Mod-III Microwave Sources – Ch-5 – Reflex klystron

Sources for generation of microwave are classified mainly into 2 categories.

1) Tubes
2) Microwave Solid State devices

Tubes

- Linear Beam ~~Type~~ Tube (O-Type)
- Crossfield Beam Tube (M-Type)

Linear Beam ~~Type~~ Tube

→ In Linear-beam tube, a magnetic field whose axis coincides with that of the electron beam, is used to hold the beam together as it travels the length of the tube.

→ O-type derived from the French name TPO (tubes à propagation ~~dessous~~ des ondes) ~~for~~ or the ~~so~~ word 'original' (meaning the original type of tube)

Examples of 'O' type

→ Two Cavity Klystron
→ Reflex Klystron ✓
→ Helix Travelling-wave Tube (TWT) ✓
→ Coupled-Cavity TWT
→ Forward wave Amplifier (FWA)
→ Backward wave Amplifier & Oscillator (BWA / BWO)

Cross-field Beam Tube :-

→ Here ~~the~~ the d.c electric field and the d.c magnetic field are perpendicular to each other.

→ Derived from the french TPOM (Tubes a' Champs magnétique: propagation des ondes a' Tubes for propagation of waves in a magnetic field)

Example of M-type:-

→ Magnetron

→ Forward wave cross field amplifier (FWCFA)

→ Dematron

→ Amplitron

→ Carcinotron

→ Gyrotrons.

2) Solid State devices

- Microwave Transistor
  - Ex:- Microwave BJT
  - HBT
  - Tunnel diode
  - HBT → Hetro Junction Bipolar Transistor
- FET
  - Ex:- JFET, MESFET
  - MOSFET,
  - NMOS, PMOS, CMOS,
  - CCD → Charge Coupled devices.
- Transferred Electron devices
  - Ex:- Gunn diode,
  - LSA diode (Limited space charge accumulation diode)
  - In P diode (Indium Phosphide diode)
- Avalanche transit-time devices
  - Ex:- Read diode,
  - IMPATT
  - TRAPATT
  - BARITT
  - Read → Name of the inventor.

- IMPATT → Impact ionization avalanche transit time diode

TRAPATT → Trapped Plasma " triggered transit-time diode

BARITT → Barrier injected transit-time diodes.

Reflex Klystron :- (Liao Book)

If a fraction of the O/P power is fed back to the I/P Cavity and rf loop gain has a magnitude of unity with a phase shift of multiple of 2π, the klystron will oscillate.

However, a two-cavity klystron oscillator is usually not constructed because, when the oscillation freq is varied, the resonant freq of each cavity and feedback path must be readjusted for a +ve phase shift feedback.

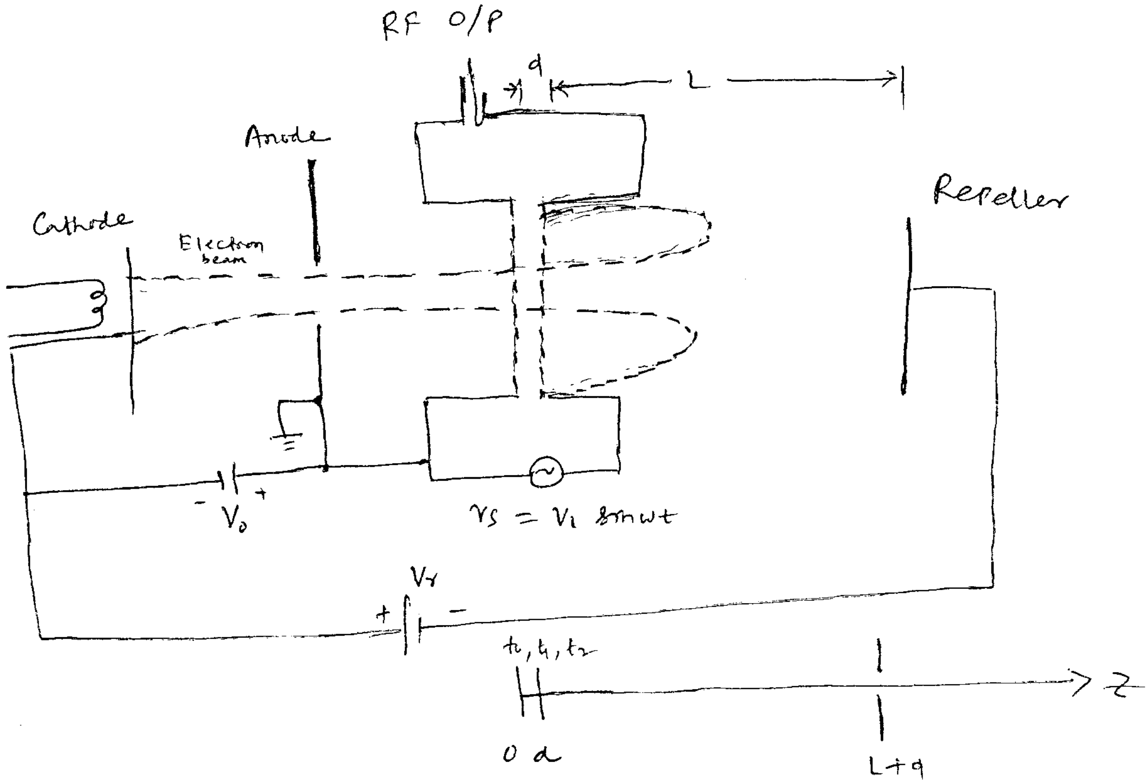
The Reflex Klystron ~~overcomes the~~ is a single Cavity klystron that over comes the disadvantage of 2-Cavity klystron ~~amplifier~~ oscillator.

→ It is a low power generator (10 - 500 mW) at a freq range (1 to 25 GHz)

→ Efficiency (20 to 30 %)

→ Widely used in laboratory for microwave measurements and in microwave receivers as local oscillators in commercial, military and airborne Doppler radars as well as missiles.

→ A schematic diagram of the reflex klystron is shown in fig 51. [Name Reflex → Electron beams are reflected back]



t_0 → time for electron entering cavity gap at $Z=0$

t_1 → time for same electron leaving cavity gap at $Z=d$

t_2 → time for same electron returned by retarding field $Z=d$ and collected on walls of cavity.

fig :- 51 : Schematic diagram of a reflex klystron.

The electron beam injected from the cathode is first velocity — modulated by the cavity gap voltage (V_s). Some electrons [(Point A) fig 52] accelerated by the accelerating field enter the repeller space with greater velocity than those with unchanged velocity. [(Point B) fig:-52]. Some electrons decelerated [Point (C) fig:52] by the retarding field enter the repeller region with less velocity.

→ All electrons turned around by the repeller voltage then pass through the cavity gap in bunches that occur once per cycle.

→ On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field and give up their kinetic energy to the electromagnetic energy of the field in the cavity.

→ Oscillator o/p energy is then taken from the cavity. The electrons are finally collected by the walls of the cavity or other grounded metal part of the tube.

→ Fig (52) shows the Applegate diagram ~~of~~ for $1\frac{3}{4}$ mode of a Reflex klystron.

