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Wave Motion (Unit 5)

Effect of temperature on the velocity of sound is $v \propto \sqrt{T}$

ie. $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

Problem - (a) At what temp, the velocity of sound will be $\frac{3}{2}$ times the velocity at $7^\circ C$?

(b) v will be doubled of its value at $0^\circ C$

(c) v " " reduced to half of its value at $0^\circ C$.

Ans → $357^\circ C$, $819^\circ C$, $-204.75^\circ C$

Soln

1. Given that $V_2 = \frac{3}{2} V_1$

$T_1 = 7^\circ + 273^\circ = 280^\circ K$

We know that

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{V_1}{\frac{3}{2}V_1} = \sqrt{\frac{280}{T_2}}$$

$$\Rightarrow \frac{2\cancel{V_1}}{3\cancel{V_1}} = \sqrt{\frac{280}{T_2}}$$

$$\Rightarrow \frac{4}{9} = \frac{280}{T_2}$$

$$\Rightarrow T_2 = \frac{9 \times \cancel{280}^{280}}{4} = 630^\circ K$$

$$\therefore T_2 = 630 - 273 = 357^\circ C$$

(b) According to question

$$V_2 = 2V_1$$

$$T_1 = 0^\circ + 273^\circ = 273^\circ K$$

we Know that

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{\cancel{V_1}}{2\cancel{V_1}} = \sqrt{\frac{273}{T_2}}$$

$$\Rightarrow \frac{1}{4} = \frac{273}{T_2}$$

$$\Rightarrow T_2 = 1092^\circ K$$

$$\therefore T_2 = 1092^\circ K - 273 = 819^\circ c$$

(iii) According to question

$$V_2 = \frac{1}{2} V_1$$

$$T_1 = 0 + 273 = 273K$$

We Know that

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{V_1}{\frac{1}{2}V_1} = \sqrt{\frac{273}{T_2}}$$

$$\Rightarrow 2 = \sqrt{\frac{273}{T_2}}$$

$$\Rightarrow 4 = \frac{273}{T_2}$$

$$\Rightarrow 4T_2 = 273$$

$$\Rightarrow T_2 = \frac{273}{4} = 68 \cdot 25^\circ k$$

$$\therefore T_2 = 68 \cdot 25^\circ - 273 = -204 \cdot 75^\circ C$$

Wave Motion

Velocity of Sound

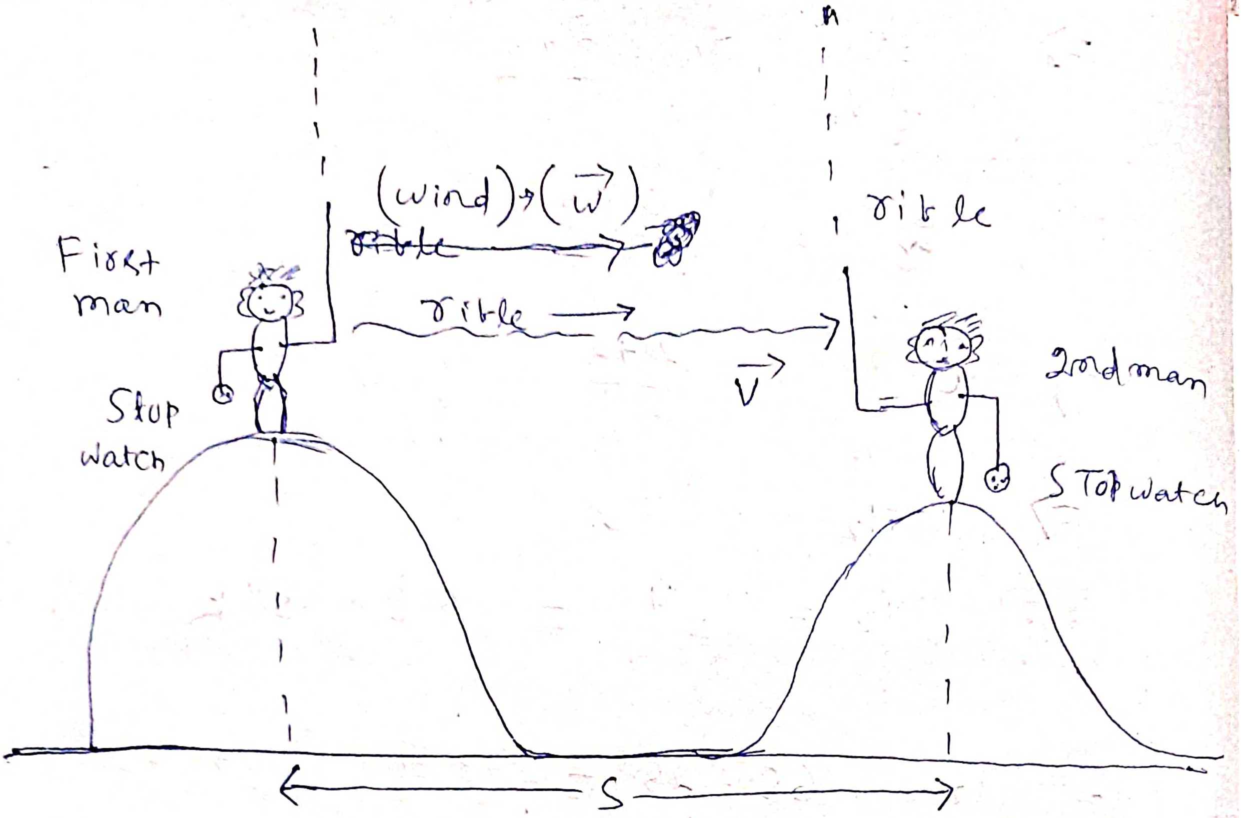
(1) Earlier Method

Two persons are to be provided with a gun and a stop watch each. The first man will fire the gun and the second man has to note the time (t_1)

$$\therefore t_1 = \frac{S}{V+W} \quad \text{————— (i)}$$

The second man will fire the gun and the first man will note the time (t_2)

$$t_2 = \frac{S}{V-W} \quad \text{————— (ii)}$$



where V = velocity of sound.

w = velocity of wind.

From eqn (i), we get

$$V + w = \frac{S}{t_1} \quad \text{——— (iii)}$$

From eqn (ii), we get

$$V - w = \frac{S}{t_2} \quad \text{——— (iv)}$$

Adding eqns (iii), and (iv), we get

$$2V = \frac{S}{t_1} + \frac{S}{t_2} =$$

$$\Rightarrow V = \frac{S}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right) \quad \text{——— (V)}$$

Subtracting eqn (4) from eqn 3, we get

$$2w = \frac{s}{t_1} - \frac{s}{t_2}$$

$$\Rightarrow w = \frac{s}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) \quad \text{—— (vi)}$$

(2) Newton's formula for the velocity of Sound

Newton derived an expression for the velocity of sound waves propagating in an elastic medium like air by calculus method

However, an expression for v can be found out by using dimensional analysis.

From experiments, it is found that the velocity of sound depends on the bulk modulus of elasticity and density of the medium through which it passes.

$$v \propto E^a \text{, when } \rho \text{ is kept constant}$$

$$v \propto \rho^b \text{, when } E \text{ is kept constant.}$$

where a and b are unknown quantities.

Combining these two variations, we get

$$v \propto E^a \cdot \rho^b \text{, when both } E \text{ and } \rho \text{ vary.}$$

$$\Rightarrow v = K \, E^a . f^b \quad \text{———} \quad (i)$$

where K is a constant.

Putting proper dimensions for each quantity, we get

$$[M^0 L^1 T^{-1}] = \left[\frac{M^1 L^1 T^{-2}}{L^2}\right]^a . \left[\frac{M}{L^3}\right]^b$$

$$\Rightarrow [M^0 L^1 T^{-1}] = [M^1 L^{-1} T^{-2}]^a . [M L^{-3}]^b$$

$$= M^{a+b} . L^{-a-3b} . T^{-2a}$$

Comparing both the side, we get

$$0 = a+b$$

$$1 = -a-3b$$

$$-1 = -2a$$

$$\Rightarrow a = \frac{1}{2}$$

$$a+b = 0$$

$$\Rightarrow \frac{1}{2} + b = 0$$

$$\Rightarrow b = \frac{-1}{2}$$

Putting these values in eqn (i)

$$V = K \, E^{\frac{1}{2}} . f^{\frac{-1}{2}}$$

$$\Rightarrow V = K \sqrt{\frac{E}{f}} \quad \text{———} \quad (ii)$$

When actual experimental data ~~used~~ were put in eqn (ii), One get

$$K \simeq 1$$

$$\therefore \boxed{V = \sqrt{\frac{E}{\rho}}} \quad \text{--- (iii)}$$

Newton's modification

Newton assumed that the temperature of a medium does not change during the propagation of sound through the medium.

(why - see BD.S → 531 page)

That is the process is isothermal, then Boyle's law ~~is~~ will be applicable (since temp is constant)

i.e $P.V = \text{Constant}$

Sound propagates in a medium like air through compressions and rarefaction

Lets consider a compression so that pressure increases from P to $(P+\Delta P)$ and volume decrease from V to $(V-\Delta V)$

$$\therefore PV = (P+\Delta P).(V-\Delta V)$$

$$\Rightarrow \cancel{PV} = \cancel{PV} - P\Delta V + \Delta P.V - \Delta P.\Delta V$$

Neglecting the term $\Delta P.\Delta V$ as a very small quantity, we get

$$P \Delta V = V . \Delta P$$

$$\Rightarrow P = \frac{V}{\Delta V} . \Delta P = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\frac{\text{Change in force}}{\text{area}}}{\frac{\text{Change in volume}}{\text{original volume}}}$$

$$= \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$= E$$

= Bulk Modulus of elasticity.

Thus eq^n (iii) changes to

$$v = \sqrt{\frac{P}{\rho}} \quad \text{———— (iv)}$$

To test this formula, let us calculate the velocity of sound at NTP

$P = 76$ cm of mercury.

$= 76 \times 13.6 \times 980$ Dyne / cm^2

ρ = Density of air at 0° C and 76 cm of mercury

$= 0.001293$ gm/c.c

$$v = \sqrt{\frac{76 \times 13.6 \times 980}{.001293}} \text{ c.m / sec}$$

$$= \sqrt{\frac{76 \times 13.6 \times 980}{12.93}} \text{ m / sec}$$

$= 279.89$ m / sec

$\simeq$ 280 m/sec

But the standard value of velocity of sound at $0°C$ is 332 m/sec. This shows that Newton's formula is not entirely correct.

③ **Laplace's Correction of Newton's formula**

Laplace argued (why, see B.D.S) [Pg 532] that the process is adibatic, when the sound wave propagates through a medium. There is no change of quantity of heat energy of the medium. Then the corresponding eqn of state

$$P.V^{\gamma} = \text{Constant},$$ has to be used.

where γ = Ratio of the two specific heats of the gas

$$= \frac{C_p}{C_v}$$

For a compression, the pressure will change from P to $(P + \Delta P)$

And the volume will change from V to $(V - \Delta V)$

$$\therefore P.V^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

$$\Rightarrow PV^{\gamma} = (P+\Delta P)\left\{V\left(1-\frac{\Delta V}{V}\right)\right\}^{\gamma}$$

$$\Rightarrow \cancel{PV^{\gamma}} = (P+\Delta P)\,\cancel{V^{\gamma}}\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

Making binomial expansion, $(1+x)^n \simeq 1+nx$ provided $x << 1$

$$\Rightarrow P = (P+\Delta P)\left(1-\gamma.\frac{\Delta V}{V}\right)$$

$$\Rightarrow \cancel{P} = \cancel{P}+\Delta P - \gamma.P\frac{\Delta V}{V} - \gamma.\frac{\Delta P.\Delta V}{V}$$

Neglecting the term $\gamma.\frac{\Delta P.\Delta V}{V}$ as a very small quantity, we have

$$\Delta P = \gamma\frac{P\Delta V}{V}$$

$$\Rightarrow \gamma P = \frac{\Delta P\, V}{\Delta V} = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$= E = \text{Bulk modulus elasticity}$$

Thus eq^n (iii) can be written as

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{———— (V)}$$

Lets calculate the velocity of sound at NTP from eqn (V)

~~P = 76~~

Air is mainly a mixture of diatomic gases like N_2, O_2, H_2.

Hence, $\gamma = 1.4$ for air.

$$\therefore V = \sqrt{1.4} \sqrt{\frac{P}{\rho}}$$

$$= (1.18 \times 280) \text{ m/sec}$$

$$= 331.3 \text{ m/sec}$$

$$\simeq 332 \text{ m/sec.}$$

This shows that Laplace's modification of Newton's original formula gives correct result. Hence

$$\boxed{V = \sqrt{\frac{\gamma \cdot P}{\rho}}}$$ can be regarded as the correct formula.

Effect of pressure on the velocity of sound (when temp. and humidity are kept constants)

Let v_1 and v_2 be the velocities of sound in a particular medium at pressures P_1 and P_2 respectively.

From Laplace's corrected formula, we know that

$$v_1 = \sqrt{\frac{\gamma P_1}{\rho_1}}$$

and $$v_2 = \sqrt{\frac{\gamma P_2}{\rho_2}}$$

where ρ_1 and ρ_2 are the densities of the medium at pressures P_1 and P_2 respectively.

Since temp remains constant, Boyle's law is applicable.

$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow P_1 \cdot \frac{m}{\rho_1} = P_2 \cdot \frac{m}{\rho_2}$$

$$\Rightarrow \frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

$\Rightarrow v_1 = v_2$

Thus, velocity of sound is independent of the change of pressure, i.e, it is unaffected by change of pressure

Effect of temp on velocity of sound (when pressure and humidity are kept constant)

Let v_1 and v_2 be the velocities of sound at temperatures θ_1 and θ_2 °C respectively.

From Laplaces corrected formula for the velocity of sound, one can write

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}}$$

and $$v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

Dividing one by the other, we get

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \quad \text{———} (i)$$

Since pressure remains constant, Charle's law is applicable.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \frac{\cancel{m}}{\rho_1 T_1} = \frac{\cancel{m}}{\rho_2 T_2}$$

$$\Rightarrow \rho_1 T_1 = \rho_2 T_2$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \quad \text{——— (ii)}$$

Using eqn (ii) in eqn (i)

$$\boxed{\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}} \quad \text{——— (iii)}$$

Hence $v \propto \sqrt{T}$.

i.e velocity of sound is directly proportional to the square root of the absolute temperature.

Approximate formula

Let V_θ and V_0 denote velocities of sound at $\theta^\circ C$ and $0^\circ C$ respectively.

From eqn (iii), we can write

$$\frac{V_\theta}{V_0} = \sqrt{\frac{\theta + 273}{0 + 273}}$$

$$= \sqrt{1 + \frac{\theta}{273}}$$

$$= \left(1 + \frac{\theta}{273}\right)^{\frac{1}{2}}$$

$$\simeq \left(1 + \frac{1}{2} \cdot \frac{\theta}{273}\right)$$

where we have made binomial expansion $(1+x)^n \simeq 1 + nx, \; x \ll 1$.

$$\therefore \quad V_\theta = V_0 + \frac{V_0}{546} \cdot \theta \qquad \text{—— (4)}$$

In c.g.s system of units

$$V_0 = 33200 \text{ c.m/sec}$$

$$\therefore \quad \frac{V_0}{546} = \frac{33200}{546} \simeq 61 \text{ c.m/sec}$$

Using this value eqn (4) gives

$$\cancel{\frac{V_0}{546}} \quad V_\theta = V_0 + 61\theta \qquad \text{—— (5)}$$

From eqn (v) we see that velocity of sound increases by 61 c.m/sec for $1^\circ C$ rise of temp

In M.K.S ~~unit~~ system of units,

$$V_0 = 332 \text{ m/sec.}$$

$$\therefore \frac{V_0}{546} = \frac{332 \text{ m/sec}}{546} = 0.61 \text{ m/sec}$$

$\therefore$ eqn (4) becomes

$$V_\theta = V_0 + 0.61\,\theta \quad \text{——(vi)}$$

From eqn (vi) we see that velocity of sound increases by 0.61 m/sec for ~~1°C~~ θ° degree rise of temp.

In F.P.S system of units

$$V_0 = 1100 \text{ ft/sec}$$

$$\therefore \frac{V_0}{546} = \frac{1100 \text{ ft/sec}}{546} = 2.01 \text{ ft/sec}$$

$\therefore$ eqn (4) becomes

$$V_\theta = V_0 + 2.01\,\theta \quad \text{——(vii)}$$

From eqn (vii) we see that velocity of sound increases by 2.01 ft/sec for degree rise of temp.

Comparision of the velocity of sound through different gases (when pressure, P, temperature, T and humidity are kept constants)

For simplicity of calculation; lets consider hydrogen and oxygen gases.

From Laplaces corrected formula for the velocity of sound, we can write

$$V_{H_2} = \sqrt{\frac{\gamma P}{\rho_{H_2}}}$$

and $$V_{O_2} = \sqrt{\frac{\gamma P}{\rho_{O_2}}}$$

Dividing one by the other, we get

$$\frac{V_{H_2}}{V_{O_2}} = \sqrt{\frac{\rho_{O_2}}{\rho_{H_2}}} = \sqrt{\frac{16\,\rho_{H_2}}{\rho_{H_2}}} = \frac{4}{1}$$

$$\Rightarrow V_{H_2} = 4\,V_{O_2}$$

Thus, velocity of sound through H_2 is 4 times the velocity of sound through O_2.

Affect of humidity on the velocity of sound

For short question (2 MK)

(a) qualitative (when pressure and temp are kept constant)

Let V_d and V_m denote the velocities of sound through dry air and moist air under similar conditions of temp and pressure. Laplace's corrected formula for velocity of sound gives

$$V_d = \sqrt{\frac{\gamma P}{\rho_d}}$$

and $$V_m = \sqrt{\frac{\gamma P}{\rho_m}}$$

Since $\rho_m < \rho_d$ (when pressure remains the same)

$$\Rightarrow V_m > V_d$$

i.e. sound travels faster through moist air compared to that through dry air.

