Mod-III Microwave Sources – Ch-5 – Reflex Klystron

Sources for generation of microwave are classified mainly into 2 categories.

1) Tubes
2) Microwave Solid State devices

Tubes

- Linear Beam ~~Type~~ Tube (O-type)
- Crossfield Beam Tube (M-Type)

Linear Beam ~~Type~~ Tube

→ In Linear-beam tube, a magnetic field whose axis coincides with that of the electron beam, is used to hold the beam together as it travels the length of the tube.

→ O-type derived from the French name TPO (tubes à propagation ~~de son~~ des ondes) ~~Fe~~ or the ~~~~ word 'original' (meaning the original type of tube)

Examples of 'O' type

→ Two Cavity Klystron
→ Reflex Klystron ✓
→ Helix Travelling-wave Tube (TWT) ✓
→ Coupled-Cavity TWT
→ Forward wave Amplifier (FWA)
→ Backward wave Amplifier & Oscillator (BWA & BWO)

Cross-field Beam Tube :-

→ Here the d.c electric field and the d.c magnetic field are perpendicular to each other.

→ Derived from the French TPOM (Tubes a' propagation des ondes a' champs magnétique: Tubes for propagation of waves in a magnetic field)

Example of M-type:-

→ Magnetron

→ Forward wave Cross field amplifier (FWCFA)

→ Dematron

→ Amplitron

→ Carcinotron

→ Gyrotrons.

2) Solid State devices

Microwave Transistor

Ex:- Microwave BJT, HBT, Tunnel diode

HBT → Hetero Junction Bipolar Transistor

FET

Ex:- JFET, MESFET, MOSFET, NMOS, PMOS, CMOS, CCD → Charge coupled devices.

Transferred Electron devices

Ex:- Gunn diode, LSA diode (Limited space charge accumulation diode), InP diode (Indium Phosphide diode)

Avalanche transit-time devices

Ex:- Read diode, IMPATT, TRAPATT, BARITT

Read → Name of the inventor.

IMPATT → Impact Ionization Avalanche Transit time diode

TRAPATT → Trapped Plasma " triggered transit-time diode

BARITT → Barrier Injected Transit-time diodes.

Reflex Klystron:- (Liao Book)

If a fraction of the o/p power is fed back to the i/p cavity and if loop gain has a magnitude of unity with a phase shift of multiple of 2π, the klystron will oscillate.

However, a two-cavity klystron oscillator is usually not constructed because, when the oscillation freq is varied, the resonant freq of each cavity and feedback path must be readjusted for a +ve phase shift feedback.

The Reflex Klystron ~~overcomes the~~ is a single cavity klystron that over comes the disadvantage of 2-cavity klystron ~~amplifier~~. oscillator.

→ It is a low power generator (10-500 mW) at a freq range (1 to 25 GHz)

→ Efficiency (20 to 30%.)

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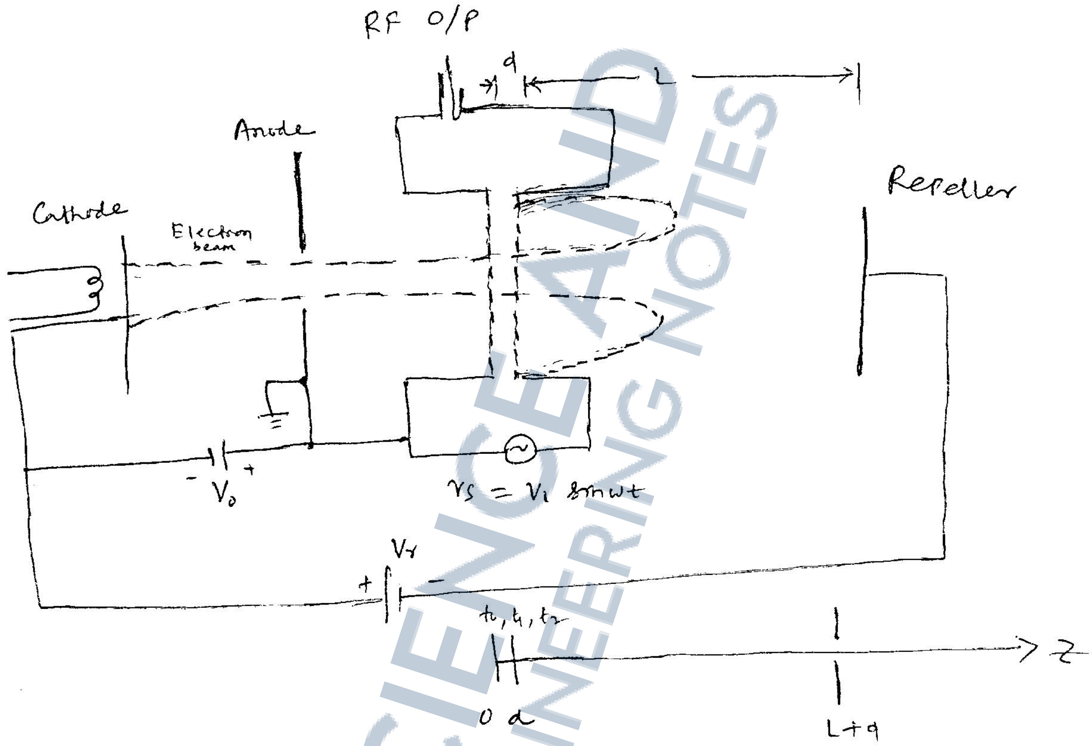
→ Widely used in laboratory for microwave measurements and in microwave receivers as local oscillators in commercial, military and airborne Doppler radars as well as missiles.

→ A schematic diagram of the reflex klystron is shown in fig 51.

