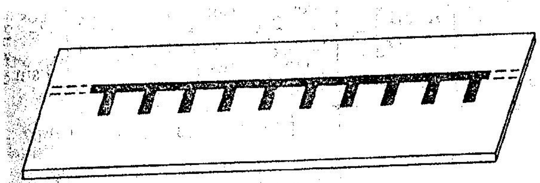
Ch-4 - Microwave Filters [Pozar Book]

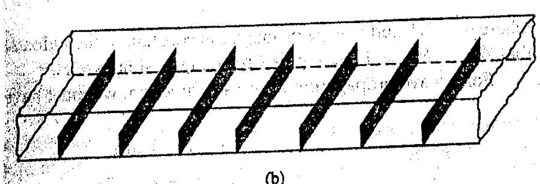
A microwave filter is a 2-port network used to control the freq. response at a certain point in a microwave system by providing transmission at frequencies within the passband of the filter and attenuation in the stopband of the filter. Typical freq responses include low-pass, high-pass, bandpass and band-reject characteristics. Appln's can be found in virtually any type of microwave communication, radar or test and measurement system.

Periodic Structures :-

An infinite transmission line or waveguide periodically loaded with reactive elements is referred to as periodic structure. As shown in fig 1, periodic structure can take various forms, depending on the transmission



(a)



(b)

fig 1:- Examples of periodic structures. (a) Periodic stubs on a microstrip line. (b) Periodic diaphragms in a waveguide.

line media being used. Often the loading elements are formed as discontinuities in the line, but in any case they can be modeled as lumped reactances across a transmission line as shown in figure 2.

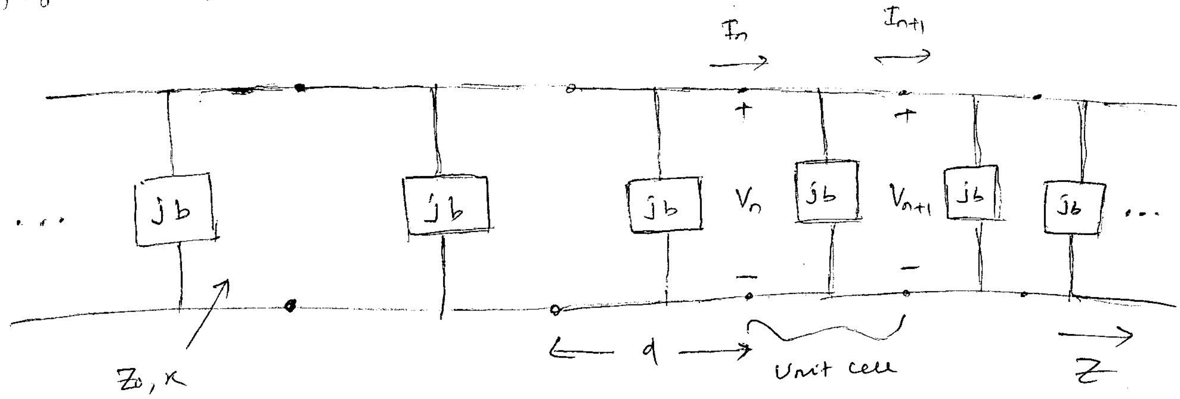


fig 2 – Equivalent circuit of a periodically loaded transmission line. The unloaded line has characteristic impedance Z_0 and propagation constant k.

→ Periodic structures support slow-wave propagation (slower than phase velocity of the unloaded line), and have passband and stopband characteristics similar to those filters; they find appl. in TWT, (Travelling wave Tube) phase shifters and antennas.

→ **Analysis of Infinite periodic Structure:-**

The analysis can be done using the propagation characteristic of the infinite loaded line shown in fig 2. Each unit cell of this line consists of length 'd' of T.L with a shunt susceptance ~~susceptance~~ across the midpoint of the line; the susceptance 'b' is normalized to the characteristic impedance, Z_0.

If we consider the infinite line as being composed of a cascade of identical two-port n/ws, we can relate the voltages and currents on either side of the n^{th} unit cell using ABCD matrix:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \quad \text{—— (1)}$$

Where A, B, C, D are matrix parameters for a cascade of a transmission line section of length '$\frac{d}{2}$' and a shunt susceptance b and another T-L section of length $\frac{d}{2}$.

In normalized form,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jb & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

Directly written in Book

$$\Rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} = {}^{*}\begin{bmatrix} \left(\cos\theta - \frac{b}{2}\sin\theta\right) & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \left(\cos\theta - \frac{b}{2}\sin\theta\right) \end{bmatrix}$$

* Proof:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jb & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} - b\sin\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} + jb\cos\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos^2\frac{\theta}{2} - b\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} - \sin^2\frac{\theta}{2} & j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} - jb\sin^2\frac{\theta}{2} + j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} \\ j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} + jb\cos^2\frac{\theta}{2} + j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} & -\sin^2\frac{\theta}{2} - b\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta - \frac{b}{2}\sin\theta & 2j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} - b\sin^2\frac{\theta}{2} \\ +2j\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} + jb\cos^2\frac{\theta}{2} & \cos\theta - \frac{b}{2}\sin\theta \end{bmatrix}$$

$$\left[\because \begin{array}{l} \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \Rightarrow \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} \\ \sin 2\theta = 2\sin\theta\cdot\cos\theta \\ \Rightarrow \sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} \end{array} \right]$$

$$= \begin{bmatrix} \cos\theta - \frac{b}{2}\sin\theta & j\sin\theta - jb\left(\frac{1-\cos\theta}{2}\right) \\ j\sin\theta + jb\left(\frac{1+\cos\theta}{2}\right) & \cos\theta - \frac{b}{2}\sin\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\cos\theta - \frac{b}{2}\sin\theta\right) & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \left(\cos\theta - \frac{b}{2}\sin\theta\right) \end{bmatrix} \quad \text{—(2)}$$

(Proved)

Where $\theta = Kd$ and K is propagation constant.

And $AD - BC = 1$, as required for reciprocal n/w.

→ For a wave propagating in $+Z$ direction, we must have

$$V(z) = V(0)\cdot e^{-\gamma z}, \quad \text{———— 3(a)}$$
$$I(z) = I(0)\cdot e^{-\gamma z}, \quad \text{———— 3(b)}$$

for a phase reference at $Z=0$. Since the structure is infinitely long, the voltage and current at the nth terminals can differ from the voltage and current at the $(n+1)$ terminals only by the propagation factor, $e^{-\gamma d}$. Thus

$$V_{n+1} = V_n \cdot e^{-\gamma d} \qquad \text{--- 4(a)}$$

$$I_{n+1} = I_n \cdot e^{-\gamma d} \qquad \text{--- 4(b)}$$

Using ~~eqn this~~ result in eqn ①,

$$\cancel{\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} e^{-\gamma d} \\ I_{n+1} e^{-\gamma d} \end{bmatrix}}$$

$\therefore$

$$V_n = V_{n+1} e^{\gamma D} \qquad \text{--- 4(c)}$$

$$I_n = I_{n+1} \cdot e^{\gamma D} \qquad \text{--- 4(d)}$$

From eqn ①,

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \qquad \text{--- 4(e)}$$

But from eqn 4(c) and 4(d),

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} V_{n+1} \cdot e^{\gamma d} \\ I_{n+1} \cdot e^{\gamma d} \end{bmatrix} \qquad \text{--- 4(f)}$$

Equating both the results, we have

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} \cdot e^{\gamma d} \\ I_{n+1} \cdot e^{\gamma d} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} e^{\gamma d} & 0 \\ 0 & e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} - \begin{bmatrix} e^{\gamma d} & 0 \\ 0 & e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = 0 \quad \text{--- (5)}$$

For non-trivial soln, the determinant of the above matrix must vanish.

$$\left(A - e^{+\gamma d}\right)\left(D - e^{\gamma d}\right) - BC = 0$$

$$\Rightarrow AD - A \cdot e^{\gamma d} - D e^{\gamma d} + e^{2\gamma d} - BC = 0$$

$$\Rightarrow AD + e^{2\gamma d} - (A+D) \cdot e^{\gamma d} - BC = 0 \quad \text{--- (6)}$$

Since for a reciprocal n/w, $AD - BC = 1$,

$$\Rightarrow 1 + e^{2\gamma d} - (A+D) \cdot e^{\gamma d} = 0$$

$$\Rightarrow 1 + e^{2\gamma d} = (A+D) \cdot e^{\gamma d}$$

$$\Rightarrow A + D = e^{-\gamma d} + e^{\gamma d} = 2\cosh \gamma d$$

$$\Rightarrow \quad \cosh \gamma d = \frac{A+D}{2} \quad \text{——— (7)}$$

From eq^n (2),

$$A = \cos\theta - \frac{b}{2}\sin\theta$$

$$D = \cos\theta - \frac{b}{2}\sin\theta$$

$$\Rightarrow \quad \frac{A+D}{2} = \cos\theta - \frac{b}{2}\sin\theta \quad \text{——— (8)}$$

Using eq^n (8) in eq^n (7), we have

$$\Rightarrow \quad \cosh \gamma d = \cos\theta - \frac{b}{2}\sin\theta \quad \text{——— (9)}$$

Now if $\gamma = \alpha + j\beta$, we have, $\cosh \gamma d$

$$\cosh(\alpha d + j\beta d) = \cosh \alpha d \cdot \cos\beta d + j \sinh \alpha d \cdot \sin\beta d \quad \text{— (10)}$$

Using eq^n (9), in eq^n (10), we have

$$\cos\theta - \frac{b}{2}\sin\theta = \cosh \alpha d \ \cos\beta d + j \sinh \alpha d \cdot \sin\beta d \quad \text{— (11)}$$

Since left hand side of eq^n (11), is purely real we must have either $\alpha = 0$ or $\beta = 0$ $[\sinh 0 = 0]$

α = Attenuation Constant

β = Propagation Constant

Case-1 :- $\alpha = 0, \ \beta \neq 0$

This case corresponds to a non attenuating, propagating wave on the periodic structure, and defines the passband of the structure. Then eq^n (11) reduces to,

$$\cos\beta d = \cos\theta - \frac{b}{2}\sin\theta \quad \text{——— (12)}$$

$\because \sinh \alpha d = 0$
$\cosh \alpha d = \cos 0 = 1$

Since $|\cos\beta a| \leq 1$

$$\Rightarrow \left|\cos\theta - \frac{b}{2}\sin\theta\right| \leq 1 .$$

[Note there are an infinite number of values of 'β' that can satisfy eqn (12)]

Case - 2 :- $\alpha \neq 0$, $\beta = 0, \pi$

In this case the wave doesn't propagate, but is attenuated along the line; this defines the stop band structure. Because the line is lossless, power is not dissipated, but is reflected back to the i/p of the line.

