$= 2 \times 10^{-24}$ kg. m. sec^{-1}

Angular momentum

$H = I \omega$

$= MR^2 \cdot \frac{V}{R}$ $\quad (\because V = R\omega)$

$= MRV$

$= (9.11 \times 10^{-31} kg) \times (.529 \times 10^{-10} m) \times (2.2 \times 10^{6} m)$

10.60×10^{-35} k.g m^2 - sec^{-1}

$= 1.06 \times 10^{-34}$ kg m^2. sec^{-1}

Q2. A fire hose nozzle has diameter = 1.5 in.

$\therefore$ 12 inch = 1 ft

1.5 inch = $\frac{1}{12} \times 1.5 = .125$ ft.

Area of the nozzle of the pipe

$= \pi r^2$

$= (3.141)\left(\frac{.125}{2}\right)^2$

$= (3.141)(.0625)^2$

$= (3.141)(.00390625)$

$= .0122499$

Volume = Area × length

$= (.0122499 \times 8.0 ft)$

$= 0.0979992$

Mass = volume x density

$= (.979 \times 62.4)$

$= 61.0896$

Force = $\frac{\Delta P}{\Delta t} = \frac{\Delta P}{1 sec}$

~~= $I\omega$~~ = $m.v$

$= 61.0896 \times 80$

= ?

After 10 page

22. Lets con

Q

A body of mass 0.5 kg makes an elastic collision with another body at rest and continues to move in the original direction with a speed equal to half of its original speed. Find the mass and velocity of the second body

(Ans : $m_2 = \frac{1}{6}$ kg, $V_2 = \frac{3}{2} U_1$)

161 Page

20. A fly has ~~mass~~ weight $= 0.1$ oz

$1 oz = \frac{1}{16} lb$

$.1 oz = \frac{1}{160} lb.$

$$Mass = \frac{w}{g} = \frac{\frac{1}{160}}{\cancel{9.8}\, 32} = \frac{1}{160 \times \cancel{9.8}\, 32}$$

$$= \frac{1}{5120} \text{ slug}$$

Centrefugal force $= \frac{mv^2}{r}$

$$= \frac{m.(r\omega)^2}{r}$$

$$= m r \omega^2$$

$$\cancel{= \frac{1}{160 \times 32} \times}$$

diameter $= 4$ in $= 2r$

$\Rightarrow r = 2 in = \frac{2}{12} = \frac{1}{6}$ ft

$$\omega = \frac{100 r}{m} = \frac{100 \times 2\pi rad}{3\ \cancel{60}} = \frac{10 \times 22}{3 \times 7}$$

$$= \frac{220}{21}$$

$$F = mr\omega^2$$

$$= \frac{1}{5120} \times \frac{1}{6} \times \left(\frac{220}{21}\right)^2$$

$$= \frac{1 \times 1 \times 220 \times 220}{5120 \times 6 \times 21 \times 21}$$

$$= \frac{48400}{13547520}$$

$$= \cancel{1.5733207}$$

$$= 3.57 \times 10^{-3} \text{ lb.}$$

3.12.2K

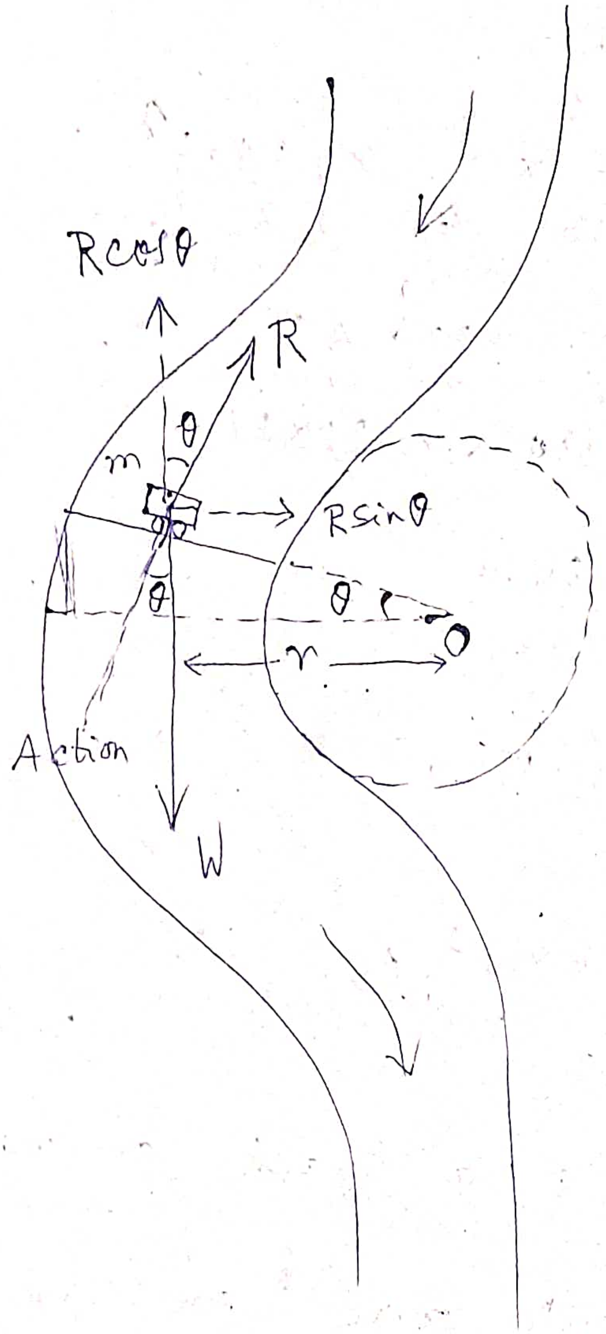
Applications

(1) Banking of a track or road

Usually vehicles move with high speed in highways. If there will be a sudden curve on the road, there is possibility of these vehicles to turn upside down.

This happens because of imbalance. The outer ~~wheel~~ wheels of the vehicle get detached from the road, because they try to provide centripetal force.

To prevent such accidents, the outer side of the road is



raised a little so that the outer wheels remain in contact with the road. The reaction of the road acts on the vehicle in a direction that makes θ with the vehicle direction.

Resolving this reaction force into two rectangular components, we see that the vertical component is R cosθ that balances the weight of the vehicle.

The horizontal component is R sinθ

which provides the necessary centripetal force.

$$\therefore \frac{mv^2}{r} = R\sin\theta \quad \text{---(i)}$$

$$mg = R\cos\theta \quad \text{---(ii)}$$

~~Both~~ Dividing eq^n (i) by eq^n (ii), we get

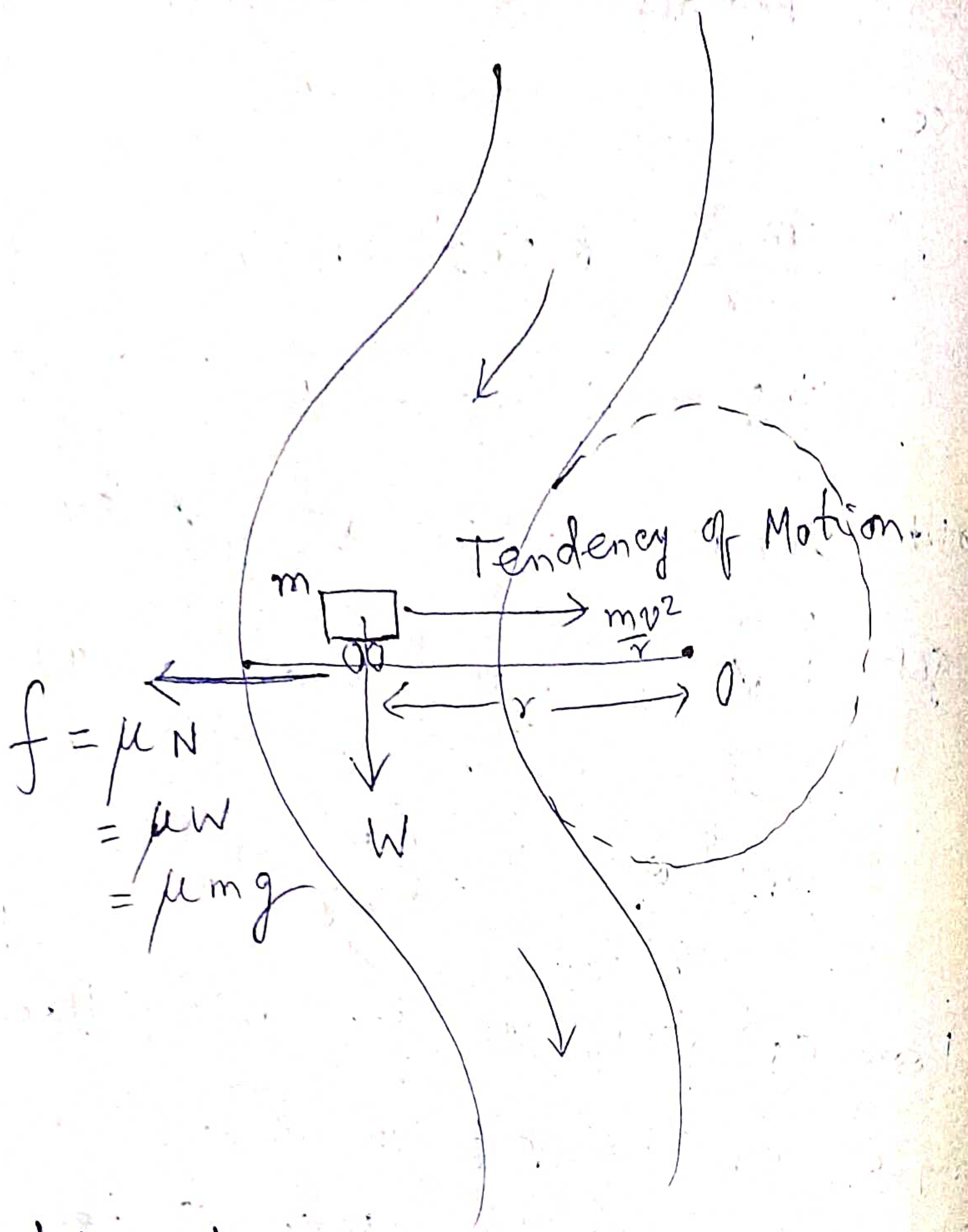
$$\boxed{\frac{v^2}{rg} = \frac{\sin\theta}{\cos\theta} = \tan\theta} \quad \text{---(3)}$$

or $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

From eq^n (iii), we see that the "angle of balancing" is independent of the mass of the vehicle. θ has to be increased when r decreased. with increase of v, θ has to be increased further..

(2) Motion of a vehicle on a rough curved road (without banking)

At the turning point, the vehicle tries to provide centripetal force of magnitude $\frac{mv^2}{r}$, directed towards the centre of curvature. This is prevented



by the frictional force. For no accident or dynamical equilibrium, these two forces must balance

i.e $\frac{mv^2}{r} = \mu N = \mu W = \mu mg$

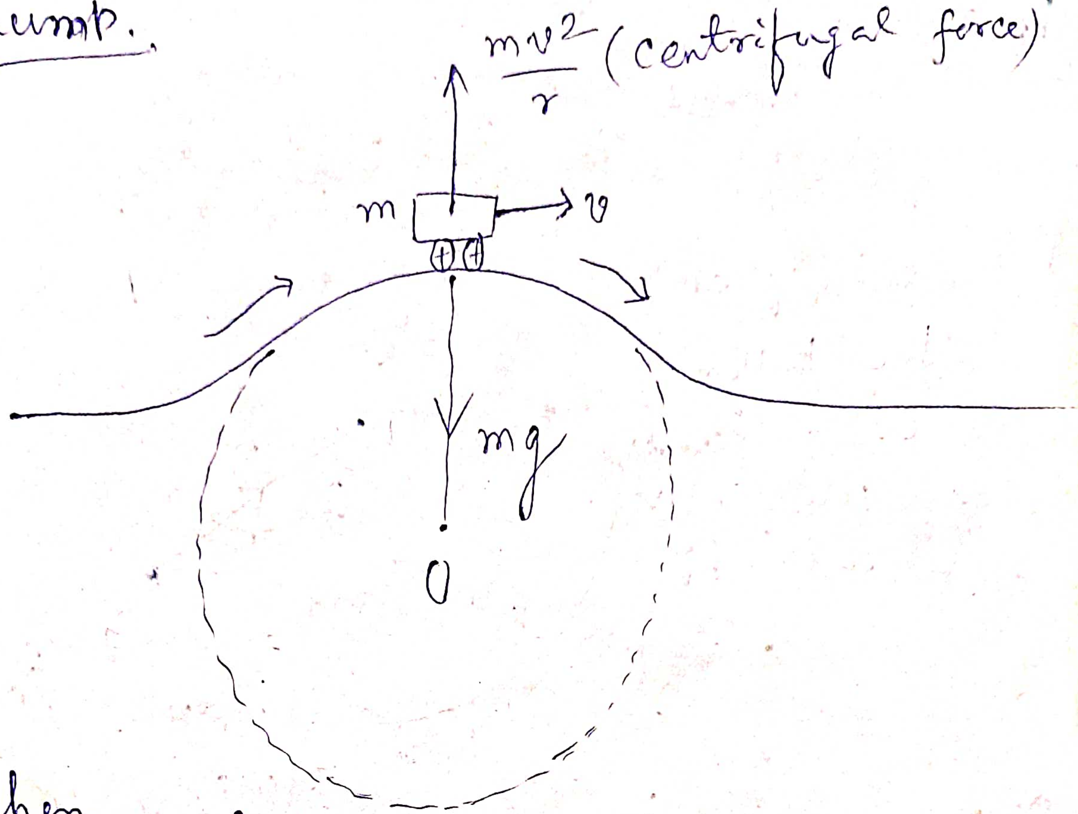
or $\frac{v^2}{r} = \mu g$

$\Rightarrow v^2 = \mu r g$

or $v = \sqrt{\mu r g}$

This is the upper limit to the velocity of the vehicle at the turning point.

(3) Motion of a vehicle on ~~the~~ a hump.



When the vehicle moves on the hump with a velocity $\vec{v}$, it experiences a centrifugal reaction force by the center of curvature 'O'. If this will be balanced by the weight of the vehicle (mg), then there will be dynamic equilibrium.

$$\therefore \frac{mv^2}{r} = mg$$

$$\Rightarrow v^2 = rg$$

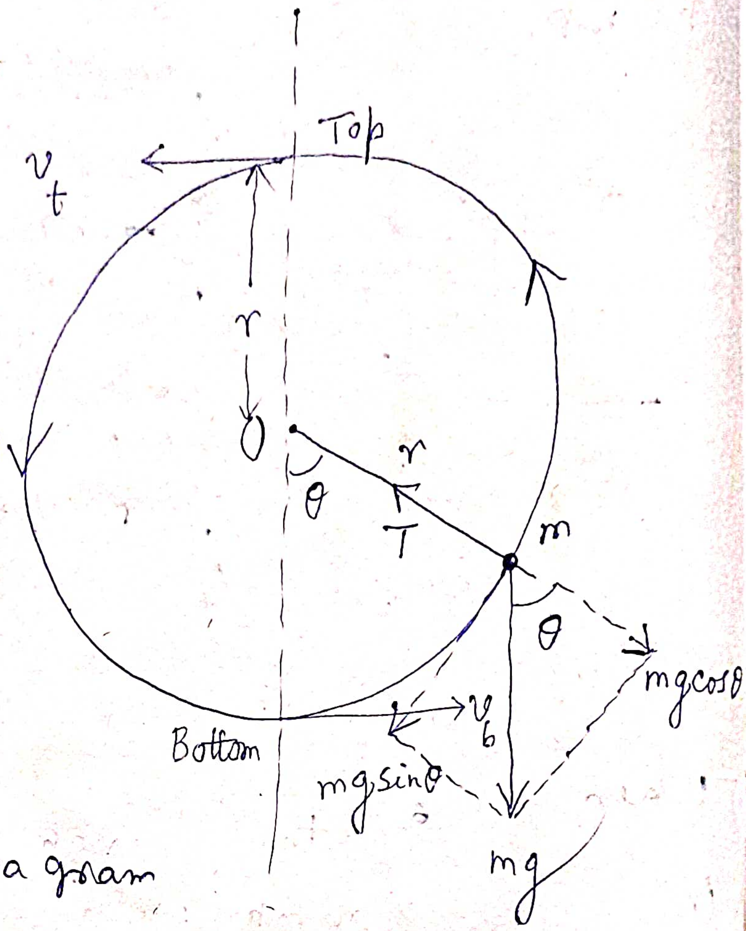
$$\Rightarrow v = \sqrt{rg}$$

This is the upper limit to the velocity of the vehicle on the hump.

4. Motion of a body on a vertical circle.

V_t = velocity at the topmost point.

V_b = velocity at the bottom of the vertical circle of radius r.



In the diagram any position of the particle rotating on the vertical circle has been shown. The radius vector joining the center with the particle makes an angle 'θ' with the vertical.

