Quine - McCluskey (QM) Technique

The K-Map technique is convenient & effective in simplifying a Boolean expression. However, for increased number of I/P variables, QM method can be adopted. QM method is more effective since digital computer can be used to simplify the given Boolean switching function using QM method. i.e it is programmable.

→ It is also known as Tabular method.

Ex- 1) ~~Consider~~ Minimize the following SOP expression using QM method

$$f(A,B,C,D) = \Sigma m(0, 5, 7, 8, 9, 10, 11, 14, 15)$$

Procedure:-

Step 1:- Represent each term (minterm) of the SOP form by binary code.

0 → 0000 ✓	8 → 1000 ✓	11 → 1011 ✓
5 → 0101 ✓	9 → 1001 ✓	14 → 1110 ✓
7 → 0111 ✓	10 → 1010 ✓	15 → 1111

Step 2:- Arrange all minterms in groups of the same number of 1s in their binary representation in Column 1. Start with the least number of 1s group and continue with groups

of increasing number of 1s. The number of 1s in a term is called the Index of the term.

Table: 1 → Table of prime implicants

Column 1

Index	Minterm	Binary representation
0	0	0000 ✓
1	8	1000 ✓
2	5	0101 ✓
	9	1001 ✓
	10	1010 ✓
3	7	0111 ✓
	11	1011 ✓
	14	1110 ✓
4	15	1111 ✓

Column 2

Pairs	First Reduction	
0, 8	−000	V
8, 9	100− ✓	
8, 10	10−0 ✓	#
5, 7	01−1	IV
9, 11	10−1 ✓	
10, 11	101− ✓	
10, 14	1−10 ✓	
7, 15	−111	III
11, 15	1−11 ✓	
14, 15	111− ✓	

Column 3

Quads	A B C D	
8, 9, 10, 11	10−−	II
~~8, 10, 9, 11~~	~~10−−~~	
10, 11, 14, 15	1−1−	I
~~10, 14, 11, 15~~	~~1−1−~~	

Note:-

Since the quads are repeated we can cancel them.

Further reduction is not possible. Because both 10−− } and the 1−1− } dash positions are not in same position.

Refer Table 1.

Step 3: Compare each term of the lowest index group with every term in the succeeding group. Two terms from adjacent groups can be combinable, if their binary representation differ by just a single digit in the same position; the combined terms consist of the original fixed representation with the differing one replaced by a dash (−).

Place a check mark (✓) next to every term, which has been combined with at least one term (each term may be combined with several terms, but only a single check is required) and write the combined term in column 2.

Repeat this by comparing each term in a group of index 'i' with every term in the group of index 'i+1', until all possible applications of the combining them have been exhausted.

Step 4:-

Now compare the terms generated in step 3 in the same fashion; Combine two terms which differ by only a single 1 and whose dashes are in the same position to generate a new term. Two terms with dashes in different positions can't be combined. Put a check mark next to each term which has been combined in column 2. Continue the process with terms in column 3, 4 etc. until no further combinations are possible. The remaining unchecked terms constitute the set of prime implicants of the expression. They are called prime implicants because they are not covered by other term with fewer literals.

Step 5:- List all the prime implicants and draw the prime implicant chart. (Table 2)

In this chart, Column are the minterm decimal values and rows are the prime implicants designated by I, II, III, IV, V.

Step 6:- Put the Cross marks under the decimal numbers that the prime implicant associated with.

→ Find all the Columns that contains a single Cross, then make a circle over the Cross. Place an asterisk to the left of those rows in which you have circled a Cross.

The rows marked with an asterisk are the essential prime implicant.

Step 7:- **Table 2:-** prime implicants Chart

Prime implicants	minterm 0	5	7	8	9	10	11	14	15
I*						×	×	⊗	×
II*				×	⊗	×	×		
III			×						×
IV*		⊗	×						
V*	⊗			×					

Step-7:- Now, first draw a horizontal line corresponding to Essential prime implicant which crushes the 'X' marks. Then draw vertical line at the places of 'X' of that Essential prime implicant. Repeat the steps for all Essential prime implicants.