[Name Reflex → Electron beams are reflected back]



t_0 → time for electron entering cavity gap at $z=0$

t_1 → time for same electron leaving cavity gap at $z=d$

t_2 → time for same electron returned by retarding field $z=d$ and collected on walls of cavity.

fig :- 51 : Schematic diagram of a reflex klystron.

The electron beam injected from the cathode is first velocity — modulated by the cavity gap voltage (V_s). Some electrons [(point A) fig 52] accelerated by the accelerating field enter the repeller space with greater velocity than those with unchanged velocity. [(point B) fig:-52]. Some electrons decelerated [point (C) fig:52] by the retarding field enter the repeller region with less velocity.

→ All electrons turned around by the repeller voltage then pass through the cavity gap in bunches that occur once per cycle.

→ On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field and give up their kinetic energy to the electromagnetic energy of the field in the cavity.

→ Oscillator o/p energy is then taken from the cavity. The electrons are finally collected by the walls of the cavity or other grounded metal part of the tube.

→ Fig (52) shows the Applegate diagram for $1\frac{3}{4}$ mode of a Reflex klystron.

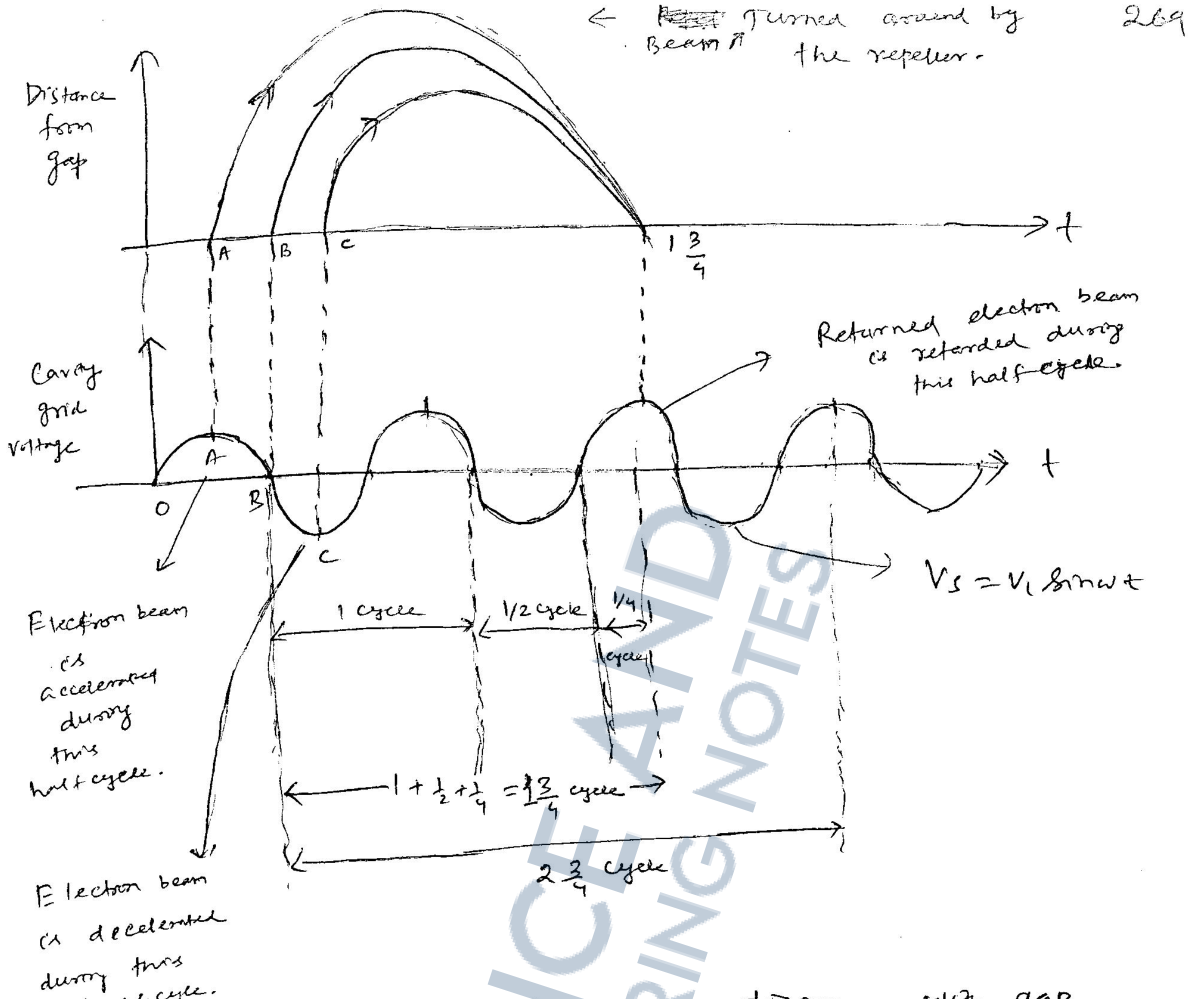


Fig 52:- Applegate diagram with gap voltage for a ($1\frac{3}{4}$ mode of) reflex klystron.

Note :-

→ By adjusting repeller voltage for a given dimension of reflex klystron, the bunching can be made to occure at $N = n + \frac{3}{4}$ ~~positive half~~ cycles.

→ Accordingly the mode of oscillation is named as

$$N = \frac{3}{4},\ 1\frac{3}{4},\ 2\frac{3}{4} \text{ etc}, \text{ for } n = 0, 1, 2, 3 \ldots$$

→ Lowest order mode $\frac{3}{4}$ occure for a maxm value of repeller voltage.

→ Higher order modes occure at lower repeller voltages.

→ Since at the highest repeller voltage, the acceleration of the bunched electron on return is maxm, the power o/p of the lowest mode is maxm.

