Module – II

Ch-3 – Power Dividers and Couplers

Pozar Book

Power dividers and directional couplers are passive microwave components used for power division or power combining, as shown in figure Figure (27). In power division, an i/p signal is divided by the coupler into two (or more) signals of lesser power. The coupler may be a three port component, with or without loss or may be a four-port component.

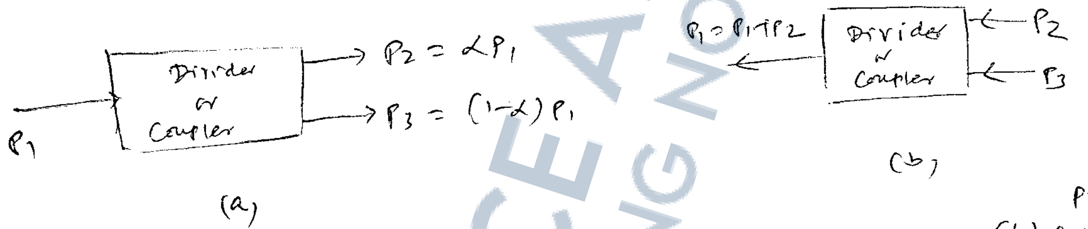


fig 27:- Power division & combining (a) power division (b) power combining

→ Three port n/w take the form of T-junctions and other power dividers, while four-port n/ws take the form of directional couplers & hybrids.

→ Three-port n/w (T-junctions)

The simplest type of power divider is a T-junction, which is a 3-port n/w with two i/ps and one output. The scattering matrix of an arbitrary 3-port n/w has 9 independent elements.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

In general, A specific element of [S] matrix can be determined as

$$S_{ij} = \frac{V_i^-}{V_j^+}\Bigg|_{V_k^+ = 0}, \quad k \neq j$$

i.e S_{ij} is the ratio of reflected ~~incident~~ ~~voltage~~ wave amplitude (V_i^-), coming out of port i, to the incident wave of voltage V_j^+. At that time, the incident waves on all ports except the jth port are set to zero, which means that all ports should be terminated in matched load to avoid reflection.

→ ∴ $S_{11} = \frac{V_1^-}{V_1^+}$ = Reflection coefficient seen looking at port 1.

$S_{22} = \frac{V_2^-}{V_2^+}$ = Reflection coefficient " " " " 2.

$S_{33} = \frac{V_3^-}{V_3^+}$ = Reflection coefficient " " " " 3.

when all other ports are terminated in matched load.

Similarly, $S_{12} = \frac{V_1^-}{V_2^+}$, $S_{13} = \frac{V_1^-}{V_3^+}$, $S_{21} = \frac{V_2^-}{V_1^+}$ and so on.

→ If the component is passive and contains no anisotropic materials, then it must be reciprocal and its scattering matrix must be symmetric ($S_{ij} = S_{ji}$).

Usually, to avoid power loss, we would like to have a junction that is lossless and matched at all ports. Practically, it is impossible to construct such a 3-port lossless reciprocal n/w i.e. matched at all ports.

Ideally, if all ports are matched $S_{ii} = 0$, and if the n/w is reciprocal the scattering matrix reduce to

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Some commonly used T-junction using waveguide are E-plane tee, (3-port) H-plane tee (3-port) and Magic Tees (4-port).

E-plane Tee (Short Note) (Line Box)

→ An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E-field of the main guide.

→

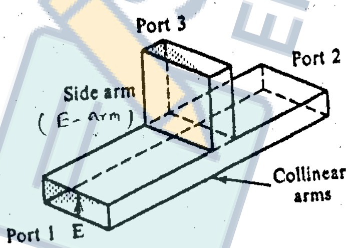


Figure ~~4-4-4~~ 28 E-plane tee

→ If the collinear arms are symmetric about side arm, there are two different transmission characteristics.

(a) I/P from main arm

(b) I/P from side arm.

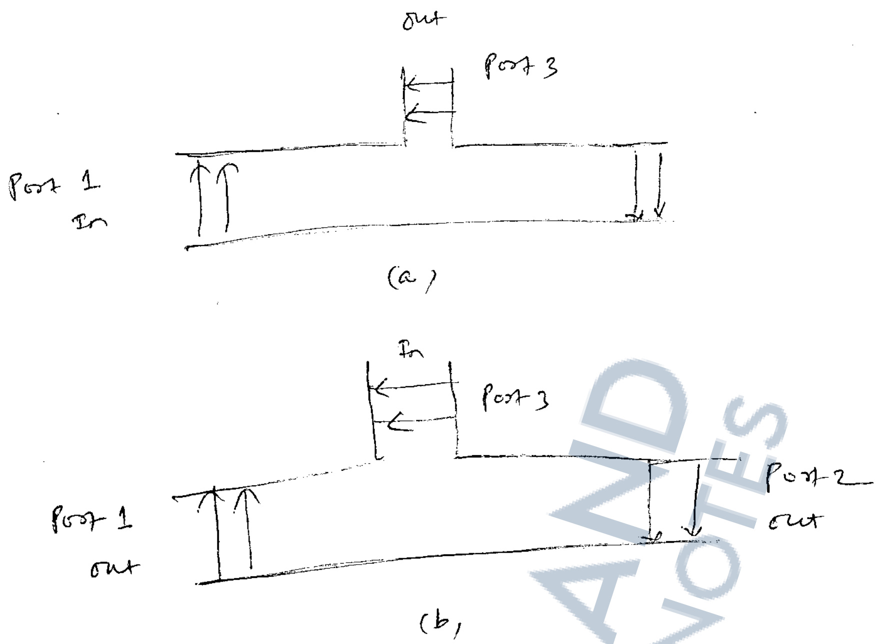


Fig 29:- Two-way transmission of E-plane Tee

(a) I/P ~~for~~ through main arm (b) I/P from side arm.

→ When the waves are fed into the side arm (Port 3), the waves appearing at Port 1 and Port 2 of the collinear arm will be in same magnitude and in opposite phase

Therefore, scattering matrix parameter

$$S_{13} = -S_{23}. \quad \text{—(1)}$$

→ The Port (1) & (2) are called collinear arms and port (3) is called side arm or E-arm.

→ For E-plane Tee since Port 1 & Port 2 are collinear

$$S_{11} = S_{22} \quad \text{—(2)}$$

The S-matrix of E-plane Tee

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{12} & S_{33} \end{bmatrix}$$

$\because$ $S_{21} = S_{12}$ (due to symmetry)

$S_{22} = S_{11}$

$S_{23} = -S_{13}$ (From eqn (1))

$S_{31} = S_{13}$ (due to symmetry)

$S_{32} = S_{23} = -S_{13}$ (due to symmetry)

→ If the waves entering from Port 1 and Port 2, then the resulting field through port 3 is proportional to the difference betn the instantaneous field from port 1 and port 2.

H-Plane -Tee

An H-plane tee is a waveguide tee in which the axis of its side arm is parallel to the H-field of the main guide.

As shown in fig,

→ Port 1 & Port 2 are called Collinear arms and port 3 is called H-arm.

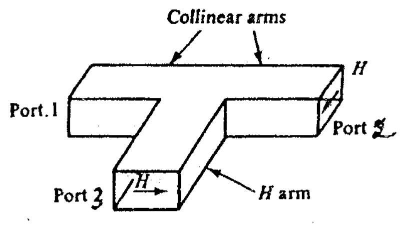


fig 3c :- (b) H-plane tee

→ It can be seen that if two i/p waves are fed into port 1 and port 2 of the collinear arm, the o/p wave at port 3 will be in phase and additive.

→ On the other hand if the i/p is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude.

→ Therefore,

$$S_{13} = S_{23}.$$

The S-matrix for H-plane becomes.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix}$$

[Similar to E-Plane Tee, only difference $S_{13} = S_{23}$]

Imp
Magic Tees (Hybrid Tees) – (Short note)

→ A magic Tee is a combination of E-Plane Tee and H-plane,

→ It is a 4 port device

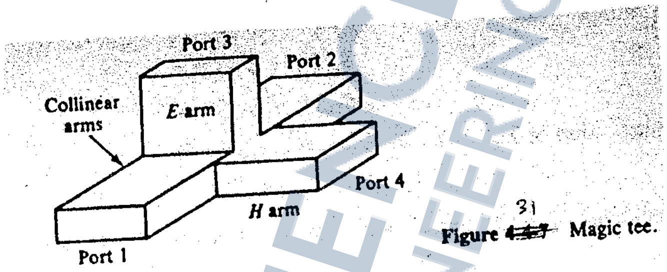


Figure ~~4.17~~ 31 Magic tee.

→ The port 1 & port 2 are collinear arms. port 3 is called E arm & Port 4 is called H-arm.

→ The magic Tee has several characteristics:

1. If two waves of equal magnitude and same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4.

2. If the wave is fed into port 4 (the H arm),

it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3 (the E arm)

3. If a wave is fed into port 3 (E arm), it will produce an o/p of equal magnitude and opposite phase at port 1 and port 2. The output at port 4 is zero.

From (2) and (3), $S_{34} = 0$, $S_{43} = 0$ — (3)

4. If a wave is fed into one of the collinear arms at port 1 ~~and~~ or port 2, it will not appear on the other collinear arm at port 2 or port 1 because the E-arm causes a phase delay while the H-arm causes a phase advance.

So $S_{12} = S_{21} = 0$ — (4)

5. Therefore S-matrix of a magic Tee can be expressed as

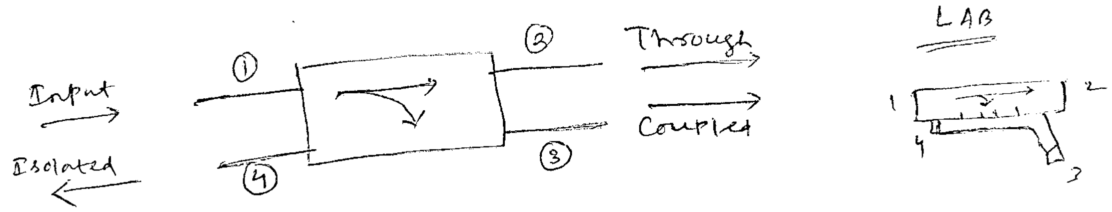
$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

[$\because$ $S_{11} = S_{22} = S_{33} = S_{44} = 0$
$\because$ Reflection at each port 0.
$S_{34} = S_{43} = 0$
$S_{12} = S_{21} = 0$]

6. The magic Tee is commonly used for mixing, $(\xrightarrow{1} \downarrow_4 \xleftarrow{2})$ duplexing $(\xleftarrow{1} \uparrow_4 \xrightarrow{2})$ and impedance measurement.

Waveguide Directional Couplers: (Pozar Book)

A directional coupler is a four-port waveguide junction as shown in figure 38.