(b) Quantitative (when temp. remains constant, Pressure may change)

Let V_d = velocity of sound through dry air at a pressure 76 c.m of mercury and $0°c$ temp.

V_m = velocity sound through moist air at a pressure P c.m of mercury and $0°c$ temp. ($P > 76$ c.m), say)

ρ_d = density of dry air at a pressure 76 c.m of mercury and $0°c$ temp.

ρ_m = density of moist air at a pressure P cm of mercury and $0°c$ temp.

From Laplaces corrected formula for the velocity of sound, we can write

$$V_d = \sqrt{\frac{\gamma \, 76 \times 13.6 \times 980}{\rho_d}} \quad \text{——— (i)}$$

$$V_m = \sqrt{\frac{\gamma \, P \times 13.6 \times 980}{\rho_m}} \quad \text{——— (ii)}$$

Dividing eqn (i) by eqn (ii) we get

$$\frac{V_{d}}{V_{m}} = \sqrt{\frac{76}{P} \cdot \frac{f_m}{f_d}} \quad \text{———— (iii)}$$

Now f_m = Mass of 1 c.c of moist air at 0°C and P c.m of mercury pressure

= Mass of 1 c.c dry air at a pressure of (P − f) c.m mercury and 0°C temp. + mass of 1 cc of moisture at a pressure of f c.m of mercury and 0°C.

———— (iv)

where f = ~~Aquet~~ Acqueous tension

= ~~Moisture~~ Partial pressure of moisture present in moist air.

Since temp remains constant, Boyle's law is applicable.

$$\therefore P_1 . V_1 = P_2 . V_2$$

$$\Rightarrow (P-f). 1\,c.c = (76). V_2$$

$$\Rightarrow V_2 = \frac{P-f}{76}\ c.c$$

Mass of V_2 cc of dry air at 76 c.m of mercury and 0°C temperature

$$= V_2 . f_d$$

$$= \frac{p-f}{76} \times \rho_d \qquad \text{———— (V)}$$

Density of moisture $= 0.622 \times$ density of dry air

$= 0.622\ \rho_d$ at 76 c.m of Hg and pressure and $0^\circ C$ temp.

Using boils law for the moisture, we have

$$P_1' V_1' = P_2' \cdot V_2'$$

$$\Rightarrow f \cdot (1) = 76 \times V_2'$$

$$\Rightarrow V_2' = \frac{f}{76} \text{ c.c}$$

Mass of V_2' c.c of moisture

$$= V_2 \times (0.622\,\rho_d)$$

$$= \frac{f}{76} \times (0.622\,\rho_d)$$

Using eqn s (V) and (Vi) in eqn (ii),

we get

$$\rho_m = \frac{p-f}{76}\,\rho_d + \frac{f}{76}(\cancel{0} \times .622\rho_d)$$

$$= \frac{\rho_d}{76}\left(p - f + .622f\right)$$

$$= \frac{\rho_d}{76}\left(p - 0.378f\right)$$

$$\Rightarrow \frac{\rho_m}{\rho_d} = \frac{1}{76}(p - 0.378f) \quad \text{—— (vii)}$$

using eqn (vii) in eqn (iii)

$$\frac{V_d}{V_m} = \sqrt{\frac{\cancel{76}}{p} \cdot \frac{1}{\cancel{76}}(p - 0.378f)}$$

$$= \sqrt{\frac{p - 0.378f}{p}}$$

$$= \sqrt{1 - \frac{0.378f}{p}}$$

$$\Rightarrow \boxed{V_d = V_m \cdot \sqrt{1 - \frac{0.378f}{p}}} \quad \text{—— (viii)}$$

$$\simeq V_m \left(1 - \frac{1}{2}\frac{0.378f}{p}\right)$$

$$= V_m - \frac{V_m}{2} \cdot \frac{0.378f}{p}$$

This also shows that velocity of sound through moist air is always ~~air~~ higher than the velocity of sound through dry air.

Problem

If the velocity of sound through H_2 4100 ft/sec what will be the velocity of sound through a mixture of 2 parts by volume of hydrogen and 1 part by volume of oxygen

Sol^n From ~~Lat~~ Laplaces corrected formula for the velocity of sound for hydrogen and mixtures, we can write

$$V_{H_2} = \sqrt{\frac{\gamma . P}{\rho_{H_2}}}$$

$$V_{Mix} = \sqrt{\frac{\gamma P}{\rho_{mix}}}$$

Dividing we get

$$\frac{V_{H_2}}{V_{mix}} = \sqrt{\frac{\rho_{mix}}{\rho_{H_2}}} \quad \text{—— (i)}$$

Lets Calculate the density of the mixture, $\rho_{mix} = \frac{\cancel{\text{density}}\ \text{Mass of mixture}}{\text{volume of mixture}}$

$$\rho_{mix} = \frac{\text{Mass of } H_2 + \text{mass of } O_2}{\text{Volume of } H_2 + \text{volume of } O_2}$$

$$= \frac{\cancel{2V\rho_{H_2}} \; 2V \cdot \rho_{H_2} + V \cdot \rho_{O_2}}{\cancel{3}\,2V + V}$$

$$= \frac{2V \cdot \rho_{H_2} + V \cdot 16\,\rho_{H_2}}{3V}$$

$$= \frac{\cancel{2V} \; \cancel{18}^{6}\cancel{V}\rho_{H_2}}{\cancel{3V}}$$

$$= 6\,\rho_{H_2} \qquad \text{------ (iv)}$$

Putting this value in eqn (i), we get

$$\frac{V_{H_2}}{V_{mix}} = \sqrt{\frac{6\,\rho_{H_2}}{\rho_{H_2}}} = \sqrt{6}$$

$$\Rightarrow \frac{4100 \text{ ft/sec}}{V_{mix}} = \sqrt{6}$$

$$\Rightarrow V_{mix} = \frac{4100}{\sqrt{6}} = \frac{4100 \times \sqrt{6}}{6} \text{ ft/sec}$$

$$= \frac{4100 \times 2.449}{6} \text{ ft/sec}$$

$$= 1673.483 \text{ ft/sec}$$

16. When the object drops it
to reach the water surface
~~covers the distance~~

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow S = 0 + \frac{1}{2} \cdot (\cancel{32}^{16}) \cdot t^2$$

$$\Rightarrow S = 16t^2$$

$$\Rightarrow 500 = 16t^2$$

$$\Rightarrow t = \sqrt{\frac{500}{16}} = 5.59$$

Time taken ~~of~~ splash to reach the

Observer $= \frac{\text{Distance}}{\text{Speed}} = \frac{500}{1140} = 0.43$

Total time taken to come to the

observer $= 5.59 + 0.43$
$= 6.02s.$

~~8~~.20,

Time taken to go the bottom of ~~sea~~ water

= Time taken to come back.

It takes total = 5 ~~secs~~ s

∴ Time taken to reach the bottom = 2.5 sec

$$\text{Time} = \frac{\text{distance}}{\text{velocity}}$$

$\Rightarrow$ distance = time × velocity

$= 2.5 \times 1450$

$= 3625$ m

2. We know that velocity of sound increases by 0.61 m/sec for degree rise up temp.

At 0°C velocity of sound = 332 m/sec.

For 20° increased velocity = 20 × .61

= 12.20 m/sec

So velocity of sound = 332 + 12.2

= 344.2 m/sec

$$\text{velocity} = \frac{\text{Distance}}{\text{time}} \Rightarrow$$ Distance = velocity × time

= 344.2 × 6

= 2065.2 m

3. Velocity increased = .61 × 24

= 14.64 m/sec

Velocity of sound = 332 + 14.64

= 346.64 m/s

~~Distance~~ Lightening is far away

= 346.64 × 2 sec = 693.28

17. Depth of the lake = 30m,

It takes time to reach the bottom, can be know by this eqn

$$S = ut + \frac{1}{2}gt^2$$

$$\Rightarrow S = 0 + \frac{1}{2}(9.8)\cdot t^2$$

$$\Rightarrow 30 = 4.9t^2$$

$$\Rightarrow t^2 = \frac{30}{4.9} = 6.122$$

$$\Rightarrow t = \sqrt{6.122}\ \text{sec} = 2.477\ \text{sec}$$

Time taken to sound to reach near the observer

$$t = \frac{\text{distance}}{\text{velocity of sound}}$$

$$= \frac{30}{332}$$

$$= .09036\ \text{sec.}$$

Total time taken

$$= 2.47435 + .09036$$

$$\boxed{\begin{array}{r} 2.47435 \\ +.09036 \\ \hline 2.56471\ \text{sec} \end{array}}$$

$$= 2.56471 \text{ (Ans)}$$

18.

Velocity of sound in gas $v = \sqrt{\frac{\gamma RT}{M}}$

because $PV = nRT$

$$\Rightarrow P = \frac{nRT}{V} = \frac{nRT}{\frac{m}{\rho}} = \frac{nRT\rho}{m}$$

$$\Rightarrow \frac{P}{\rho} = \frac{nRT}{m} = \frac{m}{M}\cdot\frac{RT}{m} = \frac{RT}{M}.$$

From Lablaces corrected formula

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma . RT}{M}}$$

$$= \sqrt{\frac{1.4 \times 8.31 \times (0+273)}{\frac{32}{1000}}}$$

$$= \sqrt{\frac{1.4 \times 8.31 \times 273 \times 1000}{32}}$$

$= 315.04$ m/sec

19. Density of Oxygen = 16 time density of hydrogen

$$\rho_{O_2} = 16\, \rho_{H_2}$$

In oxygen speed of sound = 317.5 m/sec at $0^\circ C$

velocity of sound in H_2

$$V_{H_2} = \sqrt{\frac{\gamma P}{\rho_{H_2}}} \quad \text{——— (i)}$$

velocity of sound in O_2

$$V_{O_2} = \sqrt{\frac{\gamma P}{\rho_{O_2}}} \quad \text{——— (ii)}$$

Dividing one with another

$$\frac{V_{O_2}}{V_{H_2}} = \sqrt{\frac{\rho_{H_2}}{\rho_{O_2}}}$$

$$\Rightarrow \frac{317.5}{V_{H_2}} = \sqrt{\frac{1\,\cancel{\rho_{H_2}}}{16\,\cancel{\rho_{H_2}}}}$$

$$\Rightarrow \frac{317.5}{V_{H_2}} = \frac{1}{4}$$

$$\Rightarrow V_{H_2} = \frac{317.5 \times 4}{1}$$

$$= 1270.0 \text{ m/sec}$$

Wave motion

Vibratory motion

Lets attach a load to a spring suspended from a rigid support. If the load be dragged a little in the downward direction it is found that the spring moves in the upward direction when released. Due to the inertia of motion the load moves through some distance in the upward direction as shown as s' in figure (c). Again a ~~restor~~ restoring force is developed as a case (B) due to which the load has a tendency to come back to the equilibrium position.

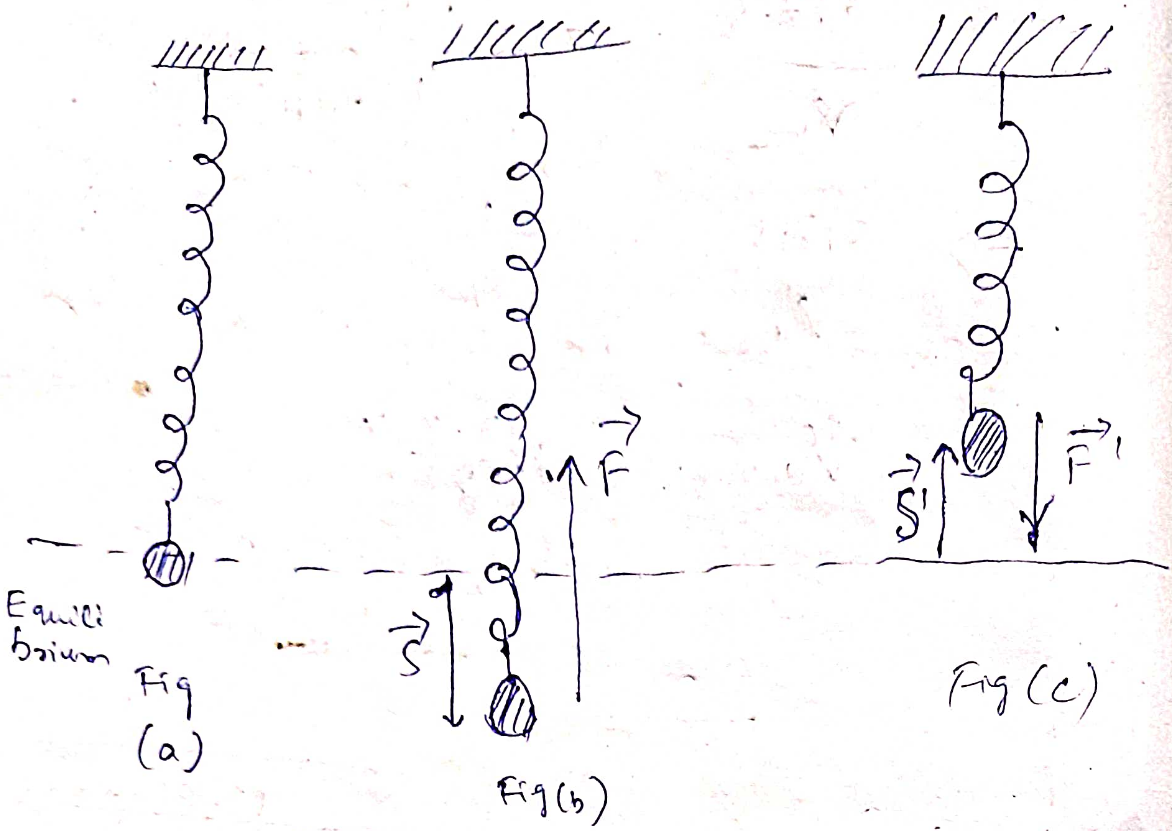


Fig (a) Fig (b) Fig (c)

In all such cases like (b) and (c) the restoring force is directly proportional to the displacement and acts in the opposite direction.

$$\therefore \vec{F} \propto (-\vec{s})$$

$$\Rightarrow \boxed{\vec{F} = -K\vec{s}}$$

where K is a constant called <u>force constant.</u>

Dividing both the sides by m, we get

$$\frac{\vec{F}}{m} = -\frac{K}{m}\vec{s}$$

$$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{s}$$ where $\frac{K}{m}$ = Constant,

Sometimes called spring constant.

$$\vec{a} \propto (-\vec{s})$$

Definition of simple harmonic motion (S.H.M)

It is a type of to and fro motion executed by a particle, when its accⁿ is directly proportional to the displacement and always directed towards the equilibrium position.

Some definition associated with simple harmonic motion

1. One Complete Vibration or Oscillation

It is the motion ~~and~~ undertaken by a particle making to and fro motion after which it is just repeated.

(2) Time period (T)

It is the time taken by a body

to make a complete Oscillation or Vibration

3. Amplitude (A)

It is defined as the maxm displacement of a particle, from the equilibrium position on any side.

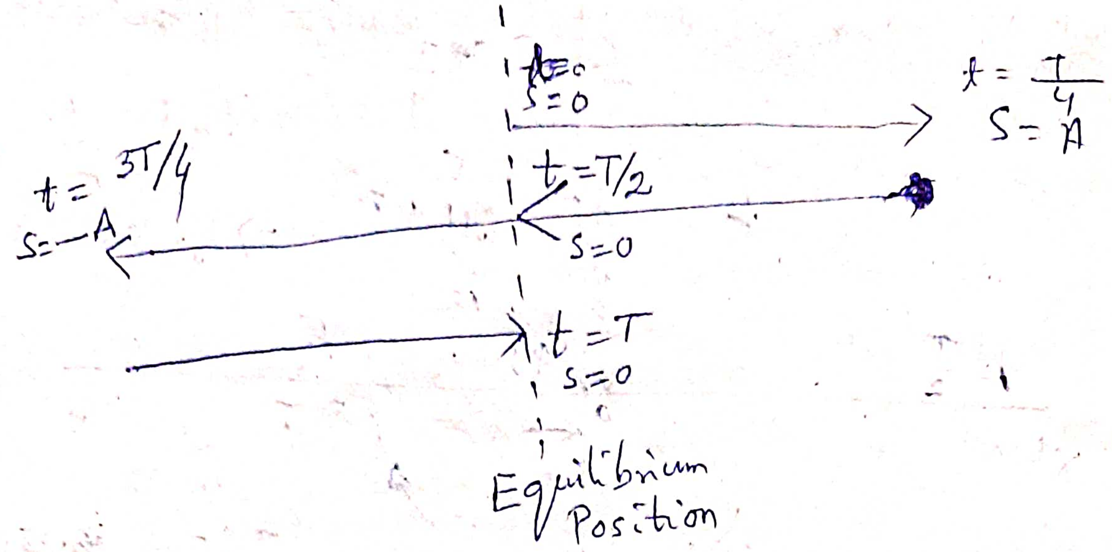
(4). Frequency (f or n)

It is defined as the number of vibrations or oscillations made per second by the vibrating body.

$$\therefore \boxed{f = \frac{1}{T}}$$

imp Derivation of expressions for displacement, velocity, accⁿ of a particle under going S.H.M

Lets consider a small particle making S.H.M about some equilibrium position. If the time period be T sec, the displacements of the particle at different times are found to vary. In table (1) we have shown these displacements

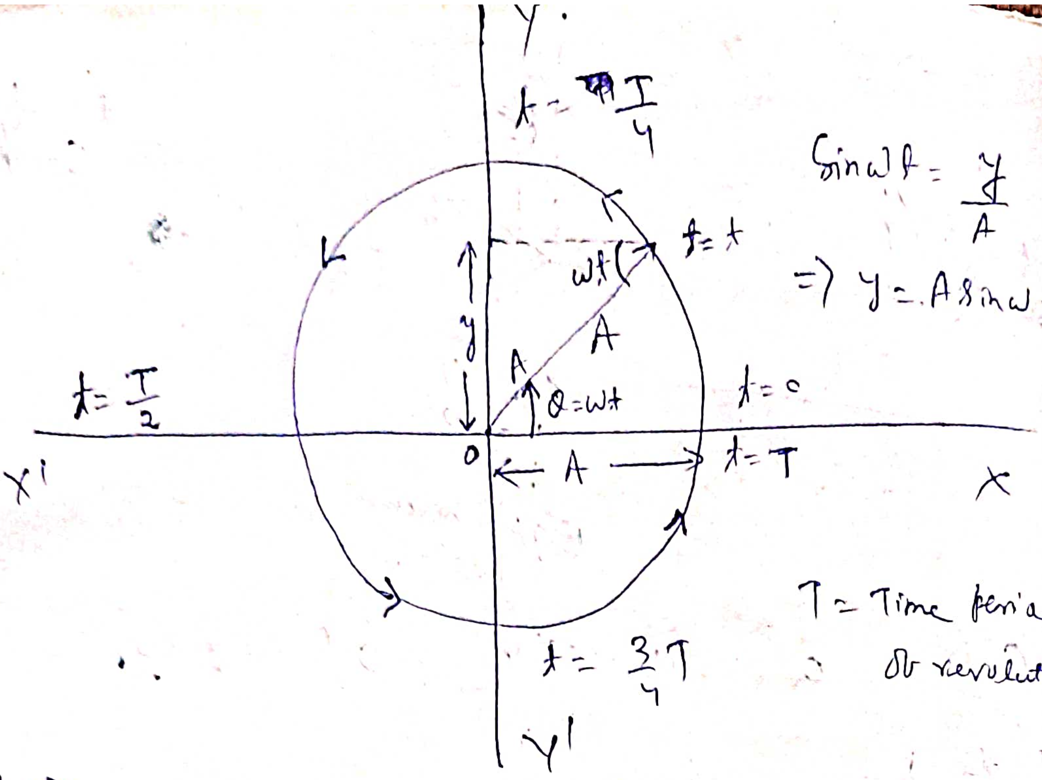


as a function of time.