Resolving the weight of the body into two rectangular components, we see that the component $W\cos\theta$ acts in a direction opposite ~~to~~ the tension. Hence the net force towards the Center is

$T - W\cos\theta =$ centripetal force

$= \frac{mv^2}{r}$ ——(i)

The other Component is $W\sin\theta$ which provides the restoring force.

At the bottom of the vertical circle, $\theta = 0°$ Hence eq^n(i) gives

$$T - W\cos 0° = \frac{mv_b^2}{r}$$

$$\text{or } T = W + \frac{mv_b^2}{r} \quad \text{——(2)}$$

$$= \text{max}^m.$$

At the top most point of the vertical circle $\theta = 180°$. Hence eqn becomes

$$T - W\cos 180° = \frac{mv_t^2}{r}$$

$$\Rightarrow T + W = \frac{mv_t^2}{r}$$

$$\Rightarrow T = \frac{mv_t^2}{r} - W \quad \text{——(3)}$$

$$= \text{min}^m.$$

If V_t be gradually decreased, then tension will reduce and ultimately it will be zero. The velocity at that instant is called critical velocity (V_c)

$$\therefore \quad 0 = \frac{mv_c^2}{r} - w$$

$$\Rightarrow mg = \frac{mv_c^2}{r}$$

$$\Rightarrow v_c^2 = rg$$

$$\Rightarrow \boxed{v_c = \sqrt{rg}} \quad \text{——— (4)}$$

This is the min^m velocity that must be provided at the topmost point of the vertical circle so that the particle can complete the motion.

The corresponding min^m velocity at the bottom is obtained from the principle of conservation of total energy.

At position I $\quad P.E = 0$

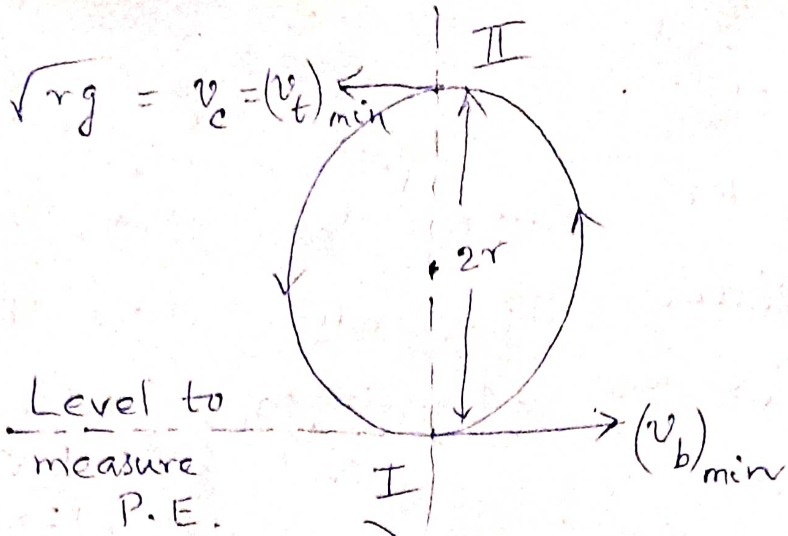
$$K.E = \frac{1}{2} m (V_b)^2_{min^m}$$

$\therefore$ Total energy at position

$$I = \frac{1}{2} m (V_b)^2_{min^m}$$

At position II

$$P.E = mg\,2r$$



$K.E = \frac{1}{2} \cdot m rg$ $(\because v^2 = rg)$

$\therefore$ Total energy at position II

$= P.E + K.E$

$= \frac{2}{2} \cdot mg\,2r + \frac{1}{2} mrg$

Equating these two expressions for the total energy at position I and II, we get

$\frac{1}{2} \cancel{m} (V_b)^2_{minm} = \cancel{m} g\,2r + \frac{1}{2} \cancel{m} r g$

$\Rightarrow V_b^2\ minm = 4rg + rg = 5rg$

$\Rightarrow (V_b)_{min} = \sqrt{5rg}$ ——— (V)

—o—

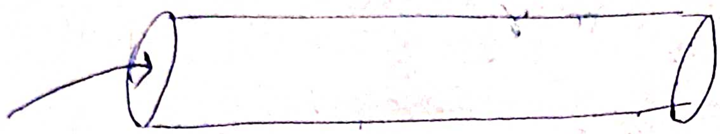
160, —3, 4, 9, 10, 12, 17, 24

581N 36.28m

22. Page 148

Let us consider the amount of water striking the wall in 1 sec. Volume of water contained in a cylinder of length 80 ft and area of cross section $\pi \left(\frac{1.5}{12 \times 2}\right)^2$

$\pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{1.5}{12 \times 2}\right)^2$



$$A = \frac{22}{7} \times \left(\frac{15}{240}\right)^2 ft^2$$

$$= \frac{11}{7} \times \frac{1}{8} \times \frac{1}{8} \times \frac{1}{2} \; ft^2$$

Volume of the water

$$= \text{Area} \times \cancel{\text{Volume}} \text{ height (length)}$$

$$= \left(\frac{11}{56 \times 16} \times 80\right) ft^3$$

$$= \frac{110}{7 \times 16} ft^3 = \frac{110}{112} ft^3$$

weight density $= 62.4 \, lb/ft^3 = \frac{\text{weight}}{\text{volume}}$

weight of the water

$$= 62.4 \frac{lb}{\cancel{ft^3}} \times \frac{110}{7 \times 16} \cancel{ft^3}$$

$$= \frac{62.4 \times 110}{16 \times 7} \; lb$$

mass of water $= \dfrac{weight}{g} = \dfrac{62.4 \times 110}{16 \times 7 \times 32}$ slug.

Momentum with which the water stricks the wall

$= $ mass $\times$ velocity

$= \dfrac{62.4 \times 110}{32 \times 7 \times 16} \times 80$ slug-ft/s

$= \dfrac{62.4 \times 110 \times 5}{32 \times 7}$ slug ft/sec

Final momentum of the water $= 0$

$\Delta p = p_i = \dfrac{62.4 \times 110 \times 5}{32 \times 7}$ slug ft/sec.

Force $F = \dfrac{\Delta p}{\Delta t} = \dfrac{\left(\dfrac{62.4 \times 110 \times 5}{32 \times 7}\right)}{1}$ lb

$= 153.214$ lb (Ans)

for next question

Ans First body has mass $= m_1 = 0.5$ kg.

Its initial velocity $= U_1$

According to question final velocity $V_1 = \frac{1}{2} U_1$

For the second body initial velocity $U_2 = 0$

final velocity $= V_2$

Since it is a elastic collision

co-efficient of restitution $= 1$

$\Rightarrow e = 1$.

$$\Rightarrow -\left(\frac{v_1 - v_2}{u_1 - u_2}\right) = 1$$

$$\Rightarrow \quad v_1 - v_2 = u_2 - u_1$$

$$\Rightarrow \quad \frac{1}{2}u_1 - v_2 = 0 - u_1$$

$$= \quad v_2 = \frac{3}{2}u_1 \quad \text{——— (Ans)}$$

~~From the principle of conservation of linear momentum, we know that~~

$$\cancel{\vec{P_i} = P_f}$$

In elastic collision kinetic energy of colliding particles after collision is equal to the kinetic energy before collision.

$$(E_K)_f = (E_K)_i$$

$$\Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m$$

In elastic collision linear momentum is found to be conserved.

$$\vec{P_f} = \vec{P_i}$$

$$\Rightarrow m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{u_1} + m_2\vec{u_2}$$

$$\Rightarrow \left(\cdot 5 \times \frac{1}{2}u_1\right) + m_2\left(\frac{3}{2}u_1\right) = (\cdot 5 \times u_1) + m_2 \cdot 0$$

$$\Rightarrow \frac{5}{20}u_1 + \frac{3}{2}m_2 u_1 = \frac{1}{2}u_1$$

$\Rightarrow \frac{1}{2}v_1 - \frac{5}{20}v_1 = \frac{3}{2}m_2 v_1$

$\Rightarrow \frac{10v_1 - 5v_1}{20} = \frac{3}{2}m_2 v_1$

$\Rightarrow \frac{\cancel{5}\cancel{v_1}}{\cancel{20}4} = \frac{3}{2}m_2 \cancel{v_1}$

$\Rightarrow \frac{1}{4} = \frac{3}{2}m_2$

$\Rightarrow m_2 = \frac{1 \times \cancel{2}}{\cancel{4}2 \times 3} = \frac{1}{6}$ kg.

or $m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$

$\Rightarrow m_1 v_1 - m_1 u_1 = \cancel{m_2 u_2 - m_2 v_2}$ $m_2 u_2 - m_2 v_2$

$\Rightarrow m_1(v_1 - u_1) = m_2(u_2 - v_2)$

$\Rightarrow \frac{m_1}{m_2} = \frac{u_2 - v_2}{v_1 - u_1} = \frac{0 - \frac{3}{2}u_1}{\frac{1}{2}u_1 - u_1}$

$\Rightarrow \frac{m_1}{m_2} = \frac{\cancel{-}\frac{3}{\cancel{2}}\cancel{u_1}}{\cancel{-}\frac{1}{\cancel{2}}\cancel{u_1}} = 3$

$\Rightarrow \frac{.5}{m_2} = 3 \Rightarrow m_2 = \frac{.5}{3} = \frac{5}{10 \times 3} = \frac{1}{6}$ kg

(Ans)

Assign stage

College physics page 160 Arrow → velocity.

3. The circle has diameter = 120 m

radius of circle = r = ~~$~~ 60 m.

$v = 192$ km/hour.

4.

weight of the boy = 445 N

mass of the body = $\frac{445}{9.8}$ = 45.4 kg

r of the circle = length of

string = 3 m.

velocity = 3 m/s.

Total force given the force for holding the swing

$$T = W + \frac{mv_h^2}{r}$$

$$= 445 + \frac{45.4 \times \cancel{9}\,3}{\cancel{3}}$$

$$= 445 + 136.2$$

$$= 581.2 \text{ Newton}$$

~~B~~.9. Coefficient of friction between tire and roadway ~~B~~ $\mu = 0.50$.

$V = 48$ km/hour.

The upper limit of velocity

$$V = \sqrt{\mu r g}.$$

$$\Rightarrow (48)^2 = (.5) \times (r) \times (9.8)$$

$$\Rightarrow r = \frac{(48)^2}{4.9}$$

$$V = \sqrt{\mu r g}$$

$$\Rightarrow \left(\frac{\cancel{48}^{24} \times \cancel{10^3}^{10^3}}{\frac{\cancel{3600}}{18}\cancel{9}}\right)^2 = (.5) \times (r).(9.8)$$

$$\Rightarrow \left(\frac{40}{3}\right)^2 \times \frac{1}{.5 \times 9.8} = r$$

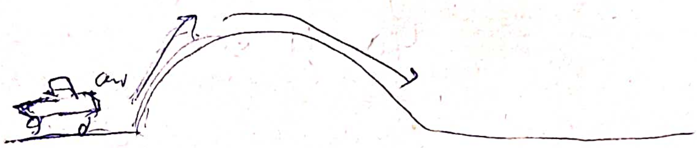
$$\Rightarrow \frac{1600}{9} \times \frac{1}{4.9} = r$$

$$\Rightarrow r = \frac{1600}{44.1} = 36.28 \text{ meter.}$$

10.

Mass of car = 3,200 lbs

radius of Knoll = 75 ft.



~~Max~~^{m} upper limit of velocity of the car $v = \sqrt{rg}$

$$\Rightarrow v^2 = rg$$

$$\Rightarrow v^2 = (75)(32) = 2400$$

$$\Rightarrow v = \sqrt{2400} = 48.99 = \cancel{49.5}$$

$$= 49 \text{ ft/sec.}$$

12. ~~Mass~~ weight of the car = ~~1200~~ 1500 lb.

radius of the curve = 100 ft

Co-efficient of friction = 0.5

Here the car is moving on a rough curved Road.

If there is no collision So centripetal force was given by the car = the frictional force

$$\Rightarrow \frac{\cancel{m}v^2}{r} = \mu \cancel{m} g$$

$$\Rightarrow v^2 = \mu r g$$

$$\Rightarrow v^2 = (.5)(100)(32)$$

$$= 1600$$

$$\Rightarrow v = 40 \text{ ft/sec.}$$

Thus the car can go maxm 40ft/sec before skidding.

14. A boy has weight = 100 lb.

He is standing 10 ft from the center of merry go round plat form.

$$r = 10 \text{ ft}$$

$$\omega = \frac{4r}{\text{min}} = \frac{\cancel{4} \times 2\pi \text{ rad}}{\cancel{60}_{15}} = \frac{2}{15}\pi \text{ rad/sec}$$

$$\text{Linear speed } V = r\omega = \cancel{10}^{2} \text{ ft} \times \frac{2}{\cancel{15}_{3}} \pi \text{ rad/sec}$$

$$= \frac{4 \times 3.141}{3} \text{ ft/sec}$$

$$= 4 \times 1.047$$

$$= 4.188 \text{ ft/sec.}$$

radial accelⁿ

$= \frac{v^2}{r} = \longrightarrow \frac{(4.2)^2}{10}$

$f = \frac{mv^2}{r} = \frac{100}{32}$

$\mu m g = \ldots$

$= \frac{17.64}{10}$

$= 1.764 \ ft/s^2$

frictional force needed to prevent him from slipping must be equal to the centripetal force given by the body $= \frac{mv^2}{r}$.

$\therefore$ frictional force is needed

$$= \frac{\left(\frac{100}{32}\right) \cdot (4.2)^2}{10}$$

$$= \frac{\overset{10}{\cancel{100}} \times 17.64}{32 \times \cancel{10}}$$

$$= \frac{176.4}{32}$$

$$= 5.5125$$

$\therefore$ frictional force $= 5.6$ lb

If he is on the verge of slipping at this speed.

$\mu_s N = 5.6$ lb

$\Rightarrow \mu_s W = 5.6$ lb

$\Rightarrow \mu_s 100 = 5.6$ lb

24. The designers of ~~automobile~~ expressway wish to have automobiles round a curve at 70 mile/hour.

we know 15 mile/hour = 22 ft/sec

1 mile/hour = $\frac{22}{15}$ ft/sec

70 mile/hour = $\left(\frac{22}{15} \times 70\right)$ ft/sec

$= \left(\frac{22 \times 14}{3}\right)$ ft/sec

The roadway is not banked.

Coefficient friction between tire and road = 0.3

weight ~~mass~~ of car = 4800 lb.

If the ~~system~~ car turn safely then

The centripetal force given by the car must be balance by friction.

$$\frac{\cancel{m} v^2}{r} = \mu \cancel{m} g$$

$$\Rightarrow v^2 = \mu \cdot r \cdot g$$

$$\Rightarrow \left(\frac{22 \times 14}{3}\right)^2 = (.3) \times (r) \times (32)$$

$$\Rightarrow r = \frac{484 \times 196}{9 \times 9.6}$$

$$\boxed{\Rightarrow r = 1097.96 \text{ ft}}$$

3. An aviator loops the loop in a circle 120m in diameter.

r = radius = 60m.

Minm velocity at the bottom

$V_b = \sqrt{5rg}$

According to question

$V_b = 192$ Km/hour.