So eqn (11), reduces to

$$\cosh \alpha d = \left|\cos\theta - \frac{b}{2}\sin\theta\right| \geq 1$$

which has only one solution ($\alpha > 0$) for positively travelling waves; ($\alpha < 0$) applies for -vely travelling waves.

Thus, depending on the freq and normalized susceptance values, the periodically loaded line will exhibit either passband or stop bands, and so can be considered as a type of filter.

Note :- The voltage and current waves defined on eqn (3) & (4), similar to the elastic waves (Bloch waves) that propagates through periodic crystal lattice.

Characteristic Impedance

We can define characteristic impedance at the unit cell terminal as

$$Z_B = Z_0 \cdot \frac{V_{n+1}}{I_{n+1}} , \qquad \text{—— (13)}$$

Since V_{n+1} and I_{n+1} in the above derivation are normalized quantities. This impedance is also referred to as Bloch impedance.

From eqn (5),

$$\begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad (A - e^{\gamma d}) V_{n+1} + B I_{n+1} = 0 \qquad \text{—— (14)}$$

$$\Rightarrow \quad (A - e^{+\gamma d}) V_{n+1} = -B I_{n+1}$$

$$\Rightarrow \quad \frac{V_{n+1}}{I_{n+1}} = \frac{-B}{(A - e^{\gamma d})} \qquad \text{—— (15)}$$

Putting eqn (15), in eqn (13), we have

$$Z_B = Z_0 \cdot \left(\frac{-B}{A - e^{\gamma d}} \right) = \frac{-B Z_0}{A - e^{\gamma d}} \qquad \text{—— (16)}$$

Eqn (6) is a quadratic eqn

$$an^2 + bx + c = 0$$

$$(e^{\gamma d})^2 - (A+D) e^{\gamma d} + (AD - BC) = 0$$

at

$$\Rightarrow e^{\gamma d} = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4.(1).(AD-BC)}}{2.(1)}$$

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$$\Rightarrow e^{\gamma d} = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4}}{2} \quad \text{—— (17)}$$

($\because AD-BC=1$ for reciprocal n/w as discussed after eqn (6))

Then Bloch Impedance, Z_B will be given by, [Putting eqn (17) in eqn (16)]

$$Z_B = \frac{-B.Z_0}{A - \left[\frac{(A+D) \pm \sqrt{(A+D)^2 - 4}}{2}\right]}$$

$$= \frac{-2BZ_0}{2A - A - D \pm \sqrt{(A+D)^2 - 4}}$$

$$= \frac{-2BZ_0}{(A-D) \pm \sqrt{(A+D)^2 - 4}}$$

$\therefore$ Bloch Impedance has 2 solⁿ, given by

$$Z_B^{\pm} = \frac{-2BZ_0}{(A-D) \mp \sqrt{(A+D)^2 - 4}} \quad \text{—— (18)}$$

(16)

For symmetrical cell [As shown in fig 2], we will always have $A=D$. In this case,

$$Z_B^{\pm} = \frac{\pm 2BZ_0}{\sqrt{A^2-1}} \quad \text{—— (19)}$$

The $\pm$ solutions corresponds to the characteristic impedance for +vely and -vely travelling waves, respectively. For symmetrical n/ws these impedances are the same except for the sign;

The characteristic impedance for a -vely travelling wave turns out to be -ve because we have defined I_n in fig 2, as always being in the +ve direction,

From eqn ②, we see that 'B' is Purely imaginary.

If $\alpha = 0$, $\beta \neq 0$ [Pass band] then

$\cosh \gamma d = \cos\theta - \frac{b}{2}\sin\theta$ [from eqn 9]

$\cosh \gamma d = A$ [From eqn 2]

$\therefore A = \cos\theta - \frac{b}{2}\sin\theta \leq 1$

$\therefore Z_B = \dfrac{\pm B Z_0}{\sqrt{A^2 - 1}}$ [Since B is imaginary and $A < 1$]

Z_B is ~~purel~~ real.

If $\alpha \neq 0$, $\beta = 0$ [Stop band].

$\cosh \gamma d = \cosh \alpha d = A \geq 1$

and B is imaginary.

Z_B is imaginary.

Filter design by image Parameter method :-

The image parameter method of filter design involves the specification of passband and stopband characteristics for a cascade two-port n/ws. [Similar to periodic structure, as discussed]

Disadvantage :- Although the method is relatively simple but has disadvantage that an arbitrary freq response can't be incorporated into the design.

Image Impedances and transfer f^n for 2-port n/ws :-

Consider the arbitrary 2-port n/w shown in fig 3, where the n/w is specified by its ABCD parameters.

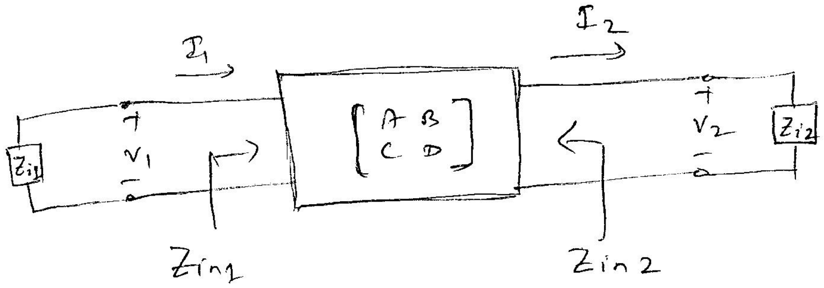

fig 3:- A 2-port n/w terminated in its image impedances.

The image impedances, Z_{i1} and Z_{i2} are defined as,

Z_{i1} = i/p impedance at port 1 when port 2 is terminated with Z_{i2}.

Z_{i2} = i/p impedance at port 2 when port 1 is terminated with Z_{i1}.

Thus both ports are matched when terminated in their image impedances.

The port voltages and currents are related as

$$V_1 = AV_2 + BI_2, \quad \text{------ } 20(a)$$

$$I_1 = CV_2 + DI_2 \quad \text{------ } 20(b)$$

The i/p impedance at port 1, with port 2 terminated in Z_{i2}, is

$$Z_{in1} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} = \frac{A.(Z_{i2}\cancel{I_2}) + B\cancel{I_2}}{C(Z_{i2}\cancel{I_2}) + D\cancel{I_2}}$$

$$\left[\because Z_{i2} = \frac{V_2}{I_2} \Rightarrow V_2 = Z_{i2} I_2\right.$$

$$\Rightarrow Z_{in1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad \text{------ } (21)$$

<u>To find Z_{in2}</u>,

$$\text{Inverse of } \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}}{AD - BC}$$

[To find inverse of 2x2, matrix short cut method is exchange the main diagonal elements, multiply -1 in other 2 diagonal elements. Now ~~the~~ divide the new matrix with determinant of original 2x2 matrix]

For a reciprocal n/w, $AD - BC = 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

So

$$V_2 = DV_1 - BI_1, \quad \text{------ } 22(a)$$

$$I_2 = -CV_1 + AI_1, \quad \text{------ } 22(b)$$

$\Rightarrow$

Then i/p impedance at port 2, with port 1 terminated with Z_{i1}, can be found as

$$Z_{in2} = \frac{-V_2}{I_2} = -\left[\frac{DV_1 - BI_1}{-CV_1 + AI_1}\right]$$

Since $Z_{i1} = -\frac{V_1}{I_1} \Rightarrow V_1 = -Z_{i1} I_1$

$$\therefore Z_{in2} = -\left[\frac{D(-Z_{i1}\cancel{I_1}) - B\cancel{I_1}}{-C(-Z_{i1}\cancel{I_1}) + A\cancel{I_1}}\right] = \frac{DZ_{i1} + B}{CZ_{i1} + A} \quad (23)$$

We desire that,

$Z_{in1} = Z_{i1}$ and $Z_{in2} = Z_{i2}$

So Eqn (21) & (23) becomes,

$$Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad \text{and} \quad Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

$$\Rightarrow Z_{i1}(CZ_{i2} + D) = AZ_{i2} + B \quad (24)$$

and $Z_{i2}(CZ_{i1} + A) = DZ_{i1} + B \quad (25)$

Solving eqn (24) & (25),

$$Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} = \frac{A\cdot\left(\frac{DZ_{i1} + B}{CZ_{i1} + A}\right) + B}{C\cdot\left(\frac{DZ_{i1} + B}{CZ_{i1} + A}\right) + D}$$

$$\Rightarrow Z_{i1} = \frac{ADZ_{i1} + AB + BCZ_{i1} + AB}{CDZ_{i1} + BC + CDZ_{i1} + AD}$$

$$\Rightarrow CDZ_{i1}^2 + Z_{i1}\cancel{BC} + CDZ_{i1}^2 + AD\cancel{Z_{i1}} = AD\cancel{Z_{i1}} + AB + \cancel{BC Z_{i1}} + AB$$