Table-1

Time (t)	Displacement (S)
0	0 (Min^m)
T/4	A (Max^m)
T/2	0 (min^m)
3T/4	-A (Max^m in the opposite direction)
T	0 (Min^m)

Thus we see that the displacement is a function of time to find that function, lets analyse the motion of a small particle on a circle of radius A with constant speed. This circle is called reference circle. The projections of the radius vector joining the particle



$\sin \omega t = \frac{y}{A}$

$\Rightarrow y = A \sin \omega t$

T = Time period of revolution.

with the center on the YY' axis be found out. Lets make a table for the projection (y) as a function of time.

Table-2

Time (t)		Projection (y)
0	$\longrightarrow$	0 (Minm)
t	$\longrightarrow$	$y = A \sin \omega t$
$\frac{T}{4}$	$\longrightarrow$	A (Maxm)
$\frac{T}{2}$	$\longrightarrow$	0 (Minm)
$\frac{3T}{4}$	$\longrightarrow$	$-A$ (Maxm in the opposite direction)
T	$\longrightarrow$	0 (Minm)

Comparing these two tables, we see that the displacement is identical to the projection.

Therefore, the expression for the projection can be taken as the expression for the displacement.

$$\therefore \boxed{S = A \sin \omega t}$$

$$\therefore S_{max} = A$$

Differenciating both the sides of the eqn $S = A \sin \omega t$ with ~~respect~~ respect to time, we get

$$\frac{dS}{dt} = A \frac{d}{dt} \sin \omega t.$$

$$\Rightarrow \boxed{V = A\omega \cos \omega t}$$

= velocity of the particle undergoing S.H.M.

$$\therefore V_{max} = A\omega$$

Differenciating both the sides of the expression for velocity with respect to time, we get for expression for accⁿ

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t)$$

$$= A\omega \frac{d}{dt} \cos \omega t$$

$$= A\omega (-\sin \omega t).\omega$$

$$= -\omega^2 A \sin \omega t$$

$$\therefore \boxed{a = -\omega^2 s}$$

$$\therefore a_{max} = -\omega^2 s_{max}$$

$$= -\omega^2 A$$

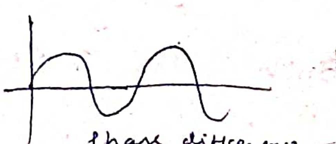
Comparing the expression $a = -\omega^2 s$ for the accⁿ with the expression $a = -\frac{K}{m}.s$, we get

$$\omega^2 = \frac{K}{m}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{K}{m}}}$$

= Constant angular speed with which the particle has to move on the reference circle.

Path difference



Phase difference

For Path diff λ → Phase diff 2π

$1 \to \frac{2\pi}{\lambda}$

$x \to \frac{2\pi}{\lambda} x$

Relation among S, v, ω, A

For a particle undergoing S.H.M, the displacement and velocity are given by the expressions

$$S = A \sin \omega t$$

$$V = A\omega \cos \omega t$$

$$\therefore \quad V^2 = A^2 \omega^2 \cos^2 \omega t$$

$$= A^2\omega^2 (1 - \sin^2 \omega t)$$

$$= A^2\omega^2 - A^2\omega^2 \sin^2 \omega t$$

$$= A^2\omega^2 - \omega^2 \cdot S^2$$

$$\therefore \quad \boxed{V^2 = \omega^2 (A^2 - S^2)}$$

Problem

1. A body is executing S.H.M. When its displacement is 6 c.m the speed is 16 cm/sec. & when the displacement is 8 cm the speed is 12cm/sec. Find Amplitude and period of the motion.

2. Derive expressions for the amplitude, time period and frequency for a particle undergoing S.H.M such that

for displacement x_1, speed is v_1, for displacement x_2, speed is v_2.

Q3. 8, 10, 13, 16

12. 15.7 cm/sec, −24.6 cm/sec², 7.85 cm/sec, −21.306 cm/sec², 7.2·13 × 10⁵ dyne

— 0 —

1. We know the formula,

$$v^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow 256 = \omega^2 (A_1^2 - 36)$$

And when displacement 8 cm, velocity 12 cm/sec

$$\Rightarrow 144 = \omega^2 (A_2^2 - 64)$$

Dividing both the eqn both the sides,

$$\Rightarrow \frac{\cancel{256}}{\cancel{144}} \frac{16}{9} = \frac{A^2 - 36}{A^2 - 64}$$

$$\Rightarrow 16A^2 - 1024 = 9A^2 - 324$$

$$\Rightarrow 7A^2 = \frac{1024 \\ 324}{700}$$

$$\Rightarrow A^2 = \frac{700}{7} = 100$$

$\Rightarrow A = \pm 10$

$\therefore A = 10$ c.m

Putting this value in first eqn

$$256 = \omega^2 (100 - 36)$$

$$\Rightarrow \cancel{256} = \omega^2 \cancel{(64)}$$

$$\Rightarrow \omega^2 = 4$$

$$\Rightarrow \omega = 2 \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{2} = 3.14 \text{ sec.}$$

2. we know the formula

$$v^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow v_1^2 = \omega^2 (A^2 - x_1^2)$$

and $v_2^2 = \omega^2 (A^2 - x_2^2)$

Dividing both the sides we get

$$\left(\frac{v_1^2}{v_2^2}\right) = \frac{A^2 - x_1^2}{A^2 - x_2^2}$$

$$\Rightarrow A^2 v_1^2 - v_1^2 x_2^2 = A^2 v_2^2 - v_2^2 x_1^2$$

$$\Rightarrow A^2 (v_1^2 - v_2^2) = v_1^2 x_2^2 - v_2^2 x_1^2$$

$$\Rightarrow A^2 = \frac{V_1^2 x_2^2 - V_2^2 x_1^2}{V_1^2 - V_2^2}$$

$$\Rightarrow A = \sqrt{\frac{V_1^2 x_2^2 - V_2^2 x_1^2}{V_1^2 - V_2^2}}$$

Putting this value in ~~eqn~~ first eqn we get

$$V_1^2 = \omega^2 (A^2 - x_1^2)$$

$$\Rightarrow \cancel{V_1^2 = \omega^2}$$

$$\Rightarrow \omega^2 = \frac{V_1^2}{A^2 - x_1^2}$$

$$= \frac{V_1^2}{\dfrac{V_1^2 x_2^2 - V_2^2 x_1^2}{V_1^2 - V_2^2} - x_1^2}$$

$$= \frac{V_1^2}{\dfrac{V_1^2 x_2^2 - V_2^2 \cancel{x_1^2} - V_1^2 x_1^2 + V_2^2 \cancel{x_1^2}}{V_1^2 - V_2^2}}$$

$$\Rightarrow \omega = \sqrt{\frac{\cancel{V_1^2}(V_1^2 - V_2^2)}{\cancel{V_1^2}(x_2^2 - x_1^2)}}$$

$$\Rightarrow \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\sqrt{v_1^2 - v_2^2}}{\sqrt{x_2^2 - x_1^2}}}$$

$$= \frac{2\pi\sqrt{x_2^2 - x_1^2}}{\sqrt{v_1^2 - v_2^2}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}} = \frac{1}{2\pi}\sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

213 Page

8. $m = 10\,lb$

$A = 10$ in

$\cancel{t} = 5\,s$

We know $S = A \sin\omega t$

$\Rightarrow S = 10 \sin\omega 5$

We know $\omega = \frac{2\pi}{t} = \frac{\cancel{2} \times 3.14}{\cancel{5}\ 2.5} = 1.256$ rad/sec

(i). Radius of the related circle

= Amplitude of particle

= 10 in

(ii) Speed at the mid point $= A\omega$ (because at the mid point displacement is min^m and speed is max^m so V_{max} is taken)

$= r.\omega$

$= 10 \times 1.256$

$= 12.56$ in/sec.

(iii) Speed at the end point $= 0$

because at end point $s = A$, we know $v^2 = \omega^2(A^2 - s^2)$

$= \omega^2(A^2 - A^2)$

$= 0$

(iv) Accel^n at the mid point

$= \cancel{\omega^2 . A} = -\omega^2 s$

$= \cancel{(12.56)^2} \times 10 = -\omega^2 . 0$

$= 0$

(iv) Accel^n at the end point

$= -\omega^2 s = -\omega^2 A = -(1.256)^2 \times 10$

$= -(1.5775) \times 10$

$= 15.775$ in/sec^2

(iv) frequency $= \frac{1}{T} = \frac{1}{5} = .2$ sec^{-1}

(ii) or

10. $a = 8.0$ m/sec^2

$S = 1$ m

We know $a = -\omega^2 s$

$\Rightarrow 8 = -\omega^2 . 1$

$\Rightarrow -\omega^2 = 8$

$\Rightarrow \omega = 2\sqrt{2}$

we know ~~$S = A \sin \omega t$~~

We know ~~$\omega = \frac{2\pi}{T}$~~

$T = \frac{2\pi}{\omega} = \frac{\cancel{2} \times 3.14}{-\cancel{2} \times \sqrt{2}}$

$= \frac{3.14}{-(1.414)} = -2.2206$ sec

$= 2.2206$

modulus term

13.

$W = 10 Kg$ $\quad \omega = \frac{2\pi}{T} = \frac{\cancel{2} \times 3.14}{\cancel{4} 2} = 1.57$

$T = 4 sec$

$A = 10 c.m$

$a_{max} = -\omega^2 A = -(1.57)^2 \times 10$

$= 2.4649 \times 10$

$= -24.649$ cm/sec2

$V_{max} = A\omega$

$= 10 \times 1.57 = 15.7$ c.m/sec

When a body is $\frac{1}{6}$ period from equilibrium position. $t = \frac{T}{6} = \frac{4}{6} = \frac{2}{3}$

$V = A\omega \cos \omega t$ ~~$= 10 \times 1.57 \times \sin 1.57 \times \frac{2}{3}$~~

$=$ ~~$10 \times \frac{2 \times 3.14}{}$~~ $A\omega \cos \frac{2\pi}{T} \cdot \frac{T}{6}$

$= A\omega \cos \frac{\pi}{3}$

$= A\omega \cos 60°$

$= v_{max} \cdot \frac{1}{2} = \frac{15.7}{2}$ cm/s

$= 7.85$ cm/sec

Accn at that time period

$= -\omega^2 S$

$= -(\omega^2 A \sin \omega t)$

$= -(\omega \cdot A\omega \sin \omega t)$

$= -(1.57 \times 15.7 \times \sin \frac{2\pi}{T} \times \frac{T}{6})$

$= -(1.57 \times 15.7 (\sin 60°))$

$= (1.57 \times 15.7 \times \frac{\sqrt{3}}{2})$

$= 21.3$ cm/s2

or $a = a_{max} \times \sin 60°$

$= 24.649 \times \frac{\sqrt{3}}{2}$

$= 21.3$ cm/s2

The net force $F = ma = 10 \times 10^{3} \times (-21.3) = 10^{4} \times 2.13 \times 10$
$= -2.13 \times 10^{5}$ dyne

16.

Given that

$A = 5$ c.m

$V = 50$ cm/sec

$S = 3$ c.m

$T = ?$

we know that

$$V^2 = \omega^2 (A^2 - S^2)$$

$$\Rightarrow 2500 = \omega^2 (25 - 9) = \omega^2 (16)$$

$$\Rightarrow \omega^2 = \frac{2500}{16} = 156.25$$

$$\Rightarrow \omega = 12.5 \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.141 \text{ rad}}{12.5 \text{ rad/sec}}$$

$$= \left(\frac{6.282}{12.5}\right) \frac{\cancel{\text{rad}} \text{ sec}}{\cancel{\text{rad}}}$$

$$= .50256 \text{ sec.}$$

To prove that for a particle undergoing S.H.M, velocity is max^m when displacement is min^m and vice versa

For a particle undergoing S.H.M, the displacement and velocity are given by the expressions

$$S = A \sin\omega t$$

$$V = A\omega \cos\omega t$$

At time $t = 0$

$$S = A . \sin 0$$
$$= 0$$
$$= \text{Min}^m$$

$$V = A\omega \cos 0$$
$$= A\omega$$
$$= \text{Max}^m$$

At time $t = \frac{T}{4}$

$$S = A \sin \omega . \frac{T}{4}$$
$$= A \sin \frac{2\pi}{T} . \frac{T}{4}$$
$$= A \sin \frac{\pi}{2}$$

$S = A$

$= Max^m$

$V = A\omega \cos\omega t$

$= A\omega \cos \frac{2\pi}{T} \cdot \frac{T}{4}$

$= A\omega \cos\frac{\pi}{2}$

$= A\omega \cdot 0$

$= 0$

$= Min^m$

At time $t = \frac{T}{2}$

$S = A \sin\omega t$

$= A \cdot \sin \cdot \frac{2\pi}{T} \cdot \frac{t}{2}$

$= A \sin\pi$

$= A \cdot 0$

$= 0$

$= Min^m$

$V = A\omega \cos\omega t$

$= A\omega \cos \frac{2\pi}{T} \cdot \frac{T}{2}$

$= A\omega \cos\pi$

$= A\omega \cdot (-1)$

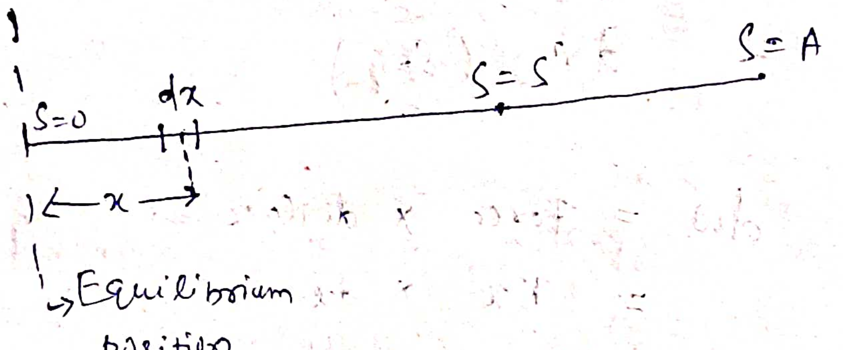
$= -A\omega$

$= Max^m$ in the opposite direction.

Thus, we see that for a particle undergoing S.H.M, velocity is max^m when displacement is min^m and velocity is min^m when displacement is max^m.

212 page -11

Expression for the potential energy of a particle undergoing S.H.M



Let's devide the distance S in to large number of small segments. One such segment has been shown in the diagram which is of thickness dx and its mid point is at a distance x from the equilibrium position.

The restoring force acting on the particle $= -kx$,

The (−ve) sign shows that the

restoring force is directed opposite to the displacement.

If we want to displace the particle away from the equilibrium position; then a minm force of magnitude Kx is to be applied.

The amount of work done on the body to displace it by an amount dx away from the equilibrium position = dw (say)

$$\therefore \quad dw = \text{Force} \times \text{displacement}$$
$$= Kx \times dx$$

Integrating both the sides with proper limits, we get

$$\int_0^w dw = K \int_0^s x \, dx$$

$$\Rightarrow (w)\Big|_0^w = K \cdot \left(\frac{x^2}{2}\right)\Big|_0^s$$

$$\Rightarrow w - 0 = K \left(\frac{s^2}{2} - 0\right)$$

$$\Rightarrow w = \frac{Ks^2}{2} = \frac{1}{2} K s^2$$

$$\boxed{\text{Potential energy of the particle} = W = \frac{1}{2} K s^2}$$

$$= \frac{1}{2} \cdot m\omega^2 \cdot s^2$$

Minim. $P.E = \frac{1}{2} K s^2_{minm}$

$= \frac{1}{2} \cdot K \cdot 0$

$= 0$

Maxm $P.E = \frac{1}{2} \cdot K s^2_{max}$

$= \cancel{\frac{1}{2} \cdot K \cdot s^2}$

$= \frac{1}{2} \cdot K A^2$

Average potential energy

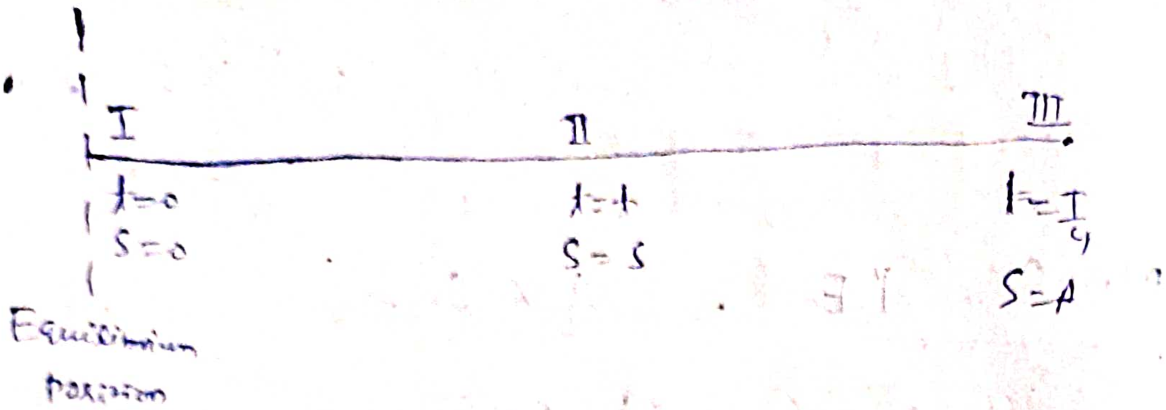
$$= \frac{(P.E)_{minm} + (P.E)_{max}}{2}$$

$$= \frac{0 + \frac{1}{2} K A^2}{2}$$

$$= \frac{1}{4} K A^2$$

Total energy of a particle undergoing S.H.M

Lets consider 3 different positions of the particle undergoing S.H.M



as Shown in the diagram.