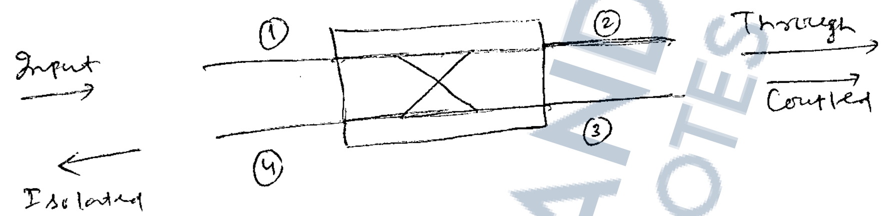


fig 38:- Two commonly used symbol for directional couplers and power flow conventions.

→ power incident at port 1 will couple to port 2 (the through port) and to port 3 (the coupled port), but not to port 4 (the isolated port).

→ Similarly, power incident on port 2 will couple to ports 1 & 4, but not 3.

→ Thus, port 1 and 4 are decoupled as are port 2 and 3.

→ The fraction of power coupled from port 1 to port 3 is given by 'C', coupling coefficient,

$$\boxed{C = 10 \log\left(\frac{P_1}{P_3}\right) \text{ dB}} \quad —(1)$$

→ The leakage of power from port 1 to port 4 is given by $\underline{I}$, the isolation,

$$\boxed{I = 10 \log\left(\frac{P_1}{P_4}\right) \text{ dB}} \quad -(2)$$

→ Another quantity that can be used to characterize a coupler is the directivity,

$$\boxed{D = 10 \log \frac{P_3}{P_4} \text{ dB}} \quad -(3)$$

, which is the ratio of power delivered to the coupled port and the isolated port.

__Ideal case__

$P_4 = 0$, So Directivity & Isolation are __infinite__ for a ideal directional coupler.

→ The ideal coupler is also lossless and matched at all ports.

→ Practically, a well designed coupler have directivity __30 to 35 dB.__

__e.g.__ ..

$$10 \log_{10} \frac{P_3}{P_4} = 30$$

$$\Rightarrow \log_{10} \frac{P_3}{P_4} = 3$$

$$\Rightarrow \frac{P_3}{P_4} = 10^3 = 1000$$

$$\Rightarrow P_3 = 1000 P_4$$

That means power at P_3 is 1000 times the power at P_4, i.e. at Port 4 the power is very negligible as compared to Port 3.

Reqn between $\boxed{C, I \& D}$:-

$$C = 10 \log \left(\frac{P_1}{P_3}\right)$$

$$I = 10 \log \left(\frac{P_1}{P_4}\right)$$

$$D = 10 \log \left(\frac{P_3}{P_4}\right).$$

$$\boxed{D = I - C} \text{ dB. or } \boxed{I = C + D \text{ dB}} \quad -(4)$$

Trick to remember

A B C D E F G H I

C+D → I

C → I−D

D → I−C

Proof :-

$$I - C \quad (\text{R.H.S})$$

$$= 10 \log \left(\frac{P_1}{P_4}\right) - 10 \log \left(\frac{P_1}{P_3}\right)$$

$$= 10 \left[\log \frac{P_1}{P_4} - \log \frac{P_1}{P_3} \right]$$

$$= 10 \left[\log \left(\frac{P_1}{P_4}\right) / \left(\frac{P_1}{P_3}\right)\right]$$

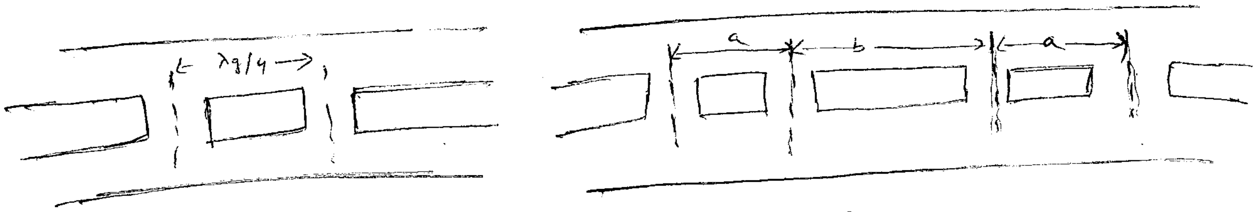
$$= 10 \log \frac{\cancel{P_1}}{P_4} \times \frac{P_3}{\cancel{P_1}}$$

$$= 10 \log \frac{P_3}{P_4}$$

$$= D \quad (\text{L.H.S})$$

Types

Several types of directional couplers exist, such as 2-hole directional coupler, four-hole directional coupler, multi-hole directional coupler, Bethe-hole directional coupler etc.



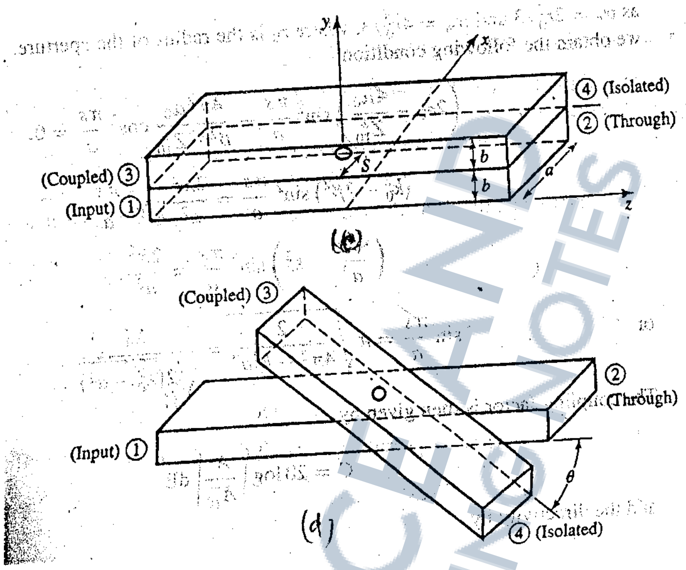


fig 39:- Different types of directional couplers. (a) Two-hole directional coupler (b) Four-hole directional coupler (c) Bethe-hole directional coupler using parallel-guide (d) using skewed guides.

Scattering Matrix of a directional Coupler :-

In directional coupler all 4 ports are perfectly matched. Thus the diagonal elements of scattering matrix are zero.

[Note:- Due to matching at all ports, reflection coefficients at all ports are zero]

$$\therefore \quad S_{11} = S_{22} = S_{33} = S_{44} = 0. \quad \text{—⑤}$$

As discussed earlier, there is no coupling between port 1 and port 4, similarly

214 between port 2 and port 3. 215

$$S_{14} = S_{41} = S_{23} = S_{32} = 0 \quad \text{— (6)}$$

Consequently, 'S' matrix of directional coupler becomes.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Using eqn (5) & (6),

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{21} & 0 & 0 & S_{24} \\ S_{31} & 0 & 0 & S_{34} \\ 0 & S_{42} & S_{43} & 0 \end{bmatrix}$$

-hole
ne-hole
les.

Wave guide Bends & Corners

In microwave circuits, often a waveguide is required to be bent to achieve desired flexibility for connection with adapters, terminators or other loads.

However, any abrupt change in the shape of the guide will cause reflection of waves from the discontinuity. Therefore, special care must be taken to form bends. Depending on the geometry, waveguide bends may be classified as E-plane, H-plane or sharp bends, as shown in figure 29. Out of these, E- and H-plane bends are gradual bends.

E-plane bends distort only the electric field distribution, whereas H-plane bends distort only the magnetic field distribution. To achieve a satisfactory performance, the minimum radius of curvature for small reflection is given by

$$R = 1.5\,b \quad \text{for an E-bend} \quad — ①$$

$$R = 1.5\,a \quad \text{for a H-bend} \quad — ②$$

where a & b are the dimension of the waveguide. [Refer figure 30].

Since sharp 90° bends create total reflection resulting in infinite SWR. Therefore bends have to be gradual.

At lower frequency a bend may ~~be~~ have to be very long & in such cases, a corner would be preferred. [Refer Fig 31].

A mitered 90° bend is a Corner. [Fig 31].

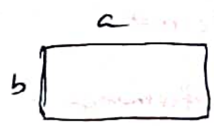
In order to minimize reflection from the discontinuities, it is desirable to have the mean length L between continuities equal to an odd number of Quarter-wave lengths. That is

$$L = (2n+1)\frac{\lambda_g}{4} \quad \text{——— (3)}$$

where $n = 0, 1, 2, 3, ---$, and λ_g is the wavelength in the waveguide.

If the mean length L is an odd number of quarter wavelengths, the reflected waves from both end of the waveguide section are completely canceled.

Note:- In H-Plane bend, the bend is in the direction of the wide dimension (a)



In E-plane bend, the bend is in the direction of narrow band dimension (b). [Refer fig 29]

* Height of the bend is more in case of E-plane bend than H-plane bend [as $a > b$]

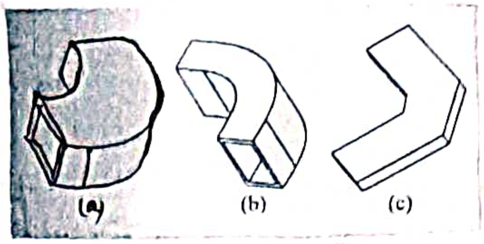


Fig. 29:- Wave guide bends (a) H-Plane (b) E-Plane (c) 45° (sharp bend)

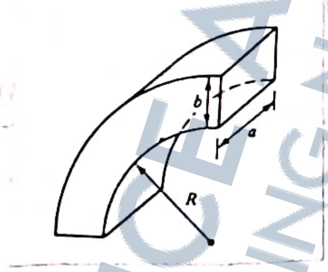


Fig. 30: Radius of curvature of wave guide bend.

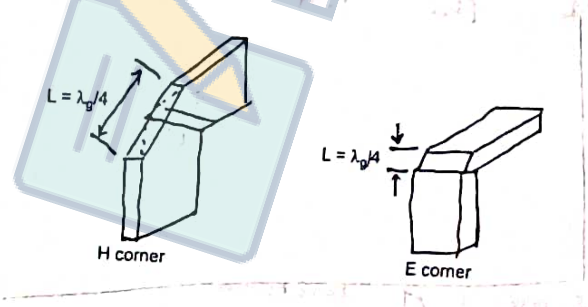


Fig 31:- H-plane Corner & E-Plane Corner

Microwave Posts

In any waveguide system, when there is a mismatch in impedance there will be reflection. In transmission line, in order to overcome this mismatch lumped impedances or stubs of required value are placed at precalculated points. In waveguides too, some discontinuities are made use of for matching purposes.

Any susceptance appearing across the guide, causing mismatch (and production of standing waves) needs to be cancelled by introducing another susceptance of same magnitude but of opposite nature. Microwave posts are used for this purpose.

When a metallic cylindrical post is introduced into the broader dimension of waveguide, it produces the same effect as lumped reactance at that point.

If the post extends only a short distance $\left(< \frac{\lambda_g}{4} \right)$ into the waveguide, it behaves capacitively [Fig. 32 (a)] and this capacitive susceptance increases with depth of penetration. When the depth of penetration is equal to $\lambda_g/4$, the post acts as a series resonant circuit [Fig. 32 (b)], if it is $(> \lambda_g/4)$, the post behaves inductively. [Fig. 32 (c)]. and this inductive ~~susce~~ susceptance decreases when the post is moved further away from the center of the wave guide.

