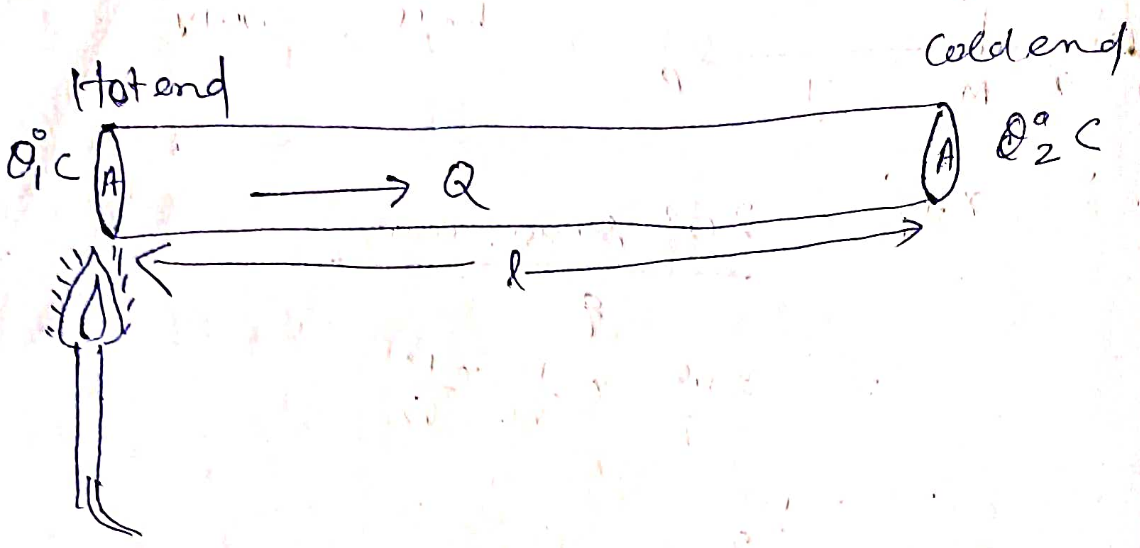
Thermal conductivity



During steady state, the amount of heat flowing through a bar or rod heated at one end is experimentally found to be directly proportional to the

(i) Area of cross section of the bar or rod,

(ii) Difference of temp between the hot end and the cold ~~end~~ end.

(iii) Time for which the flow of heat is consider

iv Inversely proportional to the length of the bar or rod

Thus, $Q \propto A$, when $(\theta_1-\theta_2)$, t, l are kept constants.

$Q \propto (\theta_1-\theta_2)$, when A, t, l are kept constants

$Q \propto t$, when A, $(\theta_1-\theta_2)$, l are kept constants.

$Q \propto \frac{1}{l}$, when $A, (\theta_1-\theta_2), t$ are kept constants.

Combining these four variations, we have

$Q \propto \frac{A.(\theta_1-\theta_2).t}{l}$, when all the quantities vary

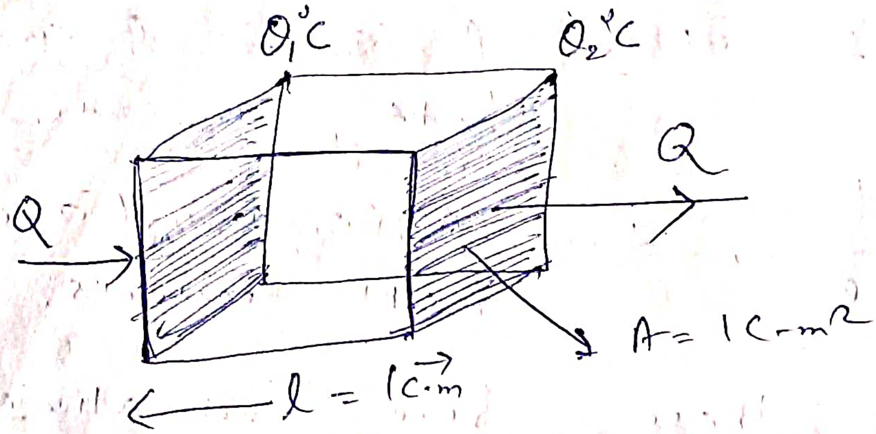
$$\boxed{\Rightarrow Q = K . \frac{A(\theta_1-\theta_2).t}{l}}$$

where K is a constant called ~~therm~~ co-efficient of thermal conduction or thermal conductivity.

Def^n of K

$Q = K$, when $A = 1$ sq unit, $(\theta_1-\theta_2) = 1\,°C$
$t = 1$ sec, $l = 1$ unit

Thus thermal conductivity can be defined as the amount of heat flowing per second between the opposite faces of a unit cube made out of the substance, provide the two faces are kept at a difference of temp. of 1 unit.



Units of K

1. C.G.S unit = $K = \frac{Ql}{A.(\theta_1-\theta_2).t}$

$$= \frac{\text{Calorie. Cm}}{\text{C.m}^2 . {}^\circ C . \text{Sec}}$$

$$= \text{Cal Cm}^{-1} . {}^\circ C^{-1} . \text{sec}^{-1}$$

2. M.K.S unit $= K = \text{Kilocalorie} . m^{-1} . {}^\circ C^{-1} . \text{sec}^{-1}$

3. F.P.S unit (~~BTU~~) = $\text{Btu} . ft^{-1} . {}^\circ F^{-1} . \text{sec}^{-1}$

Dimension $= [K] = \frac{[ML^2T^{-2}].[L]}{[L^2][K][T]}$

$$= \boxed{M L T^{-3} K^{-1}}$$

Problem (odd one)

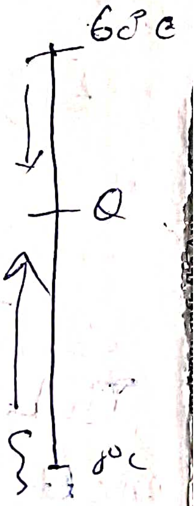
Q. Calculate the temp of the mixture when 10 gm of water at 60°C is mixed with 10gm of ice at 0°C.

10. Calculate the temp of the mixture when 1 gm of ~~ice~~ ice at 0°C is mixed with 1 gm of steam at 100°C

Ans :-

~~Correct Ans 9. Let~~

9. Heat lost by water = Heat gained by ice



$\Rightarrow 10\times1\times(60-\theta)=10\times80+10\times1\times(\theta-0)$

$\Rightarrow 600\theta = 800+10\theta$

~~$\Rightarrow 100+$~~

$\Rightarrow 600-10\theta = 800-1100$

$\Rightarrow 20\theta = -200$

$\Rightarrow \theta = -10°C$

Correct Ans :- Let first of all check wheather all the heat supplied by the warm water is sufficient to melt all the ice or not.

Even if the temp of the warm water drops from 60°C to 0°C, the maxm amount of heat that will be available

$= m s \Delta\theta$

$= ~~m~~ 10 . (1) . (60-0)$

$= 600$ cal

but, the amount of heat necessary to convent 10 gm of ice at 0°C into 10 gm of water at 0°C;

$= m . \cancel{s} L_f$

$= 10 \times 80$

$= 800$ cal.

Thus the amount of ice that melts into water $\frac{600}{800} = \frac{3}{4}$ part

of 10 gm. = 7.5 gm

Hence the temp of the mixture will be 0°C when ice and water will [illegible] Co-exist. The mixture contains 17.5 gm of water at 0°C and 2.5 gm of ice at 0°C.

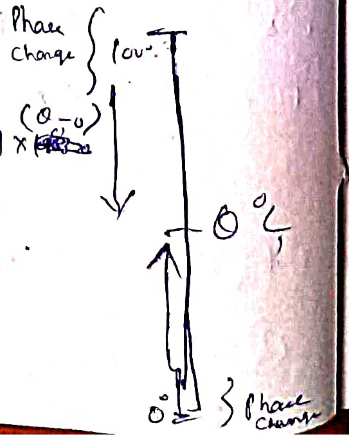
10. Heat lost by ~~ice~~ steam = Heat gained by ice

$\Rightarrow 1 \times 540 + 1 \times 1 \times (100° - \theta) = 1 \times 80 + 1 \times 1 \times (\theta - 0)$

$\Rightarrow 540 + 100 - \theta = 80 + \theta$

$\Rightarrow 2\theta = 540 + 100 - 80$

$= 560$



$\Rightarrow \theta = \frac{560}{2} = 280^\circ C$

Let's first calculate the amount of heat necessary to raise the temp of ice from 0°C to 100°C

$$Q = 1 \times 80 + 1 \times 1 (100-0)$$
$$= 80 + 100$$
$$= 180 \text{ Cal}$$

This amount of heat can be supplied by steam by condensing only $\frac{1}{3}$ gm. i.e. $\frac{180}{540} = \frac{1}{3}$

The rest $\frac{2}{3}$ gm of steam remains as steam.

Therefore the temp of the mixture will be 100°C when $\frac{2}{3}$ gm of steam at 100°C and $\overset{(1+(\frac{1}{3}))}{\frac{4}{3}}$ gm of water at 100°C will co-exist.

Kinetic theory of gases

This is a model for underesting the behaviour of gas molecules. It is based on certain simple assumptions which is called postulates. An expression for the pressure due to the gas on the sides of of the container will be found out. From this expression for pressure all the well know laws for gases can be derived. These indirectly support the postulates of the kinetic theory of gases.

imp Postulates of kinetic theory of gases

imp 1. A gas consists of a large number of identical particles called atoms or molecules. These are identical in all respects like mass, diameter, shape etc.

2) imp These molecules or atoms are constantly in motion with all possible velocities in all possible directions.

imp 3. The molecules or atoms behave as perfectly elastic spheres. That is there is no loss of kinetic energy during collision.

4) The molecules or atoms are very small in size and their total volume is negligible compared to the volume of the container. Hence they don't they collide among themselves, however they collide with the walls of the container and give rise to pressure.

5. The molecules or atoms don't attract each other.

6. Between two collisions ~~the~~ a molecules moves in a straight line obeying Newton's second law of motion.

7. Due to ([illegible]) random motion of the molecules the density of a gas at any point inside the container remains constant.

8. The time during which a molecule remains in contact with the wall is negligible com~~pound~~pared to the time taken by it to move between the two walls.

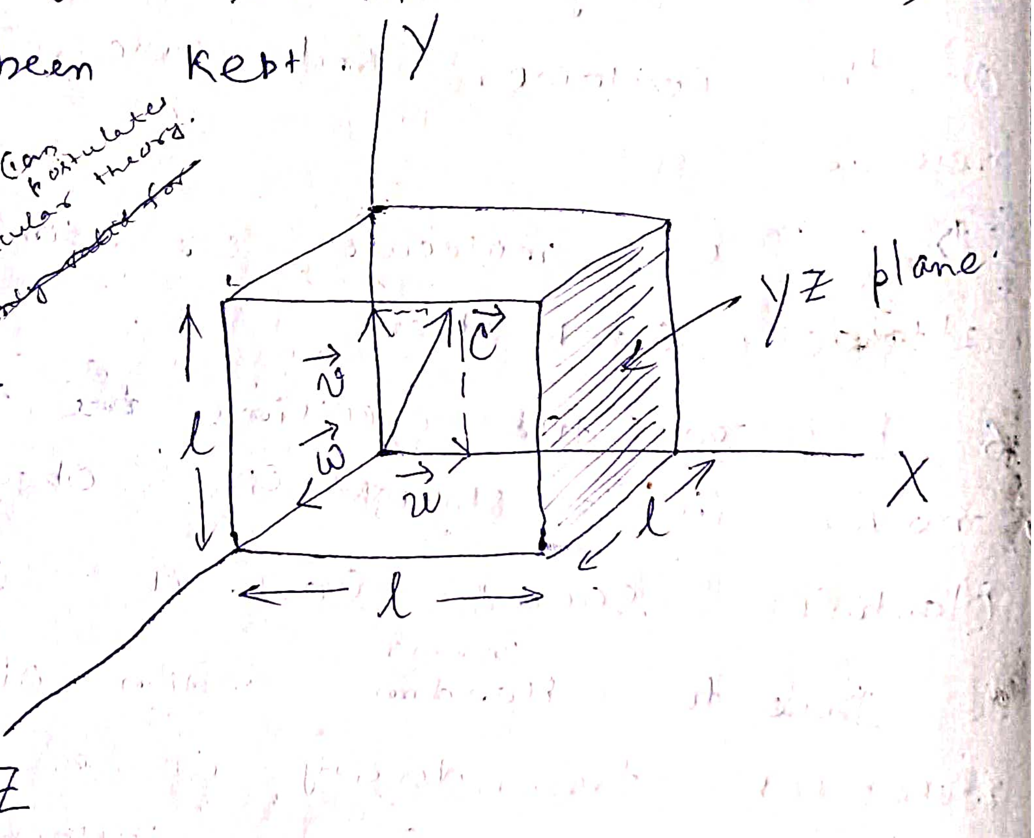
9. The temp of a gas is dependent on the velocity of the gas molecules.

Imp

Derivation of an expression for the pressure due to a gas

For simplicity of calculation, let us consider a cubical box of side length l in which the gas has been kept.

Introduction → Kinetic gas eqn can be derived from postulates of kinetic molecular theory. ~~[illegible] is only [illegible] for ideal gas~~



Let there be N number of gas molecules which are moving with velocities $\vec{C_1}, \vec{C_2}, \vec{C_3}, ---- \vec{C_N}$.

One such molecule with velocity $\vec{c}$ has been shown in the diagram. Its rectangular components are $\vec{u}, \vec{v}$, and $\vec{w}$ along X, Y, Z axes respectively.

$$\vec{C} = \hat{i}u + \hat{j}v + \hat{k}w$$

$$\Rightarrow C^2 = u^2 + v^2 + w^2 \quad \text{------} (i)$$

A molecule can be imagine to be moving with a velocity $\vec{u}$ along the x direction to strike the YZ plane.

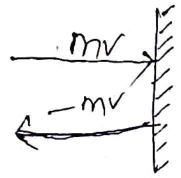
The momentum with which it strikes the YZ plane

$$= m\vec{u} = mu\hat{i}.$$

Since the collision is elastic, it returns back with a velocity $-\vec{u}$ and final momentum

$$= -m\vec{u}$$

$$= mu(-\hat{i})$$



See the modified ans in chemistry note

$$\Delta\vec{P} = \text{Change in linear momentum}$$

$$= \vec{P_i} - \vec{P_f}$$

$$= mu\hat{i} - mu(-\hat{i})$$

$$= 2mu\hat{i}$$

$$\Delta p = \text{magnitude of linear momentum}$$

$$= |\Delta P|$$

$$= 2mu|\hat{i}|$$

$$= 2mu$$

Time taken by molecule to strike the same wall again $= \frac{2l}{u}$

Force imparted to the wall during One collision $= \frac{\Delta p}{\Delta t}$

$$= \frac{2mu}{\frac{2l}{u}}$$

$$= \frac{mu^2}{l}$$

If the components of velocities of other molecules along the X-axis be $u_1, u_2, u_3, \text{------}, u_N$ then, the total force on the YZ plane due to all the molecules

$$= \frac{m u_1^2}{l} + \frac{mu_2^2}{l} + \frac{mu_3^2}{l} + \text{----} + \frac{mu_N^2}{l}$$

$$F_{total} = \frac{m}{l}\left(u_1^2 + u_2^2 + u_3^2 + \text{----} + u_N^2\right) \text{ ——(ii)}$$

Defining mean square average as the mean of the sum of the squares of diffrent velocities, we have

$$\overline{u}^2 = \frac{u_1^2 + u_2^2 + \text{------} u_N^2}{N}$$

$$\Rightarrow N.\overline{u}^2 = u_1^2 + u_2^2 + \text{------} + u_N^2 \text{ -(iii)}$$

Using eq^n (iii) in eq^n (ii), we get

$$F_{total} = \frac{m}{l}\left(N\,\overline{u^2}\right)$$

Pressure on the yz plane

$$P = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{F_{total}}{l^2}$$

$$= \frac{\frac{m}{l}(N\overline{u^2})}{l^2}$$

$$= \frac{mN\overline{u^2}}{l^3}$$

$$= \frac{mN\overline{u^2}}{V} \quad \text{———} \quad (iv)$$

where, $V = l^3$ = volume of the Cubical container.

Averaging all the terms of eqn (i) we get,

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

But $\overline{u^2} = \overline{v^2} = \overline{w^2}$

because large number of molecules are there moving at random.

Hence $\overline{c^2} = 3\overline{u^2}$

$$\Rightarrow \overline{u^2} = \frac{1}{3}\overline{c^2} \quad \text{———} \quad (v)$$

Using eqn (v) in eqn (iv), we get

$$P = \frac{mN\overline{c^2}}{3V} \quad \text{———} \quad (vi)$$

Defining $n = \frac{N}{V}$ = Number of molecules per unit volume, We have

$$P = \frac{1}{3} mn\overline{c^2} \text{ --- } \widehat{vii}$$

But $mn = \rho$ = density of the gas

[illegible] ($m\left(\frac{N}{V}\right)$ = mass of 1 number of molecule / volume)

$$\therefore P = \frac{1}{3}\rho\overline{c^2} \text{ --- } \widehat{viii}$$

~~Defining root mean square velocity~~ or ~~(r.m.s) as the square root of the mean~~ square velocity.

~~As 1~~

Defining root mean square velocity or (r.m.s) as the square root of the mean square velocity

$$\text{i.e.} = C_{rms} = \sqrt{\overline{C^2}} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 \cdots + c_N^2}{N}}$$

$$\Rightarrow C_{rms}^2 = \overline{C^2} \text{ --- } \widehat{ix}$$

using eqn $\widehat{ix}$ $\widehat{vi}$, $\widehat{vii}$, $\widehat{viii}$ we get

$$P = \frac{mN\,C_{rms}^2}{3V} \text{ --- } \widehat{x}$$

$$P = \frac{1}{3} mn\,C_{rms}^2 \text{ --- } \widehat{xi}$$

$$P = \frac{1}{3}\rho\,C_{rms}^2 \text{ --- } \widehat{xii}$$

Conclusion → This is the fundamental eqn of Kinetic theory of gas & knowns as kinetic eqn, Since it is derived from postulates of Kinetic theory, it is only valid for ideal gas.