<u>At Position I</u>

$$P.E = \frac{1}{2} \cdot K \cdot S^2$$
$$= \frac{1}{2} \cdot K \cdot 0^2$$
$$= 0$$
$$= \text{Min}^m$$

K.E of the particle

$$= \frac{1}{2} \cdot m \cdot v^2$$
$$= \frac{1}{2} \cdot m \cdot (A\omega \cos\omega.0)^2$$
$$= \frac{1}{2} \cdot m \; A^2 \omega^2 \; (1)$$
$$= \frac{1}{2} \cancel{m} A^2 \frac{K}{\cancel{m}}$$
$$= \frac{1}{2} K A^2$$

∴ Total energy of the particle

at position I = P.E + K.E

$$= 0 + \frac{1}{2} KA^2 = \frac{1}{2} KA^2 \; (i)$$

At position II

$$P.E = \frac{1}{2} \cdot K \cdot s^2$$

$$= \frac{1}{2} \cdot K \cdot A^2 \sin^2 \omega t$$

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \cdot m \cdot A^2 \omega^2 \cos^2 \omega t$$

$$= \frac{1}{2} \cdot \cancel{m} \cdot A^2 \cdot \frac{K}{\cancel{m}} \cdot \cos^2 \omega t$$

$$= \frac{1}{2} K A^2 \cos^2 \omega t$$

Total energy at the particle at position II

$$= P.E + K.E$$

$$= \frac{1}{2} K A^2 \sin^2 \omega t + \frac{1}{2} K A^2 \cos^2 \omega t$$

$$= \frac{1}{2} K A^2 (\sin^2 \omega t + \cos^2 \omega t)$$

$$= \frac{1}{2} K A^2 \quad \text{———— (ii)}$$

At position III

$$P.E = \frac{1}{2} \cdot K \cdot s^2$$

$$= \frac{1}{2} \cdot K A^2$$

$$= \text{Max}^m$$

Kinetic energy

$= \frac{1}{2} . mv^2$

$= \frac{1}{2} . m . 0$

$=$ minm

Total energy at position (ii)

$= P.E + K.E$

$= \frac{1}{2}KA^2 + 0$

$= \frac{1}{2} KA^2$ ——— (iii)

Thus, we see that the total energy of the vibrating body remains a constant being equal to $\frac{1}{2}KA^2$ at any position.

Expressions for the time period of vibration or oscillation ~~Oscillation~~ of a ~~an~~ particle undergoing S.H.M

1. Loaded Spring.

If the load be displaced by an amount s, a restoring force of magnitude Ks acts on it, but the direction is opposite

$\therefore F = -KS$

$\Rightarrow ma = -K.S$

$\Rightarrow a = -\frac{K}{m}.S$

Comparing this expression for the accen with $a = -\omega^2.S$,

we get

$$\omega^2 = \frac{K}{m}$$

$$\Rightarrow \omega = \sqrt{\frac{K}{m}}$$

$T =$ Time period of vibration

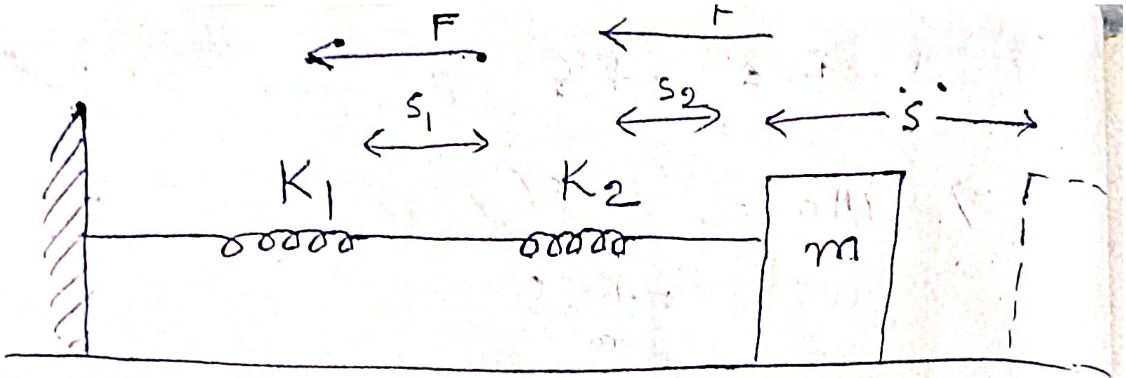
$$= \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{\sqrt{\frac{K}{m}}}$$

$$= 2\pi.\sqrt{\frac{m}{K}}$$

2. Two springs are connected in series along with a mass attach to them and the body makes S.H.M over a frictionless table.

Let the body be displaced by an amount S units. The first



Frictionless Table

Spring gets displaced by an amount S_1 whereas the second spring gets displaced by an amount S_2 units.

But the restoring force remains the same in both the cases

$$\therefore \quad F = -K_1 S_1 = -K_2 S_2$$

$$\Rightarrow \quad S_1 = -\frac{F}{K_1} \quad \text{and} \quad S_2 = -\frac{F}{K_2}$$

$$\therefore \quad S_1 + S_2 = -\frac{F}{K_1} - \frac{F}{K_2}$$

$$= -F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$S = -F\left(\frac{1}{K_1} + \frac{1}{K_2}\right) \quad \text{—— (i)}$$

If the equivalent force constant of the two springs be K, then

$$F = -KS$$

$$\Rightarrow S = -\frac{F}{K} \quad \text{—— (ii)}$$

Equating eqn (i) and (ii), we get

$$\cancel{F}\left(\frac{1}{K_1}+\frac{1}{K_2}\right)=\frac{\cancel{F}}{K}$$

$$\Rightarrow \frac{1}{K_1}+\frac{1}{K_2}=\frac{1}{K}$$

$$\Rightarrow \frac{K_1+K_2}{K_1K_2}=\frac{1}{K}$$

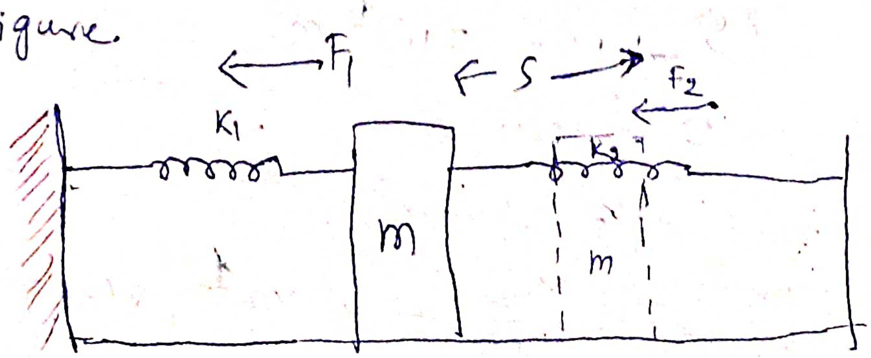
$$\Rightarrow K=\frac{K_1K_2}{K_1+K_2}$$

$$T=2\pi\sqrt{\frac{m}{K}}$$

$$=2\pi\sqrt{\frac{m}{\frac{K_1K_2}{K_1+K_2}}}$$

$$=2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$$

3. Time period of vibration of a body attached with two springs on its two sides as shown in the figure.



Frictionless table.

In this case, the displacements are the same (i.e S). The restoring forces are F_1 and F_2 directed along the same direction.

$\therefore$ Total restoring force = F

$$= F_1 + F_2$$

$$= -K_1 S + (-K_2 S)$$

$$= -(K_1+K_2)S$$

$$= -KS \quad \text{(Say)}$$

where K = Equivalant force constant

$$= K_1+K_2.$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$= 2\pi \sqrt{\frac{m}{K_1+K_2}}$$

— o —

212

(2.)

$S = 10 \text{ cm} = 10^{-1} \text{ m}$

$F = 50 \text{ N}$

$F = K.S$

$\Rightarrow K = \frac{F}{S} = \frac{50}{10^{-1}} = 50 \times 10^{1} \text{ N/M}$

$= 500 \text{ N/M}$

Period of vibration $\cancel{W}F = \cancel{50}\,100$

$\Rightarrow mg = 100$

$\Rightarrow m = \frac{100}{9.8} = \cancel{5\ldots} = 10.2$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2 \times 3.14 \times \sqrt{\frac{\cancel{\ldots}\,10.2}{\cancel{\ldots}\,500}}$$

$$= 2 \times 3.14 \times .142$$

$$= .896 \text{ sec.}$$

6) $W = 10\,lb$, $m = \frac{10\,lb}{32\,ft} =$ slug

$A = 12\,inch = 1\,ft$

$T = .60\,s$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$\Rightarrow T^2 = \frac{4\pi^2 m}{K} = \frac{4 \times (3.14)^2 \times 10}{32 \times K}$$

$$\Rightarrow 32 \times .36\,K = 4 \times (3.14)^2 \times 10$$

$$\Rightarrow K = \frac{4 \times 3.14^2 \times 10}{32 \times .36} = 34.12$$

Kinetic energy $= \frac{1}{2} K A^2$

$$= \frac{1}{2}(34.12)(1)$$

$$= 17.06 \; ft^2\,lb.$$

15. $A = 6\,in = \cancel{0.5\,ft} \frac{6}{12}$

$T = .50\,s$

(i)

$s = 5.2 = \frac{5.2}{12}$

We know $v^2 = \omega^2 (A^2 - s^2)$

$\Rightarrow \cancel{v^2 = \frac{4\pi^2}{T^2} (36 - 25}$

$\Rightarrow v^2 = \frac{4 \times (3.14)^2}{.25} \left\{ \left(\frac{6}{12}\right)^2 - \left(\frac{5.2}{12}\right)^2 \right\}$

$= \frac{4 \times (3.14)^2}{.25} \left(\frac{1}{144} (36 - 27.04) \right)$

$= \frac{4 \times (3.14)^2}{.25} \left(\frac{8.96}{144} \right)$

$= \frac{4 \times (3.14)^2}{.25} (.0622)$

$= 9.815$

$v = 3.133\ ft/sec$

(ii) $a_{max} = -\omega^2 A$

$= -\left(\frac{4\pi^2}{T^2} . A \right)$

$= -\left(\frac{4 \times (3.14)^2}{.25} \times \frac{6}{12} \right) = \frac{19.7192}{.25} = 78.87\ ft/s$

(c) $A = .6\text{ m} = \frac{1}{2}\text{ ft} = .5\text{ ft}$

$f = ?$

$F = ma$

$\Rightarrow \cancel{m}g = ma = \cancel{m}\omega^2 A$

$\Rightarrow \omega^2 = \frac{g}{A} = \frac{32}{0.5}$

$\Rightarrow \omega = \sqrt{\frac{32}{0.5}} = 8$

$f = \frac{1}{T} = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi} = \frac{8}{2 \times 3.14} = 1.27\text{ sec}^{-1}$ or

212. Page

11. ~~Pro~~ To prove that in SHM the accl^n is zero when velocity max^m and velocity is zero when accl^n is max^m.

For a particle undergoing SHM, the ~~displacement~~ accl^n and velocity are given by the expressions

$$a = -\omega^2 s = -\omega^2 A \sin \omega t$$

$$V = A\omega \cos \omega t$$

At time $t=0$

$$a = -\omega^2 A \sin \omega t$$
$$= -\omega^2 A . \sin \omega . 0$$
$$= -\omega^2 . 0$$
$$= 0$$
$$= \text{Min}^m$$

$$V = A\omega \cos \omega t$$
$$= A\omega . \cos \omega . 0$$
$$= A\omega . (1)$$
$$= A\omega$$
$$= \text{Max}^m.$$

At time $t = \frac{T}{4}$

$$a = -\omega^2 A \sin \omega t$$
$$= -\omega^2 A . \sin \frac{\cancel{2}\pi}{\cancel{T}} . \frac{\cancel{T}}{\cancel{4}_2}$$
$$= -\omega^2 A . \sin \frac{\pi}{2}$$
$$= -\omega^2 A (1)$$
$$= -\omega^2 A$$
$$= \text{Max}^m$$

$$V = A\omega \cos \omega t$$
$$= A\omega \cos \frac{\cancel{2}\pi}{\cancel{T}} . \frac{\cancel{T}}{\cancel{4}_2}$$
$$= A\omega \cos \frac{\pi}{2}$$
$$= A\omega . 0$$
$$= 0$$
$$= \text{Min}^m$$

At time $t = \frac{T}{2}$

$$a = -\omega^2 A \sin \omega t$$

$$= -\omega^2 A \sin \frac{2\pi}{T} \cdot \frac{T}{2}$$

$$= -\omega^2 A \sin \pi$$

$$= -\omega^2 \cdot A \cdot (0)$$

$$= 0$$

$$= \text{Min}^m$$

$$V = A\omega \cos \omega t$$

$$= A\omega \cdot \cos \frac{2\pi}{T} \cdot \frac{T}{2}$$

$$= A\omega \cos \pi$$

$$= -A\omega$$

$= \text{Max}^m$ in the opposite direction.

Thus, we see that the particle undergoing SHM when velocity is max^m, ~~when~~ accl^n is zero. and when velocity is min^m accel is max^m.

213 Page   Problems

4.

mass $m = 16$ K.g

Additional 1K.g is added.

The spring streachchedes 0.80m.

We know that

$$F = -K\vec{S}$$

$$\Rightarrow mg = -KS$$

$$\Rightarrow \overset{1}{\cancel{17}} \times 9.8 = -K.(.8)$$

$$\Rightarrow \cancel{K = \frac{\overset{21.25}{\cancel{17}} \times 9.8}{\cancel{.8}} = \overset{208.25}{\cancel{196}}}$$

$$\Rightarrow x = \frac{9.8}{.8} = 12.25$$

When the 1K.g mass is released

then remaining mass = 16 K.g.

We know that time period of vibration

$$= 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\times 3.14\times\sqrt{\frac{16}{\overset{12.25}{\cancel{208.25}}}}$$

$$= 17.17 \text{ sec} \quad (\text{Ans})$$

1\. Problems

A bullet is fired from a rifle with a velocity of 2240 ft/sec is heard to strike a target 3 secs later. Find the distance of the target from the Marks man.

Velocity of sound - 1120 ft/sec

Ans 2240 ft.

Let the distance between the marks man and the target be x feet.

It is fired with a velocity = 2240 ft/sec

we know

~~Velo~~ Distance = ~~veloc~~

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \text{Time} = \frac{\text{Distance}}{\text{velocity}}$$

$$= \frac{x}{2240} \text{ sec}$$

Time taken to reach the target $= \frac{x}{2240}$.

Time taken to reach near the marks man $= \frac{x}{1120}$

As per the question $\frac{x}{2240} + \frac{x}{1120} = 3$

$\Rightarrow \frac{x + 2x}{2240} = 3$

$\Rightarrow \cancel{3}x = \cancel{3}\ 2240$

$\Rightarrow x = 2240$ ft ~~sec~~.

2. Prove that the min^m distance necessary for the formation of ~~eco~~ echo is 56 feet or 17 metre between the source of sound and reflector.

Hints: Any sound that reaches the observer within $\frac{1}{10}$ sec will not be distinguished by the brain

Given $V = 1120$ ft/sec or 340 m/sec.

Proof :-

Distance covered by sound waves during $\frac{1}{10}$ sec

$S = Vt = 1120$ ft/sec $\times \frac{1}{10}$ sec
$= 112$ ft

or $S = vt = 340$ m/sec $\times \frac{1}{10}$ sec
$= 34$ metre

This includes the ~~to~~ distance to go and to come back.

So distance between the source of sound and reflector will be half of the above distance

$= \frac{112 ft}{2} = 56$ ft

or $\frac{34 m}{2} = 17$ meter.

2. Calculate the velocity of sound in a gas in which two waves of lengths 1m and 1.01 m produced 10 beats in 3 sec

Ans (336.7 m/sec)　　$V = n\lambda$

Hints :

Number of beats per sec $= \frac{10}{3} =$ frequency difference

We know that $V = n\lambda$

~~#~~ where $V =$ velocity of sound.

$n =$ frequency.

$\lambda =$ wave length.

For ~~Note~~ ~~$\lambda =$~~ The given first wave length $\lambda = 1$ m

So

$$n_1 = \frac{v}{\lambda} = \frac{v}{1} = v$$

And the second wave length

is $\lambda = 1.01$ m.

$$n_2 = \frac{v}{1.01}$$

According to question

$$v - \frac{v}{1.01} = \frac{10}{3}$$

$$\Rightarrow \frac{1.01v - v}{1.01} = \frac{10}{3}$$

$$\Rightarrow 3.03 - 3v = 10.1$$

$$\Rightarrow .03v = 10.1$$

$$\Rightarrow v = \frac{10.1}{.03} = \frac{1010}{3} = 336.66 \text{ m/s}$$

3. A particle executes SHM along a line of 8 cm length. Its velocity at mean position is 16 cm/sec. Find the time period. (Ans: 1.57 sec)

Ans

$$V_{max} = 16 \text{ cm/sec}$$

$$\Rightarrow A\omega = 16 \text{ cm/sec}$$

~~Ansf~~

Given $2A = 8$ c.m

$\Rightarrow A = 4$ c.m.

