the Gaussian surface $\phi$




$$\phi_e = \oint_S \vec{E}\cdot\hat{n}\, ds$$

$$= \int_{I} \vec{E}\cdot\hat{n}\, ds + \int_{II} \vec{E}\cdot\hat{n}\, ds + \int_{III} \vec{E}\cdot\hat{n}\, ds$$

(curved)

$$= \int_{I} |\vec{E}|.|\hat{n}|\cos 0\, ds + \int_{II} |\vec{E}|.|\hat{n}|\cos 90^\circ\, ds$$

$$+ \int_{III} \vec{E}.|\hat{n}|.\cos 0^\circ.\, ds$$

$$= E\int_{I} ds + 0 + E\int_{III} ds$$

$$= E\pi r^2 + E\pi r^2$$

$$= E.2\pi r^2$$

From Gauss theorem, we know that

$$\phi_e = \frac{\text{Charge enclosed}}{\epsilon_0}$$

$$\phi_e = \frac{\sigma \cdot 2\pi r^2}{\epsilon_0}$$

Thus
$$E \cdot \cancel{2\pi r^2} = \frac{\sigma \cdot \cancel{2\pi r^2}}{\epsilon_0}$$

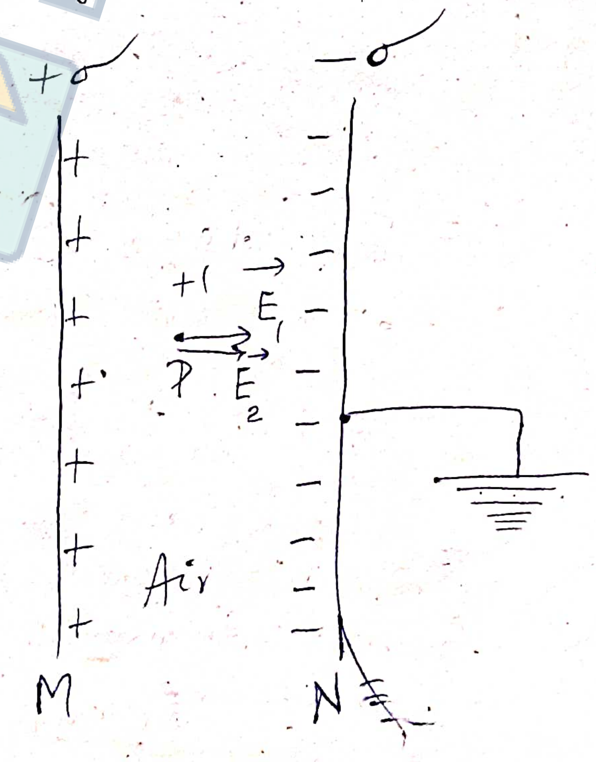
$$\Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

Supposing charge is present on one side of the metallic sheet, then
$$\phi_e = \frac{\sigma \cdot \pi r^2}{\epsilon_0} = E \cdot 2\pi r^2$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$ (Applicable for one plate of capacitor)

Corollary

To find $\vec{E}$ at any point inside a charge capacitor



$\vec{E_1}$ = Electric field intensity at P due to +Q charge distributed on one side of the plate M so that

$$\sigma = \frac{Q}{A}$$

= Surface charge density

Thus $\vec{E_1} = \frac{\sigma}{2\epsilon_0}$, directed towards N

$\vec{E_2}$ = Electric field intensity at P due to −Q charge distributed on the plate N

$= \frac{\sigma}{2\epsilon_0}$, directed along $\vec{MN}$

Hence $\vec{E_p} = \vec{E_1} + \vec{E_2}$, directed along $\vec{MN}$

$= \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$, "

$= \frac{\sigma}{\epsilon_0}$, "