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$$\Rightarrow \quad 2 CD Z_{i1}^2 = 2 AB$$

$$\Rightarrow \quad Z_{i1}^2 = \frac{AB}{CD}$$

Imp

$$\Rightarrow \quad \boxed{Z_{i1} = \sqrt{\frac{AB}{CD}}} \qquad (26)$$

and $$Z_{i2} = \frac{D Z_{i1} + B}{C Z_{i1} + A}$$

$$= \frac{D \times \sqrt{\frac{AB}{CD}} + B}{C \times \sqrt{\frac{AB}{CD}} + A}$$

$$Z_{i2} = \frac{D\sqrt{AB} + B\sqrt{CD}}{C\sqrt{AB} + A\sqrt{CD}}$$

$$= \frac{\sqrt{D}\sqrt{B}\ \cancel{(\sqrt{D}.\sqrt{A} + \sqrt{B}\sqrt{C})}}{\sqrt{C}\sqrt{A}\ \cancel{(\sqrt{C}\sqrt{B} + \sqrt{A}\sqrt{D})}}$$

Imp

$$\Rightarrow \quad \boxed{Z_{i2} = \sqrt{\frac{BD}{AC}}} \qquad (27)$$

$$\therefore \quad \frac{Z_{i1}}{Z_{i2}} = \frac{\sqrt{AB}}{\sqrt{CD}} \times \frac{\sqrt{AC}}{\sqrt{BD}} = \frac{A\sqrt{BC}}{D\sqrt{BC}} = \frac{A}{D}$$

$$\Rightarrow \quad Z_{i1} = \frac{A}{D} Z_{i2}$$

If the n/w is symmetric, $A = D$ and

$Z_{i1} = Z_{i2}$ as expected.

Voltage Transfer function

Consider a 2-port n/w terminated in its image impedances, from eqn 22(a), we have (fig 4)

$$V_2 = DV_1 - BI_1 \quad \cancel{= D(V_1 -)}$$

$$= \left(D - B \cdot \frac{I_1}{V_1}\right) V_1$$

$$V_2 = \left(D - \frac{B}{Z_{i1}}\right) V_1 \quad \text{——— (28)}$$

$$\left(\because Z_{i1} = \frac{V_1}{I_1}\right)$$

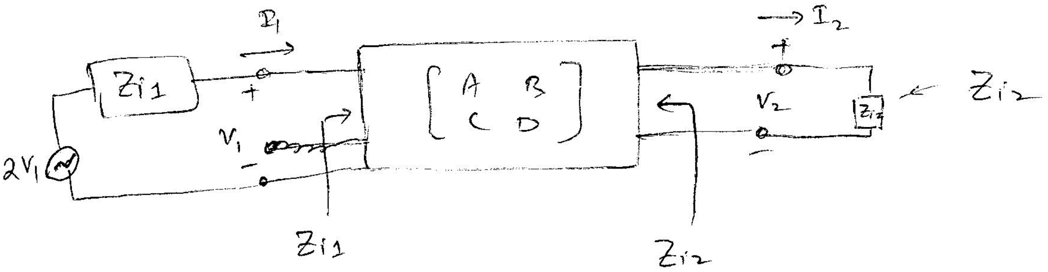


fig:- 4:- A 2-port n/w terminated on its image impedances and driven with a voltage generator

From eqn (28),

$$\frac{V_2}{V_1} = D - \frac{B}{Z_{i1}} = D - B \cdot \sqrt{\frac{CD}{AB}} \quad \left[\text{From eqn } 26\right]$$

$$\Rightarrow \frac{V_2}{V_1} = \sqrt{\frac{D}{A}} \left[\sqrt{AD} - \sqrt{BC}\right] \quad \text{——— (29)}$$

$\frac{A}{D}$

Similarly, from eqn 22 (b),

$$I_2 = I_1 \left(-C \frac{V_1}{I_1} + A\right) = I_1 \left(-C \cdot Z_{i1} + A\right)$$

$$\Rightarrow \frac{I_2}{I_1} = -C \cdot \sqrt{\frac{AB}{CD}} + A = \sqrt{\frac{A}{D}} \left[\sqrt{AD} - \sqrt{BC}\right] \text{— (30)}$$

The factor $\sqrt{\frac{D}{A}}$ occurs in reciprocal pusn in (29) & (30) and so can be interpreted as transformer turns ratio.

The propagation factor for the n/w can be defined as,

$$e^{-\gamma} = \sqrt{AD} - \sqrt{BC} \quad , \text{ where } \gamma = \alpha + j\beta \qquad \text{—(31)}$$

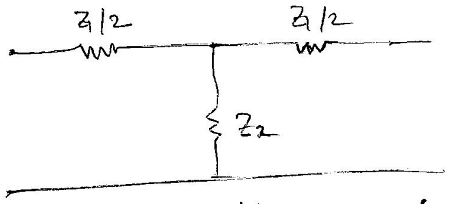
$$\text{Since, } e^{\gamma} = \frac{1}{\sqrt{AD} - \sqrt{BC}} = \frac{AD - BC}{\sqrt{AD} - \sqrt{BC}} = \sqrt{AD} + \sqrt{BC} \quad \text{—(32)}$$

$$\left[\because AD - BC = (\sqrt{AD} + \sqrt{BC})(\sqrt{AD} - \sqrt{BC}) \right]$$

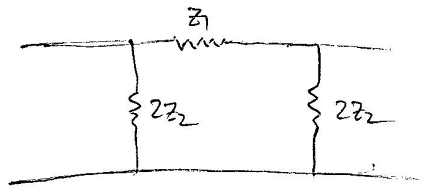
$$\cosh \gamma = \frac{e^{\gamma} + e^{-\gamma}}{2} = \sqrt{AD} \qquad \text{—— (33)}$$

Two important types of 2-port n/ws are T and π circuits.

Image parameters for T and π networks :- Table 1.1



T N/w



π N/w

fig 5:-

T N/w	π N/w
ABCD Parameters	**ABCD parameters**
$A = 1 + Z_1/2Z_2$	$A = 1 + Z_1/2Z_2$
$B = Z_1 + Z_1^2/4Z_2$	$B = Z_1$
$C = 1/Z_2$	$C = 1/Z_2 + Z_1/4Z_2^2$
$D = 1 + Z_1/2Z_2$	$D = 1 + Z_1/2Z_2$
Z- Parameters:	**Y- parameters :-**
$Z_{11} = Z_{22} = Z_2 + Z_1/2$	$Y_{11} = Y_{22} = \frac{1}{Z_1} + \frac{1}{2Z_2}$

$Z_{12} = Z_{21} = Z_2$

Image Impedance

$Z_{iT} = \sqrt{Z_1 Z_2}\,\sqrt{1 + Z_1/4Z_2}$

Propagation Constant :-

$e^{\gamma} = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1^2/4Z_2^2}$

$Y_{12} = Y_{21} = \dfrac{1}{Z_1}$

Image Impedance

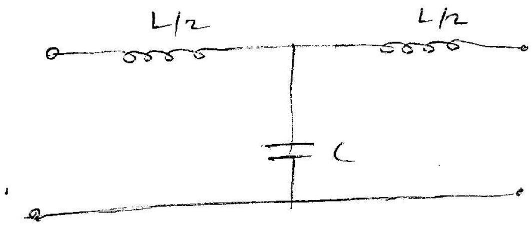
$Z_{i\pi} = \sqrt{Z_1 Z_2}\,/\sqrt{1 + Z_1/4Z_2} = \dfrac{Z_1 Z_2}{Z_{iT}}$

Propagation Constant :-

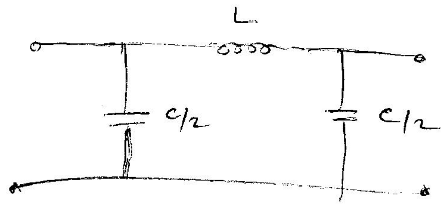
$e^{\gamma} = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1^2/4Z_2^2}$

Constant -K filter sections :-

Now we can develop low-pass and high-pass filter sections. Consider the 'T' n/w shown in figure 5.



(a)



(b)

fig 6 :- Low pass Constant - k filter sections in T and π form (a) T-section (b) π - section.

→ We can see that this is a low pass filter n/w because the series inductors and shunt capacitor tend to block high-freq signals while passing low-freq signals.

Comparing fig 5 & 6, $Z_1 = jωL$

$Z_2 = \dfrac{1}{jωC}$

From Table 1 :-

and $Z_{iT} = \sqrt{Z_1 . Z_2}\,\sqrt{1 + \dfrac{Z_1}{4Z_2}} = \sqrt{jωL . \dfrac{1}{jωC}}\,\sqrt{1 + \dfrac{jωL \times jωC}{4}}$

$Z_{iT} = \sqrt{\dfrac{L}{C}}\,\sqrt{1 - \dfrac{ω^2 LC}{4}}$ — (34)

To find cutoff freq

$$1 - \frac{\omega_c^2 LC}{4} = 0$$

$$\Rightarrow \quad 1 = \frac{\omega_c^2 LC}{4}$$

$$\Rightarrow \quad \omega_c^2 = \frac{4}{LC}$$

$$\Rightarrow \quad \boxed{\omega_c = \frac{2}{\sqrt{LC}}} \quad \text{--- (35)}$$

Defining a nominal characteristic impedance

$$R_0 = \sqrt{\frac{L}{C}} = K, \quad \text{where } K \text{ is a constant},$$

then $\quad Z_{iT} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}}$ — (36) $\quad \because \omega_c = \frac{2}{\sqrt{LC}}$

$\therefore \quad \boxed{Z_{iT} = R_0}$, for $\omega = 0$

→ Propagation factor, $e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}}$

$$\therefore e^{\gamma} = 1 + \frac{j\omega L(j\omega C)}{2} + \sqrt{j\omega L(j\omega C) + \frac{j^2\omega^2 L^2 (j^2\omega^2 C^2)}{4}}$$

$$\Rightarrow e^{\gamma} = 1 - \frac{\omega^2 LC}{2} + \sqrt{-\omega^2 LC + \frac{\omega^4 L^2 C^2}{4}}$$

$$= 1 - \frac{2\omega^2}{\left(\frac{4}{LC}\right)} + \sqrt{\omega^2 LC} \sqrt{\frac{\omega^2 L^2 C}{4} - 1}$$

$$= 1 - \frac{2\omega^2}{\omega_c^2} + \frac{2\omega}{\left(\frac{2}{\sqrt{LC}}\right)} \sqrt{\frac{\omega^2}{\left(\frac{4}{LC}\right)} - 1}$$

$$\Rightarrow e^{\gamma} = 1 - \frac{2\omega^2}{\omega_c^2} + \frac{2\omega}{\omega_c}\sqrt{\frac{\omega^2}{\omega_c^2} - 1}$$

$$\therefore e^{\gamma} = 1 - \frac{2\omega^2}{\omega_c^2} + \frac{2\omega}{\omega_c}\sqrt{\frac{\omega^2}{\omega_c^2} - 1} \quad \text{--- (37)}$$

Now consider 2 freq regions:-

1) For $\omega < \omega_c$: This is the passband of the filter section. Eq^n (36) shows that Z_{iT} is real, and eq^n (37) shows that e^{γ} is imaginary.

