Ch. 5 # Small Signal Analysis of BJTs

→ The small signal means 'less amplitude' signal where 'small' indicates small amplitude (mV) only not small frequency.
[milli volt]

How conservation of energy takes place in Amplification?

The d.c power supply (battery) is given in amplifier ckt. So there is an 'exchange' of d.c power to the a.c domain that permits establishing a higher output ac power.

→ The superposition theorem is applicable for analysis & design of dc and a.c components of a BJT network.

So one can make a complete d.c analysis of a system before considering the a.c response. Then independent a.c analysis is done. The complete analysis is done by using the superposition theorem. ~~considering~~ taking these independent analysis in to account.

→ BJT transistor Modelling:-

→ The key to transistor small-signal analysis is use of equivalent circuits (models).

→ A model is a combination of ckt elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.

D.C analysis

Steps

1) Reduce the a.c sources to zero. This means that a voltage source is replaced by short ckt and current source is replaced by open ckt.

2) Make the Capacitor open ckted.

$(X_C = \frac{1}{2\pi f c}, \quad f=0 \quad , X_C = \infty)$

A.C Analysis:-

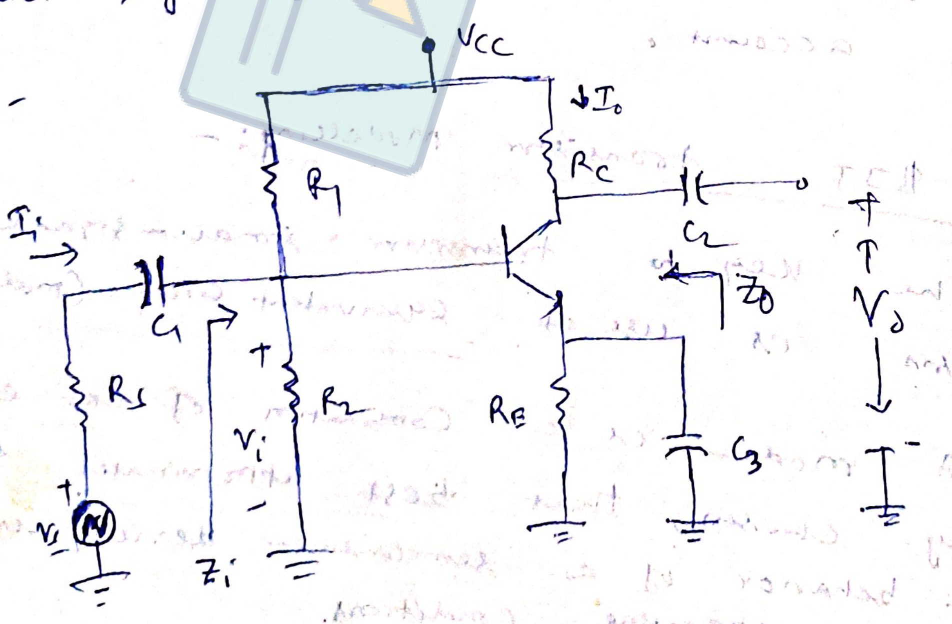
1) Set all d.c sources to zero and replace them by short-ckt equivalent.

2) Replace all capacitors by short-ckt equivalent.

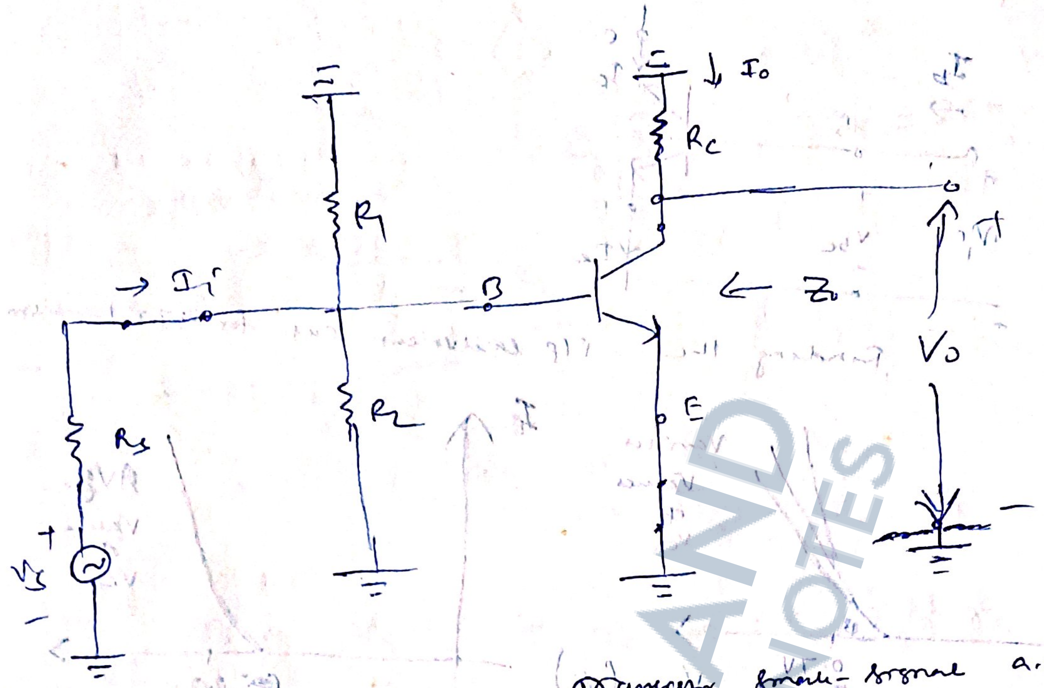
3) Remove all the elements bypassed by the short-ckt equivalent introduced by step 1 and 2.

4) Redraw the n/w in more convenient and logical form.

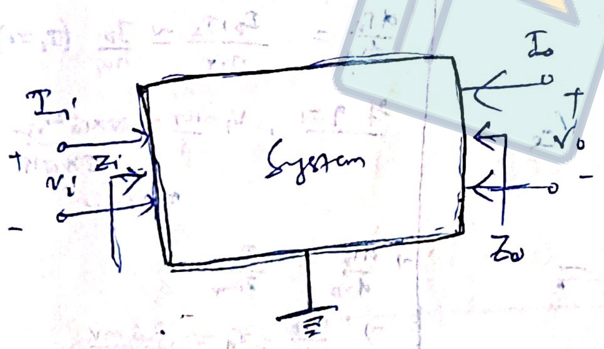
Ex:-



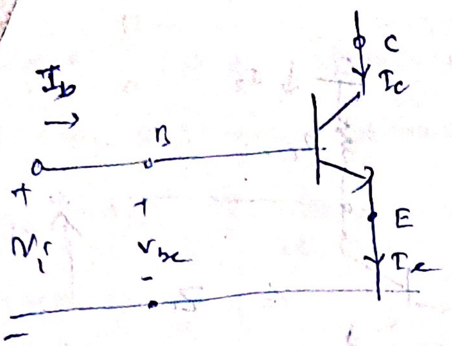
Now, for A.C analysis, d.c sources should be removed, Capacitors short ckted.



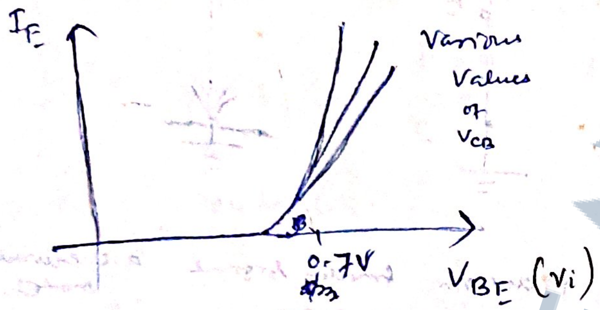
Transistor small-signal a.c equivalent model.

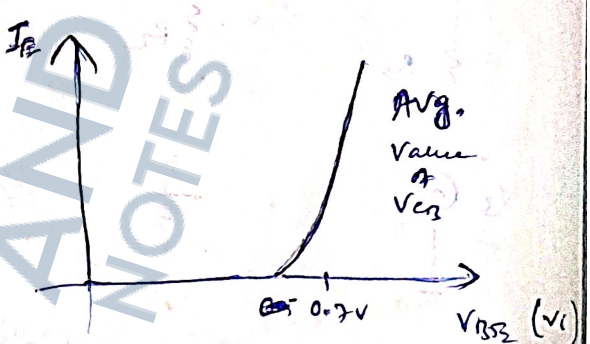


The r_e transistor model :-



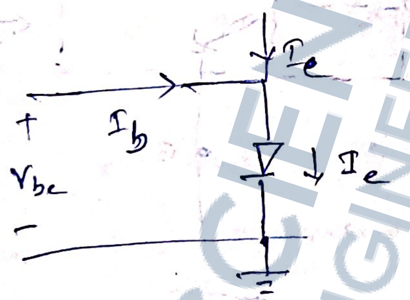
Finding the i/p equivalent ckt for BJT transistor





(CE Configuration) (Looks as simple forward-biased diode)

So



Note :-

$I_D = I_s\left(e^{\frac{V_D}{\eta V_T}} - 1\right)$ — (1)

$\Rightarrow I_D = I_s \cdot e^{\frac{V_D}{\eta V_T}} - I_s$

$\Rightarrow I_s \cdot e^{\frac{V_D}{\eta V_T}} = I_D + I_s$ — (2)

From eqn (1)

$\frac{dI_D}{dV_D} = I_s \cdot e^{\frac{V_D}{\eta V_T}} \cdot \frac{1}{\eta V_T}$ — (3)

Putting eqn (2) in eqn (3)

$\frac{dI_D}{dV_D} = \frac{I_D + I_s}{\eta V_T} \simeq \frac{I_D}{\eta V_T}$ $(I_s \simeq 0)$

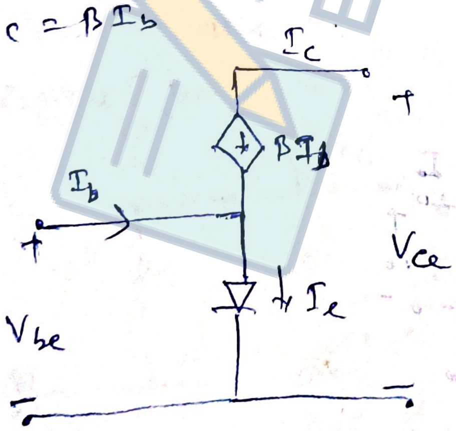
If $\eta = 1$, $V_T = \frac{KT}{q} = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}}$

$\simeq 26mV$

$\Rightarrow \frac{dI_D}{dV_D} = \frac{I_D}{26mV}$

$\Rightarrow \frac{dV_D}{dI_D} = r_d = \frac{26mV}{I_D}$

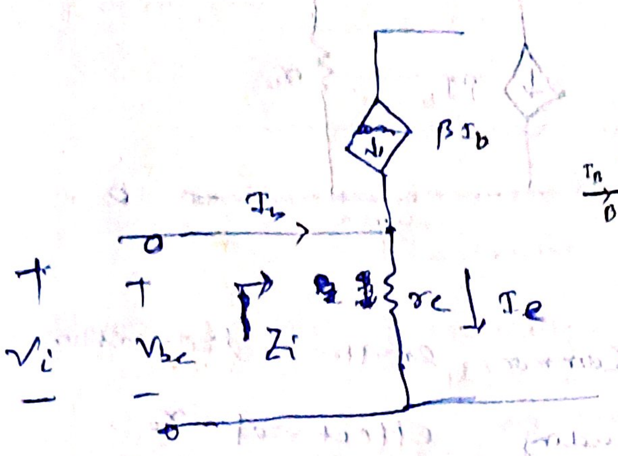
∴ $I_c = \beta I_b$

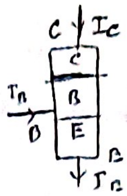


Diode can be replaced by a resistor, r_e.

diode's a.c resistance (Dynamic resistance) $r_D = \frac{26mV}{I_D}$

for emitter current, ~~r_D~~ $\rightarrow \begin{cases} r_D \rightarrow r_e \\ I_D \rightarrow I_e \end{cases}$





$\therefore r_e = \frac{26mV}{I_E}$

Now for i/p side $Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$

But $V_{be} = I_e r_e = (I_c + I_b) r_e$

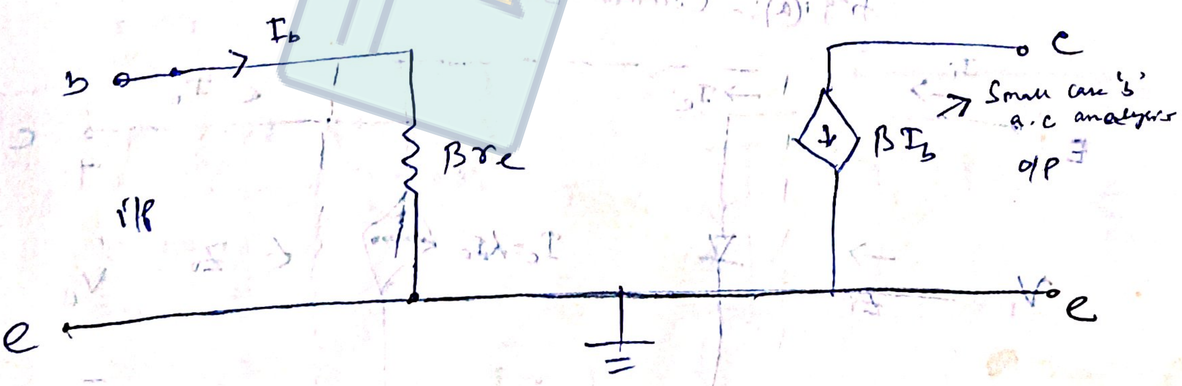
$= (\beta I_b + I_b) r_e$

$= (\beta + 1) I_b r_e$

$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta+1)\cancel{I_b} r_e}{\cancel{I_b}} = (\beta+1) r_e \simeq \beta r_e$

(Looking into the base of the n/w).

∴ The improved BJT equivalent ckt



To include the o/p ckt of BJT, we have to consider o/p resistance r_o. Using avg. value for the o/p resistance, we have

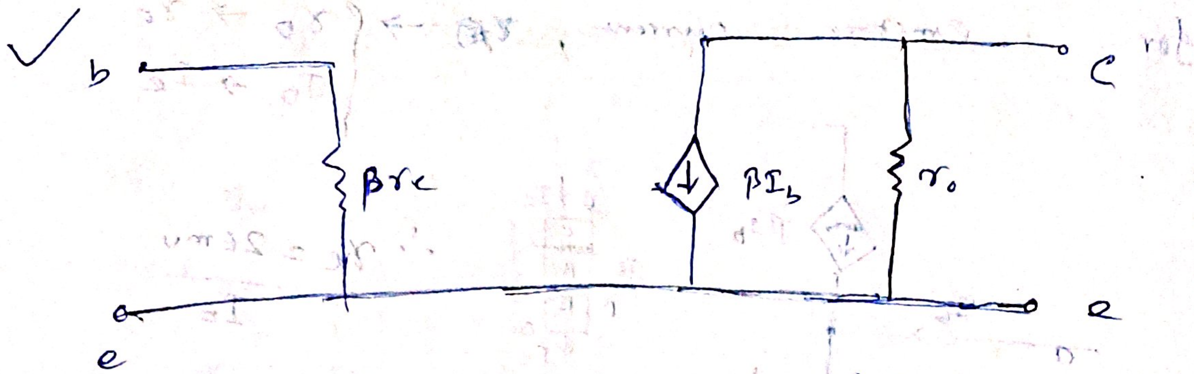
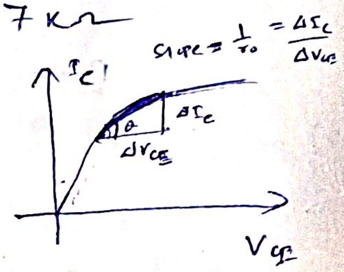


fig:- r_e model for Common-emitter transistor configuration including effect of r_o.

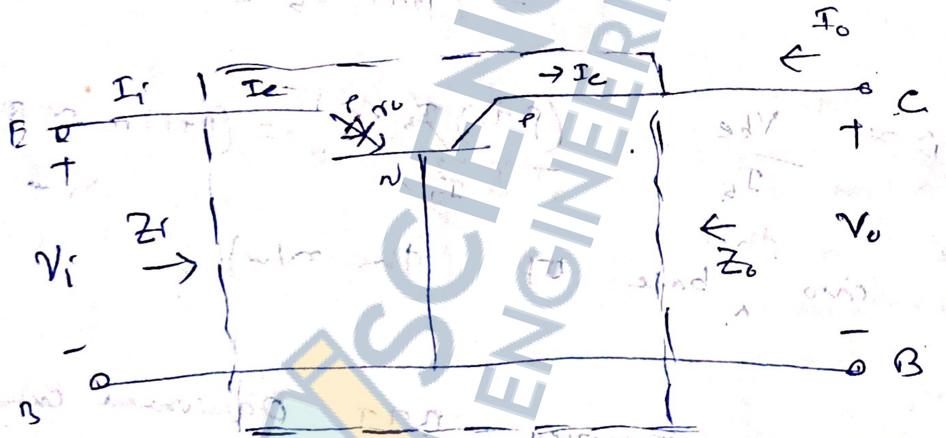
$\beta \simeq 50$ to 200

$\beta r_e \simeq$ few hundred ohm – $7\,k\Omega$

$r_o \simeq 40k - 50k\Omega$



Common-Base Configuration :-



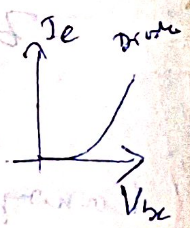


fig 1(a):- Common-Base BJT transistor.

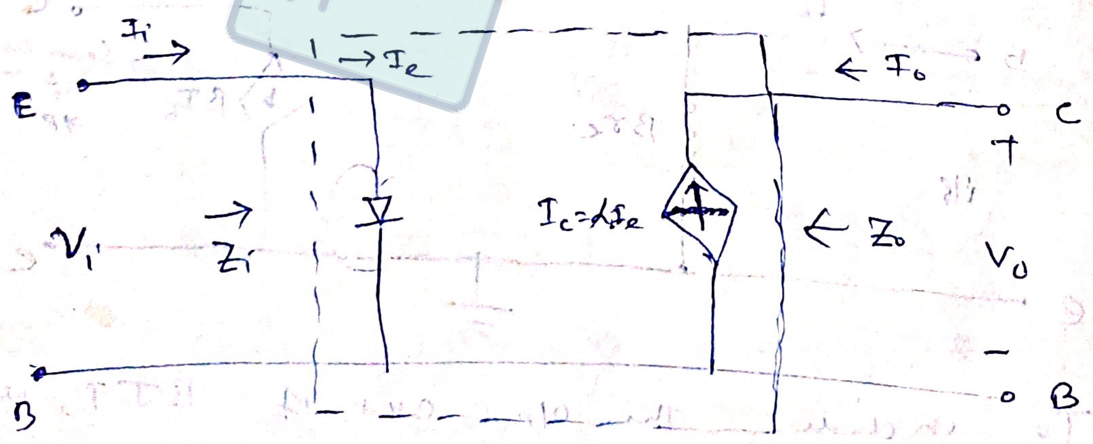
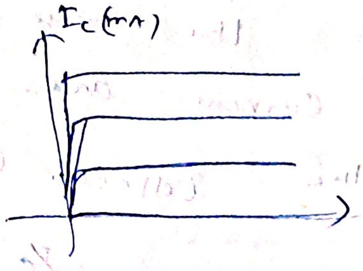


fig:(b) Equivalent ckt for configuration of (a)



Slope $= \frac{1}{r_o}$

Note:-

$Z_i = \frac{V_{be}}{I_e} = \frac{I_e r_e}{I_e}$

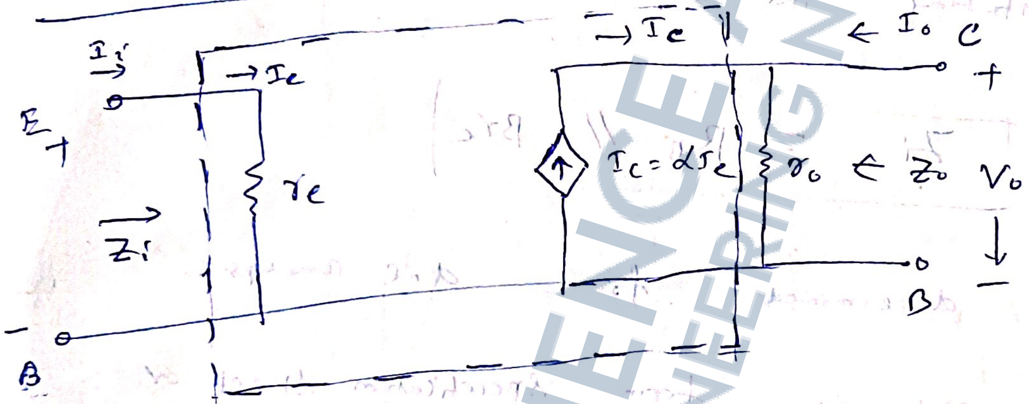
$= r_e$

∴ The almost horizontal lines clearly reveal that o/p resistance r_o will be quite high.

(Slope ≃ 0, but slope $= \frac{1}{r_o}$ ⇒ $r_o \simeq \frac{1}{0} \simeq \infty$

r_o = in the order of Mega ohm)

Taking r_o into account

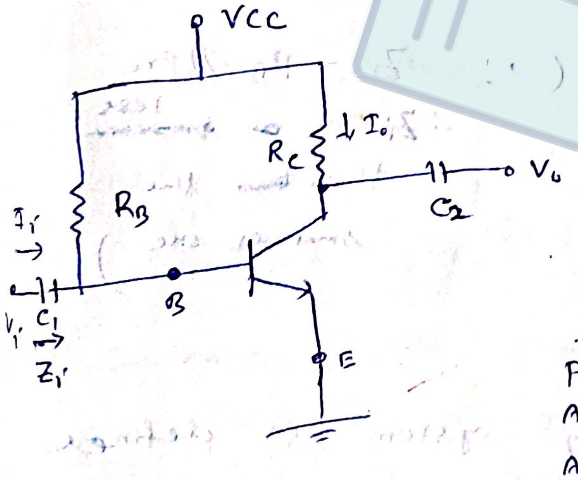


∴ Common base r_e equivalent ckt

Fig:- Common base r_e equivalent ckt

Common - emitter fixed bias configuration :-

For A.C Analysis



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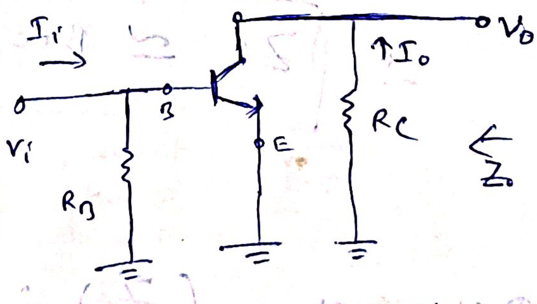
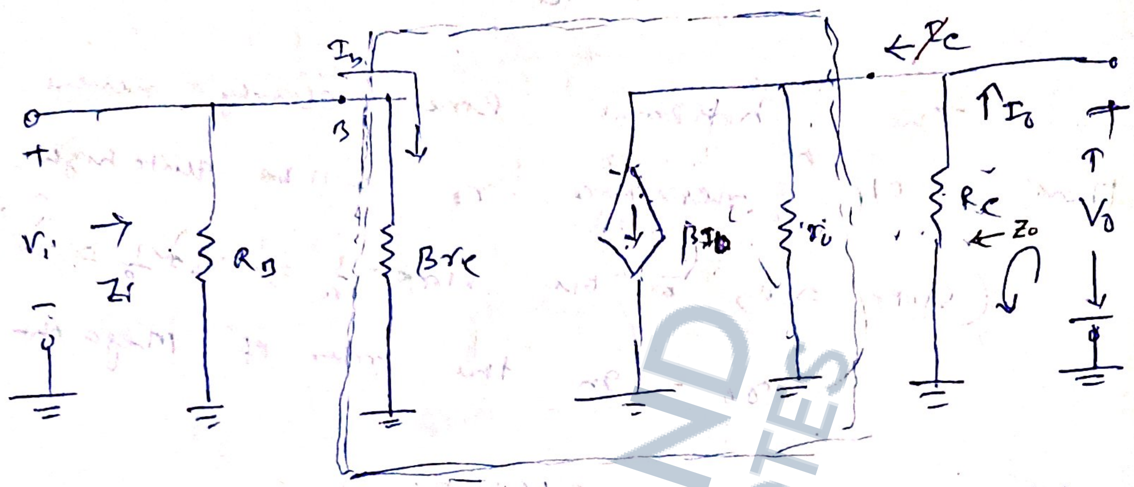


For A.C Analysis } Removing Vcc, Short Ckting C_1 & C_2

$I_i \neq I_b$

Here I_i is not the base current but the source ~~[illegible]~~ current and the output current I_o is the collector current.



Substituting the r_e model into the n/w,

$$\boxed{Z_i = R_B \,//\, \beta r_e}$$

$r_e \rightarrow$ determined from d.c analysis.

$r_o \rightarrow$ obtained from specification sheet or characteristics.

For majority situations,

R_B is greater than βr_e by more than a factor 10,

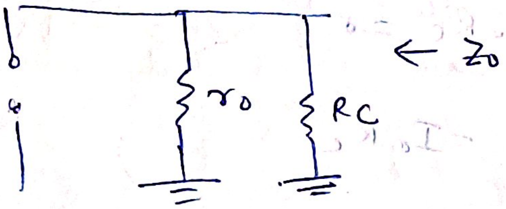
$\therefore$ $\boxed{Z_i \simeq \beta r_e}$ ($\because$ $Z_i = R_B // \beta r_e$
$\therefore Z_i$ is ~~a~~ less ~~[illegible]~~
than ~~[illegible]~~ the
smallest one)

Determining (Z_o) :-

O/P impedance of any system is defined as the impedance Z_o determined when $V_i = 0$.

When $V_i = 0$, $I_i = 0$, $I_b = 0$

Current source open ckted.



$$\boxed{Z_o = R_C // r_o} \text{ ohms}$$

If $r_o \geq 10R_C$,

$$\boxed{Z_o \simeq R_C}$$

Voltage Gain :- (A_V)

∵ KVL
$\cancel{V_o + I_o R_c = 0}$
$\Rightarrow \cancel{V_o = -I_o R_c}$
$\cancel{V_o = -I_c R_c}$

$$V_o = -(\beta I_B)(R_C // r_o)$$

But $I_B = \dfrac{V_i}{\beta r_e}$

$$\therefore V_o = \cancel{(\beta I_B)}\ (-\beta) \times \left(\frac{V_i}{\beta r_e}\right)(R_c // r_o)$$

(Putting the value of $I_b = \dfrac{V_i}{\beta r_e}$)

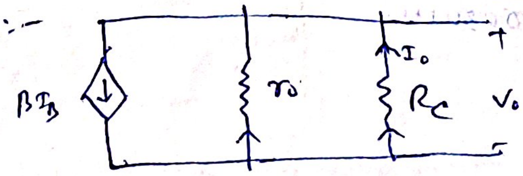
$$V_o = -\frac{V_i}{r_e}(R_c // r_o)$$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{(R_c // r_o)}{r_e}$$

$$\therefore \boxed{A_V = -\frac{R_c // r_o}{r_e}}$$

When $r_o \geq 10R_c$, $\boxed{A_V = -\dfrac{R_C}{r_e}}$

Derivation :-



By current division rule,

$$I_o = \frac{\beta I_B \times r_o}{r_o + R_c}$$

Applying KVL

$v_0 + I_0 R_C = 0$

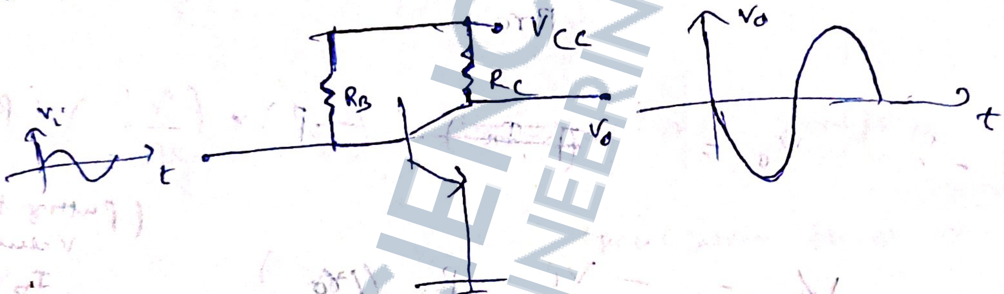
$\Rightarrow v_0 = -I_0 R_C$

$= -\left(\dfrac{\beta I_B r_0 R_C}{r_0 + R_C}\right)$

$v_0 = -\beta I_B (r_0 \,||\, R_C)$ (proved)

Phase relⁿ ship :-

The -ve sign in A_v $\left[A_v = -\dfrac{(R_C \,||\, r_0)}{r_e}\right]$ reveals that 180° phase shift occurs between the I/P and O/P signals.



Ex:1- 1) (a) Determine r_e

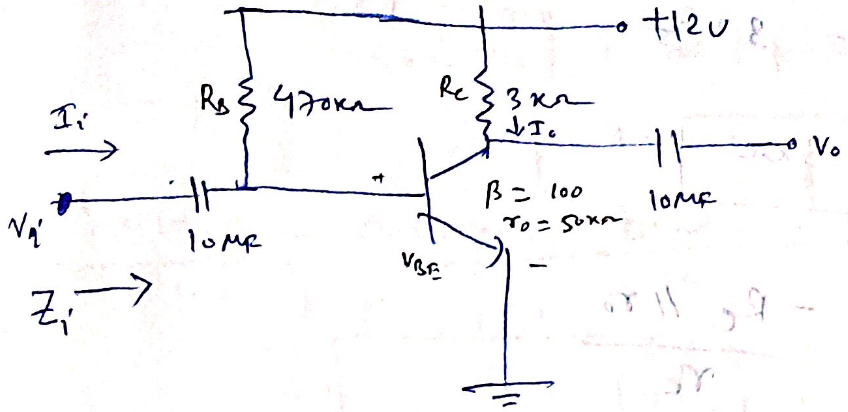
(b) Find Z_i (with $r_0 = \infty\Omega$)

(c) Calculate Z_0 (with $r_0 = \infty\Omega$)

(d) Determine A_v (with $r_0 = \infty\Omega$)

(e) Repeat part (c) & (d) including $r_0 = 50\ k\Omega$ in all calculations & compare results.

Soln :-



To determine r_e, we have to perform DC analysis. Applying KVL in the i/p loop,

$$12 - I_B \times 470 \times 10^3 - V_{BE} = 0$$

$$\Rightarrow I_B = \frac{12 - 0.7}{470 \times 10^3} = 24.04\ \mu A.$$

$$I_E = (\beta + 1) I_B = (100 + 1) \times 24.04\ \mu A$$

$$\boxed{I_E = 2.428\ mA}$$

$$r_e = \frac{26\ mV}{I_E} = \frac{26 \times 10^{-3}}{2.428 \times 10^{-3}} = 10.71\ \Omega.$$

$$\boxed{r_e = 10.71\ \Omega}$$

(a)

(b) $\beta r_e = 100 \times 10.71 = 1071 = 1.07\ k\Omega.$

$$Z_i = \beta r_e \,//\, R_B = (1.07 \,//\, 470)\ k\Omega$$

$$= \frac{1.07 \times 470}{1.07 + 470} = 1.067\ k\Omega$$

$$\boxed{Z_i \simeq 1.07\ k\Omega}$$

(c) $Z_o = R_C \,//\, r_o = \cancel{3k\Omega \,//\, 50k\Omega}$

But $r_o = \infty$,

$Z_0 \simeq R_C = 3\,k\Omega$

$$\boxed{Z_0 = 3\,k\Omega}$$

(d) $A_v = \dfrac{-R_C \parallel r_o}{r_e}$

$\simeq \dfrac{-R_C}{r_e}$

$= -\dfrac{3\,k\Omega}{10.71\,\Omega}$

$= \dfrac{-3\times10^3}{10.71}$

$$\boxed{A_v = -280.11}$$

(e) Repeat c & d using $r_o = 50\,k\Omega$

(c) $Z_0 = R_C \parallel r_o$

$= 3\,k\Omega \parallel 50\,k\Omega$

$= \dfrac{3\times50}{3+50}$

$= 2.83\,k\Omega$ (But with $r_o=\infty$, $Z_i = 3\,k\Omega$)

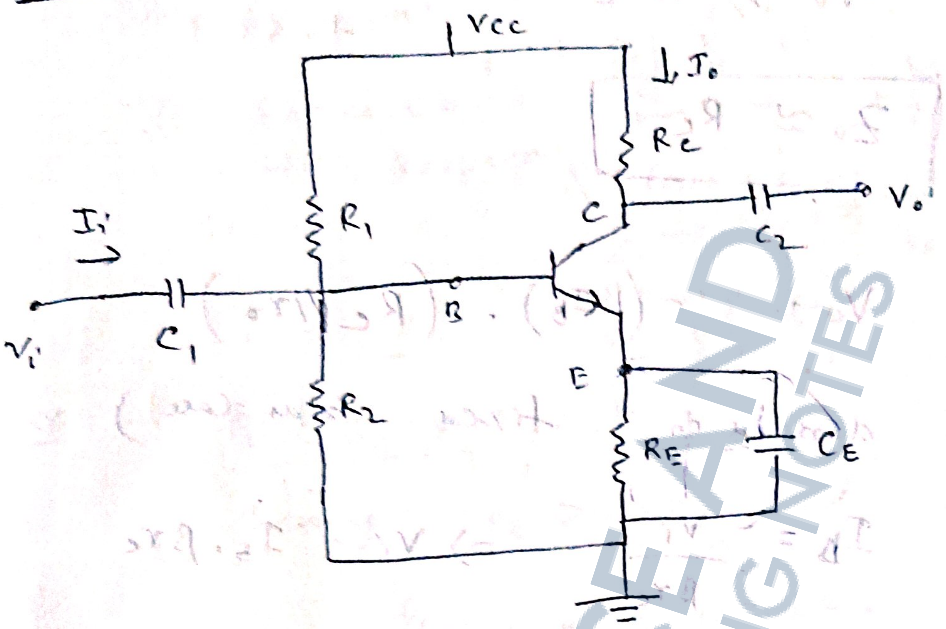
(d) $A_v = \dfrac{-R_C \parallel r_o}{r_e}$

$R_C \parallel r_o = 3k \parallel 50\,k\Omega = 2.83\,k\Omega$

$A_v = -\dfrac{2.83\,k\Omega}{10.71\,\Omega} = -264.24$

$$\boxed{A_v = -264.24}$$ as compared to -280.11 when $r_o \to \infty$.

Voltage-divider bias :- (with Bypass Capacitor)



For A.C analysis, $V_{CC} = 0$, Capacitor C_E is short cktd, so R_E will be absent.