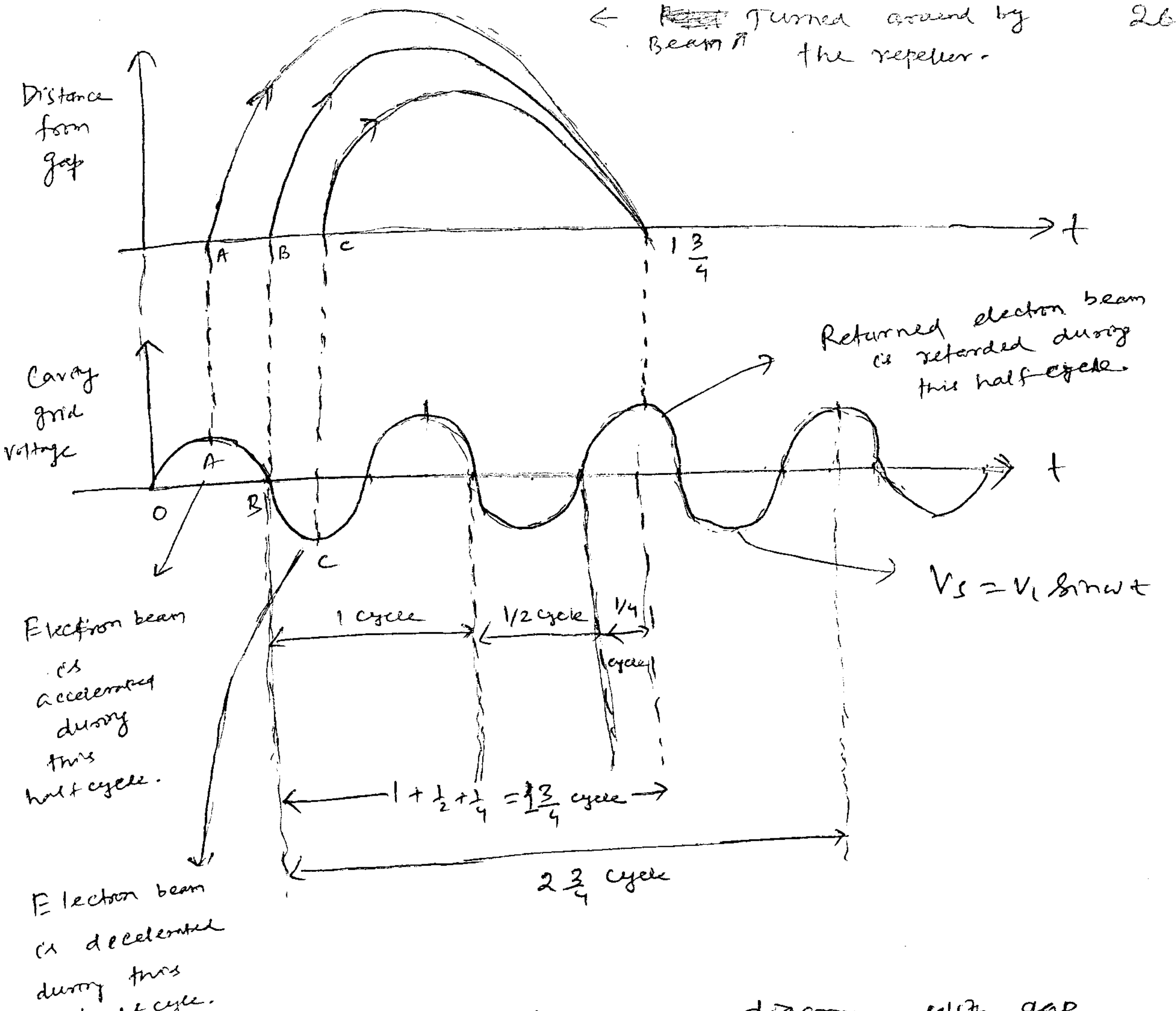


Fig 52:- Applegate diagram with gap voltage for a ($1\frac{3}{4}$ mode of) reflex klystron.

Note :-

→ By adjusting repeller voltage for a given dimension of reflex klystron, the bunching can be made to occure at $N = n + \frac{3}{4}$ ~~Positive half~~ cycles.

→ Accordingly the mode of oscillation is named as $N = \frac{3}{4}, 1\frac{3}{4}, 2\frac{3}{4}$ etc, for $n = 0, 1, 2, 3$ ---

→ Lowest order mode $\frac{3}{4}$ occure for a maxm value of repeller voltage.

→ Higher order modes occure at lower repeller voltages.

→ Since at the highest repeller voltage, the acceleration of the bunched electron on return is maxm, the power o/p of the lowest mode is maxm.

Velocity Modulation :-

The electron entering the cavity gap from Cathode, at $Z=0$ and time 't_0' is assumed to have uniform velocity

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad \text{--- (1)} \qquad \left| \begin{array}{l} \because \frac{1}{2}mv^2 = eV_0 \\ \Rightarrow v = \sqrt{\frac{2eV_0}{m}} \end{array} \right.$$

$$\Rightarrow v_0 = \sqrt{\frac{2\times1.6\times10^{-19}}{9.1\times10^{-31}}}\ V_0$$

$$\Rightarrow v_0 = 0.593\times10^{6}\sqrt{V_0} \quad \text{--- (1A)}$$

In eqn (1) it is assumed that electron leave the Cathode with zero velocity. When an sinusoidal i/p signal is applied, the gap voltage between the buncher ~~...~~ the grids appears as

$$V_s = V_1 \sin(\omega t) \quad \text{--- (2)}$$

where V_1 is the amplitude of the signal and $V_1 << V_0$ assumed.

In order to find the modulated velocity

in the cavity in terms of either the entering time 't_0' or exiting time 't_1' and the gap transit angle 'θ_g' as shown in fig 51, it is necessary to determine the average microwave voltage on the cavity gap as indicated in fig 53.

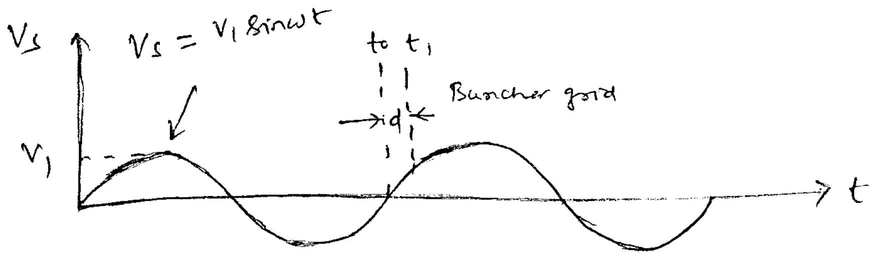


fig 53:- Signal voltage in the buncher gap.

Since $V_1 << V_0$, the average transit time through the buncher gap distance 'd' is

$$\tau = \frac{d}{v_0} = t_1 - t_0 \quad \text{——} \; (3)$$

The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \quad \text{——} \; (4)$$

The average microwave voltage in the gap can be found in the following way

$$\langle V_s \rangle = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt$$

$$= \frac{1}{\tau} V_1 \left[-\frac{\cos\omega t}{\omega}\right]_{t_0}^{t_1} = -\frac{V_1}{\omega\tau}\left[\cos\omega t_1 - \cos\omega t_0\right]$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos\omega t_1\right]$$

$$\Rightarrow \langle V_s \rangle = \frac{V_1}{\omega\tau}\left[\cos \omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{V_0}\right)\right] \quad \text{— (5)}$$

$$\left[\because \; \omega(t_1 - t_0) = \frac{\omega d}{V_0} \quad \text{eqn (3) \& (4)}\right]$$

$$\left[\Rightarrow \omega t_1 = \omega t_0 + \frac{\omega d}{V_0}\right]$$

Let

$$\omega t_0 + \frac{\omega d}{2V_0} = \omega t_0 + \frac{\theta_g}{2} = A \quad \left[\text{From eqn (4)}\right]$$

and

$$\frac{\omega d}{2V_0} = \frac{\theta_g}{2} = B$$

$$\therefore \cos \omega t_0 = \cos(A - B)$$

$$\cos\left(\omega t_0 + \frac{\omega d}{V_0}\right) = \cos(A + B)$$

$$\left[\begin{aligned} &\because A + B = \omega t_0 + \frac{\omega d}{2V_0} + \frac{\omega d}{2V_0} \\ &\Rightarrow A + B = \omega t_0 + \frac{\omega d}{V_0} \\ &A - B = \omega t_0 + \cancel{\frac{\omega d}{2V_0}} - \cancel{\frac{\omega d}{2V_0}} = \omega t_0 \end{aligned}\right]$$

$\therefore$ Eqn (5) becomes,

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau} \, 2 \sin A \cdot \sin B$$

$$\Rightarrow \langle V_s \rangle = \frac{2V_1}{\omega\tau} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) \cdot \sin\left(\frac{\omega d}{2V_0}\right)$$

$$= \frac{2V_1}{\omega\left(\frac{d}{V_0}\right)} \sin\left(\frac{\omega d}{2V_0}\right) \cdot \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right)$$

$$\langle V_s \rangle = V_1 \cdot \frac{\sin\left(\frac{\omega d}{2V_0}\right)}{\left(\frac{\omega d}{2V_0}\right)} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right)$$

$$\langle V_s \rangle = V_1 \frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\theta_g/2\right)} \cdot \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right) \quad \text{— (6)}$$

Lets take $\beta_i = \dfrac{\sin(\theta_g/2)}{(\theta_g/2)} = \dfrac{\sin\left(\frac{\omega d}{2v_0}\right)}{(\omega d/2v_0)}$ —— (7)

eqn (3) & (4)

β_i is known as the beam coupling coefficient of the input cavity gap.