(since $\frac{\omega}{\omega_c^2} - 1$ is $-ve$)

$\frac{2}{\sqrt{LC}}$

2) For $\omega > \omega_c$:

This is the stop band of the filter. Eq^n (36) shows Z_{iT} is imaginary and eq^n (37) shows that e^{γ} is real.

Z_2^2

$j^2\omega^2 c^2)$

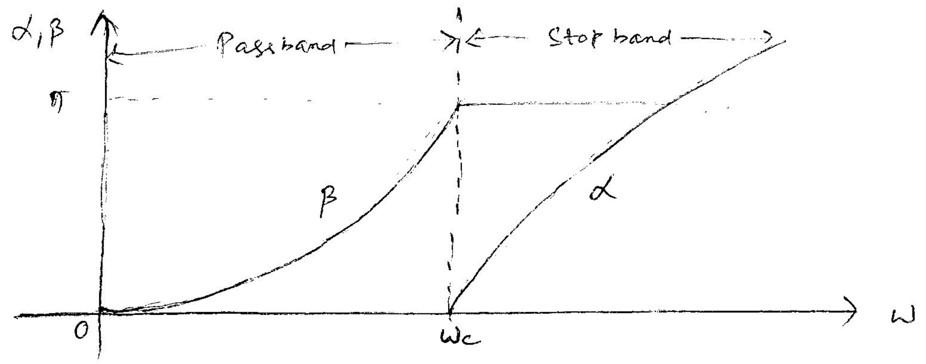


fig 7:- Typical passband and stop band characteristics of low-pass constant-K section

Typical phase and attenuation constants are sketched in fig 7. We observe that the attenuation, 'α', is zero or relatively small near the cutoff freq and $\alpha \to \infty$ as $\omega \to \infty$,

This type of filter is known as a constant-k low pass prototype. There are only two parameters to choose (L and C), which are determined by ω_c, the cutoff freq and R_0, the image impedance at zero freq.

→ The above results are valid only when the filter section is terminated on its image impedance at both ports.

→ This is a major weakness of the design, because the image impedance is a function of freq, which is not likely to match a given source or load impedance.

→ This disadvantage can be eliminated with modified (m-derived) filter section.

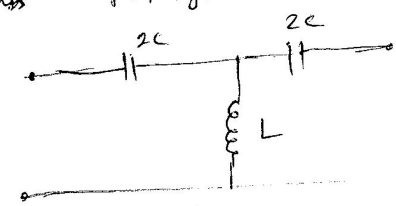
→ For low pass π n/w $Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$

So propagation factor is same as that of 'T' n/w.

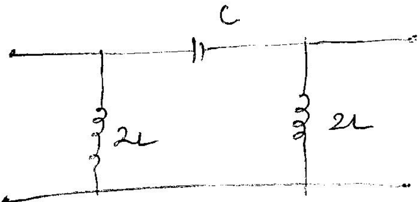
Also the 'ω_c' and 'R_0' are same as that of 'T' n/w.

→ At $\omega = 0$, $Z_{iT} = Z_{i\pi} = R_0$ where $Z_{i\pi}$ is the image impedance of low-pass-π n/w, but Z_{iT} and $Z_{i\pi}$ are generally not equal at other frequencies.

→ High pass const-K section are shown in fig 8. The posn of inductor and capacitor are reversed from that of low-pass prototype.



(a)



(b)

Fig 8 :- High pass Const-k filter section in 'T' and 'π' form (a) T- section (b) π- section

The design eqn can be shown to be

$$R_o = \sqrt{\frac{L}{C}} \quad , \quad \omega_c = \frac{1}{2\sqrt{LC}}$$

Note : R_o is same as that Low-pass type

but $\omega_c = \dfrac{2}{\sqrt{LC}}$ in low pass type.

Filter Design by the Insertion loss method

→ The perfect (ideal) filter would have zero insertion loss in the passband, infinite attenuation in the stop band and a linear phase response (to avoid signal distortion) in the passband.

→ The image parameter method may yield a usable filter response, but there is no clear-cut way to improve the design, But the insertion loss method, allows a high degree of control over the passband and stop band amplitude and phase characteristics, with a systematic way to synthesize a desired response.

→ e.g

Minm insertion loss method — Binomial response could be used.

Chebyshev response would satisfy a requirement for sharp cutoff.

linear phase filter → for better phase response.

→ In all cases, insertion loss method allows filter performance to be improved in a straightforward manner, at the expense of higher order filter. [order of filter = Number of reactive components]

Characterization by power loss ratio :-

In the insertion loss method a filter response is defined by its insertion loss, or power loss ratio, P_{LR} :

$$P_{LR} = \frac{\text{Power Available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1-|\Gamma(\omega)|^2}$$

$$\boxed{P_{LR} = \frac{1}{1-|\Gamma(\omega)|^2}} \quad \text{—— (38)}$$

$\Gamma(\omega)$ = Reflection Coefficient

The insertion loss (IL) in dB,

$$IL = 10 \log P_{LR} \quad \text{—— (39)}$$

→ But we know that $|\Gamma(\omega)|^2$ is a even function of 'ω' [Derivation given in Pozar Book sec-4.1], therefore it can be expressed as polynomial in ω^2. Thus, we can write

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)} \quad \text{—— (40)}$$

where M & N are real polynomials in ω^2.

∴ Using eqn (40) in eqn (38), we have

$$P_{LR} = \frac{1}{1-\frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}} = \frac{M(\omega^2)+N(\omega^2)}{\cancel{M(\omega^2)}+N(\omega^2)-\cancel{M(\omega^2)}}$$

$$\Rightarrow P_{LR} = 1+\frac{M(\omega^2)}{N(\omega^2)} \quad \text{—— (41)}$$

Thus, for a filter to be physically realizable, its power loss ratio must be of the form in eq^n (41).

Some practical filter responses:

Maximally flat

The characteristic is also called binomial or Butterworth response, and is optimum in the sense that it provides the flattest possible passband response for a given filter complexity or order.

For a low pass filter, it is specified by

$$P_{LR} = 1 + K^2 \left(\frac{\omega}{\omega_c}\right)^{2N} \quad \text{--- (42)}$$

where N is the order of the filter and 'ω_c' is the cutoff freq. The passband extends from $\omega = 0$ to $\omega = \omega_c$. At the band edge the power loss ratio is $1+K^2$.

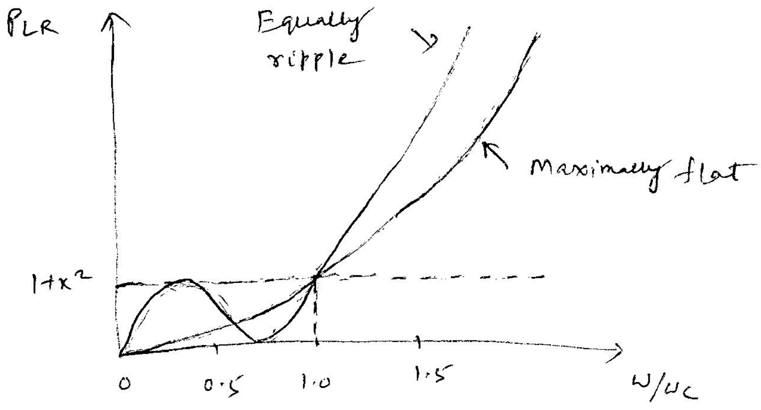


fig:- 9:- Maximally flat and equal-ripple low-pass filter responses ($N = 3$)

→ For -3 dB point, $P_{LR} = 2$ ($\because$ Power delivered to load $= \frac{1}{2}$ of power incident)

$$\Rightarrow 1 + K^2 = 2$$

$$\Rightarrow \boxed{K = 1}$$

For $\omega > \omega_c$, the attenuation increases monotonically with freq, [shown in fig 9].

For $\omega >> \omega_c$, $\boxed{P_{LR} \simeq K^2 \left(\frac{\omega}{\omega_c}\right)^{2N}}$, — 42(a) which shows that insertion loss increases at the rate of 20N dB/decade.