Problem : Calculate the r.m.s velocity of Oxygen molecules kept under standard atomspheric pressure. Density of oxygen gas = 0.00.13 gm/c

Soln : From Kinetic theory of gases, we know that,

$$P = \frac{1}{3}\rho C_{rms}^2$$

$$\Rightarrow C_{rms}^2 = \frac{3P}{\rho}$$

$$\Rightarrow C_{rms} = \sqrt{\frac{3P}{\rho}}$$

Here P = pressure exerted by a mercury column of height 76 Cm

$= hdg$

$= 76 \times 13.6 \times 980$ dyne/C.m²

$$\therefore C_{rms} = \sqrt{\frac{3 \times 76 \times 13.6 \times 980}{.0013}} \text{ Cm/sec}$$

$$= 483.47 \times 10^2 \text{ Cm/sec}$$

$$= 483.47 \text{ m/sec}$$

Page 207 : 2, 3, 6, 8

Ans : 3 → 1.186×10^{-23} cc

6. Hints : From Charles law we know that

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

$$\Rightarrow \frac{m\rho_1 T_1}{m\rho_2 T_2} \quad \frac{T_1\rho_1}{T_2\rho_2} \Rightarrow \frac{m}{\rho_1 T_1} = \frac{m}{\rho_2 T_2}$$

$$\Rightarrow T_1\rho_1 = T_2\rho_2$$ [illegible]

~~[illegible]~~

College Physics 267 page 2, 3, 6, 8

2. ✓ Formula of Ethane = C_2H_6

Gm Molecular mass = 24+6 = 30 gm.

30 gm contains $= 6.023\times10^{23}$ molecules.

1 gm " $= \dfrac{6.023\times10^{23}}{30}$ "

1 mg " $= \dfrac{6.023\times10^{23}}{30} \times \dfrac{1}{1000}$

$= \dfrac{6.023}{3} \times \dfrac{10^{23}}{10^{4}}$

$= 2\times10^{19}$ molecules.

3. ✓ Copper has atomic mass = 63.6

Specific gravity = 8.9

Volume $= \dfrac{\text{mass}}{\text{S. gravity}} = \dfrac{63.6}{8.9} = \dfrac{636}{89} = 7.146$ c.c

6.023×10^{23} ~~[illegible]~~ atoms occupy volume = 7.146 c.c

1 " " $" = \dfrac{7.146}{6.023\times10^{23}}$

$= 1.186\times10^{-23}$ c.c/~~mole~~ atom

~~$6.023\times10^{23}\times$~~

4/ gm molecular ~~mass of~~ nitrogen = 28 gr.

In STP

22,400 ml of N_2 carry = 6.023×10^{23} molecule

1 ml of N_2 carry

$$= \frac{6.023 \times 10^{23}}{224 \times 10^{2}}$$

$$= .0268 \times 10^{21}$$

$$= 2.68 \times 10^{19} \text{ molecular.}$$

→ Here $\rho_1 = .089$ gm/litre = .000089 gm/c.c

6. From Charle's law Know

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

$$\Rightarrow \frac{m}{\rho_1 T_1} = \frac{m}{\rho_2 T_2}.$$

$$\Rightarrow \rho_1 T_1 = \rho_2 T_2$$

~~⇒ (0+273).~~

$$\Rightarrow (\overset{.000089}{\cancel{.0089}})(0+273) = \rho(20+273).$$

$$\Rightarrow \overset{.000089}{\cancel{.0089}} \times 273 = \rho_2\, 293$$

$$\Rightarrow \rho_2 = \frac{.000089 \times 273}{293}$$

~~60.0082.92~~

$= .00008292$ gm/c.c

P = Pressure, exerted by a mercury column or height = 76 c.m

= hdg

= 70 × 13.6 × 980 dyne/cm 2

P = 932960

~~$C_{rms} = \sqrt{\frac{3P}{\rho}}$~~

$$P = \frac{1}{3} \rho C_{rms}^2$$

$$\Rightarrow C_{rms}^2 = \frac{3P}{\rho}$$

$$\Rightarrow C_{rms} = \sqrt{\frac{3P}{\rho}}$$

$$= \sqrt{\frac{3 \times 932960}{.00008292}} = \sqrt{\frac{2798880}{.00008292}}$$

$$= \sqrt{3.375 \times 10^{10}} = 1.837 \times 10^{5}\ \text{cm/s}$$

~~$= 33.75 \times 10^{10}$~~

8. $T = 0°C$

$T = 273K$, $P = 76$ c.m

density of oxygen = 1.429 g/litre

~~$= .00014$~~

$= .001429$ gm/c.c

$P = h.dg$

$= 76 \times \overset{13.6}{\cancel{.001429}} \times 980$

~~$= 106.43192$~~ $= 1012928$

$$C_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times \overset{1012928}{\cancel{106.43192}}}{.001429}}$$

~~$= \sqrt{\frac{319.29576}{.001429}}$~~

~~$= \sqrt{223440} = 472.69$ c.m/c.c~~

~~$= 4.7269 \times 10^{4}$~~

$$= \sqrt{\frac{3038784}{.001429}} = \sqrt{2126510847}$$

$= 46114.106$ cm/c.c

$= 4.6\cancel{\times}14 \times 10^4$ cm/c.c (Ans)

Doubt : root mean square or

here. $C_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{c_1^2 + c_2^2 + \dots + c_N^2}{N}}$

Problems on thermal Conductivity

4. Q: A spherical glass of 14 c.m radius is filled with water at 100°C and immersed in melting ice. At what rate must heat be generated inside the globe, if the temp of water in it is to be maintained constant?

Thickness of glass = 1 mm, K of glass = 0.0025

Ans: 6160 cal/sec , Hints: $A = 4\pi r^2$

5. Suppose ice 10 c.m thick to have already been formed on a pond and the air is at −5°C. How long will it take for the next 1 m.m of ice to form?

K of ice = .004 and L_f = 80 cal/gm.

Ans: 30 min, 43.17 sec.

ρ of ice = .917 gm/c.c.

Ans :

4. Length of the glass = 1 m.m = .1 C.m.

K of glass = .0025

Area of the glass $A = 4\pi r^2$

Radius of the glass = 14 c.m

$$\text{Area} = 4 \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 14 = 2464 \text{ Cm}^2$$

Temp of the ice = $0°C = \theta_2$

Temp of the water inside the glass

$= 100°C = \theta_1$

$\therefore \theta_1 - \theta_2 = 100°C - 0°C = 100°C$

Rate at which heat is generated

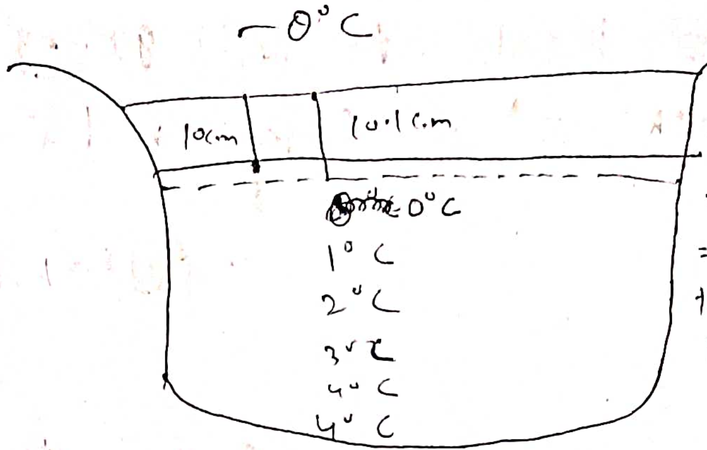
$= \frac{Q}{t}$

From the formula

$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$, we get

$$\frac{Q}{t} = \frac{.0025 \times 2464 \times 100}{.1} = 6160 \text{ cal/sec}$$

$\therefore$ 6160 Cal/sec heat ~~to~~ must be generated inside the globe, if the temp of water on it to be maintained constant, 5.



$$\frac{10+10\cdot1}{2}$$

= 10·05 c.m

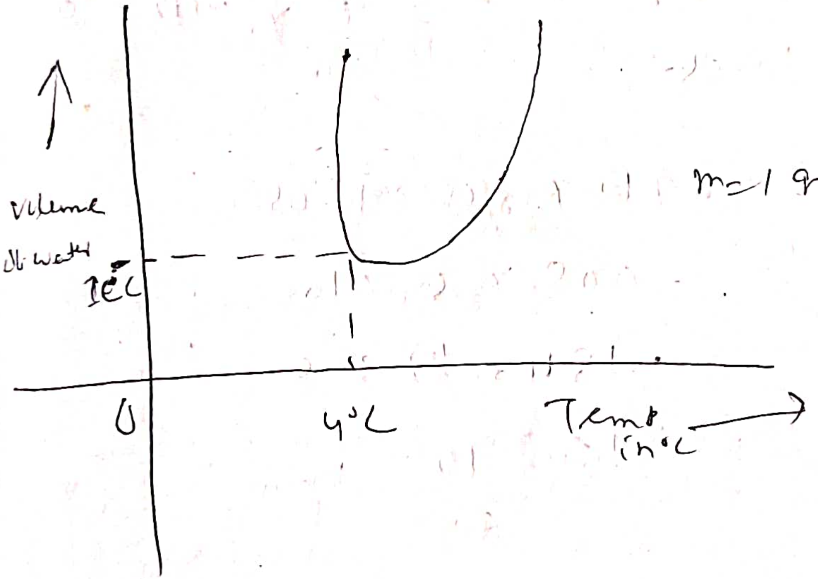
= Average thickness of the ice layer.

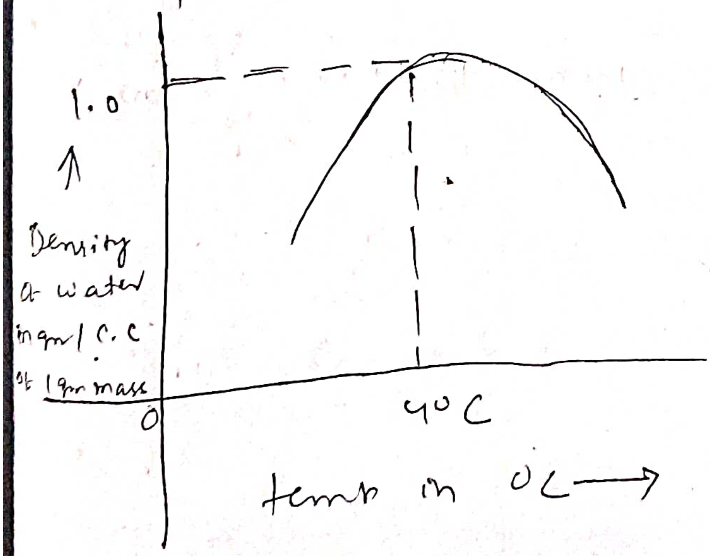
Let the surface area of the pond be A sq c.m.

The volume of ice layer of thickness

1 m.m = A × 0.1 C.C

Mass of the ice = volume × density = $\frac{A}{10}$ × ·917 gm





Anomalous Expension of water

Aquatic animals can survive during sever winter ∴ temp at the bottom of lake become, 4°C

Amount of heat to be given out by this much of ice layer = $m L_f = \frac{A}{10} \times .917 \times 80$ ca

$= Q$ (say)

Thus

Q = Amount of heat flowing through the ice ~~lay~~ layer of average thickness 10.05 c.m to the surrounding medium

$$= \frac{KA \{0 - (-5)\} t}{~~tro~~\ 10.05}$$

Thus $$\frac{0.008 \times A \times 5 \times t}{10.05} = \frac{A}{10} \times .917 \times 80$$

$$\Rightarrow t = \frac{.917 \times 80 \times 10.05}{.008 \times 5 \times 10}$$

$= 1843.17$ sec

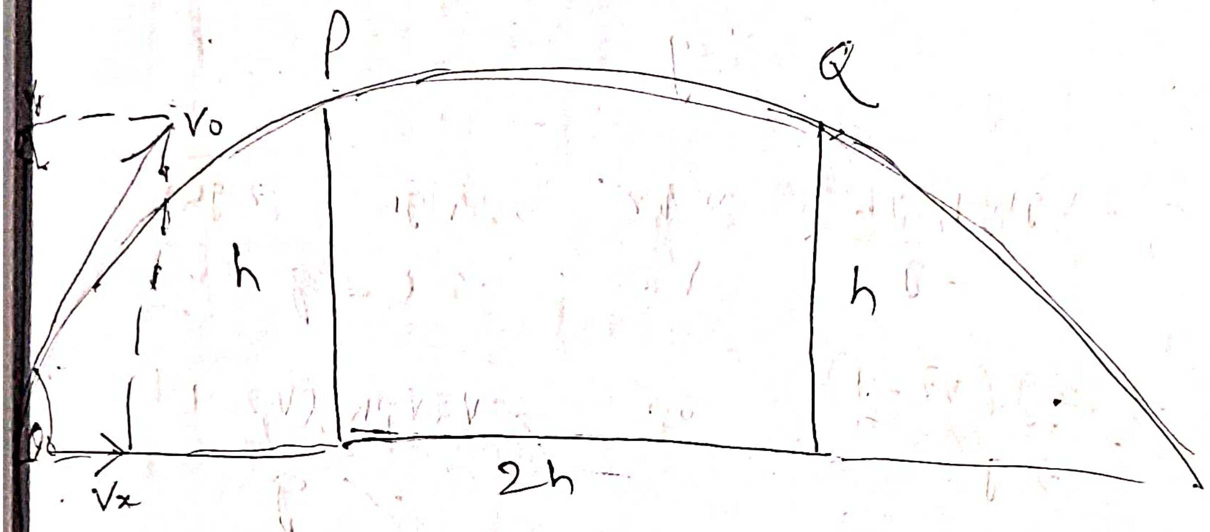
$= \frac{1843.17}{60}$ min

2

$\therefore t = 30$ min 43.17 sec (Ans.)

2000 Annual

7. A particle is projected with velocity $2\sqrt{gh}$ so that it just clears two walls of equal height h which are at a distance $2h$ from each other. Prove that the time of flight between the two walls is $2\sqrt{h/g}$.



$V_0 = 2\sqrt{gh}$, $(v_y)_2 = v_0 \sin\theta_0 = 2\sqrt{gh}\sin\theta_0$

Distance between the walls $= 2h$.

Time taken to Distance covered OPQ we have to use the formula

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow h = 2\sqrt{gh}\,t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \frac{1}{2}(-g)t^2 + 2\sqrt{gh}\,t - h = 0$$

$$t = \frac{-2\sqrt{gh} \pm \sqrt{(2\sqrt{gh})^2 - 4\cdot\left(\frac{-g}{2}\right)\cdot(-h)}}{2\cdot\left(\frac{-g}{2}\right)}$$

$$= \frac{-2\sqrt{gh} \pm \sqrt{4gh - 2gh}}{-g}$$

$$= \frac{-2\sqrt{gh} \pm \sqrt{2}\sqrt{gh}}{-g}$$

$$t = \frac{-2\sqrt{gh} + \sqrt{2gh}}{-g} \quad \text{or} \quad \frac{-2\sqrt{gh} - \sqrt{2gh}}{-g}$$

$$= \frac{\cancel{-}\sqrt{2}\sqrt{gh}(\sqrt{2}-1)}{\cancel{-}g} \quad \text{or} \quad \frac{\cancel{-}\sqrt{2}\sqrt{gh}(\sqrt{2}+1)}{\cancel{-}g}$$

$$= \frac{\sqrt{2gh}(\sqrt{2}-1)}{g} \quad \text{or} \quad \frac{\sqrt{2gh}(\sqrt{3}+1)}{g}$$

$$= \frac{\sqrt{2h}(\sqrt{2}-1)}{\sqrt{g}} \quad \text{or} \quad \frac{\sqrt{2h}(\sqrt{2}+1)}{\sqrt{g}}$$

Time taken to cover vertical distance, using the formula

$$S = ut + \frac{1}{2}at^2$$ we get

$$\Rightarrow h = 2\sqrt{gh}\sin\theta_0 t + \frac{1}{2}(-g)t^2 \quad \left(\because u = v_0\sin\theta_0 = 2\sqrt{gh}\sin\theta_0\right)$$

$$\Rightarrow 2h = 4\sqrt{gh}\sin\theta_0 . t - gt^2$$

$$\Rightarrow gt^2 - 4\sqrt{gh}\sin\theta_0 . t + 2h = 0$$

$$\xcancel{t = \frac{-(-4\sqrt{gh}) \pm \sqrt{(-4\sqrt{gh})^2 - 4.(g).2h}}{2.(g)}}$$

$$\xcancel{= \frac{4\sqrt{gh} \pm \sqrt{16gh - 8gh}}{2g}}$$

$$\xcancel{= \frac{4\sqrt{gh} \pm 2\sqrt{2gh}}{2g}}$$

$$\xcancel{= 2(\sqrt{gh} \pm \sqrt{2gh}}$$

$$t = \frac{-(-4\sqrt{gh}\sin\theta_0) \pm \sqrt{(-4\sqrt{gh}\sin\theta_0)^2 - 4.(g).2h}}{2.g}$$

$$= \frac{4\sqrt{gh}\sin\theta_0 \pm \sqrt{16gh\sin^2\theta_0 - 8gh}}{2g}$$

$$= \frac{2\sqrt{gh}\sin\theta_0 \pm \sqrt{4gh\sin^2\theta_0 - 2gh}}{g}$$

$$= 2\sqrt{\frac{h}{g}}\sin\theta_0 \pm \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}$$