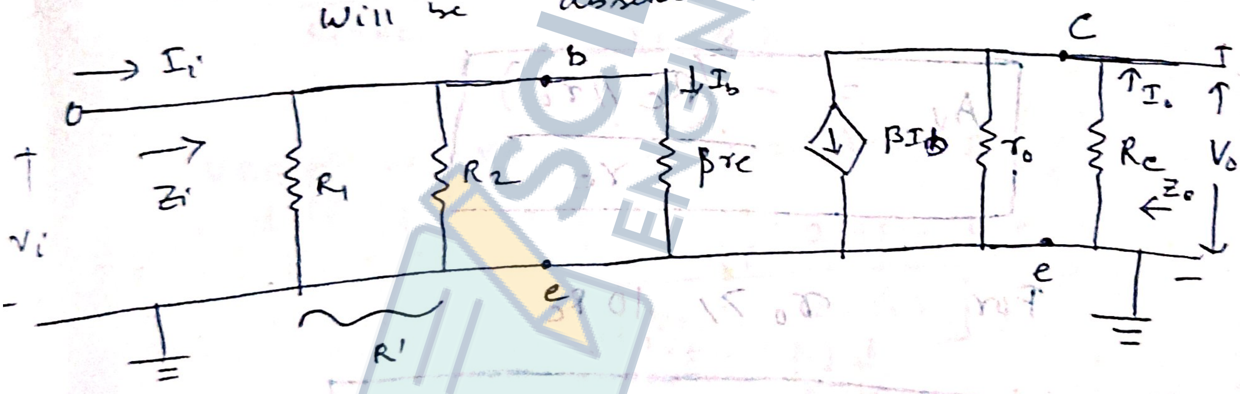


fig:- Substituting r_e equivalent circuit into a.c equivalent network.

$$R' = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{Z_i = R' // \beta r_e}$$

O/P impedance (Z_o) determined when $V_i = 0$

When $V_i = 0V$, $I_B = 0mA$, $\beta I_B = 0mA$.

$$\boxed{Z_o = r_o // R_C}$$

If $r_o \geq 10R_C$

$$\boxed{Z_o \simeq R_C}$$

<u>Av</u> :-

$$V_o = -(\beta I_B) \cdot (R_C // r_o)$$

(As derived in fixed bias case)

$$I_B = \frac{V_i}{\beta r_e} \Rightarrow V_i = I_b \cdot \beta r_e$$

$$\therefore \frac{V_o}{V_i} = \frac{-(\beta \cancel{I_b}) \cdot (R_C // r_o)}{(\cancel{I_b} \cdot \cancel{\beta} r_e)}$$

$$\boxed{A_V = \frac{-(R_C // r_o)}{r_e}}$$

For $r_o \geq 10 R_C$

$$\boxed{A_V \simeq \frac{V_o}{V_i} = \frac{-R_C}{r_e}}$$

Phase rel^n ship :- The -ve sign of the above equation shows a 180° phase shift bet^n V_o & V_i.

Ex-2 :- For the n/w given below, determine

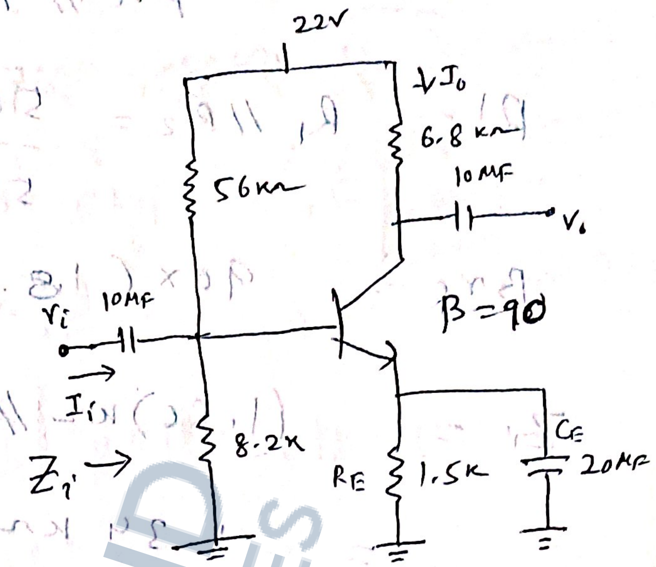
(a) r_e

(b) Z_i

(c) $Z_o (r_o = \infty)$

(d) $A_v (r_o = \infty)$

(e) Repeat c & d with $r_o = 50k\Omega$



Ans :- D.C analysis

Testing $\beta R_E \geq 10 R_2$

$$90 \times 1.5K \geq 10 \times 8.2K$$

$$135k\Omega \geq 82k\Omega \quad \text{(satisfied)}$$

$$V_B = \frac{V_{CC}}{R_1 + R_2} \times R_2 = \frac{22}{56 + 8.2} \times 8.2 = 2.81 \text{ Volt.}$$

$$V_{BE} = V_B - V_E \Rightarrow V_E = V_B - V_{BE}$$

$$\Rightarrow 2.81 - 0.7$$

$$\boxed{V_E = 2.11 V}$$

$$\Rightarrow I_E R_E = 2.11$$

[Note:- In D.C analysis, R_E can't be short, because C_E is open ckt, not short ckt]

$$\Rightarrow I_E = \frac{2.11}{1.5 \times 10^3} = 1.406 mA \simeq 1.41 mA$$

$$r_e = \frac{26 mV}{1.41 mA} = 18.44 \Omega$$

(a) $\therefore \boxed{r_e = 18.44 \Omega}$

(b) $\underline{Z_i} = \beta r_e \,//\, R'$

$$R' = R_1 // R_2 = \frac{56 \times 8.2}{56 + 8.2} = 7.15\,k\Omega.$$

$$\beta r_e = 90 \times (18.44) = 1.66\,k\Omega$$

$$Z_i = (1.66)\,k\Omega \,//\, 7.15\,k\Omega$$

$$= 1.34\,k\Omega$$

(c) $Z_o \simeq R_C$ (for $r_o \geq 10 R_C$)

$$Z_o = 6.8\,k\Omega$$

(d) $A_v \simeq \frac{-R_C}{r_e}$ for $r_e \geq 10 R_C$

$$A_v = \frac{-6.8\,k\Omega}{18.44\,\Omega} = -368.76$$

(e) $Z_i = 1.34\,k\Omega$

$$Z_o = R_C \,//\, r_o$$

$$= (6.8\,k\Omega) // 50\,k\Omega$$

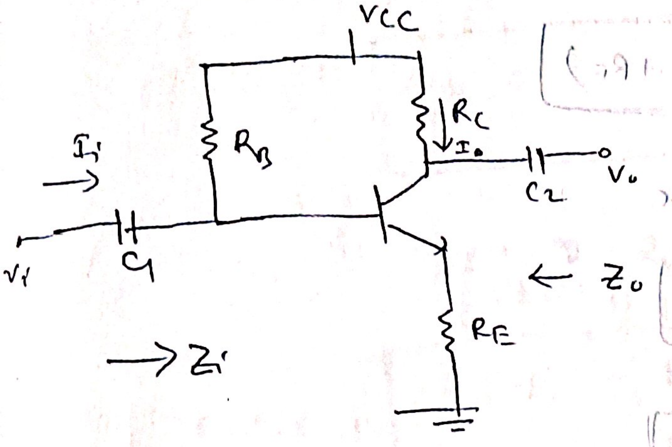
$$= 5.98\,k\Omega$$ [Compared to 6.8 kΩ]

$$A_v = \frac{-R_C // r_o}{r_e} = \frac{5.98\,k\Omega}{18.44\,\Omega} = -324.3$$

as compared to [−368.76]

Common-Emitter [Emitter Bias] Configuration:-

Unbypassed (No bypass capacitor C_E is there)



(With bypass capacitor emitter bias behaves as fixed bias)

fig:- CE emitter bias configuration

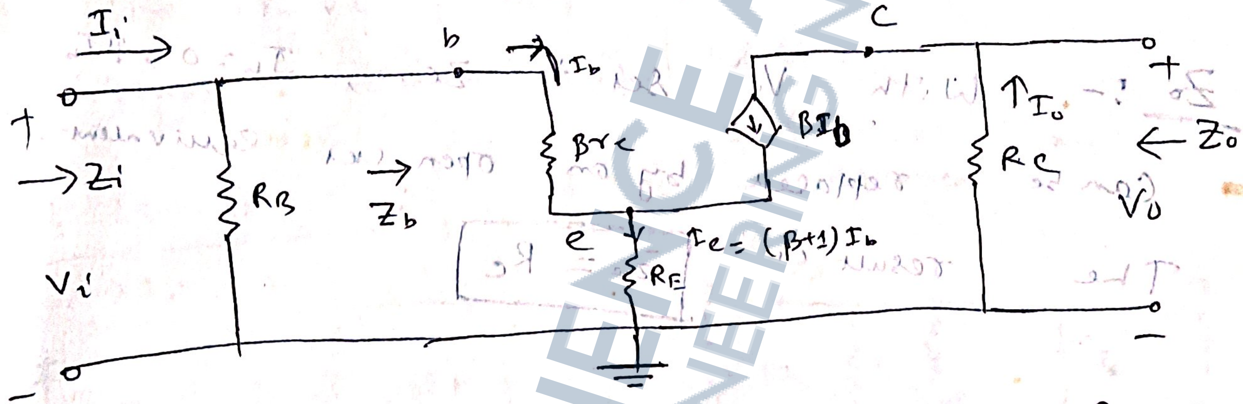


fig:- Substituting the re equivalent circuit into the ac equivalent network.

Applying KVL to the i/p side

$$V_i - I_b(\beta r_e) - (\beta+1) I_b R_E = 0$$

$$\Rightarrow V_i = I_b \beta r_e + (\beta+1) I_b R_E$$

The i/p impedance looking into the network to the right of R_B

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta+1) R_E$$

$$\therefore \boxed{Z_b = \beta r_e + (\beta+1) R_E}$$

Since $\beta+1 \simeq \beta$

$$Z_b \simeq \beta r_e + \beta R_E$$

$$\boxed{Z_b \simeq \beta(r_e + R_E)}$$

$$R_E >> r_e$$

$$\boxed{Z_b \simeq \beta R_E}$$

$$\boxed{Z_i = R_B \,//\, Z_b}$$

Z_o :- With V_i set to zero, $I_b = 0$, βI_b can be replaced by an open-ckt equivalent. The result is $\boxed{Z_o = R_C}$

A_v :- $$I_b = \frac{V_i}{Z_b}$$

and $$V_o = -I_o R_C = -\beta I_b R_C$$

$$V_o = -\beta\left(\frac{V_i}{Z_b}\right) R_C$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-\beta R_C}{Z_b}$$

$$\Rightarrow \boxed{A_v = \frac{-\beta R_C}{Z_b}}$$

But $$Z_b = \beta(r_e + R_E)$$

$$\therefore \quad A_V = \frac{-\cancel{\beta} R_C}{\cancel{\beta}(r_e + R_E)}$$

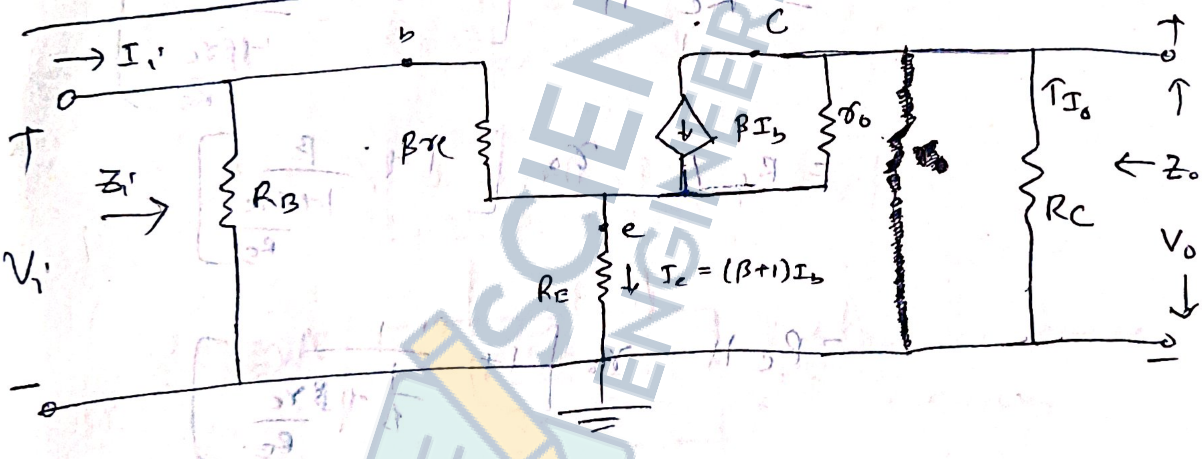
$$\Rightarrow \boxed{A_V = \frac{-R_C}{r_e + R_E}}$$

If $R_E >> r_e$

$$\boxed{A_V = \frac{-R_C}{R_E}}$$

Phase :- The -ve sign shows that 180° phase shift between V_o & V_i.

Effect of r_o :- (Not required)



Additional complexity resulting from including 'r_o' in the analysis. However, when certain conditions are met, the equations return to the form just derived.

Zi :-

$$Z_b = \beta r_e + \left[\frac{(\beta + 1) + \frac{R_C}{r_o}}{1 + \left(\frac{R_C + R_E}{r_o}\right)} \right] R_E$$

The ratio $\frac{R_C}{r_o} << \beta+1$

$$Z_b = \beta r_e + \frac{(\beta+1)R_E}{1+\frac{R_C+R_E}{r_o}}$$

$$r_o \geq 10(R_C+R_E)$$

$$\boxed{Z_b = \beta r_e + (\beta+1)R_E}$$ → Same as derived earlier.

Z_o :

$$Z_o = R_C \parallel \left[r_o + \frac{\beta(r_o+r_e)}{1+\frac{\beta r_e}{R_E}}\right]$$

$r_o >> r_e$

$$\approx R_C \parallel \left[r_o + \frac{\beta r_o}{1+\frac{\beta r_e}{R_E}}\right]$$

$$= R_C \parallel r_o\left[1+\frac{\beta}{1+\frac{\beta r_e}{R_E}}\right]$$

$$\approx R_C \parallel r_o\left[1+\frac{1}{\frac{1}{\beta}+\frac{r_e}{R_E}}\right]$$

For $\beta = 100$, $r_e = 10\,\Omega$, $R_E = 1\,k\Omega$

$\cancel{\approx R_C \parallel}$

$$r_o\left[1+\frac{1}{\frac{1}{100}+\frac{10}{1000}}\right] = r_o\left[1+\frac{1}{.02}\right] \approx 51 r_o$$

$$R_C \parallel 51 r_o$$

$$\boxed{Z_o \simeq R_C}$$ → Same as derived earlier

$$A_V = \frac{V_o}{V_i} = \frac{-\frac{\beta R_C}{Z_b}\left[1+\frac{r_e}{r_o}\right] + \frac{R_C}{r_o}}{1+\frac{R_C}{r_o}}$$

$$r_o \gg r_e$$

$$\Rightarrow \frac{r_e}{r_o} \ll 1 .$$

$$A_V = \frac{-\frac{\beta R_C}{Z_b} + \frac{R_C}{r_o}}{1+\frac{R_C}{r_o}}$$

If $r_o \geq 10 R_C$

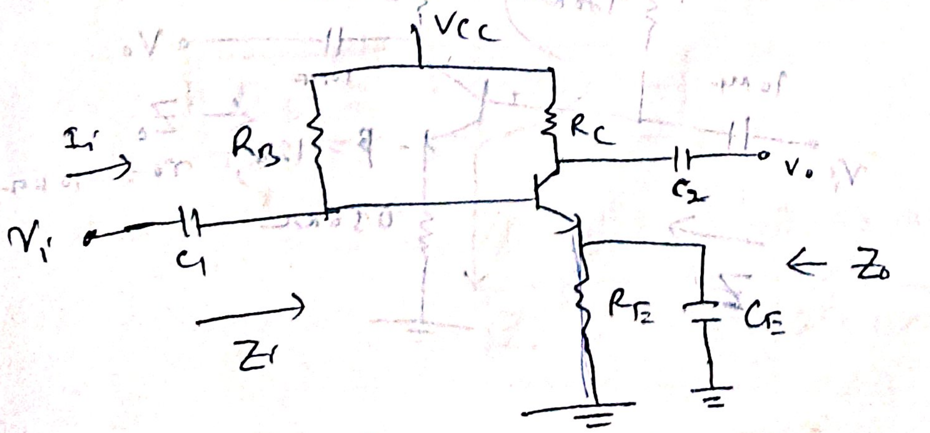
$$\boxed{A_V = \frac{-\beta R_C}{Z_b}}$$

$$\boxed{A_V = \frac{-\beta R_C}{\cancel{\beta}(r_e+R_E)}}$$

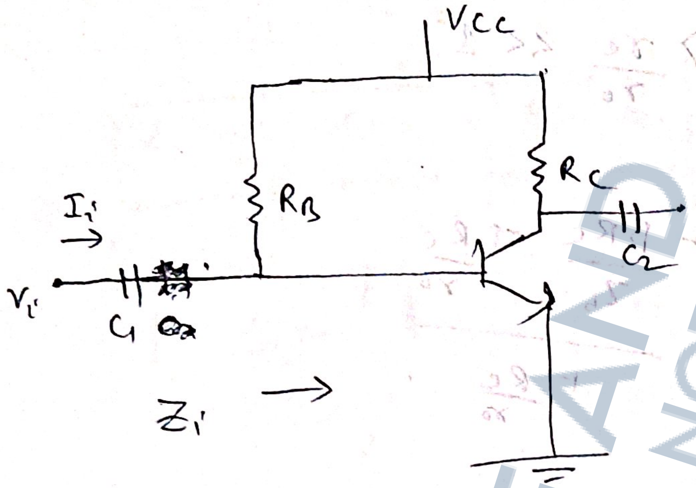
$$\Rightarrow \boxed{A_V = \frac{-R_C}{r_e+R_E}}$$ Same as derived earlier

---

Imp CE - Emitter bias [Bypass Capacitor]



Since for A.C analysis, capacitor is short ckted ~~CE with~~ capacitor (C_E) will be short ckted. So R_E will be short. So the emitter bias becomes fixed bias.



So fixed bias eqns are applicable

$$\boxed{Z_i = R_B \,//\, \beta r_e} \quad \text{or,} \quad \boxed{Z_i \simeq \beta r_e}$$

$$\boxed{Z_o = R_C \,||\, r_o} \quad \text{or} \quad \boxed{Z_o \simeq R_C}$$

$$\boxed{A_v = -\frac{(R_C \,||\, r_o)}{r_e}} \quad \text{or} \quad \boxed{A_v = -\frac{R_C}{r_e}}$$

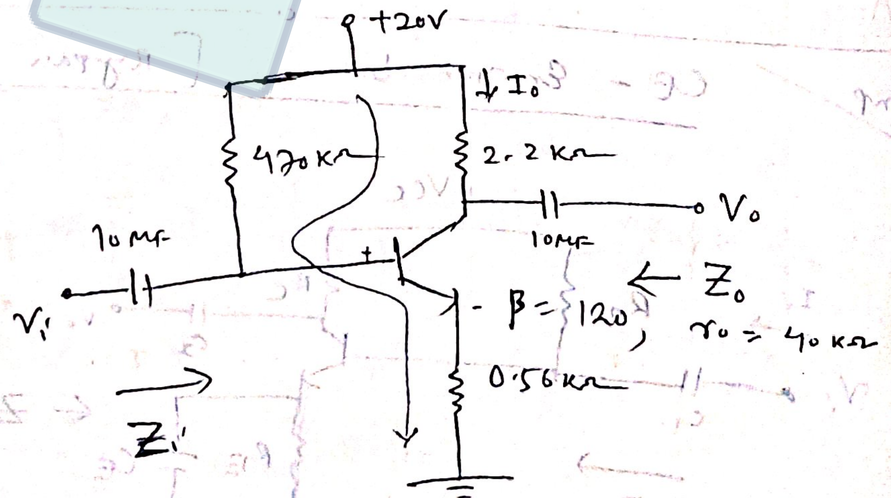
Ex-3 :- For the n/w determine

(a) r_e

(b) Z_i

(c) Z_o

(d) A_v



Soln :- To determine r_e, we need d.c analysis.

Applying KVL in i/p loop

$$20 - I_b \times 470 \times 10^3 - 0.7 - (\beta+1) I_b R_E = 0$$

$$\Rightarrow \quad I_B = \frac{20 - 0.7}{470 \times 10^3 + (121) \times 0.56 \times 10^3}$$

$$\boxed{I_B = 35.89 \ \mu A}$$

$$I_E = (\beta+1) I_B = 121 \times 35.89 \times 10^{-6}$$

$$\boxed{I_E = 4.34 \ mA}$$

(a) $$r_e = \frac{26 \times 10^{-3} \ V}{4.34 \times 10^{-3} \ A} = 5.99 \ \Omega$$

(b) ~~Ans~~ Testing condition

$$r_o \geq 10 (R_C + R_E)$$

$$40 k\Omega \geq 10 (2.2 + 0.56) k\Omega$$

$$40 k\Omega \geq 27.6 \ k\Omega$$

(Satisfied)

Then effect of r_o can be neglected.

$$Z_b = \beta (r_e + R_E)$$

$$= 120 (5.99 + 560)$$

$$\boxed{Z_b = 67.92 \ k\Omega}$$

$$Z_i = Z_b \,//\, R_B = 67.92 \ k\Omega \,//\, 470 \ k\Omega$$

$$\boxed{Z_i = 59.34 \ k\Omega}$$

(c) $Z_o = R_C = 2.2\,k\Omega$

(d) $r_o \geq 10 R_C$

$40\,k\Omega \geq 10 \times 2.2\,k\Omega$

$\therefore\ A_v = \dfrac{-\beta R_C}{Z_b}$

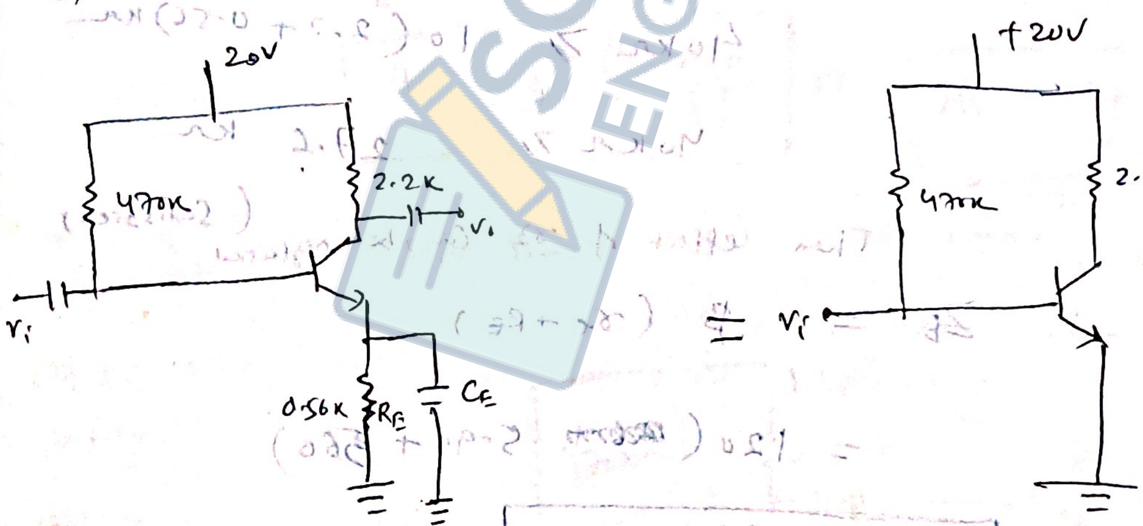
$= \dfrac{-120 \times 2.2\,k\Omega}{67.92\,k\Omega}$

$$\boxed{A_v \simeq -3.89}$$

or. $A_v = \dfrac{-R_C}{R_E} = \dfrac{-2.2\,k\Omega}{0.56\,k\Omega} \simeq -3.93$

Ex:-4 Repeat the analysis with bypass Capacitor C_E in place.

(a)



R_E is short, because capacitor is short ckted.

So we can apply the formulas of fixed bias

(a) $r_e = 5.99 \; (\Omega)$ [Same] [Because for d.c analysis, R_E will be there]

[capacitor open circuit for D.C analysis]

(b) $Z_i = R_B \parallel \beta r_e = 470\,k\Omega \parallel (120).(5.99)$

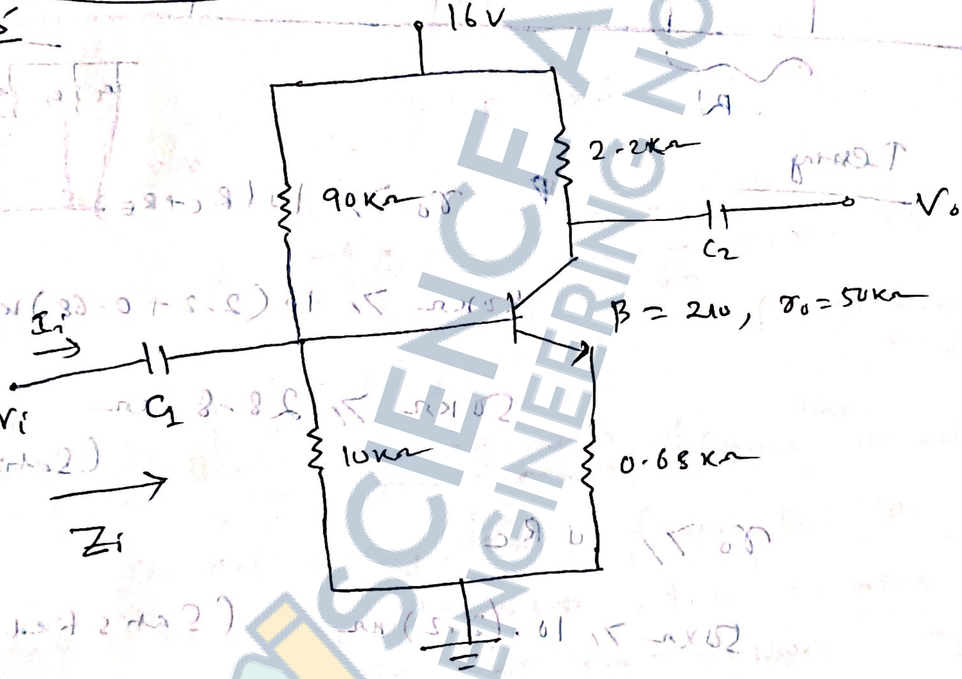
$= 717.70\,\Omega$

(c) $Z_o = R_C = 2.2\,k\Omega$

(d) $A_v = \dfrac{-R_C}{r_e} = \dfrac{-2.2\,k\Omega}{5.99} = -367.28$

Voltage divider biasing with unbypass capacitor

Ex:-5



Determine r_e, Z_i, Z_o, A_v.

Ans :- Testing $\beta R_E > 10 R_2$

$210 \times 0.68 \times 10^3 > 10 \times 10\,k\Omega$

$142.8\,k\Omega > 100\,k\Omega$.

$\therefore V_B = \dfrac{V_{CC}}{R_1 + R_2} \times R_2 = \dfrac{16}{90+10} \times 10 = 1.6\,V$.

$V_{BE} = V_B - V_E \Rightarrow 0.7 = 1.6 - V_E$

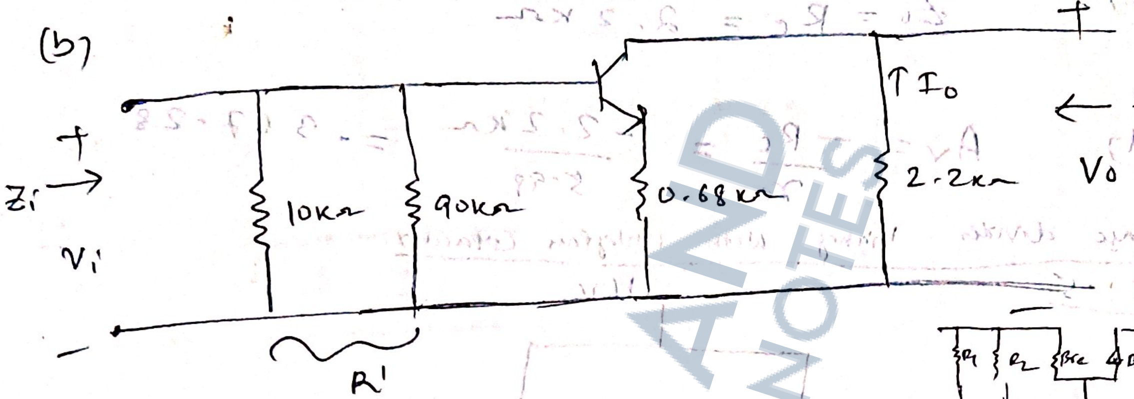
$\Rightarrow V_E = 0.9\,V$

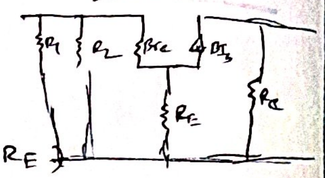
$$I_E R_E = V_E$$

$$\Rightarrow I_E = \frac{V_E}{R_E} = \frac{0.9\,V}{0.68\,k\Omega} = 1.324\,mA$$

(a) $r_e = \dfrac{26\,mV}{1.324\,mA} = 19.64\,\Omega$

(b)





Testing

$r_o \geq 10(R_C + R_E)$

$50k\Omega \geq 10(2.2 + 0.68)\,k\Omega$

$50k\Omega \geq 28.8\,k\Omega$ (Satisfied)

$r_o \geq 10 R_C$

$50k\Omega \geq 10\cdot(2.2)\,k\Omega$ (Satisfied)

$Z_b = \beta R_E = 210 \times 0.68 = 142.8\,k\Omega$

(Derivation is similar to emitter bias without bypass capacitor)

$Z_i = R' \parallel Z_b = \left(\dfrac{10\times 90}{10+90}\right) \parallel 142.8\,k\Omega$

$= 9 \parallel 142.8\,k\Omega$

$$\boxed{Z_i = 8.47\,k\Omega}$$

(c) $Z_o = R_C = 2.2\,k\Omega$

(d) $A_v = \dfrac{-R_C}{R_E} = \dfrac{-2.2\,k\Omega}{0.68\,k\Omega} = -3.23$

Ex-6 :- Repeat example 5 with bypass Capacitor (C_E).

→ It will be like Voltage divider bias Configuration (in ex-2)

Ans ÷ (a) $r_e = 19.64\,\Omega$

(b) $Z_i = \beta r_e \,//\, R'$

$= (210)(19.64) \,//\, 9\,k\Omega$

$= 4.12 \,//\, 9\,k\Omega$

$Z_i = 2.83\,k\Omega$

(c) $Z_o = R_C = 2.2\,k\Omega$

(d) $A_v = \dfrac{-R_C}{r_e} = -\dfrac{2.2\,k\Omega}{19.64} = -112.02$ (Significant increase)

Emitter-Follower Configuration [Also Common Collector]

When output is taken from the emitter terminal of the transistor, the network is referred to as an emitter-follower. The output voltage is always & slightly less than the i/p signal due to the drop from base to emitter, but approximately $\boxed{A_v \simeq 1}$, is good.

→ Unlike Collector voltage, emitter voltage is in Phase with signal V_i. That is, both V_o & V_i attain their positive & -ve peak values at the same time. The fact that V_o "follows" the magnitude of V_i with an in-phase relationship accounts for the terminology emitter-follower,

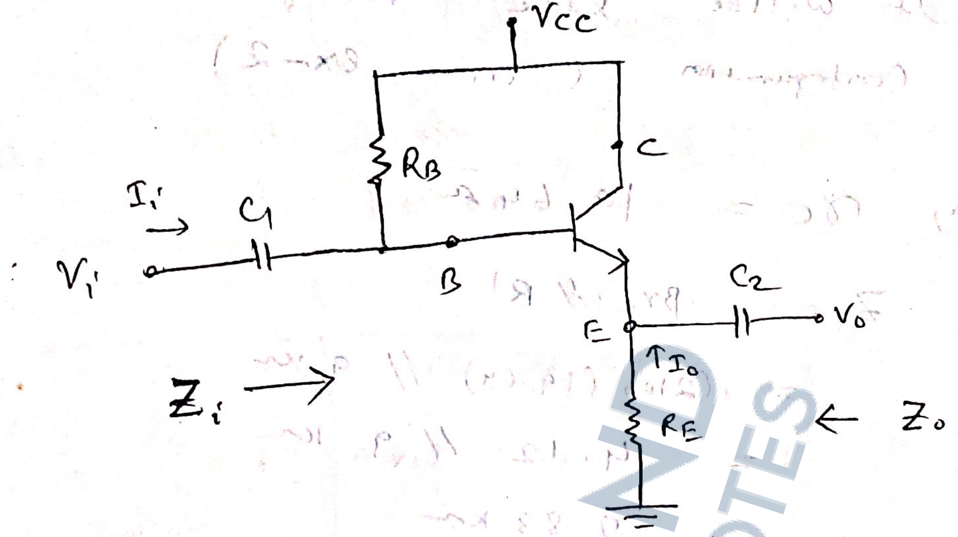


fig 1:- Emitter follower configuration

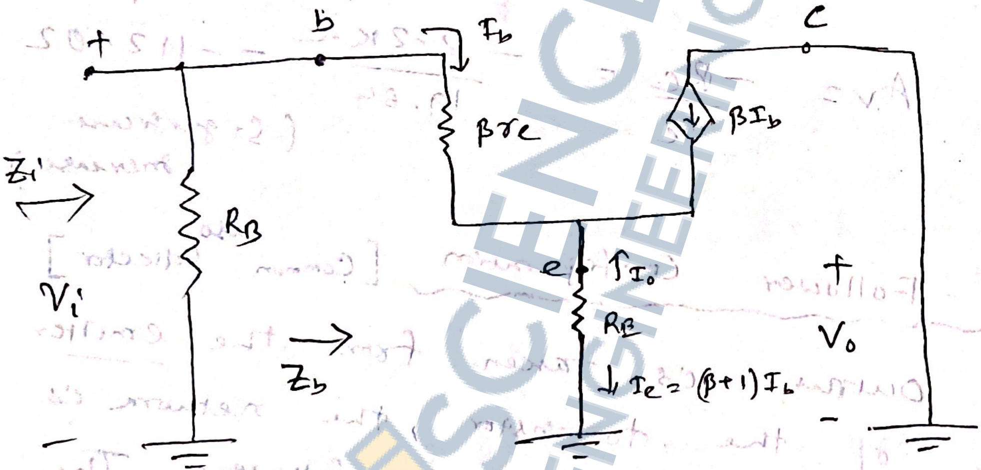


fig 2:- Substituting the re equivalent ckt into ac equivalent n/w of fig 1,

$$Z_i = R_B \parallel Z_b$$

with $Z_b = \beta r_e + (\beta+1) R_E$

$$Z_b \simeq \beta (r_e + R_E)$$

$\boxed{Z_b \simeq \beta R_E}$ $\because R_E >> r_e$

Z_o: The o/p impedance is best described by first writing the equation for the current I_b,

$$I_b = \frac{V_i}{Z_b}$$

$$\text{(1)} \quad \Rightarrow (\beta+1) I_b = (\beta+1)\frac{V_i}{Z_b} \quad \left[\text{multiplying } (\beta+1) \text{ in both the sides}\right]$$

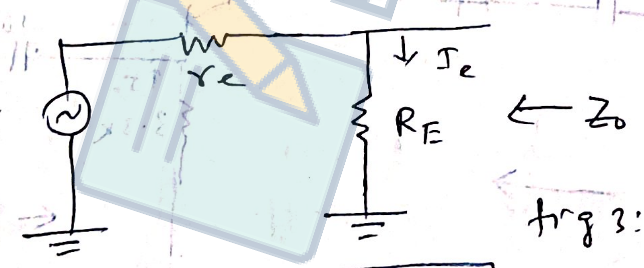
$$\Rightarrow I_e = \frac{(\beta+1) V_i}{\beta r_e + (\beta+1) R_E} \quad \left[\text{Substituting value of } Z_b\right]$$

$$= \frac{V_i}{\frac{\beta}{\beta+1} \cdot r_e + R_E}$$

$$\beta+1 \simeq \beta, \quad \frac{\beta r_e}{(\beta+1)} = \frac{\beta r_e}{\beta} = r_e$$

$$\boxed{I_e \simeq \frac{V_i}{r_e + R_E}}$$

Now we construct the n/w, defined by the above equation



Voltage source short ckt'd.

fig 3:- Defining o/p impedance for emitter-follower configuration

$$\boxed{Z_o = R_E \,//\, r_e}$$

Since $R_E >> r_e$

$$\boxed{Z_o \simeq r_e}$$

A_v :- Fig (3) (~~last~~ previous page) can be used to determine the voltage gain through an appl^n of the voltage-divider rule.

$$V_o = \frac{V_i}{r_e + R_E} \times R_E$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{R_E}{r_e + R_E} \quad \text{——} \quad (1)$$

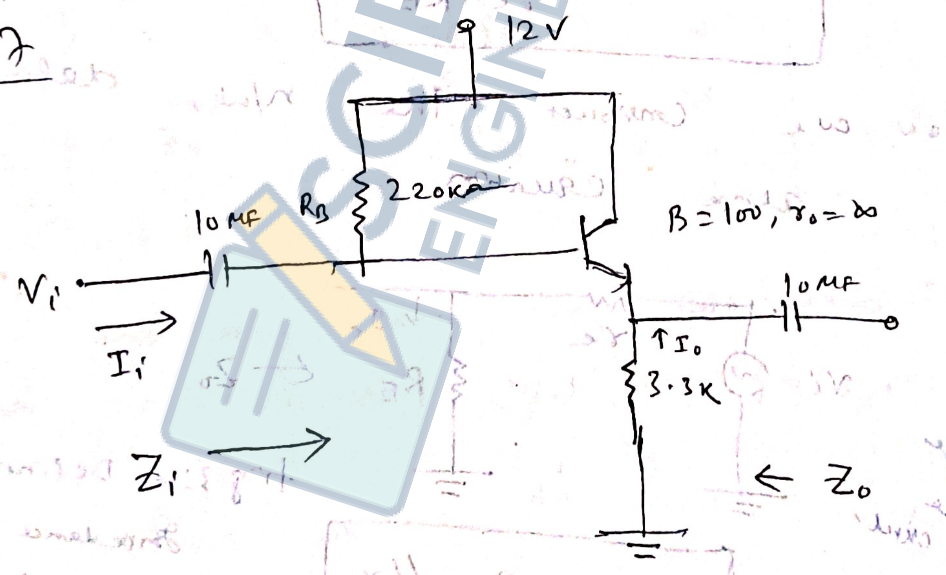
Since $R_E >> r_e$, $\frac{V_o}{V_i} \simeq \frac{R_E}{R_E}$

$$\Rightarrow \boxed{A_v \simeq 1}$$

Phase rel^n ship:-

From eq^n (1), the V_o & V_i are in phase for emitter-follower configuration.