— ○ —

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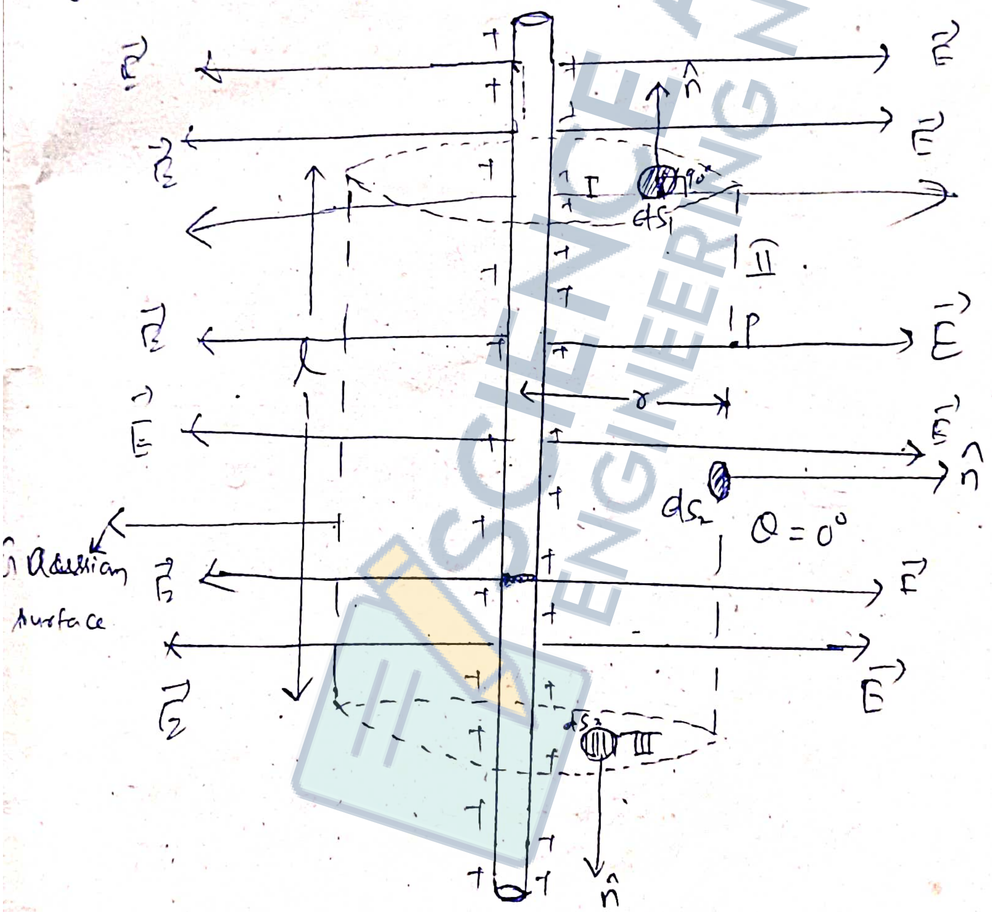
## To find $\vec{E}$ near a long, straight, wire charged uniformly

Let $\lambda$ be the linear charge density

$$\therefore \lambda = \frac{\text{Charge}}{\text{Unit length}} = \frac{Q}{L}$$

Let P be point near the wire at a distance $r$ from its axis.




Let's imagine a cylinder (shown dotted) Of length $l$ and radius $r$ which Contains the Charged wire as its axis.

Charge enclosed by the ~~gaus~~ Gaussian surface (Cylinder)

$= \lambda . l$

$= \Sigma q$

Total electric flux over the Gaussian surface $= \oint_S \vec{E} . \hat{n} \, ds$

$= \int_{I} \vec{E} . \hat{n} \, ds + \int_{II} \vec{E} \hat{n} \, ds + \int_{III} \vec{E} . \hat{n} \, ds$

(Curved surface)

$= \int_{I} |\vec{E}| \, |\hat{n}| \cos 90^\circ \, ds + \int_{II} |\vec{E}| \, |\hat{n}| \cos 0^\circ \, ds$