(4)

$\therefore \langle V_s \rangle = V_1 \cdot \beta_i \cdot \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right)$ —— ~~(6)~~

$\langle V_s \rangle = \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$ —— (8)

$\frac{\omega d}{2v_0} + \frac{\omega d}{2v_0}$

$\frac{\omega d}{2v_0} = \omega t_0$

Immediately after velocity modulation, the exit velocity from the buncher gap is given as $v(t_1)$

From eqn (1),

$$v_s(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + \langle V_s \rangle\right)}$$

$$= \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$v_s(t_1) = \sqrt{\frac{2e}{m} V_0 \left\{1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]} \quad \text{—— (9)}$$

where the factor $\frac{\beta_i V_1}{V_0}$ is called the depth of velocity modulation

Using the binomial expansion under the assumption of $\beta_i V_1 \ll V_0$

$(1+x)^n \simeq 1 + nx$ where $x \ll 1$

—— (6)

Eqn (9) becomes,

$$v_s(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{—— (10)}$$

$\left(\because \sqrt{\frac{2eV_0}{m}} = v_0\right.$ and $\left[1+\frac{\beta_i V_1}{V_0}\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]^{\frac{1}{2}}$

$$= 1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0+\frac{\theta_g}{2}\right)$$

(By Binomial expansion $(1+x)^n \simeq 1+nx$]

Eq^n (10) is equation of velocity modulation.

From eq^n (10),

$\cancel{\omega t_0+\frac{\theta_g}{2} = \omega t_0 + \frac{\omega(t_1-t_0)}{2}}$ $\quad$ $\left(\because \theta_g = \omega(t_1-t_0) \text{ eq}^n\ 4\right]$

$\Rightarrow \theta_g = \omega t_1 - \omega t_0$

$\Rightarrow \omega t_0 = \omega t_1 - \theta_g$

$\cancel{= \omega t_0 + \frac{\omega t_1}{2} - \frac{\omega t_0}{2}}$

$\cancel{= \frac{\omega t_1}{2} + \frac{\omega t_0}{2}}$

From eq^n (10), $\omega t_0 + \frac{\theta_g}{2} = \omega t_1 - \theta_g + \frac{\theta_g}{2}$

$$\Rightarrow \omega t_0 + \frac{\theta_g}{2} = \omega t_1 - \frac{\theta_g}{2} \quad \text{—— (11)}$$

Using eq^n (11) in eq^n (10), ~~we have~~

∴ The velocity modulation can alternatively written as,

$$v(t_1) = v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \text{—— (12)}$$

The same electron is forced back to the cavity $z=d$ at time t_2, by retarding electric field E, which is given by

$$E = \frac{V_r + V_0 + V_1\sin\omega t}{L} \quad \text{—— (13)} \quad \left(\because E = \frac{V}{l}\right)$$

This retarding field E is assumed to be constant in the Z-direction.

The force equation for one electron in the repeller region is

$$m\frac{d^2 z}{dt^2} = -eE = -e\,\frac{V_r + V_0}{L} \qquad (14)$$

where $E = -\nabla V$ is used in the Z-direction only, V_r is the magnitude of repeller voltage and $V_1 \sin\omega t \ll V_r + V_0$, is assumed.

Integrating eqn (14), we have

$$\frac{dz}{dt} = -\frac{e}{mL}(V_r + V_0)\cdot\int_{t_1}^{t} dt$$

$$\frac{dz}{dt} = \frac{-e(V_r+V_0)}{mL}\,(t - t_1) + K_1 \qquad (15)$$

At $t = t_1$, $K_1 = \frac{dz}{dt} = v(t_1)$ — (16)

Integrating eqn (15), we have

$$z = \frac{-e(V_r+V_0)}{mL}\int_{t_1}^{t}(t - t_1)\,dt + \int_{t_1}^{t} K_1\,dt$$

$$\Rightarrow z = \frac{-e(V_r+V_0)}{mL}\left[\frac{t^2}{2} - t_1 . t\right]_{t_1}^{t} + K_1\left[t\right]_{t_1}^{t}$$

Note: $\left[\frac{t^2}{2} - t_1 . t\right]_{t_1}^{t} = \left(\frac{t^2}{2} - t_1 . t\right) - \left(\frac{t_1^2}{2} - t_1^2\right)$

$$= \frac{t^2}{2} - t t_1 + \frac{t_1^2}{2} = \frac{1}{2}\left[t^2 - 2t . t_1 + t_1^2\right]$$

$$\therefore \quad Z = \frac{-e(V_r+V_0)}{mL} \cdot \frac{(t-t_1)^2}{2} + v(t_1)(t-t_1) + K_2 \qquad (17)$$

$[\because \; K_1 = v(t_1) \;\; eq^n \; 16]$

At $t = t_1$, $\quad Z = K_2$, but at $t = t_1$, $\boxed{Z = d}$

[see fig 51]

$$\therefore \quad \boxed{K_2 = d}$$

Eqn (17) becomes,

$$Z = \frac{-e(V_r+V_0)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1) + d \qquad (18)$$

On the assumption that the electron leaves the cavity gap at $Z = d$ and time t_1 with a velocity of $v(t_1)$ and returns to the gap at $Z = d$ and time t_2, at $\underline{t = t_2,\; Z = d}$. $\qquad (19)$

Putting eqn (19) in eq^n (18), we have,

$$\cancel{d} = \frac{-e(V_r+V_0)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1) + \cancel{d}$$

$$\Rightarrow \quad \frac{e(V_r+V_0)}{2mL}(t_2-t_1)^{\cancel{2}} = v(t_1)\,\cancel{(t_2-t_1)}$$

$$\Rightarrow \quad (t_2-t_1) = \frac{2mL\, v(t_1)}{e(V_r+V_0)} \qquad (20)$$

The round-trip transit time in the repeller region is given by,

$$T' = t_2 - t_1 = \frac{2mL\, v(t_1)}{e(V_r + V_0)} \qquad \left[\text{from eq}^n\ 20\right]$$

$$= \frac{2mL}{e(V_r+V_0)}\left[v_0\left(1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right)\right]$$

$$\left[\text{From eq}^n\ 12\right]$$

$$= \frac{2mL\,v_0}{e(V_r+V_0)}\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$T' = T_0'\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad (21)$$

21(A)

where $T_0' = \dfrac{2mL\,v_0}{e(V_r+V_0)}$, is the round-trip dc transit time of the center-of-the bunch electron.

($\because$ v_0 = uniform velocity of electron, without ~~any~~ any a.c signal applied)

From eq^n (21),

$$t_2 - t_1 = T_0' + \frac{T_0'\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

Multiplication of above eq^n through by a radian frequency results in,

$$\omega(t_2 - t_1) = \omega T_0' + \frac{\beta_i V_1}{2V_0}\cdot \omega T_0' \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

$$\Rightarrow \boxed{\omega(t_2 - t_1) = \theta_0' + X'\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)} \qquad (22)$$

where $\theta_0' = \omega T_0'$ ——22(A), is the round-trip dc transit angle of the center of the bunch electron

and $X' = \frac{\beta_i V_1}{2V_0}\theta_0'$ ——22(B), is the bunching parameter of the reflex klystron oscillator.

Power O/P and Efficiency :-

In order for the electron beam to generate a maximum amount of energy to the oscillation, the returning electron beam must cross the cavity gap when the gap field is maximum retarding. In this way, a maximum amount of kinetic energy can be ~~transmitted~~ transferred from the returning electrons to the cavity walls.

It can be seen from fig 52 that for a maximum energy transfer, the round trip transit angle, referring to the center of the bunch, must be given by,

$$\omega(t_2-t_1) = \omega T_0' = \left(n-\frac{1}{4}\right)2\pi = 2n\pi - \frac{\pi}{2} \quad -22(C)$$

where $V_1 \ll V_0$ is assumed, n = any +ve integer for cycle number, and $N = n-\frac{1}{4}$ is the number of modes.

$n=1$, $N = \frac{3}{4}$, $n=2$, & $N = 2-\frac{1}{4} = \frac{7}{4} = 1\frac{3}{4}$ and so on.