Velocity Modulation :-

The electron entering the cavity gap form cathode, at $Z=0$ and time 't_0' is assumed to have uniform velocity

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad \text{—— (1)}$$

$\because \frac{1}{2}mv^2 = eV_0$

$\Rightarrow v = \sqrt{\frac{2eV_0}{m}}$

$$\Rightarrow v_0 = \sqrt{\frac{2\times1.6\times10^{-19}}{9.1\times10^{-31}} V_0}$$

$$\Rightarrow v_0 = 0.593\times10^{6}\sqrt{V_0} \quad \text{—— (1A)}$$

In eqn (1) it is assumed that electron leave the cathode with zero velocity. When an sinusoidal i/p ~~...~~ signal is applied, ~~...~~ between the buncher ~~...~~ the gap voltage grids appears as

$$V_s = V_1 \sin(\omega t) \quad \text{—— (2)}$$

where V_1 is the amplitude of the signal and $V_1 << V_0$ assumed.

In order to find the modulated velocity

in the cavity in terms of either the entering time 't_0' or exiting time 't_1' and the gap transit angle 'θ_g' as shown in fig 51, it is necessary to determine the average microwave voltage on the cavity gap as indicated in fig 53.

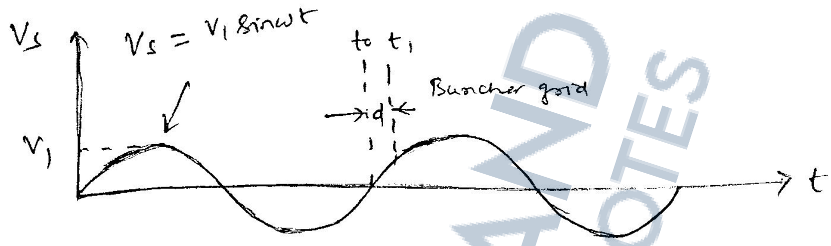


fig 53:- Signal voltage in the buncher gap.

Since $V_1 << V_0$, the average transit time through the buncher gap distance 'd' is

$$\tau = \frac{d}{v_0} = t_1 - t_0 \quad \text{— (3)}$$

The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \quad \text{— (4)}$$

The average microwave voltage in the gap can be found in the following way

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt$$

$$= \frac{1}{\tau} V_1 \cdot \left[-\frac{\cos \omega t}{\omega} \right]_{t_0}^{t_1} = -\frac{V_1}{\omega\tau} \left[\cos \omega t_1 - \cos \omega t_0 \right]$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau} \left[\cos \omega t_0 - \cos \omega t_1 \right]$$

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega\tau}\left[\cos \omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{V_0}\right)\right] \quad \text{--- (5)}$$

$$\left[\because \omega(t_1 - t_0) = \frac{\omega d}{V_0} \quad \text{eqn (3) \& (4)}\right]$$

$$\left[\Rightarrow \omega t_1 = \omega t_0 + \frac{\omega d}{V_0}\right]$$

Let

$$\omega t_0 + \frac{\omega d}{2V_0} = \omega t_0 + \frac{\theta_g}{2} = A \quad [\text{From eqn (4)}]$$

and

$$\frac{\omega d}{2V_0} = \frac{\theta_g}{2} = B$$

$$\therefore \cos \omega t_0 = \cos(A - B)$$

$$\cos\left(\omega t_0 + \frac{\omega d}{V_0}\right) = \cos(A + B)$$

$$\left[\begin{aligned} &\because A+B = \omega t_0 + \frac{\omega d}{2V_0} + \frac{\omega d}{2V_0} \\ &\Rightarrow A+B = \omega t_0 + \frac{\omega d}{V_0} \\ &A-B = \omega t_0 + \cancel{\frac{\omega d}{2V_0}} - \cancel{\frac{\omega d}{2V_0}} = \omega t_0 \end{aligned}\right]$$

$\therefore$ Eqn (5) becomes,

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau} 2 \sin A \cdot \sin B$$

$$\Rightarrow \langle V_s \rangle = \frac{2V_1}{\omega\tau} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) \cdot \sin\left(\frac{\omega d}{2V_0}\right)$$

$$= \frac{2V_1}{\omega\left(\frac{d}{V_0}\right)} \sin\left(\frac{\omega d}{2V_0}\right) \cdot \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right)$$

$$\langle V_s \rangle = V_1 \cdot \frac{\sin\left(\frac{\omega d}{2V_0}\right)}{\left(\frac{\omega d}{2V_0}\right)} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right)$$

$$\langle V_s \rangle = V_1 \frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\theta_g/2\right)} \cdot \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) \quad \text{--- (6)}$$

eqⁿ ③ & ④

(4)

Lets take $\beta_i = \dfrac{\sin(\theta_g/2)}{(\theta_g/2)} = \dfrac{\sin\left(\frac{\omega d}{2V_0}\right)}{(\omega d/2V_0)}$ —— ⑦

β_i is known as the **beam coupling coefficient** of the input cavity gap.

$$\therefore \langle V_s \rangle = V_1 \cdot \beta_i \cdot \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) \quad \text{—— ⑥}$$

$$\frac{\omega d}{2V_0} + \frac{\omega d}{2V_0}$$

$$\frac{\omega d}{2V_0} = \omega t_0$$

$$\langle V_s \rangle = \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad \text{—— ⑧}$$

Immediately after velocity modulation, the exit velocity from the buncher gap is given as $v(t_1)$

From eqⁿ ①,

$$v_s(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + \langle V_s \rangle\right)}$$

$$= \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$v_s(t_1) = \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]} \quad \text{—— ⑨}$$

where the factor $\dfrac{\beta_i V_1}{V_0}$ is called the **depth of velocity modulation**

using the binomial expansion under the assumption of $\beta_i V_1 \ll V_0$

Eqⁿ ⑨ becomes, —⑥

$$v_s(t_1) = V_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{—— ⑩}$$

$(1+x)^n \simeq 1+nx$

where $x \ll 1$

$$\left(\because \sqrt{\frac{2eV_0}{m}} = v_0 \quad \text{and} \quad \left[1 + \frac{\beta_i V_1}{V_0} \sin(\omega t_0 + \frac{\theta_g}{2})\right]^{\frac{1}{2}} \right.$$

$$= 1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_0 + \frac{\theta_g}{2})$$

(By Binomial expansion $(1+x)^n \approx 1 + nx$]

Eqn (10) is equation of velocity modulation.