$+ \int_{III} |\vec{E}| . \hat{n} \, . \cos 90^\circ \, . \, ds$

$= 0 + |\vec{E}| \int_{II} ds + 0$

$= E . 2\pi r l$

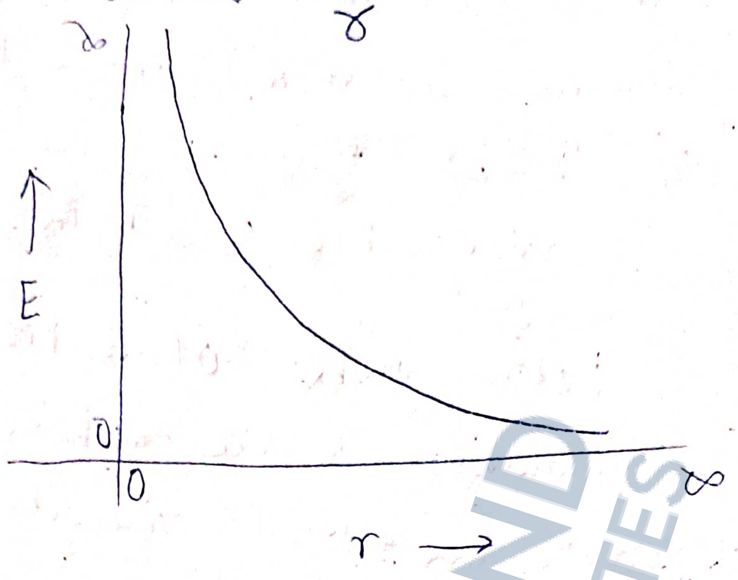
From Gauss theorem we know that

$$\phi_e = \frac{\Sigma q}{\epsilon_0}$$

$$\Rightarrow E \, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi \epsilon_0} . \frac{1}{r}}$$

$\therefore \quad E \propto \frac{1}{r}$

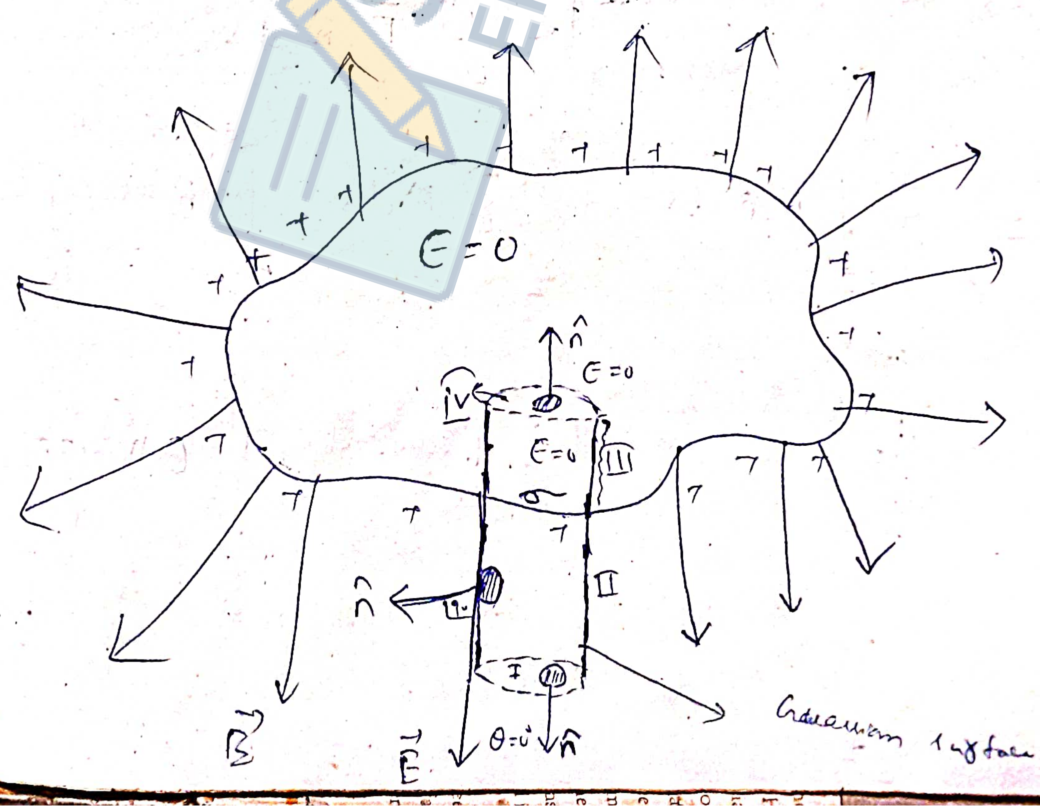



5. ## Coulomb's theorem

To show that $\vec{E}$ near a charged conductor of any shape is $\frac{\sigma}{\epsilon_0}$

$\sigma$ = Surface Charge density

$= \frac{\text{Charge}}{\text{Unit surface area}}$



We know that charges reside on the surface of conductors. Taking a Gaussian surface (spherical in shape) inside the body, it can be proved that $\vec{E} = 0$ at all points inside the conductor.