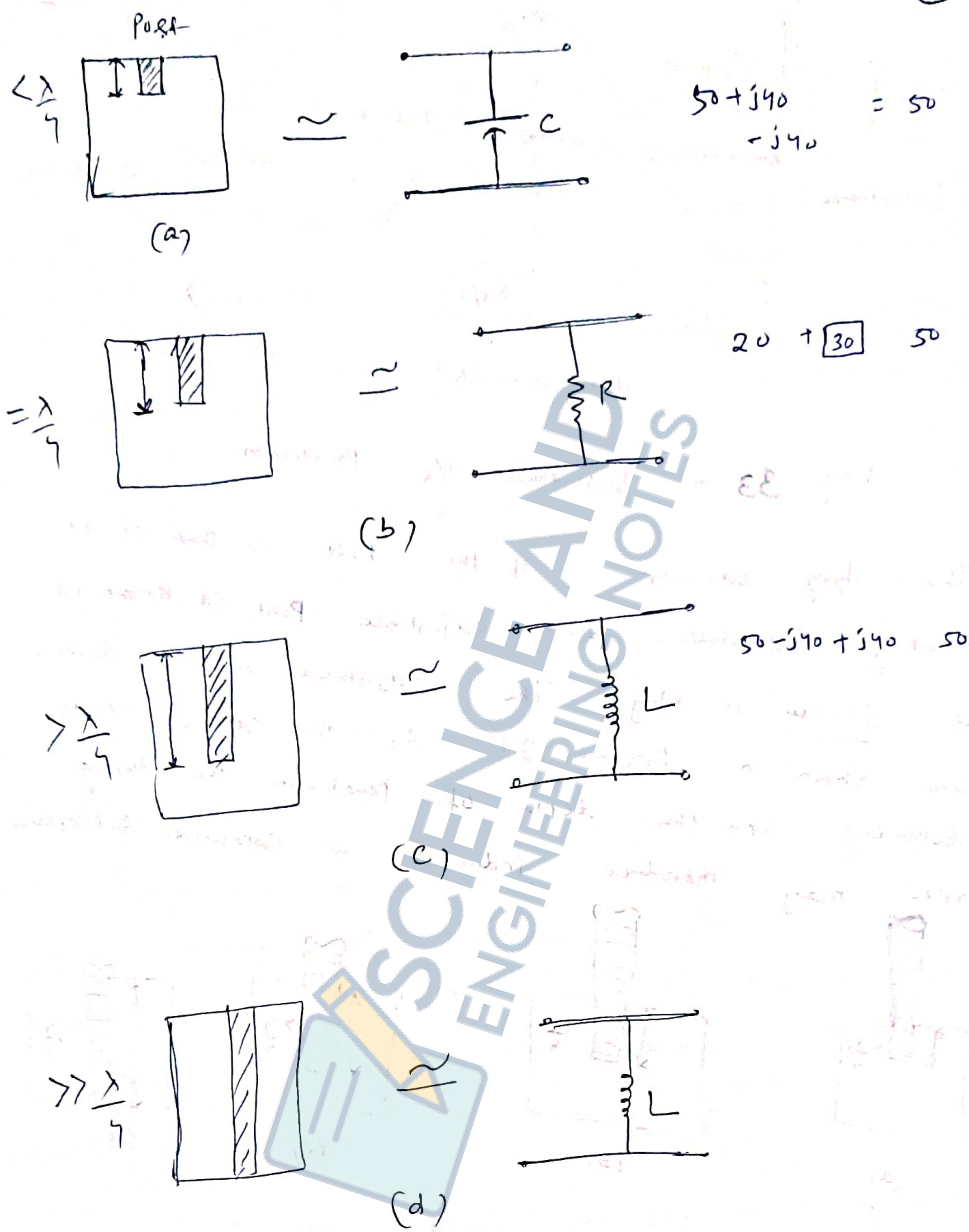


Fig 32:- Waveguide Posts.

When the post is extended completely across the waveguide, the post becomes inductive [Fig 32 (d)].

The susceptance V/s penetration (h) characteristics is shown in figure 33.

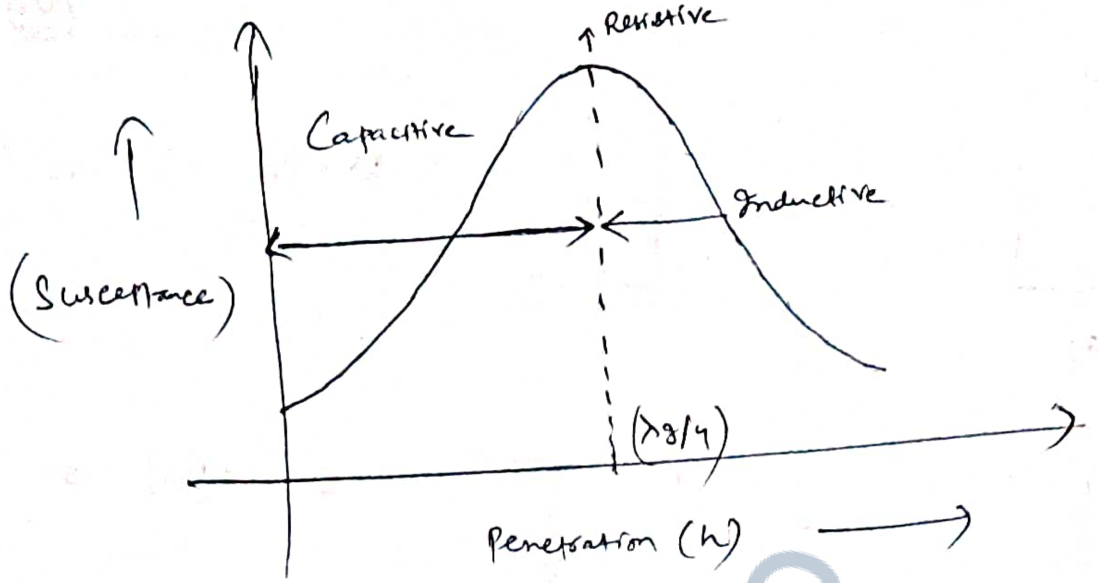


Fig 33:- Susceptance Vs penetration

The big advantage of the post is that it is readily adjustable. An adjustable post is known as a screw or slug. The adjustable or tuning screws are shown in figure 34. As in case of posts, the tuning screw may introduce inductive or capacitive susceptance depending upon the depth of penetration.

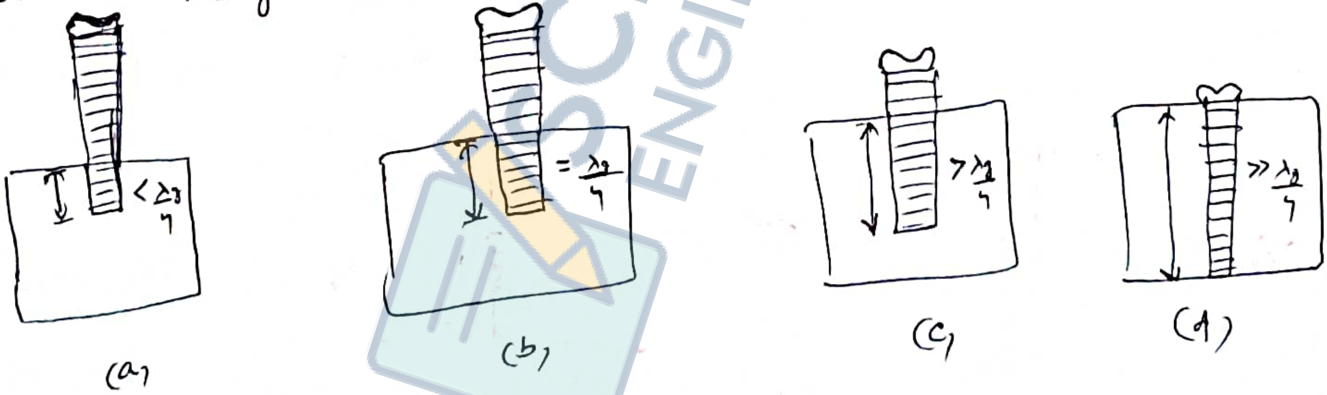


Fig 34:- Tuning Screw

Slide Screw Tuner (S.S. Tuner)

A tuning screw is a metal rod or probe inserted ~~Perpendicularly~~ perpendicularly into a rectangular waveguide through a center of a broad wall. This probe provides a reactance across the guide which can be varied from capacitive to inductive by changing the depth of penetration as shown on figure 35.

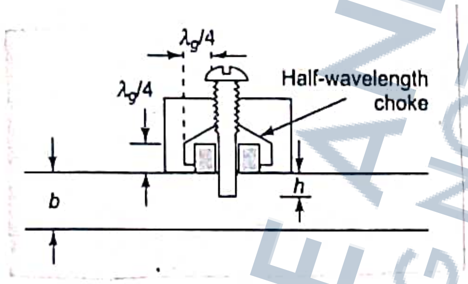


Fig 35:- Tuning screw nature

The theoretical & experimental analysis shows that a thin screw of diameter $<< \lambda g/4$ possesses a susceptance of following nature: (1) capacitive when $h < \lambda g/4$, (2) Infinite when $h \approx \lambda g/4$, (3) Inductive when $h > \lambda g/4$.

$\left(B = \frac{1}{X} = \frac{1}{0} = \infty \right.$, $Z = R + jX$, Resistive, X = Reactance, $Y = G + jB$, B = Susceptance$\left. \right)$

A tuning screw is used as tuning device for impedance matching on account of this reactive nature of the screw. To avoid power leakage through the screw gap, a half-wavelength choke is used at the screw insertion junction, as shown in fig 35.

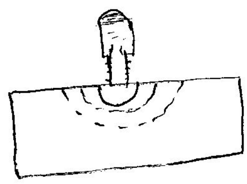
The above tuning screw can be slided along the axis of the waveguide through a narrow longitudinal

Slot, Centred in the broad wall. This helps
Varying both the penetration & position of the tuning Screw along a longitudinal distance of the half-guide wavelength for better matching with ease. Such devices are called Slide Screw tuners.

Attenuators :- Short

Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave. They are of 2 types

LAB :-



(i) Fixed

(ii) Variable

Both fixed and variable attenuators can be designed using resistive films (aquadag).

Fixed attenuator :-

This type attenuator reduces the power by a fixed amount.

e.g - 3dB attenuator reduces the i/p power by half amount.

A co-axial fixed attenuator uses a film with losses on the centre conductor to absorb some of the power as shown in figure 42(a)

— The fixed waveguide type consists of a thin dielectric strip coated with resistive film and placed at the center of the waveguide parallel to the maximum E field.