$= \frac{192 \times \cancel{1000}^{5}}{\cancel{3600}_{18}}$

$\frac{192 \times 5}{18} = \sqrt{5 \times 60 \times g}$

$\Rightarrow \left(\frac{192 \times 5}{18}\right) \times \left(\frac{192 \times 5}{18}\right) = 300 \cdot g$

$\Rightarrow g = \frac{\cancel{192}^{96} \times \cancel{192}^{96} \times \cancel{25}^{9}}{\cancel{18 \times 18}_{9 \quad 9}} \times \frac{1}{\cancel{300}_{12}}$

$T = W + \frac{m v_b^2}{r}$ = Apparent weight $= mg'$

$\Rightarrow g' = g + \frac{v_b^2}{r}$

No. of g's $= \frac{g'}{g} = 1 + \frac{v_b^2}{rg} = 1 + \frac{\left(\frac{192 \times 5}{18}\right)^2}{60 \times 9.8}$

$= 1 + 4.83$

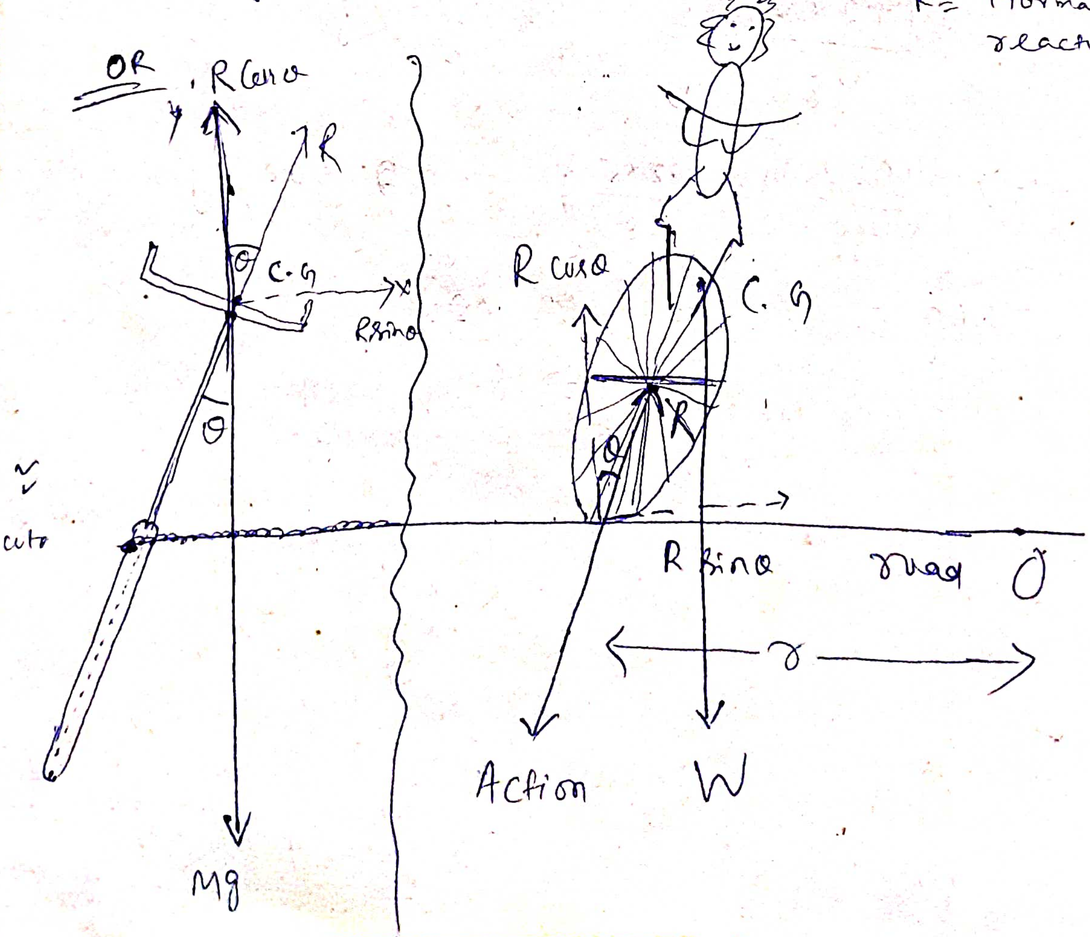
$= 5.83$ times (Ans)

---

(5) Motion of a cyclist on a curved road

When a cyclist makes a turn, the angle made by him with the vertical be θ. Resolving the reaction force on the cycle by the road into two rectangular components we see that the vertical component is $R\cos\theta$ which balances weight of the cycle and the man.

R = Normal reaction,



The horizontal component is $R\sin\theta$ that provides the necessary centripetal force.

$$\frac{mv^2}{r} = R\sin\theta \quad \text{——— (i)}$$

$$mg = R\cos\theta \quad \text{——— (ii)}$$

Dividing eqn (i) by eqn (ii), we get

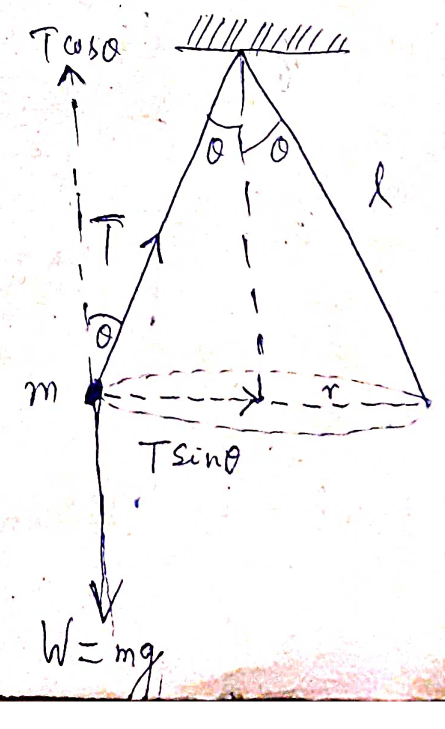
$$\boxed{\frac{v^2}{rg} = \tan\theta} \rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

This shows that the cyclist has to bend himself through an angle greater than θ when r is made smaller and v is made larger.

6th application (Conical pendulum)

$$\sin\theta = \frac{r}{l}$$

$$\Rightarrow r = l\sin\theta$$



It is just a point mass attached to a string of some length (l). The pendulum bob is made to describe a horizontal circle. The two forces acting on the body are the tension (T) and weight (W).

Resolving the tension into two rectangular components, we see that The vertical component is $T\cos\theta$ which balances the weight. The horizontal component is ~~$\$$~~ $T\sin\theta$ which provides necessary ~~to~~ centripetal force.

$$\therefore \frac{mv^2}{r} = T\sin\theta \quad \text{—(i)}$$

$$mg = T\cos\theta \quad \text{—(ii)}$$

Dividing eq^n (i) by eq^n (ii), we get

$$\boxed{\frac{v^2}{rg} = \tan\theta} \quad \text{—(iii)}$$

B.T. is not necessary here

Time period of revolution of the pendulum bob is given by

$$T = \frac{2\pi r}{v}$$

$$\Rightarrow \quad \tau^2 = \frac{4\pi^2 r^2}{v^2} \quad \text{——— (iv)}$$

From eqn (iii), we get

$$v^2 = rg \cdot \tan\theta \quad \text{——— (v)}$$

Using eqn (v) in eqn (iv), we get

$$\tau^2 = \frac{4\pi^2 r^2}{rg \tan\theta}$$

$$\Rightarrow \tau^2 = \frac{4\pi^2 r}{g \tan\theta} \quad \cancel{= 4\pi}$$

$$= \frac{4\pi^2 \, l \sin\theta}{g \cdot \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{4\pi^2 \, l \cos\theta}{g}$$

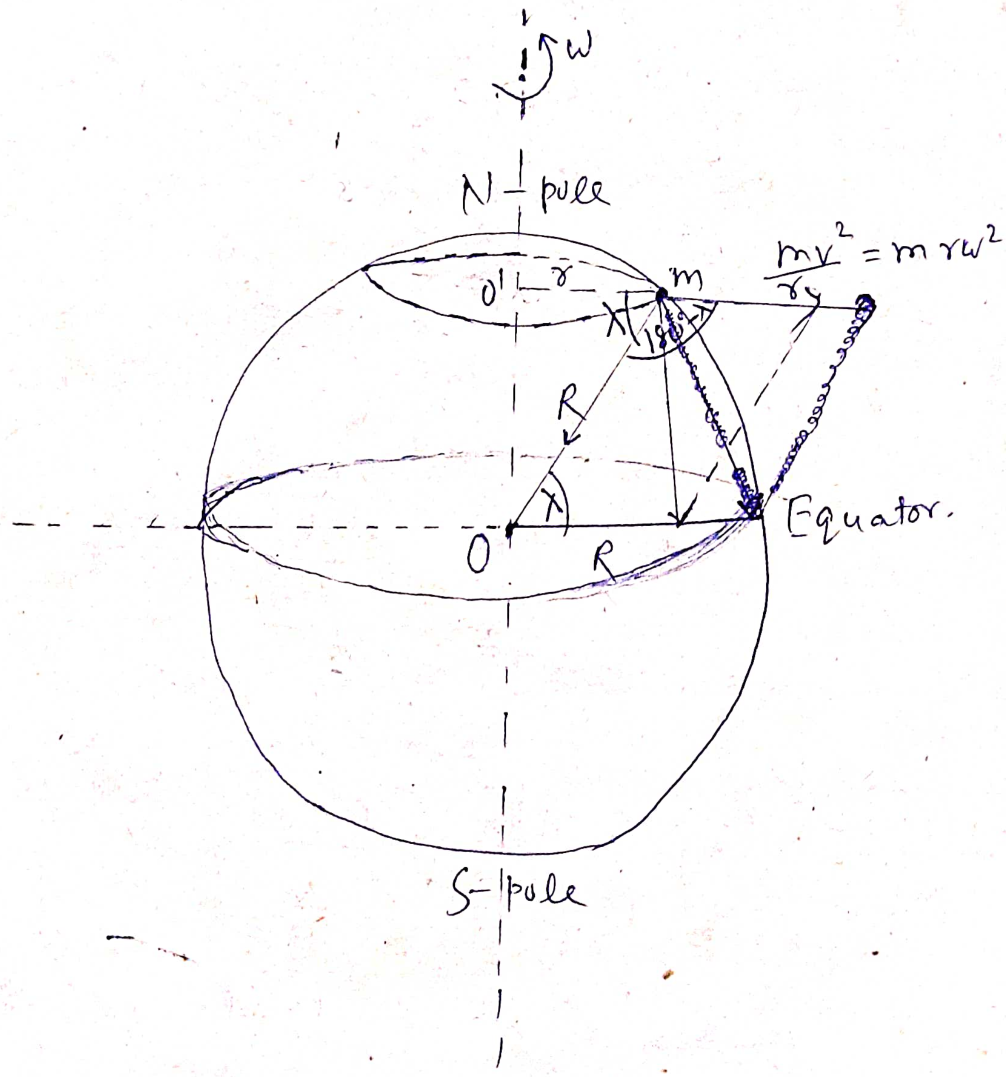
$$\Rightarrow \boxed{\tau = 2\pi \cdot \sqrt{\frac{l\cos\theta}{g}}} \quad \text{——— (vi)}$$

This expression shows that τ is independent of mass of the bob. It also resembles the expression for the time period or oscillation of simple pendulum $\left(T = 2\pi\sqrt{\frac{l}{g}} \right)$

(7)

Variation of g due to rotation of the earth

Since the period of rotation of the earth around its own axis is 24 hour, the different (x)



λ = Latitude of the place.

$$\cos\lambda = \frac{r}{R} \Rightarrow r = R\cos\lambda$$

(8) particles on the earth posses different speeds. At the pole, it is min^m and at the equator it is max^m.

Let's consider a body of mass m at a place on the earth where latitude is X. As shown in the diagram, this particles experiences an attraction towards the center of the earth and a centrifugal force. Resultant of these two forces be denoted by mg'.

From the law of parallelogram of vectors we can write

$$mg' = \sqrt{m^2g^2 + (m\omega^2 r)^2 + 2\,mg.m\omega^2 r.\cos(180°-x)}$$

$$mg' = \sqrt{m^2 g^2\left(1 + \frac{\omega^4 r^2}{g^2} - 2\frac{\omega^2 r}{g}\cos x\right)}$$

Mytinits

$$\Rightarrow \cancel{m}g' = \cancel{m}g\sqrt{1 + \left(\frac{\omega^2 r}{g}\right)^2 - 2\frac{\omega^2 r}{g}.\cos x}$$

$$\Rightarrow g' = g\sqrt{1 + \left(\frac{\omega^2 R}{g}\right)^2 \cos^2 x - 2\frac{\omega^2 R}{g}.\cos^2 x}$$

$(\because r = R\cos x)$

Calculation of $\frac{\omega^2 R}{g}$ for the earth can be done which is nearly equal to $\frac{1}{288}$

neglecting the second term under the square root sign as a very small quantity, we get

$$g' = g \cdot \sqrt{1 - \frac{2\cos^2 \lambda}{288}}$$

$$= g \left(1 - \frac{2\cos^2 \lambda}{288}\right)^{\frac{1}{2}}$$

Making Binomial expansion ~~for~~

$(1+x)^n \simeq 1+nx$, we get

$$g' = g \left(1 - \frac{1}{2} \cdot \frac{2\cos^2 \lambda}{288}\right)$$

$$\boxed{g' = g\left(1 - \frac{\cos^2 \lambda}{288}\right)}$$

Special Cases ~~Hol~~

(1) At the pole

$\lambda = 90^\circ$, $\cos \lambda = 0$

$$\therefore g' = g_{pole} = g(1-0) = g$$

$$= max^m$$

(2) At the equator

$\lambda = 0^\circ$, $\cos \lambda = 1$

$$\therefore \; g_{equator} = g\left(1 - \frac{1}{288}\right)$$

$$= g_{pole} - \frac{g_{pole}}{288}$$

$$= 980\ cm/s^2 - \frac{980}{288}\ cm/s^2$$

$$= (980 - 3.4)\ cm/s^2$$

$$= 976.6\ cm/s^2$$

Problem

1. How fast would the earth have to turn to make the apparent weight of the body to be zero at a point on the ~~equator~~ equator.

Ans :- Present angular speed of rotation of the earth $= \omega_0$ (say)

$$= \frac{2\pi\ rad}{24\ hour}$$

$$= \frac{2 \times 3.148}{24 \times 3600}\ rad/sec.$$

If the body is to become weightless,

weightless

$$w' = 0 = mg' \qquad g' = g\sqrt{\left(1 - 2\frac{\omega' R}{g}(1)\right)} = g\left(1 - \frac{2\omega' R}{g}\right)^{\frac{1}{2}}$$

$$= g\left(1 - \frac{1}{2} \cdot 2 \cdot \frac{\omega' R}{g}\right)$$

$$\Rightarrow g' = 0 = g\left(1 - \frac{(\omega')^2 R}{g} \cdot \cos^2 0^\circ\right)$$

$$\left(\because \lambda = 0^\circ \text{ at the equator}\right)$$

$$\Rightarrow 0 = 1 - \frac{\omega_1'^2 R}{g}$$

$$\Rightarrow \frac{\omega_1'^2 R}{g} = 1$$

$$\Rightarrow \omega_1'^2 = \frac{g}{R}$$

$$\Rightarrow \omega_1' = \sqrt{\frac{g}{R}}$$

Thus $$\frac{\omega'}{\omega} = \frac{\sqrt{\frac{g}{R}}}{\frac{2 \times 3.14}{24 \times 3600}} =$$

$$= \frac{\sqrt{\frac{9.8}{6.37 \times 10^6}}}{\frac{2 \times 3.141}{24 \times 36 \times 10^2}} = \sqrt{\frac{980}{637}} \times \frac{1}{10^3} \times \frac{24 \times 36}{6.28}$$

$$= \sqrt{\frac{980}{637}} \times \frac{24 \times 3.6}{6.28}$$

$$= 17.05$$

$$\therefore \omega' = 17\omega$$

Thus the body will be weightless if the ~~body~~ earth will rotate with a angular speed 17 times of the present angular speed.

15. Time period of revolution of the pendulum bob is given by

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\alpha}{g}}$$

$$\Rightarrow \frac{1}{\omega} = \sqrt{\frac{l\cos\theta}{g}}$$

Squaring both the sides

$$\Rightarrow \frac{1}{\omega^2} = \frac{l_0\cos\alpha}{g}$$

$\Rightarrow$ Here $l = 0.6\,m$,

$$\omega = 72\,\frac{rev}{min} = \frac{\cancel{72}^{36} \times \cancel{2}\pi}{\cancel{60}\,\cancel{30}\,15\ sec} = \frac{36\pi}{15\ sec}$$

$$= 7.5384\ rad/sec$$

$$\Rightarrow \frac{1}{\omega^2} = \frac{l_0\cos\alpha}{g}$$

$$\Rightarrow \frac{1}{(7.5384)^2} = \frac{(.6).\cos\alpha}{9.8}$$

$$\Rightarrow (7.5384)^2.(.6).\cos\alpha = 9.8$$

$$\Rightarrow \cos\alpha = \frac{9.8}{(7.5384)^2 \times (.6)}$$

$$= .2874$$

$$\Rightarrow \theta = \cos^{-1}(.2874) = 73.3^\circ$$

From the theory of conical pendulum we know that

$$mg = T \cos\alpha$$

$$\Rightarrow 35.6 = T.\ \cancel{(\ldots)}\ (.2874)$$

$$\Rightarrow T = \frac{35.6}{.2874} = 123.86 \text{ Newton.}$$

5. We know ~~from~~ from the theory of rotation of earth value of accn due to gravity is given by

$$g' = g\left(1 - \frac{\cos^2\lambda}{288}\right)$$

$$\Rightarrow g' = 9.8\left(1 - \frac{\cos 0^\circ}{288}\right)$$

$$\Rightarrow g' = 9.8\left(1 - \frac{1}{288}\right)$$

$$= 9.8 - \frac{9.8}{288}$$

$$= 9.8 - .034$$

$$= 9.766 \text{ m/sec}^2$$

9.800
$.034$
$\cancel{9.766}$

(b) Similarly at 40° latitude

$$g' = g\left(1 - \frac{\cos^2 40^\circ}{288}\right)$$

$$= g\left(1 - \frac{(.7660)^2}{288}\right)$$

$= \cancel{\ldots}$

$= g(1 - .0026597)$

$= \cancel{9.8 - .026065}$

$$\begin{array}{r} 980.000000 \\ 1.996554 \\ \hline 978.003446 \end{array}$$

$=$

$= 980\left(1 - \frac{(.7660)^2}{288}\right)$

$= 980(1 - .0020373)$

$= 980 - 1.9965534$

$= 978.003446$

Change in accel^n

$$= \begin{array}{r} 980.000000 \\ -978.003446 \\ \hline 1.996554 \end{array} \text{ cm/sec}^2$$

(c) At the poile

$g' = g\left(1 - \frac{\cos^2 90^\circ}{288}\right)$

$= 980\left(1 - \frac{0}{288}\right)$

$= 980(1)$

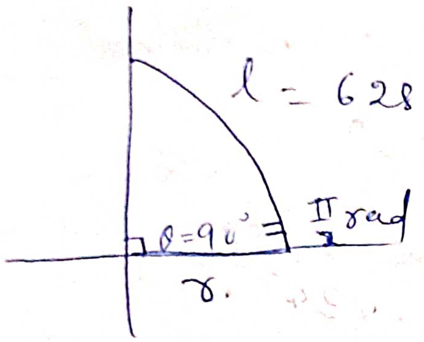
$= 980$ cm/s^2

$\therefore$ The value of g is maxm at pole and minm in equator.