Ex 7



Determine

(a) r_e, b) Z_i, c) Z_o, d) A_v

Ans :-
$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta+1) R_E}$$

$(\because \; V_{CC} - I_B R_B - V_{BE} - (\beta+1) I_B R_E = 0 \;)$

$$\Rightarrow \quad I_B = \frac{12 - 0.7}{220 \times 10^3 + (101) \times 3.3 \times 10^3}$$

$$\boxed{I_B = 20.42 \,\mu A}$$

$$I_E = (\beta+1) I_B$$
$$= (101) \times 20.42 \times 10^{-6}$$

$$\boxed{I_E = 2.062 \text{ mA}}$$

$$r_e = \frac{26 \times 10^{-3}}{2.062 \times 10^{-3}} = 12.61 \,\Omega$$

(b) $$Z_b = \beta r_e + (\beta+1) R_E$$
$$= 100 \times 12.61 + 101 \times 3.3 \times 10^3$$
$$= (1.261 + 333.3) \, k\Omega$$
$$= 334.56 \, k\Omega$$

$$Z_i = Z_b \,||\, R_B$$
$$= (334.56 \,||\, 220) \, k\Omega$$

$$\boxed{Z_i = 132.72 \, k\Omega}$$

(c) $$Z_o = R_E \,||\, r_e = 3.3 \, k\Omega \,||\, 12.61 \,\Omega$$
$$= 12.56 \,\Omega \simeq r_e$$

(d) $$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} = \frac{3.3 \times 10^3}{3.3 \times 10^3 + 12.61} = 0.996 \simeq 1$$

Note :-

(e) $$A_I = \frac{I_o}{I_i} = \frac{\beta R_B}{R_B + Z_b}$$

$I_o = \beta I_b$

$I_b = \frac{I_i \times R_B}{R_B + Z_b} \Rightarrow I_i = \frac{I_b (R_B + Z_b)}{R_B}$

$\frac{I_o}{I_i} = \frac{\beta I_b \times R_B}{I_b (R_B + Z_b)} = \frac{\beta R_B}{R_B + Z_b}$

Brief form of re-model of CE Configuration

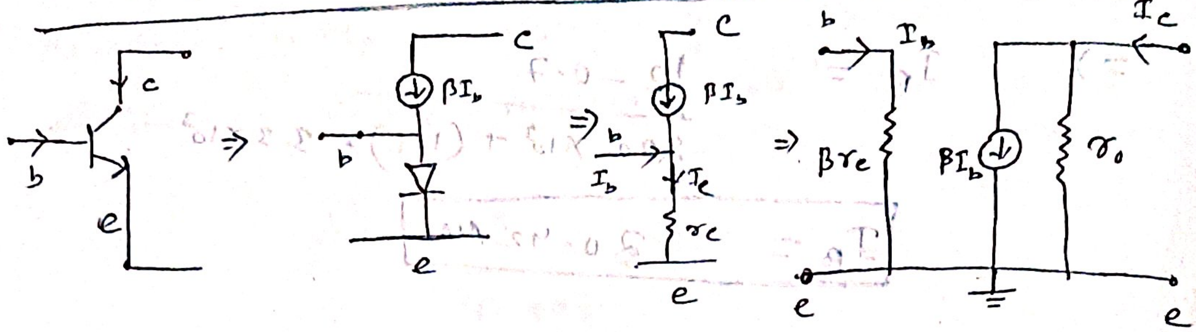
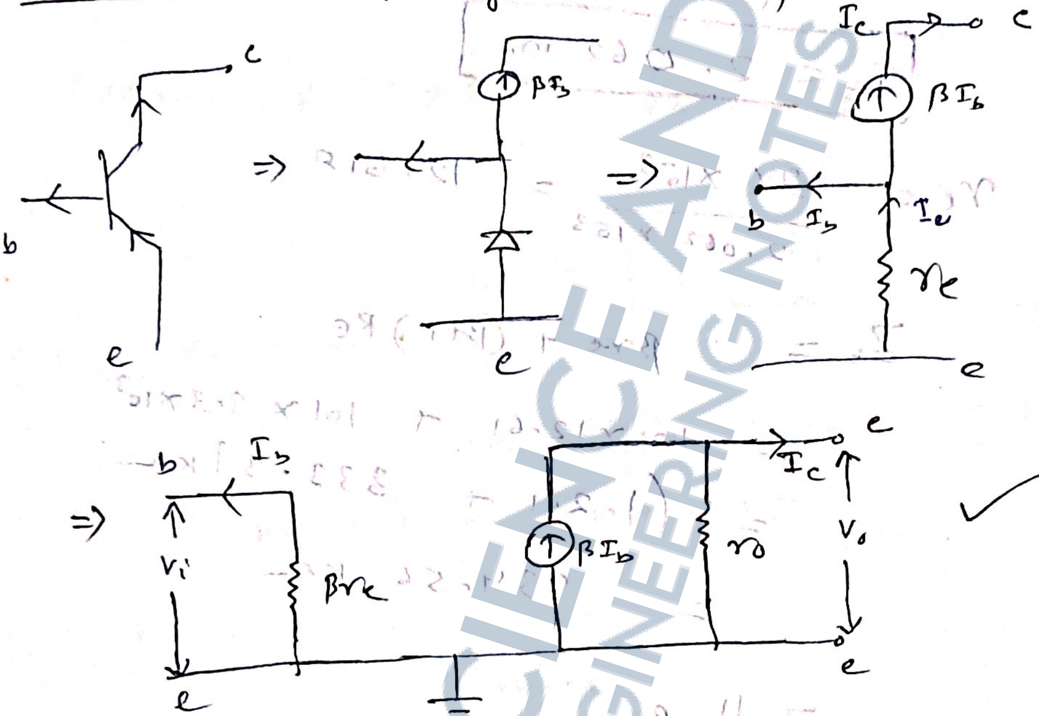


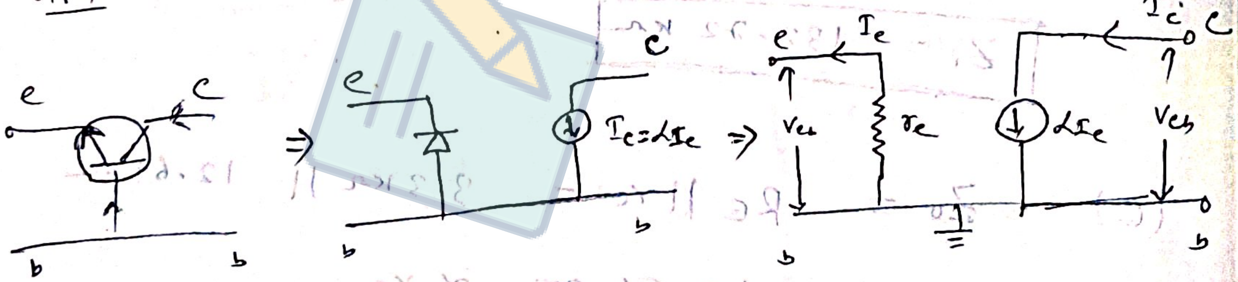
fig:- re model for n-p-n

For pnp (only change in direction)

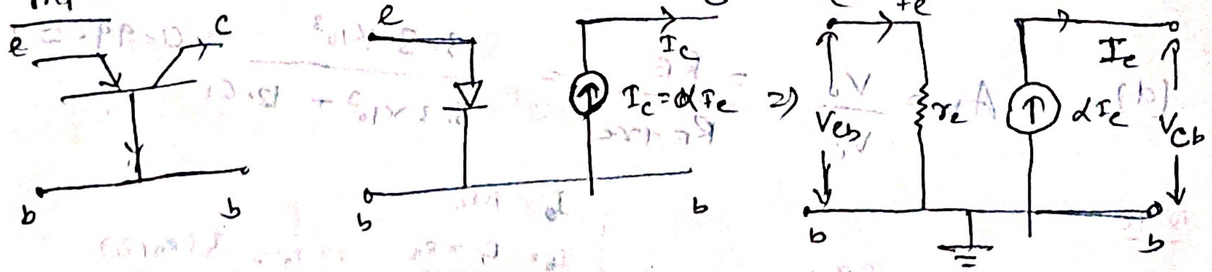


re - model for CB Configuration:-

npn



pnp



Common-Base Configuration :-

The common-base configuration is characterized as having a relatively low i/p and high o/p impedance and current gain less than 1. The voltage gain, however can be quite large.

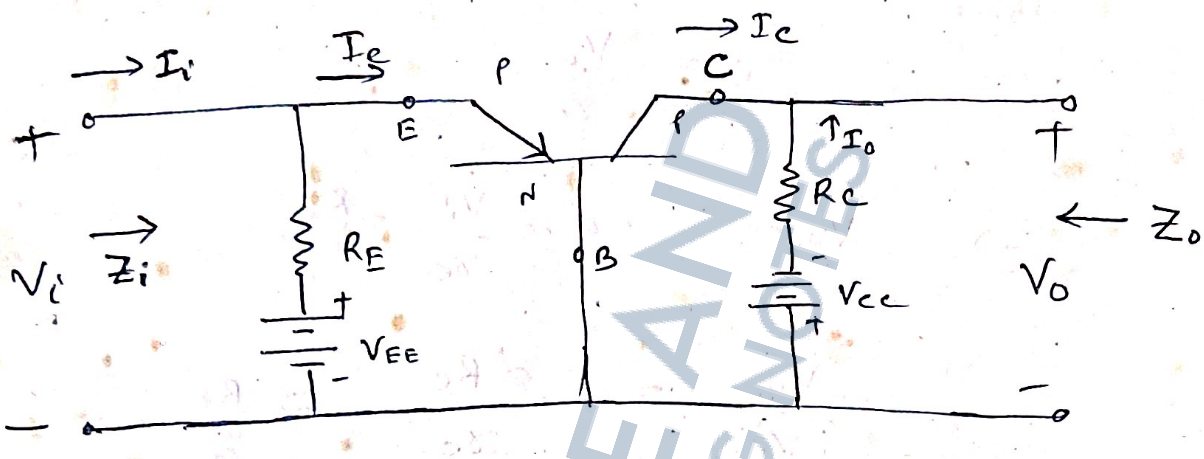
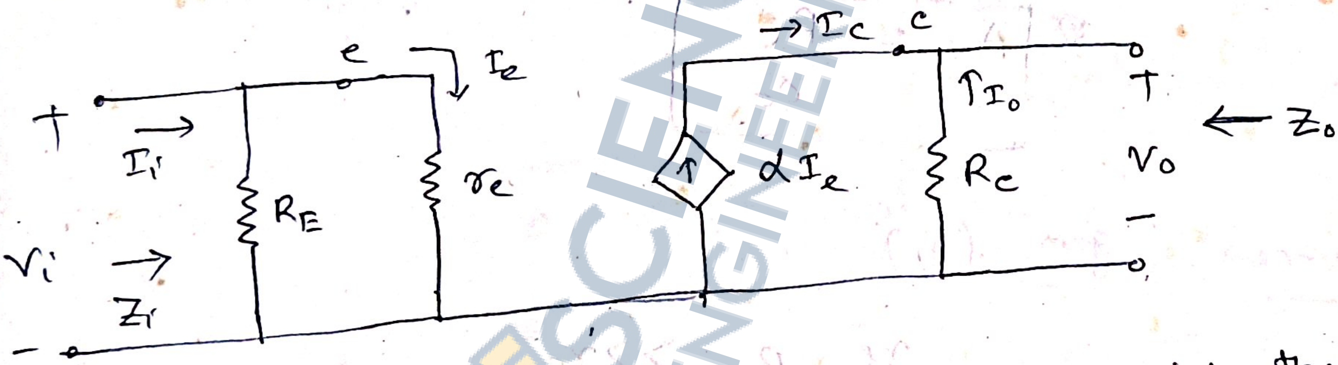


fig:- Common base configuration



Substituting the 'r_e' equivalent circuit into the ac equivalent network

→ The transistor output impedance 'r_o' is not included for the common-base configuration because it is typically in the mega ohm and can be ignored in parallel with resistor R_c.

Z_i :-

$$\boxed{Z_i = R_E \,//\, r_e}$$

$$\boxed{Z_o = R_c}$$

A_v :- (Voltage gain)

$$V_o = -I_o R_c = -(-I_e) R_c$$

$$V_o = I_c R_c = \alpha I_e R_c$$

But

$$I_e = \frac{V_i}{r_e}$$

$$\Rightarrow V_o = \alpha \cdot \frac{V_i}{r_e} \cdot R_c$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{\alpha R_c}{r_e} \simeq \frac{R_c}{r_e}$$

$(\because \alpha \simeq 1)$

$$\boxed{A_v \simeq \frac{R_c}{r_e}}$$

Current gain (A_i)

If $R_E >> r_e$,

$$I_i = I_e \quad \text{—(1)}$$

and

$$I_o = -\alpha I_e \quad \text{—(2)} \qquad (\because I_o = -I_e)$$

$$\Rightarrow \frac{I_o}{I_i} = -\alpha \simeq -1$$

$$\boxed{A_i \simeq -1}$$

[−ve sign shows, actual I_o is out of the system not into the system.]

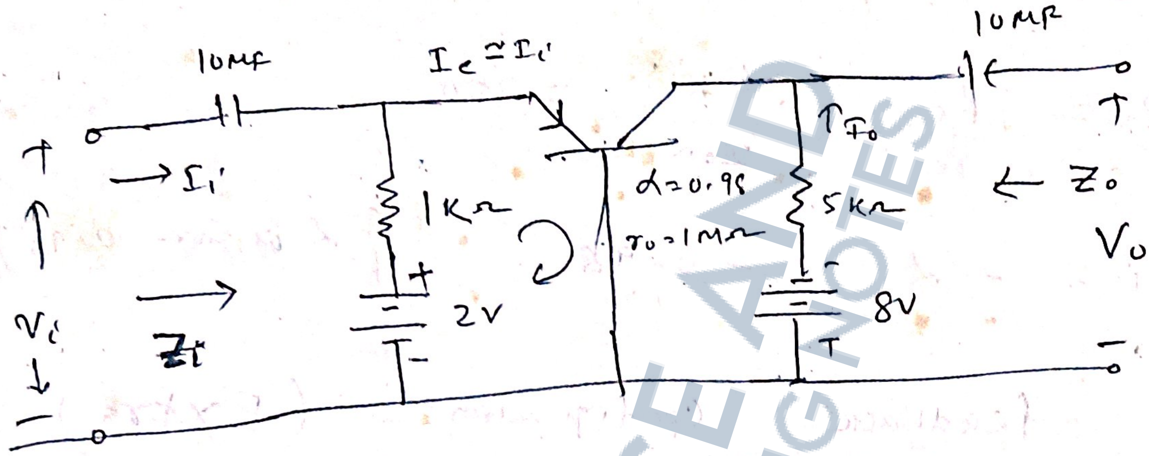
Phase :- The fact that A_v is a +ve number shows that V_o & V_i are in phase for

CB Configuration.

→ Effect of 'r_o' is neglected, because 'r_o' in megaohm. $r_o \| R_C \simeq R_C$.

Ex:- 8 : Determine

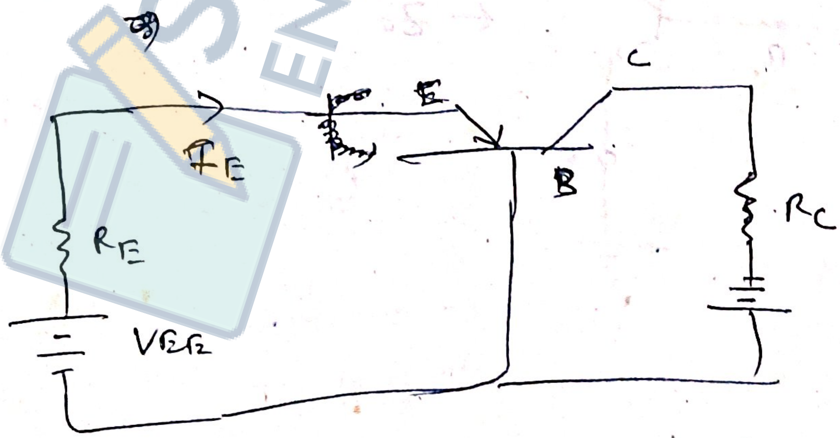
(a) r_e (b) Z_i (c) Z_o (d) A_v (e) A_i



Ans :- To determine r_e

KVL in i/p loop (D.C Analysis)

$$V_{EE} - I_E \cdot R_E - V_{BE} = 0$$



$$\Rightarrow I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{2 - 0.7}{1\,k\Omega} = 1.3\,mA$$

(a) $$r_e = \frac{26\,mV}{I_E} = \frac{26 \times 10^{-3}}{1.3 \times 10^{-3}} = 20\,\Omega$$

(b) $Z_i = R_E \parallel r_e$

$= 1\,k\Omega \parallel 20\Omega$

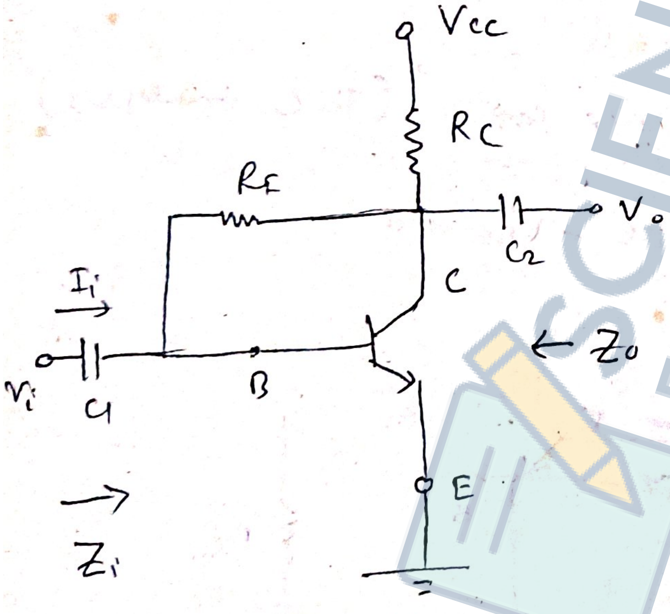
$= 19.61\Omega \simeq r_e$

(c) $Z_o = R_C = 5\,k\Omega$

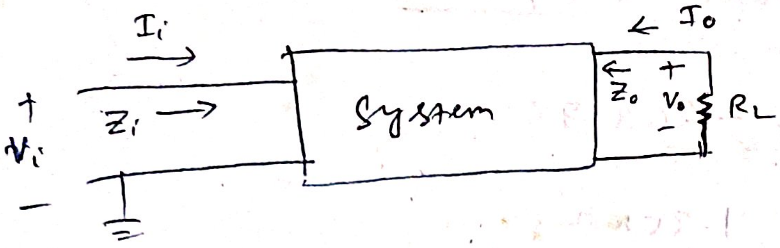
(d) $A_v = \dfrac{R_C}{r_e} = \dfrac{5\,k\Omega}{20\Omega} = 250$

(e) $A_i = -\alpha = -0.98$ (α is given 0.98)

*Collector feedback Configuration :- (Extra)



Determing the current gain :-



Current gain, $A_i = \dfrac{I_o}{I_i}$

Applying Ohm's law to the i/p & o/p ckts result in

$$I_i = \frac{V_i}{Z_i} \quad \text{and} \quad I_o = \frac{-V_o}{R_L}$$

The minus sign associated with the o/p equation is simply there to indicate that the polarity of the o/p voltage is determined by an output current having the opposite direction.

$$A_{iL} = \frac{I_o}{I_i} = \frac{\frac{-V_o}{R_L}}{\frac{V_i}{Z_i}} = -\frac{V_o}{R_L} \times \frac{Z_i}{V_i} = -\frac{V_o}{V_i} \times \frac{Z_i}{R_L}$$

(Current(i) Gain with load(L))

$$\Rightarrow \boxed{A_{iL} = -A_{VL} \times \frac{Z_i}{R_L}}$$

L → Load.

If source resistance is considered (R_s)

$$A_{is} = \frac{I_o}{I_s} = \frac{\frac{-V_o}{R_L}}{\frac{V_s}{R_s+Z_i}} = -\frac{V_o}{R_L} \times \frac{R_s+Z_i}{V_s} = -\frac{V_o}{V_s} \times \frac{R_s+Z_i}{R_L}$$

$$\boxed{A_{is} = -A_{Vs} \cdot \frac{R_s+Z_i}{R_L}}$$

Ex:- In example -2, page - 177. (of Note)

Voltage divider configuration

$$A_V = -368.76$$

$$Z_i = 1.35\,k\Omega$$

$$R_L = 6.8\,k\Omega$$

$$A_{iL} = -\left(-368.76\right) \times \frac{1.35\,k\Omega}{6.8\,k\Omega}$$

$$= \cancel{368.76}$$

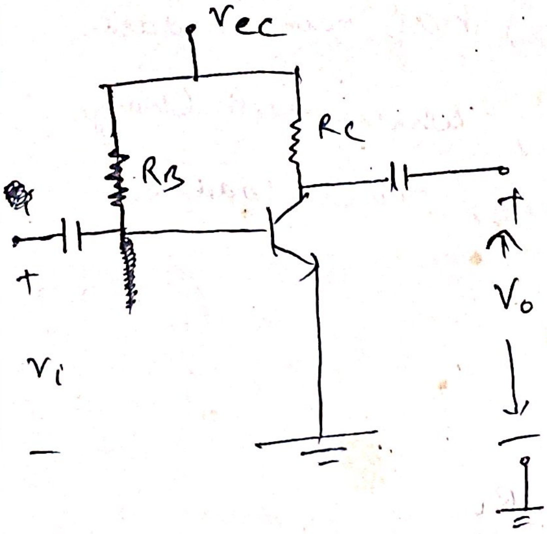
$$\boxed{A_{iL} = 73.2}$$

Effect of R_L & R_s :-

All the parameters determined so far, have been for an unloaded amplifier with i/p voltage connected directly to a terminal of the transistor. Now the effect of applying a load to the output terminal and effect of using a source with an internal resistance will be investigated.

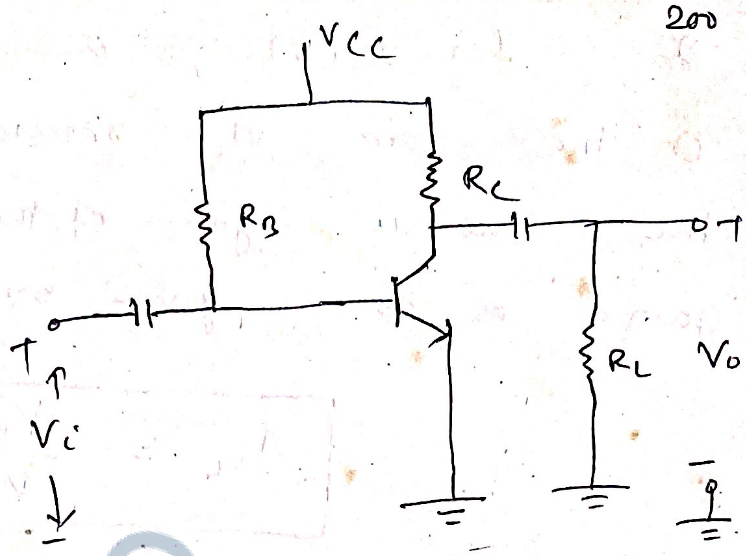
Referring to fig(2),(a), Because a resistive load is ~~was~~ not attached to the output terminal, the gain is commonly referred to as the no-load gain & is given as

$$\boxed{A_{V_{NL}} = \frac{V_o}{V_i}}$$



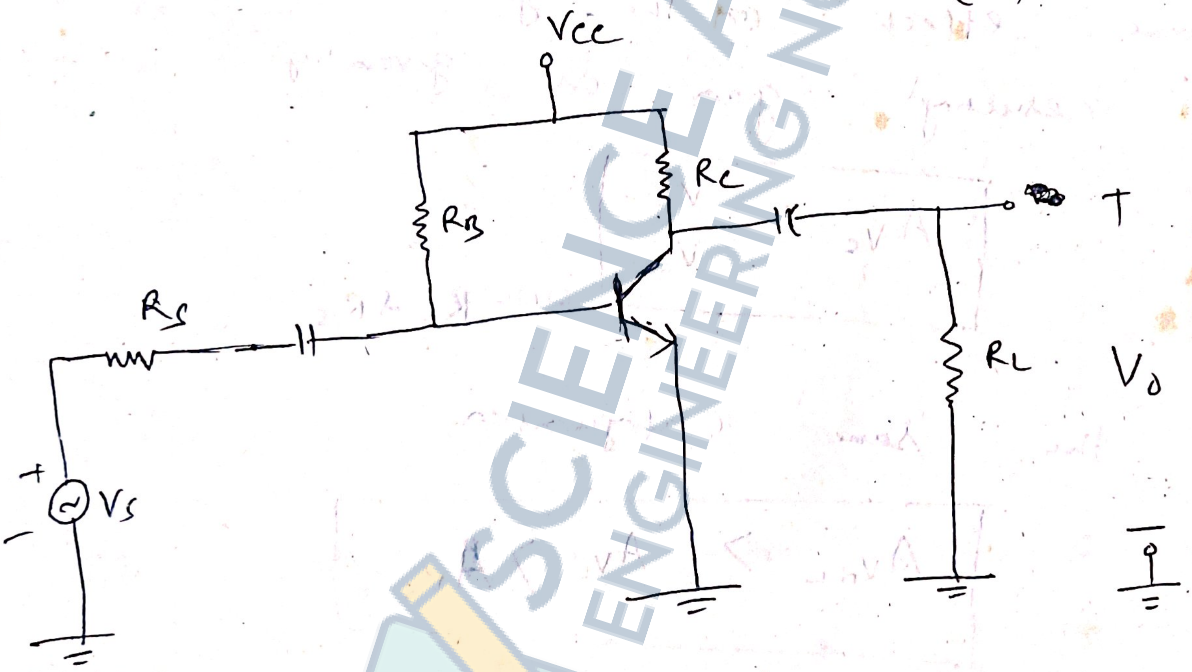
$A_{V_{NL}} = \frac{V_o}{V_i}$

(a)



$A_{V_L} = \frac{V_o}{V_i}$

(b)



(c) $A_{V_s} = \frac{V_o}{V_s}$

fig:2 Amplifier Configurations (a) unloaded (b) loaded (c) loaded with a source resistance.

In fig-2,(b), a load has been added in the form of resistor R_L, which will change the overall gain of the system, This loaded gain ~~of~~ is given by,

$$\boxed{A_{V_L} = \frac{V_o}{V_i}} \text{ with } R_L$$

In fig-2,(c) both a load & source resistance have been introduced, which will have an additional effect on the gain of the system..

The resulting gain is given by

$$\boxed{A_{V_s} = \frac{V_o}{V_s}} \text{ with } R_L \& R_s$$

For the same configuration,

$$\boxed{A_{V_{NL}} > A_{V_L} > A_{V_s}}$$

i.e. The load voltage gain of an amplifier is always less than the no-load gain.

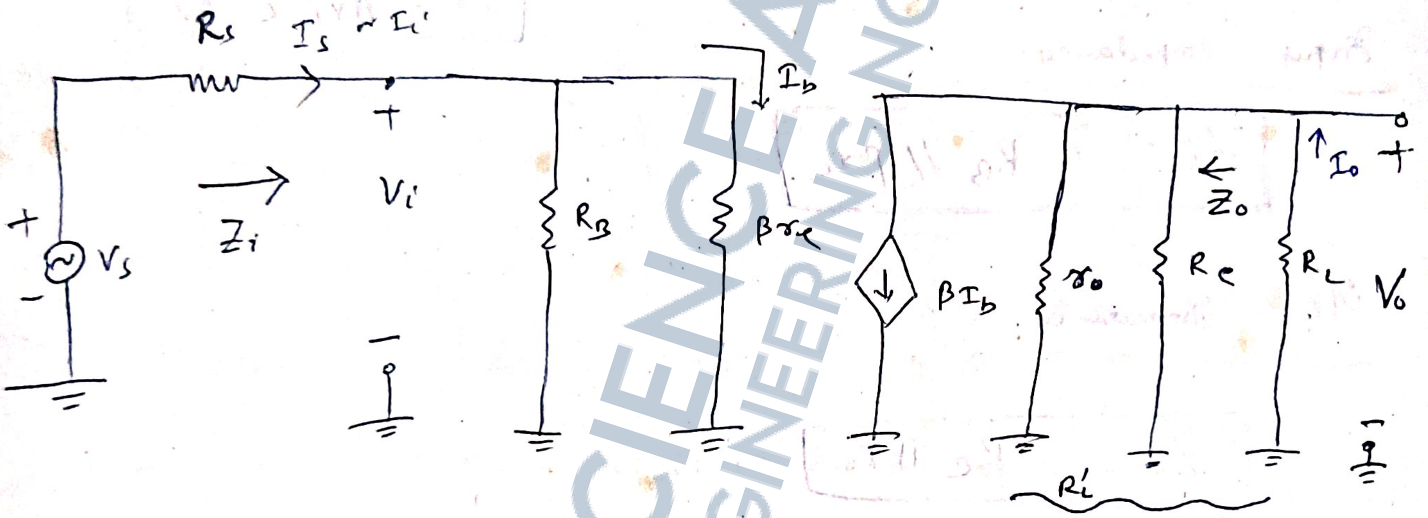
Further, The gain obtained with a source resistance in place will always less than that obtained under loaded or unloaded conditions.

→ Larger the load resistance, greater is the level of ac gain. When R_L is very high, it is approximate to open circuited, i.e no load gain. ($A_{V_{NL}}$) and gain is max^m.

→ Smaller is the source resistance, greater is the overall gain, when R_s is very small, it is equivalent to short circuited, so effect of R_s will be eliminated.

Fixed bias Configuration with R_L & R_s

For the fixed bias transistor amplifier, the r_e equivalent circuit for the transistor [previous page, fig(2)c]



→ The only changes from the a.c equivalent ckt in fixed bias, is R_L is parallel with R_C and R_s is series with V_s.