Prime implicants	minterms								
	0	5	7	8	9	10	11	14	15
I*					×	×	×	×	×
II*				×	×	×	×		
III			×	×					
IV*	×	×	×						
V*	×			×					

Step: 8

Since Essential prime implicant covers all the minterm of the function, it is the minimal form. Thus, no further steps need to be carried out

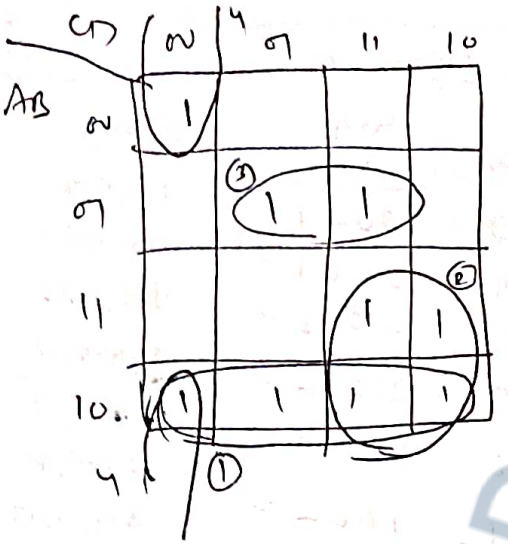
$$f(A,B,C,D) = I^* + II^* + IV^* + V^*$$

$$\boxed{f(A,B,C,D) = AC + A\bar{B} + \bar{A}BD + \bar{B}\bar{C}\bar{D}}$$

(Ans)

It can be verified by K-Map

$f(A,B,C,D) = \Sigma(0,5,7,8,9,10,11,14,15)$



① → $A\bar{B}$

② → AC

③ → $\bar{A}BD$

④ → $\bar{B}\bar{C}\bar{D}$

Step 9:- Suppose in a case in which all the Cross (x) are not Covered, in that case we have to Check intuitively which prime implicants (i.e not marked Asterisk) can be taken such a way that all (x) are Covered.

So the final function will be

$f(A,B,C,D)$ = (Asterisks) essential prime implicants + prime implicant which covers the (x) marks.

Ex-2) $f = \sum m(1,2,3,5,6,7,8,9,12,13,15)$

Obtain the minimal expression using tabular method.

Ans:

1 → 0001 ✓	5 → 0101 ✓	8 → 1000 ✓	13 → 1101 ✓
2 → 0010 ✓	6 → 0110 ✓	9 → 1001 ✓	15 → 1111
3 → 0011 ✓	7 → 0111 ✓	12 → 1100 ✓	

Table 1:- Table of prime implicant

Column 1

Index	Minterm	Binary
1	1	0001 ✓
	2	0010 ✓
	8	1000 ✓
2	3	0011 ✓
	5	0101 ✓
	6	0110 ✓
	9	1001 ✓
	12	1100 ✓
3	7	0111 ✓
	13	1101 ✓
4	15	1111 ✓

Column 2

Pairs	First reduction
1,3	00-1 ✓
1,5	0-01 ✓
1,9	-001 ✓
8,3	0 01- ✓
2,6	0-10 ✓
8,9	100- ✓
8,12	1-00 ✓
3,7	0-11 ✓
5,7	01-1 ✓
5,13	-101 ✓
6,7	011- ✓
9,13	1-01 ✓
12,13	110- ✓
7,15	-111 ✓
13,15	11-1 ✓

Column 3

Quads	A B C D	
1,3,5,7	0 - - 1	V
~~1,5,3,7~~		
1,5,9,13	- - 0 1	IV
~~1,9,5,13~~		
2,3,6,7	0 - 1 -	III
~~2,6,3,7~~		
8,9,12,13	1 - 0 -	II
~~8,12,9,13~~		
5,7,13,15	- 1 - 1	I
~~5,13,7,15~~		

[No further reduction is possible because 5,7,13,15 quad has dash at 1st and 3rd position that can n't be combined with any of the upper quads.]

Table 2:- Prime Implicant Chart

Prime implication	Minterm 1	2	3	5	6	7	8	9	12	13	15
I*				×		×	×		×	×	×
II*							×	×	×		
III*			×		×	×					
IV	×			×				×		×	
V	×		×	×		×					

Only minterm '1' is not covered. 35

Either row ③ $\overline{IV}$ or $\overline{V}$ can cover c7.

and both have same number of literals.

Thus 2 minimal expressions are possible.