Equal ripple :-

If a Chebyshev polynomial is used to specify the insertion loss of an N-order low-pass filter as

$$P_{LR} = 1 + K^2 T_N^2 \left(\frac{\omega}{\omega_c}\right) \quad \text{(43)}$$

then a sharper cutoff will result, although the passband response will have ripples of amplitude $(1+K^2)$, [fig 9],

Since $T_N(x)$ oscillates betn ± 1 for $|x| \leq 1$. Thus, K^2 determines the passband ripple level. For, larger x, $T_N(x) \simeq \frac{1}{2}(2x)^N$, so for $\omega >> \omega_c$ the insertion loss becomes,

$$P_{LR} \simeq \frac{K^2}{4} \left(\frac{2\omega}{\omega_c}\right)^{2N} \quad \text{(44)}$$

$T_N(x) = T_N\left(\frac{\omega}{\omega_c}\right)$

$x = \frac{\omega}{\omega_c}$

$2x = \frac{2\omega}{\omega_c}$

which also increases at the rate of 20N dB/decade. But the insertion loss for Chebyshev case is $\left(\frac{2^{2N}}{4}\right)$ greater than the binomial response, at any given freq, $\omega >> \omega_c$. [Comparing Eqn 42(a) & (44)]

Filter Transformations:-

(a) The low-pass filter prototypes, we have studied, were normalized designs having a source impedance of $R_s = 1\Omega$ and a cutoff freq $\omega_c = 1$. Now we show how these designs can be scaled in terms of impedance and freq and converted to give high-pass, bandpass, or band-stop characteristics.

sertion

Impedance and freq scaling:-

Impedance Scaling:-

In prototype design, the source and load resistances are unity. A source resistance of R_0 can be obtained by multiplying the impedances of the prototype design by R_0.

$\leq 1.$ → If we let primes (′) denotes impedance scaled quantities, we have the new filter component values given by

R_0,

$$L' = R_0 \cdot L \quad \text{——— (45)}$$

$$C' = \frac{C}{R_0} \quad \text{——— (46)}$$

$$R_s' = R_0 \quad \text{——— (47)}$$

$$R_L' = R_0 \cdot R_L \quad \text{——— (48)}$$

where L, C, and R_L are the component values for the original prototype

Frequency Scaling for low-pass filter:-

To change the cutoff freq of a low-pass prototype from unity to 'ω_c' requires that we scale the freq dependence of filter by the factor $\frac{1}{\omega_c}$, which is accomplished by replacing ω by $\frac{\omega}{\omega_c}$:

$$\omega \leftarrow \frac{\omega}{\omega_c} \quad \text{——— (49)}$$

Then new power loss ratio will be

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right) \quad \text{——— (50)}$$

where ω_c is the new cutoff freq; cutoff occurs when $\frac{\omega}{\omega_c} = 1$ or $\omega = \omega_c$. This transformation can be viewed as stretching or expansion of the original passband, as shown in fig 10 a, b.

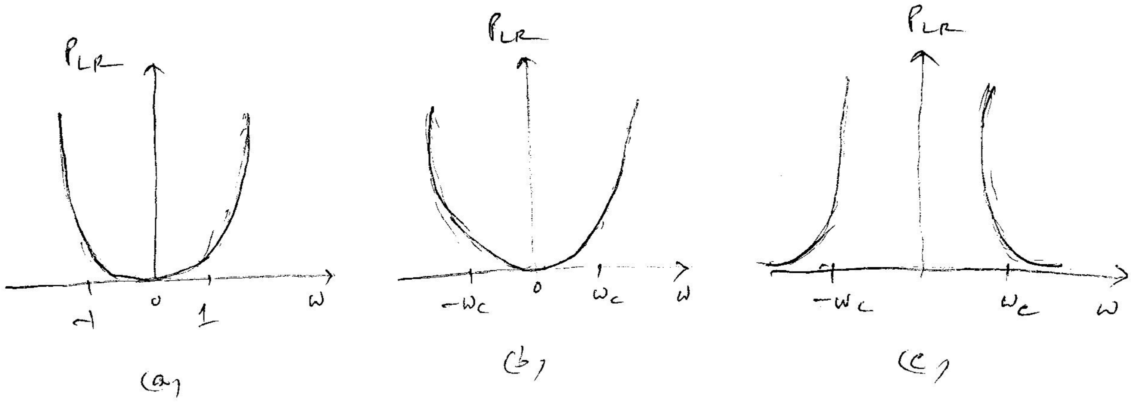


Fig 10:- Frequency scaling for low-pass filters and transformation to a high-pass response. (a) Low-pass filter prototype response for $\omega_c = 1$ (b) Frequency scaling for lowpass response (c) Transformation to high-pass response

→ The new element values are determined by applying the substitution of eq (49) to the series reactances, $j\omega L_k$ and shunt susceptance, $j\omega C_k$, of the prototype filter.

Thus,

$$jX_k = j\frac{\omega}{\omega_c} \cdot L_k = j\omega L'_k$$

$$jB_k = j\frac{\omega}{\omega_c} \cdot C_k = j\omega C'_k$$

which shows that the new element values are given by,

$$L'_k = \frac{L_k}{\omega_c} \qquad (51)$$

$$C'_k = \frac{C_k}{\omega_c} \qquad (52)$$

When both impedance & freq scaling are required, the results of (51) & (52) can be combined with (45) & (46), to give

$$L'_k = \frac{R_0 L_k}{\omega_c} \qquad (53)$$

$$C'_k = \frac{C_k}{\omega_c R_0} \qquad (54)$$

Low-pass to high-pass transformation:-

The freq substitution where

$$\omega \leftarrow -\frac{\omega_c}{\omega} \qquad (55)$$

can be used to convert a low-pass response to a high pass response, shown in fig 10(c).

The substitution maps $\omega = 0$ to $\omega = \pm\infty$ and vice versa; cut off occurs when $\omega = \pm\omega_c$.

The $-ve$ sign is needed to convert inductors (and capacitor) to realizable capacitors (and inductors).

Applying eqn (55) to series reactances, $j\omega L_k$ and shunt capacitances, $j\omega C_k$, of the prototype filter gives,

$$jX_k = j\left(-\frac{\omega_c}{\omega}\right) \cdot L_k = \frac{1}{j\omega C'_k}, \qquad (56)$$

$$jB_k = -j\frac{\omega_c}{\omega} \cdot C_k = \frac{1}{j\omega L_k'} \quad \text{--- (57)}$$

which shows that series inductors L_k must be replaced with capacitors C_k' and shunt capacitor must be replaced by inductor L_k'.

From eqn (56) & (57), we have

$$C_k' = \frac{1}{\omega_c L_k} \quad \text{--- (58)}$$

$$L_k' = \frac{1}{\omega_c C_k} \quad \text{--- (59)}$$

Impedance scaling can be included by using,

$$C_k' = \frac{1}{R_0 \omega_c L_k} \quad \text{--- (60)}$$

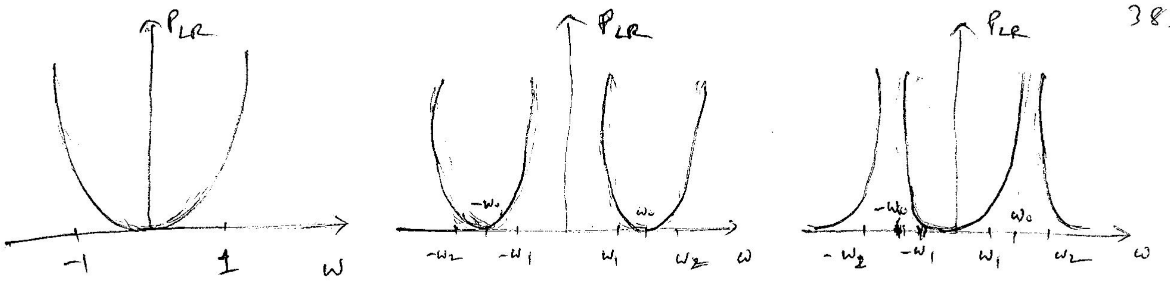
$$L_k' = \frac{R_0}{\omega_c L_k} \quad \text{--- (61)}$$

Bandpass & Band Stop transformations :-

→ Low pass prototype filter designs can also be transformed to have the bandpass or bandstop response illustrated in fig 11. If ω_1 & ω_2 denote the edges of the passband, then a bandpass response can be obtained using the following freq substitution:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \quad (62)$$

where $\Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}$ is the fractional BW of the passband — (63)



(a) (b) (c)

Fig 11:- Bandpass and band stop freq transformation (a) Low-pass filter prototype response for $\omega_c = 1$ (b) Transformation to bandpass response (c) Transformation to bandstop response.

→ The Center freq, ω_0, could be choosen as the arithmatic mean of ω_1 and ω_2, but the eq^n are simpler if it is chosen as the geometric mean

$$\omega_0 = \sqrt{\omega_1 \omega_2} \quad \text{——— (64)}$$

Then the transformation of eq^n (62), maps the bandpass characteristics of figure 11 (b) to the low-pass response of fig 11 (a) as follows

when $\omega = \omega_0$, $\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 0$

$\omega = \omega_1$, $\frac{1}{\Delta}\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = \frac{1}{\Delta}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0\omega_1}\right) = -1$;

when $\omega = \omega_2$, $\frac{1}{\Delta}\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right) = \frac{1}{\Delta}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0\omega_1}\right) = 1$;

→ The new filter elements are determined by using eq^n (62), in the expression for series reactances and shunt capacitances. Thus

$$jX_K = j \cdot \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) L_K = \frac{j\omega L_K}{\Delta\omega_0} - \frac{j\omega_0 L_K}{\Delta\omega}$$

$$= j\omega L_K' - j\frac{1}{\omega C_K'}$$

which shows that series inductor, L_K, is transformed to a series LC ckt with element values,

$$L_K' = \frac{L_K}{\Delta\omega_0} \quad \text{——— (65)}$$

$$C_K' = \frac{\Delta}{\omega_0 L_K} \quad \text{——— (66)}$$

Similarly,

$$jB_K = j \cdot \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) C_K = \frac{j\omega C_K}{\Delta\omega_0} - \frac{j\omega_0 C_K}{\Delta\omega}$$

$$= j\omega C_K' - j\frac{1}{\omega L_K'},$$

which shows that a shunt capacitor, C_K, is transformed to a shunt LC ckt with element values

$$L_K' = \frac{\Delta}{\omega_0 C_K} \quad \text{——— (67)}$$

$$C_K' = \frac{C_K}{\Delta \cdot \omega_0} \quad \text{——— (68)}$$

→ The low-pass filter elements are thus converted to series resonant ckts (low impedance at resonance) in series arms and to parallel resonant ckts (high impedance at resonance) in the shunt arms.