∴ t_1 = time taken to reach the first wall

$$= 2\sqrt{\frac{h}{g}}\sin\theta_0 - \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}$$

t_2 := time taken to reach the second wall

$$= 2\sqrt{\frac{h}{g}}\sin\theta_0 + \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}$$

$$\Delta t = t_2 - t_1 = \left(2\sqrt{\frac{h}{g}}\sin\theta_0 + \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}\right) - \left(2\sqrt{\frac{h}{g}}\sin\theta_0 - \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}\right)$$

$$= \cancel{2\sqrt{\frac{h}{g}}\sin\theta_0} + \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2} - \cancel{2\sqrt{\frac{h}{g}}\sin\theta_0} + \sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2}$$

$$= 2\sqrt{\frac{h}{g}}\sqrt{4\sin^2\theta_0 - 2} \text{ ——— (i)}$$

Again $\Delta t = \frac{2h}{V_x} = \frac{2h}{V_0\cos\theta_0} = \frac{2h}{2\sqrt{gh}\cos\theta_0}$ (ii)

equating eqn (i) and (ii)

$$\cancel{2}\sqrt{\frac{h}{g}}\cdot\sqrt{4\sin^2\theta_0 - 2} = \frac{\cancel{2}h}{2\sqrt{gh}\cos\theta_0}$$

$$\Rightarrow \frac{\cancel{\sqrt{h}}}{\cancel{\sqrt{g}}}\sqrt{4\sin^2\theta_0 - 2} \times 2 \times \cancel{\sqrt{g}}\cdot\cancel{\sqrt{h}}\cdot\cos\theta_0 = \cancel{h}\cdot 1$$

$$\Rightarrow \sqrt{4\sin^2\theta_0 - 2} = \frac{1}{2\cos\theta_0}$$

$$\Rightarrow \left(\sqrt{4\sin^2\theta_0 - 2}\right)^2 = \left(\frac{1}{2\cos\theta_0}\right)^2$$

$$\Rightarrow 4\sin^2\theta - 2 = \frac{1}{4\cos^2\theta_0} = \frac{1}{4(1-\sin^2\theta_0)}$$

$$\Rightarrow 4\sin^2\theta_0 - 2 = \frac{1}{4-4\sin^2\theta_0}$$

$$\Rightarrow 16\sin^2\theta_0 - 8 - 16\sin^4\theta_0 + 8\sin^2\theta_0 - 1 = 0$$

$$\Rightarrow 16\sin^4\theta_0 - 24\sin^2\theta_0 + 9 = 0$$

$$\Rightarrow (4\sin^2\theta_0)^2 - 2\cdot(4\sin^2\theta_0)\cdot(3) + (3)^2 = 0$$

$$\Rightarrow (4\sin^2\theta_0 - 3)^2 = 0$$

$$\Rightarrow 4\sin^2\theta_0 - 3 = 0$$

$$\Rightarrow 4\sin^2\theta_0 = 3$$

$$\Rightarrow \sin^2\theta_0 = \frac{3}{4} \text{ ——————— (iii)}$$

Putting the value of $\sin^2\theta_0$ in eq^n (i)

we get

$$\Delta t = 2\sqrt{\frac{h}{g}}\,\sqrt{4\cdot\frac{3}{4} - 2}$$

$$= 2\sqrt{h/g}$$

∴ time ~~taken~~ of flight between the two walls $= 2\sqrt{h/g}$ (Proved)

(Relation with the c_{rms}) ↗ ([illegible])

Interpretation of temperature

Equation of State for an ideal gas is known to be

$$PV = n'RT \text{ ——— (i)}$$

where n' = number of moles of the gas.

R = Universal gas constant.

= 8.31 Joule . $mole^{-1}$. $°K^{-1}$

T = Absolute temp.

from kinetic theory of Gases, we know that

$$PV = \frac{1}{3} mNc^2 \text{ ——— (ii)}$$

where c stands for c_{rms}

Equating the R.H.S of eqn s (i) and (ii), we get

$$n'RT = \frac{1}{3} mNc^2 \text{ ——— (iii)}$$

1 mole of any gas contains Avogadro number (N_0) of molecules

∴ ~~dot~~ n' will contain = $n' N_0$ number of molecules.

$$\therefore N = n' N_0 \text{ ——— (iv)}$$

Using this express of N in eqn (iii) we get

$$n' RT = \frac{1}{3} m n' N_0 c^2$$

$$\Rightarrow RT = \frac{1}{3} m N_0 c^2$$

$$\Rightarrow mc^2 = \frac{3RT}{N_0} \text{ ——— (v)}$$

Defining Boltzmann constant (K) as

$$K = \frac{R}{N_0} = \frac{8.31 \text{ Joule. mol}^{-1}\, {}^{0}K^{-1}}{6.023 \times 10^{23}}$$

$$= 1.38 \times 10^{-23} \text{ Joule. mole}^{-1}\, {}^{0}K^{-1}$$

Thus $mc^2 = 3KT$ ——— (vi)

$$\Rightarrow c^2 = \frac{3KT}{m} = \text{constant} \cdot T$$

$$\Rightarrow c^2 \propto T$$

$$\Rightarrow c \propto \sqrt{T}$$

Thus, the r.m.s velocity of a gas molecule is directly proportional to the square root of the absolute temp of the gas

From eqn (vi) we get

$$\boxed{\frac{1}{2} mc^2 = \frac{3}{2} KT} \text{ ——— (vii)}$$

Thus the average kinetic energy of a gas molecule is directly proportional to the absolute temp of the gas.

Corollary -1

Two different gases kept at the same temp posses the same amount of kinetic energy.

i.e $\frac{1}{2} m_1 c_1^2 = \frac{1}{2} m_2 c_2^2 = \text{'}------$

$$= \frac{3}{2} KT$$

Corollary - 2

A relation between r.m.s velocity and molecular weight of a gas can be obtained

from the eqn

$$mc^2 = 3KT, \text{ we get}$$

$$c^2 = \frac{3KT}{m} = \frac{3RT}{m.N_o}$$

$$= \frac{3RT}{M}$$

where M = molecular weight of the gas

$$= mN_o$$

$$\boxed{\therefore c = \sqrt{\frac{3RT}{M}}}$$ ..

Problem

1 At which temp the r.m.s velocity of a gas molecule will be (a) doubled.

(b) reduced to half of the velocity at 0°C

Ans = 819°C, −204.75°C

---

1. R.m.s velocity of a gas molecule at 0°C (273 K).

$$C_1 = \sqrt{\frac{3.R.(273)}{M}} \quad \text{—(i)}$$

R.M.S velocity of a gas molecule at θ°C (θ+273 K)

$$C_2 = \sqrt{\frac{3.R.(\theta+273)}{M}} \quad \text{—(ii)}$$

According to question

$$\frac{C_2}{C_1} = 2$$

$$\Rightarrow \frac{\frac{\sqrt{3R(\theta+273)}}{\sqrt{\cancel{M}}}}{\frac{\sqrt{3R\,273}}{\sqrt{\cancel{M}}}} = 2$$

$$\Rightarrow \frac{\cancel{3R}(\theta+273)}{\cancel{3R}(273)} = 4$$

$$\Rightarrow \theta + 273 = 1092$$

$$\Rightarrow \theta = 1092 - 273 = 819^\circ C$$

Or

$$\frac{\cancel{C_1}}{C_2} =$$

$$C_2 = 2C_1$$

$$\Rightarrow \cancel{\sqrt{3}} \cdot \cancel{\sqrt{R}} \cdot \sqrt{T_1} = 2 \cdot \cancel{\sqrt{3}} \cdot \cancel{\sqrt{R}} \cdot \sqrt{T_1}$$

$$\Rightarrow T_2 = 4T_1$$

$$\Rightarrow T_2 = 4T_1 = 4 \times 273 = 1092$$

$$\Rightarrow \theta + 273 = 1092$$

$$\Rightarrow \theta = 819^\circ C$$

II According to question

$$\frac{C_2}{C_1} = \frac{1}{2}$$

$$\Rightarrow \frac{\frac{\sqrt{3R(\theta+273)}}{\cancel{\sqrt{M}}}}{\frac{\sqrt{3R\,273}}{\cancel{\sqrt{M}}}} \quad 2$$

$$\Rightarrow \frac{3\cancel{R}(\theta+273)}{3\cancel{R}\cdot 273} = \frac{1}{4}$$

$$\Rightarrow 4\theta + 1092 = 273$$

$$\Rightarrow 4\theta = 273 - 1092 = -819$$

$$\Rightarrow \theta = \frac{-819}{4} = -204.75^\circ C$$

14. C_{rms} of $H_2 = \frac{\sqrt{3RT}}{\sqrt{M}} = \sqrt{\frac{3R\cdot 300}{2}}$

ও

C_{rms} of $O_2 = \frac{\sqrt{3RT}}{\sqrt{M}} = \sqrt{\frac{3\cdot RT}{32}}$

According to question.

$$\sqrt{\frac{3R\,300}{2}} = \sqrt{\frac{3RT}{32}}$$

$$\Rightarrow \frac{3\cdot\cancel{R}\,300}{2} = \frac{3\cancel{R}T}{32}$$

$$\Rightarrow 900 \times 32 = 6T$$

$$\Rightarrow T = \frac{900\times 32}{6}$$

$$= \frac{28800}{6}$$

$$= 4800^\circ K$$

22.

Molecular weight of $NH_3 = 17$.

17 gm NH_3 contains N_0 molecule

1 gm of NH_3 " $\frac{N_0}{17}$ "

Average Kinetic energy of a single gas molecule

$$= \frac{1}{2}mc^2 = \frac{3}{2}KT$$

∴ Total energy of all the molecules present in 1 gm

$$= \frac{\cancel{N_0}}{17} \times \frac{3}{2}\frac{R}{\cancel{N_0}}.(27+273)$$

$$= \frac{3}{34}(8.31)(27+273)$$

$$= \frac{3 \times 8.31 \times 300}{34}$$

$= 219.97$ Joule

$= 220$ Joule.

Calculate the K.E of 1 gm of He at N.T.P. what will be the energy at 127°C. Ans : 850.7 Joule, 1246 Joule.

Ans : Atomic weight of He = 4

4 gm contains N_0 atoms.

1 gm contains $\frac{N_0}{4}$ atoms.

Average Kinetic energy of single molecule

$$= \frac{1}{2} mc^2 = \frac{3}{2} KT$$

$$= \frac{3}{2} \cdot \frac{R}{N_0} \cdot (0+273) \quad \left(\because N.T.P = 0°C = 273 K \right)$$

$$= \frac{R \; 273 \times 3}{2 N_0} = \frac{R \, 819}{2 N_0}$$

Total energy of all molecules

$$= \frac{\cancel{N_0}}{4} \times \frac{R \; 819}{2 \cancel{N_0}}$$

$$= \frac{R}{2} \times \frac{819}{4}$$

$$= \frac{8.31 \times 819}{8} = \frac{6805.89}{8} = 850.736 \text{ Joule}$$

Total Kinetic energy at 127°C :

$$= \frac{N_0}{4} \times \frac{3}{2} KT$$

$$= \cancel{\frac{N_0}{4}} \times \frac{3}{2} \times \frac{R}{\cancel{N_0}} T$$

$$= \frac{3 \times 8.31 \times (127+273)}{8}$$

$$= \frac{3 \times 8.31 \times \cancel{400}^{50}}{\cancel{8}} = 1246.5 \text{ Joule}$$

(proof)

Deduction of gas laws from Kinetic theory of gases.

From the kinetic theory of gases

$$P = \frac{1}{3V} m N c^2$$

we know the ~~$PV = RT$ for 1 mole gas.~~

$\frac{1}{2} m N c^2 = K.E. = \frac{3}{2} KT$

From the interpretation of temp

we got $mc^2 = 3KT$ (Derive it from book chemistry)

Thus $P = \frac{1}{3V} N \, 3KT$

$$= \frac{NKT}{V}$$

$$\Rightarrow \boxed{PV = NKT}$$

(i) Boyle's Law :

If the temp of the gas are be kept constant then PV = constant

∴ Thus $\cancel{P}V \propto \frac{1}{P}$ when T is kept constant

i.e ~~Pressure~~ Volume of gas is inversely proportional to the ~~volume~~ pressure. when temp is kept constant. This is called Boyle's law.

ii) Charle's law :-

From the relation $PV = NKT$,

we have $\frac{V}{T} = \frac{NK}{P}$

If the pressure of the gas be kept constant then $\frac{V}{T} =$ constant.

$\Rightarrow V \propto T$ when P is kept constant.

Thus volume of the gas is directly proportional to the absolute temp of the gas when pressure of the gas is kept constant. This ~~is~~ is called Charles law.

Regnault's law

From the relation $PV = NKT$, we have

$$\frac{P}{T} = \frac{NK}{V}$$

If volume of the gas be kept constant then $\frac{P}{T}$ = constant

$$\Rightarrow P \propto T \quad \text{when } V \text{ is kept constant}$$

i.e Pressure of a gas is directly proportional to the absolute temp when volume of the gas is kept constant. This is called Regnault's law

Avogadro's law :-

Let there be two different gases kept under the same pressure P, same temp T and same volume V.

From the relation

$$PV = NKT$$

, we have

for the first gas

$$P_1V_1 = N_1KT_1$$

and for the second gas

$$P_2V_2 = N_2KT_2$$

But $P_1 = P_2$, $V_1 = V_2$ and $T_1 = T_2$

⇒ $N_1 = N_2$

Thus two different gases kept under identical conditions of temp , pressure and volume contain equal number of molecules. This is called Avogadro's law.

Dalton's law of partial pressure:

Let there be a mixture of non- reacting gases like air. If the total number of molecules be N, pressure of the gas be V and Pressure be P, volume. Temp be T, then

$$PV = \cancel{NKT} \quad NKT$$

$$= (N_1 + N_2 + N_3 + \text{-----})KT$$

where N_1, N_2, N_3, ---- are molecule of first kind, 2nd kind, 3rd kind etc.

Thus $$p = \frac{N_1 KT}{V} + \frac{N_2 KT}{V} + \frac{N_3 KT}{V} + \text{----}$$

$$= P_1 + P_2 + P_3 + \text{-----}$$

where P_1, P_2, P_3 are the partial pressure of the component gases. i.e the pressure exerted by a component gas when it is allowed to occupy the entire volume. Thus the pressure of a mixture of a gas is equal to the sum of the partial pressures of the component gases. This is called Dalton's law of partial pressure.

Graham's law of diffusion

It is a common experience to find that lighter gas molecules diffuse much quicker compared to heavier gas molecules. This can be quantitatively shown from kinetic theory of gases in the form of graham's law of diffusion.

From kinetic theory of gases the expression for pressure was found to be

$$P = \frac{1}{3} \rho c^2$$

For the first gas

$$P_1 = \frac{1}{3} \rho_1 c_1^2$$

and $P_2 = \frac{1}{3} \rho_2 c_2^2$ for the second gas.

But $P_1 = P_2$ and $T_1 = T_2$

So that the two gases will be kept under the same condition of temp and pressure.

$$\therefore \frac{1}{3}\rho_1 C_1^2 = \frac{1}{3}\rho_2 C_2^2$$

$$\Rightarrow \frac{C_1^2}{C_2^2} = \frac{\rho_2}{\rho_1}$$

$$\Rightarrow \frac{C_1}{C_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \frac{\frac{K}{\sqrt{\rho_1}}}{\frac{K}{\sqrt{\rho_2}}}$$

Where K is some constant.

$$\therefore C_1 = \frac{K}{\sqrt{\rho_1}}$$

$$C_2 = \frac{K}{\sqrt{\rho_2}}$$

Thus $C_1 \propto \frac{1}{\sqrt{\rho_1}}$

and $C_2 \propto \frac{1}{\sqrt{\rho_2}}$

i.e The rate of diffusion of a gas molecule is directly proportional to the r.m.s velocity and inversely proportional to the square root of density of gas.

i.e $r_1 \propto c_1 \propto \frac{1}{\sqrt{\rho_1}}$

and $r_2 \propto c_2 \propto \frac{1}{\sqrt{\rho_2}}$

This is called graham's law of diffusion.

Degrees of freedoms.

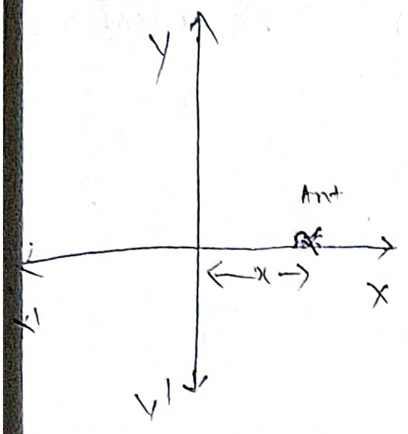
The number of independent (quantities) parameters necessary to specify the position (moving, changing position) and configuration of a dynamical system is called number of degrees of freedom.

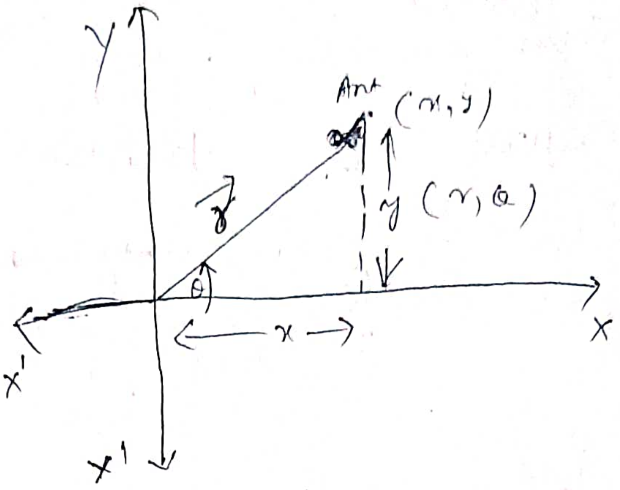
Ex:- 1)

An ant ~~rest~~ restricted to move on a straight wire or line requires only one co-ordinate to have its position specified. Hence, the number of degrees of freedom=1 (restricted)

Ex-2: An ant constrained to move on a plane requires two co-ordinates (x,y) or (r,θ)

for its position to be specified. Hence number of degrees of freedom = 2





Ex-3 To locate the position of a fly (মাছি) in space or an atom or molecule, we require three co-ordinates. (x, y, z) or (r, θ, ϕ)

Therefore, the number of degrees of freedom = 3

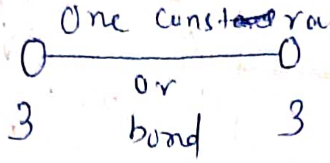
~~2. Def^n of deg. Freedom~~

Ex-4

A diatomic gas molecule like H_2, O_2, N_2, HCl, NO, CO, etc consists of two atoms which are bound together by a strong bond. Therefore, the number of degrees of freedom $= (3 \times 2) - 1$

$= 5$

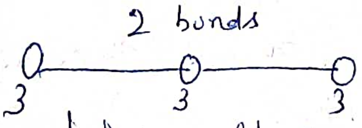
Out of these five, three are (linear) translational and two are rotational degrees of freedom.