From eqn (10),

$$\cancel{\omega t_0 + \frac{\theta_g}{2} = \omega t_0 + \frac{\omega(t_1 - t_0)}{2}}$$

$$\cancel{= \omega t_0 + \frac{\omega t_1}{2} - \frac{\omega t_0}{2}}$$

$$\cancel{= \frac{\omega t_1}{2} + \frac{\omega t_0}{2}}$$

$(\because \theta_g = \omega(t_1 - t_0)$ eqn 4]

$\Rightarrow \theta_g = \omega t_1 - \omega t_0$

$\Rightarrow \omega t_0 = \omega t_1 - \theta_g$

From eqn (10), $\omega t_0 + \frac{\theta_g}{2} = \omega t_1 - \theta_g + \frac{\theta_g}{2}$

$$\Rightarrow \omega t_0 + \frac{\theta_g}{2} = \omega t_1 - \frac{\theta_g}{2} \quad \text{--- (11)}$$

$\therefore$ Using eqn (11) in eqn (10), ~~we have~~ The velocity modulation can alternatively written as,

$$v(t_1) = V_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \frac{\theta_g}{2}) \right] \quad \text{--- (12)}$$

The same electron is forced back to the cavity $z = d$ at time t_2, by retarding electric field E, which is given by

$$E = \frac{V_r + V_0 + V_1 \sin\omega t}{L} \quad \text{--- (13)} \quad \left(\because E = \frac{V}{d} \right)$$

This retarding field E is assumed to be constant in the Z- direction.

The force equation for one electron in the repeller region is

$$m\frac{d^2 z}{dt^2} = -eE = -e\frac{V_r+V_0}{L} \quad \text{—(14)}$$

where $E = -\nabla V$ is used in the Z-direction only, V_r is the magnitude of repeller voltage and $V_1 \sin\omega t << V_r + V_0$, is assumed.

Integrating eqn (14), we have

$$\frac{dz}{dt} = -\frac{e(V_r+V_0)}{mL}\cdot\int_{t_1}^{t} dt$$

$$\frac{dz}{dt} = -\frac{e(V_r+V_0)}{mL}(t-t_1)+K_1 \quad \text{—(15)}$$

Integrating eqn (15), we have

At $t = t_1$,

$$K_1 = \frac{dz}{dt} = v(t_1) \quad \text{(16)}$$

$$Z = -\frac{e(V_r+V_0)}{mL}\int_{t_1}^{t}(t-t_1)\,dt + \int_{t_1}^{t} K_1\,dt$$

$$\Rightarrow Z = -\frac{e(V_r+V_0)}{mL}\left[\frac{t^2}{2}-t_1.t\right]_{t_1}^{t} + K_1\left[t\right]_{t_1}^{t}$$

Note :- $\left[\frac{t^2}{2}-t_1.t\right]_{t_1}^{t} = \left(\frac{t^2}{2}-t_1.t\right)-\left(\frac{t_1^2}{2}-t_1^2\right)$

$$= \frac{t^2}{2}-tt_1+\frac{t_1^2}{2} = \frac{1}{2}\left[t^2-2t.t_1+t_1^2\right]$$

$$\therefore Z = \frac{-e(V_r+V_0)}{mL} \cdot \frac{(t-t_1)^2}{2} + v(t_1)(t-t_1) + K_2 \quad \text{--- (17)}$$

$[\because K_1 = v(t_1) \text{ eq}^n \ 16]$

At $t = t_1$, $Z = K_2$, but at $t = t_1$, $\boxed{Z = d}$ [see fig 51]

$$\therefore \boxed{K_2 = d}$$

Eqn (17) becomes,

$$Z = \frac{-e(V_r+V_0)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1) + d \quad \text{--- (18)}$$

On the assumption that the electron leaves the cavity gap at $Z = d$ and time t_1 with a velocity of $v(t_1)$ and returns to the gap at $Z = d$ and time t_2, at $t = t_2$, $Z = d$. --- (19)

Putting eqn (19) in eq^n (18), we have,

$$\cancel{d} = \frac{-e(V_r+V_0)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1) + \cancel{d}$$

$$\Rightarrow \frac{e(V_r+V_0)}{2mL}(t_2-t_1)^{\cancel{2}} = v(t_1)\cancel{(t_2-t_1)}$$

$$\Rightarrow (t_2-t_1) = \frac{2mL\, v(t_1)}{e(V_r+V_0)} \quad \text{--- (20)}$$

The round-trip transit time in the repeller region is given by,

$$T' = t_2 - t_1 = \frac{2mL\, v(t_1)}{e(V_r+V_0)} \quad \left[\text{from eq}^n\ 20\right]$$

$$= \frac{2mL}{e(V_r+V_0)}\left[v_0\left(1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right)\right]$$

[From eq^n 12]

$$= \frac{2mLv_0}{e(V_r+V_0)}\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]$$

$$T' = T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right] \quad \text{— (21)}$$

21(A)

where $T_0' = \dfrac{2mLv_0}{e(V_r+V_0)}$, is the round-trip dc transit time of the center-of-the bunch electron. ($\because$ v_0 = uniform velocity of electron, without any a.c signal applied)