$A\omega = 16$ c.m / sec

$\Rightarrow 4 \cdot \omega = 16$ c.m /sec

$\Rightarrow \omega = 4 \text{ sec}^{-1}$

$$T = \frac{2\pi}{\omega} = \frac{\cancel{2} \times 3.14}{\cancel{4}\,2} = 1.57 \text{ sec.}$$

4. Find the ratio of displacement to amplitude of a particle executing S.H.M at a point where its velocity is half ~~of~~ the max^m velocity.

(Ans: 0.866)

Ans

$$V = \frac{V_{max}}{2} = \frac{A\omega}{2}$$

~~Displace~~ $\Rightarrow \cancel{A\omega} \cos \omega t = \frac{\cancel{A\omega}}{2}$

$\Rightarrow \cos \omega t = \frac{1}{2} = \cos 60°$

$\Rightarrow \omega t = 60°$

$S = A \sin \omega t$

$\Rightarrow \frac{S}{A} = \sin \omega t = \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{1.732}{2}$

$= 0.866$ (Ans)

5. A body of mass 10kg executes SHM and has a period of 5 secs. Its velocities is 5 m/sec after 1 sec of crossing the mean position. Find the potential and kinetic energy (Ans : 1184 Joule, 125 J)

$m = 10 kg, \quad T = 5 sec$

Kinetic energy of the particle

$= \frac{1}{2} m v^2$

$= \frac{1}{2} \cdot 10 \times 25$ .2974

$= 125 J$

$\omega t = \frac{2\pi}{T} \cdot t = \frac{2\pi}{5} \cdot 1 = \frac{360}{5} = 72^\circ$

$v = 5 m/sec. = A\omega \cos \omega t = A \cdot \frac{2\pi}{5} \cdot \cos 72^\circ$

$= A \cdot \frac{2 \times 3.14}{5} \cdot (.3090)$

$$\Rightarrow 5 = A \times \frac{(2 \times 3.14)}{5} (.3090)$$

$$\Rightarrow A = \frac{25}{2 \times 3.14 \times .3090}$$

$$= \frac{25}{1.94052}$$

$$\simeq 12.88 \text{ m}$$

$$\frac{K}{m} = \omega^2$$

$$\Rightarrow K = m\omega^2$$

$$\Rightarrow K = 10 \text{ kg} \times \frac{4\pi^2}{T^2} = \frac{\cancel{10}^{2} \times 4 \times (3.14)^2}{\cancel{25}_{5}}$$

$$= \frac{8 \times (3.14)^2}{5}$$

$$\text{Total energy} = \frac{1}{2} K A^2$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{8}^{4} \times (3.14)^2}{5} \times (12.88)^2$$

$$= \frac{4 \times 9.85 \times 165.89}{5} = 1309.62 \text{ J}$$

potential energy = Total energy — Kinetic energy

$$= 1309.62 \quad 125 \text{ J}$$

$$= \cancel{1185.2}$$

$$= 1184.8 \text{ J}$$

6. A body executing SHM has an amplitude of 5 cm. Its velocity is 50 cm/sec. when the displacement is 3 cm. what is the frequency.

(Ans:- 2hz)

we know that $V^2 = w^2 (A^2 - s^2)$

$\Rightarrow 2500 = w^2 (25 - 9)$

~~$\Rightarrow w^2 = \frac{2500}{16} = \frac{1225}{8}$~~

~~$\Rightarrow \omega = \sqrt{\frac{1225}{8}} = \frac{35}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$~~

~~$= \frac{35\sqrt{2}}{4}$~~

~~$= \frac{35 \times 1.414}{}$~~

~~$= 12.37$~~

$\Rightarrow w = \sqrt{\frac{2500}{16}} = \frac{50}{4} = 12.5$

$f = \frac{1}{T} = \frac{1}{\frac{2\pi}{w}} = \frac{\omega}{2\pi} = \frac{12.5}{2 \times 3.14}$

$= \frac{12.5}{6.28}$

$= 1.99$

$\simeq 2hz$

Application
4.
SHM

Simple pendulum

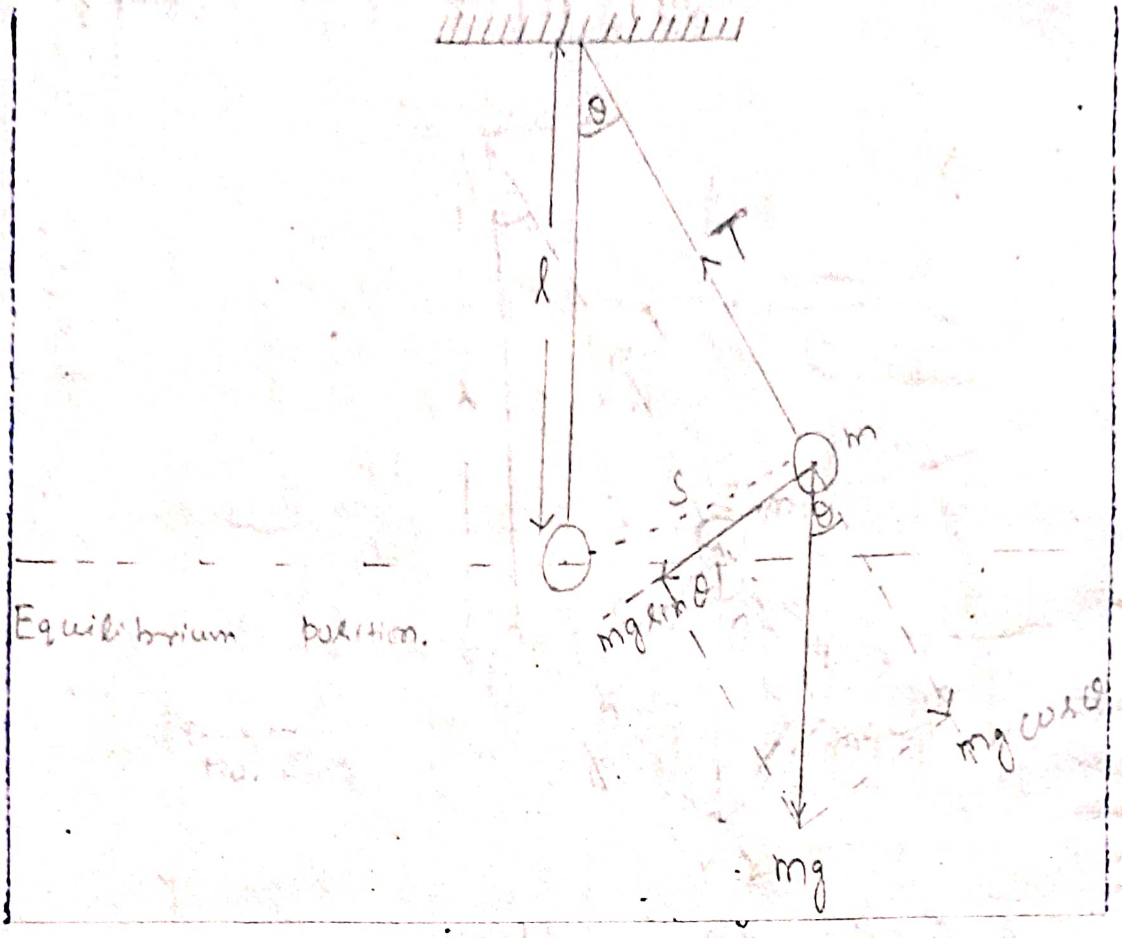
It is a point heavy mass suspended from a rigid support by means of an weightless inextensible, but perfectly flexible thread so as to make a to and fro motion about the equilibrium position.

In the diagram, it has been indicated that weight of the pendulum bob can be resolved into two rectangular components. One component $mg\cos\theta$ balances the tension in the string. where as the other component $mg\sin\theta$ provides the restoring force. The restoring force is always directed towards the equilibrium position. which is indicated by giving a -ve sign.

$$F = -mg\sin\theta$$

$$\Rightarrow ma = -mg\theta$$

where we have approximated



$\sin\theta \simeq \theta$

where θ is small and expressed in radian ($\theta < 4^\circ$)

$$\therefore \quad a = -g\theta$$

$$= -g \cdot \frac{S}{l} \quad \left(\text{because } \theta \text{ in radian} = \frac{\text{Arc length}}{\text{radius}} = \frac{S}{l} \right)$$

$\frac{l}{r}$ or

Displacement from the equilibrium position almost a straight line since θ is very small.

$$a = -\frac{g}{l} \cdot S \equiv -\omega^2 S$$

$$\omega^2 = \frac{g}{l}$$

$$\Rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

= Time period of oscillation.

~~of the~~ of the simple pendulum.

It follows from this expression that

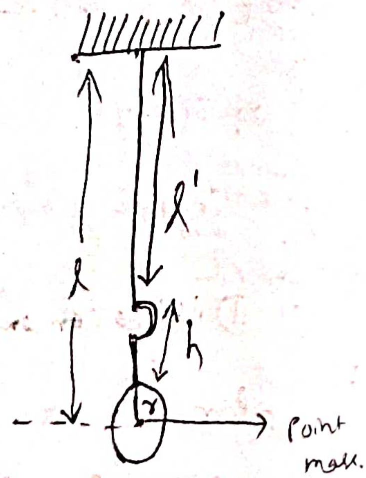
$T \propto \sqrt{l}$ (Law of length)

where l = effective length of the pendulum

= length of the string + length of the hook + radius of the bob.

i.e. $l = l' + h + r$

The time period is independent of the mass of the bob and the amplitude of oscillation.



Seconds pendulum

It is a simple pendulum having time period 2 secs. Length of a second's pendulum can be calculate from the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or From the graph (l verses T^2)

$$\therefore\ 2 = 2\pi\sqrt{\frac{l}{980}}$$

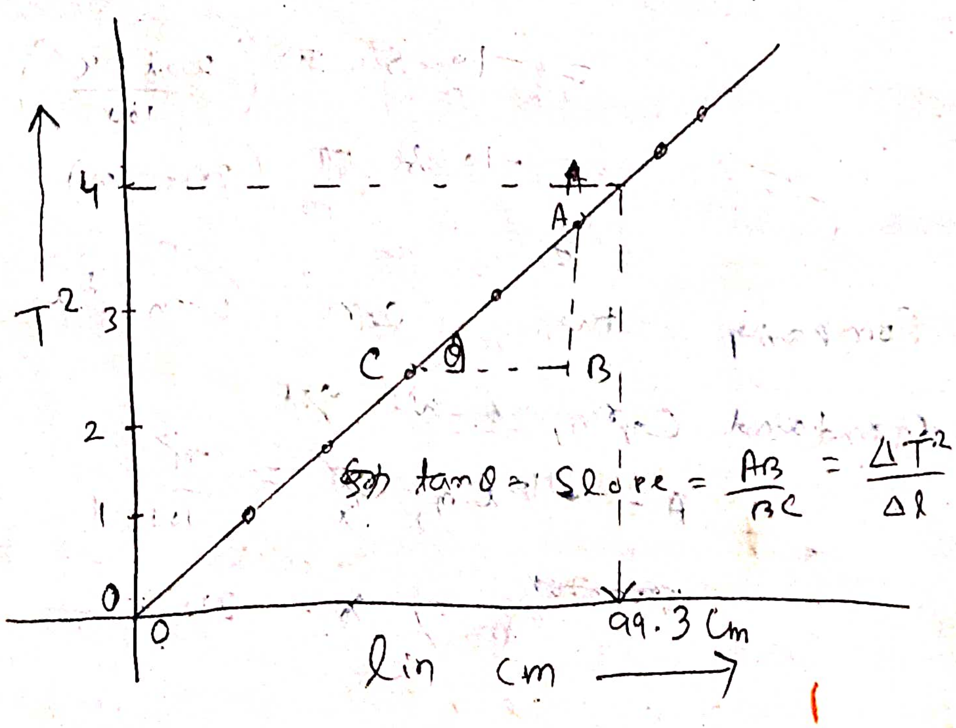
$$\Rightarrow 1 = \pi^2 \cdot \frac{l}{980}$$

$$\Rightarrow l = \frac{980}{\pi^2} = \frac{980}{9.859} = 99.3\ cm$$

From the graph, slope can be found out.

$$T^2 = 4\pi^2\frac{l}{g}$$

$$\Rightarrow g = \frac{4\pi^2 l}{T^2} = 4\pi^2 \cdot \frac{1}{slope}$$



24.1.21

Page 363 Problem

19. Standard progressive wave eqn is

$$y = A \sin \frac{2\pi}{\lambda}(vt - x)$$

Ans:

The given eqn has to be modified so that it will resemble the standard wave eqn $y = A \sin \frac{2\pi}{\lambda}(vt - x)$, so then we can find the amplitude, velocity, frequency, wavelength etc.

Using the formula $\sin(-\theta) = -\sin\theta$, we get

$$y = 10 \sin\left[\pi\left(\frac{x}{100} - 2t\right)\right]$$

$$= -10 \sin \pi\left(\frac{2t - x}{100}\right)$$

$$= -10 \sin \pi\left(\frac{200t - x}{100}\right)$$

$$= -10 \sin \frac{\pi}{100}(200t - x)$$

Comparing this eqn with the standard eqn, we get

$A = 10$ cm, $\frac{2\pi}{\lambda} = \frac{\pi}{100}$,

$\Rightarrow \lambda = 200$ cm

$V = 200$ cm/sec,

using the formula

$V = n\lambda$, we get

$\Rightarrow 200 = n \times 200$

$\Rightarrow n = 1$ hz

~~Table~~ 9, 10, 18, 20

9. During T second, the particle on the reference circle covers an angle of 2π rad

Thus, for the first wave having frequency 20 per sec 2π rad is covered in $\frac{1}{20}$ sec.

During 1 sec, the angle covers is

$\frac{2\pi}{\frac{1}{20}} = 40\pi$ rad

During .75 sec the angle covered is

$= 40 \times .75\pi$

$= \cancel{30}\pi$

$= 30\pi$ rad

For the second wave having frequency 30/sec 2π rad is cover in $\frac{1}{30}$ sec.

During 1sec, the angle cover

$$\frac{2\pi}{\frac{1}{30}} = 60\pi \text{ rad}$$

During .75 sec, the angle covered.

$$60 \times .75 = 45\pi \text{ rad},$$

$\Delta\phi$ = Phase difference

$= \phi_2 - \phi_1$

$= 45\pi\text{ rad} - 30\pi \text{ rad}$

$= 15\pi \text{ rad}.$

$= 2\pi \times 7 + \pi$

= 7 complete oscillation $+\pi$

$= \pi$ rad

$= 180^\circ$

10. Task

after 15 min

335 pm

31. → Similar problem

For both the waves using the formula

$$v = n\lambda$$

$\Rightarrow 330 = 540\,\lambda$

$\Rightarrow \lambda = \frac{33\cancel{0}}{54\cancel{0}} = \frac{33}{54} = \frac{11}{18}$ meter.

Path difference $= (4.4 - 4) = .4$ meter.

We know that for path difference

λ, Phase difference $= 2\pi$

For path difference for 1 phase

difference $= \frac{2\pi}{\lambda}$

For path difference $\varphi(4)$ phase difference

$= \frac{2\pi}{\lambda} \times \frac{4}{18} = \frac{2\times180 \times 2}{18} \cdot \frac{2\pi}{\lambda}(4)$

$= \frac{2\times180\times2\times18}{11\times5} = \frac{72\times36}{11} = 235.44 \quad \frac{2\pi}{\lambda}(4.4)$

Application path difference 4.4 phase difference $= \frac{2\pi}{\lambda}(4.4-4) = \frac{2\pi}{\lambda}(.4) = 235.44$

phase difference $= \frac{2\pi}{\lambda}$

5. Time period of Oscillation of a liquid in a 'U' tube.

(Mercury inside the glass 'U' tube)

Let the density of the liquid be 'ρ'

and the area of cross-section of the tube be 'A' units. Volume of the

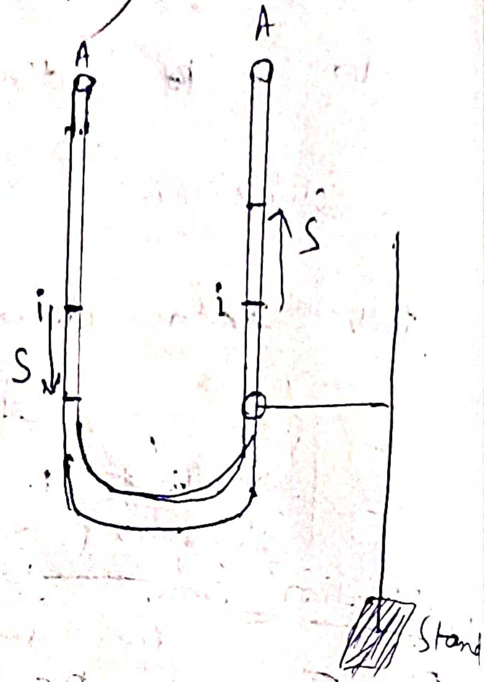
liquid $= AL$ c.c.

where L = Length of the liquid in the 'U' tube.

m = Mass of the liquid in the 'U' tube

$= AL\rho$ gm.

By some external agent like piston let the liquid be displaced by an amount 'S' from the equilibrium position (indicated by 'i')



Actual volume of the liquid displaced by $= A \cdot 2s$

$\therefore$ Mass of the displaced liquid $= 2As\rho$ gm.

~~Resolving force~~ Restoring force is due to gravity which is equal to the weight of the liquid column.