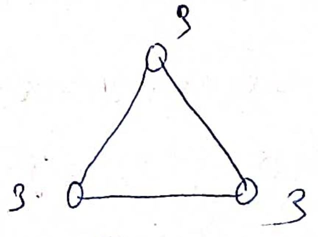
Ex-5

A tri atomic gas molecule like CO_2, NO_2, O_3, H_2S, H_2O. etc Consists of 3 atoms in it. If they will be arranged as a linear chain then there will be two bonds and the number of degrees of freedom will be $(3\times3)-2 = 7$

B · If they will be arranged at the vertices of a triangle then there will be three bonds. and the number of degrees of freedom will be $(3\times3)-3 = 6$



Linear Chain



Three bonds.

In general, for N particles with K constraints, The number of degrees of freedom

$= 3 \times N - K$

~~Lw~~ Law of equipartion of energy

Statement : The total energy of a system is equally divided among all the degrees of freedom and the energy per degree of freedom is equal to $\frac{1}{2} KT$.

Ex: From kinetic theory of gases, we have seen that the energy of a single gas molecule $= \frac{3}{2} KT$.

The number of degrees of freedom of a gas molecule $= 3$ because rotational and vibrational degrees of freedom are not consider.

Thus, energy per degree of freedom $= \frac{\frac{3}{2} KT}{3} = \frac{1}{2} KT$

Specific heats of a gas -:

A gas has two specific heats Namely (i) Specific heat at constant volume (C_v)

(ii) Specific heat at constant pressure (C_p)

Defn of C_v

It is defined as the amount of heat required to raise the temp of 1 gm of a gas through 1°C at constant volume.

Defn of C_p

It is defined as the amount of heat required to raise the temp of 1 gm of a gas through 1°C at constant pressure.

Molar specific heat at constant volume (C_V)

It is defined as the amount of heat required to raise the temp of 1 mole of a gas through 1°C at constant volume.

$\therefore C_V = M \cdot c_v$

Molar Specific heat constant pressure (C_P

It is defined as the amount of heat reauired to raise the temp of 1 mole of gas through 1°C at constant pressure.

$$\therefore C_P = M . c_p .$$

Relation Between C_p and C_V (Mayer's formula)

The relation is

$$\boxed{C_p - C_v = R}$$

where R = Universal gas constant
= 8.31 Jole mol⁻¹ °K⁻¹

$$\Rightarrow M c_p - M c_v = R$$

$$\Rightarrow (c_p - c_v) = \frac{R}{M}$$

Calcalation of Specific heat of a monoatomic ideal gas

Let's consider 1 mole of Monoatomic gas like He, Ne, Ar, Kr etc. Total number of atoms present in 1 mole of the gas

$= N_0$ (Avogadro number)

Each atom has three degrees of freedom. Thus, total number of degrees of freedom of all the atoms present in 1 mole of the gas $= 3N_0$

From the law of equipartition of energy, we know that energy per degree of freedom

$$\frac{1}{2} KT$$

Thus the total energy of all the atoms present in mole.

$$= \frac{1}{2} KT \times 3N_0$$

$$= \frac{1}{2} \cdot \frac{R}{N_0} \cdot T \times 3N_0$$

$$= \frac{3}{2} RT$$

If the temp be raised by 1° K or 1° C then the total energy will be

$\frac{3}{2} R (T+1)$

ΔE = energy required to make the temp of 1 mole of the gas through 1°C or 1°K

$$= \frac{3}{2} R (T+1) - \frac{3}{2} R (T)$$

$$= \frac{3}{2} R \cancel{(}$$

✓ $= C_v$ by def^n

From mayer's formula we know that $C_p - C_v = R$

$$\Rightarrow C_p = C_v + R$$

$$= \frac{3R}{2} + R$$

$$= \frac{5R}{2}$$

$\boxed{\cancel{\gamma =}}\ \gamma$ = ratio of the specific heat at constant pressure and specific heat at constant volume

$$= \frac{C_p}{C_v} = \frac{\frac{5R}{2}}{\frac{3R}{2}} = \frac{5}{3} = 1.666$$

Calculation of specific heats of diatomic gas

Let's consider 1 mole of diatomic gas like H_2, O_2, N_2, CO, NO etc. ~~To~~

Total number of molecule ~~atoms~~ present in 1 mole of gas = N_0 (Avogadro number

Each molecule has ~~three~~ 5 degrees of freedom. Thus, total number of degrees of freedom of all the molecules present in 1 mole of the gas = $5 N_0$

From the law of equipartition of energy, we know that energy per degrees of freedom

$$= \frac{1}{2} KT$$

Thus total number of energy of all the molecules present in 1 mole

$$= \frac{1}{2} KT \times 5N_0$$

$$= \frac{1}{2} \frac{R}{N_0} . T \times 5 N_0$$

$$= \frac{5}{2} RT$$

If the temp be raised by 1° K or 1° c then the total energy will be $\frac{5}{2}R(T+1)$.

ΔE = Energy required to raise the temp of 1 mole of the gas

$$= \frac{5}{2}R(T+1) - \frac{5}{2}RT$$

$$= \frac{5}{2}R$$

$= C_v$ by def^n.

From Mayer's formula we know

$$C_p - C_v = R$$

$$\Rightarrow C_p = R + C_v$$

$$= \frac{5R}{2} + R$$

$$= \frac{7R}{2}$$

γ = ratio of the specific heat at constant pressure and specific heat at constant volume

$$= \frac{C_p}{C_v} = \frac{\frac{7R}{2}}{\frac{5R}{2}} = \frac{7}{5} = 1.4$$

Calculation of specific heat of a triatomic gas.

Let's consider 1 mole of triatomic gas molecule like CO_2, NO_2, O_3 H_2S, H_2O etc.

Total number of molecules present in 1 mole gas = N_0 (Avogadro's number)

If the molecules will be arranged as a linear chain then the number of degrees of freedom = 7

If the molecules will be arranged at the vertices of a triangle then the number of degrees of freedom = 6

~~First consider about first case~~

Let's consider about the first case.

Total number of degrees of freedom of all the molecules present in 1 mole of gas = $7N_0$

From the law of equipartition of energy, we know that energy per degree of freedom = $\frac{1}{2}KT$

Thus total energy of all molecules

present in 1 mole

$$= \frac{1}{2} KT \times 7N_o$$

$$= \frac{1}{2} \frac{R}{N_o} . T \times 7N_o$$

$$= \frac{7RT}{2}$$

If the temp be raise by 1°K or 1°C then the total energy will be

$$\frac{7}{2} \cancel{8} R(T+1)$$

ΔE = Energy required ~~[illegible]~~ to raise the temp of 1 mole gas through 1°C or 1°K /

$$= \frac{7}{2} R(T+1) - \frac{7}{2} RT$$

$$= \frac{7}{2} R(T+1-T)$$

$$= \frac{7R}{2} = C_v \text{ by def}^n.$$

from Mayer's formula

$$C_p - C_v = R$$

$$\Rightarrow C_p = R + C_v$$

$$= R + \frac{7R}{2}$$

$$= \frac{9R}{2}$$

γ = Ratio of specific heat at constant pressure and specific heat at constant volume

$$= \frac{C_p}{C_v} = \frac{\frac{9R}{2}}{\frac{7R}{2}} = \frac{9}{7} = 1.285$$

Let's consider about a second case

After 6 Page

Relation between Average speed and r.m.s speed

Maxwell has derived expressions for the r.m.s speed and Average speed of a gas molecule based on his theory called Maxwell- Boltzmen distribution law.

$$\text{Average speed} = \bar{c} = \sqrt{\frac{8KT}{m\pi}}$$

$$= \sqrt{\frac{8}{3.14}} \cdot \sqrt{\frac{KT}{m}}$$

$$= 1.59 \cdot \sqrt{\frac{KT}{m}}$$

$$C_{rms} = \sqrt{\frac{3KT}{m}} = 1.732 \cdot \sqrt{\frac{KT}{m}}$$

Thus $\frac{C_{rms}}{\bar{c}} = \frac{1.732}{1.59} = 1.0893081$

✓ $\Rightarrow C_{rms} > \bar{c}$

Mean free path (λ)

It is defined as the average distance covered a molecule or atom to collide with another molecule or atom. The best expression for λ is

$$\lambda = \frac{1}{\sqrt{2}} \cdot \frac{1}{n \pi \sigma^2}$$

where n = number of molecules per unit volume.

σ = diameter of a molecule.

With the increase of pressure mean free path of a gas decreases because the molecule collide more often.

$\therefore P_1 \lambda_1 = P_2 \lambda_2 =$ constant

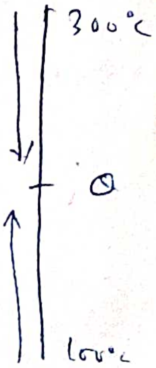
Problem

1. Two moles of a monoatomic gas at 300°C is mixed with 1 mole of a diatomic gas at 100°C. Calculate the final temp of the mixture, assuming that the mixing takes place at constant volume and that there is no exchange of heat with the environment.

Ans

Heat lost by monoatomic gas

Heat gained by diatomic gas



$$\Rightarrow 2 \cdot (C_v)_{mono} \cdot (300 - \theta) = 1 \cdot (C_v)_{di} \cdot (\theta - 100)$$

=) $2 \cdot \frac{3R}{\cancel{2}} \cdot (300 - \theta) = \frac{5R}{\cancel{2}} \cdot (\theta - 100)$

=) $6(300 - \theta) = 5\theta - 500$

=) $1800 - 6\theta = 5\theta - 500$

=) $11\theta = 2300$

=) $\theta = \frac{2300}{11} = 209.09°C$

Q. Problems on first law of thermodynamics

1. Find the difference of temp between the top and the bottom of a water fall of height 210 meter.

(Ans ÷ 0.49°C)

Ans: Let consider m gm of water at the top most point.

Potential energy possessed by it = mgh which gets converted to kinetic energy and hence to heat energy at the bottom.

$\therefore P.E = mgh$

$= (m \cdot 980 \times 21000)$ erg

$= \frac{m \times 98 \times 21 \times 10^{4}}{10^{7}}$ Joule

$= 2058m \times 10^{-3}$ Joule.

From the first law of thermodynamic

$W = JH$

where W = the work done in joule.

H = Heat produced in calory.

J = mechanical equivalent of heat

$= 4.2$ J / Cal.

Amount of heat obtained at the buttom

$= \frac{2058m \times 10^{-3}}{4.2}$

~~$= 490 \times 10^{-3}$~~ ~~$= m s \Delta\theta$~~

$\Rightarrow$ ~~$.49 = 1.$~~

$= \frac{2058m \times 10^{-3}}{4}$

$= 490m \times 10^{-3} = m s \Delta\theta$

$\Rightarrow 0.49 \cancel{m} = \cancel{m} . 1 . \Delta\theta$

$\Rightarrow \Delta\theta = 0.49°C$

[illegible]

$\frac{1 \text{ Cal}}{\text{gm}}$

[illegible]

2. A lead bullet at 15°C Strikes a target. If the lead is all just melted, determine the velocity with which the bullet heats the target.

Melting point of lead = 325°C.

Specific heat of lead = 0.03 cal.gm^{-1}

Latent heat of fusion = 5 cal/gm.

Ans 346.58 m/s.

Hints : $H = ms\Delta\theta + mL_f$

After space

3. Calculate the height from which a piece of ice at 0°C be allowed to fall so that 20% of the ice just melts into water at 0°C

Ans :)

---

2. Let A lead ball has mass = m gm.

~~Attach~~ It strikes a target and it is just melted.

Heat had been required to melt a the

lead ball

$$H = m.S\,\Delta\theta + mL_b$$

$$= m.(0.03).(325-15) + m.5$$

$$= m\left(9.3+5\right)$$

$$= 14.3m \text{ Cal} \; \text{~~erg~~} \longrightarrow (i)$$

$$\text{~~= \frac{14.3}{10^3} m~~} \text{ kilocal} \; \text{~~Joules~~}$$

From the first law of thermodynamics

we know that

$$W = J.H$$

$$\Rightarrow H = \frac{W}{J}$$

Here work done by the bullet

= its kinetic energy $= \frac{1}{2}mv^2$

$$\Rightarrow H = \frac{\frac{1}{2}mv^2}{4.2 \text{ Joule Cal}^{-1} \times 10^7} \quad (ii)$$

eqn (i) and (ii)

$$\frac{14.3\,\cancel{m}}{} = \frac{\frac{1}{2}\cancel{m}v^2}{4.2\times10^7} = \frac{v^2}{2\times4.2\times10^7}$$

$$\Rightarrow v^2 = \frac{14.3\times2\times4.2\times10^7}{}$$

$$= \frac{120.12\times10^7}{}$$

$$= \frac{1201.2\times10^6}{}$$

$$\Rightarrow v = \sqrt{\frac{1201.2\times10^6}{}} = 34.658\times10^3 = 34658 \text{ Cm/sec}$$

$$= 346.58 \text{ m/sec.}$$

The number ob degrees ob freedom ob a molecule = ~~3~~ 6

Total number ob degrees of freedom ob all the molecules present in 1 mole ob gas = 6 No

Fom the law ob equipartion of energy, we know that energy per degree of freedom = $\frac{1}{2}KT$

Thus total energy ob all the molecules present in 1 mole

$$= \frac{1}{2}KT \times 6N_0$$

$$= \frac{1}{2}\frac{R}{N_0}\cdot T \times 6N_0$$

$$= 3RT$$

Ib the temp be raised by 1°K or 1°C then the total energy

$$= 3R(T+1)$$

ΔE = Energy required to raise the temp ob 1 mole gas through 1°C or 1°K

$$= 3R(T+1) - 3RT$$

$= 3R$

$= C_v$ by defn

From Mayer's formula we know that $C_p - C_v = R$

$\Rightarrow C_p = R + C_v$

$= R + 3R$

$= 4R$

γ = Ratio of Specific heat at constant pressure and specific heat at constant volume.

$$= \frac{C_p}{C_v} = \frac{4R}{3R} = \frac{4}{3} = 1.33$$

3. Let the mass of ice = m gm, It is allowed to fall from a height.

After falling 20% ice is melted.

So melted ice $= \frac{m \times \cancel{20}}{\cancel{100}_{5}} = \frac{m}{5}$ gm

So heat ~~heat~~ has been required

$= \frac{m}{5} \times 80$ Cal./gm.

$= 16m$ Cal

$\underline{W = JH}$ from first law of thermodynamics

$\Rightarrow H = \frac{W}{J} = \frac{W}{4.2 \text{ Joule/Cal}}$

$\Rightarrow 16m = \frac{W}{4.2}$

$\Rightarrow W = 67.2m$ Joule.

$= 67.2m \times 10^7$ erg.

It is the potential energy which was present on the ice.

$mgh = 67.2m \times 10^7$ erg

$\Rightarrow \cancel{m} \times 980 \times h = 67.2 \cancel{m} \times 10^7$

$\Rightarrow h = \frac{67.2 \times 10^7}{980} = \frac{6720 \times 10^5}{980}$

$= 6.857 \times 10^5$ c.m

$$= \frac{6.857 \times 10^5}{10^2} = 6.857 \times 10^3 = 6857 \text{ meter}$$

∴ ~~The~~ The ice fall from 6857 meter height

Thermodynamics

It is a branch of ~~P~~ Physics which deals with the relation between work and Heat and internal energy.

Thermodynamics variables are Pressure (P), Volume (V), Absolute temperature (T), Internal Energy (U) and Entropy (S) etc.

System : A particle or a group of particles undergoing some changes with time is called system.

Ex: Molecules of a gas taken in a container.

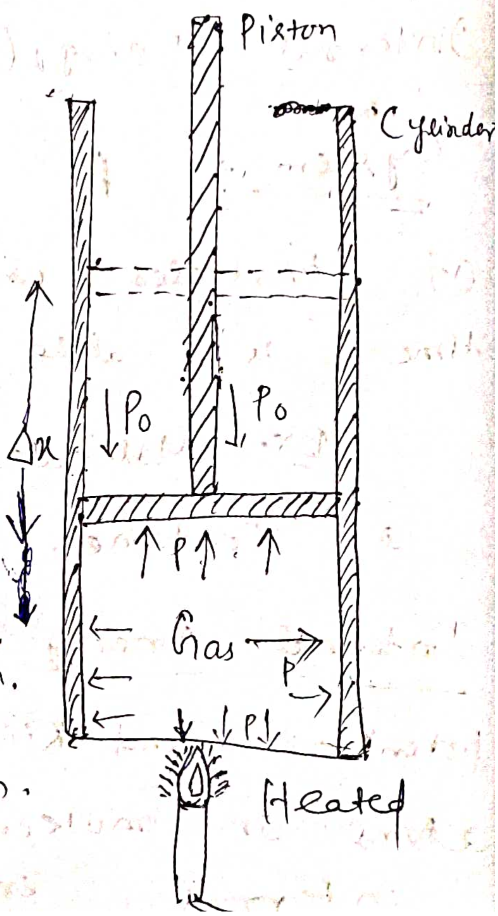
Internal Energy : The sum total of potential and kinetic energies of all the atoms or molecules of a gas is called internal energy.

It is impossible to measure the absolute value of the internal energy of a gas. But change in the internal energy can be measured as shown below.

Relation between work and Heat

Heat is a form of energy and work is said to be done when a force acting on a body displaces it through a certain distance.

To establish a relation between work and heat, let us consider an ideal cylinder Ⓧ.



$$P = P_0 + \frac{W}{A}$$

where P = Pressure due to the gas.

P_0 = atmospheric pressure.

W = weight of the piston.