From eq^n (21),

$$t_2 - t_1 = T_0' + \frac{T_0'\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)$$

Multiplication of above eq^n through by a radian frequency results in,

$$\omega(t_2-t_1) = \omega T_0' + \frac{\beta_i V_1}{2V_0}\cdot\omega T_0'\sin\left(\omega t_1-\frac{\theta_g}{2}\right)$$

$$\Rightarrow \boxed{\omega(t_2-t_1) = \theta_0' + X'\sin\left(\omega t_1-\frac{\theta_g}{2}\right)} \quad \text{— (22)}$$

where $\theta_0' = \omega T_0'$, is the round-trip dc transit angle of the center of the bunch electron — 22(A)

and $X' = \frac{\beta_i V_1}{2V_0}\theta_0'$, is the bunching parameter of the reflex klystron oscillator. — 22(B)

Power O/P and Efficiency :-

In order for the electron beam to generate a maximum amount of energy to the oscillation, the returning electron beam must cross the cavity gap when the gap field is maximum retarding. In this way, a maximum amount of kinetic energy can be ~~transmitted~~ transferred from the returning electrons to the cavity walls.

It can be seen from fig 52 that for a maximum energy transfer, the round trip transit angle, referring to the center of the bunch, must be given by,

$$\omega(t_2-t_1) = \omega T_0' = \left(n-\frac{1}{4}\right)2\pi = 2n\pi - \frac{\pi}{2} \quad -22(C)$$

where $V_1 << V_0$ is assumed, n = any +ve integer for cycle number, and $N = n-\frac{1}{4}$ is the number of modes.

$n=1$, $N = \frac{3}{4}$, $n=2$, $N = 2-\frac{1}{4} = \frac{7}{4} = 1\frac{3}{4}$ and so on.

[Similar to velocity modulation] The current modulation of the electron beam as it reenters the cavity from the repeller region can be determined ~~the~~ as follows;

$$i'_{2t} = -I_0 - \sum_{n=1}^{\infty} 2I_0 J_n(nx') \cos[n(\omega t_2 - \theta_0' - \theta_g)] \quad \text{— (23)}$$

(-ve sign, due to the beam current injected from the repeller into the cavity gap from the -ve Z- direction) region flows in the ~~buncher~~ cavity.

Where, I_0 = d.c current in the reflex klystron

i'_{2t} = beam current of a reflex oscillator.

$J_n(nx')$ = nth order Bessel function of the first kind

The fundamental component of the current induced in the cavity by the modulated electron beam is given by $i_f = \beta_i [-2I_0 J_1(x') \cdot \cos(\omega t_2 - \theta_0')]$

$$i_2 = -\beta_i i_f = \beta_i 2I_0 J_1(x') \cos(\omega t_2 - \theta_0') \quad \text{— (24)}$$

[∵ From eqn (23), we have taken $n=1$, and θ_g is neglected as a small quantity compared to θ_0'.]

β_i = coupling coefficient of the cavity.

The magnitude of the fundamental component is

$$|i_2| = I_2 = 2I_0 J_1(x') \cdot \beta_i \quad \text{— (25)}$$

The d.c power supplied by the beam voltage V_0 is

$$P_{dc} = V_0 I_0 \quad \text{— (26)}$$

The a.c power delivered to the load is given by

$$P_{ac} = \frac{V_1 I_2}{2} = \frac{V_1}{\cancel{2}} \cdot \left[\cancel{2} I_0 \beta_i J_1(x')\right] \quad \text{— (27)}$$

$$\Rightarrow P_{ac} = V_1 I_0 \beta_i J_1(x') \quad \text{— (27)}$$

From eqn 22 (B),

$$x' = \frac{\beta_i V_1}{2V_0} \theta_0'$$

Putting eqn 22 (A), in the above eqn, we have

$$x' = \frac{\beta_i V_1}{2V_0} \cdot \omega T_0'$$

Putting eqn 22 (C), in the above eqn, we have

$$x' = \frac{\beta_i V_1}{2V_0} \cdot \left(2n\pi - \frac{\pi}{2}\right) \quad \text{— } \cancel{(28)}$$

$$\Rightarrow \frac{V_1}{V_0} = \frac{2x'}{\beta_i (2n\pi - \pi/2)} \quad \text{— (28)}$$

Putting eqn (28) in eqn (27), we have

$$P_{ac} = \left[\frac{2x' V_0}{\cancel{\beta_i} (2n\pi - \pi/2)}\right] \cdot \left[I_0 \cancel{\beta_i} J_1(x')\right]$$

$$\Rightarrow P_{ac} = \frac{2 V_0 I_0 x' J_1(x')}{\left(2n\pi - \frac{\pi}{2}\right)} \quad \text{— (29)}$$

Therefore the electronic efficiency of a reflex klystron oscillator is defined as

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2X' J_1(X')}{2n\pi - \frac{\pi}{2}}$$ [Dividing eqn (29) by eqn (26)] (30)

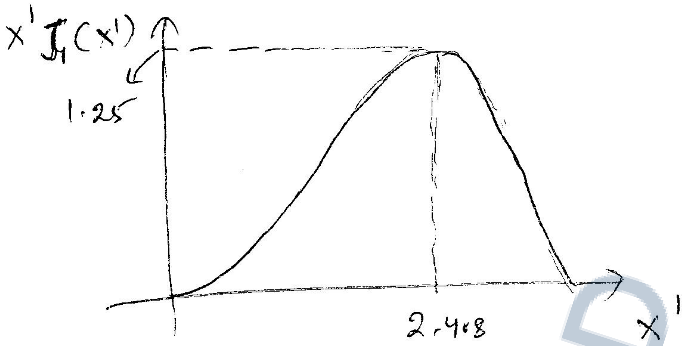


fig 54:- $X'J_1(X')$ Vs X'

The factor $X' J_1(X')$ reaches a maximum value of $\boxed{1.25}$ at $X' = 2.408$.