To find A_{V_L} (Loaded gain)

$$R_L' = r_o \| R_C \| R_L \simeq R_C \| R_L \quad (\because r_o \text{ is very high})$$

and $V_o = -I_o R_L' = -(+\beta I_b) \cdot R_L' = -(\beta I_b) \cdot (R_C \| R_L)$ ——— (1)

$(\because I_o = \beta I_b)$

But $I_b = \dfrac{V_i}{\beta r_e}$ ——— (2)

Putting eqn ② in eqn ①

$$V_o = -\cancel{\beta} \cdot \left(\frac{V_i}{\cancel{\beta} r_e} \right) \cdot (R_C // R_L)$$

$$\Rightarrow \frac{V_o}{V_i} = - \frac{(R_C // R_L)}{r_e}$$

$$\Rightarrow \boxed{A_{VL} = - \frac{R_C // R_L}{r_e}}$$

[when 'R_L' is not considered, $A_V = \frac{-R_C}{r_e}$]

[So $A_{VL} < A_V$]

Input Impedance

$$\boxed{Z_i = R_B // \beta r_e}$$

O/P Impedance

$$\boxed{Z_o = R_C // r_o}$$

A_{Vs} (Overall gain from signal source 'V_s' to o/p voltage 'V_o'

Using voltage divider rule,

$$V_i = \frac{V_s}{R_s + Z_i} \times Z_i$$

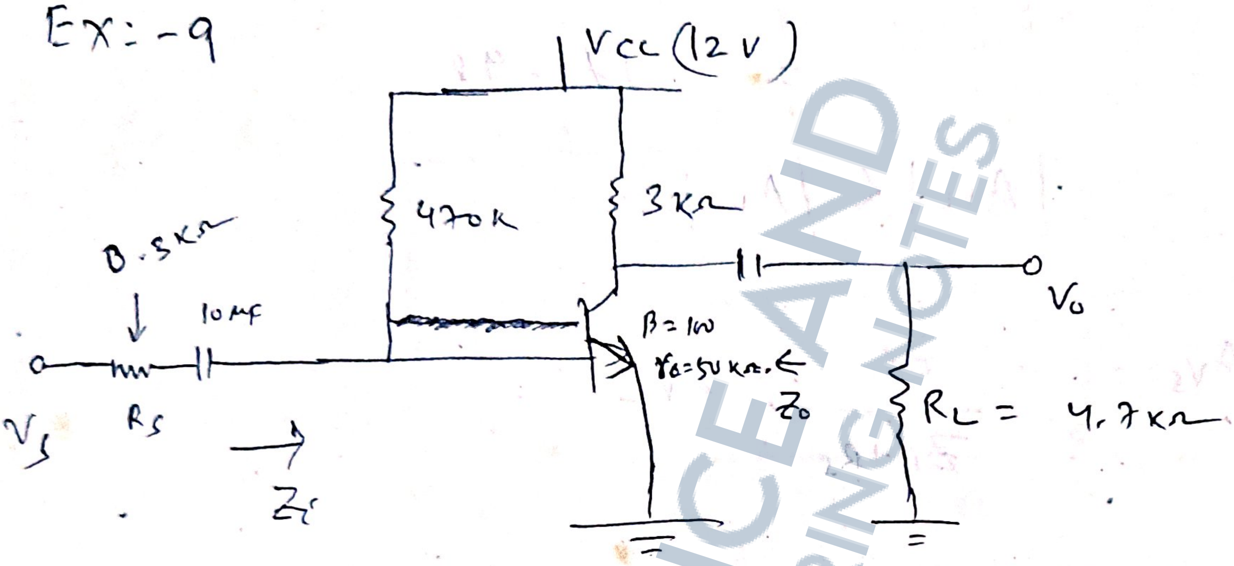
$$\Rightarrow \frac{V_i}{V_s} = \frac{Z_i}{R_s + Z_i}$$

$$A_{Vs} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_i}{V_s} = A_{VL} \times \frac{Z_i}{R_s + Z_i}$$

$$\therefore \quad \boxed{A_{V_s} = \frac{Z_i}{Z_i + R_s} \cdot A_{V_L}}$$

Since $\frac{Z_i}{Z_i + R_s} < 1$, $\boxed{A_{V_s} < A_{V_L}}$

Ex:-9



Determine A_{V_L}, A_{V_s}, Z_i, Z_o, A_{i_L} & Compare with A_V, (no ~~load~~ load gain)

Ans: [See the example of Fixed bias A-C analysis]

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{470\,k\Omega} = 24.04\,\mu A$$

$$I_E = (\beta + 1) I_B = 101 \times 24.04 \times 10^{-6} = 2.428\,mA.$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26}{2.42} = 10.71\,\Omega.$$

$$\beta r_e = 100 \times 10.71 = 1.071\,k\Omega$$

(a) $A_v = \frac{-r_o \| R_C}{r_e} \simeq \frac{-R_C}{r_e} = \frac{-3k\Omega}{10.71} = -280.11$

$A_{VL} = -\frac{R_C \| R_L}{r_e} = \frac{3k\Omega \| 4.7k\Omega}{10.71\Omega} = \frac{-1.831k\Omega}{10.71\Omega}$

$= -170.98$

$\therefore \quad |A_{VL}| < |A_v|$

(b) $A_{Vs} = \frac{Z_i}{Z_i + R_s} \cdot A_{VL}$

$Z_i = \beta r_e \| R_B$

$= 1.071 k\Omega \| 470 k\Omega$

$Z_i = 1.07 k\Omega$

$A_{Vs} = \frac{1.07}{1.07 + 0.3} \times (-170.98)$

$\boxed{A_{Vs} = -133.54}$

$\therefore \quad |A_{Vs}| < |A_{VL}| < |A_v|$

$133.54 < 170.98 < 280.11$

(c) $A_{iL} = \frac{I_o}{I_i} = \frac{I_o}{I_i} = -A_{VL} \times \frac{Z_i}{R_L} = -(-170.98) \times \frac{1.07k\Omega}{4.7k\Omega}$

$= 38.92$

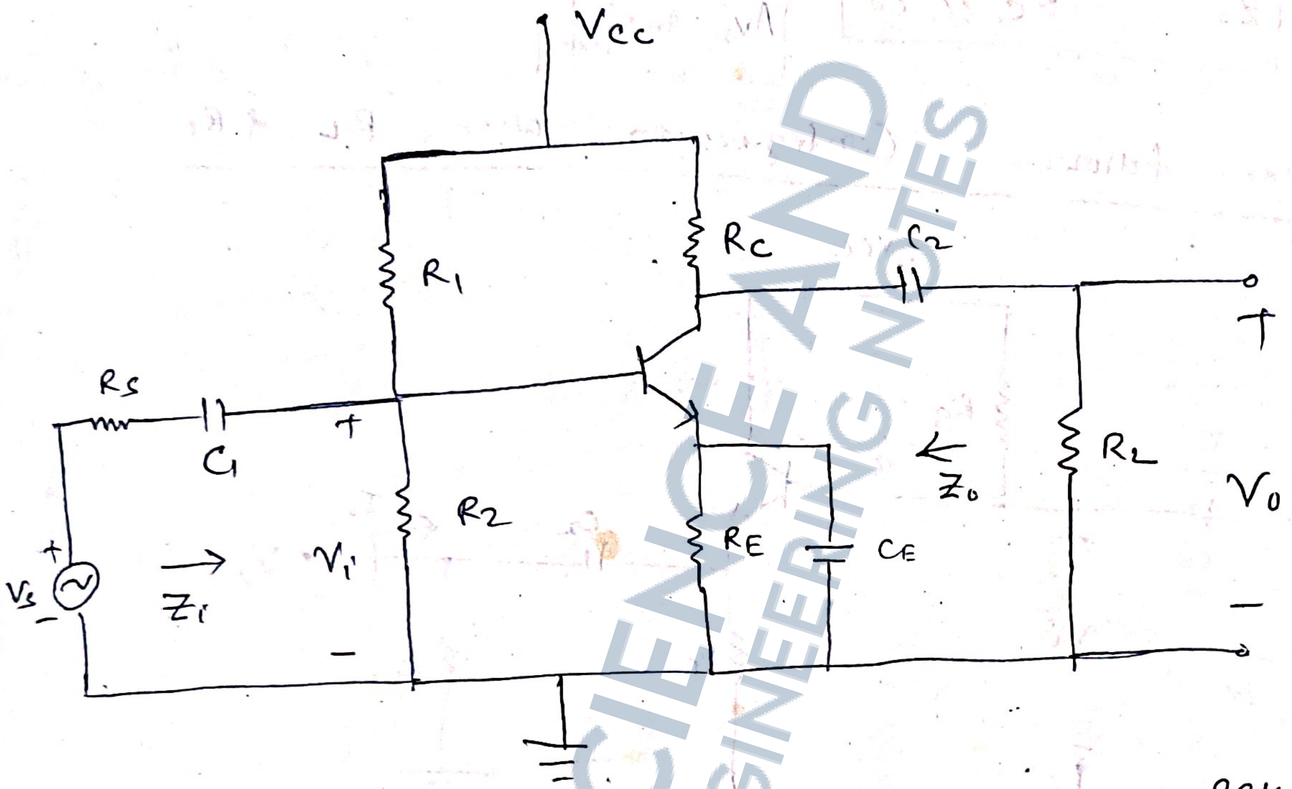
(d) . $Z_i' = 1.07\ k\Omega$

(Here $A_{iL} = A_{is}$ because $I_i = I_s$)

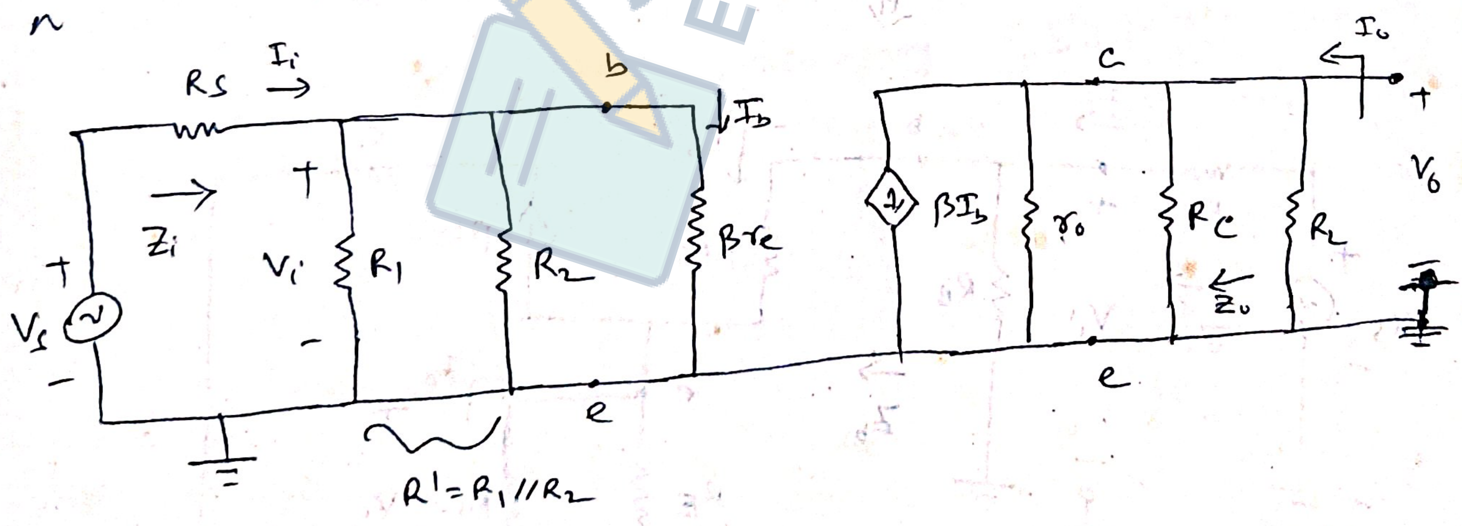
(e) $Z_0 = R_C \,//\, r_o \simeq R_C = 3\ k\Omega$

(f) $A_{is} = \frac{I_o}{I_s} = -A_{vs} \times \frac{R_s + Z_i'}{R_L} = -(-133.54) \times \left(\frac{0.3+1.07}{4.7}\right) = 38.92$

Voltage-divider bias configuration with R_L & R_s



Substituting the r_e equivalent circuit into a.c equivalent n/w



→ The figure as similar to fixed bias, only difference is $\underline{R_1 // R_2}$ instead of R_B.

$$\boxed{A_{VL} = \frac{V_o}{V_i} = -\frac{R_C \,//\, R_L}{r_e}}$$

$$\boxed{Z_i = R_1 \,//\, R_2 \,//\, \beta r_e}$$

$$\boxed{Z_o = R_C \,//\, r_o}$$

$$\boxed{A_{Vs} = A_{VL} \times \frac{Z_i}{R_s + Z_i}}$$

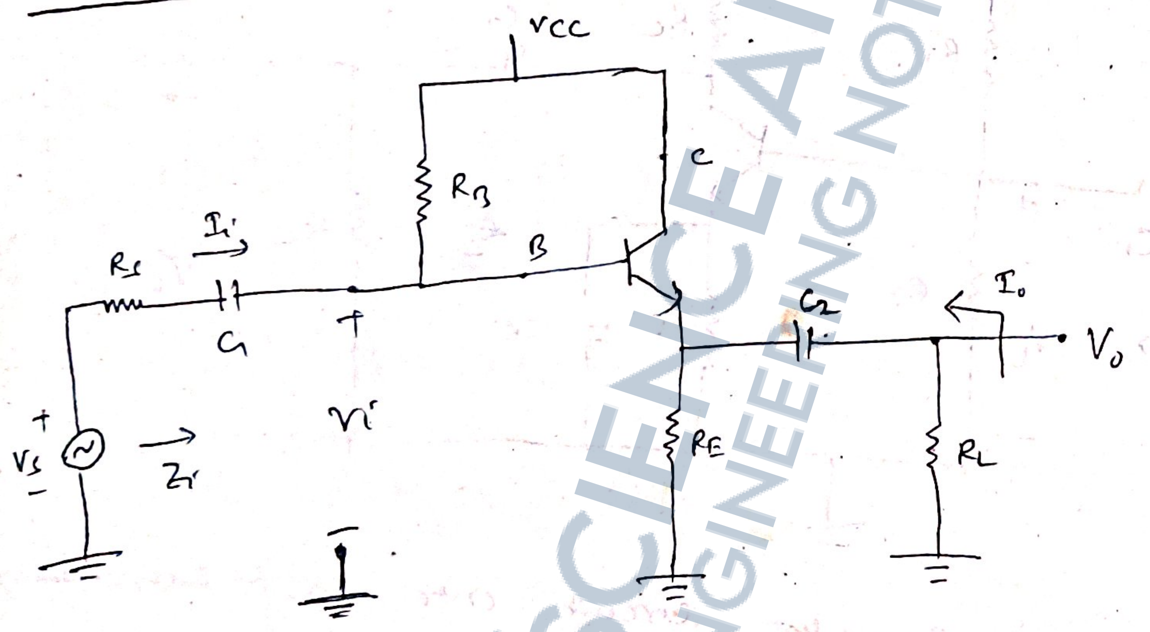
Without Bypass Capacitor [Refer 187 Page. EX-5]

$$A_{VL} = -\frac{R_C \,//\, R_L}{R_E}$$

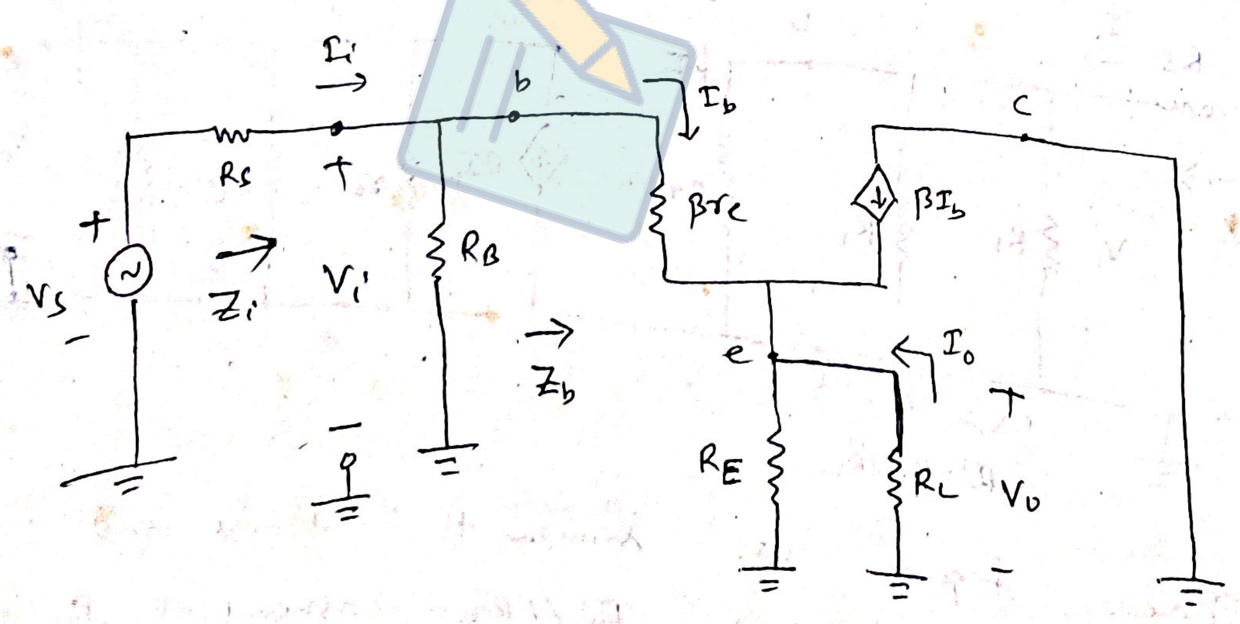
$$Z_i = R_1 \,//\, R_2 \,//\, \beta(r_e + R_E)$$
$$\simeq R_1 \,//\, R_2 \,//\, \beta R_E$$

$$Z_o = R_C \,//\, r_o \simeq R_C$$

Emitter-follower Configuration with R_L & R_s



⇓



$$\boxed{A_{VL} = \frac{V_o}{V_i} = \frac{R_E // R_L}{R_E // R_L + r_e}}$$

$\left(\because A_V = \frac{R_E}{R_E + r_e} \right)$

Replace $R_E \rightarrow R_E // R_L$

The difference betn Emitter-follower without R_L & with R_L is that, parallel combination of R_E & R_L. Whenever R_E was appearing in no load case now will be replaced by $\boxed{R_E // R_L}$

$$\boxed{Z_i = R_B // Z_b}$$

where

$$\boxed{Z_b \simeq \beta (R_E // R_L)}$$

$$\boxed{Z_o \simeq r_e}$$

$\left(\because \text{In Emitter follower without } R_L, \; Z_b \simeq \beta R_E \right)$

&

$\left(\because \text{without } R_L \; Z_o = R_E // r_e \right)$

with R_L

$Z_o = (R_E // R_L) // r_e \simeq r_e$

→ The effect of load resistor & a source impedance on the remaining BJT configuration is shown in the summary table. (Next page)

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TABLE 5.1
Unloaded BJT Transistor Amplifiers

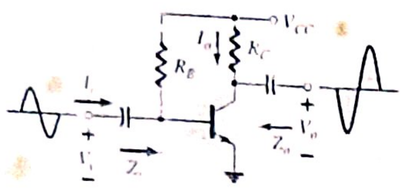
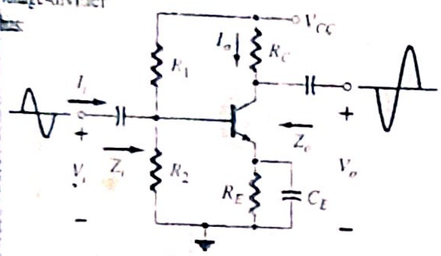
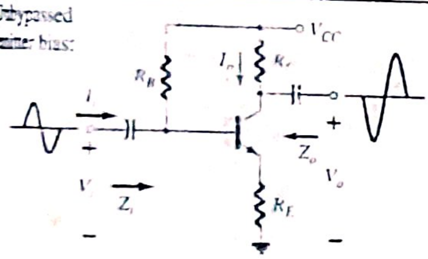
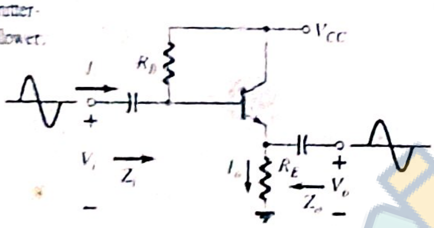
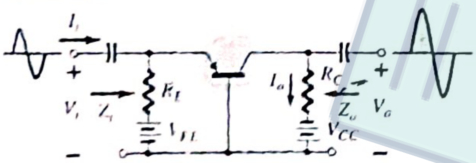
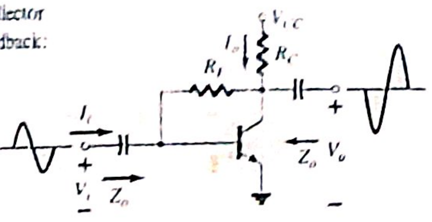
Configuration	Z_i	Z_o	A_v	A_i
Fixed-bias: 	Medium (1 kΩ) $= \boxed{R_B \| \beta r_e}$ $\cong \boxed{\beta r_e}$ $(R_B \ge 10\beta r_e)$	Medium (2 kΩ) $= \boxed{R_C \| r_o}$ $\cong \boxed{R_C}$ $(r_o \ge 10R_C)$	High (−200) $= \boxed{-\frac{(R_C \| r_o)}{r_e}}$ $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \ge 10R_C)$	High (100) $= \boxed{\frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}}$ $\cong \boxed{\beta}$ $(r_o \ge 10R_C,$ $R_B \ge 10\beta r_e)$
Voltage-divider bias: 	Medium (1 kΩ) $= \boxed{R_1 \| R_2 \| \beta r_e}$	Medium (2 kΩ) $= \boxed{R_C \| r_o}$ $\cong \boxed{R_C}$ $(r_o \ge 10R_C)$	High (−200) $= \boxed{-\frac{R_C \| r_o}{r_e}}$ $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \ge 10R_C)$	High (50) $= \boxed{\frac{\beta(R_1 \| R_2)r_o}{(r_o + R_C)(R_1 \| R_2 + \beta r_e)}}$ $\cong \boxed{\frac{\beta(R_1 \| R_2)}{R_1 \| R_2 + \beta r_e}}$ $(r_o \ge 10R_C)$
Unbypassed emitter bias: 	High (100 kΩ) $= \boxed{R_B \| Z_b}$ $Z_b \cong \beta(r_e + R_E)$ $\cong \boxed{R_B \| \beta R_E}$ $(R_E \gg r_e)$	Medium (2 kΩ) $= \boxed{R_C}$ (any level of r_o)	Low (−5) $= \boxed{-\frac{R_C}{r_e + R_E}}$ $\cong \boxed{-\frac{R_C}{R_E}}$ $(R_E \gg r_e)$	High (50) $\cong \boxed{-\frac{\beta R_B}{R_B + Z_b}}$
Emitter-follower: 	High (100 kΩ) $= \boxed{R_B \| Z_b}$ $Z_b \cong \beta(r_e + R_E)$ $\cong \boxed{R_B \| \beta R_E}$ $(R_E \gg r_e)$	Low (20 Ω) $= \boxed{R_E \| r_e}$ $\cong \boxed{r_e}$ $(R_E \gg r_e)$	Low (≅ 1) $= \boxed{\frac{R_E}{R_E + r_e}}$ $\cong \boxed{1}$	High (−50) $\cong \boxed{-\frac{\beta R_B}{R_B + Z_b}}$
Common-base: 	Low (20 Ω) $= \boxed{R_E \| r_e}$ $\cong \boxed{r_e}$ $(R_E \gg r_e)$	Medium (2 kΩ) $= \boxed{R_C}$	High (200) $\cong \boxed{\frac{R_C}{r_e}}$	Low (−1) $\cong \boxed{-1}$
Collector feedback: 	Medium (1 kΩ) $\cong \boxed{\frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}}$ $(r_o \ge 10R_C)$	Medium (2 kΩ) $\cong \boxed{R_C \| R_F}$ $(r_o \ge 10R_C)$	High (−200) $\cong \boxed{-\frac{R_C}{r_e}}$ $(r_o \ge 10R_C)$ $(R_F \gg R_C)$	High (50) $= \boxed{\frac{\beta R_F}{R_F + \beta R_C}}$ $\cong \boxed{\frac{R_F}{R_C}}$

TABLE 5.2

BJT Transistor Amplifiers Including the Effect of R_s and R_L

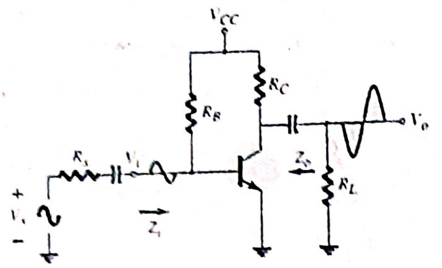
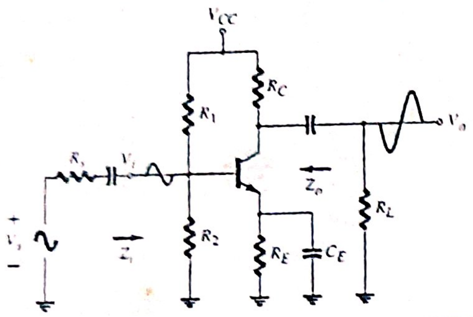
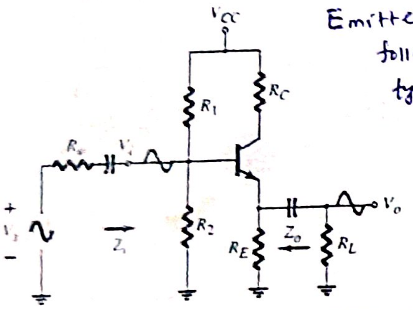
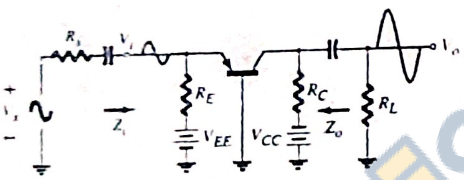
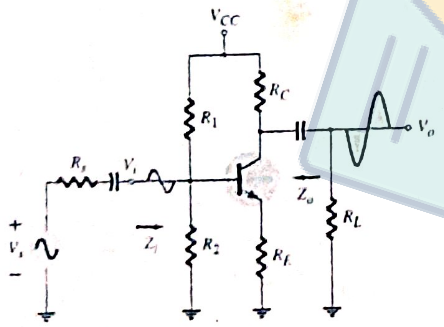
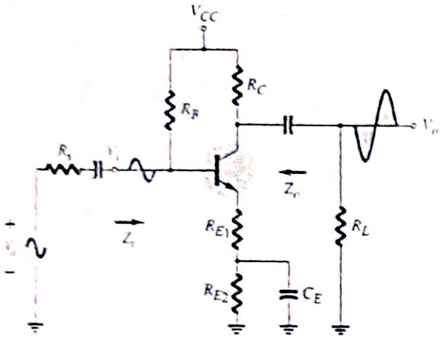
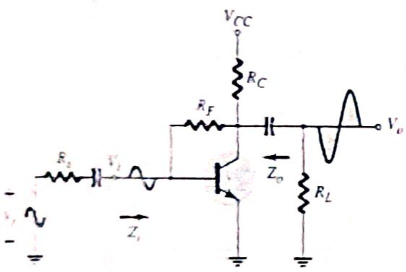
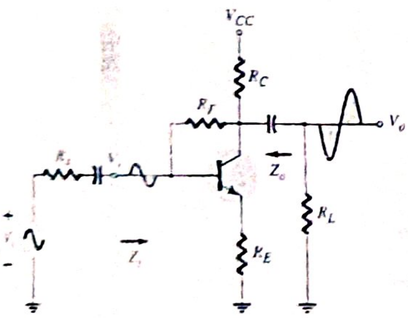
Configuration	$A_{v_L} = V_o/V_i$	Z_i	Z_o
	$\frac{-(R_L\|R_C)}{r_e}$	$R_B\|\beta r_e$	R_C
	Including r_o: $-\frac{(R_L\|R_C\|r_o)}{r_e}$	$R_B\|\beta r_e$	$R_C\|r_o$
	$\frac{-(R_L\|R_C)}{r_e}$	$R_1\|R_2\|\beta r_e$	R_C
	Including r_o: $\frac{-(R_L\|R_C\|r_o)}{r_e}$	$R_1\|R_2\|\beta r_e$	$R_C\|r_o$
 Emitter follower type	$\cong 1$	$R_E' = R_L\|R_E$ $R_1\|R_2\|\beta(r_e + R_E')$	$R_s' = R_s\|R_1\|R_2$ $R_E\|\left(\frac{R_s'}{\beta} + r_e\right)$
	Including r_o: $\cong 1$	$R_1\|R_2\|\beta(r_e + R_E')$	$R_E\|\left(\frac{R_s'}{\beta} + r_e\right)$
	$\cong \frac{-(R_L\|R_C)}{r_e}$	$R_E\|r_e$	R_C
	Including r_o: $\cong \frac{-(R_L\|R_C\|r_o)}{r_e}$	$R_E\|r_e$	$R_C\|r_o$
	$\frac{-(R_L\|R_C)}{R_E}$	$R_1\|R_2\|\beta(r_e + R_E)$	R_C
	Including r_o: $\frac{-(R_L\|R_C)}{R_E}$	$R_1\|R_2\|\beta(r_e + R_E)$	$\cong R_C$

TABLE 5.2 *(Continued)*

BJT Transistor Amplifiers Including the Effect of R_s and R_L

Configuration	$A_{v_L} = V_o/V_i$	Z_i	Z_o
	$\frac{-(R_L \| R_C)}{R_{E_1}}$	$R_B \| \beta(r_e + R_{E_1})$	R_C
	Including r_o: $\frac{-(R_L \| R_C)}{R_{E_1}}$	$R_B \| \beta(r_e + R_E)$	$\cong R_C$
	$\frac{-(R_L \| R_C)}{r_e}$	$\beta r_e \| \frac{R_F}{\|A_v\|}$	R_C
	Including r_o: $\frac{-(R_L \| R_C \| r_o)}{r_e}$	$\beta r_e \| \frac{R_F}{\|A_v\|}$	$R_C \| R_F \| r_o$
	$\frac{-(R_L \| R_C)}{R_E}$	$\beta R_E \| \frac{R_F}{\|A_v\|}$	$\cong R_C \| R_F$
	Including r_o: $\cong \frac{-(R_L \| R_C)}{R_E}$	$\cong \beta R_E \| \frac{R_F}{\|A_v\|}$	$\cong R_C \| R_F$

Cascaded Systems:-

Two-Port System Approach :-

This approach plays an important role in the design of today's systems where the designer works with packaged products rather than individual elements.

In otherwords, a particular package may house an amplifier with all the components appearing in the no-load version of configuration. Along with that package are the gain, i/p and o/p impedances. $[A_{V_{NL}}, Z_i, Z_o]$

Ex: Two port system.

If we take a "Thevenin look", at the output terminals, we find, with V_i set to Zero,

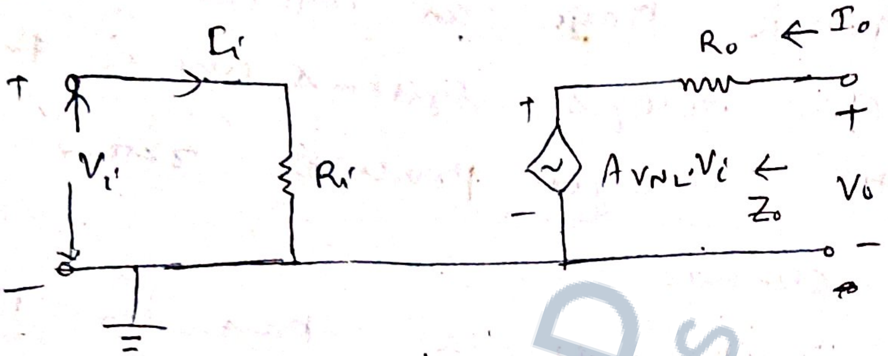
$$\boxed{Z_{Th} = Z_o = R_o}$$

E_{Th} is the open ckt voltage betn the o/p terminals, identified as 'V_o'.

$$A_{V_{NL}} = \frac{V_o}{V_i} \Rightarrow V_o = A_{V_{NL}} \cdot V_i$$

$$\Rightarrow \boxed{E_{Th} = A_{V_{NL}} \cdot V_i}$$

Substituting the Thevenin equivalent ckt betn the o/p terminals results in the o/p configuration



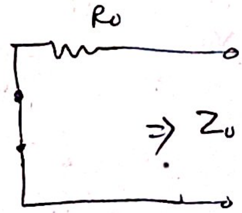
In Thevenin equivalent, voltage source with series resistor R_o.

$Z_o = R_o$ & $Z_i = R_i$

For input ckt, the parameters V_i & I_i are related by $\boxed{Z_i = R_i}$, permitting the use of R_i to represent i/p ckt.

Z_o :- To find Z_o, set $V_i = 0$, &

$\therefore A_{VNL} \cdot V_i = 0$, permitting a short-ckt for the source



$\Rightarrow Z_o = R_o$

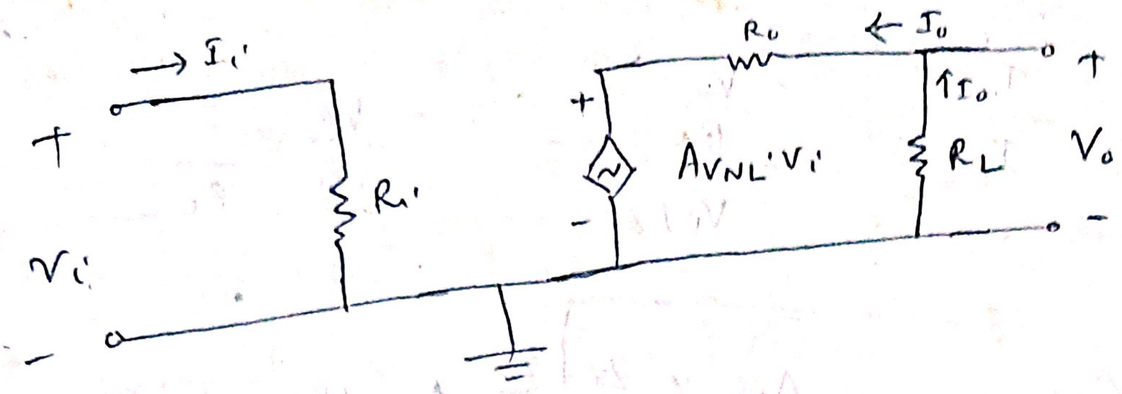
$\therefore \boxed{Z_o = R_o}$

In the absence of load, open ckted, $I_o = 0$, $I_o R_o = 0$.

$\therefore$ open ckt voltage $= A_{VNL} \cdot V_i$

$$\boxed{V_o = A_{VNL} V_i}$$

Applying load to the 2-port system, shown in fig 2. Applying voltage-divider rule



$$V_o = \frac{A_{VNL} \cdot V_i'}{R_o + R_L} \times R_L$$

($\because$ Applying KVL, in the o/p loop

$$-I_o R_o - A_{VNL} \cdot V_i' - I_o R_L = 0$$

$$\Rightarrow I_o (R_o + R_L) = -A_{VNL} \cdot V_i'$$

$$\Rightarrow I_o = -\frac{A_{VNL} \cdot V_i'}{R_o + R_L}$$

$$V_o + I_o R_L = 0$$

$$\Rightarrow V_o = -I_o R_L = -\left(-\frac{A_{VNL} \cdot V_i'}{R_o + R_L}\right) \cdot R_L$$

$$\Rightarrow \boxed{V_o = \frac{A_{VNL} \cdot V_i' \times R_L}{R_o + R_L}}$$)

$$\boxed{A_{VL} = \frac{V_o}{V_i'} = \frac{A_{VNL} \cdot R_L}{R_o + R_L}}$$

$$\because \frac{R_L}{R_o + R_L} < 1$$

$$\Rightarrow \boxed{A_{VL} < A_{VNL}}$$

Verify (Fixed Bias)

$$A_{VNL} = -\frac{R_C}{r_e}$$

$$A_{VL} = -\frac{R_C}{r_e} \times \frac{R_L}{R_C + R_L}$$

$(\because R_o \simeq R_C)$ (Z_o)

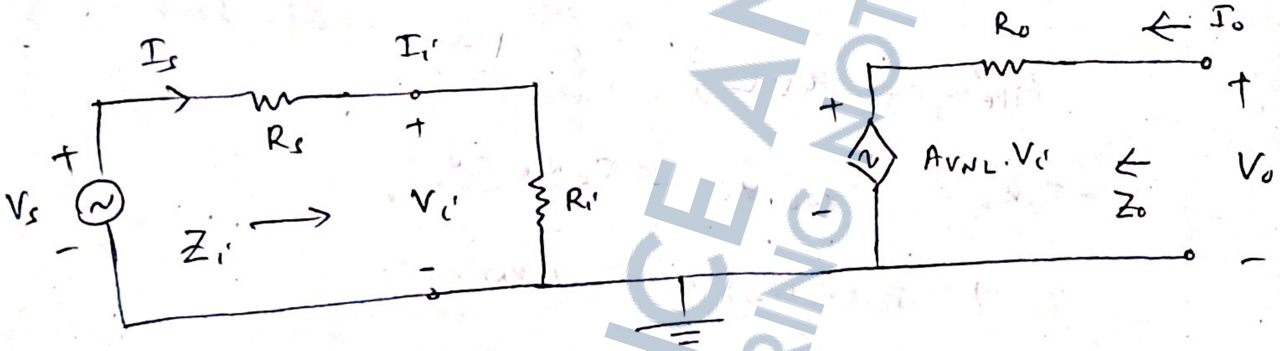
$$\Rightarrow -\frac{R_C \parallel R_L}{r_e}$$

$\therefore$ As derived earlier for effect of R_s & R_L

$$A_{iL} = \frac{I_o}{I_i} = \frac{-V_o / R_L}{V_i / Z_i} = -\frac{V_o}{V_i} \times \frac{Z_i}{R_L}$$

$$\boxed{A_{iL} = -A_{VL} \times \frac{Z_i}{R_L}} = -\frac{A_{VL} \times R_i}{R_L} \quad (\text{Here } Z_i = R_i)$$

Considering the effect of R_s (No R_L)



$$\boxed{V_i = \frac{V_s}{R_s + R_i} \times R_i} \quad \text{--- (1)}$$

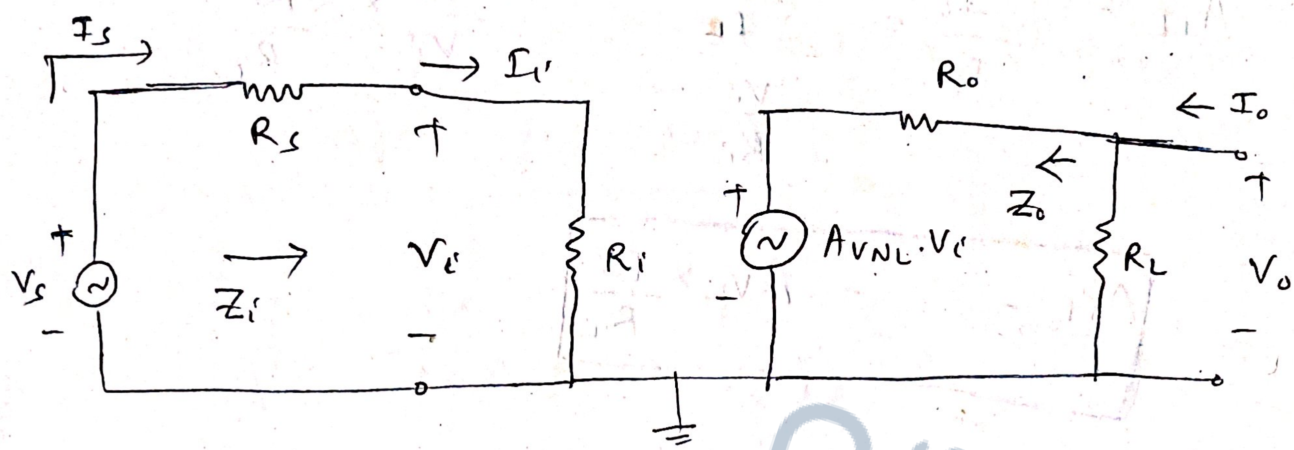
$$V_o = A_{VNL} \cdot V_i$$

$$= A_{VNL} \times \left(\frac{V_s \cdot R_i}{R_s + R_i} \right)$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} \cdot A_{VNL}$$

$$\Rightarrow \boxed{A_{Vs} = \frac{R_i}{R_i + R_s} \cdot A_{VNL}}$$

If both R_s & R_L present,



The total gain

$$A_{V_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$$

$$= A_{VL} \cdot \frac{R_i}{R_i + R_s}$$

$$\left[\because \quad \cancel{A_{VL}} = V_i = \frac{V_s}{R_s + R_i} \times R_i \right.$$

$$\left. \Rightarrow \frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} \right]$$

$$\Rightarrow A_{V_s} = \frac{R_L}{R_o + R_L} \cdot A_{VNL} \cdot \frac{R_i}{R_i + R_s}$$

$$\left(\because \quad A_{VL} = \frac{V_o}{V_i} = \frac{A_{VNL} \cdot R_L}{R_o + R_L} \right)$$

Imp

$$\boxed{A_{V_s} = \frac{R_i}{R_i + R_s} \cdot \frac{R_L}{R_L + R_o} \cdot A_{VNL}}$$

$$A_{iL}' = \frac{I_o}{I_i} = \frac{\frac{-V_o}{R_L}}{\frac{V_i'}{R_i'}} = \frac{-V_o}{V_i'} \times \frac{R_i'}{R_L}$$

$$\boxed{A_{iL}' = -A_{VL} \times \frac{R_i'}{R_L}}$$

$$A_{is}' = \frac{I_o}{I_s} = \frac{\frac{-V_o}{R_L}}{\frac{V_s}{R_s+R_i'}} = \frac{-V_o}{V_s} \times \frac{R_s+R_i'}{R_L}$$

$$\boxed{A_{is}' = -A_{Vs} \times \frac{R_s+R_i'}{R_L}}$$

Since $I_i' = I_s$ (Same current See the fig)

Mathematically

$I_i' = \frac{V_i'}{R_i'}$ — (2)

$I_s = \frac{V_s}{R_s+R_i'}$ — (3)

From eqn (1), Page 215

$\frac{V_i'}{R_i'} = \frac{V_s}{R_s+R_i'} \Rightarrow I_i' = I_s$

$$\Rightarrow \boxed{A_{iL}' = A_{is}' = \frac{I_o}{I_i'} = \frac{I_o}{I_s}}$$

Ex:-10 Determine A_{VL} & A_{Vs} using 2-port n/w method. (Ex-9, Page 205, this note)

Given $A_{VNL} = -280.11$, $Z_i = 1.07\,k\Omega$

$Z_o = 3\,k\Omega$, $R_L = 4.7\,k\Omega$, $R_s = 0.3\,k\Omega$

(a) $$A_{VL} = \frac{R_L}{R_L+R_o} \times A_{VNL}$$

$$\Rightarrow \quad A_{VL} = \frac{4.7}{4.7+3} \times (-280.11) \qquad (\because R_o = Z_o = 3\,k\Omega)$$

$$\boxed{A_{VL} = -170.98}$$

(b) $$A_{VS} = \frac{R_i}{R_i + R_s} \times \frac{R_L}{R_L + R_o} \times A_{VNL} \qquad \Big|\ R_i = Z_i$$

$$= \frac{1.07}{1.07 + 0.3} \times \frac{4.7}{4.7 + 3} \times (-280.11)$$

$$\boxed{A_{VS} = -133.45}$$

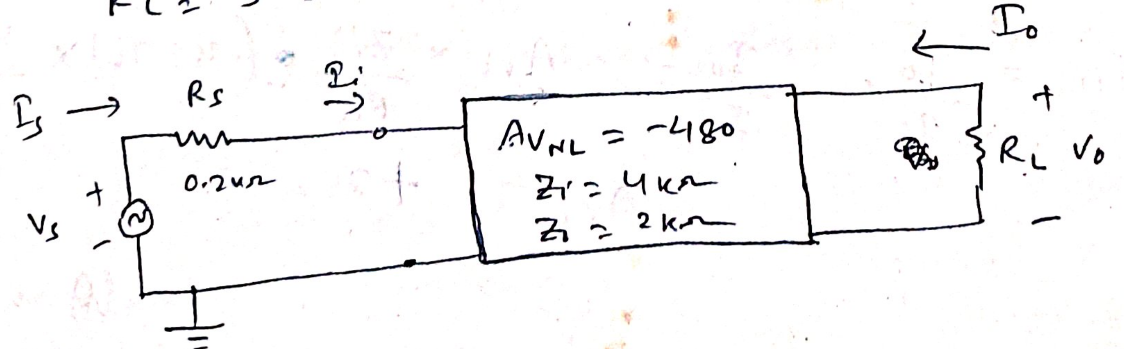
Ex-11) Given the packaged (no-entry-possible) amplifier.