$\therefore$ Restoring force $F = -2As\rho g$ dyne

which acts on the entire liquid mass $AL\rho$ gm

$$\therefore \quad ma = -2As\rho g$$

$$\Rightarrow AL\rho a = -2As\rho g$$

$$\Rightarrow a = \frac{-2As\rho g}{AL\rho} = -\frac{2sg}{L} = -\omega^2 s$$

$$\Rightarrow \omega^2 = \frac{2g}{L}$$

$$\therefore \omega = \sqrt{\frac{2g}{L}}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{L}}} = 2\pi\sqrt{\frac{L}{2g}}$$

Wave

Wave < Progressive < Longitudinal, transverse; Stationary

Energy pro-pagation is called a wave.

All types of progressive waves can be classified classified into two types.

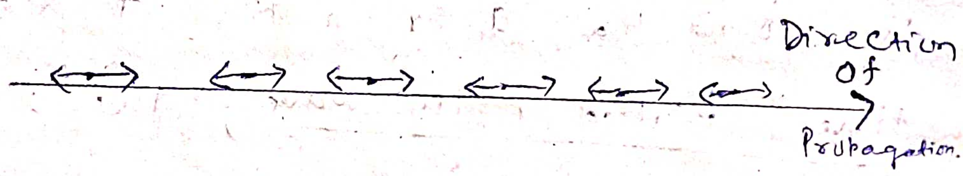
1. Longitudinal.
2. Transverse.

These waves move with time and also called progressive waves.

Longitudinal waves :

If the displacement of the particles of a medium are along the same direction of propagation of the wave, then we call it a longitudinal wave.

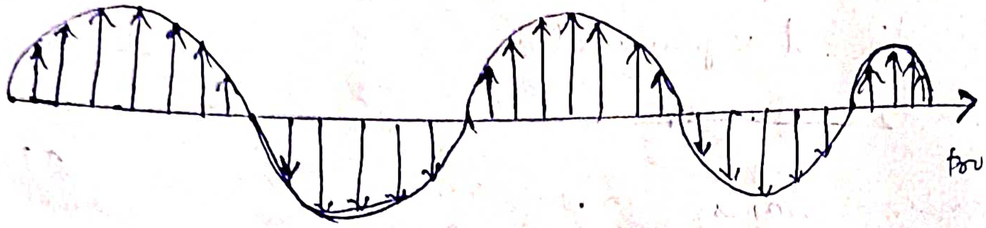
Example: Sound wave, elastic wave.



Transverse waves

If the displacements of the particles of a medium are ⊥ to the direction of propagation of the (means → perpendicular) wave, then we call it a transverse wave.

Example : Light wave, water wave.

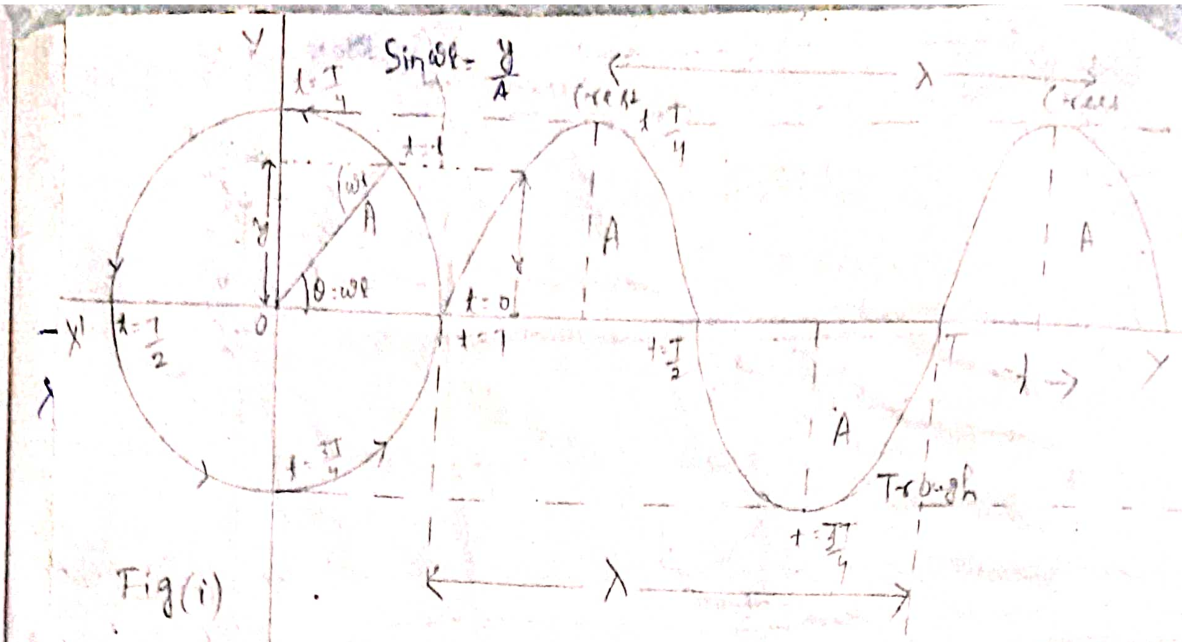


Progressive wave equations

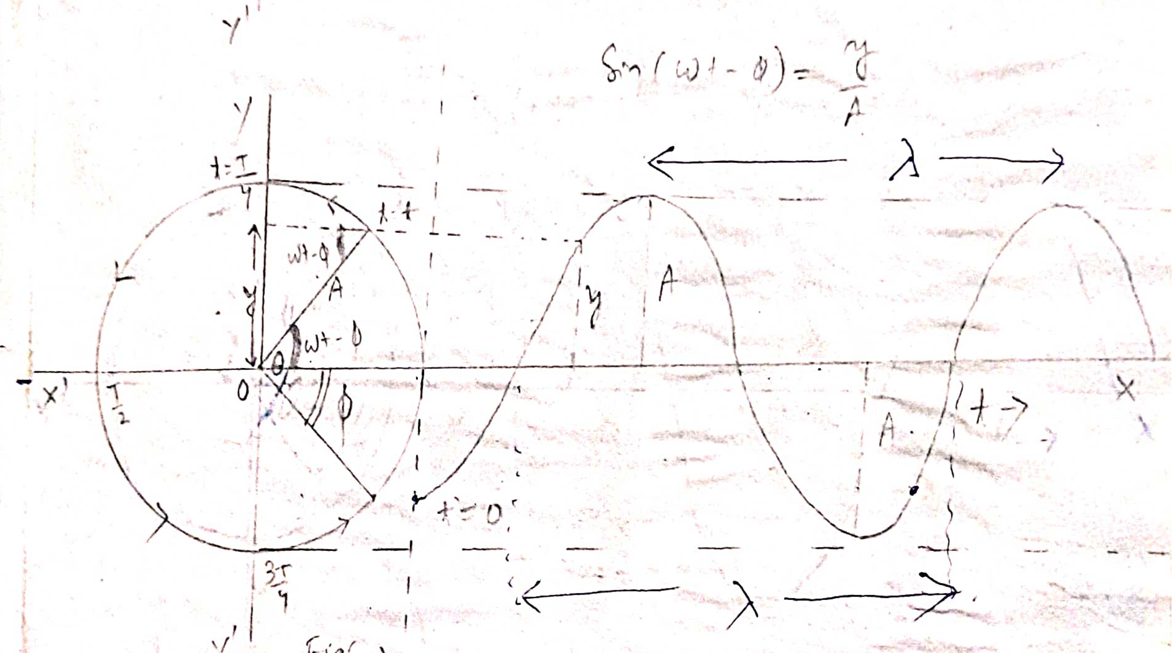
The particles of the medium get disturbed due to the propagation of wave. At any instant of time, the displacement of different particles are different. It is found that after a time T sec called time period of the wave, the displacement are identical.

Wave length (λ) is defined as the distance between two consecutive points in the same state of vibration.

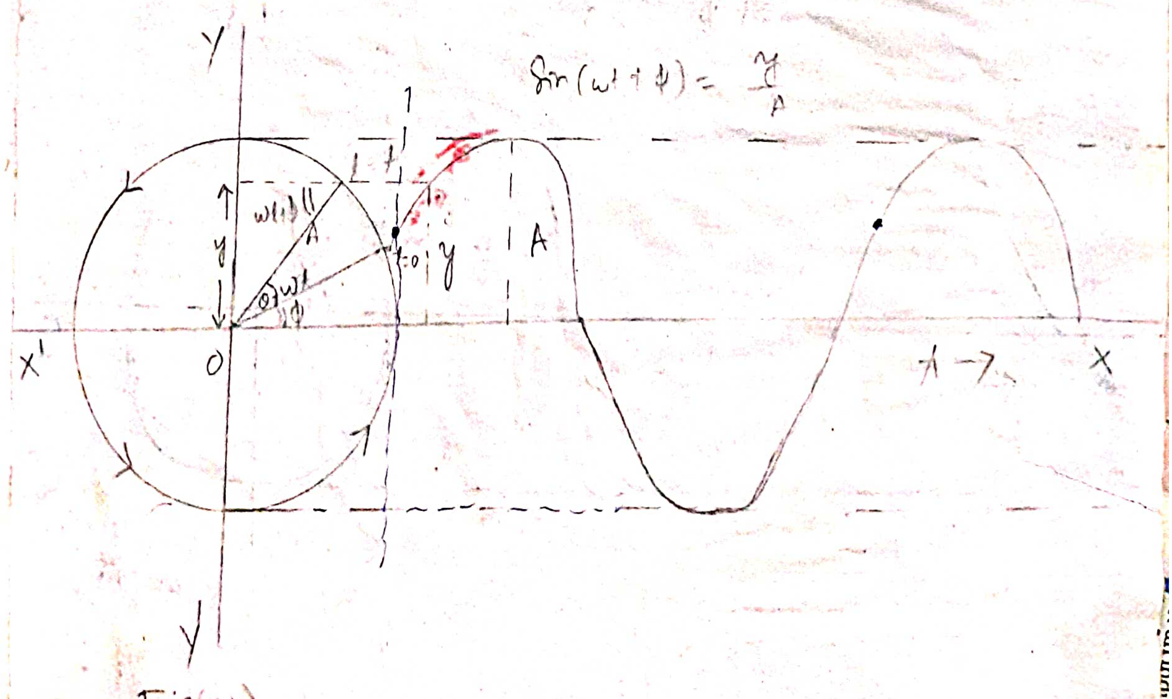
In fig (1), the displacements of different particles have been shown to ~~to~~ be identical to the projections of the radius vector ~~on~~ on the YY' axis of a



Fig(i)



Fig(ii)



Fig(iii)

Reference circle. (A circle of radius A

A = Amplitude ~~of~~ on which a particle rotates at constant speed)

$\sin \omega t = y/A$

$\Rightarrow y = A \sin \omega t$ ———— (i)

This can be regarded as an equation for the progressive wave.

If v = velocity of propagation of the wave, then.

$v = \frac{\text{Distance covered}}{\text{Time taken}}$

$= \frac{\lambda}{T} = n\lambda$

where $n = \frac{1}{T}$ = frequency of the wave.

$$\boxed{v = n\lambda}$$

Suppose the wave does not start from the point from which the particle on the reference circle starts to rotate. This has been shown in fig (ii) and (iii)

The displacement of the particle are given by the expression like

$y = A \sin(\omega t - \phi)$ in fig (ii) and $y = A \sin(\omega t + \phi)$ in fig (iii)

where ϕ = phase angle.

From fig - 1, we can write the following statement.

For path difference λ, the phase difference = 2π radian

For path difference 1, the phase difference $= \frac{2\pi}{\lambda}$ rad.

For path difference x, the phase difference

$$= \frac{2\pi}{\lambda} x \text{ rad}$$

$$= \phi$$

∴ The wave eqn will be modified like

$$y = A \sin(\omega t \pm \phi)$$

$$= A \sin\left(\omega t \pm \frac{2\pi}{\lambda} x\right)$$

$$= A \sin\left(\frac{2\pi}{T} t \pm \frac{2\pi}{\lambda} x\right)$$

$$y = A \sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right) \text{ ——— } (3)$$

Another form of the wave equations can be obtained by using $v = n\lambda$ in the above equation.

$$y = A \sin 2\pi\left(\frac{\lambda t}{\lambda T} \pm \frac{x}{\lambda}\right)$$

$$y = A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda t}{T} \pm x\right)$$

$$y = A \sin \frac{2\pi}{\lambda}(n\lambda t \pm x)$$

$$y = A \sin \frac{2\pi}{\lambda}(vt \pm x) \quad (4)$$

Another form can be obtained from eqn. (2) where,

$$K = \frac{2\pi}{\lambda} \quad \text{is substituted.}$$

$$\therefore y = A \sin\left(\omega t \pm \frac{2\pi}{\lambda} x\right)$$

$$= A \sin(\omega t \pm Kx) \quad (5)$$

where K is sometimes called wave number.

— o —

~~36.3 page~~

Problems

9. ~~Prob~~ Two waves whose frequency

$n_1 = 20$ /sec

$n_2 = 30$ /sec

10. According to question

$f_1 = \cancel{200}$ 500 /sec , $T = \frac{1}{f} = \frac{1}{500}$ sec

$f_2 = 511$ /sec. $T = \frac{1}{f} = \frac{1}{511}$ sec.

T sec. it covers the particle covers 2π radian angle

1 sec. it covers angle $= \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{500}} = 1000\pi$

For the second wave

In T sec it covers angle = 2π rad

1 sec it covers angle $= \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{511}}$

$= 1022\pi$.

During 1.4 sec the first wave covers

angle $= 1000\pi \times 1.4$

$= 1400\pi$

During 1.4 sec the second wave covers

angle $= 1022\pi \times 1.4$

$= 1430.8\pi$

$\Delta\phi$ = Phase difference

$= \phi_2 - \phi_1$

$= 1430.8\pi - 1400\pi$

$= 30.8\pi$

$= 2\pi \times 15 + .8\pi$

= 15 complete oscillation + $.8\pi$

$= .8\pi$ rad

$= .8 \times 180$

$= 144°$ (Ans)

18. The standard progressive wave eqn is

$$y = A \sin \frac{2\pi}{\lambda}(vt - x)$$

The given eqn is

$$y = \cancel{10}\ 2\sin\left[2\pi\,(2x - 100t)\right]$$

This can be modified as following,

$$y = 2\sin 2\pi\left\{-(100t - 2x)\right\}$$

$$= 2\sin -\left\{2\pi(100t - 2x)\right\}$$

$$= -2\sin 2\pi\,(100t - 2x)$$

$$= -2\sin 4\pi\,(50t - x)$$

Comparing it to the standard eqn we get

$$A = 2\ \text{c.m.}$$

$$\cancel{\frac{2\pi}{\lambda} = 4\pi}\ 2$$

$$\Rightarrow 2\lambda = 1$$

$$\Rightarrow \lambda = \frac{1}{2} = .5\ \text{c.m.}$$

$$vt = \cancel{100t}\ 50$$

$$\Rightarrow v = \cancel{100}\ 50\ \text{c.m/sec}$$

$\cancel{f = \frac{1}{}}$

$V = n\lambda$

$\Rightarrow 50 = n \times .5$

$\Rightarrow n = \frac{50}{.5} = 100 \text{ hz.}$

20. We know that the standard progressive wave eqn is

$$y = A \cos \frac{2\pi}{\lambda} (vt - x)$$

Given eqn is

$$y = 0.025 \cos (3.14x - 62.8t)$$

$$= 0.025 \cos \{-(62.8t - 3.14x)\}$$

$$= 0.025 \cos (62.8t - 3.14x)$$

$$(\because \cos(-\theta) = \cos\theta)$$

$$= 0.025 \cos (20\pi t - \pi x)$$

$$= 0.025 \cos \pi (20t - x)$$

Comparing it to the standard eqn,

we get $A = 0.025$ m.

$$\frac{2\cancel{\pi}}{\lambda} = \cancel{\pi}$$

$\Rightarrow \lambda = 2$ metre

$v = 20$ m/sec.

$v = n\lambda$ $\qquad t = \frac{1}{f} = \frac{1}{10}$ sec

$\Rightarrow 20 = n.(2)$

$\Rightarrow n = 10/$sec.

Displacement at the point $x = 0.5$ m,

$y = 0.025 \cos\pi(20t - x)$

$= 0.025 \cos\pi \cdot (20 \times \frac{1}{10} - .5)$

$= 0.025 \cos\pi \frac{3}{2}$ $\cancel{(1.5)}$

$= 0.025 \cos\frac{3\pi}{2}$

$= 0.025 \cos(\pi + \frac{\pi}{2})$

$= 0.025 \times 0 = 0$

32 $\quad v = 15$ mile/hour $= 22$ ft/sec

Distance between two crests $= \lambda = 11$ ft.

$v = n\lambda$

$\Rightarrow n = \frac{v}{\lambda} = \frac{22}{11} = 2$ per sec . (Ans)

29. $\quad f = 13/$sec, $A = 2$ inch

$t = .33$ sec , $T = \frac{1}{f} = \frac{1}{13}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{13}} = 26\pi$$
$$= 26 \times 3.14$$
$$= 81.64 \text{ rad}$$

$$\omega t = 26\pi \times \frac{1}{3} = 8\pi + \frac{2\pi}{3}$$
$$= 8\pi + 120^\circ$$

Displacement $s = A \sin\omega t$

$$= 2 \times \sin(\cancel{81.64 \times .33}) \quad 8\pi + 120^\circ$$
$$= 2 \times \sin(120^\circ)$$
$$= 2 \times \frac{\sqrt{3}}{2} = 2 \times 0.866$$
$$= 1.732 \text{ inch}$$

velocity $= A\omega \cos\omega t$

$$\cancel{= 2 \times 81.64 \cos(81.64 \times .33)}$$
$$= 2 \text{ inch} \times 26\pi \times \cos(8\pi + 120^\circ)$$
$$= \cancel{2} \times 26 \times \frac{22}{7} \times \left(\frac{-1}{\cancel{2}}\right) \text{ inch/sec}$$
$$= 81.71 \text{ in/sec}$$

33.

$f_1 = 200 /\text{sec}$

$f_2 = 220 /\text{sec}$

$v = 330 \text{ m/sec}$

Phase difference $= 2$ metre.