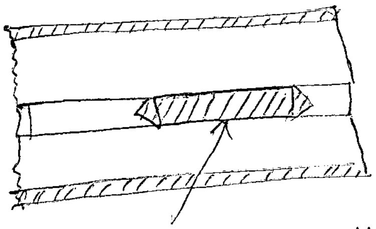
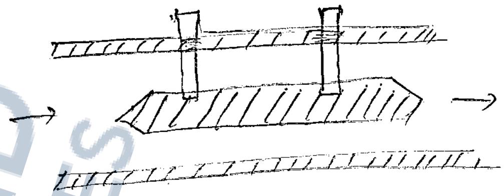


Fig.- (a) Co-axial line fixed attenuator
[42]



(b) Waveguide fixed attenuator.

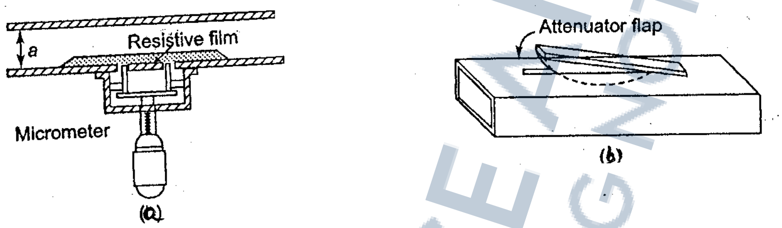
→ Induced current on the resistive film due to the incident wave results in power dissipation, leading to attenuation of microwave energy.

→ The dielectric strip is ~~tappe~~ tapered at both ends up to a length of more than half wavelength to reduce reflections.

Variable Attenuators: -

They provide continuous or stepwise attenuation, For rectangular waveguide they can be of 2 types (a) Flap type (b) Vane type

A varrasle type attenuator can be constructed by moving the resistive vane by means of micrometer screw from one side of the narrow wall to the center where the E field is max^m (fig:-43 a) or by changing the depth of insertion of a resistive disc at an E-field maximum through a longitudinal slot at the middle of the broad wall (fig-43-b).



Flap Type :-

→ The flap type attenuator consist of a resistive element or disc inserted into the longitudinal slot which is cut along the center of the wider dimension of the wave guide

fig 44:- Flap attenuator

→ This flap is mounted on a hinged arm and allowed to descend into the center of the wave guide.

→ The degree of attenuation is determined by the depth of insertion of flap and it is calibrated against a standard for precision measurement or exact measurement.

Vane type :-

It basically consist of a glass vane with coating of aquadag or carbon film similar to that fixed attenuator.

Max^m attenuation   Min^m attenuation

fig 4.5:- movable Vane attenuator.

→ If this vane ~~which at the center~~ is made movable it can be a variable attenuator.

So in this type,

→ The vane is moved laterally where it provides max^m attenuation at the center and min^m at the edge.

→ It is calibrated against a standard or exact measurement.

→ The resistive vane/flap can be shaped to give linear variation of attenuation with depth of insertion.

Phase Shifter

A Phase Shifter is a two-port passive device that produces a variable change in phase of the wave transmitted through it.

A Phase Shifter can be realised by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum E-field. A differential phase change is produced due to the change of wave velocity through the dielectric slab compared to that through an empty waveguide. Two ports are matched by reducing the reflections of the wave from the dielectric slab ~~tape~~ tapered at both ends as shown in fig 38.

$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda$

If V_p change
$\beta \rightarrow$ change

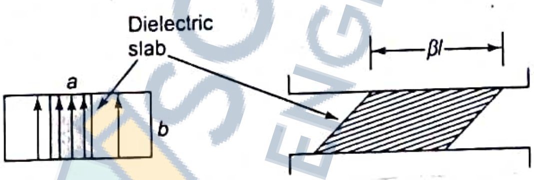


Fig 38:- Phase Shifter

The propagation constant of an empty guide is given by,

$$\beta_g = \frac{2\pi}{\lambda_g} = 2\pi \times \frac{1}{\lambda_g}$$

$$= 2\pi \sqrt{\frac{1}{\lambda_0^2} - \frac{1}{(2a)^2}}$$

$$= 2\pi \sqrt{\frac{1}{\lambda_0^2}\left[1 - \frac{\lambda_0^2}{(2a)^2}\right]}$$

We know

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

$$\Rightarrow \frac{1}{\lambda_g} = \sqrt{\frac{1}{\lambda_0^2} - \frac{1}{(2a)^2}}$$

$\because$ For TE_{10} mode

$\lambda_c = 2a$

$$\beta_g = \frac{2\pi}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \quad \text{--- (1)}$$

The propagation constant through a length 'l' of dielectric slab with relative permittivity (ϵ_r) is given by.

$$\beta_d = \frac{2\pi}{\lambda_0/\sqrt{\epsilon_r}}\sqrt{1-\left(\frac{\lambda_0}{\sqrt{\epsilon_r}\,2a}\right)^2}$$

ϵ_r = Relative permittivity of the dielectric slab.

Thus, the differential phase shift produced by the phase shifter is

$$\Delta\phi = (\beta_d - \beta_g).l$$

$\because \phi = \beta l$

By adjusting the length 'l', different phase shift can be produced. The S-matrix of an ideal phase shifter can be expressed by

$$S = \begin{bmatrix} 0 & e^{-j\Delta\phi} \\ e^{-j\Delta\phi} & 0 \end{bmatrix}$$

$S_{11} = S_{22} = 0$ (Due to perfect matching)

$S_{12} = S_{21} =$ (Symmetrical ports 1 & 2) $= e^{-j\Delta\phi}$

Short note

Microwave Isolators :- (Liao Book)

→ An isolator is a non reciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line.

→ An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called uniline.

→ Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

→ Isolator can be made by inserting a ferrite rod along the axis of rectangular waveguide as shown in fig 41. The isolator here is a Faraday-rotation isolator.

→ Its operating principle can be explained as follows. The i/p resistive card is in the Y-Z plane and the o/p resistive card is displaced 45° w.r.to i/p card.

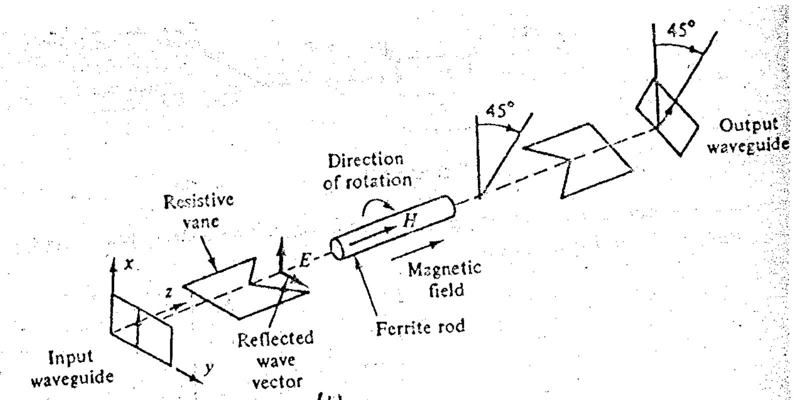


Figure ~~4-6E~~ 41 Faraday-rotation isolator.

→ The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°.

→ The degree of rotation depends on the length and diameter of the rod and on the applied dc magnetic field.

→ An i/p TE_{10} dominant mode is incident to the left end of the isolator. Since the TE_{10} mode wave is perpendicular to the i/p resistive card, the wave passes through the ferrite rod without attenuation.

→ The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the o/p resistive card.

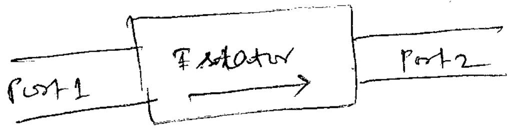
→ As a result of ~~orientation~~ rotation, the wave arrives at the o/p end without attenuation at all.

→ On the Contrary, a reflected wave from the o/p end is similarly rotated clockwise 45° by the ferrite rod.

→ However, since the reflected wave is parallel to the i/p resistive card, the wave is thereby absorbed by the i/p card.

→ The ~~typ~~ typical performance of these ~~card~~ ~~oscillator~~ isolators is about 1 dB insertion loss in forward transmission, and 20 to 30 dB isolation in reverse attenuation.

Note :- Scattering matrix of isolator can be found out as follows.



→ Since both ports are matched, the reflection coefficients at port 1 (S_{11}) and reflection coefficient at port 2 (S_{22}) are zero, → $S_{11} = S_{22} = 0$.

→ Transmission occurs only in the direction from Port 1 to port 2. But transmission does not occur in reverse direction i.e from port 2 to port 1. So ($S_{12} = 0$)

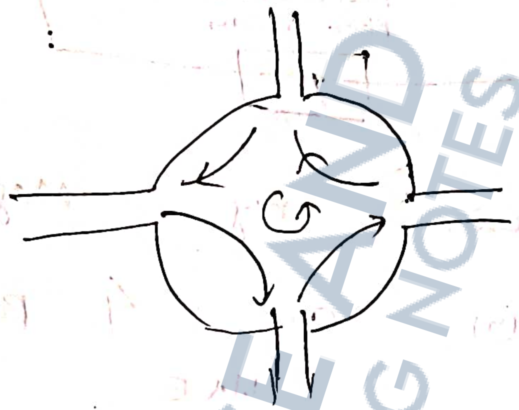
→ So S-matrix is given by

$$S = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix}$$

Circulators

A microwave circulator is a multiport waveguide junction in which the wave can flow only from n^{th} port to the $(n+1)^{th}$ port in one direction.

Port 4

Port 1  Port 3

Port 2

fig1:- Schematic diagram of a 4 port circulator.

→ Although there is no restriction on the number of ports, ~~the~~ four-port microwave circulator, and 3 port microwave circular are commonly used.

→ A four-port circulator can be constructed from two magic-T's and a non-reciprocal 180° Phase shifter.

→ In fig 2:- An i/p signal at port 1 is split into two in-phase and equal magnitude waves in the collinear arms 'b' & 'a' of the magic Tee, T_1. Then these

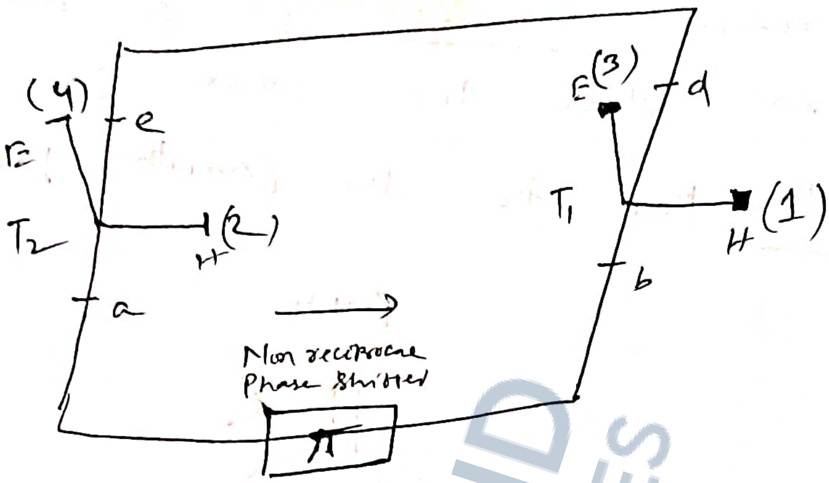


fig 2:- Circular using Magic Tee.