[Similar to velocity modulation] The current modulation of the electron beam as it reenters the cavity from the repeller region can be determined ~~the~~ as follows ;

$$i'_{2t} = -I_0 - \sum_{n=1}^{\infty} 2 I_0 \, J_n(n x') \cos\left[n(\omega t_2 - \theta_0' - \theta_g)\right] \quad \text{—— (23)}$$

(–ve sign, due to the beam current injected into the cavity gap from the repeller region flows in the –ve Z- direction) in the ~~[illegible]~~ cavity.

where, I_0 = d.c current

i'_{2t} = beam current of a reflex klystron oscillator.

$J_n(nx')$ = n^{th} order Bessel function of the first kind

The fundamental component of the current induced in the cavity by the modulated electron beam is given by i_f = ~~[illegible]~~ $[-2I_0 \, J_1(x') \cdot \cos(\omega t_2 - \theta_0')]$ ·

$$i_{2} = -\beta_i \, i_f = \beta_i \, 2 I_0 \, J_1(x') \cos(\omega t_2 - \theta_0') \quad \text{—— (24)}$$

[∵ ~~[illegible]~~ From eqn (23), we have taken $n = 1$, and θ_g is neglected as a small quantity compared to θ_0'.]

β_i = coupling coefficient of the cavity.

The magnitude of the fundamental component is

$$|i_{2}| = I_2 = 2 I_0 \, J_1(x') \cdot \beta_i \quad \text{—— (25)}$$

The d.c power supplied by the beam voltage V_0 is

$$P_{dc} = V_0 I_0 \quad \text{—— (26)}$$

The a.c power delivered to the load is given by

$$P_{ac} = \frac{V_1 I_2}{2} = \frac{V_1}{\cancel{2}} \cdot \left[\cancel{2} I_0 \beta_i J_1(x')\right] \quad \text{—} (27)$$

$$\Rightarrow P_{ac} = V_1 I_0 \beta_i J_1(x') \quad \text{—} (27)$$

From eqn 22 (B),

$$x' = \frac{\beta_i V_1}{2V_0} \theta_0'$$

Putting eqn 22 (A), in the above eqn, we have

$$x' = \frac{\beta_i V_1}{2V_0} \cdot \omega T_0'$$

Putting eqn 22 (C), in the above eqn, we have

$$x' = \frac{\beta_i V_1}{2V_0} \cdot \left(2n\pi - \frac{\pi}{2}\right) \quad \text{—} (28)$$

$$\Rightarrow \frac{V_1}{V_0} = \frac{2x'}{\beta_i (2n\pi - \pi/2)} \quad \text{—} (28)$$

Putting eqn (28) in eqn (27), we have

$$P_{ac} = \left[\frac{2x' V_0}{\cancel{\beta_i}(2n\pi - \pi/2)}\right] \cdot \left[I_0 \cancel{\beta_i} J_1(x')\right]$$

$$\Rightarrow P_{ac} = \frac{2 V_0 I_0 x' J_1(x')}{\left(2n\pi - \frac{\pi}{2}\right)} \quad \text{—} (29)$$

Therefore the electronic efficiency of a reflex klystron oscillator is defined as

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2X' J_1(X')}{2n\pi - \frac{\pi}{2}} \quad \left[\begin{array}{l}\text{Dividing eqn} \\ (29) \text{ by eq}^n (26)\end{array}\right] \quad (30)$$

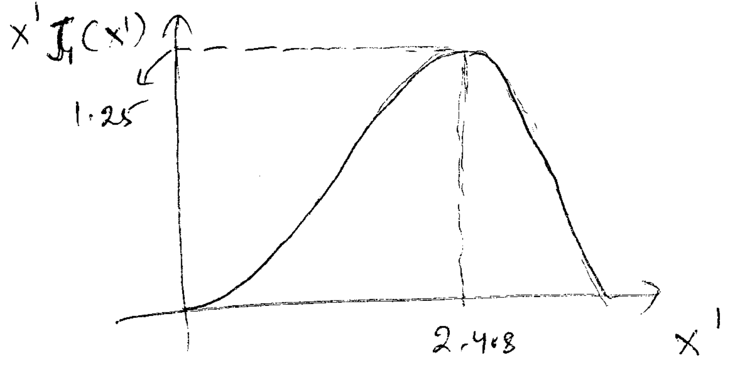


fig 54:- $X' J_1(X')$ Vs X'

The factor $X' J_1(X')$ reaches a maximum value of $\boxed{1.25}$ at $\underline{X' = 2.408}$.

In practice, for $n = 2$ or $N = 1\frac{3}{4}$ mode has the max^m power o/p.

$$\eta_{max} = (\text{Efficiency})_{max} = \frac{2 \times (1.25)}{2 \cdot 2 \cdot \pi - \frac{\pi}{2}} = \frac{2.5 \times 2}{7\pi}$$

$$\boxed{(\text{Efficiency})_{max} = 22.7\ \%} \quad \left[\begin{array}{l}\text{Theoretical} \\ \text{efficiency} \\ 20 \text{ to } 30\%\end{array}\right]$$

For Problems:-

For a given beam voltage $\underline{V_0}$, the relationship between $\underline{\text{repeller voltage}}$ and cycle number $\underline{n}$ required for oscillation, can be derived as follows.

From eqn 22 (c), $\omega T_0' = 2n\pi - \frac{\pi}{2}$ — (31)

But $T_0' = \dfrac{2mL v_0}{e(V_r+v_0)}$ from eqn ~~21~~ (A)

Putting this value in eqn (31), we have

$$\omega \times \left[\frac{2mL v_0}{e(v_r+v_0)}\right] = \left(2n\pi - \frac{\pi}{2}\right) \quad \text{— (31)}$$

Putting eqn (1) in eqn (31), we have

$$\therefore \frac{\omega \times 2mL}{e(V_r+v_0)} \times \left[\sqrt{\frac{2eV_0}{m}}\right] = \left(2n\pi - \frac{\pi}{2}\right) \quad \text{— 31 (A)}$$

$\Rightarrow$ Squaring both the sides.

$$\Rightarrow \frac{\omega^2 \times 4 \cancel{m^2}^{m} L^2}{e\,\cancel{e^2}(V_r+v_0)^2} \times \frac{2\cancel{e}V_0}{\cancel{m}} = \left(2n\pi - \frac{\pi}{2}\right)^2$$

$$\Rightarrow \boxed{\frac{V_0}{(V_r+v_0)^2} = \frac{\left(2n\pi - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \cdot \frac{e}{m}} \quad \text{— (32)}$$

The power o/p can be expressed on terms of the repeller voltage V_r, i.e

$$P_{ac} = \frac{2 V_0 I_0 X' J_1(X')}{2n\pi - \pi/2} \quad \text{— (33)}$$

Putting eqn 31(A), in eqn (33), we have

$$P_{ac} = \frac{\cancel{2}\, V_0 I_0\, X'\, J_1(X')\, \sqrt{\cancel{m}} \times \frac{\sqrt{e}}{\cancel{e}}(V_r + V_0)}{\frac{\omega \times \cancel{2}\, \cancel{m} L}{\sqrt{m}} \sqrt{2 \cancel{e} V_0}}$$

$$\Rightarrow \boxed{P_{ac} = \frac{V_0 I_0\, X'\, J_1(X_1')\,(V_r + V_0)}{\omega L} \sqrt{\frac{e}{2 m V_0}}} \quad (34)$$

→ Note :-

It can be seen from eqn (32) that for a given beam voltage V_0 and ~~the~~ cycle number 'n' or mode number 'N', the center repeller voltage V_r can be determined in terms of the center freq.

→ The power o/p at the center freq can be calculated using eqn (34). When the freq varies from the center freq, and the repeller voltage about the center voltage, the o/p power vary accordingly, assuming a bell shape.