In practice, for $n = 2$ or $N = 1\frac{3}{4}$ mode has the maxm power o/p.

$$\eta_{max} = (\text{Efficiency})_{max} = \frac{2 \times (1.25)}{2 \cdot 2 \cdot \pi - \frac{\pi}{2}} = \frac{2.5 \times 2}{7\pi}$$

$$\boxed{(\text{Efficiency})_{max} = 22.7\ \%}$$

[Theoretical efficiency 20 to 30%]

For Problems:-

For a given beam voltage V_0, the relationship between repeller voltage and cycle number n required for oscillation, can be derived as follows. From eqn 22(c), $\omega T_0' = 2n\pi - \frac{\pi}{2}$ (31)

But $T_0' = \dfrac{2mL\,V_0}{e(V_r+V_0)}$ from eqn ~~21~~ (A)

Putting this value in eqn (31), we have

$$\omega \times \left[\frac{2mL\,V_0}{e(V_r+V_0)}\right] = \left(2n\pi - \frac{\pi}{2}\right) \quad \text{— (31)}$$

Putting eqn (1) in eqn (31), we have

$$\frac{\omega \times 2mL}{e\,(V_r+V_0)} \times \left[\sqrt{\frac{2eV_0}{m}}\right] = \left(2n\pi - \frac{\pi}{2}\right) \quad \text{— 31 (A)}$$

⇒ Squaring both the sides.

$$\Rightarrow \frac{\omega^2 \times 4\cancel{m^2}^{\,m} L^2}{e\,\cancel{e^2}(V_r+V_0)^2} \times \frac{2\cancel{e}V_0}{\cancel{m}} = \left(2n\pi - \frac{\pi}{2}\right)^2$$

$$\Rightarrow \boxed{\frac{V_0}{(V_r+V_0)^2} = \frac{\left(2n\pi - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \cdot \frac{e}{m}} \quad \text{— (32)}$$

The power o/p can be expressed in terms of the repeller voltage V_r, i.e

$$P_{ac} = \frac{2\,V_0 I_0\, X'\, J_1(X')}{2n\pi - \pi/2} \quad \text{— (33)}$$

Putting eqn 31(A), in eqn (33), we have

$$P_{ac} = \frac{\cancel{2} V_0 I_0 X' J_1(X') \sqrt{\cancel{m}} \times \frac{\sqrt{e}}{\cancel{e}}(V_r+V_0)}{\frac{\omega \times \cancel{2} \cancel{m} L \sqrt{2 \cancel{e} V_0}}{\sqrt{m}}}$$

$$\Rightarrow \boxed{P_{ac} = \frac{V_0 I_0 X' J_1(X_1') (V_r+V_0)}{\omega L} \sqrt{\frac{e}{2 m V_0}}} \quad (34)$$

→ Note :-

It can be seen from eqn (32) that for a given beam voltage V_0 and ~~the~~ cycle number 'n' or mode number 'N', the center repeller voltage V_r can be determined in terms of the center freq.

→ The power o/p at the center freq can be calculated using eqn (34). When the freq varies from the center freq, and about the center voltage, the repeller voltage vary accordingly, assuming the o/p power a bell shape.

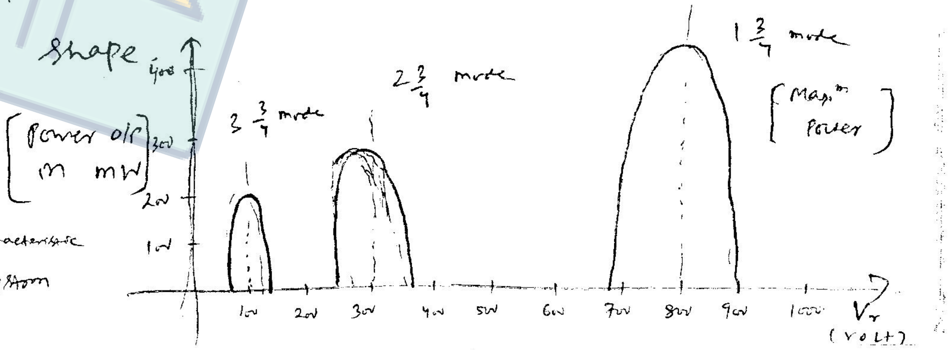


fig 55:- Power o/p characteristic of reflex klystron

Electronic Admittance:-

From eqn (24), the induced current can be written in phasor form as

$$i_2 = 2 I_0 \beta_i J_1(x') \cdot e^{-j\theta_0'} \quad \text{--- (35)}$$

The voltage across the gap at time 't_2' can also be written in phasor form

$$V_2 = V_1 \cdot e^{-j\frac{\pi}{2}} \quad \text{--- (36)}$$

$$\left[\because \; 1\tfrac{3}{4} \text{ cycle} = 1\tfrac{3}{4} \times 2\pi = \tfrac{7}{4} \times 2\pi = \tfrac{7\pi}{2} = -\tfrac{\pi}{2} \right]$$

The ratio of i_2 to V_2 is defined as the electronic admittance of the reflex klystron, i.e.

$$Y_e = \frac{2 I_0 \beta_i J_1(x') \cdot e^{-j\theta_0'}}{V_1 \cdot e^{-j\pi/2}} \quad \text{--- (37)}$$

From eqn (28), we have $V_1 = \dfrac{2x' V_0}{\beta_i' (2n\pi - \pi/2)}$

$$= \frac{2x' V_0}{\beta_i \theta_0'}$$

[From eqn 22(c) & 22(d)]

$$\therefore Y_e = \frac{\cancel{2} I_0 \beta_i J_1(x') \cdot e^{j(\frac{\pi}{2} - \theta_0')}}{\cancel{2} x' V_0} \cdot (\beta_i \theta_0')$$

$$Y_e = \frac{I_0}{V_0} \cdot \frac{\beta_i^2 \theta_0'}{2} \cdot \frac{2 J_1(x')}{x'} \cdot e^{j(\frac{\pi}{2} - \theta_0')} \quad \text{--- (38)}$$

It is evident that the electronic admittance is nonlinear, since it is proportional to the factor $\frac{2J_1(X')}{X'}$, and X' is proportional to the signal voltage $\left[\because X' = \frac{\beta_i V_1}{2V_0} \theta_0'\right]$

The term $\frac{2J_1(X')}{X'}$ is called saturation factor.