→ But both series & parallel resonator elements have a resonant freq of $\underline{\omega_0}$.

→ Inverse transformation can be used to obtain a band stop response, Thus,

$$\omega \leftarrow \Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1},$$

Δ & ω_0 are same as defined in (63) & (64)

Then series inductor of low-pass prototype are converted to parallel LC ckts having element values given by

$$L'_k = \frac{\Delta L_k}{\omega_0} \quad \text{—— (69)}$$

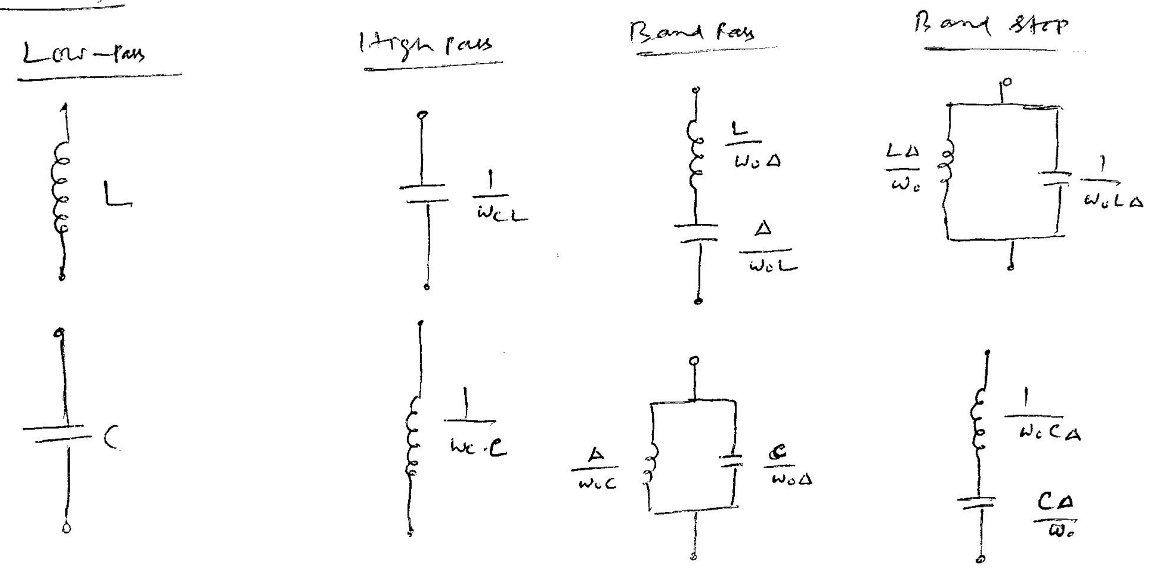
$$C'_k = \frac{1}{\omega_0 \Delta L_k} \quad \text{—— (70)}$$

The shunt capacitor of the low-pass prototype is converted to series LC ckt having elements given by

$$L'_k = \frac{1}{\omega_0 \Delta C_k} \quad \text{—— (71)}$$

$$C'_k = \frac{\Delta C_k}{\omega_0} \quad \text{—— (72)}$$

Summary



Filter Implementation :-

[discussed earlier]

The lumped-element filter design generally works well at low frequencies, but two problems arise at microwave frequencies.

First, lumped elements such as inductor and capacitors are generally available only for limited range of values and are difficult to implement at microwave frequencies, ~~In addition~~ but approximated with distributed components.

In addition, at microwave frequencies the distance betn filter components is not negligible. [But in ckt theory it is negligible].

→ Richard's transformation is used to convert lumped elements to Transmission line sections.

while Kuroda's ~~transformation~~ identities can be used to separate filter elements by using transmission line sections. Because such addition T.L sections don't affect filter response, this type of design is called redundant filter synthesis.

Richard's Transformation :

The transformation, (Capital omega)

$$\Omega = \tan\beta l = \tan\left(\frac{\omega l}{v_p}\right) \quad (73) \quad \left[\because \beta = \frac{\omega}{v_p}\right]$$

maps the 'ω' plane to the 'Ω' plane, which repeats with a period of $\omega l / v_p = 2\pi$.

This transformation was introduced by P. Richard to synthesize an LC n/w using open- and short circuited transmission lines. Thus, if we replace the freq variable 'ω' with Ω, the reactance

386 387

of an inductor can be written as

$$jX_L = j\Omega L = jL\tan\beta l \quad \text{--- (74)}$$

and the susceptance of a capacitor can be written as

$$jB_C = j\Omega C = jC\tan\beta l \quad \text{--- (75)}$$

These results indicates that an inductor can be replaced with short-circuited stubs of length 'βl' and characteristic impedance L, while a capacitor can be replaced with an open-circuited stubs of length 'βl' and characteristic impedance $\frac{1}{C}$.

Note :- Recall in T.L, $Z_{in} = jZ_0\tan\beta l$ for short ckted line

here it is $jL\tan\beta l$.

$\therefore Z_0 = L$

→ Cutoff occurs at unity freq for low-pass filter ~~prototype~~; to obtain the same cutoff freq for Richard's-transformed filter, eqⁿ (73) shows that

$$\Omega = 1 = \tan\beta l \quad \text{--- (76)}$$

which gives a stub length of $l = \frac{\lambda}{8}$, where λ

$\left[\because \tan\beta l = 1 \Rightarrow \frac{2\pi}{\lambda}\cdot l = \frac{\pi}{4} \Rightarrow l = \frac{\lambda}{8}\right]$

is the wavelength of the line at the cutoff freq, w_c.

→ At freq $w_0 = 2w_c$, the lines will be $\frac{\lambda}{4}$ long,

→ and an attenuation pole will occure

→ At freq away from w_c the impedances of the stubs will no longer match the original lumped element impedances and the filter response will differ from the desired prototype response. Also, the response will be periodic in freq, repeating every $4\omega_c$.

$\left[\because 4 \times \omega_c = 4 \times \frac{\lambda}{8 \cdot 2} = \frac{\lambda}{2} \right.$, ~~After~~ In each $\frac{\lambda}{2}$ repetition occures. (In Smith chart repetition occures after $\frac{\lambda}{2}$.) $\Big]$

→ In principle, then the inductor and capacitors of lumped-element filter design can be replaced with short-circuited and open-circuited stubs as shown in figure 12.

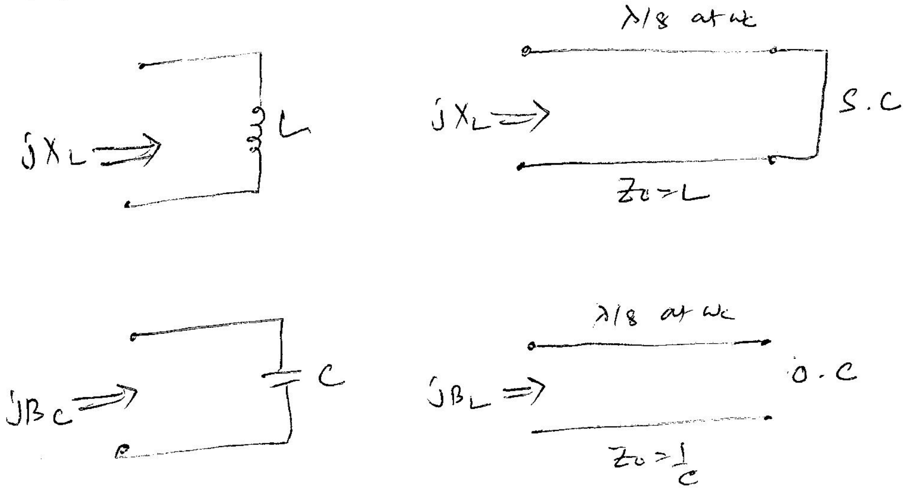


fig 12 :- Richard's transformation (a) For an Inductor to a short-circuited stub (b) For a capacitor to an open-circuited stub.

→ Since the lengths of all the stubs are same ($\frac{\lambda}{8}$ at ω_c), these lines are called Commensurate lines.

Kuroda's Identities :-

The four Kuroda's identities use redundant transmission line section to achieve a more practical microwave filter implementation by performing any of the following operations.

→ Physically separate transmission line stubs

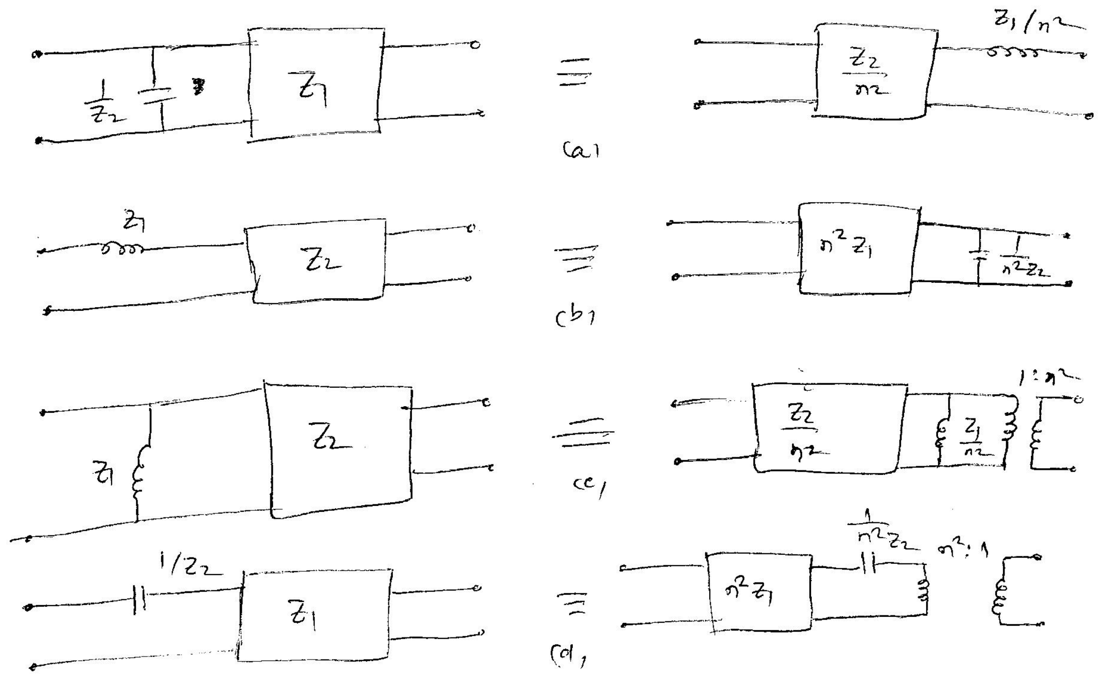
→ Transform series stubs into shunt stubs or vice versa.