(a) Determine the gain A_{VL} and compare it to the no-load value with $R_L = 1.2\,k\Omega$

(b) Repeat part (a) with $R_L = 5.6\,k\Omega$ and compare solutions.

(c) ~~Find~~ Determine A_{VS} with $R_L = 1.2\,k\Omega$

(d) Find the current gain $A_i = \frac{I_o}{I_i} = \frac{I_o}{I_s}$ with $R_L = 5.6\,k\Omega$.



(a) $$A_{VL} = \frac{R_L}{R_L + R_o} A_{VNL}$$

$$= \frac{1.2}{1.2+2} \times (-480) \quad \left(\because R_o = Z_o = 2k\Omega \right)$$

$$= -180$$

which is a dramatic drop from the no-load value.

(b) $R_L = 5.6\,k\Omega$

$$A_{VL} = \frac{5.6}{5.6+2} \times (-480)$$

$$= -353.76.$$

which clearly reveals that the larger the load resistance, better is the gain.

(c) $$A_{VS} = \frac{R_i}{R_i + R_s} \times \frac{R_L}{R_L + R_o} \cdot A_{VNL}$$

$R_i = Z_i = 4k\Omega$

$$= \frac{4}{4+0.2} \times \frac{1.2}{1.2+2} \times (-480)$$

$$= (0.952)(0.375) \times (-480)$$

$$= -171.36.$$

(d) $$A_{iL} = \frac{I_o}{I_i} = \frac{I_o}{I_s} = -A_{VL} \times \frac{Z_i}{R_L} = -(-353.76) \times \frac{4}{5.6}$$

$$A_{iL} = +252.6$$

(Ans.)

Cascaded System :

Two - Port systems approach is Particularly useful for Cascaded system, Shown in figure 1, where A_{v1}, A_{v2}, A_{v3} & so on, are the Voltage gains of each stage under loaded condⁿ.

Total gain of the system is determined by the product of individual gains

i.e

$$\boxed{A_{vT} = A_{v1} \cdot A_{v2} \cdot A_{v3} \cdot \text{-----}}$$

and total Current gain is given by,

$$\boxed{A_{iT} = - A_{vT} \cdot \frac{Z_{i1}}{R_L}}$$

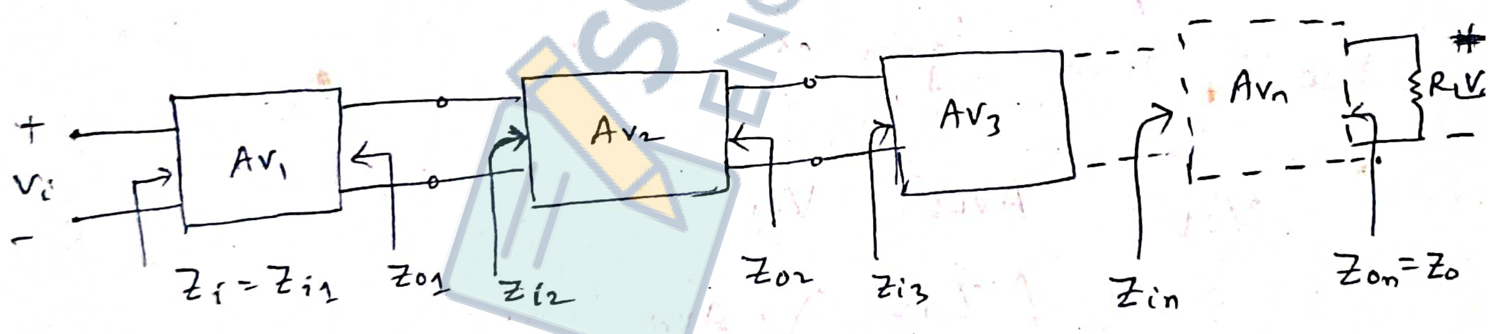


fig:- Cascaded System

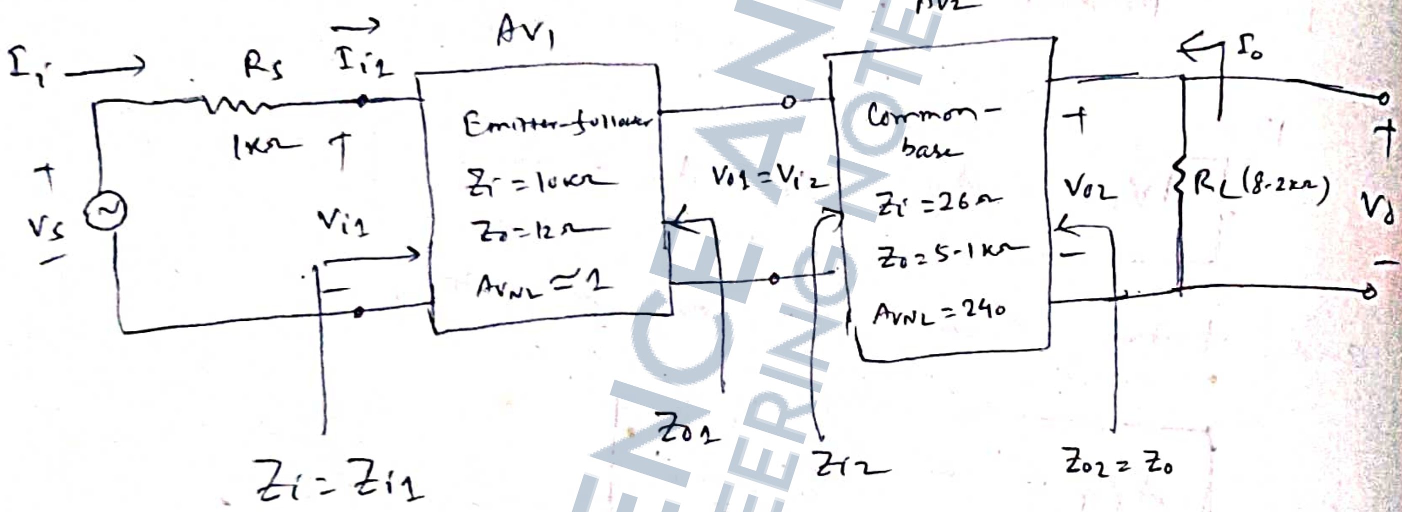
Ex :- 12. The two-stage system, shown, employs a transistor emitter-follower configuration prior to a common base configuration to ensure that the maximum percentage of applied signal appears at the i/p terminal of common base amplifier. Determine

(a) The loaded gain for each stage

(b) The total gain for the system, A_{V_T} & A_{V_S}

(c) The total current gain for the system.

(d) The total gain for the system if the emitter-follower configuration were removed.



Ans: (a) For the emitter-follower configuration, the load gain is.

$$V_{o1} = A_{VL1} \cdot V_{i1}$$

$$= \left(\frac{Z_{i2}}{Z_{i2} + Z_{o1}}\right) \cdot A_{VNL} \cdot V_{i1}$$

$$\left(\because \quad A_{VL} = \frac{R_L}{R_L + R_o} \cdot A_{VNL}\right)$$

$$V_{o1} = \frac{26}{26+12} \cdot (1) \cdot V_{i1}$$

$$\Rightarrow \frac{V_{o1}}{V_{i2}} = \frac{26}{38} = 0.684$$

For Common-base Configuration,

$$V_{o2} = A_{vL2} \cdot V_{i2}$$

$$= \left[\frac{R_L}{R_L+R_{o2}} \cdot A_{vNL}\right] \cdot V_{i2}$$

$$V_{o2} = \frac{8.2}{8.2+5.1} \times 240 \times V_{i2}$$

$$\Rightarrow A_{v2} = \frac{V_{o2}}{V_{i2}} = \frac{8.2 \times 240}{8.2+5.1} = 147.97$$

(b) Total gain,

$$A_{vT} = A_{v1} \times A_{v2}$$

$$= (0.684) \cdot (147.97)$$

$$\boxed{A_{vT} = 101.20}$$

$$A_{vs} = \frac{Z_{i1}}{Z_{i1}+R_s} \cdot A_{vT}$$

$$= \frac{10 \times 101.20}{10+1}$$

$$\boxed{A_{vs} = 92}$$

or $A_{is} = -A_{vs} \times \frac{Z_i+R_s}{R_L}$

$= -92 \times \frac{10+1}{8.2}$

$= -123.41$ (Ans)

(c) Total current gain, $A_{iT} = -A_{vT} \cdot \frac{Z_i}{R_L}$

$$= -(101.20) \cdot \frac{10}{8.2}$$

$$A_{iT} = -123.41$$

(d) If emitter-follower configuration is removed,

$$V_i = \frac{Z_{iCB}}{Z_{iCB} + R_s} \cdot V_s = \frac{26}{26+1k} \times V_s$$

CB = Common-Base

or

$$A_{VS} = \frac{R_L}{R_L+R_o} \times \frac{Z_i}{Z_i+R_s} \times A_{VNL}$$
$$= \frac{8.2}{8.2+5.1} \times \frac{0.026}{0.02691} \times 240$$
$$= 3.74$$

$$\Rightarrow \frac{V_i}{V_s} = 0.025$$

$$A_{V2} = 149.97 = \frac{V_o}{V_i}$$

$$A_{VS} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_i}{V_s} = 149.97 \times 0.025 = 3.74$$

$\therefore$ A_{VS} with emitter follower = 92
A_{VS} without emitter follower = 3.74

gain 25 times greater.

R-C Coupled BJT amplifier

The name is derived from the capacitive coupling ~~coupling~~ capacitor C_c and the fact that the load on the first stage is an RC combination. The coupling capacitor isolates the two stages from a d.c view point but acts as a short-ckt equivalent for ac response. The i/p impedance of the second stage act as a load on the first stage.

Ex:- 13

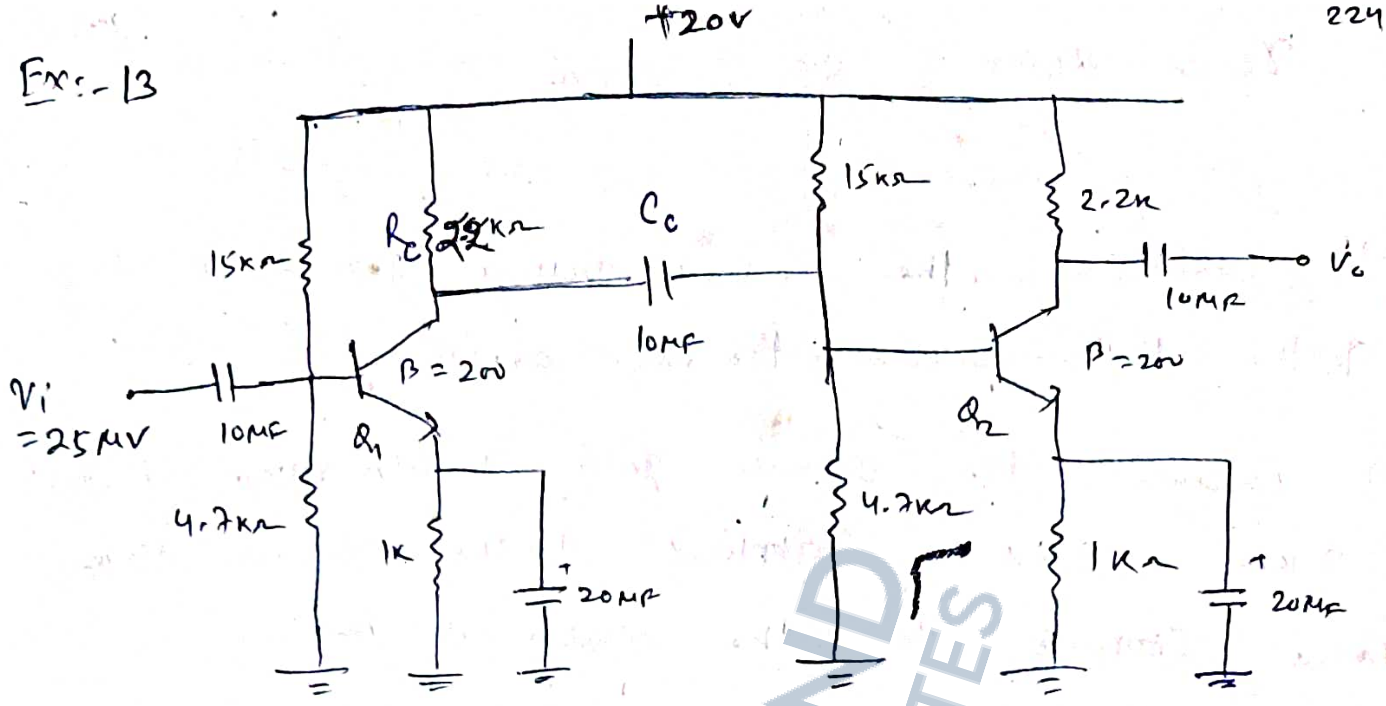


fig:- RC - Coupled BJT amplifier example.

Sol:

D.C bias analysis for each transistor

(Both transistor Configuration are exactly similar)

Since $\beta R_E > 10 R_2$

$$200 \times 1K > 10 \times 4.7K$$

$$200K > 47K$$

$$V_B = \frac{20}{15+4.7} \times 4.7$$

$\boxed{V_B = 4.7}$

$V_{BE} = V_B - V_E \Rightarrow 0.7 = 4.7 - V_E \Rightarrow \boxed{V_E = 4V}$

$$I_E R_E = 4V$$

$\Rightarrow I_E = \frac{4}{1K} = 4mA. \Rightarrow \boxed{I_E = 4mA}$

$V_C = V_{CC} - I_C R_C = 20 - 4 \times 2.2 = 11.2$ volt

$$r_e = \frac{26mV}{I_E} = \frac{26}{4} = 6.5\Omega$$

(a) Calculate the no-load voltage gain & o/p voltage of the R-C Coupled transistor amplifier

(b) Calculate the overall gain & o/p voltage of a 4.7 kΩ load is applied to the second stage and compare to the results of part (a)

(c) Calculate the i/p impedance of the first stage and o/p impedance of the second stage.

(a) The loading of the second stage is

$$Z_{i2} = R_1 \parallel R_2 \parallel \beta r_e$$

$$= (15 \parallel 4.7 \parallel 1.3)\,k\Omega \qquad \left(\beta r_e = 200 \times 6.5\right)$$

Gain of the first stage $= -\dfrac{R_C \parallel \cancel{R_o}\, R_L}{r_e}$

$$= -\frac{R_C \parallel Z_{i2}}{r_e}$$

$$= -\frac{2.2 \parallel (15 \parallel 4.7 \parallel 1.3)\,k\Omega}{6.5}$$

$$= -\frac{665.2\,\Omega}{6.5}$$

$$A_{v1} = -102.3$$

For the unloaded second stage, the gain is

$$A_{V2}(NL) = \frac{-R_C}{r_e} = \frac{-2.2\,k\Omega}{6.5} = -338.46.$$

Overall gain $= A_{V1} \times A_{V2}(NL)$

$= -102.3 \times -338.46$

$A_{VT} = 34.6 \times 10^3$

The o/p voltage is then,

$V_o = A_{VT} \times V_i$

$= 34.6 \times 10^3 \times 25\ \mu V$

$$\boxed{V_o = 865\ mV}$$

(b) The overall gain with $4.7\,k\Omega$ load,

$$A_{VT} = \frac{R_L}{R_L + R_o} \times A_{VNL} = \frac{4.7}{4.7 + 2.2} \times 34.6 \times 10^3$$

$$\boxed{A_{VT} = 23.6 \times 10^3}$$

$V_o = A_{VT} \times V_i$

$= 23.6 \times 10^3 \times 25 \times 10^{-6}$

$$\boxed{V_o = 590\ mV}$$

(c) $Z_{i1} = R_1 \,//\, R_2 \,//\, \beta r_e = (15 \,//\, 4.7 \,//\, 1.3)\,k\Omega$

$= 953.6\ \Omega$.

$Z_{o2} = R_C = 2.2\,k\Omega$

Darlington Connection :-

A very popular connection of two bipolar junction transistor for operation as one 'Super-beta' transistor is the Darlington connection shown below.

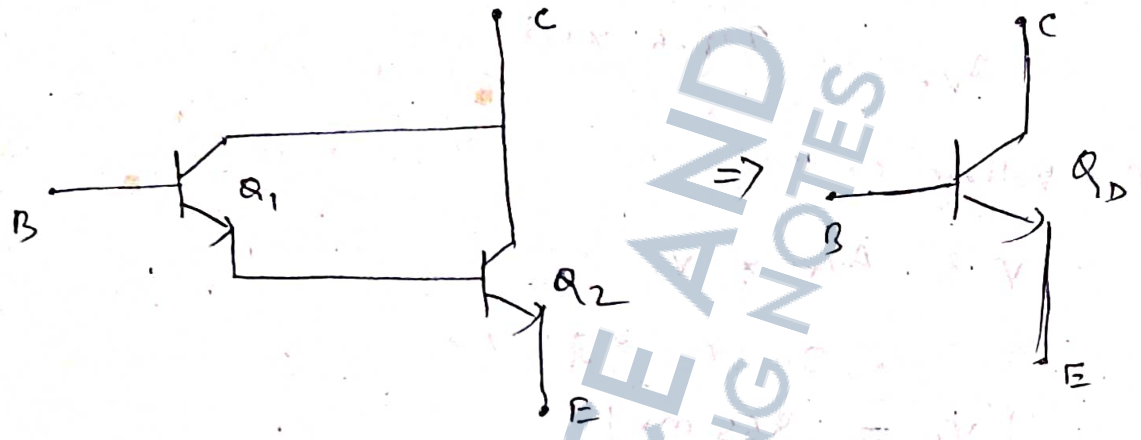


fig:- Darlington Combination

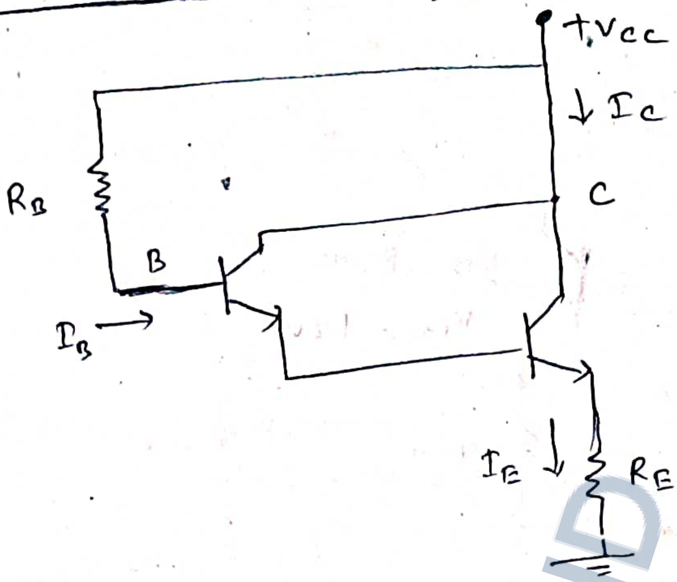
The main feature of Darlington connection is that the composite transistor acts as a single unit with a current gain that is the product of the current gains of the individual transistors.

If the connection is made using 2 separate transistors having current gains of β_1 & β_2, the Darlington connection provides a current gain of

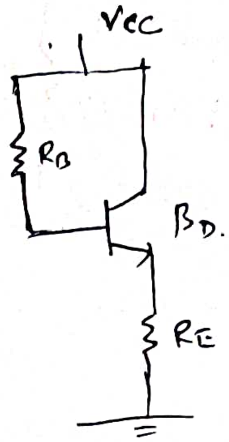
$$\boxed{\beta_D = \beta_1 . \beta_2}$$

If the two transistors are matched, so that $\beta_1 = \beta_2 = \beta$, the Darlington connection provides a current gain of $\boxed{\beta_D = \beta^2}$

D.C Bias of Darlington Circuit:-



$\Rightarrow$



A Darlington transistor having very high current gain β_D is used.

Applying KVL,

$$V_{CC} - I_B R_B - V_{BE} - \beta_D I_B R_E = 0$$

$$\Rightarrow \quad I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E} \quad \text{——— (1)}$$

Here β_D & V_{BE} are much greater.

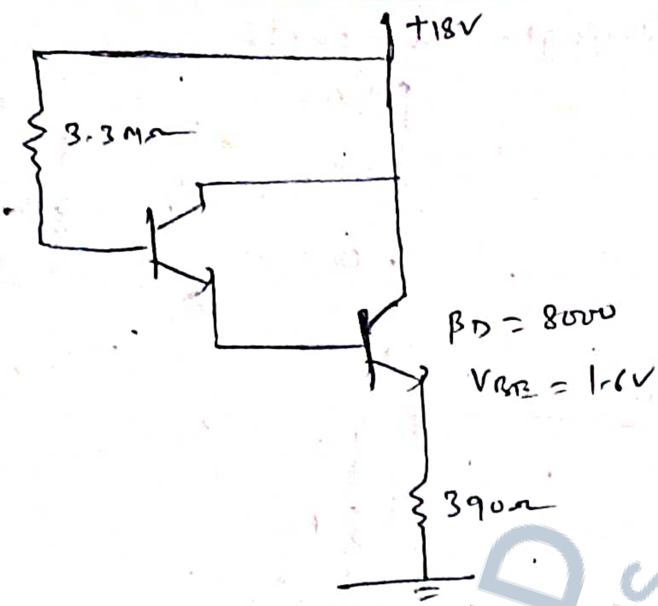
$(\beta_D = \beta_1 \cdot \beta_2, \quad V_{BE} = 1.6\,V)$

$$I_E \simeq (\beta_D + 1) I_B \simeq \beta_D I_B \quad \text{——— (2)}$$

$$V_E = I_E R_E \quad \text{——— (3)}$$

$$V_B = V_E + V_{BE} \quad \text{——— (4)}$$

Ex-14 :- Calculate the bias voltages of current shown in figure below.



Ans :-

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E} = \frac{18 - 1.6}{3.3 \times 10^6 + 8000 \times 390}$$

$$= \frac{16.4}{6.42 \times 10^6}$$

$$\boxed{I_B = 2.55\ \mu A}$$

$$I_E \simeq \beta_D I_B = 8000 \times 2.55 \times 10^{-6} = 20.4\ mA.$$

$$V_E = I_E R_E = 20.4 \times 10^{-3} \times .390 \times 10^3 = 7.96\ \text{volt}$$

$$V_B = V_E + V_{BE} = 7.96 + 1.6 = 9.56\ \text{volt}.$$

V_C = supply voltage = 18V.

AC Equivalent ckt:-

A ~~Darlington~~ Darlington emitter-follower ckt is shown. The ~~...~~ ac input signal is applied to the base of Darlington transistor through capacitor C_1, with the ac o/p V_o obtained from the emitter through capacitor C_2. Due to the absence of a load R_L, the o/p current I_o is defined through R_E.

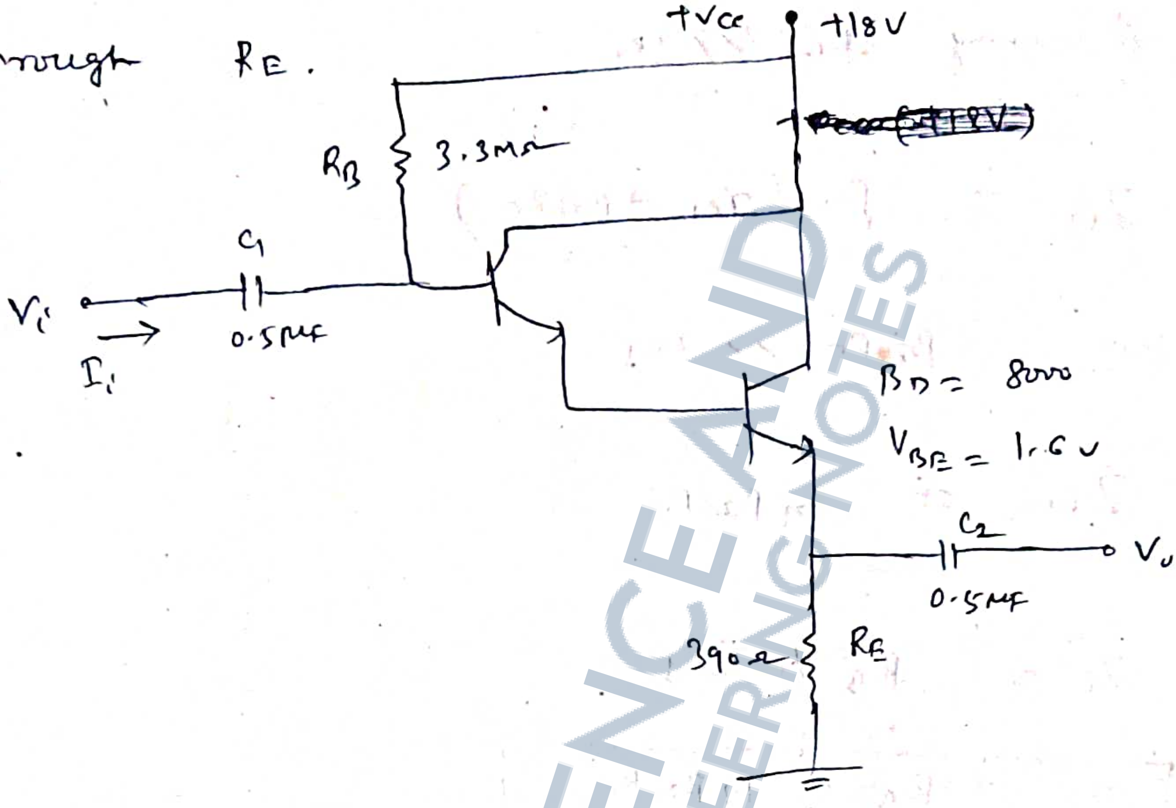
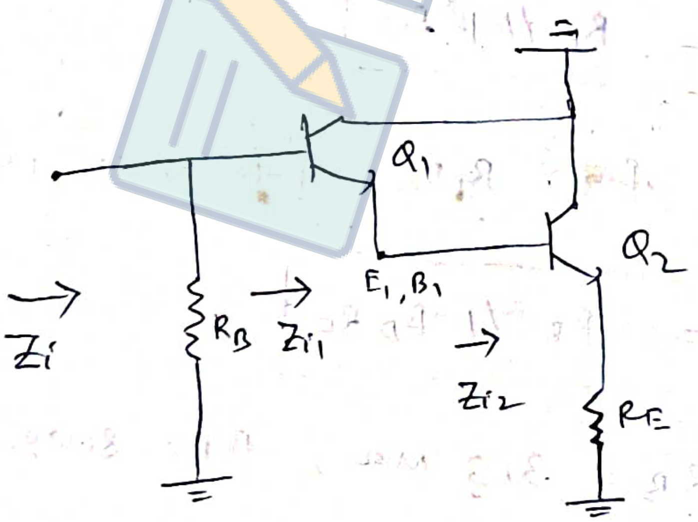


fig:- Darlington emitter-follower circuit.

I/P impedance



[∵ emitter-follower $Z_i = \beta(r_e + R_E)$]

$$Z_{i2} = \beta_2 (r_{e2} + R_E) \quad \text{—(1)}$$

$$Z_{i1} = \beta_1 (r_{e1} + Z_{i2}) \quad \text{—(2)}$$

∴ Putting eqn ①, in eqn ②, we have

$$Z_{i1} = \beta_1 (r_{e1} + \beta_2 (r_{e2} + R_E))$$

Assuming $R_E >> r_{e2}$

$$Z_{i1} = \beta_1 (r_{e1} + \beta_2 R_E)$$

Again $\beta_2 R_E >> r_{e1}$

$$Z_{i1} \simeq \beta_1 \beta_2 R_E$$

AND $Z_i = R_B \,//\, Z_{i1}$

$$\boxed{Z_i = R_B \,//\, \beta_1 \beta_2 R_E}$$

For $\beta_1 = \beta_2 = \beta$,

$$\boxed{Z_i = R_B \,//\, \beta^2 R_E}$$

or if $\beta_D = \beta_1 \beta_2$ [from specification sheet]

$$\boxed{Z_i = R_B \,//\, \beta_D R_E}$$

Ex: If $R_B = 3.3\ M\Omega$, $\beta_D = 8000$, $R_E = 390\Omega$

$$Z_i = 3.3\ M\Omega \,//\, (8000 \times 390)$$
$$= 3.3\ M\Omega \,//\, 3.12\ M\Omega = 1.6\ M\Omega$$

(Ans)

* Current gain

$$\boxed{A_i = \frac{I_o}{I_i} \simeq \frac{\beta_1 \beta_2 R_B}{R_B + \beta_1 \beta_2 \cdot R_E}}$$

or

$$\boxed{A_i = \frac{\beta_D \cdot R_B}{R_B + \beta_D \cdot R_E}}$$

* Voltage gain :-

$$A_v = \left[\frac{\beta_D \cdot R_B}{R_B + \beta_D \cdot R_E}\right] \cdot \left[\frac{\frac{R_E}{\beta_D R_B R_E}}{R_B + \beta_D R_E}\right]$$

$A_v \simeq 1$ (in reality less than 1)

* Output Impedance

$$\boxed{Z_o = \frac{r_{e1}}{\beta_2} + r_{e2}}$$

* → Extra (Not required) -
  → Derivation in Boylestad.

Current Mirror Circuits :-

A current mirror circuit provides a constant current and used primarily in integrated ckts. (IC) The constant current is obtained from an o/p current, which is the reflection or mirror of a constant current developed on one side of the ckt.

The ckt is particularly suited to IC manufacture because the ckt requires that the transistors used have identical base-emitter voltage (V_{BE}) drops and identical values of beta (β) results best achieved when transistors are formed at the same time in IC manufacture.

In fig 1, the current I_x set by transistor Q_1 and resistor R_x is mirrored in the current I through transistor Q_2.

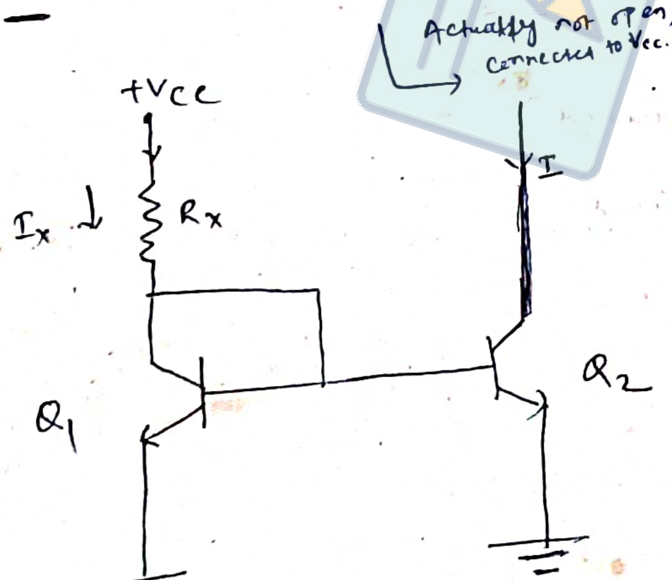


fig 1:- Current Mirror ckt.

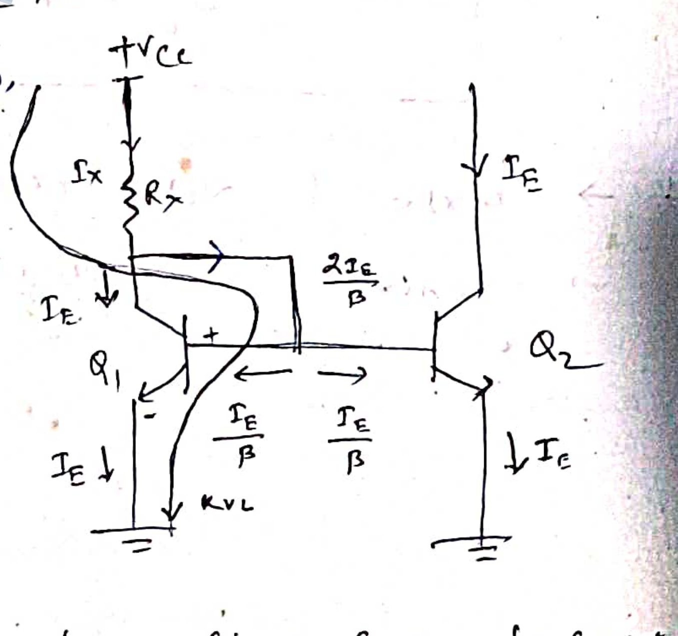


fig 2:- Circuit currents for current mirror circuit.