A = Area of the piston.

ⓧ fitted with ideal piston and encloses a gas. The bottom of the cylinder is conducting and the sides are made out of perfectly non-conducting material.

The piston is also perfectly non conducting and fits perfectly with the cylinder when it moves, friction is absent.

Out of ΔQ amount of heat supplied to the gas, a ~~point~~ part is used to increase the internal energy of the gas because of rise of temp.

The rest energy is ~~[illegible]~~ utilized in moving the piston by an amount Δx. The piston moves through Δx only because the pressure of the gas becomes equal to the sum of atmospheric pressure and the pressure due to the piston.

ΔW = Amount of work done by the piston in moving through a distance Δx.

$= F . \Delta x$

$= PA \cdot \Delta x$ $\left(\because \text{Pressure} = \frac{\text{Force}}{\text{Area}}\right)$

$= P \cdot \Delta V$

where ΔV = Change in volume of the gas.

Thus, we can write

$$\boxed{\Delta Q = \Delta U + \Delta W}$$

where ΔQ = Heat energy supplied to the gas.

ΔU = Change in the internal energy of the gas

ΔW = External work done by the gas in moving the piston.

This statement is also called <u>First Law of Thermodynamics</u> which follows from the principle of conservation of energy.

→ Refⁿ from → 6.85 Page → Basic. Rath, Sharma

Isothermal Process

If the temperature will be kept constant during a process, then we will call it an isothermal process. Since temperature does not change, Kinetic energy of the gas molecules will not change and internal energy will not change.

$$\therefore \quad \Delta U = 0$$ (except when there is change in phase)

Thus $\Delta Q = \Delta W$

⇒ All the heat energy supplied to the gas is utilized for doing external work.

Work done during an isothermal Process

We have seen that

$$dW = P . dV$$

But equation of state for an ideal gas is known to be $PV = nRT$

or $$P = \frac{nRT}{V}$$

Thus $$dW = \frac{nRT}{V} dV$$

Integrating both the sides ~~p~~ with proper limits, we get,

$$\int_0^W dW = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

or $$W \Big|_0^W = nRT \left(\log_e V\right)\Big|_{V_i}^{V_f}$$

or $$W - 0 = nRT \left(\log_e V_f - \log_e V_i\right)$$

$$W = nRT \log_e \frac{V_f}{V_i}$$

$$W = nRT \times 2.3026 \times \log_{10} \frac{V_f}{V_i}$$

Adiabatic Process

If a system is kept completely isolated from the surrounding such that no heat can enter into the system or can come out of the system, then we call the process as adiabatic.

i.e $\Delta Q = 0$

$\Rightarrow \Delta U + \Delta W = 0$

$\Rightarrow \Delta W = -\Delta U$

i.e Any amount of external

work done by the system involves the decrease of its ~~external~~ internal energy. Hence the system will be cooled.

Examples: Sudden expansion of a cloud and formation of rain drop.

Work done during an adiabatic process

The appropriated equation of state for a gas undergoing adiabatic process is

$$P.V^{\gamma} = \text{Constant} = K \text{ (say)}$$

where γ = ratio of specific heats ~~of a gas~~ of a gas

$$= \frac{C_p}{C_v}$$

$$\Rightarrow P = \frac{K}{V^{\gamma}}$$

Small amount of workdone is given by

$$dW = P.dV$$

$$\Rightarrow dW = \frac{K}{V^{\gamma}} \cdot dV$$

Integrating both the sides of the

above eqn with proper limits, we get

$$\int_0^{W} dW = K\int_{V_i}^{V_f} V^{-\gamma}\, dV$$

$$\Rightarrow (W)\Big|_0^{W} = K\left(\frac{V^{-\gamma+1}}{-\gamma+1}\right)\Bigg|_{V_i}^{V_f}$$

$$\Rightarrow W - 0 = \frac{K}{1-\gamma}\left[V_f^{-\gamma+1} - V_i^{-\gamma+1}\right]$$

or $$W = \frac{1}{(1-\gamma)}\left[K V_f^{-\gamma} . V_f^{1} - K V_i^{-\gamma} . V_i^{1}\right]$$

$$= \frac{1}{1-\gamma}\left[P_f V_f - P_i V_i\right]$$

$\left(P = \frac{K}{V^{\gamma}} \Rightarrow P_f = \frac{K}{V_f^{\gamma}}\right)$

$$= \frac{1}{1-\gamma}\left[nRT_f - nRT_i\right]$$

$$W = \frac{nR}{1-\gamma}\left(T_f - T_i\right)$$

Isochoric Process

It is a process where volume of the gas is kept constant.

$$\Delta V = 0$$

$$\Delta W = P . \Delta V = 0$$

Hence

$$\Delta Q = \Delta U$$

i.e All the heat supplied from outside is utilized in increasing the internal energy of the gas.

Ex :- Pressure cooker.

Problem :-

Four moles of an ideal gas at 375 K is compressed isothermally from an initial volume of 7.5 litres to a final volume of 1.3 litres Calculate the workdone in this process.

(Ans :- 2.006×10^4 J)

Ans :-

$$\Delta W = nRT \times 2.3026 \log_{10} \frac{V_f}{V_i}$$

$$= 4\,\text{mole} \times 8.31 \frac{\text{Joule}}{\text{mole}\,\cancel{K}} \times 375\cancel{K} \times 2.3026 \times \log_{10} \frac{1.3}{7.5}$$

$$= 4 \times 8.31 \times \overset{375}{\cancel{35}} \times 2.3026\,(.1139 - .8751)$$

$$= 4 \times 8.31 \times 3575 \times 2.3026\,(\cancel{.1139}\ \overset{-.7612}{\cancel{.7612}})\ \text{Joule}$$

$$= -2.1847 \times 10^4\ \text{Joule}$$

Problem :2

A Certain quantity of air at 76 c.m of Hg pressure is compressed adiabatically to two-third of its initial volume. what is the final pressure? (Ans - 134 cm of Hg)

Ans:

Equation of state of the gas undergoing adiabatic process gives

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

or $$(76 \times 13.6 \times 980) V_1^{\gamma} = (h_2 \times 13.6 \times 980)\left(\frac{2}{3}V_1\right)^{\gamma}$$

$$76 \times V_1^{\gamma} = h_2 \times \left(\frac{2}{3}\right)^{\gamma} . V_1^{\gamma}$$

$$\Rightarrow h_2 = 76 \times \left(\frac{3}{2}\right)^{\gamma} = 76 \times (1.5)^{1.4}$$

When $\gamma = 1.4$ taken for air since it is a mixture of diatomic gas like N_2, O_2, H_2 mainly.

Taking Logarithm of both the sides, we get

$$\log_{10} h_2 = \log_{10} 76 + 1.4 \log_{10} 1.5$$

$\therefore \log h_2 = 1.8808 + 1.4 \times 0.1761$

$= 1.8808 + 0.24654$

$= 2.12734$

$\therefore h_2 =$ Antilog of 2.12734

$= 0.1341 \times 10^3$

$= 134.1$ c.m of Mecrury.

Reversible Process

A reversible process is that which an be retraced in the opposite dinection. so that the working substance passes through exactly the same condition as it does in the direct process

It may be defined as a process that is performed in such a way that at the end of the process, ~~the~~ both the system and the local surrounding may be restored to their initial state without producing any change in the rest of the Universe.

Even a cyclic process will not be reversable unless dissipative ([illegible]) forces like friction, viscocity will be absent.

Ex:1 All isothermal and adiabatic processes when performed slowly can be called reversable. A Carnot heat engine is a reversable engine because the piston moves as a frictionless piston.

Ex-2 : One gram of ice at 0°C absorbs 80 Cal to become 1 gm of water at 0°C. On the other hand, one gram of water at 0°C rejects 80 Cal to become 1 gm of ice at 0°C

In nature no process is perfectly reversable because there is always loss of energy through friction etc which can not be brought back.

Irreversable Process

If the whole system including the surrounding can not be completely restored to its initial condition even with the use of all possible physical means, the process is irreversible.

Ex:- 1

Work done against friction is irreversible because, in reversing the direction of motion, again work has to be done against friction.

Ex-2

Heat produced by the passage of current through a resistance wire is irreversible because in reversing the direction of current, again heat is produced.

Ex-3 Processes like diffusion, conduction, radiation etc are irreversible processes.

First Law Of Thermodynamics

Statement

1. Whenever work is converted into heat or heat into work, the quantity of work is mechanically equavalant to the quantity of heat.

If W is the work done in generating an amount of heat H, then

$$W \propto H$$

$$\Rightarrow \boxed{W = JH}$$

where J is a constant called mechanical equivalent of heat

Its value is 4.186 $\simeq$ 4.2 Joule/cal.

2. Energy can not be created or destroyed, but can be transformed from one form into the other.

(given)

Whenever heat is imparted

to a body, a part is used
to increase the internal energy
and the rest is used to do external work.

$$\therefore \boxed{\Delta Q = \Delta U + \Delta W}$$

where ΔQ = heat energy absorbed by the system

ΔU = increase in the internal energy

$\Delta W = P.\Delta V$ = External work done by the gas or system.

([illegible])

Significance of the first law

1. It establishes a relation between heat and work.

2. According to this law, heat can be produced only by the (use) expenditure of energy in some other form.

It is impossible to make a (continuous working) "perpetual motion machine" without expenditure of energy.

4

Debiciency Ob the first law

1. It does not specify the condition under which a body can use its heat energy to produce work.

2. It does not specify the exact amount ob heat i.e converted in to work.

Applications ob first law ob thermodynamics

1. Relation between C_p and C_v (Mayer's formula)

Let's have n mole ob any gas enclosed in between the piston and the cylinder.

If the piston be held tight and the gas be heated so that the temp will rise by ΔT °K.

$$\therefore \Delta Q = \Delta U = n . C_v . \Delta T \quad -(1)$$

The gas be cooled to the original temp and again heated while the piston is left free. The heating be continued till temp again rises to ΔT °K. The amount ob heat absorbed by the gas will be more because the internal energy has to be increased and external work has to be done.

$$\therefore \Delta Q' = \Delta U + \Delta W$$

$$\Rightarrow n\,C_p \cdot \Delta T = n\,C_v \Delta T + P \cdot \Delta V \text{ — (ii)}$$

because pressure of the gas remains constant while the piston moves.

Eqn of state for an ideal gas is $PV = nRT$

Differentiating both the sides, we get

$$P \cdot \Delta V + V \cdot \Delta P = nR \cdot \Delta T \text{ ——— (iii)}$$

But P = constant so, that $\Delta P = 0$

Thus eqn (3) because

$$P \cdot \Delta V = nR\,\Delta T \text{ ——— (iv)}$$

Using eqn (IV) in eqn (ii) gives

$$n\,C_p\,\Delta T = n\,C_v\,\Delta T + nR\Delta T$$

$$\Rightarrow C_p = C_v + R$$

$$\Rightarrow \boxed{C_p - C_v = R}$$

It is called Mayer's formula

Corollary:-

If the amount of heat energy supply to a gas be expressed in Calory whereas work done by the gas be expressed in Joule then we have to use the formula

$$W = JH \text{ in eq}^n \text{ (ii)}$$

$$\therefore \; nC_p\Delta T = nC_v\Delta T + \frac{P\Delta V}{J}$$

$$\Rightarrow nC_p\Delta T = nC_v\Delta T + \frac{nR\Delta T}{J}$$

$$\Rightarrow C_p = C_v + \frac{R}{J}$$

$$\Rightarrow \boxed{C_p - C_v = \frac{R}{J}}$$

2 mark ✓

To derive the eqn of State ~~of~~ for an ideal gas undergoing adiabatic process.

An adiabatic process is the process where the system is completely isulated from the surrounding so that no heat can enter into the system or can come out of the system

system e.i

$$\therefore \; \Delta Q = 0$$

$$\Rightarrow \Delta U + \Delta W = 0$$

$\Rightarrow$ ~~$\Delta U =$~~

$\Rightarrow n\, C_v \,.\, \Delta T + P \,.\, \Delta V = 0$ —— (i)

Eqn of state for an ideal gas is

$$PV = nRT$$

Differenciating both the sides of the above eqns, we get

$$P \,.\, \Delta V + V \,.\, \Delta P = nR\,\Delta T$$

$\Rightarrow \Delta T = \dfrac{P\Delta V + V\Delta P}{nR}$ —— (ii)

Using eqn (ii) in eqn (i), we get

$$\cancel{n}\, C_v \left(\frac{P\Delta V + V\Delta P}{\cancel{n} R} \right) + P\Delta V = 0$$

$\Rightarrow \dfrac{C_v P\Delta V + C_v V\Delta P + PR\Delta V}{R} = 0$

$\Rightarrow C_v P\Delta V + C_v V\Delta P + PR\Delta V = 0$

Using Mayer's formula, $C_p - C_v = R$

in the above eqn, we get

$$C_v P\Delta V + C_v V\Delta P + P(C_p - C_v)\Delta V = 0$$

$\Rightarrow \cancel{C_v P\Delta V} + C_v V\Delta P + C_p P\Delta V - \cancel{C_v P\Delta V} = 0$

$\Rightarrow \quad C_v V \Delta P + C_p P \Delta V = 0$

Dividing by $C_v . P . V$ through out,

we get

$$\frac{\Delta P}{P} \text{ or } \frac{C_v V \Delta P}{C_v P V} + \frac{C_p P \Delta V}{C_v P V} = \frac{0}{C_v P V}$$

$$\Rightarrow \frac{\Delta P}{P} + \frac{C_p}{C_v} . \frac{\Delta V}{V} = 0$$

$$\Rightarrow \frac{\Delta P}{P} + \gamma . \frac{\Delta V}{V} = 0 \quad \text{—(ii)}$$

where, γ = ratio of the specific heats

$= \dfrac{C_p}{C_v} = \dfrac{\text{Molar specific heat at constant pressure}}{\text{Molar specific heat at constant volume}}$

Integrating both the sides of eqn (ii), we get

$$\int \frac{dP}{P} + \nu \int \frac{dV}{V} = \text{Constant} = K_1 \text{ (Say)}$$

$\Rightarrow \log_e P + \nu \log_e V = K_1$

$\Rightarrow \log_e P + \log_e V^{\nu} = K_1$

$\Rightarrow \log_e (P . V^{\nu}) = \log_e K$

(where K is some other constant)

$$\Rightarrow \boxed{P.V^{\gamma} = K}$$

This is the eqⁿ of ~~State~~ ~~state~~ for ~~[illegible]~~ an ideal gas undergoing adiabatic process.

(3) Boiling Process

Suppose ~~M~~ m gm of a liquid at its boiling point be heated till it gets converted into a gas or vapour.

$$\Delta Q = \Delta U + \Delta W$$ becomes

$$m.L_v = \Delta U + \frac{P.\Delta V}{J}$$ where L_v = Latent heat of vapourisation

Thus $\Delta U = mL_v - \frac{P\Delta V}{J}$ = Change in the internal energy due to change in the interaction potential energy between mulecules.

Spacing between mulecules changes to a great extent.

(4) Freezing Process

Here $\Delta Q = mL_f = \Delta U + \Delta W$

where L_f = Latent heat of fusion.

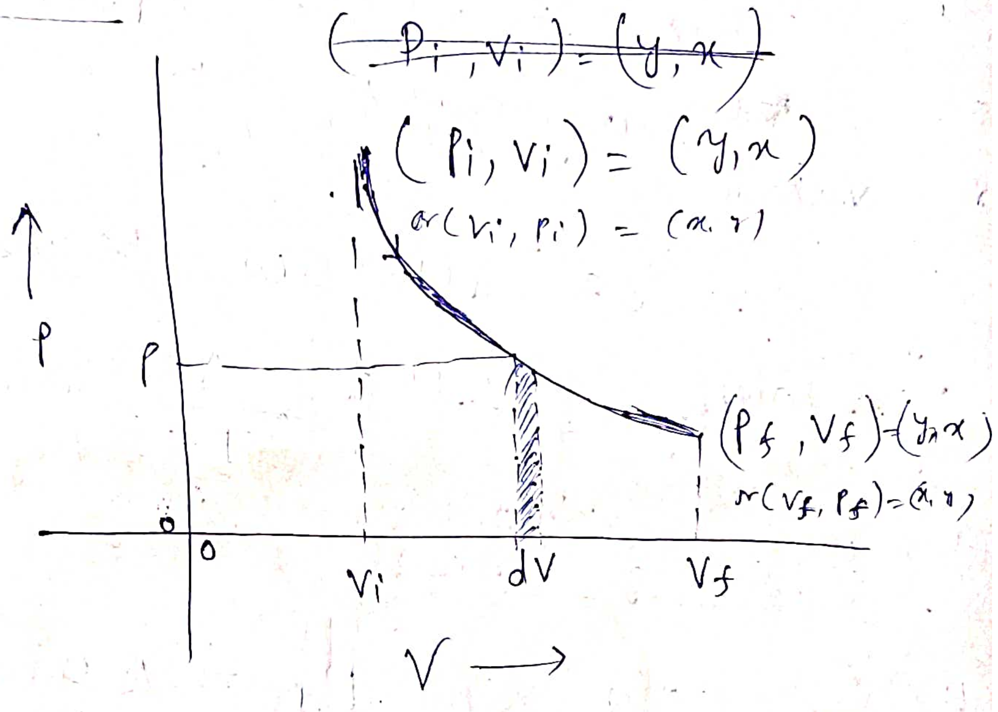
$\therefore \Delta U = [ML_f - \frac{P\Delta V}{J})$ Cal

Indicator Diagram

It is a graph obtained when pressure is plotted along y-axis and Volume along x-axis. A curve is obtained. The area under the curve represents the amount of work done during the process.

1. Indicator diagram
2. 2 Problems
3. doubt

~~$(P_i, V_i) = (y, x)$~~



Area of the strip $= P \cdot dV = dW$

Integrating both the sides, we get

$$W = \int_0^W dW = \int_{V_i}^{V_f} P \, dV$$ = Sum of the areas all such strips

= Area made by the curve with the x-axis.

problems

1. 1 gm of water having a volume of 1 c.c becomes 1671 c.c of steam when boiled at 1 atmospheric pressure. If $L_v = 540$ Cal/gm at 1 atm, how much gain in internal energy takes place?