$V = f\lambda$

$\Rightarrow \lambda = \frac{V}{f}$

$$\lambda_1 = \frac{330}{200} = 1.65 \text{ metre}$$

$$\lambda_2 = \frac{330}{220} = \frac{3}{2} = 1.5 \text{ meters.}$$

We know that for path difference

λ_1, phase difference $= 2\pi$

$\therefore$ for path difference 1, phase ~~difference~~ difference

$= \frac{2\pi}{\lambda_1}$ (ϕ_1) say

for path difference ~~[illegible]~~ 2, phase ~~difference~~ difference

$= \frac{2\pi}{\lambda_1}(2)$

for path difference λ_2, phase ~~difference~~ difference $= 2\pi$

" " " 1, " " $= \frac{2\pi}{\lambda_2}$

" " " 2, " " $= \frac{2\pi}{\lambda_2}(2)$

(ϕ_2 say)

$\Delta\phi =$ phase difference ~~Phase difference~~ $= \phi_1 - \phi_2$

$= \frac{4\pi}{\lambda_2} - \frac{4\pi}{\lambda_1}$

$= 4\pi\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$

$= 4\pi\left(\frac{1}{1.5} - \frac{1}{1.65}\right)$

$= 4\pi\left(\frac{1.65 - 1.5}{1.5 \times 1.65}\right)$

$= 4\pi\left(\frac{.15}{1.5 \times 1.65}\right)$

10

$= 4\pi \left(\frac{1}{16.5} \right)$

$= 4 \times \cancel{18}^{36} \times \frac{10}{\cancel{165}_{33}}$

$= \cancel{4 \times 1.16 \times 10}$

$= 4 \times (1.0909) \times 10$

$= 40 \times 1.0909$

$= 43.636^\circ$ (Ans)

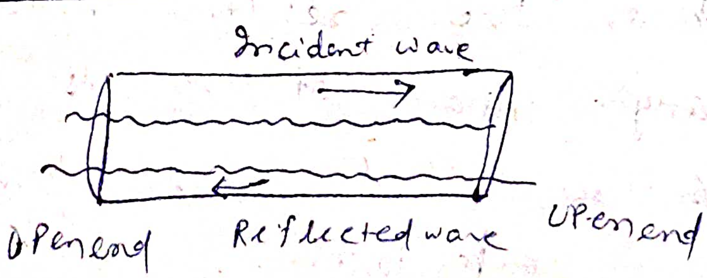
— o —

Stationary wave or standing wave

When a medium is disturbed by two waves equal in amplitude, frequency and wave length, but moving in opposite directions, then a stationary wave is formed.

Lets try to derive expressions for the eqⁿs of the stationary waves.

Case I Formation of stationary waves inside an open-end pipe.



If the incident wave be represented by the eq^n

$$y_1 = A \sin \frac{2\pi}{\lambda} (vt - x) \text{ ; then}$$

the reflected wave has to be represented by the eq^n

$$y_2 = A \sin \frac{2\pi}{\lambda} (vt + x) .$$

Principle of superposition

It states that the resultant displacement of a particle of a medium disturbed by several waves is equal to the algebraic sum of the individual displacements of the same particle by the waves separately.

$$\therefore y = y_1 + y_2 + y_3 + \cdots$$

In this case, a particle present inside the opened pipe is disturbed by two waves, one moving forward and the other moving backwards.

Thus, the resultant displacement at any instant of time is obtained ~~from the~~ from the principle of super position.

$\therefore y = y_1 + y_2$

$= A \sin \frac{2\pi}{\lambda}(vt - x) + A \sin \frac{2\pi}{\lambda}(vt + x)$

$\cancel{= A \sin \frac{2\pi}{\lambda}}$

$= A\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$

where $\alpha = \frac{2\pi}{\lambda} vt$ and $\beta = \frac{2\pi}{\lambda} x$

$\therefore y = A\left[\sin\alpha . \cos\beta - \cancel{\cos\alpha . \sin\beta} + \sin\alpha . \cos\beta + \cancel{\cos\alpha . \sin\beta}\right]$

$\cancel{= A [2A}$

$= 2A \sin\alpha \cos\beta$

$= \left[2A \cos\frac{2\pi x}{\lambda}\right] \sin\frac{2\pi}{\lambda} vt$

$= A' . \sin\frac{2\pi}{\lambda} n\lambda t$

$\boxed{y = A' \sin \omega t}$ $\left(n = \frac{1}{T}\right)$

where A' = Amplitude of the stationary wave

$= 2A \cos\frac{2\pi x}{\lambda}$ which is a variable quantity.

This shows that the amplitude depends on position. i.e the amplitude of different particles are different. The presence of $\sin \omega t$ clearly shows that all the particles vibrate with the same time period or frequency ($T = \frac{2\pi}{\omega}$ and $f = \frac{1}{T}$)

Lets calculate the amplitude of different particles.

At $x = 0$, $A' = 2A \cos 0$

$= 2A$

$=$ Max^m

At $x = \frac{\lambda}{4}$, $A' = 2A \cos \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$

$= 2A \cdot \cos \frac{\pi}{2}$

$= 2A \cdot 0$

$= 0$

$=$ Min^m

At $x = \frac{\lambda}{2}$, $A' = 2A \cos \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}$

$= 2A \cos \pi$

$= -2A$

$=$ Max^m in the opposite direction.

At $x = \frac{3\lambda}{4}$, $A' = 2A \cos \frac{2\pi}{\lambda} \cdot \frac{3\lambda}{4}$

$= 2A \cos \frac{3\pi}{2}$

$= 2A \cos \left(\pi + \frac{\pi}{2}\right)$

$= 2A \cos\left(-\frac{\pi}{2}\right)$

$= 2A \times 0$

$= \cancel{\text{2A}}\ 0$

$= \text{Min}^m$

At $x = \lambda$, $A' = 2A \cos \frac{2\pi}{\lambda} \cdot \lambda$

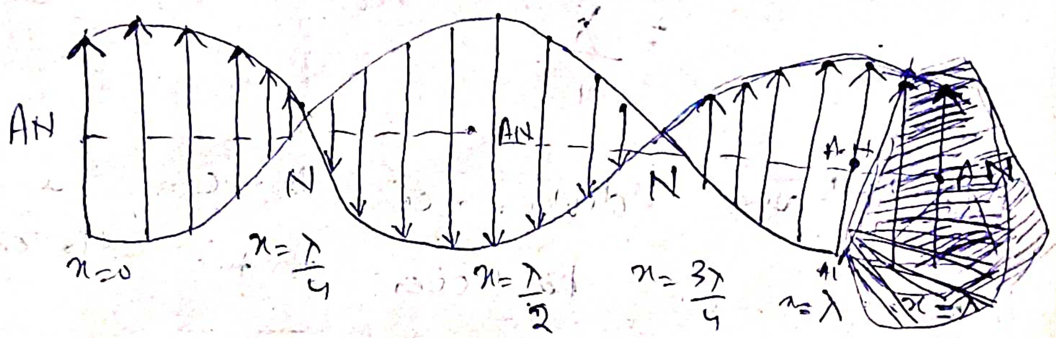
$= 2A \cos 2\pi$

$= \cancel{2A \cos \pi}$

$= 2A \times 1$

$= 2A$

$= \text{Max}^m$



From the above calculation, we see that there are some particles which are permanently at rest and we call such points as <u>nodes.</u> ~~AN~~

At positions like $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ------,

nodes are present.

Distance between ~~conjuct~~ consecutive nodes

$= \frac{\lambda}{2}$

There are some points on the Stationary wave where the particles of the medium with max^m amblitudes. This points are called antinodes.

At points like $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} ......$, antinodes are formed.

Thus, distance between two consecutive antinodes $= \frac{\lambda}{2}$

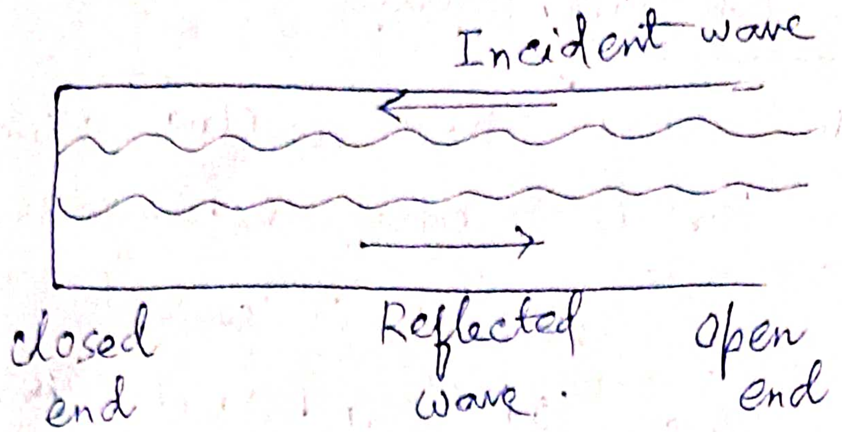
Distance between a node and nearest antinode $= \frac{\lambda}{4}$.

The phase difference between the vibration of particles on two adjacent loopes is $= \pi$ radian or 180^0

It can be shown be direct calcalation that energy transfered by a stationary wave is zero.

Case II

Formation of stationary wave inside a closed - end pipe.



If the eq^n of the incident wave be

$$y_1 = A \sin \frac{2\pi}{\lambda}(vt + x), \text{ then}$$

the eq^n for the reflected wave will be given by

$$y_2 = A \sin\left\{\frac{2\pi}{\lambda}(vt - x) + \pi\right\}$$

$$= -A \sin \frac{2\pi}{\lambda}(vt - x)$$

(2018)

Where an abrupt change of phase by π radian has been introduced as

~~Stock~~ Stoke has proved theoritically that when ever there is reflection from a denser medium, there will be change of phase by π radian or 180°.

Priciple of superposition

It states that the resultant displacement of a particle of a medium distributed by several waves is equal to the ~~particles~~ aigebric sum of individual displacement of same particle by waves seperately.

$\therefore y = y_1 + y_2 + y_3 + \cdots$

In this case the particle present inside a closed end pipe is distributed by two waves, one moving forward and other moving backward.

Thus, the resultant displacement at any instant of time is obtained from principle of superposition.

$$\therefore y = y_1 + y_2$$

$$= A \sin \frac{2\pi}{\lambda}(vt + x) - A \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= A \left\{ \sin \frac{2\pi}{\lambda}(vt + x) - \sin \frac{2\pi}{\lambda}(vt - x) \right\}$$

$$= A \left\{ \sin \left(\frac{2\pi}{\lambda} vt + \frac{2\pi}{\lambda} x \right) - \sin \left(\frac{2\pi}{\lambda} vt - \frac{2\pi}{\lambda} x \right) \right\}$$

Let $\alpha = \frac{2\pi}{\lambda} vt$ and $\beta = \frac{2\pi}{\lambda} x$

$\therefore y = A \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$

$= A \{ \sin\alpha . \cancel{\cos\beta} + \cos\alpha . \sin\beta - \sin\alpha . \cancel{\cos\beta} + \cos\alpha . \sin\beta)$

$= A \{ 2 \cos\alpha \sin\beta)$

$= (2A \sin\beta) \cos\alpha$

$= \left(2A \sin \frac{2\pi x}{\lambda}\right) . \cos \frac{2\pi}{\lambda} vt$

$= A' . \cos \frac{2\pi}{\lambda} . n\lambda . t$

$$\boxed{y = A' . \cos \omega t}$$

where A' = Amplitude of the stationary wave

$= 2A \sin \frac{2\pi x}{\lambda}$ which is a variable quantity.

This shows that the amplitude depends on position i.e the amplitude of different particles are different. The presence of $\cos \omega t$ clearly shows that all the particles vibrate with the same time period or frequency. ($T = \frac{2\pi}{\omega}$ and $f = \frac{1}{T}$)

Lets calculate the amplitude of different particles

At $x = 0$, $A' = 2A . \sin \frac{2\pi}{\lambda} (0)$

$A' = 2\pi \times 0$

$= 0$

$= \text{Min}^m$.

At $x = \frac{\lambda}{4}$, $A' = 2A \sin \frac{\cancel{2}\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{\cancel{4}_2}$

$= 2A \sin \frac{\pi}{2}$

$= 2A$

$= \text{Max}^m$

At $x = \frac{\lambda}{2}$, $A' = 2A \sin \frac{\cancel{2}\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{\cancel{2}}$

$= 2A \sin\pi$

$= 0$

$= \text{Min}^m$

At $x = \frac{3\lambda}{4}$, $A' = 2A \sin \frac{\cancel{2}\pi}{\cancel{\lambda}} \cdot \frac{3\cancel{\lambda}}{\cancel{4}2}$

$= 2A \sin\left(\frac{3\pi}{2}\right)$

$= 2A \sin\left(\pi + \frac{\pi}{2}\right)$

$= 2A - \sin\frac{\pi}{2}$

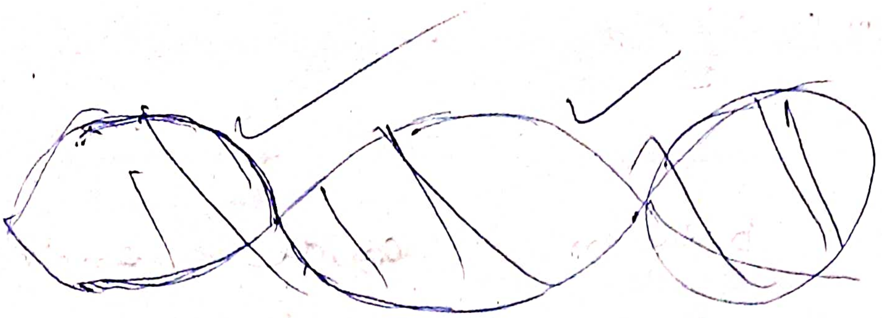
$= -2A$

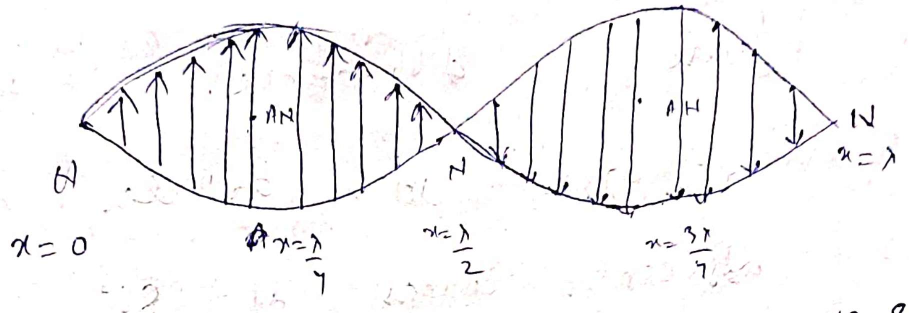
$= \text{Max}^m$ in the opposite direction.

At $x = \lambda$, $A' = 2A \sin \frac{2\pi}{\cancel{\lambda}} \cdot \cancel{\lambda}$

$= 0$

$= \text{Min}^m$





From above Calculation, we see that there are some particles which are permanately at rest and we call such as points as nodes.

At positions like $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ ------

Distance between two Consecutive nodes

$= \frac{\lambda}{2}$

There are some points on the stationary wave where the particles of medium with max amplitude. This points are called antinodes.

At points like $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ----------

antinodes are formed.

Thus distance between two Consecutive

antinodes $= \frac{\lambda}{2}$

~~Thus~~

Distance between anode and nearest antinode $= \frac{\lambda}{4}$,

The phase difference between the vibration of particles on two adjacent loopes is $= 180°$.

It can be shown by directe calculation that energy transformed by a stationary wave is zero.

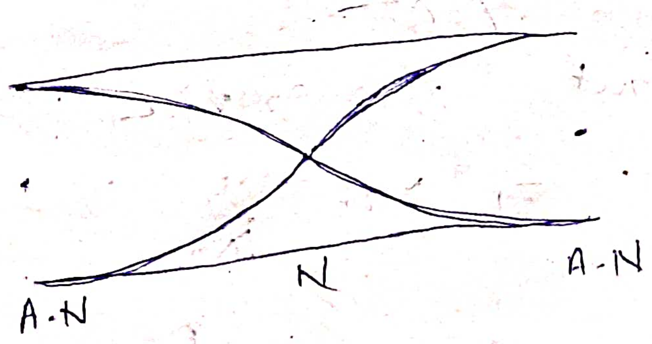
~~Vibro~~

Vibration Of air Column in an open end Organ pipe

When sound waves are introduced into an opened organ pipe, stationary wave is formed inside it due to reflection of soundwave from the open end. But, the two open ends become the antinodes. With this restriction, the air column inside the pipe can vibrate in various ways.

(1) Fundamental

In this case there is only one node at the middle of the open end pipe. However two antinodes are formed at the two open ends.



Here $l = \frac{\lambda_1}{2}$

$\Rightarrow \lambda_1 = 2l$

But $v = n_1 \lambda_1$

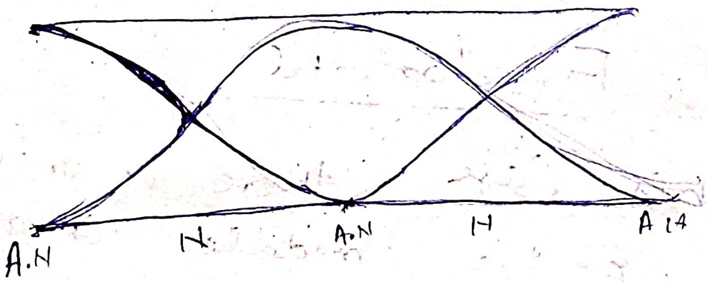
$\Rightarrow n_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$ = Fundamental frequency

= $\frac{v}{2l}$ Min^m frequency that can be emitted by the open end tube

(Higher frequency)

(2) First over tone

In this case, in addition to two antinodes at the two open ends, there are two nodes and another antinode at the middle of the pipe as shown in the figure



Here $l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2} = \lambda_2$

(3)

~~$\Rightarrow \lambda_2 =$~~

But $v = n_2 \lambda_2$

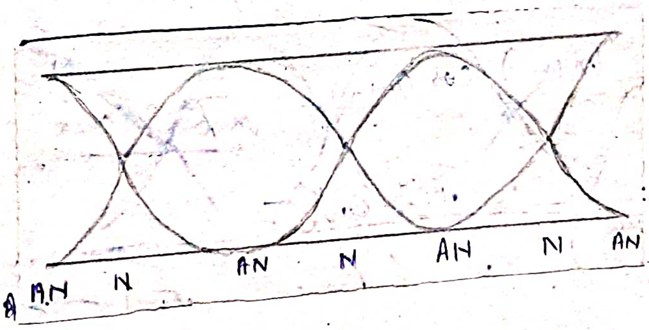
$\Rightarrow n_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2 \cdot \frac{v}{2l}$

$= 2 n_1$

Thus the frequency of the first over tone is double of the frequency double of the frequency of the fundamental. Hence, we call it second harmonic.