The current I_x and I can be obtained using the circuit currents listed in fig 2 -

We assume that the emitter current (I_E) for both ~~the~~ transistors is the same (Q_1 & Q_2 being fabricated near each other on the same chip)

The two transistor base currents are then approximately,

$$I_B = \frac{I_E}{\beta+1} \simeq \frac{I_E}{\beta}$$

The collector current of each transistor is then $I_C \simeq I_E$

Finally, the current I_x through resistor R_x is

$$I_x = I_E + \frac{2I_E}{\beta} = \frac{\beta I_E + 2I_E}{\beta} = \frac{(\beta+2) I_E}{\beta} \simeq I_E$$

$(\because \beta+2 \simeq \beta)$

Applying a KVL

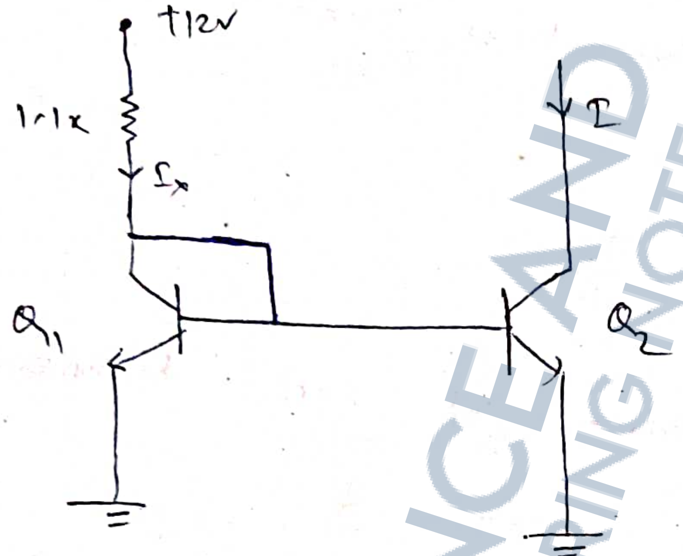
$$V_{CC} - I_x R_x - V_{BE} = 0$$

$$\Rightarrow \boxed{I_x = \frac{V_{CC} - V_{BE}}{R_x} = I_E}$$

$\therefore$ The constant current provided at the collector of Q_2 mirrors that of Q_1. The transistor

Q_1 is referred to as a diode-connected transistor because the base & collector are shorted together.

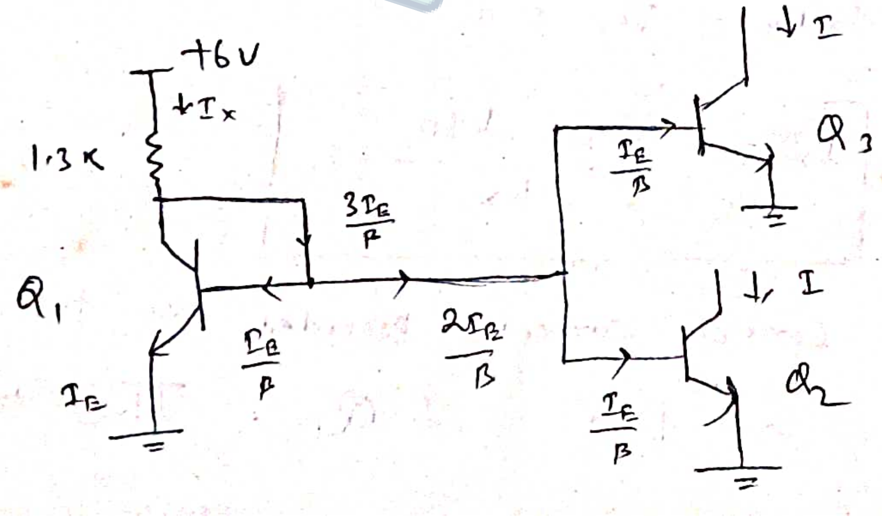
Ex:-15 Calculate the mirrored current I in the ckt shown below.



$$I = I_x = \frac{V_{CC} - V_{BE}}{R_x} = \frac{12 - 0.7}{1.1} = \frac{11.3}{1.1 \times 10^3}$$

$$\therefore \boxed{I_x = I = 10.27\ mA}$$

Ex-16:- Calculate the current I through each of the transistor Q_2 & Q_3 in the ckt, shown below



Sol^n :-

$$I_x = I_E + \frac{3I_E}{\beta} = \frac{\beta I_E + 3I_E}{\beta}$$

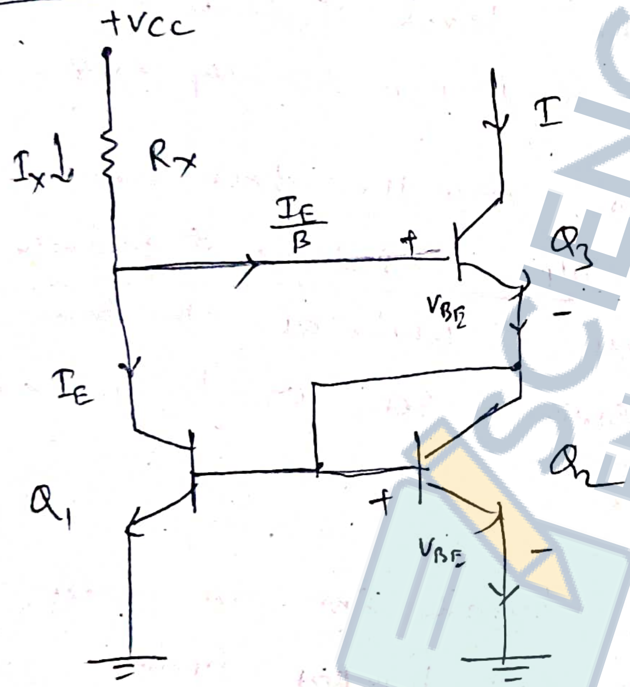
$$= \frac{(\beta+3)\, I_E}{\beta}$$

$$I_x \simeq I_E \qquad (\because \beta+3 \simeq \beta)$$

$$\therefore I = I_x = \frac{V_{CC} - V_{BE}}{R_x} = \frac{6 - 0.7}{1.3 \times 10^3}$$

$$\boxed{I = 4.08\ \text{mA}}$$

*Extra :- (Not required)



Current Mirror Ckt
With higher output impedance

$$I_x = \frac{V_{CC} - 2V_{BE}}{R_x} = \frac{I_E}{} + \frac{I_E}{\beta}$$

$$= \frac{\beta I_E + I_E}{\beta} = \frac{(\beta+1)\, I_E}{\beta}$$

$$\boxed{I_x \simeq I_E}$$

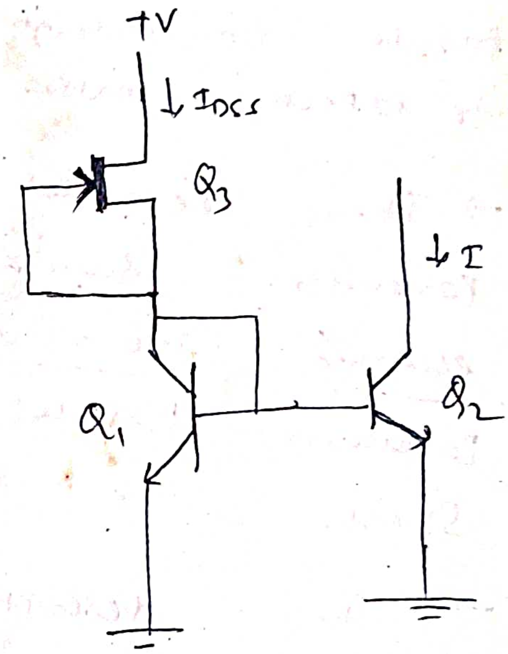


fig:- Current mirror Connection using a JFET & 2 transistor.

$$\boxed{I_{DSS} = I}$$

The Hybrid equivalent model :-

⇒ It was used in the yearly years before the popularity of the r_e model developed.

→ The 'r_e' model has the advantage that the parameters are defined by the actual operating conditions where as the parameters of the hybrid equivalent circuit are defined by general terms for any operating conditions.

→ In otherwords, the hybrid parameters may not reflect the actual operating conditions but simply provide an indication of the level of each parameter to expect, no matter what conditions actually exist.

→ The r_e model suffers from the fact that parameters such as the o/p impedance & feedback elements are n't available, where as the hybrid parameters provide the entire set on the specification sheet.

→ The description of hybrid equivalent model will begin with the general two-port system.

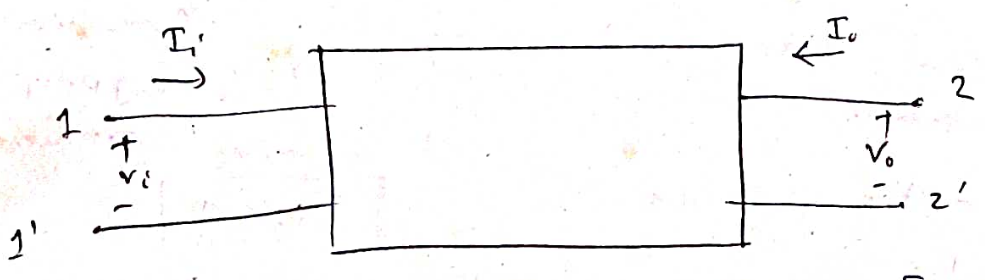


fig:- Two port system $\begin{bmatrix} V_i \\ I_o \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_i \\ V_o \end{bmatrix}$

i/p voltage & o/p current in terms of i/p current & o/p voltage

$$V_i = h_{11} I_i + h_{12} V_o \quad \text{--- (1)}$$

$$I_o = h_{21} I_i + h_{22} V_o \quad \text{--- (2)}$$

The parameters relating the four variables are called h-parameters, from the word "hybrid".

The term hybrid was chosen because the mixture of variable (V and I) in each equation results in a "hybrid" set of units of measurement for h-parameters.

From eqn (1),

If $V_o = 0$, (short ckt the o/p terminal)

$$\boxed{h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o=0}} \text{ Ohms.}$$

h_{11} = Short-ckt input impedance parameter.

If I_i is set to zero, by opening i/p leads,

$$\boxed{h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i=0}} \text{ unitless.}$$

h_{12} = Open-ckt reverse transfer voltage ratio.

Similarly,

If V_o is set equal to zero,

$$\boxed{h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0}} \text{ unitless}$$

h_{21} = ~~Forward~~ Short ckt forward transfer current ratio parameter.

The last parameter, h_{22} can be found out setting $I_i = 0$,

$$\boxed{h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0}}$$ Siemens.

h_{22} = Open ckt o/p admittance parameter.

Trick to remember.

$\boxed{i\ r\ f\ o \quad i- \quad r- \quad f- \quad o-}$

In Summary,

h_{11} = input resistance = h_i

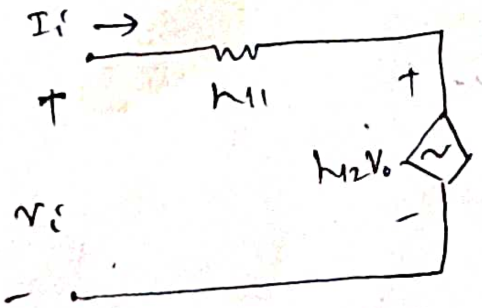
h_{12} = reverse transfer voltage ratio = h_r

h_{21} = forward transfer current ratio = h_f

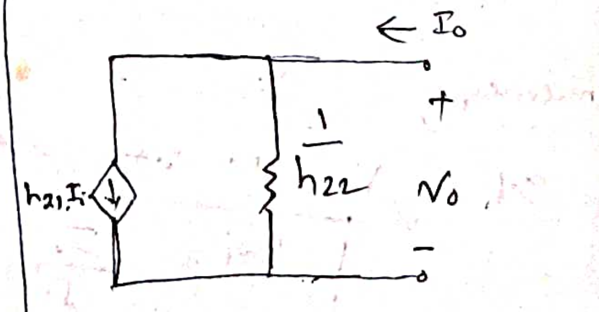
h_{22} = Output conductance = h_o

For Common-Emitter (CE)	CB	CC
h_{ie}	h_{ib}	h_{ic}
h_{re}	h_{rb}	h_{rc}
h_{fe}	h_{fb}	h_{fc}
h_{oe}	h_{ob}	h_{oc}

Hybrid i/p equivalent



Hybrid o/p equivalent



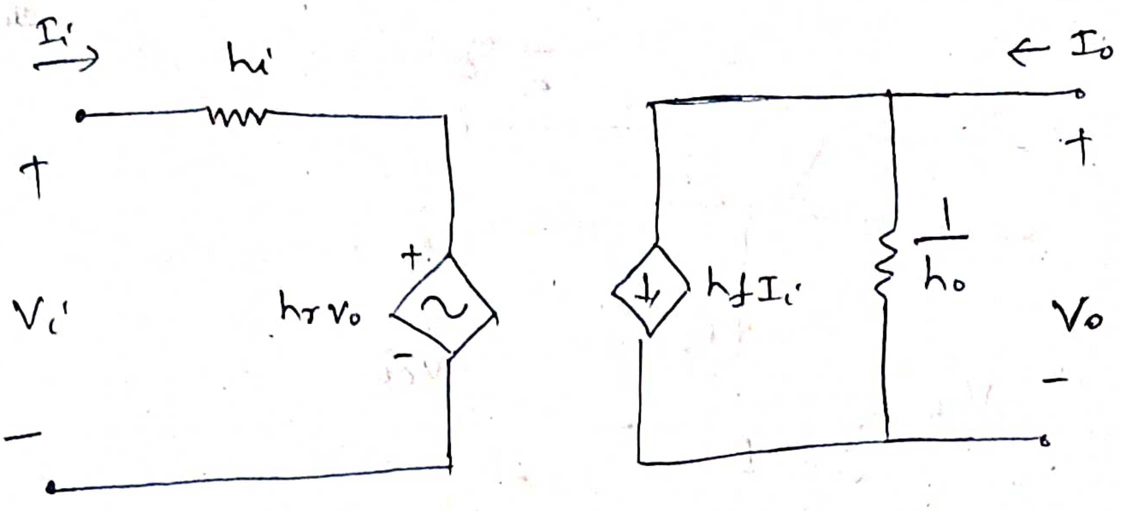
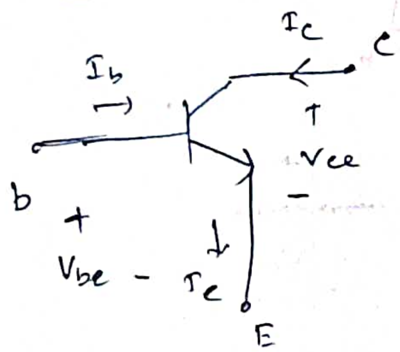
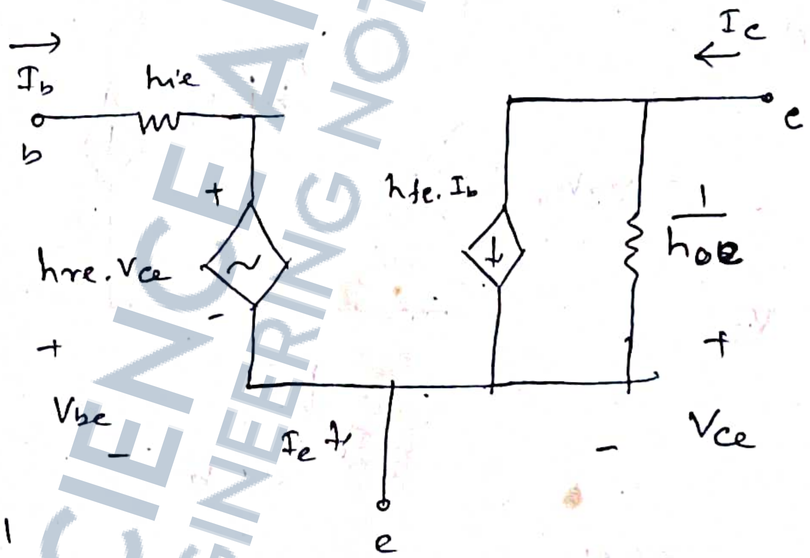


fig:- Complete hybrid equivalent circuit.

CE :-





CE - (a) Graphical Symbol

(b) Hybrid equivalent model.

Here, $I_i = I_b$, $I_o = I_c$

$I_e = I_b + I_c$

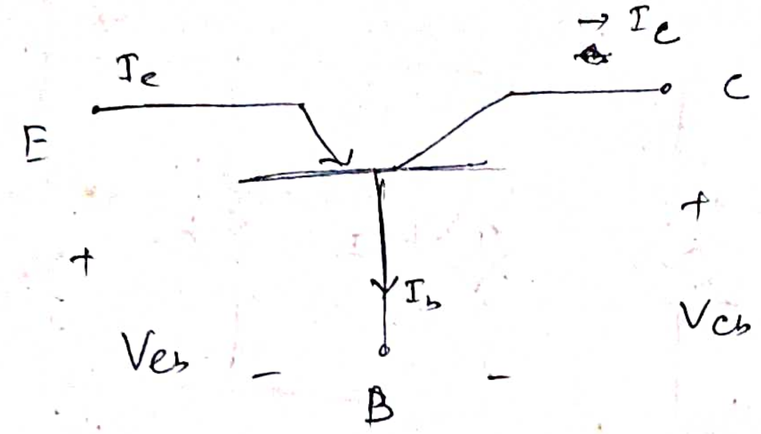
I/p Voltage $= V_{be}$, O/p Voltage $= V_{ce}$.

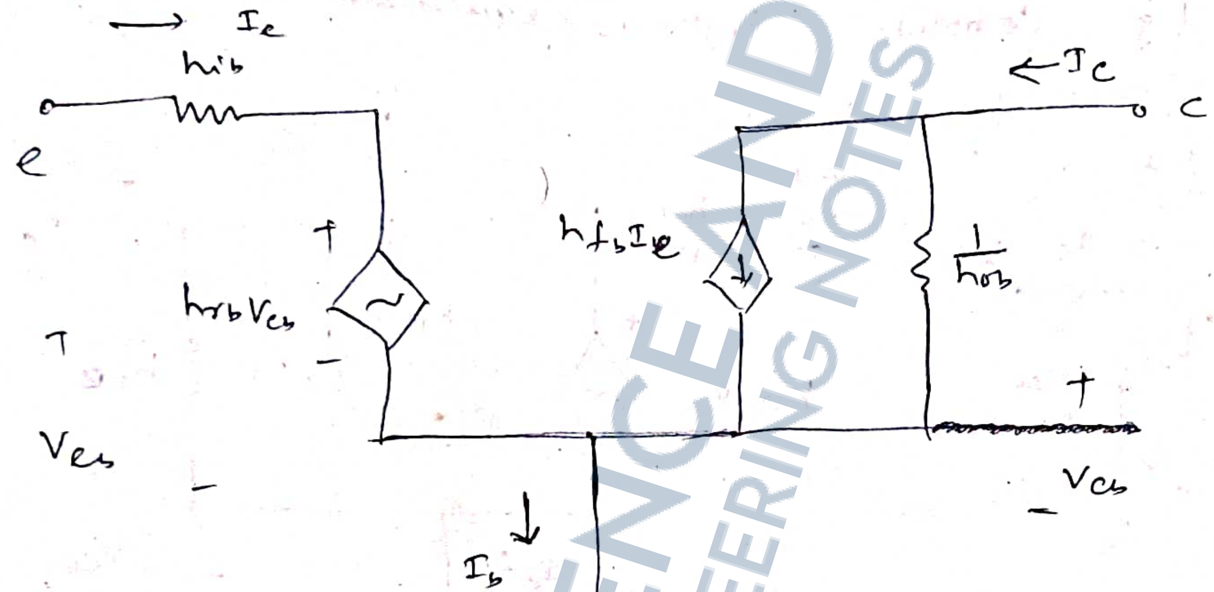
For Common-base :-

$I_i = I_e$, $I_o = I_c$, with $V_{eb} = V_i$ and

$V_{cb} = V_o$

CB





Here actual I_e, ~~in~~ direction should be outward or up, but in the two port-system or hybrid model I_o is shown into the system, i.e. I_e is shown inward. why

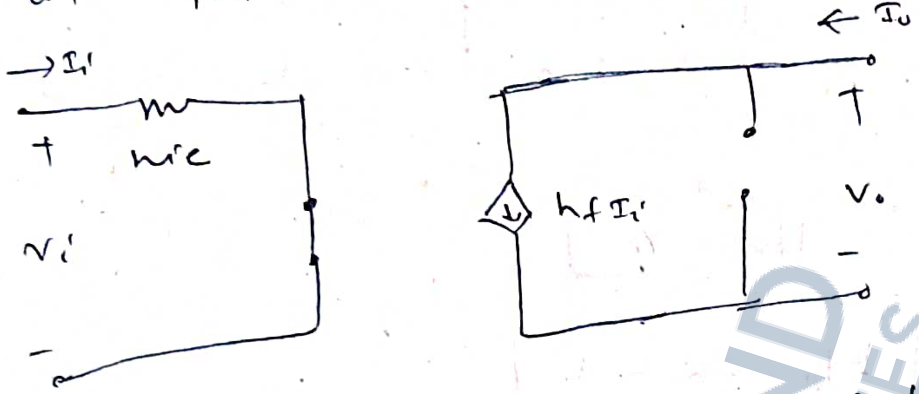
So h_{fb} will ~~also~~ automatically come $-ve$.

Note :- h_r is relatively small quantity.

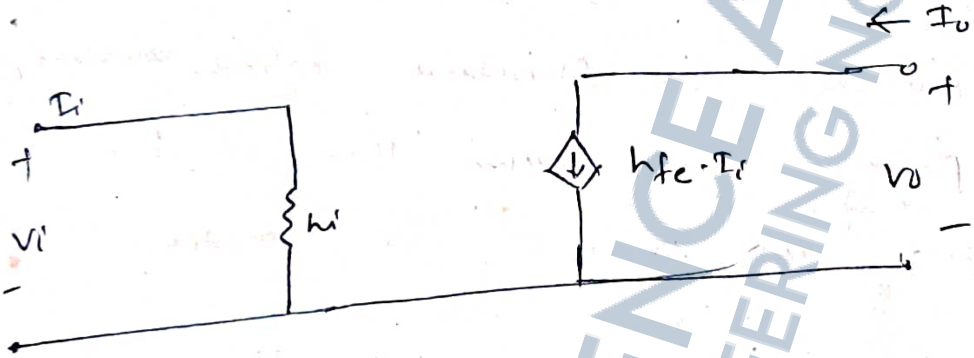
$h_r \simeq 0$

$h_r v_o \simeq 0$, resulting a short-ckt equivalent.

Since resistance determined by $\frac{1}{h_o}$ is often too large enough to be ignored, permitting by an open-circuit equivalent,

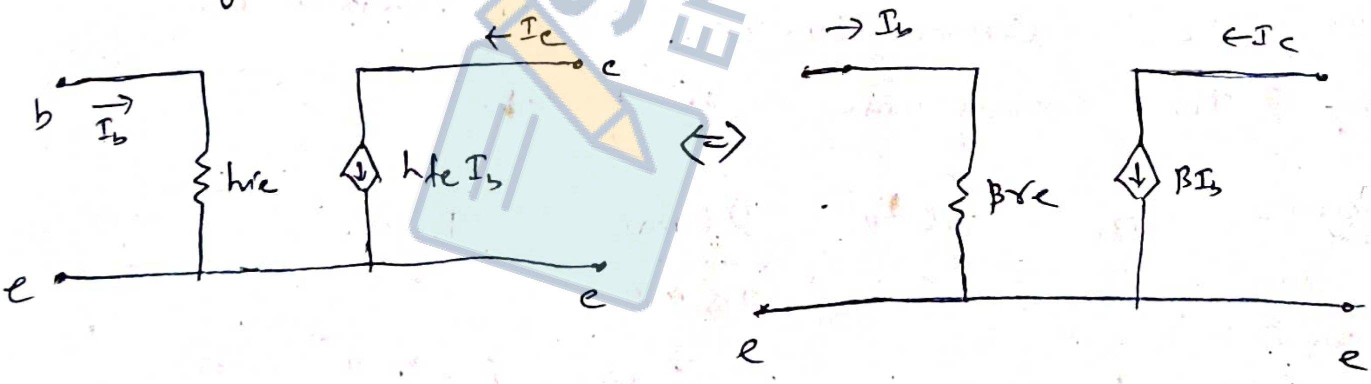


Effect of removing h_{re} & h_{oe}.



Approximate hybrid equivalent model.

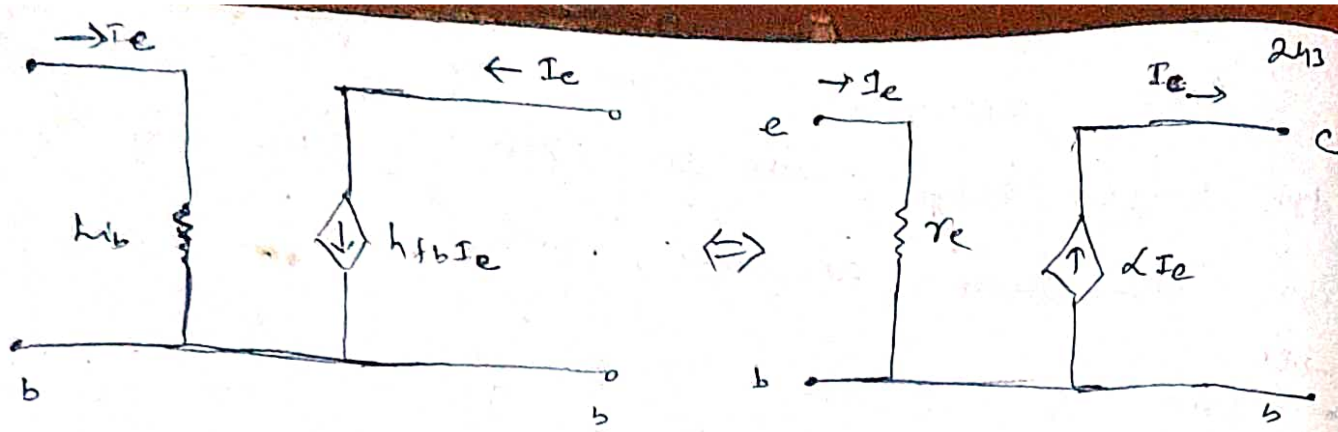
Comparing this approximate hybrid model with r_e model.



$\therefore$ $\boxed{h_{ie} = \beta r_e}$

$\boxed{h_{fe} = \beta_{ac}}$

fig:- hybrid Vs r_e model :- Common emitter configuration



$$\therefore \boxed{h_{ib} = r_e}$$

$$\boxed{h_{fb} = -\alpha \simeq -1}$$

The − sign, accounts for the fact that the current source of standard hybrid equivalent circuit is pointing down rather than in the actual direction as shown in r_e model.

Ex : 17 :- Given $I_E = 2.5\,mA$, $h_{fe} = 140$

$h_{oe} = 20\,\mu S$ (μmho), $h_{ob} = 0.5\,\mu S$

determine :

(a) The common-emitter hybrid equivalent ckt

(b) The common-base r_e model

Ans :-

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.5\,mA} = 10.4\,\Omega.$$

$h_{ie} = \beta r_e \Rightarrow h_{ie} = \beta \cdot 10.4$ —— (1)

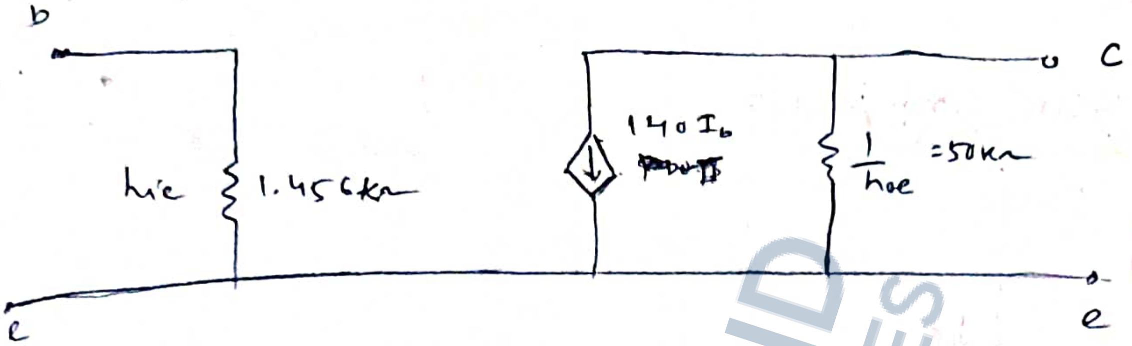
$h_{fe} = \beta \Rightarrow 140 = \beta$ —— (2)

Putting eqn (2), in eqn (1), $h_{ie} = 140 \times 10.4 = 1.456\,k\Omega.$

$$\boxed{h_{ie} = 1.456\,k\Omega}$$

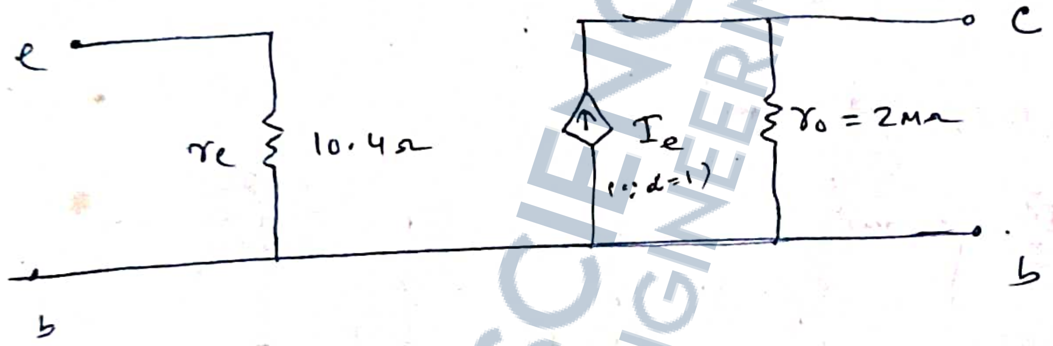
$h_{oe} = 20\mu\mho$

$$\Rightarrow \frac{1}{h_{oe}} = \frac{1}{20 \times 10^{-6}} = \frac{10^6}{20} = \frac{\cancel{10} \times 10^{\cancel{2}} \times 10^3}{\cancel{20}} = 50\,k\Omega .$$

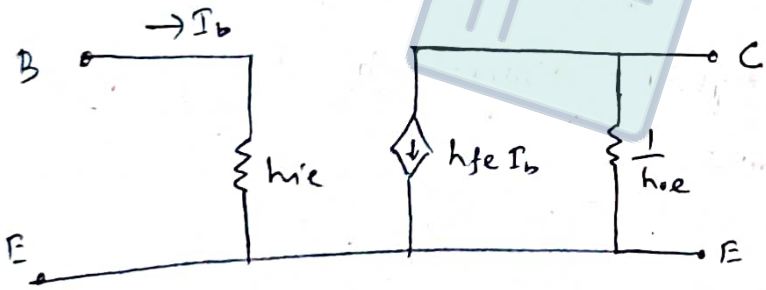


(b) $r_e = 10.4\Omega$.

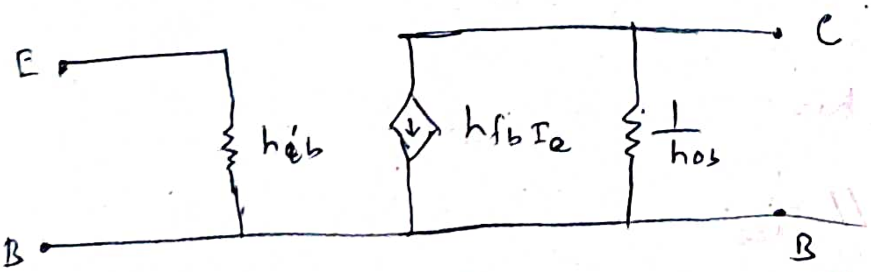
$\alpha \simeq 1$, $r_o = \frac{1}{h_{ob}} = \frac{1}{0.5 \times 10^{-6}} = 2\,M\Omega$.



Approximate Hybrid Equivalent Circuit:-



$\Rightarrow$ Approximate CE hybrid equivalent circuit



$\Rightarrow$ Approximate CB hybrid equivalent circuit.

Rel^n betn r_e & hybrid model

$h_{ie} = \beta r_e$, $h_{fe} = \beta$, $h_{oe} = \dfrac{1}{r_o}$, $h_{fb} = -\alpha$, $h_{ib} = r_e$

$h_{ob} = \dfrac{1}{r_o}$

Fixed - Bias configuration :-

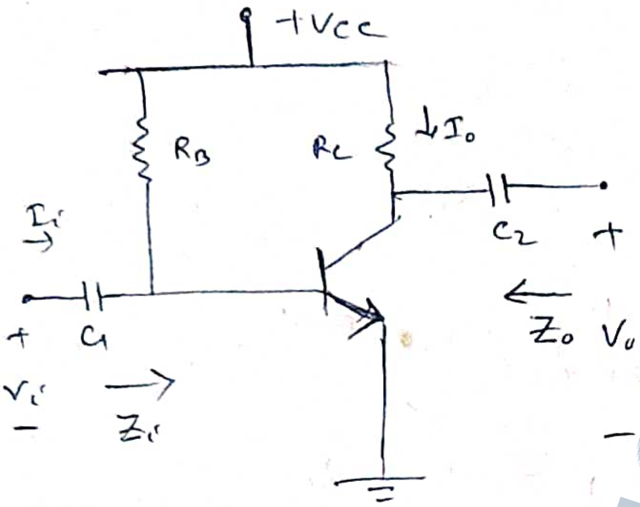


fig:- Fixed-Bias Configuration

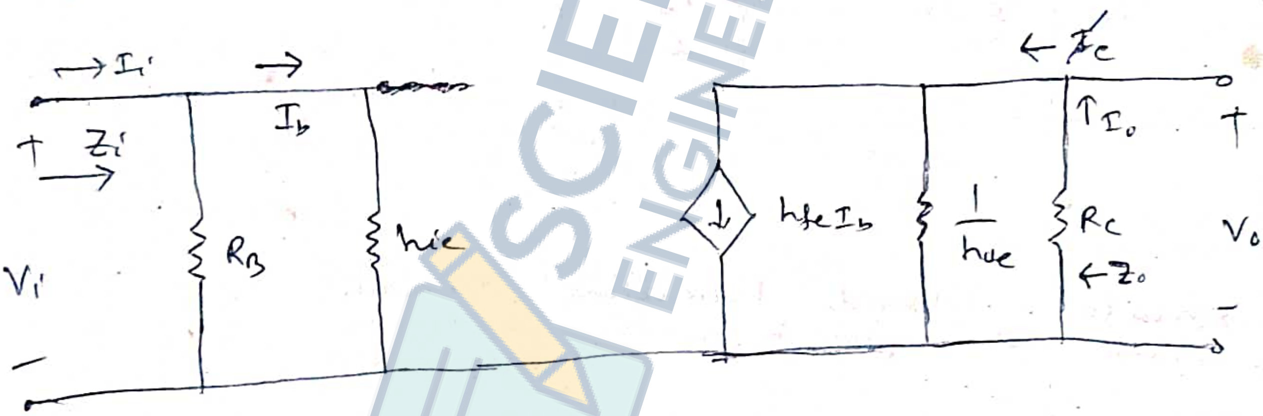


fig:- approximate hybrid model for Fixed bias configuration.