→ Change impractical characteristics impedances into more realizable ones.

The additional transmission line sections are called unit elements and are $\frac{\lambda}{8}$ long at ω_c; the unit elements are thus Commensurate with the stubs used to implement the inductors and capacitors of the prototype design.

The four identities are illustrated on table 2,

Table 2:- The 4 Kuruda identities $\left[n^2 = 1 + \frac{Z_2}{Z_1}\right]$.



Here each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\frac{\lambda}{8}$ at ω_c). The inductors and capacitors represent short-ckt and open-ckt stubs, respectively.

* Equivalent ckt of Kuroda's identity (a)

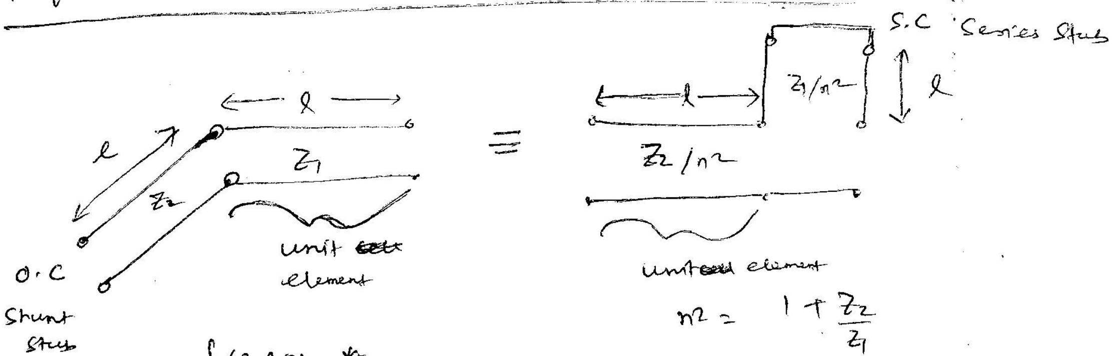


fig 35:- *

i.e

Coupled line filters :-

→ The parallel coupled T.L can also be used to construct many types of filters. Fabrication of multisection band pass or band stop coupled line filters is particularly easy in microstrip or stripline form.

⇒ Filter properties of a coupled line section :-

A parallel coupled line section is shown in fig 36 (a), with port voltage and current definitions. We will derive the open-ckt impedance matrix for this 4-port n/w by considering the superposn of even and odd mode excitations which are shown in fig 36(b). Thus the current sources i_1 and i_3 drive the line in the even mode, while i_2 and i_4 drive the line in odd mode. By superposition, we see that the total port currents, I_i can be expressed in terms of the even and odd modes current as

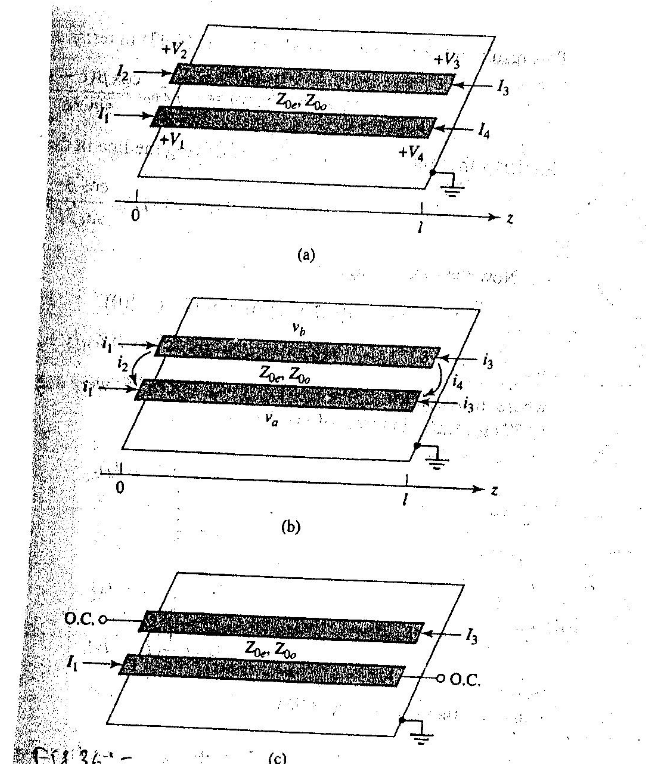


Fig 36:- Definitions pertaining to a coupled line filter section. (a) A parallel coupled line section with port voltage and current definitions. (b) A parallel coupled line section with even- and odd-mode current sources. (c) A two-port coupled line section having a bandpass response.

$$I_1 = i_1 + i_2 \quad \text{———} \quad 77(a)$$

$$I_2 = i_1 - i_2 \quad \text{———} \quad 77(b)$$

$$I_3 = i_3 - i_4 \quad \text{———} \quad 77(c)$$

$$I_4 = i_3 + i_4 \quad \text{———} \quad 77(d)$$

First consider the line as being driven in the even mode by the i_1 current sources. If the other ports are open circuited, the impedance seen at port 1 or 2 is

$$Z_{in}^{e} = -j Z_{0e} \cot\beta l \quad \text{--- (78)} \quad \left[\begin{array}{l} \text{Recall in} \\ \text{open circuited line} \\ Z_{in} = -j Z_0 \cot\beta l \end{array} \right]$$

The voltage on either conductor can be expressed as

$$V_a^1(z) = V_b^1(z) = V_e^+ \left[e^{-j\beta(z-l)} + e^{j\beta(z-l)} \right]$$

$$= 2 V_e^+ \cos\beta(l-z) \quad \text{--- (79)}$$

So the voltage at port 1 or 2 is

$$V_a^1(0) = V_b^1(0) = 2V_e^+ \cos\beta l = i_1 Z_{in}^e$$

This result and (78) can be used to rewrite (79) in terms of i_1 as

$$V_a^1(z) = V_b^1(z) = -j Z_{0e} \frac{\cos\beta(l-z)}{\sin\beta l} i_1 \quad \text{--- (80)}$$

Similarly, the voltages due to current sources i_3 driving the line in the even mode are

$$V_a^3(z) = V_b^3(z) = -j Z_{0e} \frac{\cos\beta z}{\sin\beta l} i_3 \quad \text{--- (81)}$$

Now consider the line as being driven in the odd mode by current i_2. If the other ports are open circuited, the impedance seen at port 1 or 2 is

$$Z_{in}^{o} = -j Z_{0o} \cot\beta l \quad \text{--- (82)}$$

$0 \to$ Zero
$o \to$ odd

The voltage on either conductor can be expressed as

$$V_a^2(z) = -V_b^2(z) = V_o^+ \left[e^{-j\beta(z-l)} + e^{j\beta(z-l)} \right]$$

$$= 2 V_o^+ \cos\beta(l-z) \quad \text{--- (83)}$$

Then the voltage at port 1 or port 2 is

$$V_a^2(0) = -V_b^2(0) = 2v_0^+ \cos\beta l = i_2' Z_{in}^o$$

The result and eqn (82) can be used to rewrite eqn (83) in terms of i_2' as

$$V_a^2(z) = -V_b^2(z) = -j Z_{0o} \frac{\cos\beta(l-z)}{\sin\beta l} \cdot i_2' \quad \text{—— (84)}$$

Similarly, the voltages due to current i_4' driving the line in odd mode are

$$V_a^4(z) = -V_b^4(z) = -j Z_{0o} \frac{\cos\beta z}{\sin\beta l} i_4 \quad \text{—— (85)}$$

Now the total voltage at port 1 is

$$V_1 = V_a^1(0) + V_a^2(0) + V_a^3(0) + V_a^4(0)$$

$$= -j(Z_{0e} i_1 + Z_{0o} i_2)\cot\theta - j(Z_{0e} i_3 + Z_{0o} i_4)\operatorname{cosec}\theta \quad \text{—— (86)}$$

Where the the result of (80), (81), (84), and (85) were used, and $\theta = \beta l$. Next, we solve (77) for i_s in terms of Is:

$$i_1 = \frac{1}{2}(I_1 + I_2) \quad \text{—— 87 (a)}$$

$$i_2 = \frac{1}{2}(I_1 - I_2) \quad \text{—— 87 (b)}$$

$$i_3 = \frac{1}{2}(I_3 + I_4) \quad \text{—— 87 (c)}$$

$$i_4 = \frac{1}{2}(I_4 - I_3) \quad \text{—— 87 (d)}$$

and use these results in eqn (86), we have

$$V_1 = \frac{-j}{2}(Z_{0e} I_1 + Z_{0e} I_2 + Z_{0o} I_1 - Z_{0o} I_2)\cot\theta$$

$$-\frac{j}{2}(Z_{0e} I_3 + Z_{0e} I_4 + Z_{0o} I_4 - Z_{0o} I_3)\operatorname{cosec}\theta \quad —(88)$$

This result yields the top row of the open-ckt impedance matrix [Z] that describes the coupled line section. From symmetry, all other matrix elements can be found once the first row is known.