Chang in volume of water $= 1671 - 1$
$= 1670$ c.c

Ans ⇒ We know that

$$\Delta Q = \Delta U + \Delta W$$

or $$mL_v = \Delta U + \frac{P \cdot \Delta V}{J}$$

$$\Rightarrow 1 \times 540 = \Delta U + \frac{76 \times 13.6 \times 980 \frac{dyne}{cm^2} \times 1670\,cm^3}{4.2 \times 10^7\,erg/Cal}$$

$$\Rightarrow 540\,Cal = \Delta U + 41\,Cal$$

$$\Rightarrow \Delta U = 540 - 41 = 499\ Cal.$$

Hence out of 540 Cal, 41 Cal goes to do external work and 499 Cal goes to increase of internal energy.

2. How would you expect the 80 Cal of heat needed to melt 1 gm. of melting ice having a volume of 1.09 c.c to water at 0°C and 1 atm pressure, having a volume 1 c.c. to be shared by the external work and internal energy?

We change in volume = 1.09 - 1 = .09 c.c

know from the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$M L_f = \Delta U + \frac{P\Delta V}{J}$$

$$\Rightarrow 1 \times 80 = \Delta U + \left(\frac{(76 \times 13.6 \times 980)\,\frac{dyne}{cm^2} \times (.09)\,cm^3}{4.2 \times 10^7\,erg/Cal} \right)$$

$$= \Delta U + .0021\,Cal$$

$$\Rightarrow 80 = \Delta U + .0021\,Cal$$

$$\Rightarrow \Delta U = 80 - .0021$$

$$= 79.9979\,Cal.$$

Heat engine

(যন্ত্র)

It is a device to convert heat into work or work into heat. Earlier heat engines had a very low efficiency ranging from 5% to 25%. Scientists thought that bad designing is the cause of such a low efficiency. But, with the improvement of the design, efficiency does not change

At least, a French Engineer named Sadi Carnot thought over the problem and prove that the efficiency cannot be increased ~~oo~~ indefinitely. He gave the concept of an ideal heat engine and showed that efficiency cannot reach 100 %.

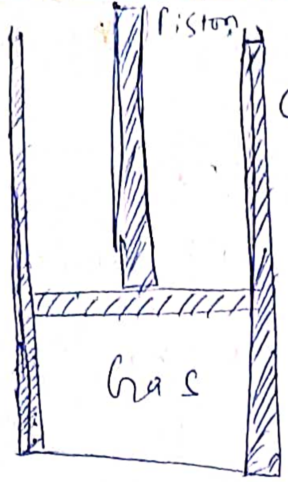
Imp

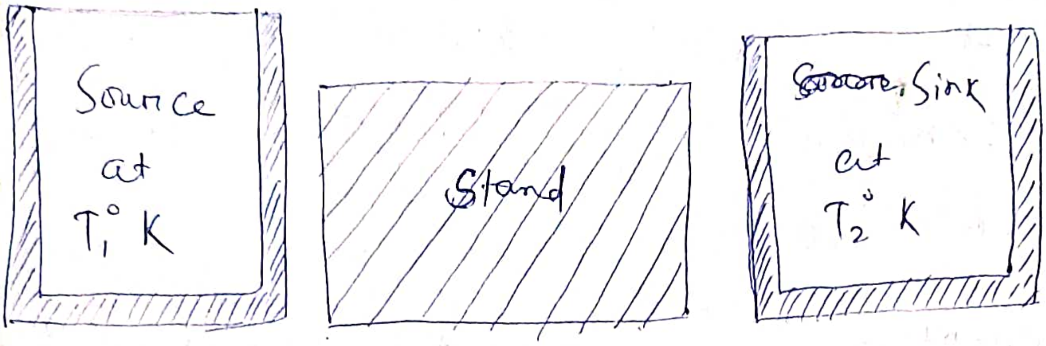
Description of Carnot's ideal heat engine

The main parts of Carnot's ideal heat engine is described below.

① Cylinder :- The cylinder has perfectly non-conducting sides and a perfectly conducting bottom.

It is ~~[illegible]~~ fitted with a non-conducting piston which moves as a frictionless object. A gas is enclosed in between the piston and the bottom of the cylinder, which is called working substance.





$T_1 > T_2$

② Source

It is a container of heat at some higher temp T_1°K such that any amount of heat taken away from it will not be able to change the temp of ~~heat~~ ~~[illegible]~~ it

(3) Stand

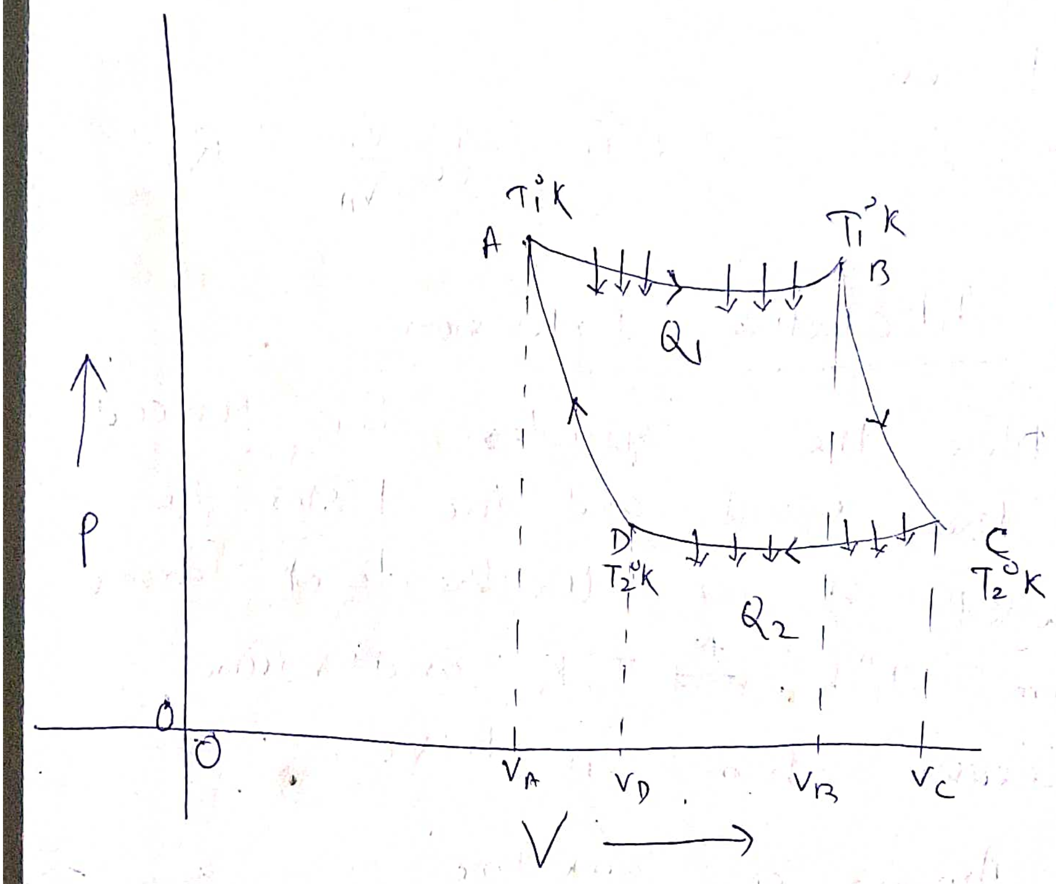
It is a perfectly non-conducting platform on which the cylinder can be placed when adiabatic process is to be performed.

(4) Sink

It is another container of heat at some lower temperature T_2°K such that any amount of heat given to it will not be able to increase the temp of heat.

The Carnot cycle

Carnot has prescribed a particular manner in which the engine has to be operated. By this, maximum efficiency can be reached. It consists of the following four process.



(1) Isothermal Expansion

To start the carnot cycle, let the cylinder be placed on the source for sometime so that the gas will attain the temperature $T_1^\circ K$. The volume of the gas be V_A. The piston be raised so that the volume changes to V_B. Some amount of heat must flow from the source so that the temperature of the gas will remain constant at T_1°K. This amount of heat entering the gas from the source be Q_1.

Amount of work done during this process

$$= \Delta W_1 = nRT_1 \log_e \frac{V_B}{V_A} = Q_1$$

② Adiabatic Expansion

Now the cylinder be placed on the stand and the piston be raised further till the temp. drops from T_1° K ~~and~~ to T_2° K and volume increases from V_B to V_C.

Amount of work done

$$= \Delta W_2 = \frac{nR}{(1-\gamma)} (T_2 - T_1)$$

(3) Isothermal Compression

The cylinder be placed on the sink and the piston be moved in to the gas so that volume decreases from V_C to V_D. The heat generated due to compression be Q_2 which is rejected into the sink.

Amount of work done

$$= \Delta W_3 = nRT_2 \log_e \frac{V_D}{V_C} = -Q_2$$

(4) Adiabatic Compression

The cylinder be placed on the stand and the piston be further moved into the gas till the temperature of the gas rises from $T_2{}^0K$ to $T_1{}^0K$. and the volume decreases from V_D to V_A.

Workdone during the process

$$= \Delta W_4 = \frac{nR(T_1 - T_2)}{(1-\gamma)}$$

Calculation of the efficiency of the Carnot engine.

Total workdone during the Carnot cycle $= \Delta W_1 + \Delta W_2 + \Delta W_3 + \Delta W_4$

$$= nRT_1 \log_e \frac{V_B}{V_A} + \cancel{\frac{nR}{(1-\gamma)}(T_2 - T_1)}$$
$$+ nRT_2 \log_e \frac{V_D}{V_C} + \cancel{\frac{nR}{(1-\gamma)}(T_1 - T_2)}$$

$= nRT_1 \log_e \frac{V_B}{V_A} - nRT_2 \log_e \frac{V_C}{V_D}$

$= Q_1 - Q_2$

Efficiency of the engine

$= \eta = \frac{\text{out put}}{\text{input}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

It can be shown that $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$$\therefore \eta = 1 - \frac{T_2}{T_1}$$

__Proof__ :- Let us develope a new equation of the state for an ideal gas undergoing adiabatic process

$$PV^{\gamma} = \text{constant} = K$$

$$\& \quad PV = nRT$$

Dividing we get

$$V^{\gamma-1} = \left(\frac{K}{nR}\right) \cdot \frac{1}{T}$$

$$\Rightarrow TV^{\gamma-1} = \text{Constant}$$

Applying the above eqn of state for

the adiabatic processes BC and DA of the Carnot cycle, we get.

$$T_1 . V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$$

$$T_1 V_A^{\gamma-1} = T_2 V_D^{\gamma-1}$$

Dividing, we get

$$\left(\frac{V_B}{V_A}\right)^{\gamma-1} = \left(\frac{V_C}{V_D}\right)^{\gamma-1}$$

$$\Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$$

$$\therefore \log_e \frac{V_B}{V_A} = \log_e \frac{V_C}{V_D}$$

Now $\dfrac{Q_1}{Q_2} = \dfrac{nRT_1 \log_e \frac{V_B}{V_A}}{nRT_2 \log_e \frac{V_C}{V_D}} = \dfrac{T_1}{T_2}$ (~~proved~~) (proved)

Discussion

① From the expression for the efficiency of the Carnot engine, we see that

$\frac{T_2}{T_1}$ = a fraction because $T_2 < T_1$

$\therefore \quad 1 - \frac{T_2}{T_1}$ = another ~~fra~~ fraction.

Thus $\eta < 1$ i.e $\eta < 100\%$.

Therefore, real engines will have efficiency still less than that of the Carnot engine.

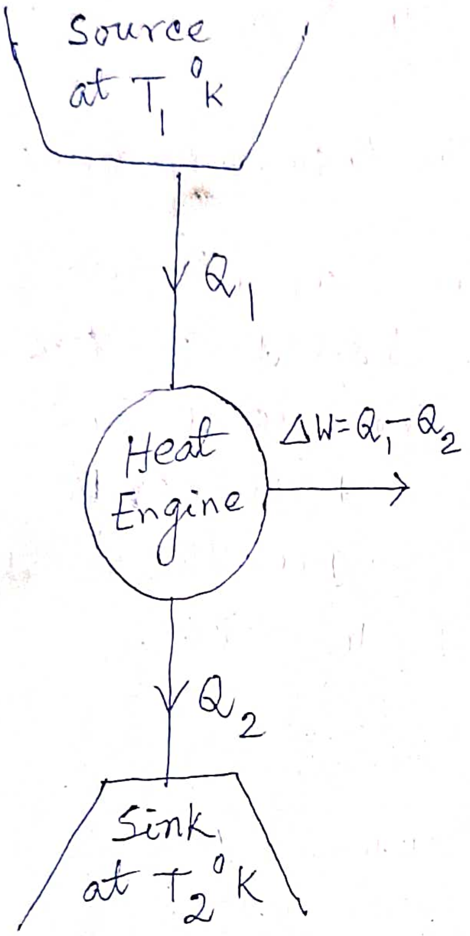
② Only when $T_2 = 0$, $\eta = 1$ or 100%.
But it is very difficult to attain absolute temp. Near 0°K, all the gases get liquefied or become solidfied.
Hence the piston can not compress or make them expand. All the lubricants get solidfied at such a low temperature. Hence the piston will not move.

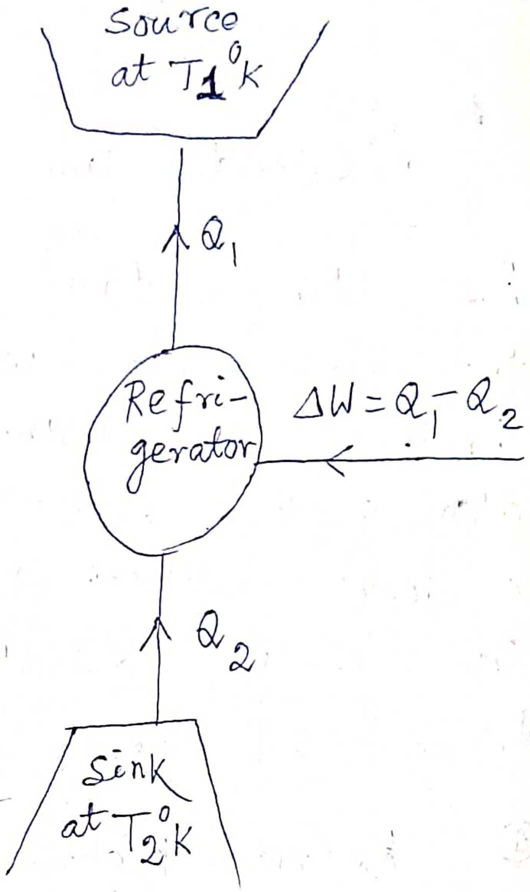
(3) Two Carnot engines operating between the same two temperatures will have the same efficiency. The size or shape is ~~immaterati~~ immaterial. ([illegible])

(4) It is independent of the nature of

The working substance.

Principle of Refrigerator





A refrigerator is nothing but a heat engine operated on reverse direction so that it will extract heat from the sink and rejects heat into the source.

To enable the refrigerator, to extract heat from a colder body, work has to be done on the engine by some external agency (electricity).

Efficiency of the refrigerat
remains the same as that of the heat engine.

$$\therefore \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

Second law of Thermodynamics

As the first law arose from the fact that man failed to derive heat out of nothing, the second law also arose as man failed to derive a 100% efficient heat engine. The second law can be stated by the following three statements..

① Kelvin-Planck Statement :

It is impossible to construct a heat engine which, operating in a ~~simple~~ complete cycle, will abstract heat from a single body, and convert the whole of it to work, without leaving changes in the working system.

② Kelvin's ~~Constant~~ Statement :

It is impossible by means of an inanimate ([illegible]) material agency to derive mechanical effect from any portion of matter

by cooling it below the source?
of the coldest of the sur

② **Clausius's Statement**

It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body ~~at lower~~ at lower temperature to another at a higher temperature.

OR

Heat can not of itself pass from a colder body to a warmer body.

Distinction between the two laws

(or Stress)

The first law asserts in general the equivalence between work and heat implying that mechanical energy and heat are only different forms of energy. Mechanical energy can be completely converted to heat by process like friction, But, it says nothing about the origin of heat, (whether) a cold body or a hot body are simultaneously necessary or not

~~The secon~~

The second law goes remains ~~state~~ step further and imposes ([illegible]) additional restrictions on the first law by asserting that when heat is converted into work only a partial conversion is possible even under the best conditions. Any such conversion essentially requires a hot body and a cold ~~body~~ body simultaneously.

The two laws are empirical (based on experi observation) as they can not be (~~cannot be derived~~) deduced ([illegible]). But, in every sphere of science there predictions ([illegible]) happen giving these laws stronger roots day by day.

Problems

② An inventor claims to have developed an engine working between 27° C and 227° C having an efficiency of 45%. Is his claim valid? why?

~~Ans: As~~ the

Q3) A Carnot engine operates between the temps. 1000°K and 500°K of another Carnot engine of the same efficiency uses the source of the first as its sinks. what will be the temp of the source?

Ans : 2500°K

4. A refrigerator works between 0°C as inside temp and 27°C as outside temp. If 1 Joule of work be done on the engine, Calculate the amount of energy rejected by the engine.

Ans : 11.11 Joule

Hints :

$$\eta = \cancel{B}\,\frac{\text{out put}}{\text{in put}} = \frac{Q_1 - Q_2}{Q_1}$$

$$= \frac{\Delta W}{Q_1} = 1 - \frac{T_2}{T_1}$$

5. A reversible engine converts $\frac{1}{6}$ of the heat input of the work. When the temp of sink is reduced by 62°K efficiency is doubled. Find the temp of source and the sink in °C.