Problem

2. An automobile has a mass of 1500 K.g. what Centripetal force is necessary for this vehicle to make a uniform 90° turn at 60 K.m/hr in a distance of 628 m along the road? (Hints $\pi = 3.14$)

Ans



$\theta = \frac{l}{r}$

$\theta = \frac{628}{r}$

$\frac{\pi}{2}\text{ rad} = \frac{628}{r}$

$\Rightarrow \frac{\cancel{3.14}}{2} = \frac{\cancel{628}\ 200}{r}$

$\Rightarrow r = 400$ meter

Centripetal force $= \frac{mv^2}{r} = (1500) \cdot \frac{(60\text{ K.m/hour})^2}{400\text{ meter}}$

$= \frac{(15\cancel{00}) \times \left(\frac{60 \times 10^3}{3600}\right)^2}{400} = \frac{15 \times 60 \times 10^3}{36 \times 400}$

$= \frac{1500 \times (600)^2}{36^2 \times 400} = \frac{15 \times 6 \times 10^6}{36^2 \times 4} \frac{10^4}{10^2}$

$$= \frac{1500 \times \left(\frac{60 \times 10^3}{\frac{3600}{6}}\right)^2}{400}$$

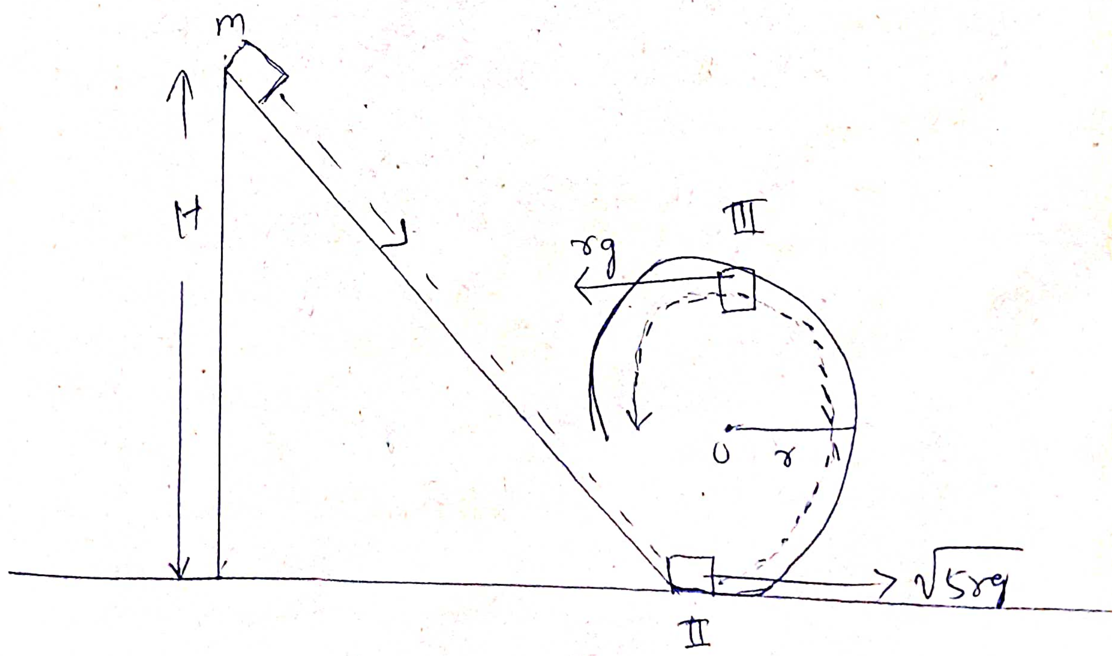
$$= \frac{1500 \times \cancel{600}\left(\frac{50}{3}\right)^2}{\cancel{400}}$$

$$= \frac{15 \times 2500}{9 \times 4}$$

$= 1041.66$ Newton.

Corollary of vertical circle

(1) To find 'H' so that the body sliding on the frame will complete the vertical circle (Frictional ~~effected~~ be neglected)



At the position I

Potential energy = mgH

Kinetic energy = 0.

So total energy at the ~~bottom~~ top = $mgH + 0$

$= mgH$

At the position II

Potential energy = 0

Kinetic energy = $\frac{1}{2} m v_b^2$

Total energy at the bottom

$= 0 + \frac{1}{2} m V_b^2 = \frac{1}{2} m V_b^2$

From the law of conservation of energy ~~we know~~

energy at position I = energy at position II

$\Rightarrow mgH = \frac{1}{2} m v_b^2$

$\Rightarrow \cancel{m} g H = \frac{1}{2} \cdot \cancel{m} \left(\sqrt{5rg}\right)^2$

$\Rightarrow \cancel{g} H = \frac{5r\cancel{g}}{2}$

$\Rightarrow H = \frac{5r}{2}$

Corollary - 2

TO find 'H' so that ~~too~~ ~~s~~ the solid sphere rolling on the frame will complete a vertical circle (Frictional effect be neglected).

Ans At the position I

Potential energy = mgH

Kinetic energy = 0

Total energy at the top = mgH

At position II

Potential energy = 0

Total Kinetic energy = $\frac{7}{10} m v^2$

Total energy = $\frac{7}{10} m v^2$

From the law of conservation of energy

$$\cancel{m}gH = \frac{7}{10} \cancel{m} v_b^2$$

$$\Rightarrow \cancel{g}H = \frac{7}{10} \cdot \frac{5 r \cancel{g}}{2}$$

$$\Rightarrow H = \frac{7r}{2}$$

26. At the top position

total energy $= mgH$

Half of the energy is extended in work against friction $= \frac{mgH}{2}$

So rest energy for the bottom $= mgH - \frac{mgH}{2}$

$= \frac{mgH}{2}$

At the bottom kinetic energy $\frac{7}{10} m v_b^2$.

From the law of conservation of energy

$= \frac{mgH}{2} = \frac{7}{10} m v_b^2$

$\Rightarrow \frac{\cancel{m}\cancel{g}H}{2} = \frac{7}{10} \cancel{m}\, 5r \cancel{g}$

$\Rightarrow H = 7r$

$\Rightarrow H = 7 \cdot (20\,cm)$

$\Rightarrow H = 140\,cm.$ — (Ans)

2. Variation of g with altitude is given by the formula

$$g' = \frac{g \cdot R^2}{(R+h)^2}$$

where R = radius of the sphere.

h = Height above the surface of the sphere.

g = Accn due to gravity on the surface of the sphere.

g' = Accn due to gravity at a height h above the surface of the sphere.

Multiplying ~~by~~ by m to both the sides of the above eqn, we get

$$mg' = \frac{mg \cdot R^2}{(R+h)^2}$$

$$\Rightarrow w' = \frac{W R^2}{(R+h)^2}$$

Given that h = 50 mile

R = 1000 mile, w = 100 lb

∴ Physical weight =

$$W' = \frac{W R^2}{(R+h)^2}$$

$$= \frac{(100) \times (1000)^2}{(1000+50)^2}$$

$$= \frac{10^8}{(1050)^2} = \frac{\cancel{100}^{\cancel{20}\,4} \times \cancel{10^6}\,10^4}{\cancel{1050}_{21} \times \cancel{105}_{21} \times \cancel{10}^2}$$

$$= \frac{\quad}{\cancel{1102500}}$$

$$= \frac{4 \times 10000}{21 \times 21}$$

$$= .0090702 \times 10^4$$

$$= 90.702 \text{ lb}$$

$$W_2' = \frac{WR^2}{(R+h)^2} = \frac{100 \cdot (1000)^2}{(1000+100)^2}$$

$$= \cancel{\frac{10^8}{(2000)^2}} \quad \frac{\cancel{10^8}\,10^4}{4 \times \cancel{10^4}}$$

$$= \frac{10^8}{(1100)^2} = \frac{10^8}{(11 \times 10^2)^2} = \frac{\cancel{10^8}\,10^4}{121 \times \cancel{10^4}}$$

$$= \frac{10^4}{121} = \frac{10,000}{121} = 82.6 \text{ lb}$$

$$w' = \frac{wR^2}{(R+h)^2} = \frac{(100)\,10^8}{(1000+500)^2}$$

$$= \frac{10^8}{(1500)^2} = \frac{\cancel{10^8}\,10^4}{225 \times \cancel{10^4}}$$

$$= \frac{10{,}000}{225} = 44.4 \text{ lb}$$

$$w' = \frac{wR^2}{(R+h)^2}$$

$$= \frac{10^8}{(1000+1080)^2}$$

$$= \frac{10^8}{(2080)^2} = \frac{\cancel{10^8}\,10^6}{(208)^2 \times \cancel{10^2}}$$

$$= \frac{\cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10^3}\ \cancel{10} \times \cancel{10} \times \cancel{10}}{\cancel{208} \times \cancel{208}}$$

104 104

52 52

26 26

$$= \frac{15625}{676} = 23.1 \text{ lb}$$

10. Initial velocity of rocket $u = 0$

$$V = \frac{966 \text{ km}}{\text{hour}}$$

$$= \frac{966 \times 10^3}{36\cancel{00}} = \frac{966 \times 5}{18}$$

$$= \frac{\cancel{9660}}{\cancel{36}} \quad \frac{966 \times 5}{18}$$

$S = 305$ m.

using the formula $v^2 - u^2 = 2as$, we get

$$\left(\frac{966 \times 5}{18}\right)^2 - 0 = 2 . (a) . (305)$$

$$\Rightarrow \frac{\cancel{9966}}{698025} =$$

$$\Rightarrow a = \frac{\left(\frac{966 \times 5}{18}\right)^2}{2 \times 305}$$

$$= \frac{\left(\cancel{966 \times 5}\right)^{483} \times \left(\cancel{966 \times 5}\right)^{483}}{\cancel{18} \times \cancel{18} \times \cancel{2} \times \cancel{305}} = 118.037 \text{ m/s}^2$$

number of times of g $\Rightarrow \frac{118.037}{9.8} = 12.044$. (Ans)

14. A ~~force~~ 5 lb ball is swung at the end of a cord in a vertical circle of radius 2 ft at the rate of $\frac{2 \text{ rev}}{\text{sec}}$. What is the tension of the cord when the ball is

(a) at the bottom.

(b) at the top.

(c) ~~at~~ Find the tension in the cord and the angle ~~beta~~ the cord makes with the horizontal when the body is at the level of the center.

Ans : 54 lb, 44 lb, 49 lb, $0°$

→ At ~~the~~ bottom, Tension

$$T_1 = \cancel{\omega} \frac{m V_1^2}{r} - \omega$$

$$\Rightarrow \cancel{T = \frac{5}{32}}$$

$m = \frac{5}{32}$, $V_1 \, \omega = \frac{2 r}{\text{sec}} = \frac{2 \times 2\pi \text{ rad}}{\text{sec}} = 4\pi \text{ rad/sec}$

$W = \cancel{5\,lb}$

$T = \cancel{m v_b^2}$

(a) At the bottom

Tension $T = W + \dfrac{m v_b^2}{r}$

$W = 5\,lb$, $m = \dfrac{5}{32}$, $\omega = 2\,\dfrac{rev}{sec}$

$= \dfrac{2 \cdot 2\pi\, rad}{sec} = 4\pi\, rad/sec$

$r = 2\,ft$

$V_b = r \cdot \omega$
$= 2 \times (4\pi)$
$= 8\pi$
$= 8 \times 3.14$

$T = W + \dfrac{m v_b^2}{r}$

$= 5 + \dfrac{5}{32} \cdot \dfrac{(8 \times 3.14)^2}{2}$

$= 5 + \dfrac{5}{64}(631.0144)$

$= 5 + 49.298$

$= 54.298\ lb$

(b) At the top

$T = \dfrac{m v_t^2}{r} - W$

$\Rightarrow T = 49.298 - 5$

$= 44.298\ lb$

(c) When the body is at the level of center $\theta = 90°$.

We know that net force towards the center $T - W\cos\theta = \frac{mv^2}{r}$

$\Rightarrow T = 5.\cos 90° + 49.298)$

$\Rightarrow T = 0 + 49.298$

$\Rightarrow T = 49.298$ lb.

(d) The angle made by the body with the horizontal $= 0°$.

6. A ball rolling on an inclined plane moves with constant accⁿ. One ball is realeased from rest, at the top of an inclined plane 18 meter long and reaches the

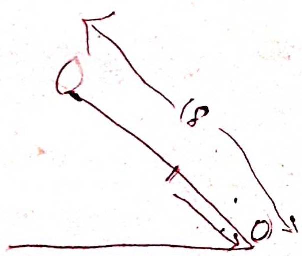
buttom 3 second later. At the same instant that the first ball reached, a second ball is projected upward along the plane from its buttom with certain initial velocity. The second ball is to travel partway up the plane, stop and return to the buttom so that it arrives simultaneously with first ball.

(a) What must be the initial velocity of the second ball

(b) How far up the plane will it travel.

[Ans 6 ms^{-1}, 4.5m]

Ans



Initial velocity of firstball $= U = 0$

$t = 3sec$.

$S = 18m$.

Using the formula $S = ut + \frac{1}{2}at^2$, we get

$\Rightarrow 18 = 0 + \frac{1}{2}a \cdot 9$

$\Rightarrow a = \frac{36}{9} = 4 m/s^2$

The two balls ~~are~~ were projected at the same time and reach the buttom same time

So time for the second ball = 3 sec.

Time was taken to reach the point = Time was taken to fall $= 1.5$ sec.

For the second ball, when it goes to the highest point

Initial velocity $u = ?$, $s = ?$ $t = 1.5$

$v = 0$, $a = -4 \, m/s^2$

using the formula $v = u + at$, we get

$0 = u + (-4).(1.5)$

$\Rightarrow -u = 6 \, m/s$.

~~$s = ut$~~ $s = ut + \frac{1}{2}at^2$

$= 6.(1.5) + \frac{1}{2}(-4).(2.25)$

$= 9 - 4.50$

$= 4.5 \, m$.

Initial velocity $6 \, m.s^{-1}$

Distance $= 4.5 \, m$,

Derivation of formulae by dimensional analysis.

① To derive $T = 2\pi\sqrt{\frac{l}{g}}$ for a simple pendulum.

From experiments it is found that the time period of a simple pendulum depends on (a) Length of it.

(b) Acceleration due to gravity.

Let $T \propto l^a$, when g is kept constant.

$T \propto g^b$ " l " " ".

Combining these two variations, we have,

$T \propto l^a . g^b$. when both l & g vary.

$\Rightarrow T = K . l^a . g^b$ ——— ①

where K is a constant. a and b are unknown parameters to be determined.

Putting proper dimensions for each quantity, we have,

$$[M^0 L^0 T^1] = [L]^a \, [L^1 T^{-2}]^b$$

$$= L^{a+b} . T^{-2b}$$

Comparing both the sides, we get,

$0 = a + b$

$1 = -2b \Rightarrow b = -\frac{1}{2}$

$a = -b = \frac{1}{2}$ Putting these values in eqn ①, we get, $T = K \cdot l^{\frac{1}{2}} \cdot g^{-\frac{1}{2}}$

$$= K\sqrt{\frac{l}{g}} \quad \text{——————} \quad (2)$$

Putting actual experimental data in eqn (2), we get,

$$K = 2\pi.$$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

(2) Derive Stoke's Law $F = 6\pi\eta r v$

where η = co-efficient of viscosity having dimension $[M^1 L^{-1} T^{-1}]$

F = Viscous force

v = Terminal velocity

r = radius of the sphere

From experiments it is found that the ~~dis~~ viscous force on a sphere depends on

(a). Co-efficient of viscosity (η)

(b) radius of the sphere (r)

(c) Terminal velocity. (v)

Let $F \propto \eta^a$, when r and v are kept constant.

$F \propto r^b$, when η and v are kept constant.

$F \propto v^c$, when η and r are kept constant.

Combining these three varations, we get

$$F \propto \eta^a \gamma^b v^c$$

when ~~~~ ~~~~ ~~~~ η, γ, v vary.

$$\Rightarrow F = K \eta^a \gamma^b . v^c \quad \text{———— (i)}$$

where K is a constant. ~~~~ and a, b, c are unknown parameters to be determined.