Let's think of a pill box as the Gaussian surface which lies partly inside the conductor and outside the conductors.

If the face area of the pill box be $A$, then charge enclosed

$$= \sigma A$$

$$= \Sigma q$$

Total electric flux over the Gaussian surface (pill box) $\phi_e = \oint_S \vec{E}.\hat{n}\, ds$

$$\phi_e = \int_{I} \vec{E}.\hat{n}\, ds + \int_{II} \vec{E}.\hat{n}\, ds + \int_{III} \vec{E}\hat{n}\, ds + \int_{IV} \vec{E}\hat{n}\, ds.$$

$$= \int_{I} E.1.\cos 0^\circ ds + \int_{II} E.1.\cos 90^\circ ds + \int_{III} 0.\hat{n}.\cos\theta\, ds$$

$$+ \int_{IV} 0.\hat{n}\cos\theta\, ds$$

$$= E\int_{I} ds + 0 + 0 + 0$$

$\phi_e = E A$

From Gauss theorem we know that

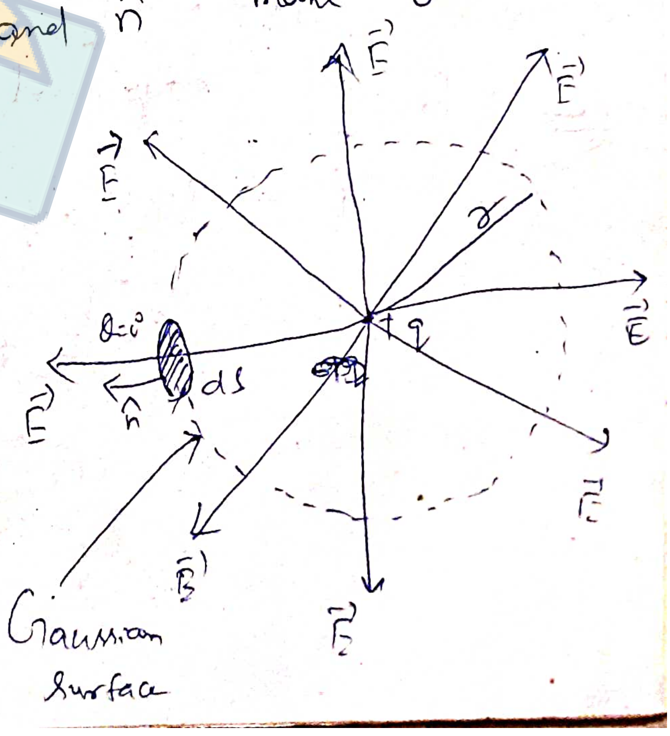
$$\oint_S \vec{E}\,\hat{n}\,ds = \frac{\Sigma q}{\epsilon_0}$$

$$\Rightarrow E \cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

## Derivation of Coulomb's law from Gauss law

Let's consider a point charge +q. We can imagine a spherical surface of radius r having the centre at the side of the charge. The lines of force emerge from +q symmetrically in all direction show that $\vec{E}$ and $\hat{n}$ make $0^\circ$ with each other.




Total electric flux over the Gaussian surface

$$\phi_e = \oint_S \vec{E}\,\hat{n}\,.ds$$

$$\phi_e = \oint_S |\vec{E}| \cdot |\vec{ds}| \cos 0^\circ \, ds$$

$$= E \oint ds$$

$$= E \, 4\pi r^2$$

From Gauss theorem we know that

$$\phi_e = \frac{\Sigma q}{\epsilon_0}$$

$$\Rightarrow E \, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} = \frac{Kq}{r^2}$$

If there is a charge $+q_0$ placed at any point on the Gaussian surface, then the ~~mag~~ magnitude of the force experienced by it is

$$F = q_0 E$$

$$F = K \frac{q q_0}{r^2}$$

This is coulomb's law.