(a) Z_i

$$Z_i = R_B \,//\, h_{ie}$$

(b) $Z_o = R_C \,//\, \dfrac{1}{h_{oe}}$

(C) $\underline{A_v}$

Let $R' = \dfrac{1}{h_{oe}} \parallel R_c$ —— (1)

~~$V_o = I_o R'$~~

By current division rule

current ↙ opposite branch resistance ↙

$$I_o = \frac{(h_{fe} \cdot I_b) \times \frac{1}{h_{oe}}}{\left(\frac{1}{h_{oe}} + R_c\right)} \leftarrow \text{Total resistance}$$

$$V_o = -I_o R_c = -\left[\frac{h_{fe} \cdot I_b \times \frac{1}{h_{oe}}}{\frac{1}{h_{oe}} + R_c}\right] \cdot R_c$$

$$= -h_{fe} I_b \cdot \left(\frac{\frac{1}{h_{oe}} \times R_c}{\frac{1}{h_{oe}} + R_c}\right)$$

$$V_o = -h_{fe} I_b \cdot R' \quad \text{—— (2)}$$

$\left(\because R' = \frac{1}{h_{oe}} \parallel R_c\right)$

But $I_b = \dfrac{V_i}{h_{ie}}$ —— (3)

Putting eqn (3), in eqn (2), we have

$$V_o = -h_{fe} \cdot \frac{V_i}{h_{ie}} \cdot R'$$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{h_{fe}}{h_{ie}} \cdot \left(R_c \parallel \frac{1}{h_{oe}}\right) \simeq \frac{-h_{fe} \cdot R_c}{h_{ie}}$$

$$\Rightarrow \boxed{A_v = \frac{-h_{fe} \cdot \left(R_c \parallel \frac{1}{h_{oe}}\right)}{h_{ie}}} \simeq -\frac{\beta \cdot R_c}{\beta r_e} = -\frac{R_c}{r_e}$$

from re model, $A_v = -\dfrac{R_c}{r_e}$

$h_{ie} = \beta r_e$

$\Rightarrow r_e = \dfrac{h_{ie}}{\beta} = \dfrac{h_{ie}}{h_{fe}}$

$\therefore -\dfrac{R_c}{r_e} = -\dfrac{R_c}{\left(\frac{h_{ie}}{h_{fe}}\right)} = -\dfrac{h_{fe} R_c}{h_{ie}}$

(d) A_i

Assuming $R_B >> h_{ie}$, $I_b \simeq I_i$

$\frac{1}{h_{oe}} >> 10 R_C$, $I_o = I_c$

Taking R_B

$A_i = \frac{h_{fe} R_B}{R_B + h_{ie}}$

(As derived in voltage divider bias next page)

$\therefore I_o = h_{fe} I_b \simeq h_{fe} \cdot I_i \quad (\because I_b = I_i)$

$\Rightarrow \frac{I_o}{I_i} = h_{fe}$

$\Rightarrow \boxed{A_i = h_{fe}}$

or

$I_b = \frac{I_i \times R_B}{R_B + h_{ie}}$

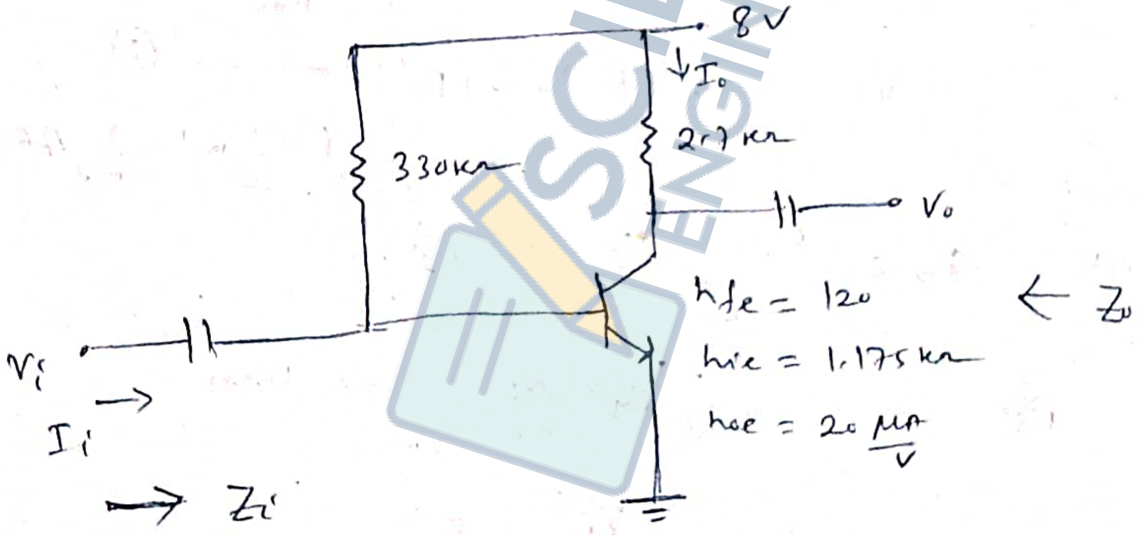
$\Rightarrow I_i = \frac{I_b (R_B + h_{ie})}{R_B}$

$I_o = h_{fe} I_b$

$\frac{I_o}{I_i} = \frac{h_{fe} \cancel{I_b} \times R_B}{\cancel{I_b}(R_B + h_{ie})} = \frac{h_{fe} R_B}{R_B + h_{ie}}$

Ex:-18 For the n/w determine

(a) Z_i (b) Z_o (c) A_v (d) A_i



Ans :-

$Z_i = R_B \parallel h_{ie} = 330\,k\Omega \parallel 1.175\,k\Omega$

$\boxed{Z_i \simeq 1.171\,k\Omega \simeq h_{ie}}$

(b) $Z_o = \frac{1}{h_{oe}} \,//\, R_C$

$$\frac{1}{h_{oe}} = \frac{1}{20\times10^{-6}} = \frac{100\times10^{4}}{20} = 50\,k\Omega.$$

$$Z_o = 50\,k\Omega \,//\, 2.7\,k\Omega \simeq 2.56\,k\Omega \simeq R_C$$

(c) $A_v = \frac{-h_{fe}}{h_{ie}} \cdot \left(R_C \,//\, \frac{1}{h_{oe}}\right)$

$$= \frac{-120}{1.175} \cdot (2.7\,k\Omega \,//\, 50\,k\Omega)$$

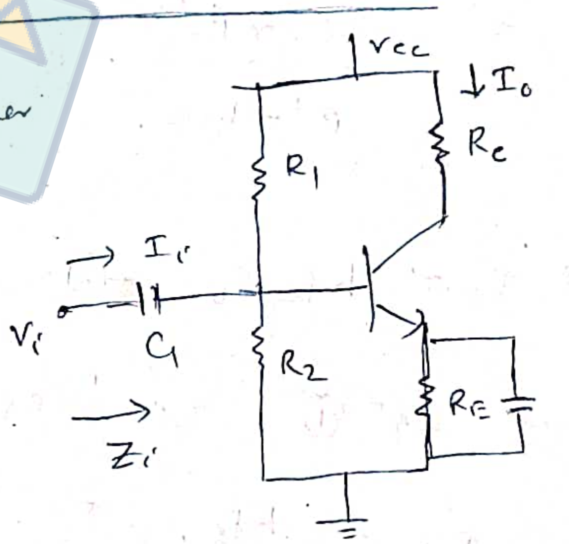
$$= \frac{-120 \times 2.56\,k\Omega}{1.175\,k\Omega}$$

$$A_v = -261.45$$

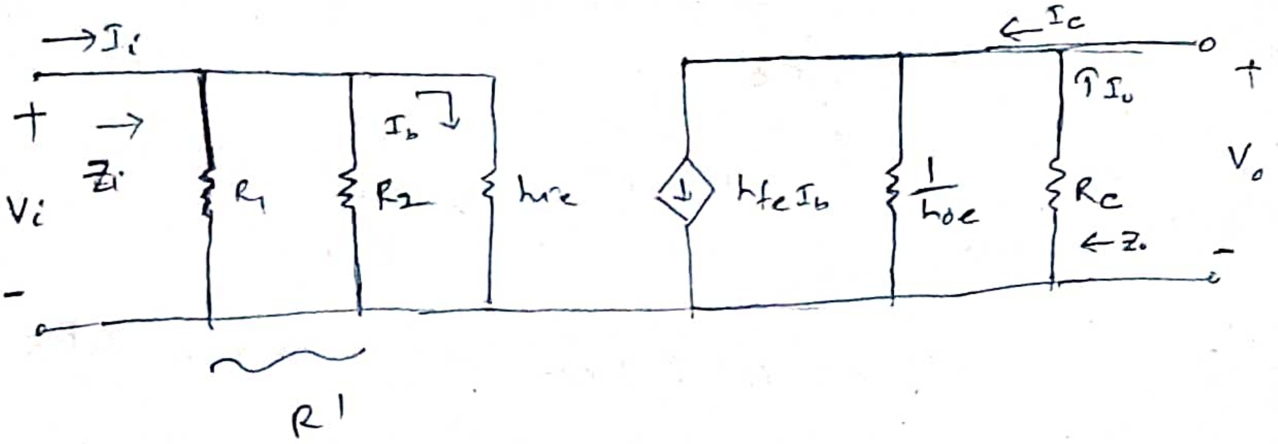
(d) $A_i \simeq h_{fe} = 120$. or $A_i = \frac{h_{fe}R_B}{R_B + h_{ie}} = 119.57$ (exact).

Voltage-divider Configuration :- (with bypass capacitor)

For voltage divider bias configuration, the a.c equivalent n/w is same as that of fixed bias except $\boxed{R_B}$ will be replaced by $\boxed{R_1 // R_2}$.



$R_B \leftrightarrow R_1 // R_2$



$\underline{Z_i}$ =

$$Z_i = R_1 \,//\, R_2 \,//\, h_{ie}$$

$\underline{Z_o}$

$$Z_o \simeq R_c \quad \left(\because Z_o = \frac{1}{h_{oe}} \,//\, R_c \right.$$

since $\frac{1}{h_{oe}} >> R_c$

$\left. Z_o \simeq R_c \right)$

$$A_v = -\frac{h_{fe}}{h_{ie}} \cdot \left(R_c \,//\, \frac{1}{h_{oe}} \right) \quad \langle \text{same as fixed bias} \rangle$$

$\underline{A_i}$ =

$$I_b = \frac{I_i \times R'}{R' + h_{ie}} \quad \text{—①} \quad (\text{By current division rule})$$

& $I_o \simeq h_{fe} I_b$ —② $\left(\because \frac{1}{h_{oe}} >> 10 R_c \right)$

Putting eq^n ① in eq^n ②,

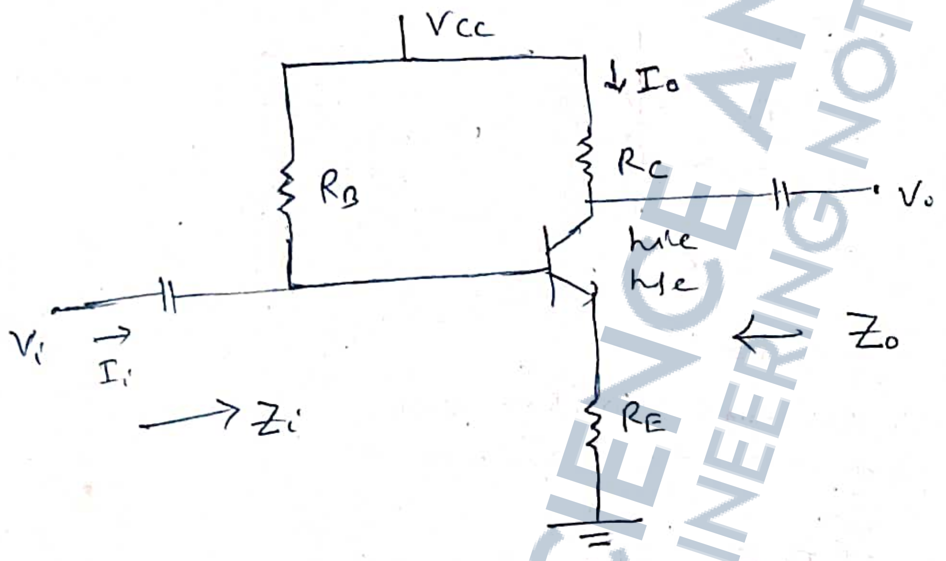
$$I_o = h_{fe} \cdot \left(\frac{I_i \cdot R'}{R' + h_{ie}} \right)$$

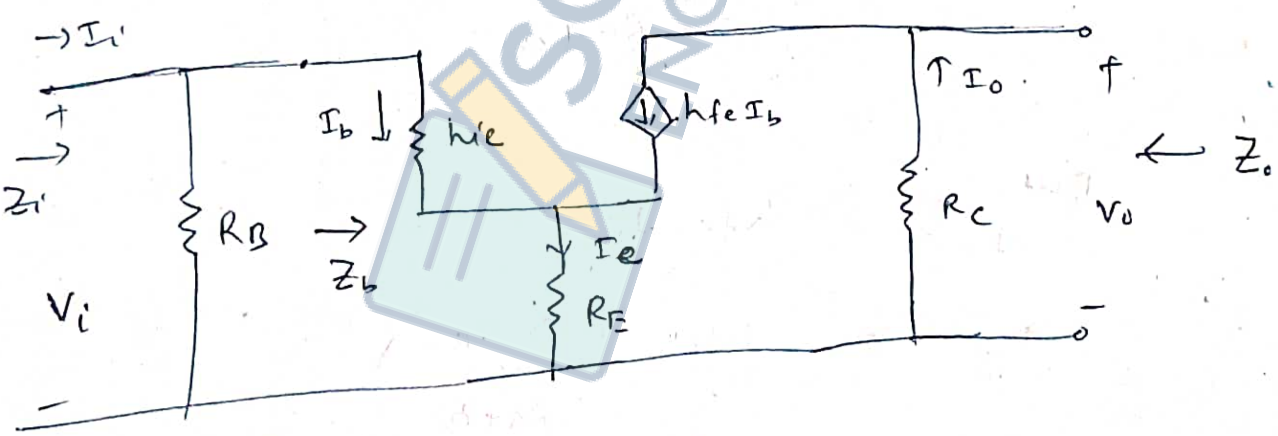
$$\Rightarrow \frac{I_o}{I_i} = \frac{h_{fe} \cdot R'}{R' + h_{ie}}$$

$$\Rightarrow \boxed{A_i = \frac{h_{fe} \cdot R'}{R' + h_{ie}}}$$

where $R' = R_1 // R_2$

Unbypassed Emitter-bias Configuration :-





Z_i :-

$$\boxed{Z_b = h_{fe} R_E}$$

($\because$ In re model, $Z_b = \beta R_E$)

$$\boxed{Z_i = R_B \,//\, Z_b}$$

Z_o

$$Z_o = R_C$$

A_v

In re model, voltage-divider

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fe} R_C}{Z_b} \qquad \left(\because \; A_v = -\frac{\beta R_C}{Z_b} \right)$$

$$= -\frac{\cancel{h_{fe}} \cdot R_C}{\cancel{h_{fe}} \, R_E}$$

$$\boxed{A_v = -\frac{R_C}{R_E}}$$

A_i

By current division rule

$$I_b = I_i \times \frac{R_B}{R_B + Z_b}$$

But

$$I_o = h_{fe} \cdot I_b$$

$$I_o = h_{fe} \cdot \left(\frac{I_i R_B}{R_B + Z_b} \right)$$

$$\Rightarrow \frac{I_o}{I_i} = \frac{h_{fe} R_B}{R_B + Z_b}$$

$$\Rightarrow \boxed{A_i = \frac{h_{fe} R_B}{R_B + Z_b}}$$

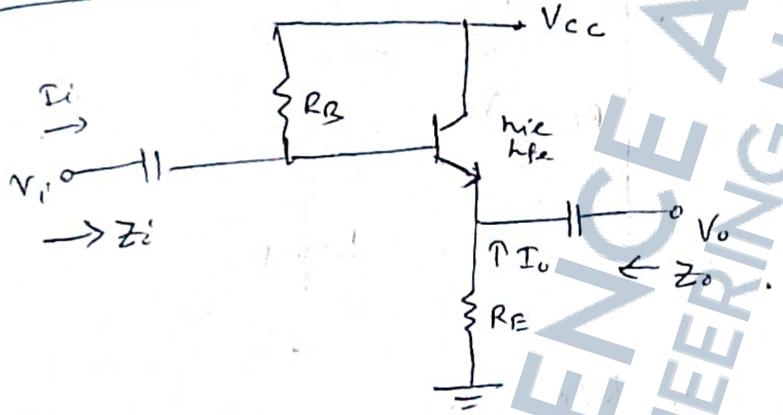
or $\boxed{A_i = -A_v \cdot \frac{Z_i}{R_C}}$ ✓ $\left(\because A_i = -A_v \cdot \frac{Z_i}{R_L} \right)$

$$\left[\because A_i = -\left(\frac{-h_{fe} \cancel{R_C}}{\cancel{Z_b}}\right) \cdot \frac{R_B \times \cancel{Z_b}}{R_B + Z_b} \cdot \frac{1}{\cancel{R_C}} \right.$$

(A_v ↓, ↓ Z_i)

$$\left. A_i = + \frac{h_{fe} R_B}{R_B + Z_b} \right]$$

Emitter - follower Configuration :-



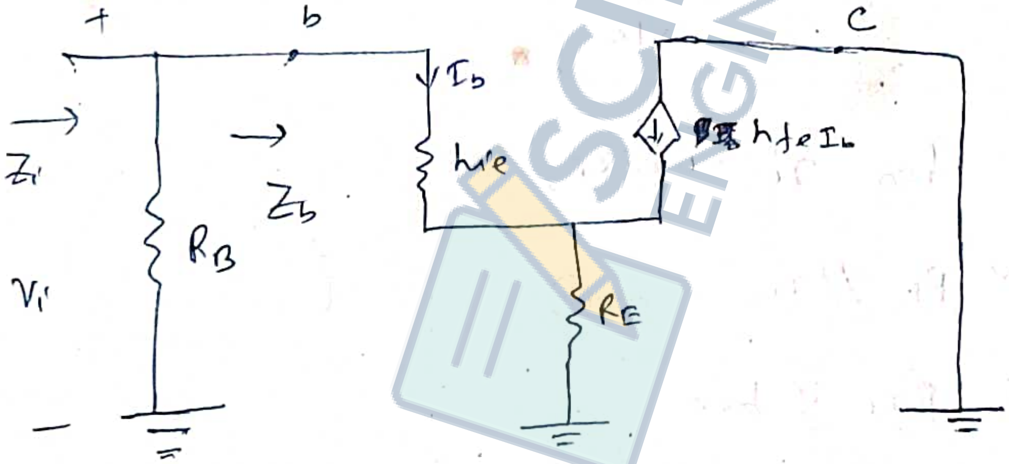
3. Voltage divider without Bypass

1) $Z_i = R' \,//\, Z_b$
$= R' \,//\, h_{fe} R_E$

2) $Z_o = R_C$

3) $A_V = -\frac{R_C}{R_E}$

4) $A_I = \frac{h_{fe} R'}{R' + Z_b}$



→ $\beta r_e = h_{ie}$, $h_{fe} = \beta$ (Same as that of r_e' model)

$\boxed{Z_i = R_B \,//\, Z_b}$

with $\boxed{Z_b \simeq h_{fe} R_E}$

r_e model

$$\left(\because Z_b = \beta(r_e + R_E) \simeq \beta R_E \right)$$

Z_o [Same approach as in r_e model]

$$I_b = \frac{V_i}{Z_b} = \frac{V_i}{\beta(r_e + R_E)}$$

$$\Rightarrow (\beta+1) I_b = \frac{(\beta+1) V_i}{\beta(r_e+R_E)}$$ [Multiplying $(\beta+1)$ in L.H.S & R.H.S]

$$\Rightarrow I_E \simeq \frac{\beta V_i}{\beta r_e + \beta R_E} = \frac{h_{fe} V_i}{h_{ie} + h_{fe} R_E} = \frac{V_i}{\frac{h_{ie}}{h_{fe}} + R_E}$$

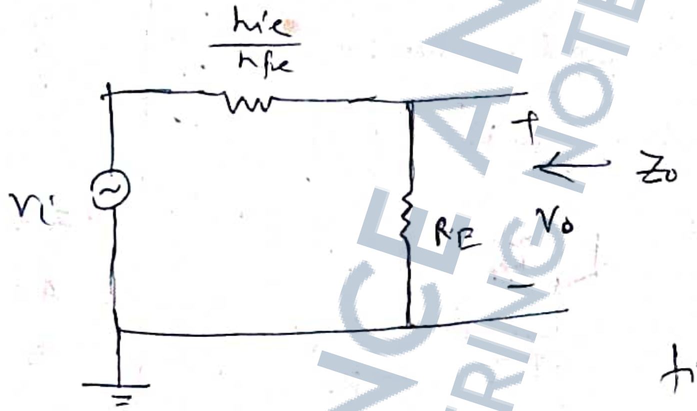


fig 1.

$$\boxed{Z_o = R_E \,//\, \frac{h_{ie}}{h_{fe}}}$$

or Directly from r_e model,

$$Z_o \simeq R_E \,//\, r_e \quad \text{—①}$$

But $\beta r_e = h_{ie}$

$$\Rightarrow r_e = \frac{h_{ie}}{\beta} = \frac{h_{ie}}{h_{fe}}.$$

$$\therefore \boxed{Z_o \simeq R_E \,//\, \frac{h_{ie}}{h_{fe}}.}$$

$\underline{A_v}$:- $\underline{\text{From fig 1}}$, (previous page)

$$V_o = \frac{V_i}{\frac{h_{ie}}{h_{fe}} + R_E} \times R_E$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{R_E}{R_E + \frac{h_{ie}}{h_{fe}}}$$

$$\Rightarrow \boxed{A_v = \frac{R_E}{R_E + \frac{h_{ie}}{h_{fe}}}}$$

or r_e model

$$A_v = \frac{R_E}{r_e + R_E}$$

$$= \frac{R_E}{\frac{h_{ie}}{h_{fe}} + R_E}$$

$\left(\because \beta r_e = h_{ie} \Rightarrow r_e = \frac{h_{ie}}{\beta} = \frac{h_{ie}}{h_{fe}} \right)$

$\underline{A_i}$

$$\boxed{A_i = \frac{h_{fe} R_B}{R_B + Z_b}}$$

[Same derivation as unbypassed Emitter-bias]

or $\boxed{A_i = -A_v \cdot \frac{Z_i}{R_E}}$

Common-Base Configuration :-

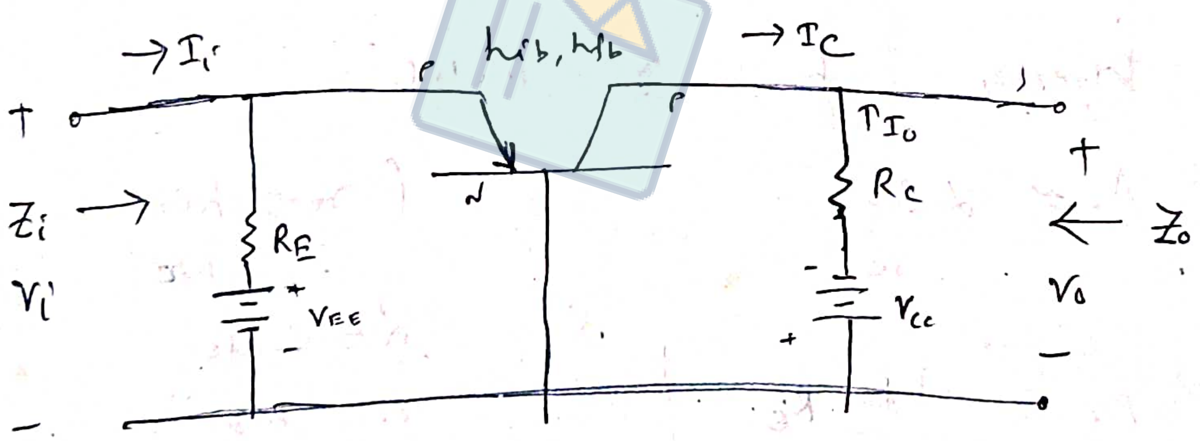
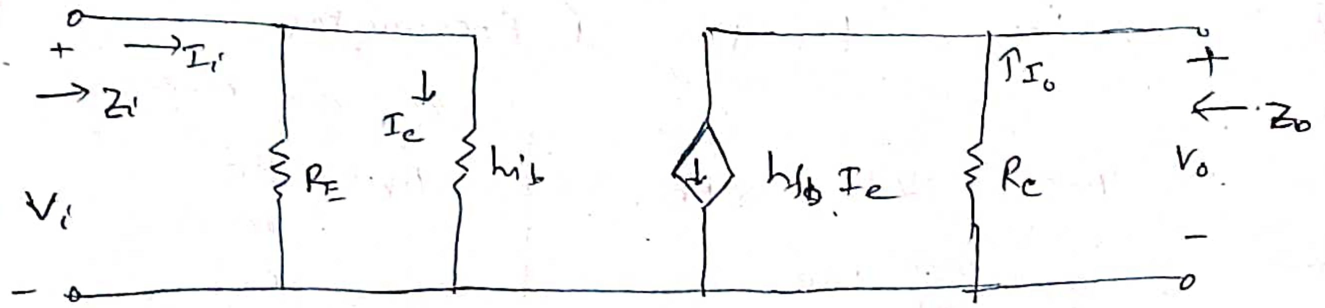


fig :- Common base Configuration



$\underline{\underline{Z_i}}$

$$Z_i = R_E \,//\, h_{ib}$$

$\underline{\underline{Z_o}}$

$$Z_o = R_C$$

$$\left[\begin{array}{l} Z_o = R_C \,//\, \dfrac{1}{h_{ob}} \\ \simeq R_C \qquad \because \dfrac{1}{h_{ob}} >> R_C \end{array} \right]$$

$\underline{\underline{A_V}}$

$$V_o = -I_o R_C = -(h_{fb} I_e) R_C$$

With $I_e = \dfrac{V_i}{h_{ib}}$, $V_o = -h_{fb} \cdot \dfrac{V_i}{h_{ib}} \cdot R_C$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-h_{fb}}{h_{ib}} \cdot R_C$$

$$\boxed{A_V = \frac{-h_{fb}}{h_{ib}} \cdot R_C}$$

$\underline{\underline{\text{or}}}$ Directly from 'r_e' model

$$A_V \simeq \frac{\alpha R_C}{r_e}$$

$\left(\cancel{r_e = \frac{h_{ie}}{h_{fe}}} \right.$

$\alpha = -h_{fb}$

$r_e = h_{ib}$

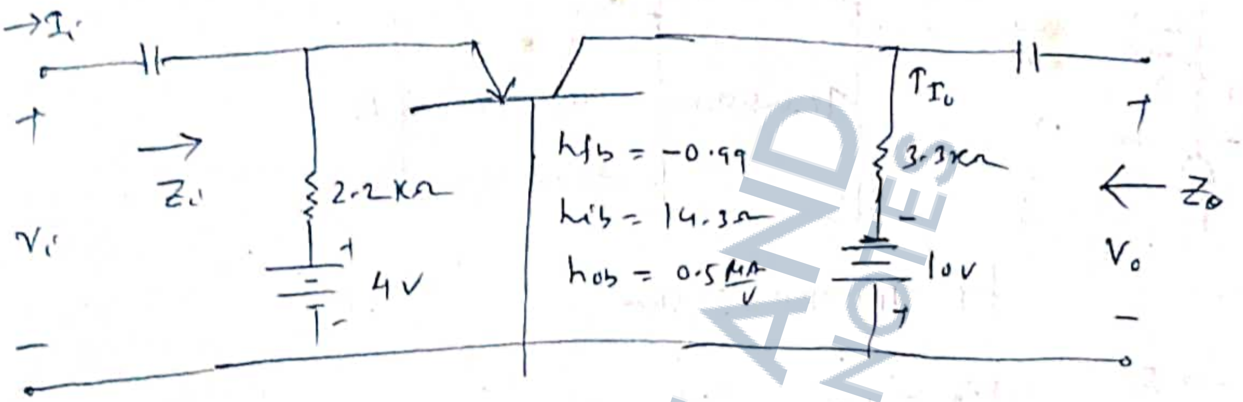
$$\boxed{A_V = \frac{-h_{fb}}{h_{ib}} R_C}$$

$\underline{A_i}$ $\quad A_i = \dfrac{I_o}{I_i} = h_{fb} \simeq -1$ (If $R_E >> h_{ib}$)

$\left(\begin{array}{l} I_o = h_{fb} I_e \\ I_i = I_e \end{array} \right)$

Ex-19 Determine

(a) Z_i (b) Z_o (c) A_v (d) A_i



Soln :- (a) $Z_i = R_E \,//\, h_{ib} = 2.2\,K\Omega \,//\, 14.3\Omega = 14.1\Omega \simeq h_{ib}$

(b) $Z_o \simeq R_c \simeq 3.3\,K\Omega$

Actually $\quad Z_o = R_c \,//\, \dfrac{1}{h_{ob}} = 3.3\,K\Omega \,//\, \dfrac{1}{0.5\mu A/V} = 3.3K\Omega \,//\, 2M\Omega$

$\simeq 3.3\,K\Omega$

(c) $A_v = \dfrac{-h_{fb}}{h_{ib}} \cdot R_c = -\dfrac{(-0.99)}{14.3} \times 3.3\,K\Omega$

$\simeq 229.91$

(d) $A_i \simeq h_{fb} \simeq -0.99 \simeq -1.$

Note :- (Previous figure)

$I_e = \dfrac{I_i \times R_E}{R_E + h_{ib}} \Rightarrow I_i = \dfrac{I_e\,(R_E + h_{ib})}{R_E}$

$I_o = h_{fb} I_e \quad \Rightarrow \dfrac{I_o}{I_i} = \dfrac{h_{fb} \cancel{I_e}}{\cancel{I_e}\,(R_E + h_{ib})} \times R_E = \dfrac{h_{fb} R_E}{(R_E + h_{ib})}$

$\Rightarrow A_I \simeq \dfrac{h_{fb} \cancel{R_E}}{\cancel{R_E}} \simeq h_{fb}.$

Complete Hybrid Equivalent Model

→ Effect of h_r & h_{oe} also taken into account.

→ Consider the general configuration

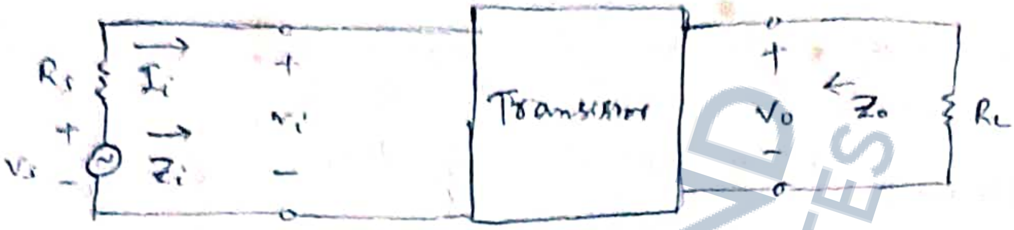


fig:- Two-port system

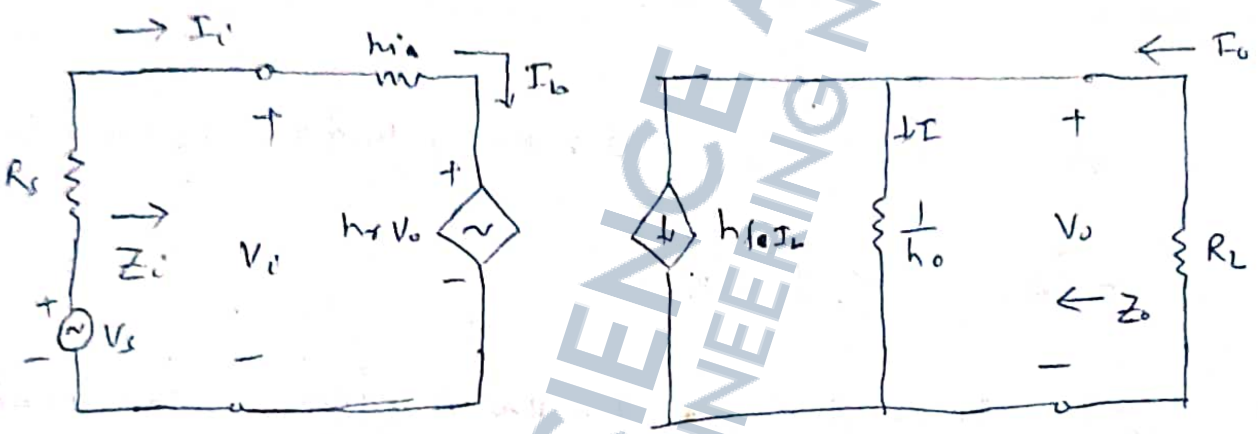


fig:- Substituting the complete hybrid equivalent circuit into the two-port system.

Current gain :- $\left(A_i = \frac{I_o}{I_i}\right)$

Applying KCL to the o/p ckt

$$I_o = h_f I_b + I = h_f I_i + \frac{V_o}{(1/h_o)} = h_f I_i + h_o V_o$$

Substituting $V_o = -I_o R_L$ in the above equation,

$$\Rightarrow I_o = h_f I_i + h_o(-I_o R_L)$$

$$\Rightarrow I_o(1 + h_o R_L) = h_f I_i$$

$$\Rightarrow \frac{I_o}{I_i} = \frac{h_f}{1+h_o R_L} \quad \text{——} (1)$$

$$\rightarrow \boxed{A_i = \frac{h_f}{1+h_o R_L}}$$

Note :- if $h_o R_L << 1$, $\boxed{A_i = h_f}$ (Approximate model) CB

Voltage gain $\left(A_v = \frac{V_o}{V_i}\right)$

KVL in the i/p ckt

$$V_i - I_i h_i - h_r V_o = 0$$

$$\Rightarrow V_i = I_i h_i + h_r V_o \quad \text{——} (2)$$

From eqn (1), $I_i = \frac{I_o(1+h_o R_L)}{h_f}$
(Given above)

Putting I_i value, in eqn (2), we have

$$\Rightarrow V_i = \frac{I_o(1+h_o R_L)h_i}{h_f} + h_r V_o \quad \text{——} (3)$$

Again $I_o = \frac{-V_o}{R_L}$

$$\therefore V_i = \frac{-V_o}{R_L} \cdot \frac{(1+h_o R_L)h_i}{h_f} + h_r V_o = V_o\left(h_r - \frac{(1+h_o R_L)h_i}{R_L h_f}\right)$$

$$\Rightarrow \frac{V_i}{V_o} = \frac{h_r \cdot R_L h_f - h_i - h_o R_L h_i}{R_L h_f} = \frac{-h_i - R_L(h_i h_o - h_f h_r)}{R_L h_f}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L}$$

$$\Rightarrow \boxed{A_v = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L}}$$

Note :- If $(h_i h_o - h_f h_r) R_L << h_i$, $\boxed{A_v = \frac{-h_f R_L}{h_i}}$

Input Impedance $(Z_i) = V_i / I_i$

Applying KVL in the I/P ckt,

$$V_i = h_i I_i + h_r V_o \quad \text{——} \quad (A)$$

Substituting $V_o = -I_o R_L$, we have

$$V_i = h_i I_i - h_r . (I_o R_L) \quad \text{——} \quad (B)$$

But $A_i = \frac{I_o}{I_i} \Rightarrow I_o = A_i I_i$

$$\Rightarrow V_i = h_i I_i - h_r . (A_i I_i) R_L$$

$$\Rightarrow V_i = I_i (h_i - h_r A_i R_L)$$

$$\Rightarrow \frac{V_i}{I_i} = h_i - h_r . A_i R_L$$

But $A_i = \frac{h_f}{1 + h_o R_L}$

$$\Rightarrow \cancel{Z_i = \frac{V_i}{I_i} = h_i + h_r \cdot \left(\frac{h_f}{1+h_oR_L}\right) \cdot R_L}$$

$$\Rightarrow \frac{V_i}{I_i} = h_i - h_r \cdot \left(\frac{h_f}{1+h_oR_L}\right) \cdot R_L$$

$$\Rightarrow \boxed{Z_i = h_i - \frac{h_r h_f R_L}{1+h_oR_L}}$$

<u>Note</u> : If $h_oR_L << 1$, $Z_i \simeq h_i$

<u>O/P Impedence Z_o (V_o/I_o)</u>

O/P Impedance of an amplifier is defined to be ratio of O/P voltage to O/P current with signal V_s set to zero. For I/P ckt with $V_s = 0$, [Short ckted]

$$-I_iR_s - I_ih_i - h_rV_o = 0$$

$$\Rightarrow I_i(R_s + h_i) = -h_rV_o$$

$$\Rightarrow I_i = \frac{-h_rV_o}{R_s + h_i} \quad \text{—— (A')}$$

<u>KCL in O/P loop</u>

$$I_o = h_f I_b + I = h_f I_i + \frac{V_o}{1/h_o} = h_f I_i + h_oV_o$$

Putting $\cancel{(A)}$, I_i value from eqn (A'),

$$I_o = h_f \cdot \left(\frac{-h_rV_o}{R_s + h_i}\right) + h_oV_o = V_o\left(h_o - \frac{h_rh_f}{h_i + R_s}\right)$$

$$\Rightarrow \frac{I_o}{V_o} = \left[h_o - \frac{h_r h_f}{h_i + R_s} \right] \cancel{= \frac{h_o(h_i+R_s) - h_r h_f}{h_i+R_s}}$$

$$\cancel{\Rightarrow \frac{V_o}{I_o} = h_i + R_s}$$

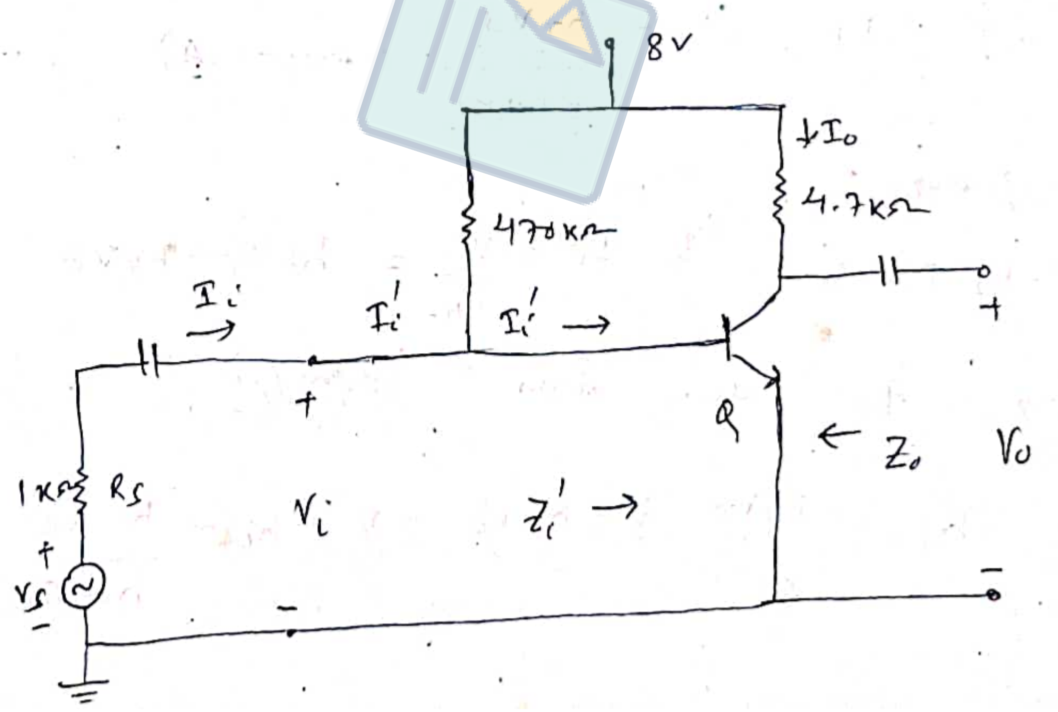
$$\Rightarrow \frac{V_o}{I_o} = \frac{1}{h_o - \frac{h_r h_f}{h_i + R_s}}$$

$$\Rightarrow \boxed{Z_o = \frac{1}{\left[h_o - \left[h_r h_f / (h_i + R_s) \right] \right]}}$$

Note :- $Z_o = \frac{1}{h_o}$ when $\frac{h_r h_f}{h_i + R_s} << h_o$

Ex-20 :- Determine the following parameters using Complete hybrid equivalent model.