Putting proper dimensions for each quantity, we have

$$[M^1 L^1 T^{-2}] = [M^1 L^{-1} T^{-1}]^a . [L]^b . [L . T^{-1}]^c$$

$$= M^a . L^{b+c-a} . T^{-a-c}$$

Comparing both the sides, we get

$1 = a$ —(i) $1 = b + c - a$ (ii), $-2 = -a - c$ —(iii)

Putting the value of a in eq^n (iii), we get

$$-2 = -a - c$$

$$\Rightarrow -2 = -1 - c$$

$$\Rightarrow -c = -1$$

$$\Rightarrow c = 1$$

Putting the value of a and c in eq^n (ii), we get

$$1 = b + c - a$$

$\Rightarrow \cancel{1 = b + \frac{1}{2} - 1}$

$\Rightarrow \cancel{b = 2 - \frac{1}{2} = \frac{3}{2}}$

$\Rightarrow 1 = b + 1 - 1$

$\Rightarrow b = 1$

$\therefore a = 1, \quad b = 1, \quad c = 1$

Putting these values in eq^n (i), we get

$$F = K \eta^1 . r^1 . v^1$$
$$= K \eta r v \quad \text{———} \quad (ii)$$

Putting actual experimental data in eq^n (ii) we get

$$K = 6\pi$$

$$\therefore F = 6\pi \eta r v$$

③ Derive the expression for the critical velocity ie. $v_c = \frac{2000 \eta}{\rho . D.}$

From experiments it is found that the critical velocity (V_c) depends on

~~depends on~~

(a) Density of the liquid (ρ)

(b) Diameter of the tube (D)

(c) Co-efficient of viscosity (η)

Let

$$V_c \propto \rho^a \text{, when } D \text{ and } \eta \text{ are kept constant.}$$

$$V_c \propto D^b \text{, when } \rho, \eta \text{ are kept constant.}$$

$$V_c \propto \eta^c \text{, when } \rho, D \text{ are kept constant.}$$

Combining these three variations, we get

$$V_c \propto \rho^a D^b \eta^c \quad \text{when } \rho, D, \eta \text{ vary.}$$

$$\Rightarrow \quad V_c = K \rho^a . D^b . \eta^c \quad \text{——— (i)}$$

where K is kept constant and a, b, c are unknown parameters to be determined.

Putting proper dimensions for each quantity, we get

$$[M^0 L^1 . T^{-1}] = [M^1 L^{-3}]^a . [L]^b . [M^1 L^{-1} T^{-1}]^c$$

$$= M^{a+c} . L^{-3a+b-c} . T^{-c}$$

Comparing both the sides, we get

$$0 = a + c \quad \text{——— (ii)}$$

$$1 = b - 3a - c \quad \text{——— (iii)}$$

$$-1 = -c \quad \text{——— (iv)}$$

$$c = 1,$$

Putting the value of c in eqⁿ (ii),

we get.

$$a + c = 0$$

$$\Rightarrow a + 1 = 0$$

$$\Rightarrow a = -1$$

Putting the value of a and c in eqⁿ (iii),

we get

$$1 = b - 3a - c$$

$$\Rightarrow 1 = b - 3(-1) - (1)$$

$$\Rightarrow 1 = b + 3 - 1$$

$$\Rightarrow b = 2 - 3 = -1$$

$$\therefore \quad a = -1, \quad b = -1, \quad c = 1$$

Putting these value in eqⁿ (i), we get

$$V_c = K \cdot f^{-1} \cdot D^{-1} \cdot \eta^{1} \quad \text{——— (v)}$$

$$= \frac{K \eta}{D \cdot f}$$

Putting actual experimental data in eqⁿ (v), we get

$$K = 2000$$

$$\therefore V_c = \frac{2000 \cdot \rho}{D \cdot \eta}$$

$$V_c = \frac{2000\,\eta}{\rho \cdot D}$$

(4) Derive Poiseuille's formula

$$V = \frac{\pi P r^4}{8 \eta l}$$

where V = volume of liquid flowing/sec through a capillary tube of length l and radius r.

P = Pressure difference between its two ends.

η = co-efficient of viscosity.

$\frac{P}{l}$ = Pressure gradient to be taken as a single variable for the purpose of the derivation.

From experiments it is found that

Volume of liquid flowing / sec through a cappilary tube of length l and radius r depends on

(a) pressure gradient

(b) radius of tube

(c) co-efficient of viscocity.

let $V \propto \left(\frac{p}{l}\right)^a$, when γ and η are kept constant.

$V \propto \gamma^b$, when $\frac{p}{l}$ and η are kept constant,

$V \propto \eta^c$, when $\frac{p}{l}$ and γ are kept constant.

Combining these three variations, we have

$V \propto \left(\frac{p}{l}\right)^a \cdot (\gamma)^b \cdot (\eta)^c$ when $\frac{p}{l}$, γ, η vary.

$$\Rightarrow V = K \left(\frac{p}{l}\right)^a \cdot (\gamma)^b \cdot (\eta)^c \text{ ——— (i)}$$

where K is a constant a, b, c are unknown parameters to be determined.

Putting proper dimensions for each quantity, we have

$$[M^0 L^3 T^{-1}] = \left[\frac{ML^{-1}T^{-2}}{L}\right]^a \cdot [L]^b \cdot [M^1L^{-1}T^{-1}]^c$$

$$= [ML^{-2}T^{-2}]^a [L^1]^b [M^1L^{-1}T^{-1}]^c$$

$$= M^{a+c} \cdot L^{-2a+b-c} \cdot T^{-2a-c}$$

Comparing both the sides, we get

$$0 = a+c \text{ ——— (ii)}$$

$$3 = -2a + b - c \quad \text{—— (iii)}$$

$$-10 = -2a - c \quad \text{—— (iv)}$$

From eqn (ii)

$$-2a = c - 1$$

$$\Rightarrow a = -\left(\frac{c-1}{2}\right)$$

Putting the value of a in eqn (i), we get

$$a + c = 0$$

$$\Rightarrow -\left(\frac{c-1}{2}\right) + c = 0$$

$$\Rightarrow \frac{-c + 1 + 2c}{2} = 0$$

$$\Rightarrow c + 1 = 0$$

$$\Rightarrow c = -1$$

Putting ~~the~~ the value of c in eqn (i), we get

$$a + c = 0$$

$$\Rightarrow a - 1 = 0$$

$$\Rightarrow a = 1$$

Putting the value of a and c in eqn (iii), we get

$$3 = -2a + b - c$$

$$\Rightarrow 3 = -2.(1) + b - (-1)$$

$$= -2 + b + 1$$

$$= b - 1$$

$\Rightarrow b = 4$

$\therefore \quad a = 1, \; b = 4, \; c = -1$

~~The~~ Putting the value of a, b, c in eqn (i), we get

$$V = K \cdot \left(\frac{P}{l}\right)^1 \cdot r^4 \cdot \eta^{-1}$$

Putting actual experimental data in eqn (i), we get

$$K = \frac{\pi}{8}$$

$$\therefore \quad V = \frac{\pi}{8} \cdot \frac{P}{l} \cdot r^4 \cdot \eta^{-1}$$

$$V = \frac{\pi P r^4}{8 \eta l}$$

(5) Frequency of a vibrating string is given by $n = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

where T = Tension in the string

μ = Mass per unit length of the string

l = length of the string

n = Frequency = $\frac{1}{\text{Time period}}$

From experiments it is found that frequency (n) of the vibrating body depends on

(a) Tension of the string (T)

(b) Mass per unit length of the string (μ)

(c) Length of the string (l)

Let

$n \propto T^a$ when μ and l are kept constant

$n \propto \mu^b$ when T and l are kept constant

$n \propto l^c$ when μ, T are kept constant

Combining these three variation, we get

$n \propto T^a \mu^b l^c$ when T, μ, l vary

$\Rightarrow \quad n = K T^a \mu^b l^c$ where K is a constant and a, b, c are unknown parameters to be determined.

$\Rightarrow$ ~~$n =$~~

Putting proper ~~diameters~~ dimensions for each quantity, we have

$$[T^{-1}] = [MLT^{-2}]^a \left[\frac{M}{L}\right]^b . [L]^c$$

$$[M^0 L^0 T^{-1}] = M^{a+b} . L^{-b+c+a} . T^{-2a}$$

Comparing both the sides, we get,

$$0 = b+a \text{ — (i)}$$

$$0 = a+c-b \text{ — (ii)}$$

$$-1 = -2a \text{ — (iii)}$$

$\therefore$ $a = \frac{1}{2}$, $b+a=0$ $\Rightarrow b+\frac{1}{2}=0$ $\Rightarrow b = -\frac{1}{2}$

putting these values in eqn (ii), we get

$a+c-b=0$ $\Rightarrow \frac{1}{2}+c-(-\frac{1}{2})=0$ $\Rightarrow \frac{1}{2}+c+\frac{1}{2}=0$ $\Rightarrow c+1=0$ $\Rightarrow c=-1$

$\therefore a=\frac{1}{2}, b=-\frac{1}{2}, c=-1$ Putting these values in eqn (i)

$$n = K.T^{\frac{1}{2}}.\mu^{-\frac{1}{2}}.l^{-1}$$

$$= K.\sqrt{\frac{T}{\mu}}.\frac{1}{l} \text{ ——— (v.)}$$

Putting actual experiment data in eqn(v),

~~we~~ we get $K = \frac{1}{2}$

$$\therefore n = \frac{1}{2}.\frac{1}{l}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

(6) Derive Newton's formula for the velocity of sound.

$$v = \sqrt{\frac{E}{\rho}}$$

where E = Bulk modulus of elasticity having the dimension of stress. $\left(\frac{\text{force}}{\text{area}}\right)$

ρ = Density of the medium.

From experiments it is found that velocity of sound depends on

(a) Bulk modulus ob elasticity.

(b) Density of medium.

Let $v \propto E^a$ where ρ is constant.

$v \propto \rho^b$ where E is kept constant.

Combining these three ~~two~~ variations, we get

$v \propto E^a . \rho^b$ when ρ, E are kept constant

$\Rightarrow$ $v = K E^a . \rho^b \longrightarrow$ (i) where K is a constant and a, b are unknown parameters to be determined.

Putting proper dimensions for each quantity, we have

$$[LT^{-1}] = \left[\frac{ML^{1}T^{-2}}{L^2}\right]^a . [ML^{-3}]^b$$

$$[M^0 L^1 T^{-1}] = [ML^{-1}T^{-2}]^a \cdot [ML^{-3}]^b$$

$$= M^{a+b} \cdot L^{-a-3b} \cdot T^{-2a}$$

Comparing both the sides, we get

$0 = a + b$ — (i)

$1 = -a - 3b$ — (ii)

$-1 = -2a$ — (iii)

$\therefore$ ~~from~~ $a = \frac{1}{2}$

$a + b = 0$

$\Rightarrow \frac{1}{2} + b = 0$

$\Rightarrow b = -\frac{1}{2}$

$\therefore a = \frac{1}{2}, \quad b = -\frac{1}{2}$

Putting these values in eqn. (1), we get

$$V = K \cdot E^{\frac{1}{2}} \cdot \rho^{-\frac{1}{2}}$$

$$= K \sqrt{\frac{E}{\rho}}$$

Putting actual experiment data in eqn (1),

we get

$$V = \sqrt{\frac{E}{\rho}}$$

24.12.2K

Gravitation Unit - 4 (a)

Newton's law of gravitation

According to this law (1), every body of the universe attracts every other body with a force called gravitational force (1) It which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$F \propto m_1 m_2$, when r is kept constant

$F \propto \frac{1}{r^2}$, when m_1 and m_2 are kept constant.

m_1 m_2

Combining these two variations, we have

$F \propto \frac{m_1 m_2}{r^2}$, when all the quantities vary.

or $\boxed{F = G \frac{m_1 m_2}{r^2}}$

where G = Constant Called universal gravitational constant having Value 6.67×10^{-11} $N\,m^2/Kg^2$

Dimension of G

$$G = \frac{F s^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M^2]}$$

$$= [M^{-1} L^3 T^{-2}]$$

Defination of G

$G = F$ when $m_1 = m_2 = 1$ Unit
$S = 1$ Unit.

Thus universal gravitational Can be be defined as numerically equal to the force between two unit masses placed unit distance aparat in the medium like air.

Units of G

$$G = \frac{F s^2}{m_1 m_2}$$

(a) C. G. S system = $\frac{Dyne \cdot cm^2}{gm^2}$

(b) M.K.S system $= \dfrac{\text{Newton} . m^2}{Kg^2}$

(c) F.P.S (absolute) $= \dfrac{\text{poundal} . ft^2}{(\text{poundmass})^2}$

(d) F.P.S (Gravitational) $= \dfrac{\text{pound} . ft^2}{(\text{slug})^2}$

If $G = 6.67 \times 10^{-11}$ N m^2/Kg^2 its value in C.G.S system will be

$$= \frac{6.67 \times 10^{-11} \times 1N \times 1m^2}{1Kg^2}$$

$$= \frac{6.67 \times 10^{-11} \times 10^5 \text{ dyne} \times (10^2 \text{ cm})^2}{(10^3 \text{ gm})^2}$$

$$= \frac{6.67 \times 10^{-11} \times \cancel{10^6} \times 10^3 \text{ dyne.cm}}{\cancel{10^6} \text{ gm}^2}$$

$$= 6.67 \times 10^{-8} \text{ dyne . cm / gm}^2$$

Application :

1. Weighing the earth

Let us consider a small body on the surface of earth. It is attracted towards the center of the earth by a force which is called weight. ($W = mg$)

From Newton's law of Gravitation, the attractive force on the body is given by $F = G\frac{Mm}{R^2}$

(where R = Radius of the earth)

Thus $F = W$ or $G\frac{M\cancel{m}}{R^2} = \cancel{m}g$

$$\therefore \boxed{M = \frac{gR^2}{G}}$$

Calculation of M

$$M = \frac{9.8 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= \frac{9.8 \times 6.37 \times 6.37 \times 10^{12}}{6.67 \times 10^{-11}}$$

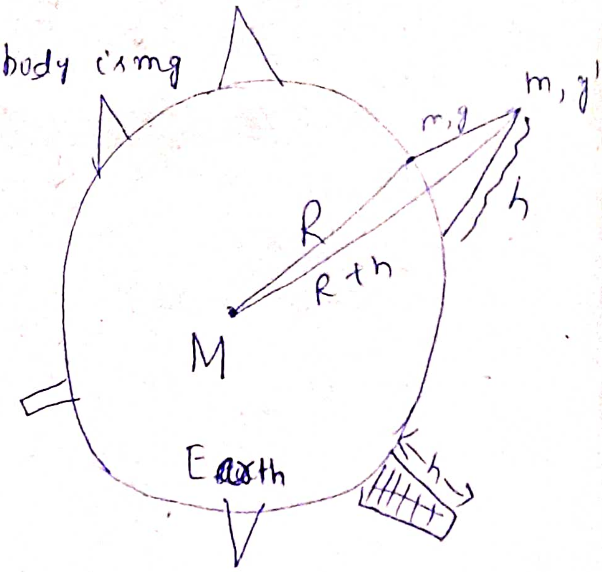
$$= 59.618 \times 10^{23} \text{ kg}$$

This value agrees fairly well with the value found of other methods.

(2). Variation of g with altitude

To find the relation between the accn due to gravity at a height h above the surface of the earth and that on the surface of the earth, let's consider a small mass on the surface of the earth at the foot of the mountain.

The weight of the body i.e. mg which is equal to the gravitational attraction of the earth.



$$\therefore \frac{GMm}{R^2} = mg \text{ ——— (i)}$$

If the same body be taken to the top of the mountain, then the force of attraction by the earth will be

$$\frac{GMm}{(R+h)^2} = mg' \text{ ——— (ii)}$$

where g' = acc^n due to gravity at the top of the mountain.

Dividing eq^n (i) by eq^n (ii), we get

$$\frac{(R+h)^2}{R^2} = \frac{g}{g'}$$

$$\Rightarrow \frac{R^2}{(R+h)^2} = \frac{g'}{g}$$

$$\Rightarrow \boxed{g' = \frac{g \cdot R^2}{(R+h)^2}} \quad \text{———— (iii)}$$

Obviously $g' < g$

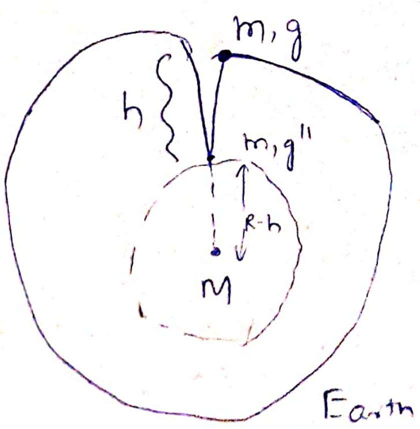
This shows that the accⁿ due to gravity decreases with the increase of height i.e weight of the body decreases with the increase of height.

(3) <u>Variation of g with depth</u>

Let's consider a body first on the surface of the earth at the mouth of the mine of depth 'h'. The value of gravity at that place be g

Then $G\frac{Mm}{R^2} = mg$ ———— (i)

If the same body be taken to the bottom of the mine, then the value of gravity becomes g'' (Say).