(3) Second overtone

In addition to the two antinodes at the two ends of the there are 3 nodes and 2 antinodes at the middle portion of the pipe as shown in the figure.



Here $l = \frac{3\lambda_3}{2}$

$\Rightarrow \lambda_3 = \frac{2l}{3}$

But $v = n_3 \lambda_3$

$\Rightarrow n_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3n_1$

Thus, the frequency of the second overtone is three times of that of the fundamental.

Hence, it is called <u>third harmonic</u>.

Proceeding like this, it can be shown that the frequencies emitted by an open end organ pipe are in the ratio 1 : 2 : 3 : 4 : 5 ---------

i.e both odd and even harmonics are present.

Vibration of air column in a closed end organ pipe

When sound waves are introduced in to a closed end organ pipe, stationary wave is formed inside it due to the reflection of sound waves from the closed end. But, the closed end becomes the node and the open end becomes the antinode. With this restriction, the air column inside the pipe can vibrate in various ways

(1) Fundamental :

In this case there is ~~always~~ one one node at the closed end and one antinode at the open end.

Here $l = \frac{\lambda_1}{4}$

$\Rightarrow \lambda_1 = 4l$

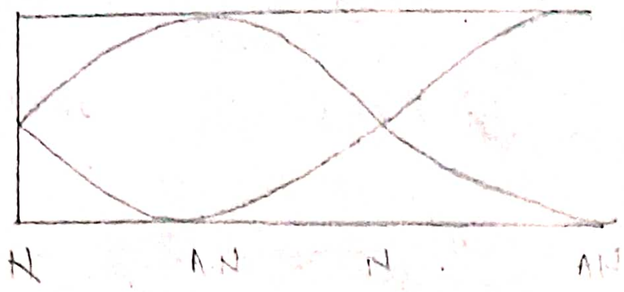
But $v = n_1 \lambda_1$

$\Rightarrow n_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$ = Fundamental frequency

= Min^m frequency that can be emitted by the closed end ~~tube~~ pipe

(2) <u>First over tone</u>

In this case, in addition ~~of~~ to a node at the closed end ~~and~~ an antinode at the open end there is another antinode a note at the middle of the pipe as shown in the figure



Here $l = \frac{3\lambda_2}{4}$

$\Rightarrow \lambda_2 = \frac{4l}{3}$

But $v = n_2 \lambda_2$

$\Rightarrow n_2 = \frac{v}{\lambda_3} = \frac{3v}{4l} = 3 n_1$

Thus the frequency of the first overtone is 3 times the of frequency of fundament, Hence it is called third harmonic.

3) <u>Second overtone</u>

~~In addition to~~ In this case, there is one antinode at the open end and one node at the closed end. There are two antinodes and two nodes present in the middle point of the pipe.

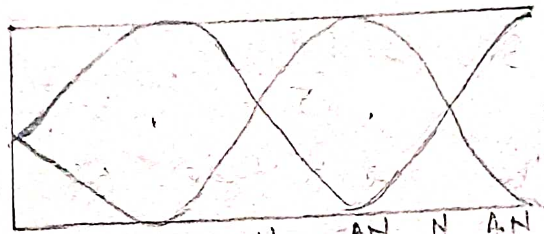
Here $l = \frac{5\lambda_3}{4}$

$$\Rightarrow \lambda_3 = \frac{4l}{5}$$

But we know that $v = n_3 \lambda_3$

$$\Rightarrow n_3 = \frac{v}{\lambda_3}$$

$$\Rightarrow n_3 = \frac{v}{\frac{4l}{5}} = \frac{5v}{4l}$$



Thus the frequency of the second overtone is 5 time of the frequency of fundamental. Hence it is called 5th harmonic.

Pouceeding like this it can be shown that the frequency is emitted by an closed end organ pipe in the ratio 1:3:5:7 - - - - -

i.e only odd harmonics are present.

That is why, sound emitted by open end organ pipe are sweeter to hear than that due to the ~~second~~ sound emitted by a closed end organ pipe.

— 0 —

1. A stationary wave is given by an eqn

$$y = 3.6 \cos \frac{2\pi}{1.5} \cdot 4.5 \cdot \sin \frac{660\pi}{1.5} \cdot t$$

what is the amplitude of the wave ?

Can you represent the two interfering waves which give out the stationary wave ?

(Ans = $3.6 \cos \frac{2\pi}{1.5} \cdot 4.5$

$$y_1 = 1.8 \sin \frac{2\pi}{1.5} \cdot (330t - 4.5)$$

$$y_2 = 1.8 \sin \left\{\frac{2\pi}{1.5}\right\} \left(330t + 4.5\right)$$

2. If l_1 and l_2 be the lengths of air column of the first and second positions of the resonance column for a tuning fork of frequency n, then the end correction is given by

$$\frac{l_2 - 3l_1}{2}$$

3. The third overtone of a closed end organ pipe is found to be in unison with the first overtone of an open end organ pipe. The ratio of the lengths of closed and opened pipe is 7 : 4

4. Two open organ pipes of length 50 and 50.5 cm produced 3 beats per sec. Calculate the velocity of sound in air

Ans:- 303 m/sec.

5. What will be the frequency of the node emitted by ~~an~~ a narrow test tube 10 cm long.

Ans:- 830 hz

6. Stationary waves of frequency 200hz are formed in air. If the velocity of waves be 1120 ft/sec, find the shortest distance between

(a) Two nodes.

(b) Two antinodes.

(c) A node and an antinode

Ans (2.8 feet, 2.8 feet, 1.4 feet, ~~[illegible]~~

7. A Closed pipe 50 cm long & fill with a gas resounds fork of frequency 200 at 0°c. What length will resound at 30°c?

Ans: 52.6 cm

8. Two tuning forks A and B give 6 beats per seconds. A resonance resounds with a Closed column of air 15 cm long, B with an open column 30.5 cm long. Calculate the frequencies

(Ans: 366 hz, 360 hz)

9. The frequency of the node given by an open organ pipe at 15°C is 156. At what temp. will the frequency be 160. (Neglect the expansion of pipe.

(Ans: 29.95 °c)

10. An open pipe is suddenly closed with the result that the second overtone at the closed pipe is found to be higher in frequency by 100 Hz. than the first overtone of the original pipe. what

is the fundamental frequency of the open pipe?

(Ans. : 200 Hz)

1. A fork of frequency 256 is held over a tube and max^m resonance is obtained when the columns of air are 31 cm and 97 cm long. Determine the end correction and the velocity of sound in air.

(Ans : 2 cm, 337.92 m/sec)

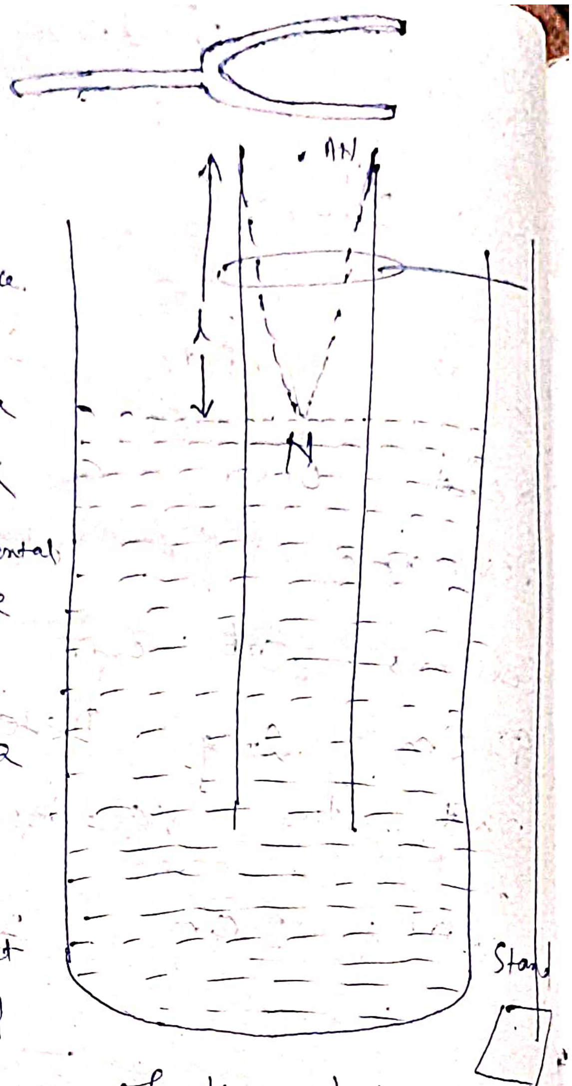
— o —

Experimental determination of velocity of sound in air in the laboratory

(1) +2 Method method

A metalic tube open at both the ends is introduced vertically into a tall jar containing water. A stand is used to hold it at the right position. A tunning fork is struck with a rubber pad and made to vibrate and it is kept at a position ~~slidely abo~~ slightly above the edge of the tube such that the sound wave with proceed towards the water. The water surface serves as the reflector. By trial and error method,

a particular length of the tube is found out for which resonance takes place.



i.e the frequency of the vibration of the tuning fork becomes equal to the fundamental frequency of the closed end pipe

The distance between a node and antinode is $\frac{\lambda}{4}$

Laplace has shown that the antinode is formed slightly above the edge of the tube and he has prescribed a correction of amount $0.6r$ for the length.

where r = internal radius of the tube.

$$\therefore l + 0.6r = \frac{\lambda}{4}$$

$$\Rightarrow \lambda = 4(l + 0.6r)$$

$$v = n\lambda = 4n(l + 0.6r)$$

where n = frequency of the tuning fork

Method-2

In this method resonance is produced twice, first with a length l_1 and with a length l_2 ~~cat~~ of the same tube with the help of same tuning fork as shown the diagram. The end correction as prescribed by Laplace is eliminated as shown by the following two eqns.

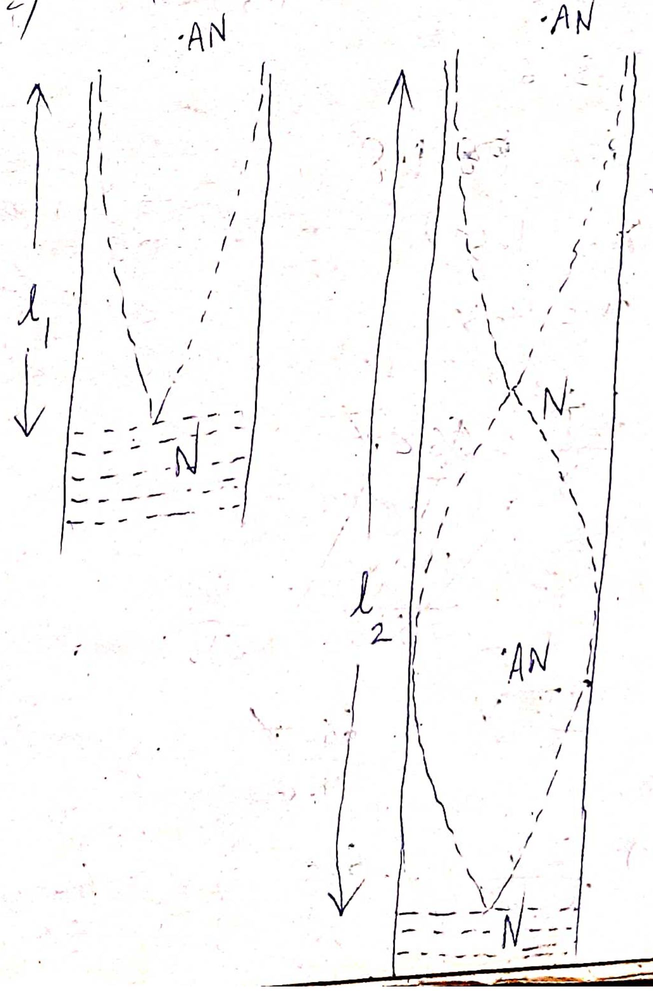
$$l_1 + 0.6r = \frac{\lambda}{4} \quad \text{———} (i)$$

$$l_2 + 0.6r = \frac{3\lambda}{4} \quad \text{———} (ii)$$

Subtracting eqn (i) form eqn (ii), we get

$$\Rightarrow l_2 - l_1 = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2(l_2 - l_1)$$



But $V = n\lambda$

$= 2n(l_2 - l_1)$

By this device the determination of end correction has been factbuley ~~also~~ avoided.

— o —

① . We know that eqn of stationary wave is

$$y = 2A \cos \frac{2\pi}{\lambda} x \cdot \sin \frac{2\pi}{\lambda} v t$$

$$= A' \, \sin \omega t$$

Here given eqn is

$$y = 3.6 \cos \frac{2\pi}{1.5}(4.5) \sin \frac{660\pi}{1.5} t$$

$2A = 3.6$

$\Rightarrow A = 1.8$

$\lambda = 1.5$

$x = 4.5$

$$\frac{2\pi}{\lambda} = \frac{660\pi}{1.5}$$

$$\Rightarrow \lambda = \frac{1.5 \times 2}{660} = \frac{3}{660}$$

$$\frac{2\pi}{\lambda} v = \frac{660\pi}{1.5}$$

$$\Rightarrow \frac{2v}{1.5} = \frac{660}{1.5} \Rightarrow v = \frac{660}{2} = 330$$

Amitude of Stationary wave = A1

$A = 3.6 \cos \frac{2\pi}{1.5} 4.5$

$y_1 = A \sin \frac{2\pi}{\lambda} (vt - x)$

$= 1.8 \sin \frac{2\pi}{1.5} (330t - 4.5)$

$y_2 = 1.8 \sin \frac{2\pi}{1.5} (330t + 4.5)$

(2) The end correction eqn & for the first air column = $l_1 + 0.6r = \frac{\lambda}{4}$

for the second length $l_2 + 0.6r = \frac{3\lambda}{4}$

$\Rightarrow l_2 - l_1 = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2(l_2 - l_1)$

From eqn (i), we get

$0.6r = \frac{\lambda}{4} - l_1$

$= \frac{\cancel{2}(l_2 - l_1)}{\cancel{4}\,2} - l_1$

$= \frac{l_2 - l_1 - 2l_1}{2}$

$= \frac{l_2 - 3l_1}{2}$

(3) The frequencies of Closed end pipe 1st overtone, second overtone, are given by 1 : 3 : 5 : 7

So frequency of 3-rd overtone = $7n_1$

" " 1st overtone or opened = $2n_1$

$$7n_1 = 2n_1$$

$$\Rightarrow 7\times\frac{v}{4l_1} = 3\times\frac{v}{4l_2}$$

$$\Rightarrow 7\times\frac{v}{4l} = 2\times\frac{v}{2l}$$

$\Rightarrow$

$$\Rightarrow 7\times\frac{v}{4l_1} = 2\times\frac{v}{2l_2}$$

$$\Rightarrow \frac{7}{2l_1} = \frac{1}{l_2}$$

$$\Rightarrow 7\times\frac{v}{4l_1} = \cancel{2}\times\frac{v}{\cancel{2}l_2}$$

$$\Rightarrow 7l_2 = 4l_1$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{7}{4} = 7:4$$

4. $l_1 = 50$ } open organ

$l_2 = 50.5$ cm

$n_1 - n_2 = 3$

$$n_1 = \frac{V}{2l_1} = \frac{V}{2\times 50} = \frac{V}{100}$$

$$n_2 = \frac{V}{2l_2} = \frac{V}{2\times 50.5} = \frac{V}{101.0}$$

According to question

$$n_1 - n_2 = 3$$

$$\Rightarrow \frac{V}{100} - \frac{V}{101} = 3$$

$$\Rightarrow \frac{101V - 100V}{100\times 101} = 3$$

$$\Rightarrow V = 30300 \text{ cm/sec}$$

$$= 303 \text{ m/sec}$$

5. $l = 10$ c.m

$n = ?$

$V = 330$

Test tube is closed end

$$n = \frac{V}{4l} = \frac{33\cancel{2}\times 100}{4\times 10} = 830 \text{ hz}$$

(we have take velocity of sound at $0°C$)

6. $n = 200$hz, $V = \cancel{330}$ 1120 ft/sec

$$V = n\lambda$$

$$\Rightarrow 1120 = 200\lambda$$

$$\Rightarrow \lambda = \frac{1120}{200} = 5.6 \text{ ft}.$$

(a) Distance between two nodes $= \frac{\lambda}{2} = \frac{5.6}{2} = 2.8$ feet

(b) Distance between two antinodes

$= \frac{\lambda}{2} = \frac{5.6}{2} = 2.8$ feet

(c) Distance between anode and antinode $= \frac{\lambda}{4} = \frac{5.6}{4} = 1.4$ feet.

7. $l = 50$ cm Closed pipe

$f = 200$ c/sec at $0°C$

$n = \frac{\cancel{q}V}{4\cancel{q}l} = \frac{V}{4 \times 50}$

$\Rightarrow 200 = \frac{V}{200}$

$\Rightarrow V = 4 \times 10^4$ cm/sec

= velocity of sound at 0°C

In 1°C velocity of sound increase 60.6 cm

In 30°C " " " "

$= 60.6 \times 3 = 180$ cm

181.8 cm/sec

∴ velocity will be $= 40000 + 181.8$ cm/sec

$= 40181.8$ cm/sec