(a) Z_i & Z_i' (b) A_v (c) $A_i = \frac{I_o}{I_i}$ (d) Z_o' (within R_c) and Z_o (including R_c)



Given

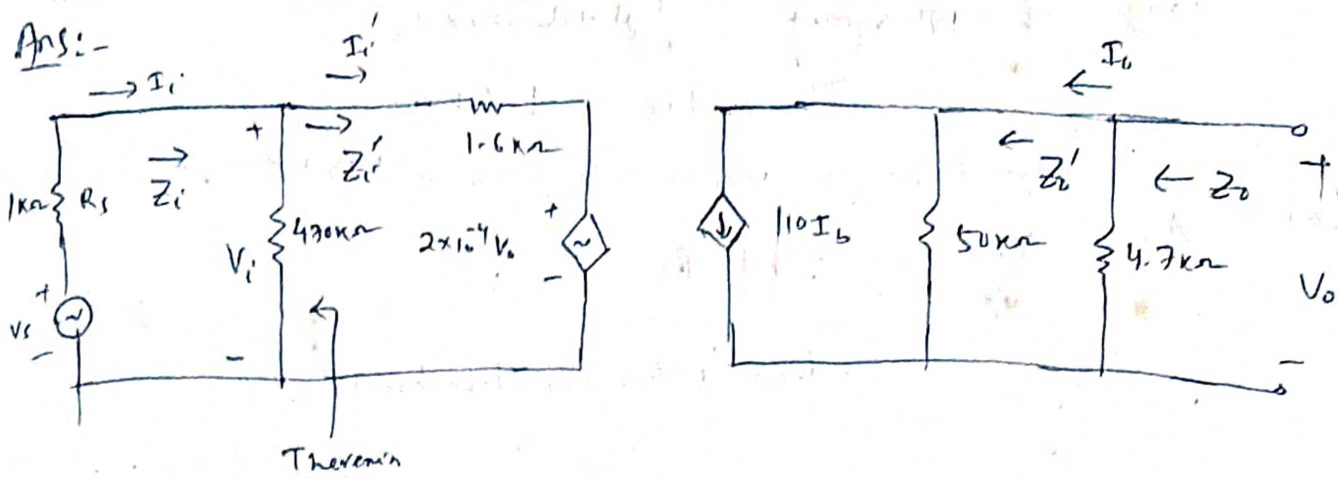
$h_{fe} = 110$

$h_{ie} = 1.6\ k\Omega$

$h_{re} = 2 \times 10^{-4}$

$h_{oe} = 20\ \frac{\mu A}{V}$

Ans:-

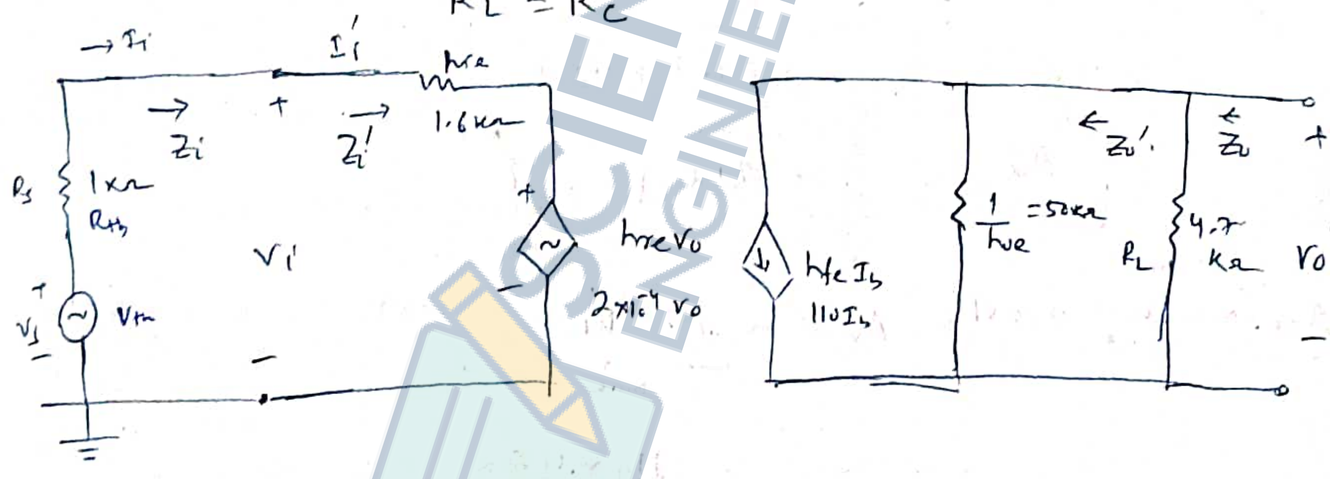


To apply the formulae, we have derived for generalized complete hybrid model, The ckt need to be converted to 'complete hybrid equivalent ckt model'.

For that $E_{Th} \simeq V_s$, [open ckt voltage Removing 470 kΩ] Th → Thevenin

$R_{th} = R_s \parallel 470\,k\Omega \simeq R_s$

$R_L = R_C$



(a) $Z_i' = \dfrac{V_i}{I_i'} = h_{ie} - \dfrac{h_{fe} h_{re} R_L}{1 + h_{oe} R_L}$

$$= 1.6\,k\Omega - \frac{(110)\times(2\times10^{-4})\times(4.7\,k\Omega)}{1 + 20\times10^{-6} \times 4.7\,k\Omega}$$

$$= 1.6\,k\Omega - 94.51\,\Omega$$

$$\boxed{Z_i' \simeq 1.51\,k\Omega}$$

Using approximate hybrid model,

$$Z_i' \simeq h_{ie} = 1.6\,k\Omega$$

$$Z_i = 470\,k\Omega \,//\, Z_i' = 470\,k\Omega \,//\, 1.6\,k\Omega = 1.51\,k\Omega \simeq Z_i'$$

(b) $$A_V = \frac{V_o}{V_i} = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_L}$$

$$= \frac{-110 \times 4.7k}{1.6\,k\Omega + \left[1.6\,k\Omega \cdot 20\times10^{-6} - 110\times 2\times10^{-4}\right] 4.7\times10^{3}}$$

$$= \frac{-517\times10^{3}}{1.6\times10^{3} + \left[0.032 - 0.0220\right]\times 4.7\times10^{3}}$$

$$= \frac{-517\times10^{3}}{1.6\times10^{3} + 47\,\Omega}$$

$$\boxed{A_V = -313.9}$$

Approximate model, $A_V \simeq -\dfrac{h_{fe} R_L}{h_{ie}}$

$$\simeq -\frac{110\times 4.7k}{1.6\,k\Omega}$$

$$A_V = -323.125$$

(C) $$A_i' = \frac{I_o}{I_i'} = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{110}{1 + 20\times10^{-6}\times 4.7\times10^{3}}$$

$$= \frac{110}{1.094} = 100.55$$

Approximate model, $A_I = h_{fe} = 110$

Since $470\,k\Omega >> Z_i'$, $I_i \simeq I_i'$

$$A_i \simeq A_i' = 100.55$$

(d) $$Z_o' = \frac{V_o}{I_o} = \frac{1}{h_{oe} - [h_{fe} h_{re} / (h_{ie} + R_s)]}$$

$$= \frac{1}{20\times10^{-6} - [110 \times 2\times10^{-4} / (1.6\,k\Omega + 1\,k\Omega)]}$$

$$= \frac{1}{20\times10^{-6} - 8.46\times10^{-6}}$$

$$= \frac{1}{11.54\times10^{-6}}$$

$$Z_o' = 86.66\,k\Omega$$

Approximate model, $Z_o' = \frac{1}{h_{oe}} = 50\,k\Omega$.

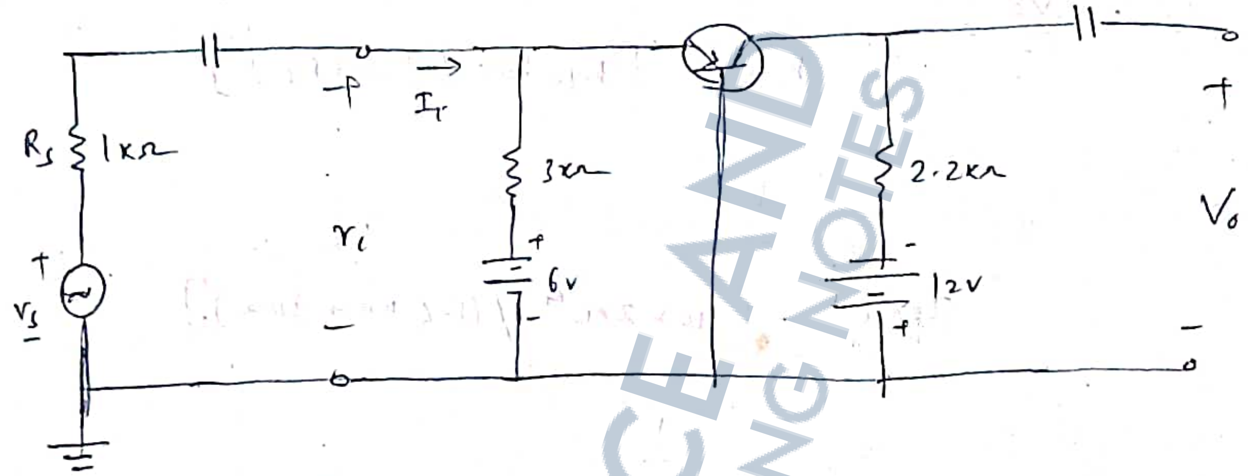
$$Z_o = Z_o' \,//\, R_C = 86.66\,k\Omega \,//\, 4.7\,k\Omega$$

$$\boxed{Z_o = 4.46\,k\Omega}$$

Approximate model, $Z_o = R_C = 4.7\,k\Omega$.

Ex-21 :- For the Common base amplifier, determine the following parameters using complete hybrid equivalent model & compare the results to those obtained using the approximate model.

(a) Z_i (b) A_i (c) A_v (d) Z_o



Given $h_{ie} = 1.6\,k\Omega$, $h_{fe} = 110$

$h_{re} = 2 \times 10^{-4}$ $h_{oe} = 20\,\mu S$

Note :- Typical parameter values of CE, CC, CB transistor configurations

Parameter	CE	CC	CB
h_i	1kΩ	1kΩ	20Ω
h_r	2.5×10^{-4}	$\simeq 1$	3.0×10^{-4}
h_f	50	-50	-0.98
h_o	$25\,\mu A/V$	$25\,\mu A/V$	$0.5\,\mu A/V$
$1/h_o$	40kΩ	40kΩ	2MΩ

Approximate conversion equations: -

CE ↔ CB

$$h_{ie} \simeq \frac{h_{ib}}{1+h_{ib}} \simeq \beta r_e$$

$$h_{re} \simeq \frac{h_{ib}\, h_{ob}}{1+h_{fb}} - h_{rb}$$

$$h_{fe} \simeq \frac{-h_{fb}}{1+h_{fb}} \simeq \beta$$

$$h_{oe} = \frac{h_{ob}}{1+h_{fb}}$$

CB ↔ CE ↔ CC

$$h_{ib} \simeq \frac{h_{ie}}{1+h_{fe}} \simeq \frac{-h_{ic}}{h_{fc}} \simeq r_e$$

$$h_{rb} = \frac{h_{ie}\, h_{oe}}{1+h_{fe}} - h_{re} \simeq h_{rc} - 1 - \frac{h_{ic} h_{oc}}{h_{fc}}$$

$$h_{fb} = \frac{-h_{fe}}{1+h_{fe}} \simeq \frac{-(1+h_{fc})}{h_{fc}} \simeq -\alpha$$

$$h_{ob} \simeq \frac{h_{oe}}{1+h_{fe}} \simeq \frac{-h_{oc}}{h_{fc}}$$

CC

$$h_{ic} \simeq \frac{h_{ib}}{1+h_{fb}} \simeq \beta r_e$$

$$h_{rc} \simeq 1$$

$$h_{fc} \simeq \frac{-1}{1+h_{fb}} \simeq -\beta$$

$$h_{oc} = \frac{h_{ob}}{1+h_{fb}}$$

Soln :- The h-parameters are given for 'CE' configuration, for 'CB' we have to convert it using the conversion formula given on last page.

$$h_{ib} \simeq \frac{h_{ie}}{1+h_{fe}} = \frac{1.6\ k\Omega}{1+110} = 14.41\,\Omega$$

or By approximate hybrid model

$$h_{ib} = r_e = \frac{h_{ie}}{\beta} = \frac{1.6\times10^3}{h_{fe}} = \frac{1.6\times10^3}{110} = 14.55\,\Omega$$

$$h_{rb} = \frac{h_{ie}h_{oe}}{1+h_{fe}} - h_{re} = \frac{1.6\times10^3 \cdot 20\times10^{-6}}{1+110} - 2\times10^{-4}$$

$$= 2.883\times10^{-4} - 2\times10^{-4}$$

$$h_{rb} = 0.883\times10^{-4}$$

$$h_{fb} = \frac{-h_{fe}}{1+h_{fe}} = \frac{-110}{1+110} = -0.991$$

$$h_{ob} = \frac{h_{oe}}{1+h_{fe}} = \frac{20\times10^{-6}}{1+110} = 0.18\ \mu S$$

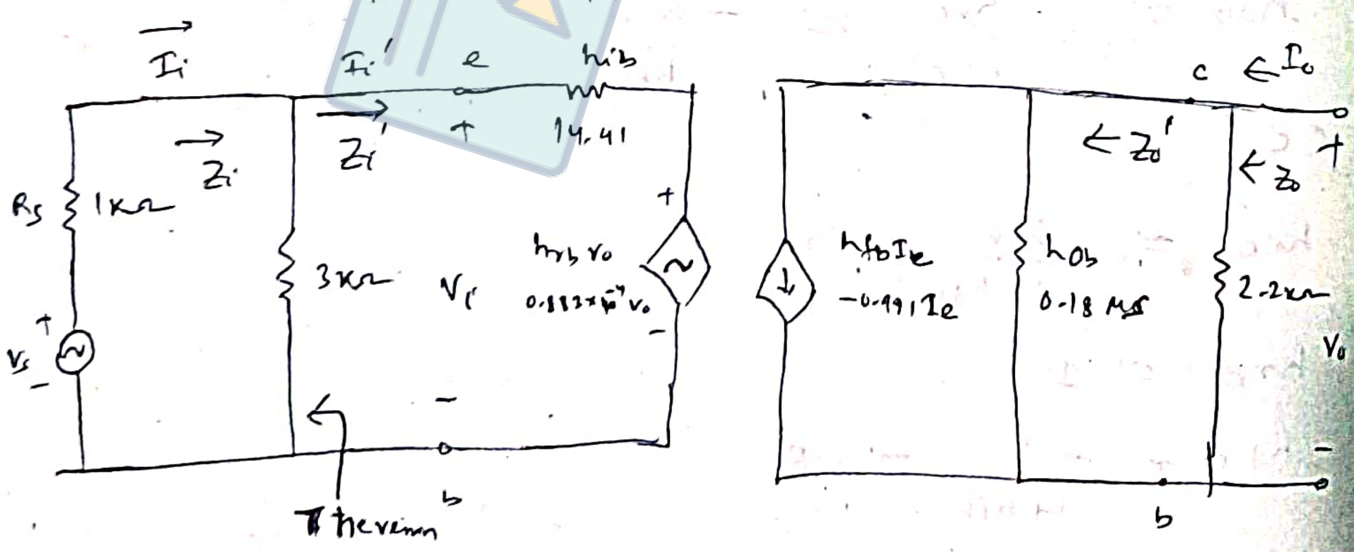


fig:- CB hybrid equivalent ckt

~~R_{Th}~~ = The thevenin equivalent n/w for i/p ckt results in $R_{th} = 3k\Omega \,||\, 1k\Omega = \underline{0.75\,k\Omega}$

(a) $\underline{Z_i'}$ $= \dfrac{v_i'}{I_i'} = h_{ib} - \dfrac{h_{fb}\, h_{rb} R_L}{1 + h_{ob} R_L}$

$$= 14.41 - \frac{(-0.991)\,.\,(0.883\times10^{-4}) \times 2.2\times10^{3}}{1 + 0.18\times10^{-6} \times 2.2\times10^{3}}$$

$$= 14.41 + \frac{.1925}{1.000396}$$

$$Z_i' = 14.60\,\Omega$$

Note:- Approximate model, $Z_i' = h_{ib} = 14.41\Omega$

and $Z_i = Z_i' \,//\, 3k\Omega = 14.60 \,//\, 3\,k\Omega \simeq 14.60\Omega$.

(b) $\underline{\underline{A_i'}}$

$$A_i' = \frac{I_o}{I_i'} = \frac{h_{fb}}{1 + h_{ob} R_L} = \frac{-0.991}{1 + 0.18\times10^{-6} \times 2.2\times10^{3}} = -0.991$$

Because, $3k\Omega >> Z_i'$ i.e $3k\Omega >> 14.60\Omega$,

$I_i' \simeq I_i$, $A_i = \dfrac{I_o}{I_i} \simeq -1$.

(c) $A_v = \dfrac{v_o}{v_i} = \dfrac{-h_{fb} R_L}{h_{ib} + (h_{ib} h_{ob} - h_{fb} h_{rb}) R_L}$

$$= \frac{-(-0.991) \times 2.2\times10^{3}}{14.41 + [14.41 \times 0.18\times10^{-6} - (-0.991)\,.\,0.883\times10^{-4}]\,2.2\times10^{3}}$$

$\therefore A_v = 149.25$

By approximate model

$$A_v = \frac{-h_{fb} R_L}{h_{ib}} = -\frac{(-0.991) \times 2.2 \times 10^3}{14.41}$$

$$A_v = 151.3$$

(d)

$$Z_o' = \frac{1}{h_{ob} - \left[h_{fb} h_{rb} / (h_{ib} + R_s) \right]}$$

$$= \frac{1}{0.18 \times 10^{-6} - \left[(-0.991) \cdot (0.883 \times 10^{-4}) / (14.41 + 0.75 \times 10^3) \right]}$$

Note: $R_s = 0.75 \times 10^3$ (R_{th}) $[1k\Omega // 3k\Omega]$

$\Rightarrow Z_o' = 3.39\ M\Omega$

By approximate model.

$$Z_o' = \frac{1}{h_{ob}} = \frac{1}{0.18 \times 10^{-6}} = 5.56\ M\Omega.$$

$$Z_o = R_C // Z_o' = 2.2\ k\Omega // 3.39\ M\Omega = \underline{2.199\ k\Omega}.$$

versus

~~vs~~ $Z_o \simeq R_C = \underline{2.2\ k\Omega}.$

Graphical Determination of Hybrid Parameters

Using Partial derivatives, magnitude of h-Parameters can be found using the following equations,

$$h_{ie} = \frac{\partial v_i}{\partial i_i} = \frac{\partial v_{be}}{\partial i_b} \simeq \frac{\Delta V_{be}}{\Delta i_b}\Bigg|_{V_{CE} = \text{Constant}} \quad (\text{ohms})$$

$$h_{re} = \frac{\partial v_i}{\partial v_o} = \frac{\partial V_{be}}{\partial V_{ce}} \simeq \frac{\Delta V_{be}}{\Delta V_{ce}}\Bigg|_{I_B = \text{Constant}} \quad (\text{unitless})$$

[Note:-

$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

i.e

$$V_{be} = h_{ie} I_b + h_{re} V_{ce}$$

$$I_c = h_{fe} I_b + h_{oe} V_{ce}$$]

$$h_{fe} = \frac{\partial i_o}{\partial i_i} = \frac{\partial i_c}{\partial i_b} \simeq \frac{\Delta i_c}{\Delta i_b}\Bigg|_{V_{CE} = \text{constant}} \quad (\text{unitless})$$

$$h_{oe} = \frac{\partial i_o}{\partial v_o} = \frac{\partial i_c}{\partial V_{ce}} \simeq \frac{\Delta i_c}{\Delta V_{ce}}\Bigg|_{I_B = \text{Constant}} \quad (\text{Siemens})$$

→ Δ refers to a small change in that quantity around the quiescent point of operation

→ hie & hre are determind from I/P or base Characteristics.

→ hfe & hoe are obtained from output or collector Characteristics.

Procedure :-

(A) hfe determination :-

1) First find the Quiescent point of operation from the o/p Characteristics V_{CE} Vs I_C.

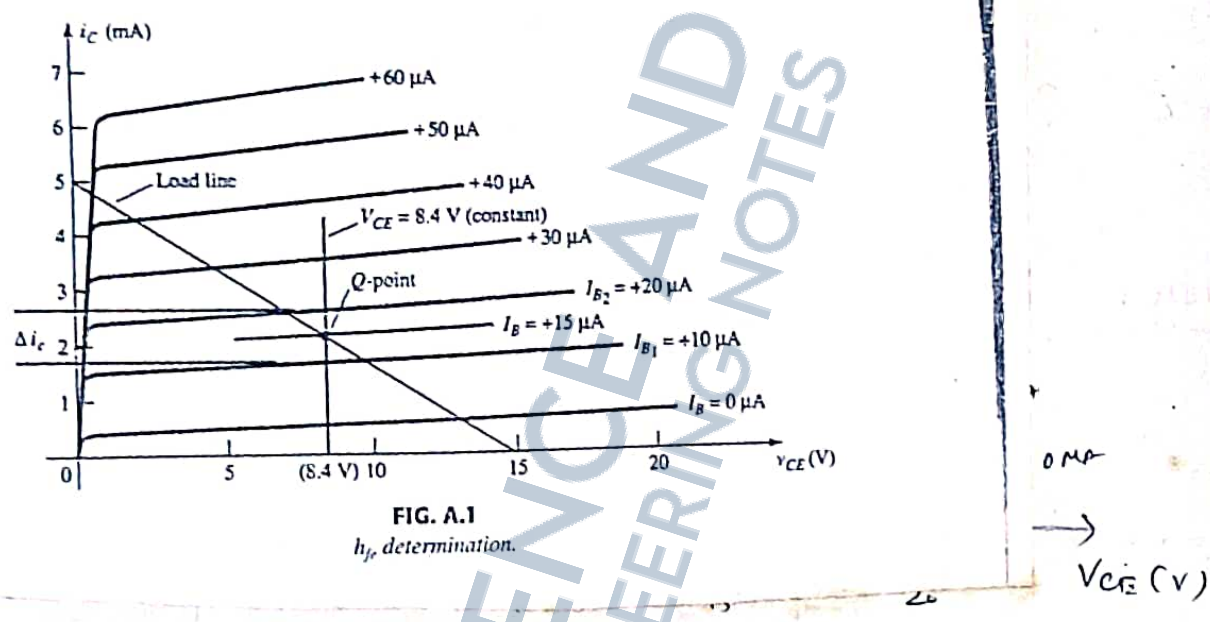


FIG. A.1
h_{fe} determination.

fig 1 :- hfe determination

2) $$h_{fe} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE} = \text{Const}} \quad \text{——— (1)}$$

So at a constant V_{CE} (Given by Q point), changes in base current and collector current can be obtained by the vertical line drawn through the Q-point.

3) Eqn (1), show small change in I_c to small change in i_b, So in fig 1, the change in i_b is chosen to extend from I_{B_1} to I_{B_2} along the perpendicular straight line at V_{CE}. The

Corresponding change in i_c is then found by drawing the horizontal lines from the intersection of I_{B1} & I_{B2}, with V_{CE} = Const to vertical axis. Then substitute the value of Δi_c & Δi_b in eqn ①,

$$|h_{fe}| = \left.\frac{\Delta i_c}{\Delta i_b}\right|_{V_{CE} = \text{constant}}$$

$$= \left.\frac{(2.7 - 1.7)\,mA}{(20-10)\,\mu A}\right|_{V_{CE} = 8.4\,V}$$

$$= \frac{1}{10} \times 10^{+3}$$

$$\boxed{|h_{fe}| = 100}$$

(B) **h_{oe} determination :-**

(i) $h_{oe} = \left.\frac{\Delta i_c}{\Delta V_{ce}}\right|_{I_B = \text{Const}}$

(i) First in the O/P characteristics, Q point is determined.

(ii) Then a straight line is drawn tangent to the curve I_B through the Q-point to establish a line I_B = Constant.

(iii) A change in V_{CE} was then chosen, and the corresponding change in i_c determined by drawing the horizontal lines to the vertical axis at the intersection on the I_B = constant, 273 line.

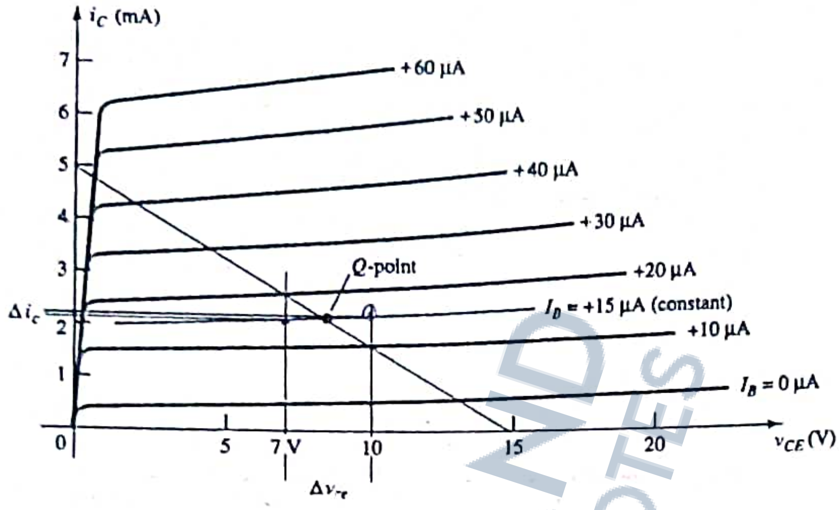


FIG. A.2
h_{oe} determination.

$$\therefore |h_{oe}| = \left.\frac{\Delta i_c}{\Delta V_{ce}}\right|_{I_B = \text{Const}} = \left.\frac{(2.2 - 2.1)\,mA}{(10 - 7)\,V}\right|_{I_B = 15\,\mu A}$$

$$= \frac{0.1 \times 10^{-3}}{3} = 33\ \frac{\mu A}{V}$$

$$\boxed{h_{oe} = 33\ \mu S}$$

(c) h_{ie} determination

(i) The Q point must be first found on the I/P or base characteristic curve [For Q point, we have a particular I_B]

(ii) $h_{ie} = \left.\dfrac{\Delta V_{be}}{\Delta i_b}\right|_{CE = \text{Const}}$.

(iii) At V_{CE} = 8.4 V, a line is drawn tangent Curve

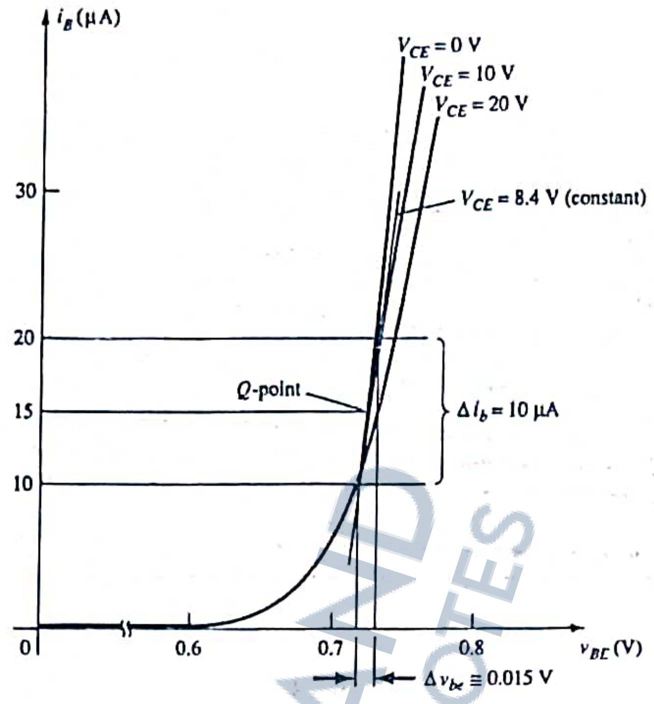


FIG. A.3
h_{ie} determination.

to the curve through the Q - point to establish a line V_{CE} = Const.

(iii) A small change in V_{be} is then chosen, resulting in a corresponding change in i_b.

$$\therefore \ |h_{ie}| = \left.\frac{\Delta V_{be}}{\Delta i_b}\right|_{V_{CE} = \text{const}} = \left.\frac{(733 - 718)\,\text{mV}}{(20 - 10)\,\mu A}\right|_{V_{CE} = 8.4V}$$

$$= \frac{15 \times 10^{-3}}{10 \times 10^{-6}}$$

$$\boxed{h_{ie} = 1.5\ K\Omega}$$

(D) <u>h_{re} determination :-</u>

(i) First Q - point is found out in I/p characteristics. [From a particular I_b corresponds to Q - point.]

(ii) $h_{re} = \left.\dfrac{\Delta V_{be}}{\Delta V_{ce}}\right|_{I_B = \text{constant}}$

(ii) A horizontal line is drawn through the Q-point at $I_B = 15\mu A$.

(iii) Then where ever the line cuts the corresponding V_{CE} curves, change in V_{CE} & change in V_{BE} is determined.

(iv) $$|h_{re}| = \left.\frac{\Delta V_{be}}{\Delta V_{ce}}\right|_{I_B = \text{Constant}}$$

$$= \frac{(733-725)\ mV}{(20-0)\ V}$$

$$= \frac{8 \times 10^{-3}}{20}$$

$$|h_{re}| = 4 \times 10^{-4}$$

~~The~~

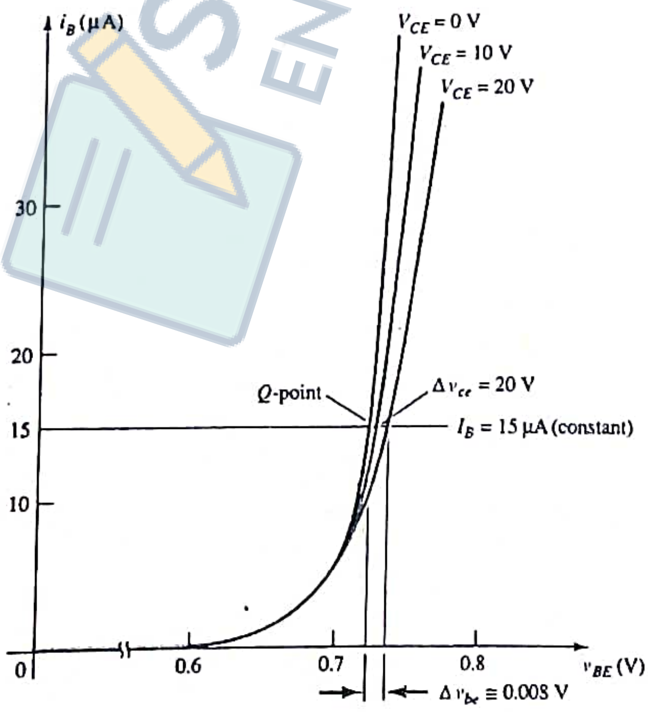


FIG. A.4
h_{re} determination.

The resultant ckt,

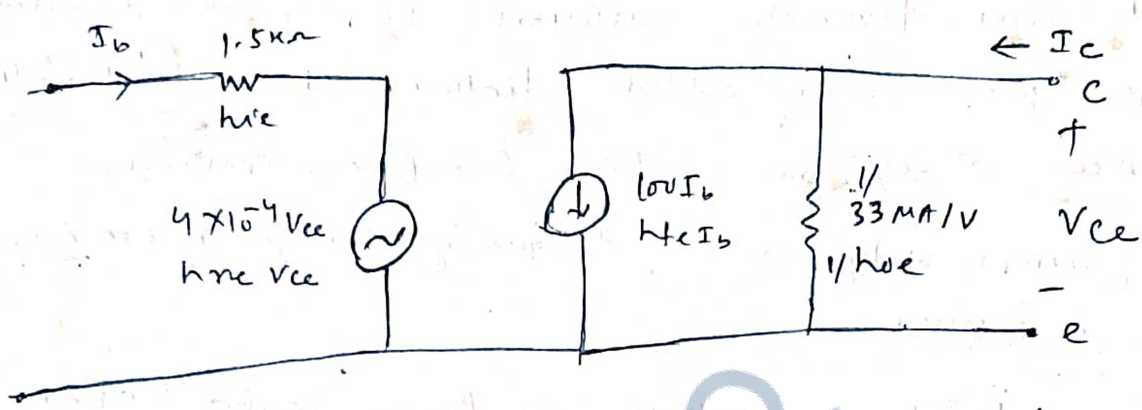


fig: 5 :- Complete hybrid equivalent ckt for a transistor having characteristics appears in fig 1-4.