Gauss has proved that the body will be attracted by the mass present in the sphere of radius $(R-h)$. If this mass be M_1, then

$$\frac{G M_1 m}{(R-h)^2} = m g'' \quad \text{---(ii)}$$

where M_1 = Mass of the sphere of radius $(R-h)$.

$$= \frac{4}{3}\pi (R-h)^3 \cdot \rho$$

(where ρ = density of the earth to be uniform.

$$= \frac{M}{\frac{4}{3}\pi R^3} \Big)$$

Dividing eqn (i) by eqn (ii)

$$\frac{M}{M_1} \cdot \frac{(R-h)^2}{R^2} = \frac{g}{g''}$$

$$\Rightarrow \frac{\cancel{\frac{4}{3}\pi R^3 \rho}\, R}{\cancel{\frac{4}{3}\pi (R-h)^3 \rho}\,(R-h)} \cdot \frac{\cancel{(R-h)^2}}{\cancel{R^2}} = \frac{g}{g''}$$

$$\Rightarrow \frac{R}{R-h} = \frac{g}{g''}$$

$$\Rightarrow \frac{R-h}{R} = \frac{g''}{g}$$

$$\Rightarrow \boxed{g'' = \frac{g(R-h)}{R}}$$

$$\Rightarrow 1 - \frac{h}{R} = \frac{g''}{g}$$

$$\Rightarrow \boxed{g'' = g\left(1 - \frac{h}{R}\right)}$$

Since $h < R$, $\frac{h}{R}$ is a fraction

and $1 - \frac{h}{R}$ is another fraction

Thus $g'' < g$

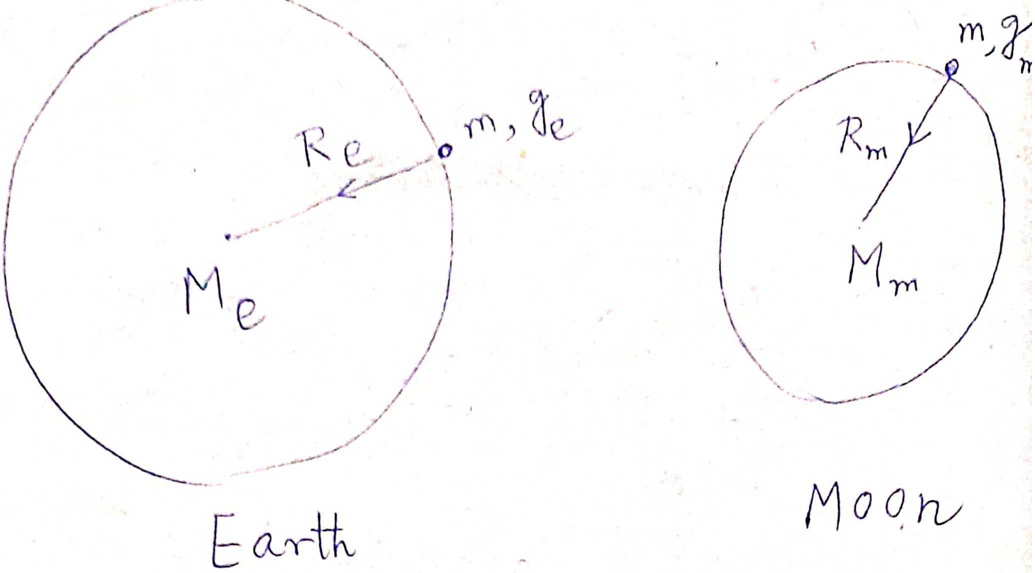
i.e the accl^n due to gravity decreases as we go more and more towards the center of the earth. Ultimately, at the center of the earth $h = R$

and $g'' = g(1-1) = g.0 = 0$

i.e the ~~at~~ accl^n due to gravity vanishes at the center of the earth and the body becomes weightless

71 page

34.



Given

$$R_m = 0.25\, R_e$$

$$M_m = 0.0127\, M_e$$

Lets place a small mass m, on the surface of the earth and then on the surface of the moon.

$$G\frac{M_e . m}{R_e^2} = m . g_e \quad \underline{\text{for the earth}} \; (i)$$

$$G\frac{M_m . m}{R_m^2} = m . g_m \quad \underline{\text{for moon}} \; (ii)$$

Dividing eqn (i) by eqn (ii), we get

$$\frac{M_e R_m^2}{M_m R_e^2} = \frac{g_e}{g_m}$$

$$\Rightarrow \left(\frac{M_e}{M_m}\right)\left(\frac{R_m^2}{R_e^2}\right) = \frac{g_e}{g_m}$$

$$\Rightarrow \left(\frac{M_e}{0.0127 M_e}\right)\left(\frac{0.25 R_e}{R_e}\right)^2 = \frac{g_e}{g_m}$$

$$\Rightarrow \frac{1}{0.0127} \times \frac{\cancel{0.25}\ 1}{16} = \frac{32 \text{ ft/s}^2}{g_m}$$

$$\Rightarrow g_m = 32 \times 16 \times 0.0127 \text{ ft/s}^2$$

$$= 6.5024 \text{ ft/s}^2 \quad \text{(Ans)}$$

This shows that $g_m \simeq \frac{g_e}{5}$.

(2). The mass of Jupiter is 314 times that of the earth and the diameter of Jupitor is 11.35 times that of the earth. If g has a value of 9.8 m/s^2 on the earth, what is its value on Jupitor.

(23.9 m/s^2)

(3). A body weighs 90 lb on the surface of the earth. How much will it ~~way~~ weigh on the surface of Mars whose mass is $\frac{1}{9}$ and radius is $\frac{1}{2}$ of that of the earth.

(Ans : 40 lb)

3. If a body weighs 10N on the surface of the earth, what will be its weight when it is taken to a height of 2000 miles above the surface of the earth?

R = 4000 miles.

(Ans : 4.44 N)

3) Variation of g with altitude is know to be

$$g' = g \frac{R^2}{(R+h)^2}$$

$$\Rightarrow mg' = mg \frac{R^2}{(R+h)^2}$$

$$\Rightarrow W' = \frac{W \cdot R^2}{(R+h)^2}$$

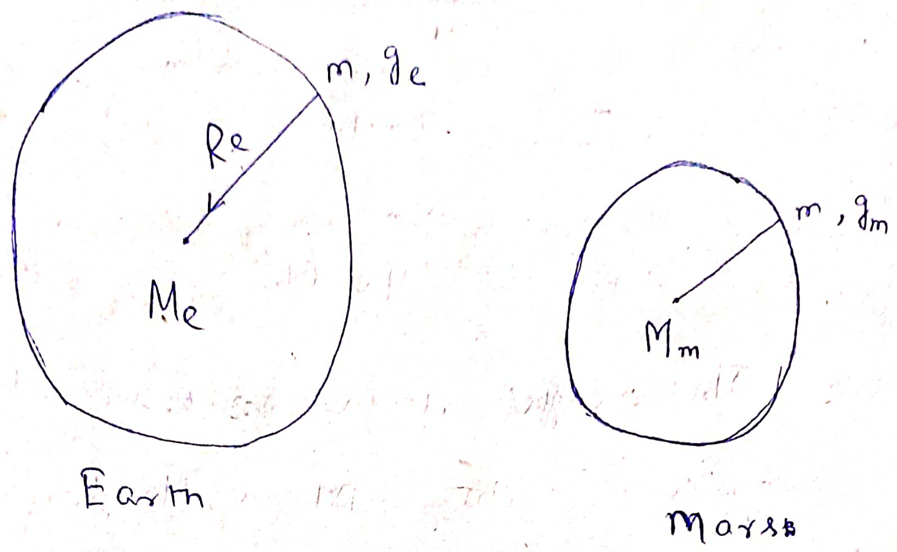
$$= \frac{10 \times (4000)^2}{(4000+2000)^2} = \frac{10 \times 16 \times 10^6}{36 \times 10^6}$$

$$= \frac{10 \times \cancel{16}^{4} \times \cancel{10^6}}{\cancel{36}_{9} \times \cancel{10^6}}$$

$$= \frac{40}{9} = 4.44 \text{ N}$$

∴ Its weight 2000 mile above the surface of the earth = 4.4N

2.



Given that $R_m = \frac{1}{2} R_e$

$M_m = \frac{1}{9} M_e$

Lets place a body 90 lbs on the surface of the earth and then on the surface of the mars.

$$G \frac{M_e m}{R_e^2} = m \cdot g_e \text{ (for earth)} = W_e \longrightarrow (i)$$

$$G \frac{M_m \cdot m}{R_m^2} = m \cdot g_m \text{ for } \cancel{\text{man}} \text{ mar} = W_m \text{ —(ii)}$$

Diviling eqn (i) by eqn (ii), we get

$$\Rightarrow \left(\frac{M_e}{M_m}\right)\left(\frac{R_m^2}{R_e^2}\right) = \frac{W_e}{W_m}$$

$$\Rightarrow \left(\frac{M_e}{\frac{1}{9}M_e}\right) \times \left(\frac{\frac{1}{2}R_e}{R_e}\right)^2 = \frac{90}{W_m}$$

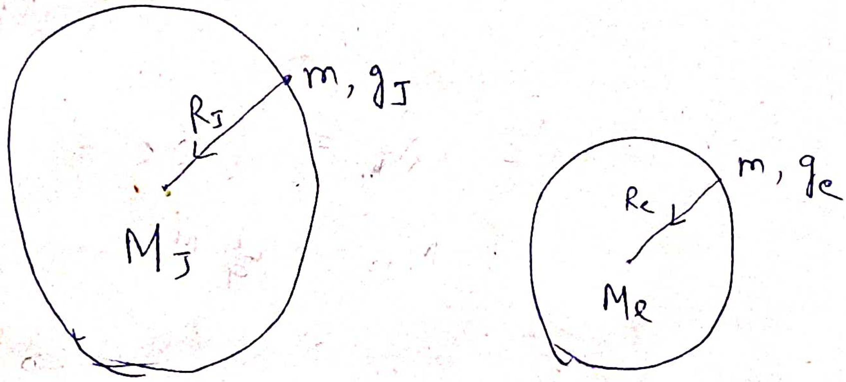
$$\Rightarrow 9 \times \frac{1}{4} = \frac{90}{W_m}$$

$$\Rightarrow \frac{\cancel{9}}{4} = \frac{\cancel{90}\;10}{W_m}$$

$$\Rightarrow W_m = 40 \text{ lb} \qquad (\text{Ans})$$

∴ ~~The~~ weight of the ~~~~ body at the surface of Mars 40 lb.

1.



Jupitor   Earth

Given $M_J = 3\cancel{0}14 \times M_e$

Diameter of Jupitor = 11.35 time of earth.

$$\Rightarrow \cancel{2} \times R_J = 11.35 \times (\cancel{2} \times R_e)$$

$$\Rightarrow R_J = 11.35\, R_e$$

Lets place a small mass m on the surface of the earth and then on the surface of Jupitor

$$G \cdot \frac{M_J \cdot m}{(R_J)^2} = m \cdot g_J \quad \text{for Jupiter} - (i)$$

$$G \cdot \frac{M_e \cdot m}{(R_e)^2} = m \cdot g_e \quad \text{for Jupiter (ii)}$$

$$\Rightarrow \left(\frac{M_J}{M_e}\right)\left(\frac{R_e^2}{R_J^2}\right) = \frac{g_J}{g_e}$$

$$\Rightarrow \left(\frac{314 \times M_e}{M_e}\right)\left(\frac{R_e}{R_J}\right)^2 = \frac{g_J}{9.8}$$

$$\Rightarrow (314)\left(\frac{R_e}{11.35 R_e}\right)^2 = \frac{g_J}{9.8}$$

$$\Rightarrow (314)\left(\frac{1}{11.35}\right)^2 = \frac{g_J}{9.8}$$

$$\Rightarrow (314)\left(\frac{1}{11.35}\right)^2 \cdot (9.8) = g_J$$

$$\Rightarrow \frac{(314) \times 1 \times 9.8}{11.35 \times 11.35} = g_J$$

$$\Rightarrow 23.88 = g_J$$

27.12.2k

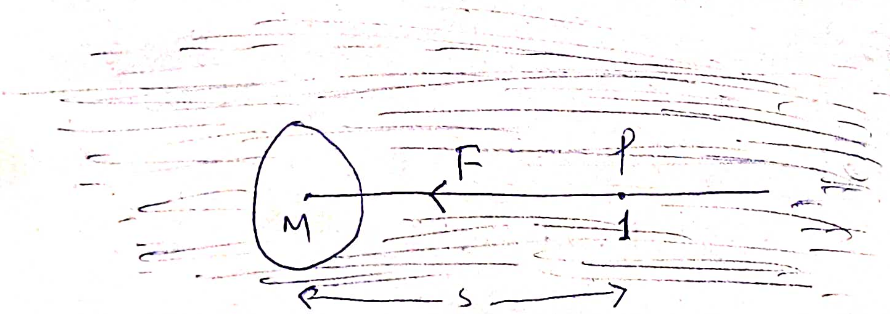
Gravitational field

It is the space ~~of~~ surrounding a mass where its influence is felt.

Gravitational field intensity

It is a vector quantity by which the intensity of the field created by some mass can be known.

Gravitational field intensity at a point in a gravitational field is defined as the force experienced by a unit mass placed at the point concerned.



$$F = \frac{G.M.1}{s^2}$$

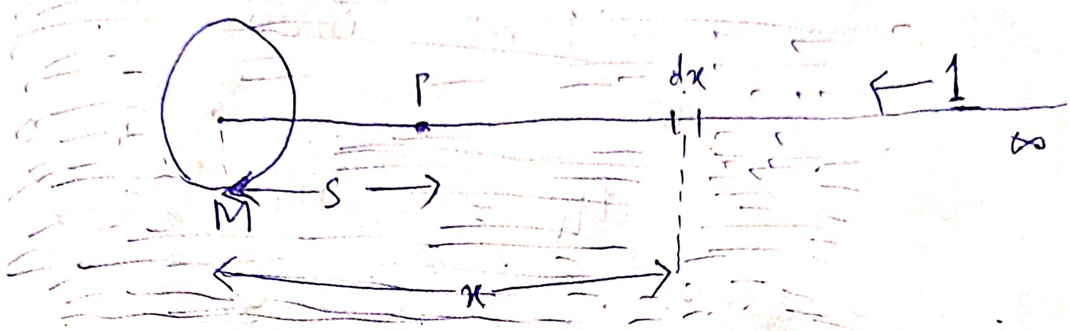
My hints
M = A isolated body of mass M.

Gravitational field intensity at ~~p~~ P

$$= \frac{G.M}{s^2}$$, directed towards the mass M

Gravitational Potential

It is a scalar quantity and defined as the amount of work done to bring a unit mass from infinite up to the point concerned.



To derive an expression for the gravitional potential at a point in a gravitational field created by a mass M, let us devide the entire distance into large number of small segments each of width dx.

dW = Small amount of work done to shift the unit mass through a small distance dx.

= Force × displacement

$$= \frac{GM}{x^2} \times dx$$

But this work is negative because the displacement is along the direction of the force of attraction by the mass M

$$\therefore \; dW = -\frac{GM}{x^2} \times dx$$

Total amount of work done to bring the unit mass from infinite up to the point P is obtained by integrating both the sides with proper limits.

$$\int_0^{W} dW = -GM \int_S^{\infty} x^{-2} \cdot dx$$

$$\Rightarrow (W)\Big|_0^{W} = -GM \left(\frac{x^{-2+1}}{-2+1}\right)\Bigg|_S^{\infty}$$

$$\Rightarrow W - 0 = -GM \left(-\frac{1}{x}\right)\Bigg|_S^{\infty}$$

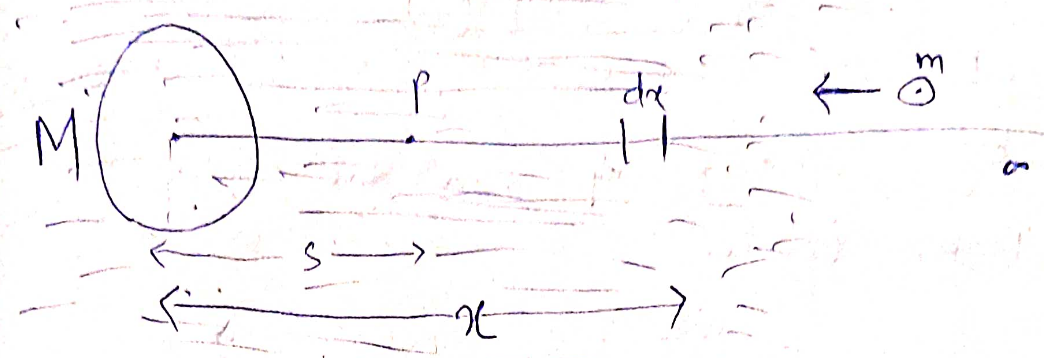
$$\Rightarrow W = -GM \left[\left(-\frac{1}{\infty}\right) - \left(-\frac{1}{S}\right)\right]$$

$$\Rightarrow \boxed{W = -\frac{GM}{S}} \quad \left(\because -\frac{1}{\infty} = 0\right)$$

= Gravitational potential at the point P (by defn)

Gravitational Potential energy.