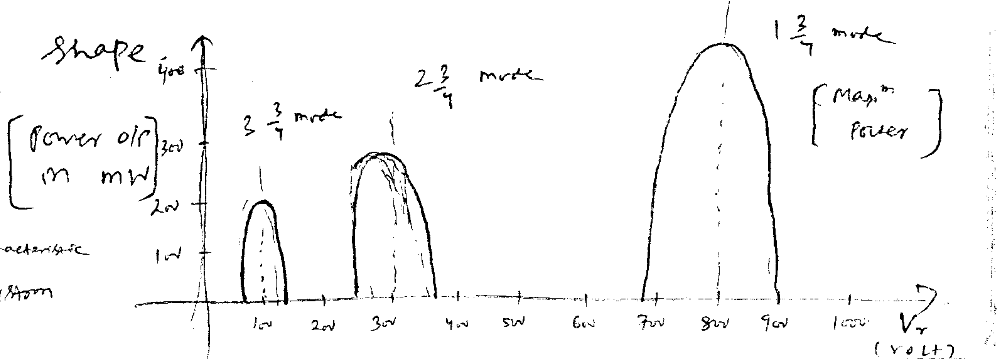


fig 55:- Power o/p characteristic of reflex klystron

Electronic Admittance:-

From eqn (24), the induced current can be written in phasor form as

$$i_2 = 2 I_0 \beta_i J_1(x') \cdot e^{-j\theta_0'} \quad \text{--- (35)}$$

The voltage across the gap at time 't_2' can also be written in phasor form

$$V_2 = V_1 \cdot e^{-j\frac{\pi}{2}} \quad \text{--- (36)}$$

$$\left[\because 1\tfrac{3}{4} \text{ cycle} = 1\tfrac{3}{4} \times 2\pi = \tfrac{7}{4} \times 2\pi = \tfrac{7\pi}{2} = -\tfrac{\pi}{2}\right]$$

The ratio of i_2 to V_2 is defined as the electronic admittance of the reflex klystron. i.e

$$Y_e = \frac{2 I_0 \beta_i J_1(x') \cdot e^{-j\theta_0'}}{V_1 \cdot e^{-j\pi/2}} \quad \text{--- (37)}$$

From eqn (28), we have $V_1 = \dfrac{2x' V_0}{\beta_i (2n\pi - \pi/2)}$

$$= \frac{2x' V_0}{\beta_i \theta_0'}$$

[From eqn 22(c) & 22(d)]

$$\therefore Y_e = \frac{\cancel{2} I_0 \beta_i J_1(x') \cdot e^{j(\frac{\pi}{2} - \theta_0')}}{\cancel{2} x' V_0} \cdot (\beta_i \theta_0')$$

$$Y_e = \frac{I_0}{V_0} \cdot \frac{\beta_i^2 \theta_0'}{2} \cdot \frac{2 J_1(x')}{x'} \cdot e^{j(\frac{\pi}{2} - \theta_0')} \quad \text{--- (38)}$$

It is evident that the electronic admittance is nonlinear, since it is proportional to the factor $\frac{2J_1(X')}{X'}$, and X' is proportional to the signal voltage $\left[\because X' = \frac{\beta_i V_1 \theta_0'}{2V_0}\right]$

The term $\frac{2J_1(X')}{X'}$ is called saturation factor.

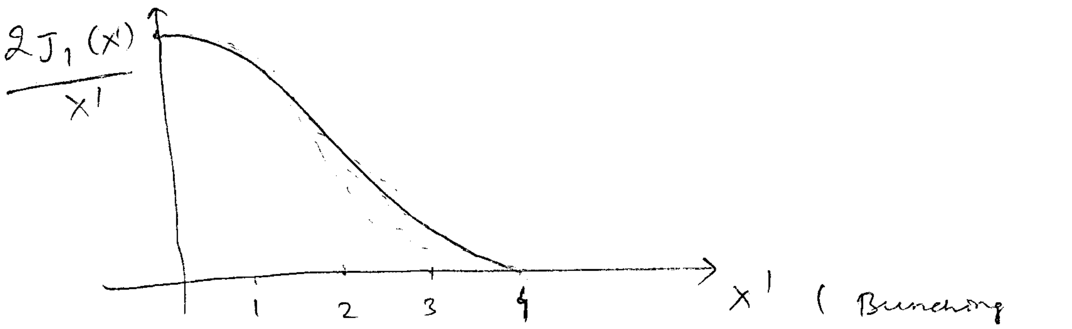


fig 2-56:- Reflex klystron saturation factor

The equivalent circuit of reflex klystron is shown in fig 57.

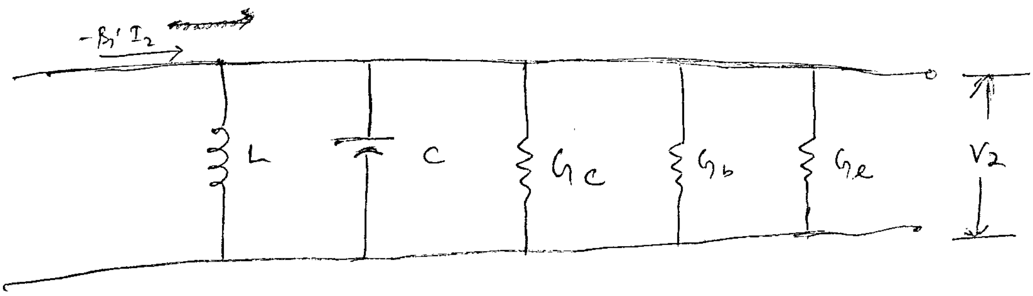


fig 57:- Equivalent circuit of a reflex Klystron

In this circuit, L & C are the energy storage elements of the cavity; G_c represents the copper cavity losses, G_b beam loading conductance and G_l the load conductance.

The necessary condⁿ for oscillation is that the magnitude of the −ve real part of electronic admittance, not less than the total conductance of the cavity circuit.

Electronic admittance in rectangular form

$$Y_e = G_e + jB_e \quad — (39)$$

Magnitude of −ve real part → $|-G_e|$

Total conductance of the cavity circuit,

$$G = G_c + G_b + G_e = \frac{1}{R_{sh}}$$ and R_{sh} is the effective shunt resistance.

∴ The necessary condⁿ for oscillation is,

$$\boxed{|-G_e| \geq G} \quad — (40)$$

→ Since the electronic admittance shown in eqⁿ (37) is in exponential form, its phase is $\frac{\pi}{2}$ when $\theta_0' = 0$,

→ The rectangular plot of the electron admittance Y_e is a spiral. Any value of θ_0' for which the spiral lies in the area to the left of the line $(-G - jB)$ will yield oscillation. i.e

$$\theta_0' = \left(n - \frac{1}{4}\right) 2\pi = N 2\pi \quad — (41)$$

Where N is mode number as indicated on the plot. [As discussed earlier about the mode]

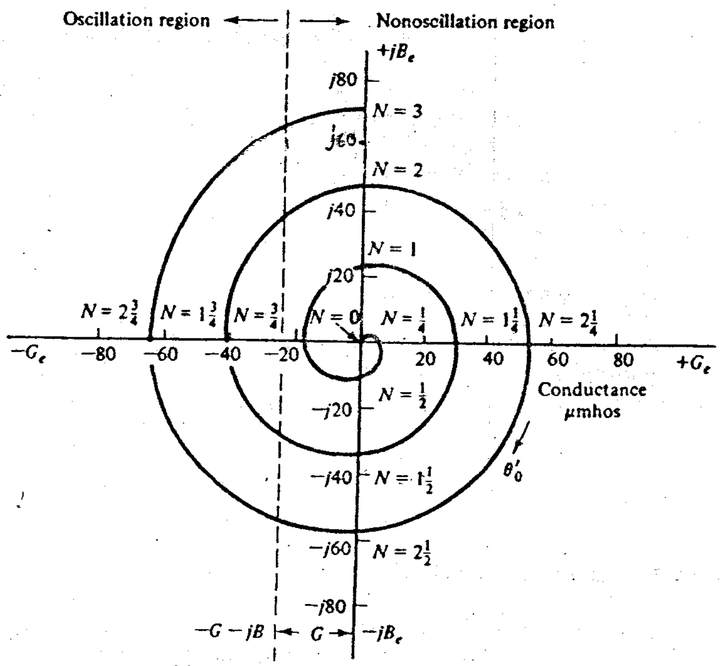


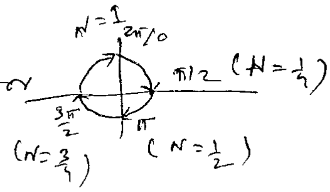
Figure ~~9-17~~ 58 Electronic admittance spiral of a reflex klystron.

Note:-

$1 \text{ Cycle} = 2\pi^c$

$2\pi^c = 1 \text{ cycle}$

$\frac{\pi}{2} = \frac{1}{4} \text{ cycle}$ [∵ For



For each quadrant,

N will be increment by $(\frac{1}{4})$.

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{4}$ ......