(1), (2), (3), (4) are 4 ports of circulator

(1) & (2) are H & E ports of Magic Tee 1.

(3) & (4) are H & E port of Magic Tee 2.

→ 2 waves are added up to emerge from port 2 in magic Tee T_2.

→ So o/p at port 1 of circulator travels to port 2 only.

→ At port 3, o/p = 0. [Isolated port for port 1]

At port 4, difference of two equal and same phase waves = 0.

→ On the other hand, a signal at port 2

will be splitted into two equal amplitude and equiphase waves in collinear arms (a & e) of magic Tee, T_2, and appears at 'b' and 'd' out of phase due to presence of non reciprocal 180° phase shifter.

→ Then out of phase waves ~~added~~ subtracted and appear from port 2 in ~~arm~~ magic Tee, T_1. $\left[\frac{V_1}{2} - \left(-\frac{V_1}{2}\right) = V_1\right]$.

and port 1 of T1, they are added

So net o/p ~~at~~ port 1 = 0 $\left(\frac{V_1}{2} + \left(-\frac{V_1}{2}\right) = 0\right)$.

→ Similarly, at port 1, o/p = 0.
I/P given to '3', (Isolated Port).

→ Splitted into ~~section~~ 2 equal & opposite phase wave through b & d. arms.

→ At port 4, they are subtracted
So o/p at port 4.

→ At port 2 they are added o/p = 0 at port 2.

→ I/P to Port 4, at port 2, o/p = 0 (Isolated port)
equal & opposite phase wave at arm 'e' 'a'.

Wave through arm 'a' gets a 180° Phase shift.

→ Now these 2 equal phase waves combined at port 1.

→ Also o/p at port 3 = 0. (Substracted 2 waves)

Summary :- waves travel form

$$\left.\begin{array}{l} 1 \rightarrow 2 \\ 2 \rightarrow 3 \\ 3 \rightarrow 4 \\ 4 \rightarrow 1 \end{array}\right] \begin{array}{l} S_{21} = 1 \\ S_{32} = 1 \\ S_{43} = 1 \\ S_{14} = 1 \end{array}$$

~~Rest~~. For a perfectly matched, lossless and non reciprocal 4 port Circulator

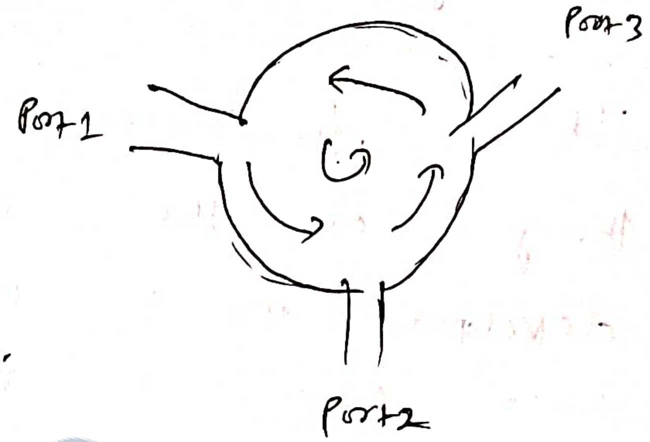
S-matrix ;

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Similarly for 3 port Circulator :-

1 → 2
2 → 3
3 → 1



$S_{21} = S_{32} = S_{13} = 1$.

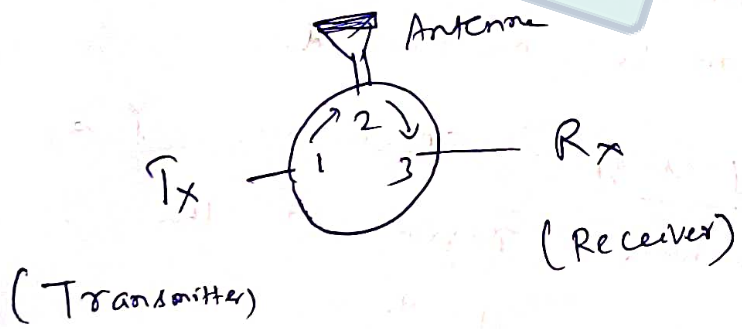
Rest all Components = 0.

$$\begin{bmatrix} S_{11} & S_{12} & \underset{}{S_{13}}^{=1} \\ S_{21}{}_{=1} & S_{22} & S_{23} \\ S_{31} & S_{32}{}^{=1} & S_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Typical Characteristic of Circulator

- Insertion loss < 1 dB
- Isolation ≃ 30 – 40 dB.
- VSWR < 1.5.

Use of Circulator



Port	Port	
1 →	2 →	Tx to Antenna.
2 →	3 →	Antenna to Rx

So one 3 Port n/w can be used in radar.

— o —

Q) 1) Why a 3-port n/w can't be matched at all ports?

[Ex- E-Plane Tee / H-Plane Tee]

Proof :- Assume that all the 3-ports are matched,

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \quad \text{—— (1)}$$

From the symmetry property of S-matrix

$$S_{12} = S_{21}, \quad S_{13} = S_{31}, \quad S_{23} = S_{32} \quad \text{—— (2)}$$

From zero property of S matrix, the sum of products of each term of any column (or row) multiplied by complex conjugate of the corresponding terms of any other column (or row) is zero and it is

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \quad \text{—— (3)}$$

Since $S_{11} = 0$ and $S_{22}^* = 0$, eq^n (3) becomes

$$S_{31} S_{32}^* = 0$$

By symmetry, $S_{13} S_{23}^* = 0$ ——— (4) $\left(\because \begin{array}{l} S_{31} = S_{13} \\ S_{32} = S_{23} \end{array} \right)$

This means that either S_{13} or S_{23}^* or both should be zero. However, from the unity property of S-matrix, the sum of the products of each term of any one row (or column) multiplied by its complex conjugate is unity; i.e

$$S_{21} S_{21}^* + S_{31} S_{31}^* = 1 \quad \text{——— (5)}$$

$$S_{12} S_{12}^* + S_{32} S_{32}^* = 1 \quad \text{——— (6)}$$

$$S_{13} S_{13}^* + S_{23} S_{23}^* = 1 \quad \text{——— (7)}$$

Substitution of eqn (2) in eqn (5), we have

$$S_{12} S_{12}^* = 1 - S_{13} S_{13}^*$$

$$\Rightarrow |S_{12}|^2 = 1 - |S_{13}|^2$$

Similarly, from eqn (6), $|S_{12}|^2 = 1 - |S_{23}|^2$

$$\therefore |S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad \text{——— (8)}$$

Eqn (7) and (8) are contradictory, for, if $S_{13} = 0$, [from eqn (7)] then from [eqn 8] then S_{23} is also zero and thus eqn (7) becomes false ($S_{23} = 1$)

Similarly, if $S_{23} = 0$, [from eqn 8] S_{13} becomes zero, eqn (7) " " .

This inconsistency proves that our assumption is false.

Tee jn [E/H] Plane Tee can n't be matched to 3 arms. i.e diagonal elements of S-matrix of Tee jn are not all zeros.

2) Why ferrite is used in circulator?

Ans :- Ferromagnetic material when placed in d.c magnetic field, electromagnetic wave propagation become non-reciprocal. This property is used for construction of circulators and isolators.

3) Write short notes on Hybrid rings (Rat-Race ring)

A hybrid ring consists of an annular line of proper electrical wavelength to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions. Fig 1. shows a hybrid ring with series junctions.

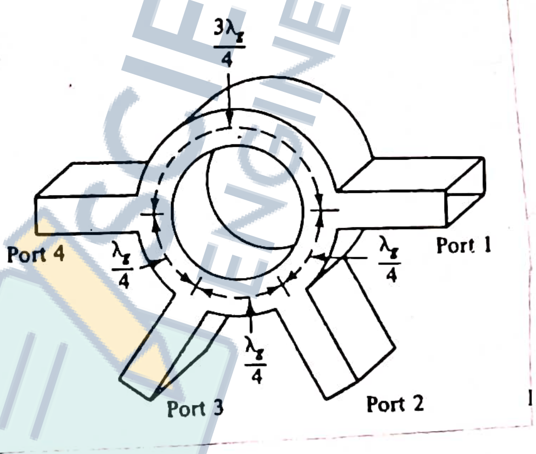
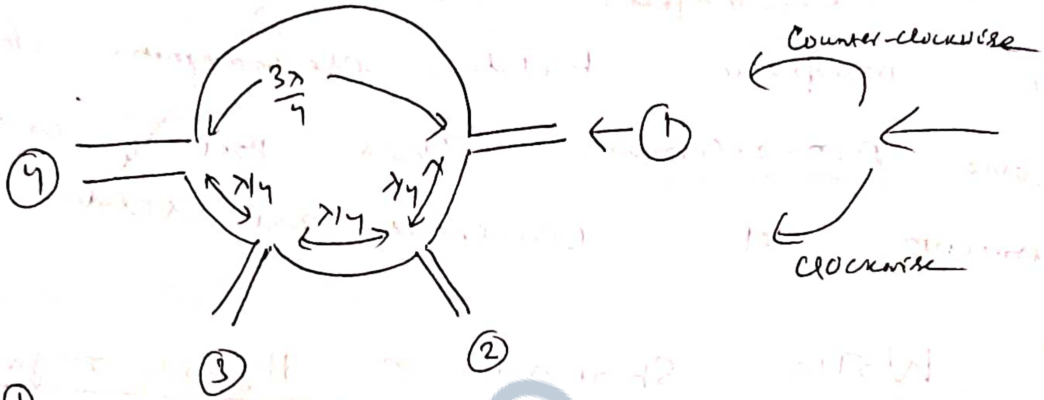


Fig 1:- Hybrid ring

The hybrid ring has characteristics similar to those of hybrid tee (magic Tee). When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves travelling in clockwise and counter clockwise directions is 180°.

Note:-



Wave fed in ①, travels in clockwise direction to port 3

Covers the path $\frac{\lambda}{4}+\frac{\lambda}{4}=\frac{\lambda}{2}$.

the wave, that travels in anticlockwise direction to port 3

Covers the path, $\frac{3\lambda}{4}+\frac{\lambda}{4}=\lambda$

λ path diff $= 360^\circ$

$\frac{\lambda}{2}$ " " $= 180^\circ$.

If wave at port 1, $= A \sin\omega t$

At port 3, clockwise path wave $= \frac{A}{2}\sin(\omega t+180^\circ) = -\frac{A}{2}\sin\omega t$

At port 3, Anticlockwise " " $= \frac{A}{2}\sin(\omega t+360^\circ) = \frac{A}{2}\sin\omega t$

Total wave at port 3 $= -\frac{A}{2}\sin\omega t+\frac{A}{2}\sin\omega t = 0$

Thus the waves are cancelled at port 3.