Gravitational potential energy of a body of mass m placed at a point in a gravitational field is defined as the amount of workdone to bring that mass from infinite up to the point concerned



To derive an expression for the gravitational potential energy at a point in a gravitational field created by mass M, let us divide the entire distance into large number of small segments each of width dx.

dW = small amount of work done to shift the mass m through a small distance dx.

= Force x displacement

$$= G\frac{Mm}{x^2} \times dx$$

But this work is negative because the displacement is along the direction of force of attraction by mass M.

$\therefore \quad dW = -\frac{GMm}{x^2} \times dx$

Total amount of work done to bring the mass (m) from infinite up to the point P is obtained by integrating both the sides with proper limits.

$$\int_0^W dW = -GMm \int_s^\infty x^{-2} \cdot dx$$

$$\Rightarrow (W)\Big|_0^W = -GMm \left(\frac{x^{-2+1}}{-2+1}\right)\Bigg|_s^\infty$$

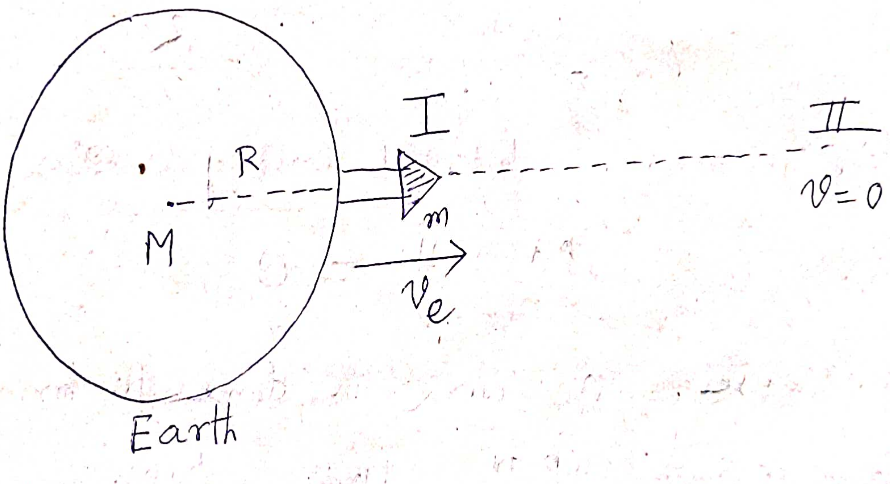
$$\Rightarrow (W - 0) = -GMm \left(-\frac{1}{x}\right)\Bigg|_s^\infty$$

$$\Rightarrow W = -GMm \left[\left(-\frac{1}{\infty}\right) - \left(-\frac{1}{s}\right)\right]$$

$$\Rightarrow \boxed{W = -\frac{GMm}{s}}$$ = Gravitational potential energy at the point P (by defn)

Escape Velocity (V_e)

It is defined as the min^m velocity with which a space vehicle like a rocket is to be projected from the surface of the earth ~~show~~ so as to enable it to reach the point with zero velocity where gravitational attraction of the earth ceases.



To derive an expression for the escape velocity, let's calculate the total energy of the rocket at position I, (on the surface of the earth) and at a point where the gravitational attraction of the earth just ceases (Position II).

Gravitational potential energy at position

$$I = -\frac{GMm}{R}$$

Kinetic energy of the rocket at position I $= \frac{1}{2} m v_e^2$

Total energy of the rocket at position I

$$= K.E + P.E$$

$$= \frac{1}{2} m v_e^2 - \frac{GMm}{R} \quad \text{——— (i)}$$

Gravitational potential energy ~~at~~ of the rocket at position II $= 0$

because no work is done to bring it infinite up to that point.

K.E of the rocket $= 0$

∴ Total energy of the rocket at position II

$$= 0 + 0 = 0 \quad \text{——— (ii)}$$

Neglecting the resistance of air, principle of conservation of energy can be used so that

Total energy of the rocket at position I

= Total energy of the rocket at position II

$$\Rightarrow \frac{1}{2} m v_e^2 - \frac{GMm}{R} = 0$$

$$\Rightarrow \frac{1}{2} \cancel{m} v_e^2 = \frac{GM\cancel{m}}{R}$$

$$\Rightarrow v_e^2 = \frac{2GM}{R}$$

$$\Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

Calculation of v_e of the earth

$$v_e = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.96 \times 10^{24}}{6.37 \times 10^{6}}} \ m/s$$

$$= 10^3 \sqrt{\frac{20 \times 6.67 \times 5.96}{6.37}}$$

$$= (10^3 \times 11.17) \ m/s$$

$$= 11.17 \ km/s.$$

Another expression for the escape velocity can be obtained by using the expression for the mass of the earth

$$\therefore v_e = \sqrt{\frac{2GM}{R}}$$

$$V_e = \sqrt{2\,\frac{G}{R}\cdot\frac{gR^2}{G}} \qquad \left(\because\ mg = \frac{GMm}{R^2} \Rightarrow M = \frac{gR^2}{G}\right)$$

$$\boxed{V_e = \sqrt{2gR}}$$

Calculation of V_e for the earth

$$V_e = \sqrt{2gR}$$

Escape velocity for moon - 2.4 km/sec.

gravity at moon - 1.[illegible]

$$= \sqrt{2\cdot(9.8)\cdot(6.37\times10^6)}\ \text{m/s}$$

$$= 11.17\times10^3\ \text{m/s}$$

$$= 11.17\ \text{K.m/s.}$$

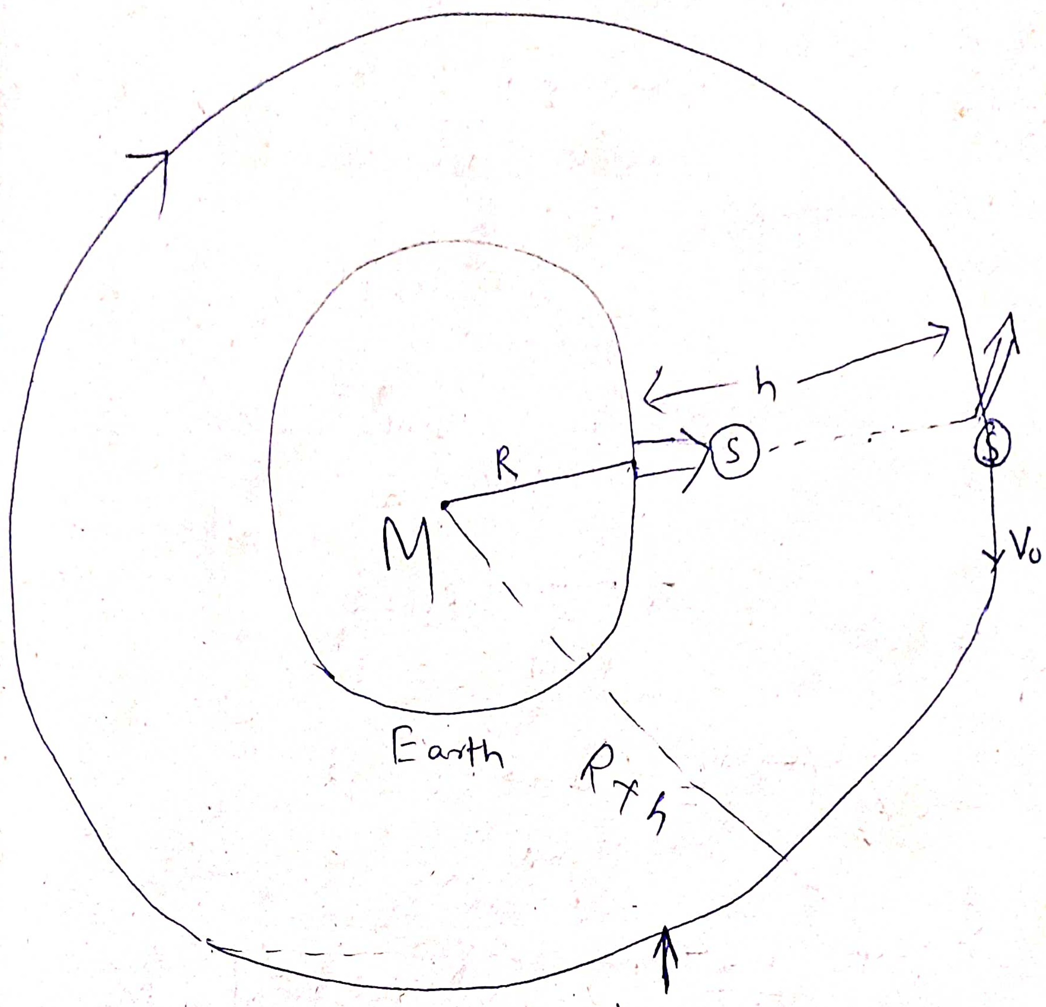
([illegible])

Launching of a satellite, Orbital velocity (V_o)

Artificial satellites are sent into space for various purposes like study of weather, (গুপ্তচরগিরি) spying, telecommunication, etc.

The satellite is lifted by a rocket which takes it up to the desired height and gives a push with a certain velocity called orbital velocity.

The satellite is found to rotate around the earth because the centripetal force necessary for the circular



Orbit of the satellite

motion is provided by the gravitational attraction of the Earth and satellite.

$$\therefore \frac{m v_0^2}{(R+h)} = \frac{G Mm}{(R+h)^2}$$

where m = mass of the satellite

M = mass of the earth.

$$\Rightarrow v_0^2 = \frac{GM}{R+h}$$

$$\Rightarrow \boxed{v_0 = \sqrt{\frac{GM}{R+h}}}$$

Time period of revolution of the satellite is given by

$$T = \frac{2\pi(R+h)}{V_0}$$

$$= \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$= \frac{2\pi}{\sqrt{GM}}(R+h)^{\frac{3}{2}}$$

$$\Rightarrow T^2 = \frac{4\pi^2}{GM}(R+h)^3$$

Since $\frac{4\pi^2}{GM}$ is a constant for all satellites, we can write

$$T^2 \propto (R+h)^3$$

— o —

181 page

12. We know that 1 mile = 1.6 K.m

500 mile = 800 K.M.

= 800×10^3 meter

$$V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11}\times5.96\times10^{24}}{(6.37\times10^6+8\times10^5)}}$$

$$= \sqrt{10^8 \cdot \sqrt{\frac{6.67 \times 5.96}{(63.7+8)}}}$$

Format.

$$= 10^4 \times x \quad m/s$$

$$= 10\,x \quad km/s$$

$$= \frac{10x}{1.6} \; mi/s = \frac{10x}{1.6} \times \frac{3600\,mi}{3600\,sec}$$

hr

$$= \frac{10x}{1.6} \times 3600 \quad mi/hr.$$

Ans $= (10^4 \times .744)\ m/s$

$= 10 \times .744\ km/s$

$= 7.44\ km/s.$

$= \dfrac{7.44}{1.6}\ mile/s$

$= 4.65\ mile/sec.$

$= \dfrac{4.65\ mile}{\frac{1}{3600}}$

$= 4.65 \times 3600$

$= 16740\ mile/hour$

at 1000 mile

we know 10 mile = 16 K.m
1000 mile = 1600 K.m

$$V_0 = \sqrt{\frac{GM}{R+h}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.96 \times 10^{24}}{(6.37 \times 10^6 + 1600 \times 10^3)}}$$

$$= \sqrt{\frac{6.67 \times 5.96 \times 10^{13}}{10^5 (63.7 + 16)}}$$

$$= 10^4 \sqrt{\frac{6.67 \times 5.96}{79.7}}$$

$$= 10^4 \times .70624 \text{ m/sec}$$

$$= 10 (.70624) \text{ km/sec}$$

$$= 7.0624 \text{ K.m/sec}$$

$$= \frac{7.0624}{1.6} \text{ mile/sec}$$

$$= \frac{7.0624}{1.6} \times 3600 \text{ mile/houre}$$

$$= 15,890.58 \text{ mile/hour.}$$

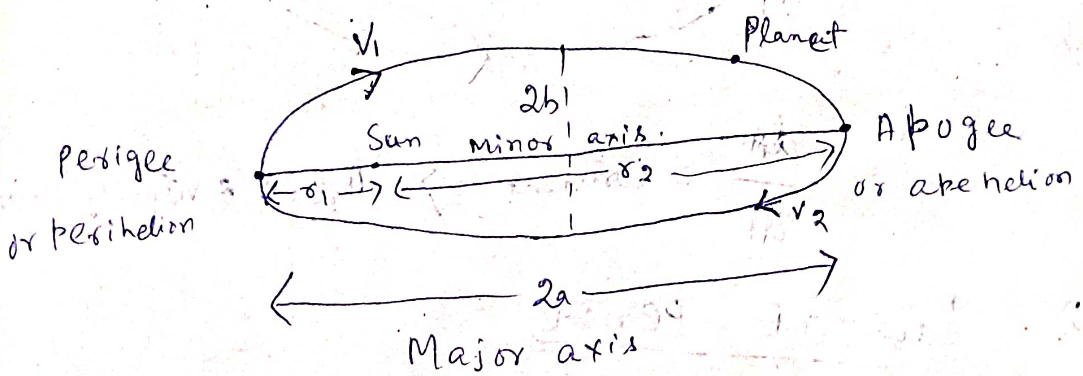
28.12.2K

Kepler's Law of Planetary motion

After analysing the experimental data for many years, Kepler arrived at the following three laws regarding the motion of planets.

1. Law of the Orbit

Every planet revolves around the sun in certain elliptical orbit with the sun placed at one of the foci (Plural of focous)



The velocity of the planet when it is nearest to the sun is greater than the velocity of the same planet when it is at farthest point.

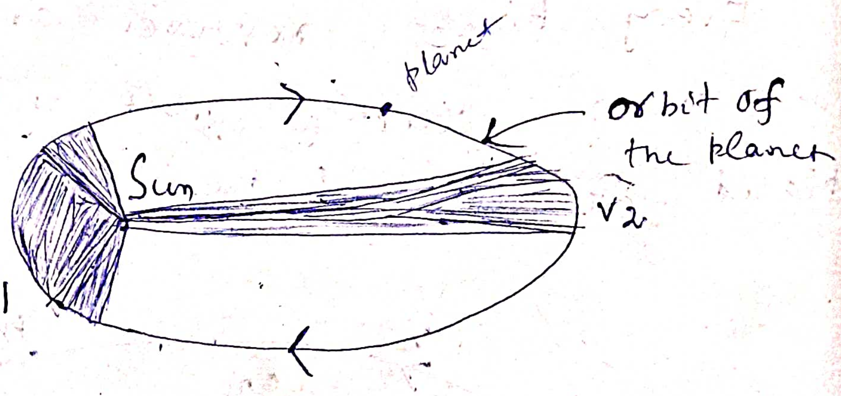
$$\frac{mv_1^2}{r_1} = \frac{GMm}{r_1^2} \quad \text{and} \quad \frac{mv_2^2}{r_2} = \frac{GMm}{r_2^2}$$

or $V_1 = \sqrt{\frac{GM}{r_1}}$ and $V_2 = \sqrt{\frac{GM}{r_2}}$

Since $r_1 < r_2 \Rightarrow V_1 > V_2$

(2) <u>Law of Area</u>

The <u>area</u> swept by the radius vector joining the sun to the planet covers equal <u>area</u> in eaual interval of time.



$\frac{dA}{dt}$ = Constant $\Rightarrow$ Areal velocity = constant.

3 <u>Law of period</u>

The square of the time period of revolution of planet is directly proportional to the cube of the mean distance of the planet from the sun.

i.e $T^2 \propto r^3$

or $T^2 = K r^3$ where K = a constant having the same ~~velocity~~ value for all the planets.

$$r = \frac{r_1 + r_2}{2} = \frac{2a}{2} = a = \text{Semi major axis.}$$

Problem page 181,

No: 4

Ans: Applying Kepler's Third law to the planet mercury and earth, we get

$$(0.241 yr)^2 = K r^3 \quad \text{——} (1)$$

$$(1 \text{ year})^2 = K.(1 a.u)^3 \quad \text{——} (2)$$

wher 1 au = Astronomical unit

$= 1.5 \times 10^8$ Km

= Average distance of the sun from the earth.