Ans : 99°C , 37°C

① Prove that $T^{\gamma} . P^{1-\gamma}$ = Constant for an ideal gas undergoing adiabatic process

Ans : $PV = nRT$ and PV^{γ} = ~~const~~ = Constant

To eliminate V,

$$(PV)^\gamma = (nRT)^\gamma$$

5. Given $\Delta W = \frac{Q_1}{6}$

$$\therefore \eta = \frac{\Delta W}{Q_1} = \frac{\frac{Q_1}{6}}{Q_1} = \frac{1}{6}$$

$$\Rightarrow 1 - \frac{T_2}{T_1} = \frac{1}{6}$$

$$\Rightarrow \frac{T_2}{T_1} = 1 \cancel{+} - \frac{1}{6} = \frac{5}{6} \quad \text{—(i)}$$

As per the question

$$\eta' = 2\eta = 1 - \frac{(T_2 - 62)}{T_1}$$

$$\Rightarrow \eta' = 2 \times \frac{1}{6} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

$$\Rightarrow \frac{1}{3} = \cancel{\frac{1 - T_2 + 62}{T_1}} \; 1 - \frac{T_2}{T_1} + \frac{62}{T_1} \quad \text{—(ii)}$$

Using eqn (i) in eqn (ii) we get

$$\frac{1}{3} = 1 - \frac{5}{6} + \frac{62}{T_1}$$

$$\Rightarrow \frac{1}{3} + \frac{5}{6} - 1 = \frac{62}{T_1}$$

$$\Rightarrow \frac{2 + 5 - 6}{6} = \frac{62}{T_1}$$

$$\Rightarrow \frac{1}{6} = \frac{62}{T_1}$$

$$\Rightarrow T_1 = 62 \times 6 = 372^\circ K$$

$\therefore \theta_1 + 273 = 372$

$\Rightarrow \theta_1 = 372 - 273 = 99^\circ C$

(Ans)

From eqⁿ (1), $\frac{T_2}{T_1} = \frac{5}{6}$

$$\Rightarrow \frac{T_2}{372} = \frac{5}{6}$$

$$\Rightarrow T_2 = \frac{\overset{62}{\cancel{372}} \times 5}{\cancel{6}} = 310^\circ K$$

$\Rightarrow \theta_2 + 273 = 310^\circ K$

$\Rightarrow \theta_2 = 310 - 273 = 37^\circ C$

(Ans)

①. Eqⁿ of state for an ideal gas is known to be $PV = nRT$

eqⁿ of state for a gas undergoing adiabatic process is

$$PV^\gamma = \text{constant} = K$$

To eliminate V,

$$(PV)^\gamma = (nRT)^\gamma$$

$$\Rightarrow P^\gamma V^\gamma = (nR)^\gamma . T^\gamma$$

Dividing both the sides by PV^γ, we get

$$\Rightarrow \frac{P^\gamma . V^\gamma}{PV^\gamma} = \frac{(nR)^\gamma T^\gamma}{PV^\gamma} = \frac{(nR)^\gamma T^\gamma}{K}$$

$$\Rightarrow P^{\gamma - 1} = \cancel{\text{constant}\ T^\gamma}\ \frac{(nR)^\gamma}{K} . T^\gamma$$

$$\Rightarrow \frac{p^{\gamma-1}}{T^{\gamma}} = \text{constant}$$

$$\Rightarrow \frac{p^{\gamma}}{p} = \cancel{\text{constant}} \; \frac{(nR)^{\gamma}}{K} \cdot T^{\gamma}$$

$$\Rightarrow \frac{p}{p^{\gamma}} = \cancel{\text{constant}} \frac{K}{(nR)^{\gamma}} \frac{1}{T^{\gamma}} = \text{constant} \frac{1}{T^{\gamma}}$$

$$\Rightarrow T^{\gamma} . p^{1-\gamma} = \text{constant}.$$

2. An inventor claims to have developed an engine working between 27° to 227°C. i.e 300°K and ~~600°K~~ 500°K and having efficiency 45%.

$$\boxed{1-\frac{T_2}{T_1} = 1-\frac{300}{500}}$$

When the Carnot working between this temp its efficiency

$$\eta = 1-\frac{T_2}{T_1} = 1-\frac{300}{500} = 1-\frac{3}{5} = \frac{2}{5}$$

% of efficiency $= \frac{2}{5} \times 100 = 40\%$

Carnot engine being the most ideal engine has efficience 40%. So the real engine has efficiency must have efficiency less than 40%.

So inventors claim is incorrect.

3. A Carnot engine operates between 100°K and 500°K, So T_1 = source = 500°K
T_2 = Sink = 100°K

Its efficiency $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{500} = 1 - \frac{1}{5} = \frac{4}{5}$

Another Carnot engine's efficiency is the same with the 1st ~~2nd~~ Carnot engine $= \frac{4}{5}$

It uses the source of first = 500°K when it sinks so T_2 = 500°K

$$\text{efficiency} = \frac{4}{5} = 1 - \frac{T_2}{T_1} = 1 - \frac{500}{T_1}$$

$$\Rightarrow 1 - \frac{4}{5} = \frac{500}{T_1}$$

$$\Rightarrow \frac{1}{5} = \frac{500}{T_1}$$

$$\Rightarrow T_1 = 2500\,°K.$$

∴ The source of 2nd engine 2500°K.

4. A refrigerator works between 0°C i.e 273°K inside temp and 27°C i.e 300°K as outside temperature.

T_2 = 273°K and T_1 = 300°K

$$\text{Efficiency} = \eta = \frac{\text{Outputs}}{\text{Input}} = \frac{Q_1 - Q_2}{Q_1}$$

$$= \frac{\Delta W}{Q_1} = 1 - \frac{T_2}{T_1}$$

1 Joule or work done on the engine

$\therefore \Delta W = 1$ Joule.

$$\Rightarrow \frac{1}{Q_1} = 1 - \frac{273}{300} = \frac{27}{300}$$

$$\Rightarrow Q_1 = \frac{300}{27} = 11.11 \text{ Joule}$$

$\therefore$ 11.11 Joule or energy is rejected by the engine.

6. A Carnot engine works between ice point and steam point. It is desired to increase the efficiency by 20%.

(a) by keeping the temp of the source constant.

(b) by keeping the temp of sink constant.

Calculate the change in temp in two cases.

(Ans : 20°C, 29.5°C)

Ans :- ~~Heat [illegible]~~

Efficiency of the Carnot engine

$$\eta = 1 - \frac{T_2}{T_1}$$

It works between ice point = 0°C = 273K
and steam point = 100°C = 373K

$$\Rightarrow \eta = 1 - \frac{273}{373} = \frac{100}{373}$$

100 will become = 120

1 will become = $\frac{120}{100}$

$\frac{100}{373}$ will become = $\frac{120}{100} \times \frac{100}{373} = \frac{120}{373} = \eta'$

for a and b.

~~By~~ The temp at source

① Constant we have $T_1 = 100 + 273 = 373\,K$

~~[illegible]~~

$$\therefore \frac{120}{373} = 1 - \frac{T_2}{373}$$

$$\Rightarrow \frac{120}{\cancel{373}} = \frac{373 - T_2}{\cancel{373}}$$

$$\Rightarrow T_2 = 373 - 120 = \cancel{27}\,253$$

$\therefore$ Change in the sink temp

$= 273 - 253 = 20°K = 20°C$

(ii) Then the temp of sink = constant

we have $T_2 = 273°K$

$$\therefore \frac{120}{373} = 1 - \frac{273}{T_1}$$

$$\cancel{\Rightarrow \frac{120}{373} = \frac{T_1 - 273}{T_1}}$$

$$\Rightarrow \frac{120}{373} + \frac{273}{T_1} = 1$$

$$\cancel{\Rightarrow \frac{120 + T_1}{373\, T_1}}$$

$$\Rightarrow \frac{273}{T_1} = 1 - \frac{120}{373} = \frac{373 - 120}{373} = \frac{253}{373}$$

$$\therefore T_1 = \frac{273 \times 373}{253} = 402.48^\circ K$$

Change in the source temp

$\Delta T_1 =$ ~~373~~ $402.48^\circ K - 373^\circ$

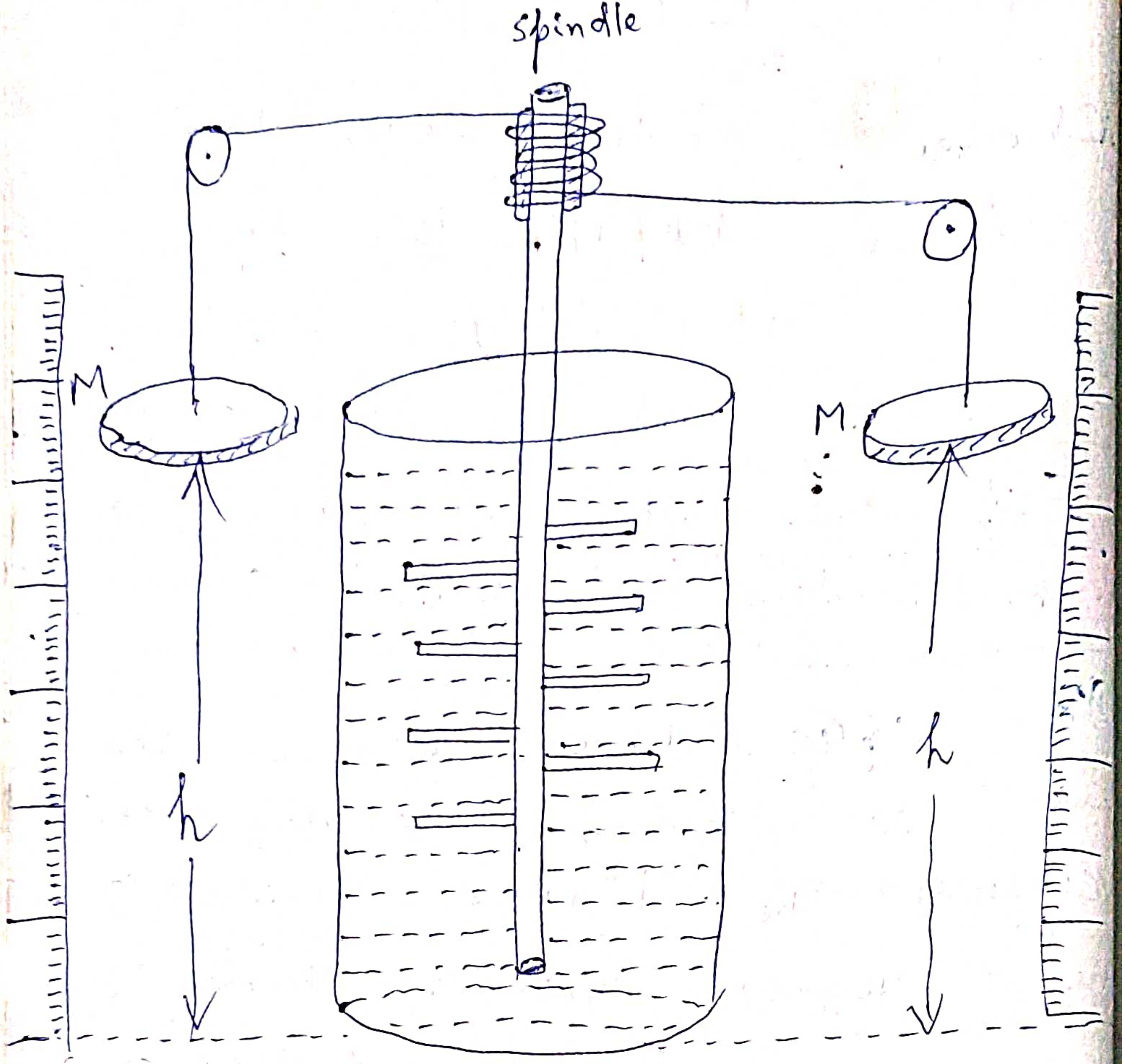
$$= 29.48^\circ K$$

$$= 29.48^\circ C$$

Experimental Determination of 'J' in the laboratory by using Joule's calorimeter

In 1849 Joule determined the value of mechanical equavalant of heat accurately. The princible involved in the experiment consists of (मथना) churning water by doing work from out side. The resulting rise in temp be noted.

A specially constructed copper calorimeter containing water was taken. A spindle having 8 vanes is placed centrally. The upper ~~of~~ part of the spindle was having a wooden frame so that heat can not conduct from the calorimeter to surrounding. A string is wound several times on the wooden frame and it passes over two pulleys and terminates



at two heavy masses M each. These two masses are kept at some height 'h' above the ground level. When these masses are allowed to fall, they will rotate the spindle and hence churned the water. This falling plate experiment has to be repeated many time to get an appreciable rise of temp.

Calculation :- ~~Let m~~

Let m = Mass' of water taken in the

Calorimeter.

M = Mass of each weight

h = height through which each mass has to fall.

n = number of falls.

v = velocity of the weights when they reach the ground.

W = Water equavalant of the Calorimeter,
$= m_1 s_1$

m_1 = mass of the Calorimeter.

s_1 = Specific heat of the Calorimeter.

$\Delta\theta$ = Rise in temp at the end of the experiment.

Potential energy of each mass at the raised position $= Mgh$

Kinetic energy of each mass just before striking the ground $= \frac{1}{2} Mv^2$

Energy supplied to the water by both the masses during one fall

~~$2(Mgh) - \frac{1}{2}Mv^2)$~~

$= 2\left(Mgh - \frac{1}{2}Mv^2\right)$

Energy supplied to the water by both the masses during n falls

$$= 2n\left(Mgh - \frac{1}{2}Mv^2\right)$$

Heat produced ~~of~~ out of this ~~Man~~ much of work

$$= \frac{2n\left(Mgh - \frac{1}{2}Mv^2\right)}{J} \text{ Cal} \quad \text{—(i)}$$

(~~because of~~ ($\because W = JH$)

This much of heat raises the temp of water and Calorimeter.

$$\therefore Q = m \cdot 1 \cdot \Delta\theta + m_1 s_1 \Delta\theta$$

$$= (m + W)\Delta\theta \quad \text{——(ii)}$$

Equating these two expressions (i) and (ii), we get

$$\frac{2n\left(Mgh - \frac{1}{2}Mv^2\right)}{J} = (m + W)\Delta\theta$$

$$\Rightarrow J = \frac{2n\left(Mgh - \frac{1}{2}Mv^2\right)}{(m+W)\Delta\theta} \text{ Joule/Ca}$$

Precautions

1. Corrections ~~per~~ for the energy converted into sound must be made.

2. Corrections should be made for the losses due to conduction, radiation and energy lost due to friction

3. Joule assumed the specific heat of water to be 1 at all temperatures. which is not correct.

4. A Thermometer having very small least count should be use because the rise of temp. is ~~extra~~ extremey small.

Result :-

The value of J found by Joule was 773.4 ft lb/BTU.

Later on more refined experiments gave the value of J as 778 ft lb/BTU

Thermodynamics scale and absolute scale of temperature.

In the investigation and measurement of temp, we have seen that all scale depend upon the properties of particular substances such as expansion of liquid or gas, change in the electrical resistance of platinum etc. Hence, the readings for any fixed temp. appeares differently in different thermometers based on differement properties or substances.

With the introduction of Carnots ideal heat engine (whose efficiency is entirely independent of the nature of the working substances) and from Carnot's theorem that the efficiency of all reversible engines working between two temps is a function of the two temps only, Kelvin realised the possibility of obtaining a non - arbitrary scale of temp. in terms of mechanical energy.

If Q_1 is the quantity of heat absorbed at a higher temp. θ_1 and Q_2 be the quantity of heat given out at a lower temp. θ_2 (where θ_1 and θ_2 are the temp measured on any arbitary scale), then efficiency η is given by

$$\eta = \frac{\Delta W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

but, efficiency of all reversible engines working between ~~m~~ two temps. θ_1 and θ_2 is a function of the two temps only.

Thus,

$$\eta = f(\theta_1, \theta_2)$$

$$\Rightarrow 1 - \frac{Q_2}{Q_1} = f(\theta_1, \theta_2)$$

$$\Rightarrow 1 - f(\theta_1, \theta_2) = \frac{Q_2}{Q_1}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{1}{1 - f(\theta_1, \theta_2)} = F(\theta_1, \theta_2) \quad \text{(i)}$$

where F denotes some other unknown function of θ_1 and θ_2

Similarly if we have another reversible heat engine working between the temps θ_2 and θ_3 $(\theta_2 > \theta_3)$, then

$$\frac{Q_2}{Q_3} = F(\theta_2, \theta_3) \quad \text{(ii)}$$

Multiplying eqns (i) and (ii),

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_2) \times F(\theta_2, \theta_3) \quad \text{(iii)}$$

If we have another reversible heat engine working between the ~~the~~ temps θ_1 and θ_3 such that it absorbs an amount of heat Q_1

at a temp. θ_1 and rejects an amount of heat Q_3 at a temp θ_3,

then $$\frac{Q_1}{Q_3} = F(\theta_1, \theta_3) \text{ —— (iv)}$$

Compareing eqn (iii) and (iv), we get

$$F(\theta_1,\theta_3) = F(\theta_1,\theta_2) \times F(\theta_2,\theta_3) \text{ ——— (v)}$$

From eqn (5) we find that

θ_2 is absent on the L.H.S but present in both the terms of the R.H.S.

∴ Therefore, the form must be given by $$F(\theta_1,\theta_2) = \frac{\psi(\theta_1)}{\psi(\theta_2)}$$

For any reversible heat engine

we have $$\frac{Q_1}{Q_2} = F(\theta_1,\theta_2) = \frac{\psi(\theta_1)}{\psi(\theta_2)} \text{ —(v}$$

If $\theta_1 > \theta_2$, then

$Q_1 > Q_2$ so that

$$\psi(\theta_1) > \psi(\theta_2)$$

i.e $\psi(\theta)$ is a <u>linear</u>

function of θ. It may be used to measure temp.

Representing $\overset{(\text{साइ})}{\psi(\theta)}$ as τ (टाउ) (~~The~~ on the thermodynamic scale), we get

$$\frac{Q_1}{Q_2} = \frac{\tau_1}{\tau_2} \qquad \text{—— (VIII)}$$

This relation can be used to define a new scale of temp which does not depend upon the properties on any particular substance. The ratio of any two temps on this scale is equal to the ratio of the heat taken in and heat rejected by an engine working reversibly between the two temps.