For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.

If all the ports are matched

$S_{11} = S_{22} = S_{33} = S_{44} = 0$ —— ①

$$S_{31} = S_{13} = 0 \quad \text{--- (2)}$$

$$S_{42} = S_{24} = 0 \quad \text{--- (3)}$$

Using (1), (2) & (3), we have the scattering matrix of hybrid ring as follows

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \quad \text{--- (4)}$$

It should be noted that the phase cancellation occurs only at designated freq for an ideal hybrid ring. In actual hybrid rings there are small leakage couplings, and therefore the zero elements in matrix of eqn (4), are not quite equal to zero.

Scattering Matrix - Properties

Defn:- The Scattering Matrix of an m-port junction is a square matrix of a set of elements which relate incident and reflected waves at the port of the junction. The diagonal elements of the S-matrix represents reflection coefficients and off diagonal elements represent transmission coefficients.

Characteristics of S-Matrix:-

→ It describes any passive microwave component

→ It exists for linear passive and time invariant networks

→ It gives complete information on reflection and transmission coefficients

Properties of S-Matrix

1) Zero diagonal elements for perfect matched network:-

i.e For an ideal N-Port network with matched terminations $S_{ii} = 0$, since there is no reflection from any.

2) ~~Symmetry~~ Property for a reciprocal network

A reciprocal device has the same transmission characteristics in either direction of pair of ports and is characterized by a symmetric scattering matrix.

$S_{ij} = S_{ji}$ which result

$S^T = S$. This property is known

as Symmetry property of S-matrix.

3) Unitary property for lossless junction

For any lossless network the sum of the products for each term of any one row or of any column of the S-matrix multiplied by its complex conjugate is a unity. ~~matrix~~.

$$\left[S \cdot (S^*)\right]^T = I$$

4) Zero property of Scattering matrix

The sum of product of each term of any column (or row) multiplied by the complex conjugate of the corresponding terms of any other column (or row) is zero.

Q) 1) Derive the Scattering matrix of E-plane Tee

In E-plane tee if the waves are fed into Port 3, the waves appearing at Port 1 & Port 2 of Collinear arm will be opposite phase & same magnitude.

$$\therefore \quad S_{13} = -S_{23} \quad \text{——— (1)}$$

For matched junctions, the

$$S_{11} = S_{22} = S_{33} = 0 \quad \text{——— (2)}$$

$\therefore$ The S-matrix is given by

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \quad \text{——— (3)}$$

From the symmetry property,

$$S_{12} = S_{21}, \quad S_{13} = S_{31}, \quad S_{23} = S_{32} \quad \text{——— (4)}$$

From Zero Property.

The sum of product of each term of any column (or row) multiplied by complex conjugate of the corresponding terms of other column (row) is zero.

Applying for first column & 2nd column

$$S_{11} \cdot S_{12}^{*} + S_{21} \cdot S_{22}^{*} + S_{31} \cdot S_{32}^{*} = 0 \quad \text{——— (5)}$$

From eqn (2), $S_{11} = 0$, $S_{22} = 0$ or $S_{22}^{*} = 0$

Putting these conditions in eqn (5), we have

$$0 + 0 + S_{31} \cdot S_{32}^{*} = 0 \quad \text{——— (6)}$$

Thus S_{13} ~~or~~ S_{31} / ~~or~~ S_{32} or S_{23} or both are zero.

From Unity property,

The sum of product of each term of any row (or column) multiplied by its complex conjugate is unity first row $\quad S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1 \quad$ — (7)

2nd row $\quad S_{21} \cdot S_{21}^* + S_{23} \cdot S_{23}^* = 1 \quad$ — (8)

3rd row $\quad S_{31} \cdot S_{31}^* + S_{32} \cdot S_{32}^* = 1 \quad$ — (9)

~~Substituting eqn (4) in eqn (7)~~

From eqn (7),

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow |S_{12}|^2 = 1 - |S_{13}|^2 \quad \text{— (10)}$$

From eqn (8)

$$|S_{21}|^2 = 1 - |S_{23}|^2$$

$$\Rightarrow |S_{12}|^2 = 1 - |S_{23}|^2 \quad \text{— (11)}$$

$(\because S_{12} = S_{21})$

From (10) & (11),

$$S_{13} = S_{23} \quad \text{— (12)}$$

If $S_{13} = 0$, then $S_{23} = 0$ — (13).

But from eqn (9), If $S_{13} = 0$

$$\boxed{S_{23} = 1} \quad \text{— (14)}$$

$\because S_{13} = 0$
$\Rightarrow S_{31} = 0$
& $S_{32} = S_{23}$

So eqn (13) & (14) are contradictory.

Similarly, If $S_{23} = 0$, $S_{13} = 0$ from eqn (12)

But from eqn (9), $\left.\begin{matrix} \\ \text{If } S_{23}=0 \end{matrix}\right\}$ $S_{13} = 1$.

$\therefore$ Contradicting.

Thus, this inconsistency proves ~~that~~ the statement that the Tee Junction cannot be matched to the three arms. In other words, the diagonal elements of S-matrix of a Tee Junction are not all zero.

Thus,

$$S_{ij} \neq 0 \quad \text{if} \quad i = j.$$

Since the collinear arm is symmetric about the side arm,

$$S_{11} = S_{22} \quad \text{—— (15)}$$

Using eqn (1), (4), (15), S matrix becomes,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$

$$S^* = \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{11}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & S_{33}^* \end{bmatrix}$$

From Unitary Property

$$[S \cdot S^*]^T = I$$

Since 'S' satisfies symmetric property, we can write

$$S \cdot S^* = I.$$

Row Column

$$\begin{array}{ll} R1 \times C1 & S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1 \quad \text{—— (16)} \\ R_2 \times C_2 & S_{12} \cdot S_{12}^* + S_{11} \cdot S_{11}^* + S_{13} \cdot S_{13}^* = 1 \quad \text{—— (17)} \\ R_3 \times C_3 & S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* + S_{33} \cdot S_{33}^* = 1 \quad \text{—— (18)} \end{array}$$

For a matched '3' port, we have

$$S_{33} = 0$$

From eqn 18,

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow 2|S_{13}|^2 = 1$$

$$\Rightarrow |S_{13}|^2 = \frac{1}{2}$$

$$\Rightarrow S_{13} = \frac{1}{\sqrt{2}} \quad \text{—— (19)}$$

Putting eqn (19) in,

~~From~~ eqn (16), $|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} = 1$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 = \frac{1}{2} \quad \text{—— (20)}$$

$$S \cdot S^* = I$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{11}^* & -S_{13}^* \\ S_{13} & -S_{13}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying 1st row row of 1st matrix . 3rd column of 2nd matrix = 0

$(R1 \times C_3)$ = S_{13}

$$S_{11} \cdot S_{13}^* + S_{12} \cdot (-S_{13}^*) + S_{13} \cdot S_{33}^* = 0$$

But $\underline{S_{33} = 0}$ (As discussed earlier)

$$S_{11} \cdot S_{13}^* + S_{12}(-S_{13}^*) = 0$$

$$\Rightarrow S_{11} \cdot \cancel{S_{13}^*} = S_{12} \cdot \cancel{S_{13}^*}$$

$$\Rightarrow S_{11} = S_{12} \quad \text{—— (21)}$$

Putting eqn (21) in (20)

$$|S_{11}|^2 + |S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow 2|S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow \quad |S_{11}|^2 = \frac{1}{4}$$

$$\Rightarrow \quad S_{11} = \frac{1}{2} \quad \text{— (22)}$$

& $S_{12} = \frac{1}{2}$ — (23) & $S_{13} = \frac{1}{\sqrt{2}}$ — (24)

(from eqⁿ (21)) (As derived earlier)

$$\therefore S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \quad \text{— (25)}$$

For H-plane Tee the derivation is similar only -ve sign is not there.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \quad \text{— (26)}$$

Now,

$$S \cdot S^* = I$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{11}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{array}{ll} R_1 \times C_1 & S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1 \quad — (27) \\ R_2 \times C_2 & S_{12} \cdot S_{12}^* + S_{11} \cdot S_{11}^* + S_{13} \cdot S_{13}^* = 1 \quad — (28) \\ R_3 \times C_3 & S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* + S_{33} \cdot S_{33}^* = 1 \quad — (29) \end{array}\right\}$$

Same as E-Plane Tee

Similarly solving, $S_{13} = \frac{1}{\sqrt{2}}$ — (30)

Multiply $R_1 \times C_3 = 0$

$$S_{11} \cdot S_{13}^* + S_{12} \cdot S_{13}^* + S_{13} \cdot S_{33}^* = 0$$

$$\Rightarrow S_{11} \cdot S_{13}^* + S_{12} \cdot S_{13}^* + 0 = 0 \quad (\because S_{33} = 0)$$

$$\Rightarrow S_{11} \cdot \cancel{S_{13}^*} = - S_{12} \cancel{S_{13}^*}$$

$$\Rightarrow \boxed{S_{11} = - S_{12}} \quad — (31)$$

Putting (30) & (31) in (27),

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} = 1$$

$$\Rightarrow \quad |S_{11}|^2 + |S_{12}|^2 = \frac{1}{2}$$

$$\Rightarrow \quad |S_{11}|^2 + |-S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow \quad 2|S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow \quad |S_{11}|^2 = \frac{1}{4}$$

$$\Rightarrow \quad S_{11} = \frac{1}{2} \quad \text{— (32)}$$

$$\& \quad S_{12} = -S_{11} = -\frac{1}{2} \quad \text{— (33)}$$

$$\& \quad S_{13} = \frac{1}{\sqrt{2}} \quad \text{— (34)}$$

$$\therefore S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

for H-Plane Tee

Q) A signal of power 32 mW is fed onto one of the Collinear port of a lossless H-plane tee. Determine the powers on the remaing ports when other ports are terminated by means of matched load?

Ans ÷ Let 32 mW power fed at Port 1.

Port ② & ③ are terminated with matched load.

$$[b] = S \cdot [a]$$

$$\begin{pmatrix} O/P \\ voltage \end{pmatrix} = S \cdot \begin{pmatrix} I/P \\ voltage \end{pmatrix} \text{ or } O/P\ Power = S^2 \cdot \begin{pmatrix} I/P \\ Power \end{pmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2}\right)^2 & \left(-\frac{1}{2}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 \\ \left(-\frac{1}{2}\right)^2 & \left(\frac{1}{2}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 \\ \left(\frac{1}{\sqrt{2}}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2}\right)^2 & \left(-\frac{1}{2}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 \\ \left(-\frac{1}{2}\right)^2 & \left(\frac{1}{2}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 \\ \left(\frac{1}{\sqrt{2}}\right)^2 & \left(\frac{1}{\sqrt{2}}\right)^2 & 0 \end{bmatrix}_{3\times 3} \begin{bmatrix} 32 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2}\right)^2 \times (32) \\ \left(-\frac{1}{2}\right)^2 \times 32 \\ \left(\frac{1}{\sqrt{2}}\right)^2 \times 32 \end{bmatrix} = \begin{bmatrix} \frac{32}{4} \\ \frac{32}{4} \\ \frac{32}{2} \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix}$$

$\therefore$ Power Coming at Port 1 = 8 mW

" " " Port 2 = 8 mW

" " " Port 3 = 16 mW.