Dividing eqn (1) by eqn (2), we get

$$(.241)^2 = \frac{r^3}{(1au)^3}$$

or $$r^3 = (1 au)^3 \times \left(\frac{241}{1000} \times \frac{241}{1000}\right)$$

or $$r = \frac{1 au}{100} (241)^{\frac{2}{3}}$$

$$= \frac{1 au}{100} x \quad \text{(say)}$$

where $x = (241)^{\frac{2}{3}}$

Taking logarithm of both the sides

we get

$$\log_{10} x = \frac{2}{3} \log 241$$

$$= \frac{2}{3} \times 2.3820$$

$$= \frac{4.7640}{3} = 1.5880$$

$$x = \text{Antilog} (1.5880)$$

$$= 0.3873 \times 10^{1+1}$$

$$= 38.73$$

$$r = \frac{1 \text{au}}{100} \times (38.73)$$

$$= 0.3873 \simeq .39 \text{ au}$$

$$= .39 \times 1.5 \times 10^8 \text{ km}$$

$$= .585 \times 10^8 \text{ km}$$

$$= 5.85 \times 10^7 \text{ k.m}$$

16 km = 10 mile

$$5.85 \times 10^7 \text{ k.m} = \frac{10}{16} \times 5.87 \times 10^7$$

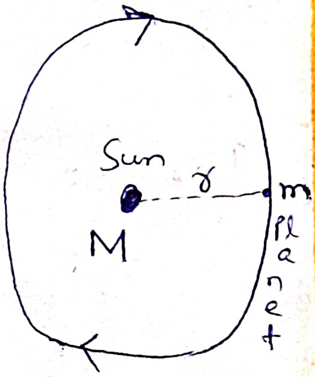
$$= 3.65 \times 10^7 \text{ mile.}$$

Derivation of Kepler's 2nd law and 3rd law from Newton's Law of Gravitation

For simplicity, let us consider the orbit of a planet to be circular with the sun placed at the centre.

The centripetal force necessary for the circular motion is provide by the gravitational attraction of Sun.

$$\therefore \frac{mv^2}{r} = G\frac{Mm}{r^2}$$



$$\text{or } v^2 = \frac{GM}{r}$$

$$\text{or } v = \sqrt{\frac{GM}{r}}$$

Time period of revolution of the planet

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}}$$

Squaring both the sides, we get

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Since $\frac{4\pi^2}{GM}$ is constant $= K$ which is same for all the planets, we have

$$\boxed{T^2 = Kr^3}$$

or $T^2 \propto r^3$ This is Kepler's 3rd law.

To derive Kepler's 2nd law, let us Calculate the areal velocity of the planet.

i.e $\frac{dA}{dt} = \frac{\pi r^2}{t} = \frac{\pi r^2}{\frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}}} = \frac{\pi r^2 \sqrt{GM}}{2\pi r^{\frac{3}{2}}}$

$= \cancel{\sqrt{\frac{GMr}{2}}}$ $= \frac{\sqrt{GMr}}{2}$

For a particular planet r is constant

$\therefore \frac{dA}{dt} = $ a constant

i.e Areal velocity of a planet is a constant quantity. This is Kepler's 2nd law.

Derivation of Newton's Law of Gravitation from Kepler's 3rd law

For simplicity, let us consider a circular orbit for the planet that rotates around the Sun. The Sun must be having some attraction for the planet and attractive force 'F' provides the necessary centripetal force.

$$\therefore \quad F = \frac{mv^2}{r} \quad \text{———} (i)$$

From Kepler's 3rd law on planetary motion, we know that

$$T^2 = Kr^3$$

$$\text{or} \quad \left(\frac{2\pi r}{v}\right)^2 = Kr^3$$

$$\text{or} \quad \frac{4\pi^2 r^2}{v^2} = Kr^3$$

$$\text{or} \quad \frac{4\pi^2}{v^2} = Kr$$

$$\text{or} \quad v^2 = \frac{4\pi^2}{Kr} \quad \text{———} (2).$$

using eqn (2) in eqn (i), we get

$$F = \frac{m}{r} \cdot \left(\frac{4\pi^2}{Kr}\right)$$

$$F = \frac{4m\pi^2}{Kr^2} = \frac{4\pi^2 m}{Kr^2} \quad \text{———} (3)$$

For the particular planet, m is constant

So that $\frac{4\pi^2 m}{K}$ = a constant = K' (Say)

$$\therefore \quad F = \frac{K'}{r^2}$$

or $F \propto \frac{1}{r^2}$ ——— (4)

This is called <u>inverse square law</u> which is a part of the Newton's law of gravitation.

To derive the complete law, we have to put the expression for K in eqn (3)

$$F = \frac{4\pi^2 m}{\frac{4\pi^2}{GM} \cdot r^2} = \frac{\cancel{4\pi^2} GMm}{\cancel{4\pi^2} r^2}$$

$\therefore$ $\boxed{F = \frac{GMm}{r^2}}$ This is the Newton's law of gravitation.

<u>Geostationary Satellite</u>

If the time period of revolution of a satellite will 24 hours, then the satellite appears to be <u>stationary at the orbit</u>. This will be made possible only when satellite is made rotate at a particular orbit. ✓

To calculate the height of

the Orbit above the surface of the earth, let us proceed as follows.

~~let~~

$$V_0 = \frac{2\pi(R+h)}{T} \quad \text{where } T = 24\text{ hr} \quad \text{---(i)}$$

$\therefore$ Centripetal accⁿ is

$$a = \frac{V_0^2}{R+h} = \frac{\frac{4\pi^2(R+h)^2}{T^2}}{R+h} = \frac{4\pi^2(R+h)}{T^2} \quad \text{---(ii)}$$

Force acting on the satellite of mass 'm' due to the gravitational attraction of the earth is $F = \frac{GMm}{(R+h)^2}$

$$= \left(\frac{GM}{R+h}\right)\frac{m}{R+h}$$

$$= \frac{V_0^2\, m}{R+h}$$

$$= m \cdot a = \frac{m\, 4\pi^2(R+h)}{T^2}$$

or $$\frac{GM\cancel{m}}{(R+h)^2} = \frac{\cancel{m}\, 4\pi^2(R+h)}{T^2}$$

$$\Rightarrow (R+h)^3 = \frac{GMT^2}{4\pi^2} \quad \text{---(3)}$$

From the expression for mass of the earth, we know that

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\Rightarrow GM = gR^2 \quad \text{——— (4)}$$

using eqn (4) in eqn (3), we get

$$(R+h)^3 = \frac{gR^2T^2}{4\pi^2}$$

$$\text{or} \quad R+h = \left(\frac{gr^2T^2}{4\pi^2}\right)^{\frac{1}{3}} \quad \text{——— (5)}$$

$$= \left\{\frac{9.8\times(6.37\times10^6)^2\times(24\times3600)^2}{4\times(3.141)^2}\right\}^{\frac{1}{3}}$$

$$= \left\{\left[\frac{9.8\times6.37\times6.37\times24\times\cancel{36}\times\cancel{36}}{\cancel{4}\times\underset{1.57}{\cancel{3.141}}\times\underset{1.57}{\cancel{3.141}}}\right]10^{16}\right\}^{\frac{1}{3}}$$

$\times 24\times9 \quad \cancel{18}\ 9$

$$= \left\{\left[\frac{98\times6.37\times6.37\times576\times81}{1.57\times1.57}\right]\times10^{15}\right\}^{\frac{1}{3}}$$

$$= \cancel{\quad} \ 10^5\times x \quad (\text{Say})$$

$$\log_{10}x = \frac{1}{3}\left[\log_{10}98 + 2\log_{10}6.37 + \log_{10}576 + \log_{10}81 - 2\log_{10}1.57\right]$$

$$= \frac{1}{3}\left[1.9912 + 2\times0.8041 + 2.7604 + 1.9085 - 2\times0.1959\right]$$

$= \frac{1}{3}\left[1.9912 + 1.6082 + 2.7604 + 1.9085 + \bar{3}.918\right]$

$= \frac{1}{3}\left[7.8765\right]$

$= 2.6255$

$x =$ antilog (2.6255)

$= 0.4222 \times 10^3$

$= 422.2$

$\therefore R+h = 10^5 \times 422.2$ m

$= 10^2 \times 422.2$

$= 42220$ K.m

$\therefore h = 42220 - R = 42220 - 6370$

$= 35850$ K.m.

To Calculate the height of the orbit above the surface of earth, lets proceed as follows.

$T = 24$ hrs, To get 'h'.

$T^2 = \frac{4\pi^2}{GM}(R+h)^3$ has to be used.

$\Rightarrow (R+h)^3 = \frac{GMT^2}{4\pi^2}$

$$\Rightarrow R+h = \left[\frac{6.67\times10^{-11}\times5.96\times10^{24}\times(24\times3600)^2}{4\times(3.14)^2}\right]^{\frac{1}{3}}$$

$$= x \quad (\text{Say})$$

$$\log_{10}x = \frac{1}{3}\Big[\log_{10}6.67 + \log_{10}5.96 + 2\log_{10}24 + 2\log_{10}36 + 17\log_{10}10 - 2\log_{10}2 - 2\log_{10}3.14\Big]$$

$$= \frac{1}{3}\Big[0.8241 + 0.7752 + 2\times1.3802 + 2\times1.5563 + 17 - 2\times0.3010 - 2\times0.4969\Big]$$

$$= \frac{1}{3}\Big[0.8241 + .7752 + 2.7604 + 3.1126 + 17 - 0.602 - .9938\Big]$$

$$= \frac{1}{3}\Big[22.8765\Big]$$

$$= 7.6255$$

$$x = \text{antilog}(7.6255) = 0.4222\times10^{8}\ m$$

$$= 42220\ Km$$

$\Rightarrow h = (42220 - 6370)$ k.m

$= 35850$ k.m.

6 ~~[illegible]~~ 2, 79 page - 33

33. Mass of the earth $M = 5.98 \times 10^{24}$ Kg.

Mass of the moon $m = .0123$ times greater.

$= .0123M$

Gravitational force between them

$$F = \frac{GMm}{R^2}$$

$$\Rightarrow F = \frac{G \times M \times .0123M}{R^2}$$

$$= \frac{G \times .0123 \times M^2}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 0.0123 \times (5.98 \times 10^{24})^2}{(3.84 \times 10^5 \times 10^3)^2}$$

$$= \left(\frac{6.67 \times .0123 \times 5.98 \times 5.98}{3.84 \times 3.84}\right) \times 10^{-11} \times 10^{48} \times 10^{-16}$$

$$= .198 \times 10^{21}$$

$= 1.98 \times 10^{20}$ Newton.

(ii) when this force is exerted by earth $F = Ma_1$

But $F = 1.98 \times 10^{20}$ Newton.

$\therefore$ $Ma_1 = 1.98 \times 10^{20}$

$\Rightarrow a_1 = \dfrac{1.98 \times 10^{20}}{5.98 \times 10^{24}}$

$= .3311 \times 10^{-4}$

$= 3.311 \times 10^{-5}$ m/sec²

(iii) when this force is exerted by the moon

$F = ma_2 = \dfrac{M}{.0123}$

But $F = 1.98 \times 10^{20}$ Newton

$\Rightarrow a_2 = \dfrac{1.98 \times 10^{20}}{\dfrac{M \times .0123}{\cancel{0.0123}}}$

$= \dfrac{1.98 \times \cancel{\quad} \times 10^{20}}{5.98 \times 10^{24} \times .0123}$

$= \cancel{4.072 \times 10^{3} \times 10^{-24} \times 10^{20}}$

$= \cancel{4.072 \times 10^{-2} \, m/s^2}$

$= 26.91 \times 10^{-4}$

$= 2.691 \times 10^{-3}$ m/s²

18) Pars

11.81

6. प्र 2

Ans: Applying Kepler's Third law to the planet Jupiter and earth, we get

$$T^2 = K.(7.78\times10^8)^3 \quad \text{——} (1)$$

$$(1\,year)^2 = K.(1\,a.u)^3 \quad \text{——} (2)$$

wher 1 a.u = Astronomical unit
= 1.5×10^8 km
= Average distance of the sun from the earth.

Dividing ~~both~~ eqn (i) by the eqn (ii), we get

$$T^2 = \frac{(7.78\times10^8)^3}{(1au)^3}$$

$$= \frac{(7.78)^3\times10^{24}}{(1.5\times10^8)^3}$$

$$= \left(\frac{7.78}{1.5}\right)^3 \times \frac{10^{24}}{10^{24}}$$

$$\Rightarrow T = \sqrt{139.529177}$$

$$= 11.8122 \text{ years.}$$

take calculator and Geometry box

Space For Problems

Gravitation

To find the velocity of a solid sphere at the ~~boot~~ bottom of an inclined plane (Loss of energy due to friction being neglected) and to find the time taken by the sphere to reach the bottom

Moment of inertia of a solid sphere rotating about any diameter = $\frac{2}{5}MR^2$.

Moment of inertia of a cylinder rotating about its own axis = $\frac{1}{2}MR^2$

Moment of inertia of a ring rotating about its own axis = MR^2

K.E of rotation and translation

K.E of translation $\frac{1}{2}Mv^2$

Hence K.E of rotation $= \frac{1}{2}I\omega^2$

If the ball ~~rotates~~ rolls, then it posses K.E of rotation and translation.

. Total K.E = $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

Ex-1 : For a solid sphere

$$K.E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{\cancel{2}}\cdot\left(\frac{\cancel{2}}{5}M\cancel{R^2}\right)\cdot\frac{v^2}{\cancel{R^2}}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

$$= \left(\frac{1}{2}+\frac{1}{5}\right)Mv^2$$

$$= \frac{7}{10}Mv^2 \quad \text{———— (a)}$$

Ex-2 :

For a solid cylinder

$$K.E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\left(\frac{1}{2}M\cancel{R^2}\right)\times\frac{v^2}{\cancel{R^2}}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$$

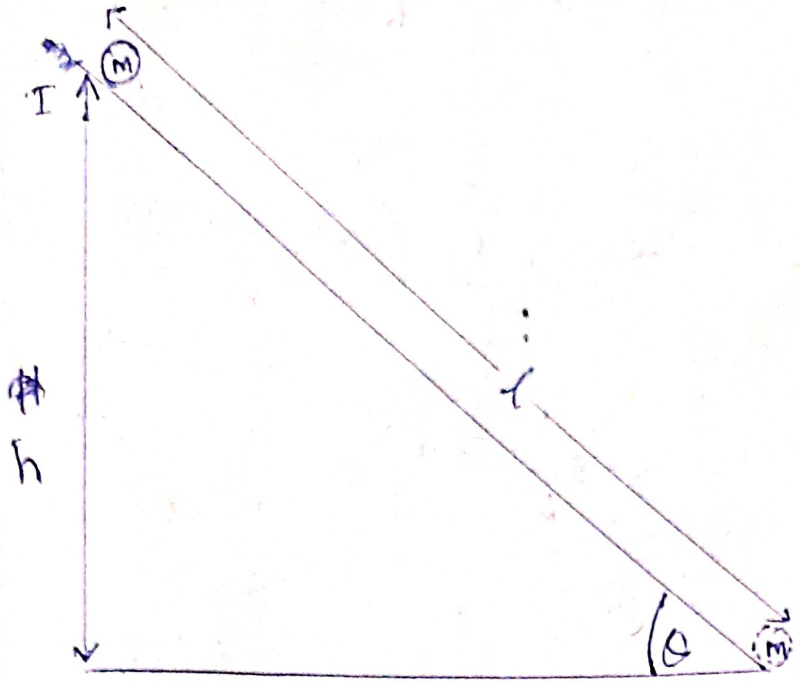
$$= \frac{3}{4}Mv^2 \quad \text{———— (b)}$$

Ex-3

For a thine ring $K.E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\cdot(M\cancel{R^2})\times\frac{v^2}{\cancel{R^2}}$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2 \quad \text{—— (c)}$$



Now consider the velocity of sphere on an inclined surface

In the first position $\hat{I}$ the body is at rest. So potential energy $= Mgh$.

Kinetic energy of the sphere $= 0$

$\therefore$ Total energy $= P.E + K.E = Mgh + 0 = Mgh$ —(i)

In position II the body is at the lowest position. So the potential energy $= 0$.

The $K.E = \frac{7}{10} Mv^2$ ——— from (a)

Total energy $= P.E + K.E = 0 + \frac{7}{10}Mv^2 = \frac{7}{10}Mv^2$ (ii)

According to principle of conservation of energy

$$\cancel{M}gh = \frac{7}{10}\cancel{M}v^2$$

$$\Rightarrow v^2 = \frac{10gh}{7}$$

$$\Rightarrow v = \sqrt{\frac{10}{7} gh}$$

Average velocity

$$= \frac{\text{Initial velocity} + \text{final velocity}}{2}$$

$$= \frac{0 + \sqrt{\frac{10}{7} gh}}{2}$$

$$= \frac{1}{2}\sqrt{\frac{10}{7} gh}$$

$$t_1 = \frac{l}{v} = \frac{l}{\frac{1}{2}\sqrt{\frac{10}{7} gh}} = \frac{2l}{\sqrt{\frac{10}{7} gh}} = \frac{2l}{\sqrt{1.42 gh}}$$

For the cylinder

$$t_2 = \frac{l}{\left(\frac{0 + \sqrt{\frac{4}{3} gh}}{2}\right)} = \frac{2l}{\sqrt{\frac{4}{3} gh}} = \frac{2l}{\sqrt{1.33 gh}}$$

For the ring

$$t_3 = \frac{l}{\left(\frac{0 + \sqrt{gh}}{2}\right)} = \frac{2l}{\sqrt{gh}} = \frac{2l}{\sqrt{1 gh}}$$

Thus $t_1 < t_2 < t_3$

Hence the sphere will reach the bottom first, followed by cylinder and lastly by ring.