The matrix elements are then

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = \frac{-j}{2}(Z_{0e} + Z_{0o})\cot\theta \quad — 89(a)$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = \frac{-j}{2}(Z_{0e} - Z_{0o})\cot\theta \quad — 89(b)$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{-j}{2}(Z_{0e} - Z_{0o})\operatorname{cosec}\theta \quad — 89(c)$$

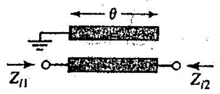
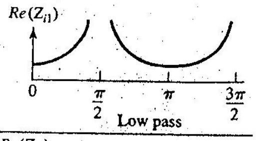
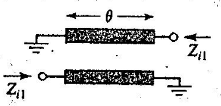
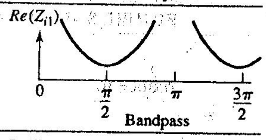
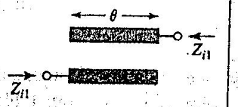
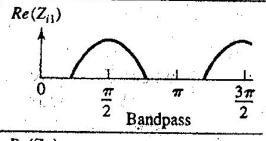
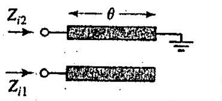
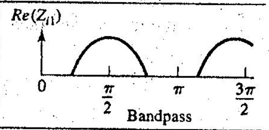
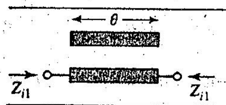
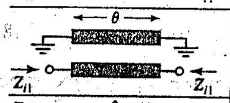
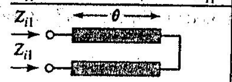
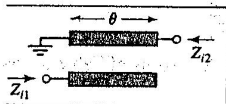
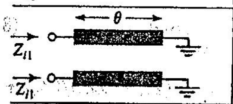
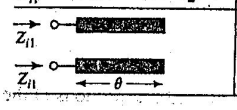
$$Z_{14} = Z_{41} = Z_{23} = Z_{32} = \frac{-j}{2}(Z_{0e} + Z_{0o})\operatorname{cosec}\theta \quad — 89(d)$$

→ A 2-port n/w can be formed from the coupled line section by terminating 2 of the 4-ports in either open or short ckts; there are 10 possible combinations as, illustrated in table 3:

→ As indicated in this table, the various ckts have different freq responses, including low-pass, bandpass, all pass and all stop. For bandpass filters, fig 36 (c) can be used, as open ckt are easier to fabricate than are short ckts.

In this case, $I_2 = I_4 = 0$, so the

TABLE 8.8 Ten Canonical Coupled Line Circuits

Circuit	Image Impedance	Response
(88) 	$Z_{i1} = \frac{2Z_{0e}Z_{0o}\cos\theta}{\sqrt{(Z_{0e}+Z_{0o})^2\cos^2\theta - (Z_{0e}-Z_{0o})^2}}$ $Z_{i2} = \frac{Z_{0e}Z_{0o}}{Z_{i1}}$	 Low pass
	$Z_{i1} = \frac{2Z_{0e}Z_{0o}\sin\theta}{\sqrt{(Z_{0e}-Z_{0o})^2 - (Z_{0e}+Z_{0o})^2\cos^2\theta}}$	 Bandpass
	$Z_{i1} = \frac{\sqrt{(Z_{0e}-Z_{0o})^2 - (Z_{0e}+Z_{0o})^2\cos^2\theta}}{2\sin\theta}$	 Bandpass
89(a) 	$Z_{i1} = \frac{\sqrt{Z_{0e}Z_{0o}}\sqrt{(Z_{0e}-Z_{0o})^2 - (Z_{0e}+Z_{0o})^2\cos^2\theta}}{(Z_{0e}+Z_{0o})\sin\theta}$ $Z_{i2} = \frac{Z_{0e}Z_{0o}}{Z_{i1}}$	 Bandpass
89(b) 	$Z_{i1} = \frac{Z_{0e}+Z_{0o}}{2}$	All pass
89(c) 	$Z_{i1} = \frac{2Z_{0e}Z_{0o}}{Z_{0e}+Z_{0o}}$	All pass
	$Z_{i1} = \sqrt{Z_{0e}Z_{0o}}$	All pass
89(d) 	$Z_{i1} = -j\frac{2Z_{0e}Z_{0o}}{Z_{0e}+Z_{0o}}\cot\theta$ $Z_{i2} = \frac{Z_{0e}Z_{0o}}{Z_{i1}}$	All stop
	$Z_{i1} = j\sqrt{Z_{0e}Z_{0o}}\tan\theta$	All stop
	$Z_{i1} = -j\sqrt{Z_{0e}Z_{0o}}\cot\theta$	All stop

ckt

Four port impedance matrix eq^n's reduces to

$$V_1 = Z_{11} I_1 + Z_{13} I_3, \quad \text{——— 90(a)}$$

$$V_3 = Z_{31} I_1 + Z_{33} I_3, \quad \text{——— 90(b)}$$

1pass,

Where Z_{ij}'s is given in eq^n (89).

We can analyze the filter characteristics of this ckt by calculating the image impedance (which is same at port 1 and port 3), and the propagation

the

Constant, From Table 1 (Page 370), the image impedance in terms of Z-parameter is

$$Z_i = \sqrt{Z_{11}^2 - \frac{Z_{11} Z_{13}^2}{Z_{33}}}$$

$$= \frac{1}{2}\sqrt{(Z_{0e} - Z_{0o})^2 \csc^2\theta - (Z_{0e} + Z_{0o})^2 \cot^2\theta} \quad -(91)$$

When the coupled line section is $\frac{\lambda}{4}$ long $(\theta = \frac{\pi}{2})$, the image impedance reduce to

$$Z_i = \frac{1}{2}(Z_{0e} - Z_{0o}) \quad -(92)$$

βl

$\because \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$\csc\frac{\pi}{2} = 1$

$\cot\frac{\pi}{2} = 0$

which is real and +ve, since $Z_{0e} > Z_{0o}$.

But when $\theta \to 0$ or π, $Z_i \to \pm j\infty$, indicating a stop band. The real part of image impedance is sketched in fig 37, where cutoff frequencies can be found

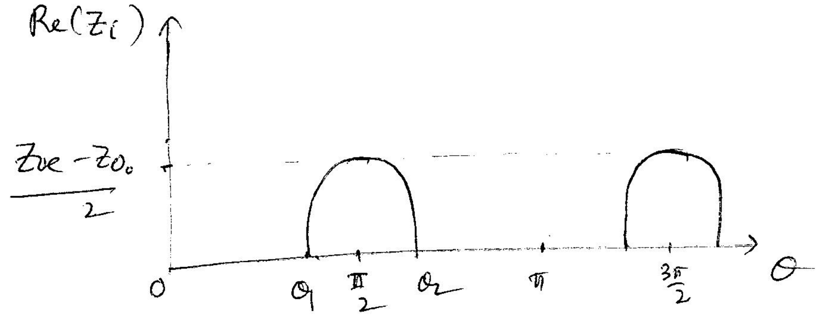


fig 37:- The real part of image impedance of bandpass n/w of fig 36 (c).

→ eqn (91), as $\cos\theta_1 = -\cos\theta_2 = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$ ———

The propagation constant can also be found from the result of table 1, as

$$\cos\beta = \sqrt{\frac{Z_{11} Z_{33}}{Z_{13}^2}} = \frac{Z_{11}}{Z_{13}}$$

$$\cos\beta = \frac{Z_{0e} + Z_{0o}}{Z_{0e} - Z_{0o}} \cos\theta \qquad \text{---} \ (93)$$

which shows β is real for $\theta_1 < \theta < \theta_2 = \pi - \theta_1$,

where $\cos\theta_1 = \dfrac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$.

— o —

Q) Why a 3-port n/w cannot be matched at all ports? [Ex- E-plane Tee / H-plane Tee]

Proof :- Assume that all the 3-ports are matched,

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \qquad \text{---} \ (1)$$

From the symmetry property of S-matrix

$$S_{12} = S_{21}, \quad S_{13} = S_{31} \quad S_{23} = S_{32} \qquad \text{---} \ (2)$$

From zero property of S matrix, the sum of products of each term of any column (or row) multiplied by complex conjugate of the corresponding terms of any other column (or row) is zero and it is

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \qquad \text{---} \ (3)$$

Since $S_{11} = 0$ and $S_{22}^* = 0$, eqn (3) becomes

$$S_{31} S_{32}^* = 0$$

By Symmetry, $S_{13} S_{23}^* = 0$ ——— (4)

This means that either S_{13} or S_{23}^* or both should be zero. However, from the unity property of S-matrix, the sum of the products of each term of any one row (or column) multiplied by its complex conjugate is unity; i.e

$$S_{21} S_{21}^* + S_{31} S_{31}^* = 1 \quad \text{——— (5)}$$

$$S_{12} S_{12}^* + S_{32} S_{32}^* = 1 \quad \text{——— (6)}$$

$$S_{13} S_{13}^* + S_{23} S_{23}^* = 1 \quad \text{——— (7)}$$

Substitution of eqn (2) in eqn (5), we have

$$S_{12} S_{12}^* = 1 - S_{13} S_{13}^*$$

$$\Rightarrow |S_{12}|^2 = 1 - |S_{13}|^2$$

Similarly, from eqn (6), $|S_{12}|^2 = 1 - |S_{23}|^2$

$$\therefore |S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad \text{——— (8)}$$

Eqn (7) and (8) are contradictory, for if $S_{13} = 0$, [eqn 8] then S_{23} is also zero and thus eqn (7) becomes false.

Similarly, if $S_{23} = 0$, [eqn 8] S_{13} becomes zero, eqn (7) " ",

This inconsistency proves that our assumption is false.

Tee jn [E/H] Plane Tee can n't be matched to 3 arms. i.e diagonal elements of S-matrix of Tee jn are not all zeros.

o

Best of Luck