Q) Scattering Matrix of Magic Tee

$$\begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

3 → H-arm

4 ⇒ E-arm

If

3 - E arm

4 - H arm

$$S = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

— o —

Q) 1) Why a 3-port n/w Can't be matched at all ports? [Ex- E-Plane Tee / H-Plane Tee]

Proof :- Assume that all the 3-ports are matched,

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \quad \text{--- (1)}$$

From the symmetry property of S- matrix

$$S_{12} = S_{21}, \quad S_{13} = S_{31}, \quad S_{23} = S_{32} \quad \text{--- (2)}$$

From Zero property of S matrix, the sum of products of each term of any Column (or row) multiplied by complex conjugate of the corresponding terms of any other Column (or row) is Zero and it is

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \quad \text{--- (3)}$$

Since $S_{11} = 0$ and $S_{22}^* = 0$, eqn (3) becomes

$$S_{31} S_{32}^* = 0$$

By symmetry, $S_{13} S_{23}^* = 0$ ——— (4) $\left(\because \begin{matrix} S_{31} = S_{13} \\ S_{32} = S_{23} \end{matrix} \right)$

This means that either S_{13} or S_{23}^* or both should be zero. However, from the unity property of S-matrix, the sum of the products of each term of any one row (or column) multiplied by its complex conjugate is unity; i.e

$$S_{21} S_{21}^* + S_{31} S_{31}^* = 1 \quad \text{——— (5)}$$

$$S_{12} S_{12}^* + S_{32} S_{32}^* = 1 \quad \text{——— (6)}$$

$$S_{13} S_{13}^* + S_{23} S_{23}^* = 1 \quad \text{——— (7)}$$

Substitution of eqn (2) in eqn (5), we have

$$S_{12} S_{12}^* = 1 - S_{13} S_{13}^*$$

$$\Rightarrow |S_{12}|^2 = 1 - |S_{13}|^2$$

Similarly, from eqn (6), $|S_{12}|^2 = 1 - |S_{23}|^2$

$$\therefore |S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad \text{——— (8)}$$

Eqn (7) and (8) are contradictory, for if $S_{13} = 0$, [eqn 8] (from eqn (4)) then from then S_{23} is also zero and thus eqn (7) becomes false ($S_{23} = 1$)

Similarly, if $S_{23} = 0$, [from eqn 8] S_{13} becomes zero, eqn (7) " ".

This inconsistency proves that our assumption is false.

Tee jn [E/H] Plane Tee can n't be matched to 3 arms. i.e diagonal elements of S-matrix of Tee jn are not all zeros.

2) Why ferrite is used in circulator?

Ans ÷ Ferromagnetic material when placed in d.c magnetic field, electromagnetic wave propagation become non-reciprocal. This property is used for construction of circulators and isolators.

Gyrator

It is a two port device that has a relative phase difference of 180° for transmission from port ① to port ② and no phase shift (0° phase shift) for transmission from port ② to port ① as shown in figure 36.

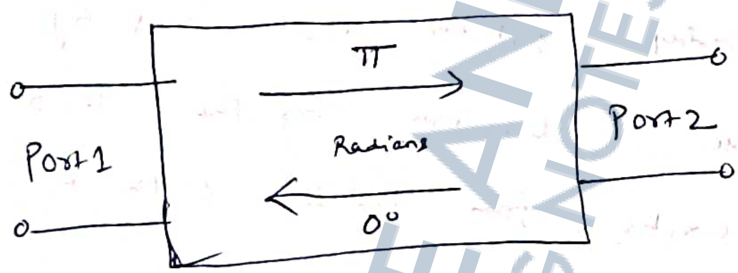


Fig 36:- Block diagram of gyrator.

The S-matrix of an ideal gyrator is as follows.

$$[S] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\because$ $S_{12} \rightarrow 0° \rightarrow 1$
o/p i/p

$S_{21} \rightarrow 180° \rightarrow -1$
o/p i/p

A ferrite gyrator is composed of 2 rectangular waveguide ports [1 & 6 of figure 37], one 90° twist [2 of figure 37], one rectangular to circular waveguide transition [3 of figure 37], a 90° Faraday rotator [4 of figure 37] and a circular to rectangular waveguide transition [5 of figure 37].

When the field moves from port 1 to port 2, it is first rotated by 90° in the anti-clockwise direction by the twist and then by another 90° in the anticlockwise direction by the Faraday rotator. It finally exits from port 2 with a 180° phase difference from the i/p signal.

When a signal is given at port 2, it is first rotated by 90° in anticlockwise direction by the Faraday rotator and next by 90° in the clockwise direction by the twist. Therefore, it ~~exist~~ exits from port 2 with a 0° phase difference.

Hence a wave at port 1 undergoes a phase shift of π radian (or 180°) but a wave fed from port 2 doesn't change its phase in a gyrator.

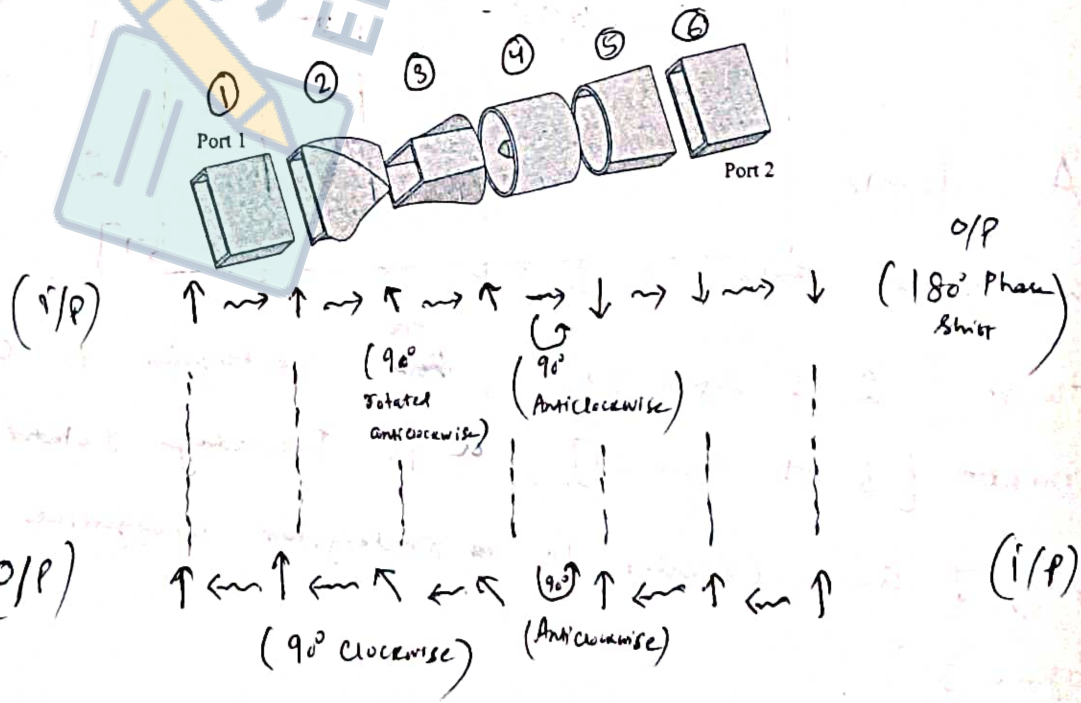


Figure: 37:- Gyrator with twist

Microwave Resonant Cavities

→ A Cavity resonator is a metallic enclosure that confines the electromagnetic energy. The stored electric and magnetic energies inside the cavity determine its equivalent inductance and capacitance. The energy dissipated by the finite conductivity of the cavity walls determines its equivalent resistance.

→ Different types

→ Rectangular - Cavity resonator
→ Circular - Cavity resonator
→ reentrant - Cavity resonator

→ At high frequencies ($\geq 100\,MHz$) the RLC ckt elements are inefficient when used as resonators because the dimension of the ckt are comparable to the operating wavelength, and consequently, there is unwanted radiation.

→ Therefore, at high frequencies RLC resonant circuits are replaced by electromagnetic cavity resonators. Such resonator cavities are used in Klystron tubes, microwave filters, frequency meter etc.

→ A microwave oven essentially consists of a power supply, a waveguide feed and an oven cavity.

→ The cavity is simply a rectangular waveguide shorted at both ends. Because of radiation loss from open-ended waveguide, waveguide resonators are usually shorted at both ends, thus forming a closed box or cavity.

→ Consider the rectangular cavity as shown in figure 46. Since the waveguide is shorted at both the ends, we therefore expect to have standing wave and also TM & TE modes of wave propagation.

Depending on how the cavity is excited, the wave can propagate in x-, y- or z- direction. We choose +z-direction as 'direction of wave propagation'. In fact, there is no wave propagation, rather there are standing waves.

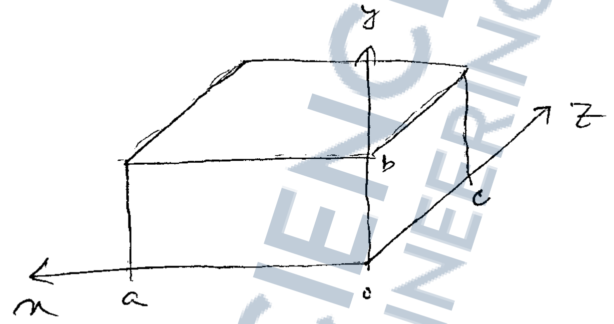


fig 46 :- Rectangular cavity

Note: - The max^m amplitude of standing wave occures when freq of impressed signal is same to the resonant freq.

TM mode :-

[Rectangular waveguide – page – 113]

As discussed earlier, the Maxwell's eqn for lossless dielectric medium are

$$\nabla^2 E_s + k^2 E_s = 0$$

$$\nabla^2 H_s + k^2 H_s = 0$$

For TM mode

$H_z = 0$

$E_z \neq 0$

Repeating the same steps, [As done for Rectangular waveguide]

We have

$$E_{z_s}(x,y,z) = X(x).\ Y(y).\ Z(z)$$

Solution

$$X(x) = C_1 \cos k_x x + C_2 \sin k_x x$$

$$Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$$

$$Z(z) = C_5 \cos k_z z + C_6 \sin k_z z$$

only change is $Z(z)$, it was $Z(z) = C_5 e^{\gamma z} + C_6 e^{-\gamma z}$ since direction of propagation was $+z$ direction.

→ Here in the closed cavity, no propagation along z-direction, only we have standing waves.