Ex:- 1 (BPUT-2009)

A reflex klystron operates under the following conditions:

$V_0 = 600V$, $L = 1\ mm$

$R_{sh} = 15k\Omega$, $\frac{e}{m} = 1.759 \times 10^{11}$

$f_r = 9\ GHz$, $J_1(x') = 0.582$

The tube is oscillating at f_r at the peak of the $n=2$ or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

(a) Find the value of the repeller voltage (V_r)

(b) Find the direct current necessary to give a microwave gap voltage of 200V.

(c) What is the electronic efficiency under their condition?

Ans :- (a) From eqn (32), we have

$$\frac{V_0}{(V_r+V_0)^2} = \frac{\left(2n\pi-\frac{\pi}{2}\right)^2}{8\,\omega^2 L^2} \cdot \frac{e}{m}$$

$$\Rightarrow \frac{V_0}{(V_r+V_0)^2} = \frac{\left(2.2.\pi-\frac{\pi}{2}\right)^2}{8\cdot(2\pi\cdot 9\times10^9)^2\times(10^{-3})^2} \times 1.759\times10^{11}$$

$$\Rightarrow \frac{V_0}{(V_r+V_0)^2} = 0.832\times10^{-3}$$

$$\Rightarrow \quad \frac{600}{(V_r+600)^2} = 0.832\times10^{-3}$$

$$\Rightarrow \quad (V_r+600)^2 = \frac{600}{.832\times10^{-3}}$$

$$\Rightarrow \quad \boxed{V_r = 249.20\ V}$$

(b) Assuming $\beta_i' = 1$,

$\because \beta_i = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\frac{\theta_g}{2}}$

$\frac{\theta_g}{2} \to 0, \quad \frac{\sin\theta_g/2}{\theta_g/2} = 1$

Since transit time/angle is neglected

$$i_2 = 2 I_0 J_1(x')$$

$$V_2 = i_2 . R_{sh}$$

$$V_2 = 2 I_0 J_1(x') \cdot 15\times10^3$$

$$\Rightarrow \quad I_0 = \frac{V_2}{2 J_1(x')\times 15\times10^3} = \frac{200}{2\times0.582\times15\times10^3}$$

$$\Rightarrow \quad \boxed{I_0 = 11.45\ mA}$$

(c) $\eta = \frac{2x' J_1(x')}{2n\pi - \frac{\pi}{2}}$ [Eqn 20]

$x' = \frac{\beta_i V_1}{2V_0} \cdot \theta_0'$ eqn (22 a)

$= \frac{1\times \cancel{200}}{2\times \cancel{600}\,3} \times 3.5\pi$

$x' = 1.83$

$\theta_0' = 2n\pi - \frac{\pi}{2}$
$= 2.2.\pi - \frac{\pi}{2} = \frac{7\pi}{2}$
$= 3.5\pi$

$$\eta = \frac{2x' J_1(x')}{2n\pi - \frac{\pi}{2}}$$

$$= \frac{2 \times 1.83 \times 0.582}{3.5 \times \pi}$$

$$= 0.1937$$

$$\boxed{\eta = 19.37\%}$$

OR

$$\eta = \frac{2x' J_1(x')}{2n\pi - \pi/2} = \frac{2x' J_1(x')}{\theta_0'}$$

$$= 2\left(\frac{\beta_i V_1}{2V_0} \cdot \cancel{\theta_0'}\right) \cdot \frac{J_1(x')}{\cancel{(\theta_0')}}$$

$$= \frac{\cancel{2} \times 1 \times \cancel{2n\pi} \times 0.582}{\cancel{2} \times \cancel{6n\pi}\, 3}$$

$$\simeq \cancel{1.94}\ 0.194$$

$$\eta = 19.4\%$$

2) BPUT 2012

A reflex klystron operates with the following conditions: d.c accelerating voltage $V_{dc} = 1.4\,KV$, Repeller voltage $= -100\,V$, Resonant freq $f_r = 8\,GHz$. Distance betn cavity and repeller $(L) = 2\,cm$. Compare (a) d.c electron velocity (b) Round trip d.c transit time

Ans ÷ (a) D.C electron velocity (v_0)

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

$$= 0.593\times10^6\sqrt{V_0}$$ (As derived earlier)

$$\Rightarrow v_0 = 0.593\times10^6\sqrt{1.4\times10^3}$$

$$\boxed{v_0 = 22.18\times10^6\ \frac{m}{sec}}$$

(b) Round trip d.c transit time

$$T_0' = \frac{2mL v_0}{e(V_r+V_0)}$$

Note ÷ Here we have to take repeller voltage magnitude. $|-100| = 100$

$$= \frac{2\times9.1\times10^{-31}\times2\times10^{-2}\times22.18\times10^6}{1.6\times10^{-19}(100+1400)}$$

$$= \frac{2\times9.1\times2\times22.18}{1.6\times15}\times10^{-10}$$

$$= 33.63\times10^{-10}$$

$$= 3.363\times10^{-9}$$

$$\boxed{T_0' = 3.36\ ns}$$

— o —

Klystron Amplifier or Two-Cavity Klystron :-

The two-Cavity Klystron is widely used microwave amplifiers operated by principles of velocity & current modulation. All ~~the~~ electrons injected from the cathode arrive at the first cavity with uniform velocity. [Refer Fig 14]. Those electrons passing the first cavity gap at zeros of gap voltage (or signal voltage) pass through with unchanged velocity; those passing through the +ve half cycles of the gap voltage undergo an increase in velocity; those passing through the -ve swings of the gap voltage undergoes a decrease in velocity.

As a result of these actions, the electrons gradually bunch together as they travel down the drift space. The variation in electron velocity in the drift space is known as <u>velocity modulation</u>.