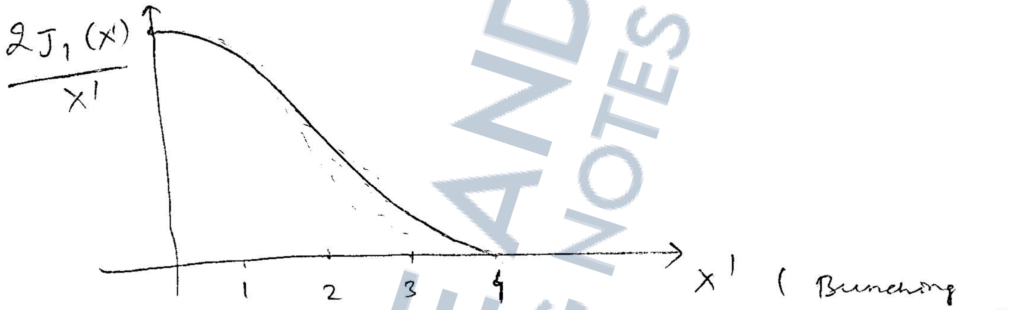


fig 2-56:- Reflex klystron saturation factor

The equivalent circuit of reflex klystron is shown in fig 57.

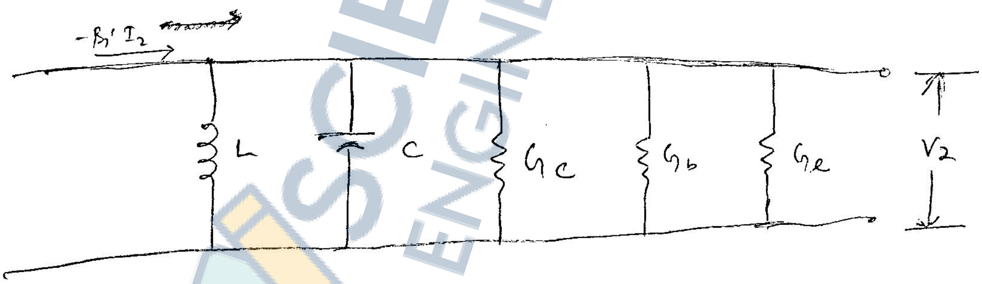


fig 57:- Equivalent circuit of a reflex klystron

In this circuit, L & C are the energy storage elements of the cavity; G_c represents the copper cavity losses, G_b beam loading conductance and G_e the load conductance.

(2)

22(A)

-(38)

The necessary condⁿ for oscillation is that the magnitude of the −ve real part of electronic admittance, not less than the total conductance of the cavity circuit.

Electronic admittance in rectangular form

$$Y_e = G_e + jB_e \quad — (39)$$

Magnitude of −ve real part → $|-G_e|$

Total conductance of the cavity circuit,

$$G = G_c + G_b + G_e = \frac{1}{R_{sh}}$$ and R_{sh} is the effective shunt resistance.

∴ The necessary condⁿ for oscillation is,

$$\boxed{|-G_e| \geq G} \quad — (40)$$

→ Since the electronic admittance shown in eqⁿ (37) is in exponential form, its phase is $\frac{\pi}{2}$ when $\theta_0' = 0$,

→ The rectangular plot of the electron admittance Y_e is a spiral. Any value of θ_0' for which the spiral lies in the area to the left of the line $(-G - jB)$ will yield oscillation. i.e

$$\theta_0' = \left(n - \frac{1}{4}\right) 2\pi = N 2\pi \quad — (41)$$

Where N is mode number as indicated on the plot. [As discussed earlier about the mode]

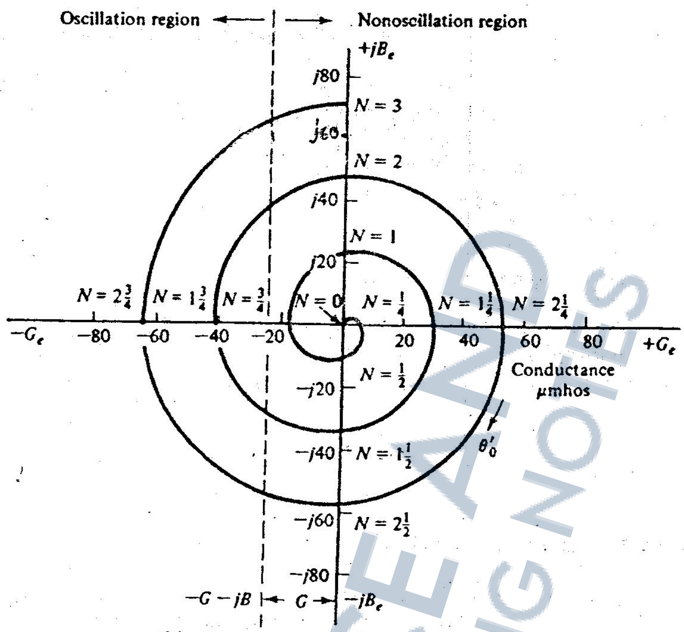


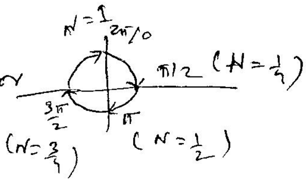
Figure 58 Electronic admittance spiral of a reflex klystron.