So eqn $Z(z) = C_5 \cos k_z z + C_6 \sin k_z z$

→ Putting the boundary conditions, and solving we have

$$\boxed{E_{z_s} = E_0 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cos\left(\frac{p\pi}{c}\right)z}$$

where $E_0 = C_2 C_4 C_5$ and phase constant (β)

$$\boxed{\beta^2 = k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = k_x^2 + k_y^2 + k_z^2}$$

k = wave number

where $m = 1, 2, 3, \ldots$
$n = 1, 2, 3, \ldots$
$p = 0, 1, 2, 3, \ldots$

But $\beta^2 = \omega^2 \mu \epsilon$

$$\Rightarrow \omega^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2$$

$$\Rightarrow \omega = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

$m \neq 0, n \neq 0$
If m or $n = 0$
E_{z_s} will be zero.

m, n, p are no. of half cycle variation along x, y, z-direction respectively.

$\therefore$ Resonant freq (f_r)

$$f_r = \frac{\cancel{\pi}}{2\cancel{\pi}\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

Imp

$$\Rightarrow \boxed{f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

$$\Rightarrow \boxed{f_r = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

where $u' = \frac{1}{\sqrt{\mu\epsilon}}$

$$\Rightarrow \boxed{\lambda_r = \frac{u'}{f_r} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}}$$

For free space

$$f_r = \frac{1}{2\sqrt{\mu_0\epsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} = \frac{3\times10^8}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

For dielectric medium

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

But $\mu_r = \frac{\mu}{\mu_0} \Rightarrow \mu = \mu_r\mu_0$

$\epsilon_r = \frac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = \epsilon_r\epsilon_0$

$$\Rightarrow \quad f_r = \frac{1}{2\sqrt{\mu_r \epsilon_r}} \cdot \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

imp

$$\Rightarrow \quad \boxed{f_r = \frac{3\times10^8}{2\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

Dominant mode [Mode having lowest resonant frequency]

In general, among the dimension of Rectangular W.G

$b < a < c$, or $a > b < c$

For this case,

The dominant mode in TM mode is,

$$\boxed{TM_{mnp} = TM_{110}}$$

TE Mode

For TE mode, $E_z = 0$, $H_z \neq 0$.

Repeating the similar steps,

$$\boxed{H_{zs} = H_0 \cos\left(\frac{m\pi}{a}\right)x \, \cos\left(\frac{n\pi}{b}\right)y \, \sin\left(\frac{p\pi}{c}\right)z}$$

Where $m = 0, 1, 2, 3 - -$

$n = 0, 1, 2, 3 - -$

$p = 1, 2, 3 - - -$

m, n, p are no. of half cycle variations along x, y, z, directions respectively.

Similar to TM mode

$f_r =$ is same as that of TM mode

$$= \frac{\omega'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

or $$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

Dominant mode [Mode having lowest resonant freq]

for $b < a < c$, or $a > b < c$

$$\boxed{TE_{mnp} = TE_{101}}$$ is the dominant mode

Overall dominant mode For Rectangular wave guide

$$(f_r)_{TE} < (f_r)_{TM}$$

$$(f_r)_{101} < (f_r)_{110}$$

$\therefore$ $\boxed{(f_r)_{101} \text{ is lowest}}$ i.e. TE_{101} is overall dominant mode.

Quality factor

Practically, resonant cavity walls are not perfect conductors ($\sigma_c \neq \infty$) and dielectric medium is lossy ($\sigma \neq 0$). Therefore, there is loss of stored energy. The Quality factor is a means of determining the loss. [Lower loss implies high Q]

Quality factor (Q) is defined as,

$$Q = \omega \cdot \frac{(\text{average energy stored})}{(\text{energy loss/second})}$$

$$Q = \omega \cdot \frac{(W_m + W_e)}{P_e} \qquad \left[\frac{\text{Energy}}{\text{Second}} = \text{Power} \right.$$

$W_m \rightarrow$ Stored magnetic energy.

$W_e \rightarrow$ Stored Electric energy.

$P_e \rightarrow$ Power loss.

At resonance $W_m = W_e$

$$\Rightarrow \boxed{Q = \frac{2 \omega W_e}{P_e}}$$

Case-I :- Quality factor of cavity with lossy conducting walls but lossless dielectric medium i.e $\sigma_c \neq \infty$, $\sigma = 0$.

Then $Q_c = \frac{2\omega_0 W_e}{P_c}$

$$Q_c = \frac{(Kac)^3 bq}{2\pi^2 R_s} \cdot \frac{1}{(2p^2a^3b + 2bc^3 + p^2a^3c + ac^3)}$$

In pozar book or in exam they give the notation

d instead of c [length of cavity]

l instead of p [p = no. of half cycles variations along z-axis $= 1, 2, 3, \ldots$]

So

$$\boxed{Q_c = \frac{(Kad)^3 bq}{2\pi^2 R_s} \cdot \frac{1}{[2l^2a^3b + 2bd^3 + l^2a^3d + ad^3]}}$$

Case-II :- Cavity with a lossy dielectric medium but perfectly conducting wall.

$$\boxed{Q_d = \frac{2\omega W_e}{P_d} = \frac{1}{\tan\delta}}$$, $\tan\delta$ = Loss tangent

where $R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$, $\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$

$K = \omega\sqrt{\mu\epsilon}$, $\eta = \sqrt{\frac{\mu}{\epsilon}}$

Imp

$$\boxed{\text{Total } Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}}$$

Pozar

Ex :- 1) A rectangular waveguide cavity is made from a piece of copper with $a = 4.755$ cm $b = 2.215$ cm. The cavity is filled with Polyethylene ($\epsilon_r = 2.25$, $\tan\delta = 0.0004$). If resonance is to occure at $f = 5$ GHz, find the required length 'd', and resulting Q for $l=1$ and $l=2$ resonant modes.

Ans :- Given

$a = 4.755$ cm , $b = 2.215$ cm , $\epsilon_r = 2.25$,

$\tan\delta = 0.0004$, $f_r = 5 \times 10^9$ Hz, $d = ?$

$Q = ?$ for $l = 1, 2$.

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

~~Where~~ Here $p = l$, $c = d$

for $l=1$, TE_{101}

$$5 \times 10^9 = \frac{1}{2\sqrt{\epsilon_r}\sqrt{\mu_0\epsilon_0}} \cdot \sqrt{\left(\frac{1}{a}\right)^2 + 0 + \left(\frac{1}{d}\right)^2}$$

$$\Rightarrow 5 \times 10^{10} = \frac{3 \times 10^{10}}{2\sqrt{2.25}} \sqrt{\frac{1}{(4.755 \times 10^{-2})^2} + \frac{1}{d^2}}$$

$$\Rightarrow \sqrt{\frac{10^4}{4.755^2} + \frac{1}{d^2}} = 50$$

$$\Rightarrow \frac{1}{d^2} = 2500 - 442.28$$

$$\Rightarrow \boxed{d = 2.20 \text{ cm}}$$

$l = 2$, TE_{102}

$$f_r = \frac{1}{2\sqrt{\epsilon_r}\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{2}{d}\right)^2}$$

$$\Rightarrow 5\times10^9 = \frac{3\times10^8}{2\sqrt{2.25}} \sqrt{\frac{1}{a^2} + \left(\frac{2}{d}\right)^2}$$

$$\Rightarrow \frac{2^2}{d^2} = 2500 - 442.28 = 2057.72$$

$$\Rightarrow \boxed{d = 4.40} \text{ cm.}$$

$\therefore$ d (at $l=2$) is double of that d (at $l=1$).

Quality factor (Q)

$$Q_d = \frac{1}{\tan\delta} = \frac{1}{0.0004} = 2500$$

Q_c ~~for $l=1$~~

$$Q_c = \frac{(kad)^3 b\eta}{2\pi^2 R_s} \cdot \frac{1}{(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3)}$$

$$k = \omega\sqrt{\mu\epsilon} = 2\pi f_r \sqrt{\mu_0 \epsilon_0}\sqrt{\epsilon_r}$$

$$= 2\pi \times 5\times10^{9} \frac{1}{3\times10^8} \times\sqrt{2.25} = 157.08 \text{ m}^{-1}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{1}{\sqrt{\epsilon_r}}$$

$$= \frac{377}{\sqrt{2.25}} = 251.33\ \Omega.$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\sigma_{cu} = 5.813 \times 10^7\ \frac{S}{m}$$

$$R_s = \sqrt{\frac{2\pi \times 5 \times 10^9 \times 4\pi \times 10^{-7}}{2 \times 5.813 \times 10^7}} = 1.84 \times 10^{-2}\ \Omega.$$

<u>For $l = 1$</u>, $d = 2.20\ \text{cm}$.

$$Q_c = \left[\frac{(157.08 \times 4.755 \times 10^{-2} \times 2.20 \times 10^{-2})^3 \times 2.215 \times 10^{-2} \times 251.33}{2\pi^2 \times 1.84 \times 10^{-2}}\right] \times$$

$$\left[\frac{1}{2 \cdot 1^2 \cdot (4.755 \times 10^{-2})^3 \cdot (2.215 \times 10^{-2}) + 2 \cdot (2.215 \times 10^{-2}) \cdot (2.20 \times 10^{-2})^3 + 1^2 \cdot (4.755 \times 10^{-2})^3 \cdot (2.20 \times 10^{-2}) + (4.755 \times 10^{-2})(2.20 \times 10^{-2})^3}\right]$$

$$Q_c = 8389$$

<u>Similarly</u>, $l = 2$, $d = 4.40\ \text{cm}$

$$Q_c = 11{,}882$$

Total Q, for $l=1$

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_D}\right)^{-1}$$

$$Q = \left(\frac{1}{8389} + \frac{1}{\cancel{11,882}\ 2500}\right)^{-1}$$

$$\boxed{Q = 1926.}$$

Total Q, for $l=2$

$$Q = \left(\frac{1}{11,882} + \frac{1}{2500}\right)^{-1}$$

$$\boxed{Q = 2065.42}$$

2) BPUT-2012

A rectangular cavity resonator has dimensions of $a = 5$ cm, $b = 2$ cm, $d = 15$ cm. Compute the

(i) Resonant freq of dominant mode for an air filled cavity.

(ii) The resonant freq of the dominant mode for a dielectric filled cavity of relative permittivity 2.56

Ans :- Given

$a = 5cm, \quad b = 2cm, \quad d = 15 cm.$

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

Dominant mode TE_{101}

'f_r' for air filled cavity

$$f_r = \frac{3\times10^8}{2} \sqrt{\frac{1}{(5\times10^{-2})^2} + 0 + \frac{1}{(15\times10^{-2})^2}}$$

$$\boxed{f_r = 3.16\ GHz.}$$

'f_r' for dielectric filled cavity

$$f_r = \frac{1}{2\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r}} \sqrt{\frac{1}{(5\times10^{-2})^2} + \frac{1}{(15\times10^{-2})^2}}$$

$$= \frac{3.16\ GHz}{\sqrt{\epsilon_r}}$$ [3.16 GHz as derived for air filled cavity]

$$= \frac{3.16}{\sqrt{2.56}}\ GHz$$

$$\boxed{f_r = 1.975\ GHz}$$

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