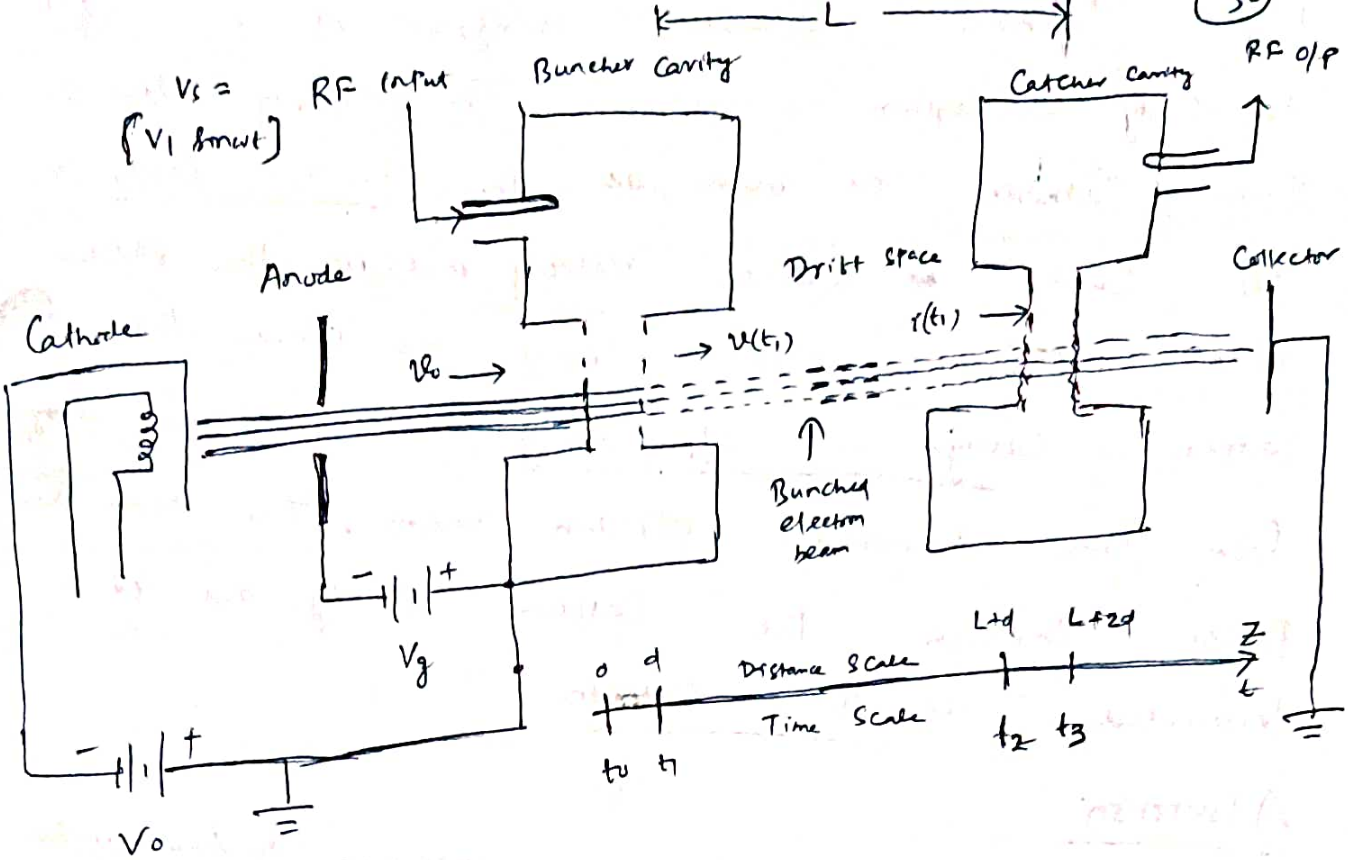


Fig 14:- Two-Cavity Klystron amplifier

The density of the electrons in the second cavity gap varies cyclically with time. The electron beam contains an ac component and is said to be current-modulated. The maxm bunching should occur approximately midway betn the secondary cavity grids during its retarding phase; thus the kinetic energy is transferred from the electrons to the field of the second cavity. The electrons then emerge from the second cavity with reduced velocity and finally terminate at the collector.

Characteristics of two-cavity klystron amplifier :-

1. Efficiency : about 40%.
2. Power o/p : avg power is up to 500 kW
3. Power gain : about 30 dB.

Fig 14. Shows Schematic diagram of a two-Cavity Klystron amplifier. The Cavity close to the Cathode is known as the buncher cavity or i/p Cavity, which velocity-modulates the electron beam. The other Cavity is called the Catcher cavity or o/p cavity; it catches energy from the bunched electron beam. The beam then passes through the Catcher cavity and is terminated at the Collector.

Assumption

1. The electron beam is assumed to have a uniform velocity on the cross section of the beam.
2. Space-charge effects are negligible
3. The magnitude of the microwave signal i/p is assumed to be much smaller than the d-c accelerating voltage

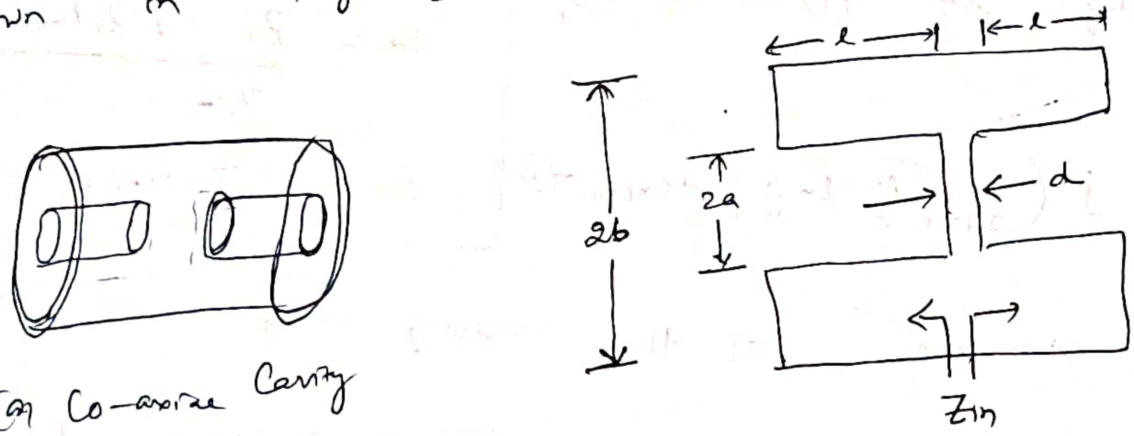
Reentrant Cavities :-

At a frequency well below the microwave range, the cavity resonator can be represented by a lumped-constant resonant circuit. When the operating freq. is increased to several tens of Mega Hertz, both the inductance & Capacitance must be reduced to a minimum in order to maintain resonance at the operating frequency. $\left[\because \uparrow f = \frac{1}{2\pi\sqrt{LC}} \downarrow\downarrow \right]$

Ultimately the inductance is reduced to a minimum by short wire. $[\because L \propto l]$ (Inductance) (length)

Therefore, the reentrant cavities are designed for use in keystrons & microwave ~~tetrodes~~ triodes.

✓A reentrant cavity is one in which the metallic boundaries extend into the interior of the cavity. One of the most commonly used reentrant cavities is the coaxial cavity as shown in fig 15.



(a) Co-axial Cavity

(b) Cross-section of co-axial cavity [Side View]

Fig 15: Coaxial Cavity & its equivalent

→ It is clear from fig 15. that not only has the inductance been considerably decreased but the resistance losses are markedly reduced as well, and the shelf shielding enclosure prevents radiation losses. It is difficult to calculate the resonant frequency of the coaxial cavity. An approximation can be made, however, using transmission-line theory. The characteristic impedance of the coaxial line is given by

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right) \text{ ohms} \quad (1)$$

The Coaxial Cavity is similar to a Coaxial line Shorted at two ends & Joined at the Center by a Capacitor. The i/p impedance to each Shorted Coaxial line is given by

$$Z_{in} = j Z_0 \tan\beta l \quad \text{—} (2)$$

Where 'l' is the length of the Co-axial line.

$$\because \; Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan\beta l}{Z_0 + j Z_L \tan\beta l} \right]$$

At Shorted load, $Z_L = 0$

$$Z_{in} = \frac{\cancel{Z_0} \cdot j Z_0 \tan\beta l}{\cancel{Z_0}}$$

$$Z_{in} = j Z_0 \tan\beta l$$

Substituting eqn (1) in eqn (2),

$$Z_{in} = j \left(\frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \frac{b}{a} \right) . \tan\beta l \quad \text{—} (3)$$

The inductance of the Cavity is given by,

$Z_{in} = R_{in} + j X_{in}$

$\because$ Z_{in} = Purely reactive

$= X_{in}$ = only storing energy

$R_{in} = 0$

$$|L| = \frac{2 X_{in}}{\omega} = \left| \frac{\cancel{2} \times j \frac{1}{\cancel{2}\pi} \sqrt{\frac{\mu}{\epsilon}} . \ln\left(\frac{b}{a}\right) . \tan\beta l}{\omega} \right|$$

$$L = \frac{1}{\pi \omega} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \tan\beta l \quad \text{—} (4)$$

and the Capacitance of the gap is given by

$$C_g = \frac{\epsilon(\pi a^2)}{d} \quad \text{—} (5)$$

& $C = \frac{\epsilon A}{d}$, $A = \pi a^2$

Circular cross section of the cylinder

At resonance

$$X_L = X_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C_g}$$

$$\Rightarrow \omega^2 L C_g = 1$$

$$\Rightarrow \omega^{\cancel{2}} \times \frac{1}{\cancel{\pi}\omega} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \cdot \tan\beta l \times \frac{\epsilon \cancel{\pi} a^2}{d} = 1$$

$$\Rightarrow \omega \cdot \frac{\sqrt{\mu}}{\cancel{\sqrt{\epsilon}}} \times \overset{\sqrt{\epsilon}}{\cancel{\epsilon}} \ln\left(\frac{b}{a}\right) \cdot \frac{\tan\beta l \cdot a^2}{d} = 1$$

$$\Rightarrow \frac{\omega}{v} \cdot \frac{\ln\left(\frac{b}{a}\right) \cdot \tan\beta l \cdot a^2}{d} = 1$$

$$\boxed{\because v = \frac{1}{\sqrt{\mu\epsilon}} \qquad \sqrt{\mu\epsilon} = \frac{1}{v}}$$

$$\Rightarrow \tan\beta l = \frac{v d}{\omega a^2 \ln(b/a)} \quad \text{---} (6)$$

where $v = \frac{1}{\sqrt{\mu\epsilon}}$ = Phase Velocity in any medium.

The solution to this above eq^n gives the resonant frequency of Co-axial Cavity. Since eq^n (6) contains the tangent function, it has infinite number of solutions with larger values of frequency. Therefore this type of reentrant Cavity can support an infinite number of resonant frequencies or mode of oscillation.

Velocity Modulation Process:-

Refer to the velocity modulation, exactly same as Reflex Klystron's velocity modulation. From equation (1) to Equation (12), so final expression is.