Hence it is called absolute scale or thermodynamic scale

Relation with perfect gas scale and Calibration of the absolute Scale.

The efficiency of the reversible engine on the thermodynamics

Scale is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{\tau_2}{\tau_1}$$

$$= \frac{\tau_1 - \tau_2}{\tau_1}$$

In order that the efficiency may be 1, τ_2 should be zero. But $\tau_2 = 0$

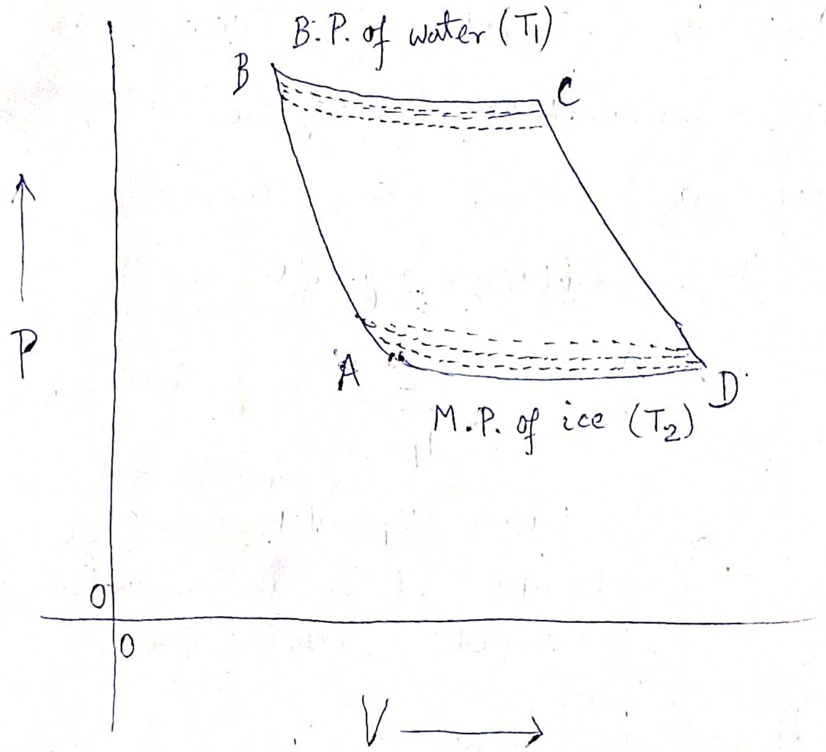
$$\Rightarrow Q_2 = 0$$

i.e all the heat taken in by the engine has been converted into work. τ_2 can not be less than zero (i.e −ve) (because) for Q_2 will be negative which implies that the engine would be drawing heat both from the source and the sink which is impossible according to the second law of thermodynamics.

$\tau_2 = 0$ is the lowest (Think) temp that can be conceived. This is the thermodynamic defn

06 absolute Zero of temp.

To determine the size of the degree on the thermodynamic scale, consider a Carnot reversible engine working between the temps of boiling water and melting ice. The Carnot cycle is represented by ABCDA and the work done by the engine is numerical eaual to the area of the loop, ABCDA.



Let this area be divided into 100 equal parts by isothermals drawn parallel to BC and AD. Then, the area of each strip represents 1° on the thermodynamic scale.

"Hence, a thermodynamic degree is defined as the difference in temperature between which a Carnot's engine should be worked to get energy equal to $\frac{1}{100}$th of the energy obtained from Carnot's engine working between temps of boiling water and melting ice."

Relation with perfect gas scale

We have already seen that for a reversible engine using a perfect gas as the working substance, the efficiency is

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 and T_2 are the temps of the source and sink measured in perfect gas scale.

but $$\eta = 1 - \frac{\tau_2}{\tau_1}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{\tau_2}{\tau_1} \quad \text{——— (viii)}$$

This relation shows that the ratio of any two temps. on the perfect gas scale and the thermodynamic scale are equal.

If $\tau_2 = 0$, Then $T_2 = 0$

Thus, the zero of the thermodynamic scale coincides with the zero of the perfect gas scale.

If T_1 be the temp of boiling water and T_2, that of melting ice, measured on the perfect gas scale, then

$$T_1 - T_2 = 100$$

Also, on the thermodynamics scale we have, for the same two fixed temps. $\tau_1 - \tau_2 = 100$ as found under calibration.

$$\text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{\tau_2}{\tau_1}$$

$$\Rightarrow \frac{T_1 - T_2}{T_1} = \frac{\tau_1 - \tau_2}{\tau_1}$$

$$\Rightarrow \frac{100}{T_1} = \frac{100}{\tau_1}$$

$\Rightarrow T_1 = \tau_1$

This means that the temps. of the boiling point of water and melting point of ice are identical on the two scales. In a similar manner, it can be shown that any other temperature will have the same value on the two scales. Hence, the two scales are identical.

It may be noted that, although the perfect gas scale cannot be realised directly, since actual gases only tend to the perfect gas at very low pressure. But scales of actual gas thermometers such as constant volume hydrogen gas thermometer, can be reduced to the perfect gas scale.

Relation between thermodynamic scale and absolute scale (2 marks)

Different thermometers based

on the properties of different materials so different reading for the same temps. Kelvin suggested the use of expression for the efficiency of the Carnot heat engine to prepare a thermodynamic scale. This thermodynamic scale is independent of nature of the working substance.

$$\text{Efficiency}, \eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{\tau_2}{\tau_1}$$

Using the above expressions, it has been proved that thermodynamic scale is identical to the absolute scale

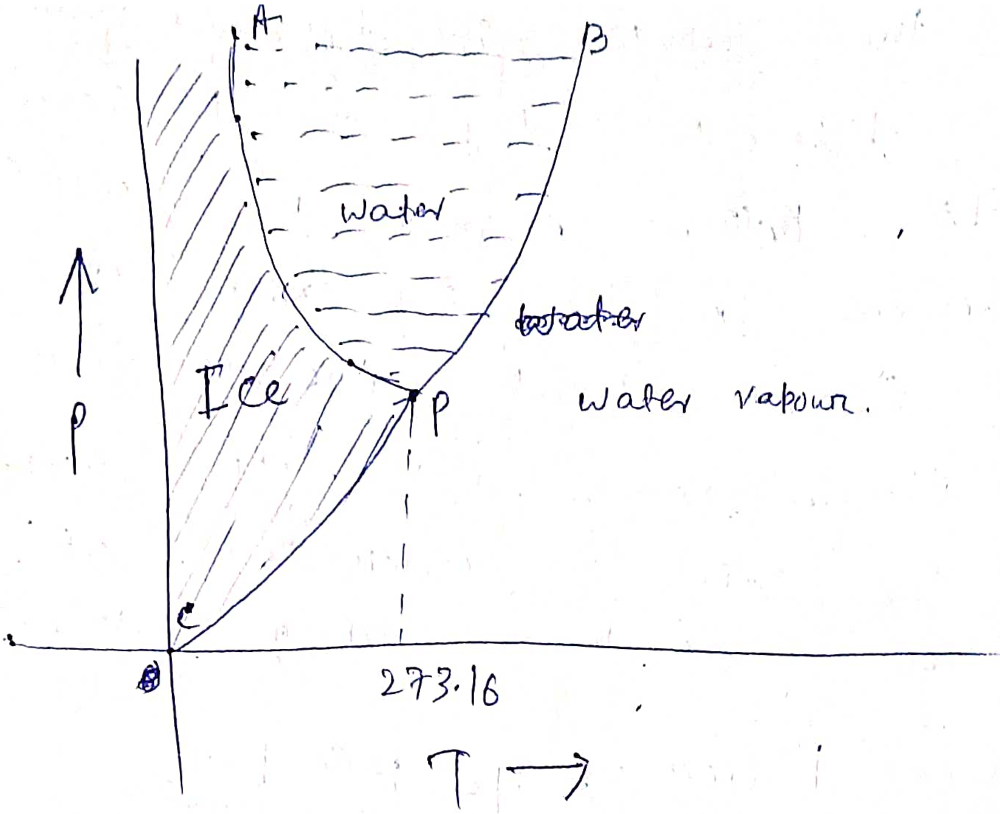
---

Triple point :-

Triple point is a temperature when the solid state, liquid state and gaseous state of a substance co-exist.

Ex:- At 0°c or 273.16°K, ice, water, and water vapour co-exist.

The line AP indicates the solid-liquid curve. It has



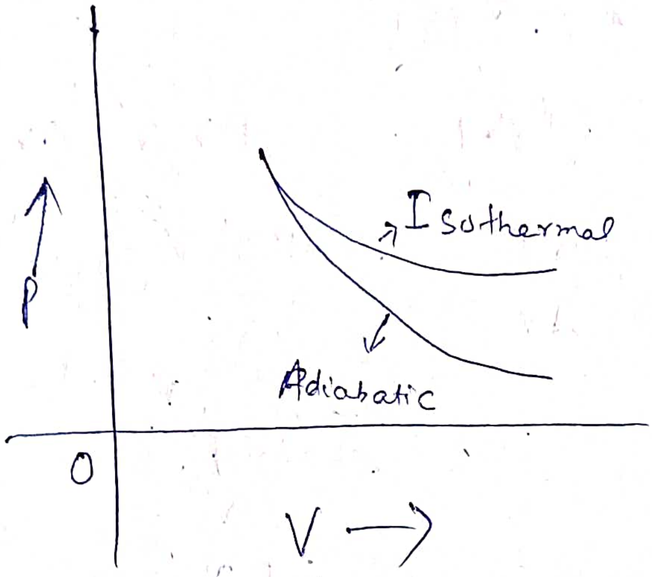
a -ve slope. This indicates that melting point of water decreases due to an increase ~~of~~ in ~~of~~ pressure.

The curve BP represents the liquid vapour curve. Its slope is +ve. This indicates that the boiling point of water increases due to an increase in pressure.

The curve CP represents the solid-vapour curve (Called Hoar-frost line). This is the sublimation curve.

Triple point of a substance is a single unique temp. and it is independent of pressure

Slope Of the isothermal and idiabatic curves



For an ideal gas undergoing Isothermal process Boyle's law holds good.

$\therefore$ PV = Constant.

Differnciating both the sides, we get

$$P.\Delta V + V.\Delta P = 0$$

~~$\Rightarrow P\Delta V =$~~

$$\Rightarrow V\,\Delta P = -P\Delta V$$

$$\Rightarrow \frac{\Delta P}{\Delta V} = \frac{-P}{V} \quad \text{———(i)}$$

$$= \text{Slope}$$

If the ideal gas undergoes idiabatic process, then the eqn of state will be

$$PV^{\gamma} = \text{Constant.}$$

Differnciating both the sides of the above eqn, we get

$$P \cdot \gamma \cdot V^{\gamma-1} \Delta V + \Delta P \cdot V^{\gamma} = 0$$

$$\Rightarrow \quad V^{\gamma} \Delta P = -P \gamma V^{\gamma-1} \Delta V$$

$$\Rightarrow \quad \frac{\Delta P}{\Delta V} = -\frac{P \gamma V^{\gamma-1}}{V^{\gamma}}$$

$$= \frac{-P \gamma \, V^{\gamma} \cdot V^{-1}}{V^{\gamma}}$$

$$= \frac{-P\gamma}{V} \quad \text{—(ii)}$$

$$= \text{Slope}$$

From eqn (i) and (ii), we have

$$\frac{\text{Slope of the adiabatic curve}}{\text{Slope of the isothermal curve}} = \frac{-\frac{P\gamma}{V}}{-\frac{P}{V}} = \gamma$$

Since $\gamma = \frac{C_P}{C_V}$ and $C_P > C_V$,

we have $\gamma > 1$

$\Rightarrow$ Slope of the adibatic curve > Slope of the isothermal curve

Problems

1. Two rods A and B are of equal length. Each rod has its ends at temperature θ_1 and θ_2 $(\theta_1 > \theta_2)$. What should be the condition for equal rates of flow of heat through the rods. If their are radii r_1 and r_2 and thermal conductivities K_1 and K_2 .. (Ans : $\frac{r_1}{r_2} = \sqrt{\frac{K_2}{K_1}}$)

2. 1 gm of steam is passed into a calorimeter containing 100 gm of ice. Calculate the quantity of ice melted. (Ans : 7.95 gm)

3. Some hot water is added to 4 times its mass of water at 10°C. Calculate the temp of hot water if the resulting temp is 25 °C.

(Ans: 85°C),

4. A Carnot engine takes 100 calory from a reservoir at 427°C and performs 60 J of work. Calculate the (a) The amount of heat absorb in Joule

(b) Heat rejected in Joule

(c) efficiency of engine

(d) Temp. of the cold reservoir.

(Ans: 420 J, 360 J, 14.28%, 327°C)

5. A liquid takes 5 minute to cool from 80°C to 50°C, How much time will it take to cool from 60°C to cool 60°C to 30°C, if the surrounding is at 20°C

(Ans: 10 minute,)

Ans 5. we know that

$$\log_e\left(\frac{\theta_1-\theta_0}{\theta_2-\theta_0}\right) = K_1 \Delta t$$

where K_1 = a constant

θ_1 = initial Temp, θ_2 = Final temp after t sec

θ_0 = Surrounding temp.

Δt = time taken to cool ~~from~~ from θ_1°C to θ_2°C.

Here $\theta_1 = 80°C$, $\theta_2 = 50°C$

$\theta_0 = 20°C$, $\Delta t = 5$ minutes

$$\therefore \log_e\left(\frac{80-20}{50-20}\right) = K_1 \times 5$$

$\Rightarrow \log_e 2 = K_1 \times 5$

$\Rightarrow K_1 = \dfrac{\log_e 2}{5}$

For second case

$\theta_1 = 60^\circ, \quad \theta_2 = 30^\circ$

$\theta_0 = 20^\circ. \quad \Delta t = ?$

$\log_e \left(\dfrac{60-20}{30-20} \right) = K_1 . \Delta t$

$\Rightarrow \log_e 4 = \dfrac{\log_e 2}{5} . \Delta t$

$\Rightarrow 2 . \cancel{\log_e 2} = \dfrac{\cancel{\log_e 2}}{5} \Delta t$

$\Rightarrow \Delta t = 10$ minutes.

1. The amount of heat passing through rod A $= Q_1 = \dfrac{K.A.(\theta_1 - \theta_2).t}{l}$

$= \dfrac{K_1 . \pi r_1^2 . (\theta_1 - \theta_2) 1}{l}$

The amount of heat passing through rod B $Q_1 = \dfrac{K_2 . \pi r_2^2 (\theta_1 - \theta_2) . 1}{l}$

But according to question $Q_1 = Q_2$

$\Rightarrow \dfrac{K_1 \cancel{\pi} r_1^2 \cancel{(\theta_1 - \theta_2)} . \cancel{1}}{\cancel{l}} = \dfrac{K_2 \cancel{\pi} r_2^2 \cancel{(\theta_1 - \theta_2)}}{\cancel{l}}$

$$\Rightarrow K_1 r_1^2 = K_2 r_2^2$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{K_2}{K_1}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{K_2}{K_1}}$$

2. Heat lost by Steam = Heat gained by ice

(Considering Calorimeter is bad conductor of heat)

$$\Rightarrow m_1 L_v + m_1 s_1 \Delta\theta_1 = m L_f$$

$$\Rightarrow 1 \times 536 + 1 \times 1 \times (100-0) = m \times 80$$

$$\Rightarrow 5.36 + 100 = m_2 \times 80$$

$$\Rightarrow 636 = m_2 \times 80$$

$$\Rightarrow m_2 = \frac{636}{80} = 7.95 \text{ gm}$$

Amount of ~~heo~~ ice melted = 7.95 gm

3. Let the mass of water = m

~~wh~~ The mass of water at 10°C = 4m

Heat lost by hot water = Heat gained by 10°C water

$$\Rightarrow \cancel{m}\,\cancel{s}\,(\theta - 25) = 4\cancel{m}.\cancel{s}(25-10)$$

$$\Rightarrow \theta - 25 = 60$$

$$\Rightarrow \theta = 85^\circ C$$

∴ The temp of hot water 85°C.

4. $\eta = 1 - \frac{Q_2}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{\Delta W}{Q_1} = 1 - \frac{T_2}{T_1}$

$Q_1 = 100$ cal, $T_1 = 427°C$
$= (100 \times 4.2) = 420 J$ $\quad = 700°K$
$\Delta W = 60 J$

$$\frac{\Delta W}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{60}{100} = 1 - \frac{T_2}{427}$$

$$\Rightarrow ~~\frac{3}{5} \quad 427~~$$

$$\Rightarrow \frac{T_2}{427} = 1 - \frac{3}{5} = \frac{2}{5}$$

Temp of sink $\Rightarrow T_2 = \frac{427 \times 2}{5} = \frac{854}{5}$

$$= 170.8°C$$
$$~~273~~$$
$$= 443.8°K$$

Efficiency of engine

$$= 1 - \frac{T_2}{T_1} = \left(1 - \frac{443.8}{700}\right) \times 100$$

$$\Rightarrow \left(\frac{700 - 443.8}{700}\right) \times 100$$

$$= \frac{256.2}{700} \times 100$$

$$\frac{\Delta W}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{60}{\ } = 1 - \frac{T_2}{700}$$

$$\Rightarrow \frac{T_2}{700} = 1 - \frac{1}{7} = \frac{6}{7}$$

$$\Rightarrow T_2 = \frac{\cancel{700}^{100} \times 6}{\cancel{7}} = 600^\circ K$$

$$= 327^\circ C$$

(d) :. temp of cold reservoir = 327°C

(a) Amount of heat absorbed

= 100 cal = 100 × 4.2 = 420 Joule.

(b) we know $\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$

$$\Rightarrow \frac{70\cancel{0}}{60\cancel{0}} = \frac{420}{Q_2}$$

$$\Rightarrow Q_2 = \frac{6 \times \cancel{420}^{60}}{\cancel{7}}$$

$$= 360 \text{ Joule}$$

(c) Efficiency $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

$$= \left(1 - \frac{600}{700}\right) \times 100\%$$

$$= \frac{100}{700} \times \cancel{100}$$

$$= 14.28\%$$