(i) $I^* + II^* + III^* + IV = BD + A\bar{C} + \bar{A}C + \bar{C}D$

or (ii) $I^* + II^* + III^* + V = BD + A\bar{C} + \bar{A}C + \bar{A}D$

$\therefore f = \Sigma(1,2,3,5,6,7,8,9,12,13,15)$

$$\begin{cases} = & BD + A\bar{C} + \bar{A}C + \bar{C}D \\ \text{or} & \\ = & BD + A\bar{C} + \bar{A}C + \bar{A}D \end{cases}$$

Ex:- 3 Using the tabular method, obtain the minimal expression for

$f = \Sigma m(6,7,8,9) + d(10,11,12,13,14,15)$

Ans :- d → don't care

In the given Boolean expression there are don't Cares. Treat the don't Care as min terms and apply the usual procedure to obtain the set of prime implicants as shown in Table 3

6 → 0110 ✓
7 → 0111 ✓
8 → 1000 ✓
9 → 1001 ✓

10 → 1010 ✓
11 → 1011 ✓
12 → 1100 ✓
13 → 1101 ✓

14 → 1110 ✓
15 → 1111 ✓

Table 3:- Prime Implicants

Column 1

Index	Minterm	Binary
1	8	1000 ✓
2	6	0110 ✓
	9	1001 ✓
	10	1010 ✓
	12	1100 ✓
3	7	0111 ✓
	11	1011 ✓
	13	1101 ✓
	14	1110 ✓
4	15	1111

Column 2

Pair	First Reduction
8,9	100– ✓
8,10	10–0 ✓
8,12	1–00 ✓
6,7	011– ✓
6,14	–110 ✓
9,11	10–1 ✓
9,13	1–01 ✓
10,11	101– ✓
10,14	1–10 ✓
12,13	110– ✓
12,14	11–0 ✓
7,15	–111 ✓
11,15	1–11 ✓
13,15	11–1 ✓
14,15	111– ✓

Column 3

Quad	Second Reduction	
8,9,10,11	10–– ✓	
~~8,9,12,13~~		
8,9,12,13	1–0– ✓	
~~8,10,9,11~~		
8,10,12,14	1––0 ✓	
~~8,12,9,13~~		
~~8,12,10,11~~		
6,7,14,15	–11–	II
~~6,14,7,15~~		
9,11,13,15	1––1 ✓	
10,11,14,15	1–1– ✓	
~~10,14,11,15~~		
12,13,14,15	11–– ✓	
12,14,13,15	11–– ✓	

Column 4

Octet	ABCD	
8,9,10,11,12,13,14,15	1–––	I
~~8,9,10,11,12,14,13,15~~		
~~8,9,12,13,10,11,14,15~~		
~~8,10,12,14,9,11,13,15~~		

Table 2:-

Prime implicant chart of $\Sigma(6,7,8,9) + d(10,11,12,13,14,15)$ all don't care minterms are omitted.

Table 2:- Prime implicant chart.

Prime Implicants	Minterm 6	7	8	9
I^*		⊗	⊗	
II^*	⊗			⊗

As seen from the table, I & II both are essential prime implicants. So the minimal expression is

Sop → $I^* + II^*$

$\overline{A} + BC$

$\therefore$ Minimized $f = \overline{A} + BC$.

Ex-1 :- Minimize $f(W,X,Y,Z) = \Sigma m(0,1,5,7,8,10,14,15)$

Ans:

0 - 0000	8 - 1000
1 - 0001	10 - 1010
5 - 0101	14 - 1110
7 - 0111	15 - 1111

Table - 1

Column 1

Index	Min term	Binary
0	0	0000 ✓
1	1	0001 ✓
	8	1000 ✓
2	5	0101 ✓
	10	1010 ✓
3	7	0111 ✓
	14	1110 ✓
4	15	1111 ✓

Column 2

Pair	First Reduction	
0,1	000–	I
0,8	–000	II
1,5	0–01	III
8,10	10–0	IV
5,7	01–1	V
10,14	1–10	VI
7,15	–111	VII
14,15	111–	VIII

Column 3

No quads are possible.

None of the terms in any group of Column 2 can be combined with other term in next group. So all of them are prime implicants.

Table 2: Prime implicant Chart

Prime Implication	Min term 0	1	5	7	8	10	14	15
I	×	×						
II	×				×			
III		×	×					
IV					×	×		
V			×	×				
VI						×	×	
VII				×				×
VIII							×	×

We can see from the prime implicant chart that there are no essential prime implicants and one possible minimal Combination of prime implicant that can cover all the min term is ~~I, IV~~, I, IV, V, VIII.

Therefore

$$f(w,x,y,z) = I + IV + V + VIII$$

$$= 000- + 10-0 + 01-1 + 111-$$

$$= \bar{x}\bar{y}$$

$$f(w,x,y,z) = \bar{w}\bar{x}\bar{y} + w\bar{x}\bar{z} + \bar{w}xz + wxy$$

(Ans)

Ex-7

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