Note:-

1 cycle $= 2\pi^c$

$2\pi^c = 1$ cycle

$\frac{\pi}{2} = \frac{1}{4}$ cycle [∵ For



For each quadrant, N will be incremented by $(\frac{1}{4})$.

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{4}$ ......

(BPUT-2009)

Ex:- 1 A reflex klystron operates under the following conditions:

$V_0 = 600V$, $L = 1\ mm$

$R_{sh} = 15k\Omega$, $\frac{e}{m} = 1.759 \times 10^{11}$

$f_r = 9\ GHz$, $J_1(x') = 0.582$

The tube is oscillating at f_r at the peak of the $n=2$ or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

(a) Find the value of the repeller voltage (V_r)

(b) Find the direct current necessary to give a microwave gap voltage of 200V.

(c) What is the electronic efficiency under this condition?

Ans :- (a) From eqn (32), we have

$$\frac{V_0}{(V_r+V_0)^2} = \frac{\left(2n\pi - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \cdot \frac{e}{m}$$

$$\Rightarrow \frac{V_0}{(V_r+V_0)^2} = \frac{\left(2.2.\pi - \frac{\pi}{2}\right)^2}{8 \cdot (2\pi \cdot 9\times10^9)^2 \times (10^{-3})^2} \times 1.759\times10^{11}$$

$$\Rightarrow \frac{V_0}{(V_r+V_0)^2} = 0.832 \times 10^{-3}$$

$$\Rightarrow \frac{600}{(V_r+600)^2} = 0.832\times10^{-3}$$

$$\Rightarrow (V_r+600)^2 = \frac{600}{.832\times10^{-3}}$$

$$\Rightarrow \boxed{V_r = 249.20\ V}$$

(b) Assuming $\beta_i' = 1$,

$\because \beta_i' = \dfrac{\sin\left(\frac{\theta_g}{2}\right)}{\frac{\theta_g}{2}}$

$\frac{\theta_g}{2} \to 0$, $\dfrac{\sin\theta_g/2}{\theta_g/2} = 1$

Since transit time/angle is neglected

$$i_2 = 2 I_0\, J_1(x')$$

$$V_2 = i_2 \cdot R_{sh}$$

$$V_2 = 2 I_0\, J_1(x') \cdot 15\times10^3$$

$$\Rightarrow I_0 = \frac{V_2}{2\,J_1(x')\times15\times10^3} = \frac{200}{2\times0.582\times15\times10^3}$$

$$\Rightarrow \boxed{I_0 = 11.45\ mA}$$

(c) $$\eta = \frac{2x'\,J_1(x')}{2n\pi - \frac{\pi}{2}}$$ [Eq^n 20]

$$x' = \frac{\beta_i V_1}{2V_0}\cdot\theta_0' \qquad \text{eqn (22 B)}$$

$\left[\ \theta_0' = 2n\pi - \frac{\pi}{2} = 2\cdot2\cdot\pi - \frac{\pi}{2} = \frac{7\pi}{2} = 3.5\pi\right.$

$$= \frac{1\times\cancel{200}}{2\times\cancel{600}_{3}}\times 3.5\times\pi$$

$$x' = 1.83$$

$$\eta = \frac{2x' J_1(x')}{2n\pi - \frac{\pi}{2}}$$

$$= \frac{2 \times 1.83 \times 0.582}{3.5 \times \pi}$$

$$= 0.1937$$

$$\boxed{\eta = 19.37\%}$$

OR

$$\eta = \frac{2x' J_1(x')}{2n\pi - \pi/2} = \frac{2x' J_1(x')}{\theta_0'}$$

$$= 2\left(\frac{\beta_i V_1}{2V_0} \cdot \cancel{\theta_0'}\right) \frac{J_1(x')}{\cancel{(\theta_0')}}$$

$$= \frac{\cancel{2} \times 1 \times \cancel{200} \times 0.582}{\cancel{2} \times \cancel{600}\ 3}$$

$$\simeq 0.194$$

$$\eta = 19.4\%$$

2) BPUT 2012

A reflex klystron operates with the following conditions: d.c accelerating voltage $V_{dc} = 1.4$ kV, Repeller voltage $= -100$ V, Resonant freq $f_r = 8$ GHz. Distance betn cavity and repeller $(L) = 2$ cm. Compare (a) d.c electron velocity (b) Round trip d.c transit time

Ans ÷ (a) D.C electron velocity (v_0)

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

$$= 0.593 \times 10^6 \sqrt{V_0}$$ [As derived earlier]

$$\Rightarrow v_0 = 0.593 \times 10^6 \sqrt{1.4 \times 10^3}$$

$$\boxed{v_0 = 22.18 \times 10^6 \ \frac{m}{sec}}$$

(b) Round trip d.c transit time

$$T_0' = \frac{2 m L v_0}{e(V_r + V_0)}$$

Note ÷ Here we have to take repeller voltage magnitude.
$|-100| = 100$

$$= \frac{2 \times 9.1 \times 10^{-31} \times 2 \times 10^{-2} \times 22.18 \times 10^6}{1.6 \times 10^{-19}(100 + 1400)}$$

$$= \frac{2 \times 9.1 \times 2 \times 22.18}{1.6 \times 15} \times 10^{-10}$$

$$= 33.63 \times 10^{-10}$$

$$= 3.363 \times 10^{-9}$$

$$\boxed{T_0' = 3.36 \ ns}$$

— o —

ge

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ry

next time