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \right] \quad \text{---} (7)$$

Output Power & Beam Loading

The maximum bunching should occur approximately midway between the Catcher grids. The phase of the Catcher gap voltage must be maintained in such a way that the bunched electrons, as they pass through the grids, encounter a retarding phase. When the bunched electron beam passes through the retarding Phase, its kinetic energy is transferred to the field of the Catcher Cavity. When the electrons emerge from the Catcher grids, they have reduced velocity & are finally Collected by the Collector.

The induced Current in the Catcher Cavity :-

Since the current induced by the electron beam in the walls of the Cavity is directly proportional to the amplitude of the microwave i/p voltage V_1, the fundamental Component of the induced microwave current in the Catcher is given by

$$i_{2\,ind}^{'} = \beta_0\, i_2 = \beta_0 .\, 2I_0\, J_1(x) \cos\left[\omega(t_2 - \tau - T_0)\right] \quad (8)$$

Where β_0 is the beam coupling Coefficient of the Catcher gap. If the buncher & Catcher Cavities are identical, then $\beta_r = \beta_0$. The fundamental Component of the current induced in the Catcher Cavity then has magnitude

$$I_{2ind} = \beta_0 I_2 = \beta_0\, 2 I_0 J_1(x) \quad (9)$$

Fig 16 shows an o/p equivalent ckt in which R_{sho} represents the wall resistance of the catcher cavity, R_B the beam loading resistance, R_L is the external load resistance & R_{sh} is the effective shunt resistance.

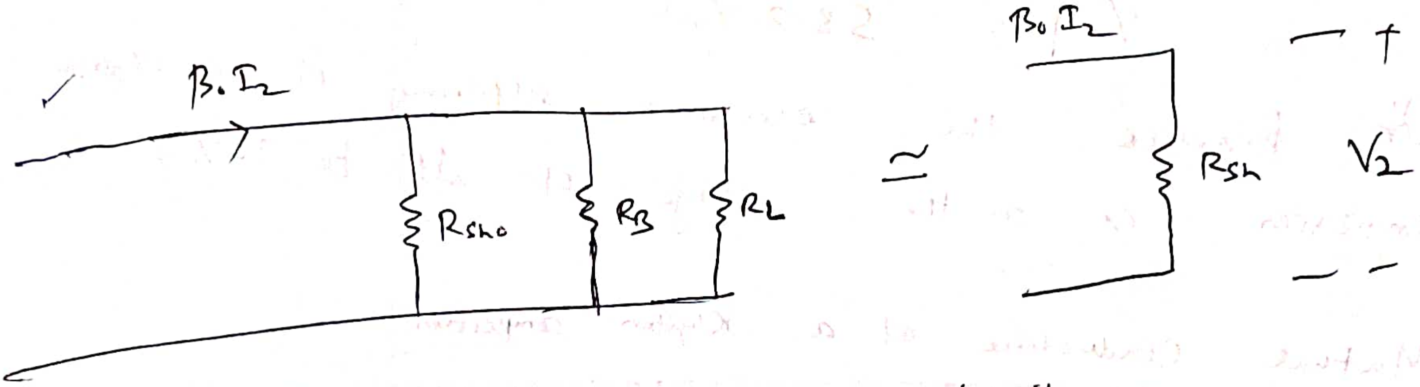


Fig 16:- O/P Equivalent Circuit

The o/p power delivered to the catcher cavity and the load is given by

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2} \quad (10)$$

$$P_{out} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = \frac{I_m R \cdot I_m}{2} = \frac{I_m^2 R}{2}$$

$\because$

$$\frac{(\beta_0 I_2)^2 R_{sh}}{2} = \frac{(\beta_0 I_2) \cdot (\beta_0 I_2 R_{sh})}{2} = \frac{\beta_0 I_2 \cdot V_2}{2}$$

Efficiency of Klystron

The electronic efficiency of the klystron amplifier is defined as the ratio of the o/p power to the i/p power.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\left(\frac{\beta_0 I_2 V_2}{2}\right)}{I_0 V_0} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} \quad (11)$$

If the coupling is perfect, $\beta_0 = 1$, the max^m beam current approaches $I_{2max} = (1) \cdot 2 \cdot I_0 \cdot (0.582)$ [From eq^n (9)]

& $J_1(X) = 0.582$
At $X = 1.841$

also, the voltage V_2 equal to V_0. [In a amplifier, a signal can be amplified max^m to the supply voltage V_0]

$$\eta_{max} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} = \frac{1 \times \cancel{2} \cancel{I_0} (0.582) \cancel{V_0}}{\cancel{2} \cancel{I_0} \cancel{V_0}} = 0.582$$

$$\% \eta = 58.2\%.$$

In practice, the electronic efficiency of a klystron amplifier is in the range of 15 to 30%.

Mutual Conductance of a Klystron amplifier :—

The equivalent mutual conductance of the klystron amplifier can be defined as the ratio of the induced o/p current to the i/p voltage. That is,

$$|G_m| \equiv \frac{i_{2ind}}{V_1} = \frac{2 \beta_0 I_0 J_1(X)}{V_1} \quad \text{—(12)}$$

We know from Reflex klystron [Eqn 22-(B)]

Bunching parameter, $X' = \frac{\beta_i V_1 \theta_0'}{2 V_0}$

For Klystron Amplifier, let $X = \frac{\beta_i V_1 \theta_0}{2 V_0}$

$$\Rightarrow V_1 = \frac{2 V_0 X}{\beta_i \theta_0} \quad \text{—(13)}$$

Putting eqn (13) in eqn (12), we have

$$G_m = \frac{2\beta_0 I_0 J_1(x)}{\left(\frac{2V_0 X}{\beta_i Q_0}\right)} = \frac{\cancel{2}\beta_0 I_0 J_1(x)}{\cancel{2} V_0 X} \cdot \beta_i Q_0$$

Assuming $\beta_i = \beta_0$, [Identical i/p & o/p cavity]

$$G_m = \left(\frac{I_0}{V_0}\right) \cdot \beta_0^2 \cdot \left(\frac{J_1(x)}{X}\right) \cdot Q_0$$

$$G_m = G_0 \; \beta_0^2 \cdot \left(\frac{J_1(x)}{X}\right) \cdot Q_0$$

$$\Rightarrow \frac{|G_m|}{G_0} = \beta_0^2 \cdot Q_0 \; \frac{J_1(x)}{X} \quad -(14)$$

where $G_0 = \frac{I_0}{V_0}$ = D C beam Conductance.

$\Rightarrow$ Normalized mutual conductance $= \beta_0^2 \; Q_0 \; \frac{J_1(x)}{X}$

$\therefore$ Normalized mutual conductance is not a constant but decreases as the bunching parameter X increases.

For a maxm o/p at $X = 1.841$, the normalized mutual conductance is

$$\frac{|G_m|}{G_0} = (0.316)\beta_0^2 Q_0 \quad -(15)$$

From eqn (14) $\frac{J_1(x)}{X} = 0.316$

The voltage gain of a Klystron amplifier is defined as

$$A_V = \frac{V_2}{V_1} = \frac{(\beta_0 I_2) \times R_{sh}}{V_1} = \frac{\beta_0 \times (2 I_0 J_1(x)) \times R_{sh}}{\left(\frac{2V_0 X}{\beta_i Q_0}\right)}$$

(From Fig 16) → From eqn (9) → From eqn (13)

$$\therefore A_v = \frac{\beta_0 \times \cancel{2} I_0 J_1(X) \times R_{sh} \times \beta_i \times \theta_0}{\cancel{2} V_0 (X)}$$

$$A_v = \frac{\beta_0^2 \theta_0}{R_0} \cdot \frac{J_1(X)}{X} \cdot R_{sh} \quad \text{—(16)}$$

$\because \beta_i = \beta_0$

$R_0 = \frac{V_0}{I_0}$

Using eqn (14), $\beta_0^2 \theta_0 \frac{J_1(X)}{X} = \frac{G_m}{G_0}$

$$A_v = \left(\frac{G_m}{G_0}\right) \times \frac{1}{R_0} \cdot R_{sh}$$

$$A_v = G_m R_{sh} \quad \text{—(17)}$$

$\because G_0 R_0 = \frac{\cancel{I_0}}{\cancel{V_0}} \times \frac{\cancel{V_0}}{\cancel{I_0}} = 1$

Example : Klystron Amplifier

1) A two-cavity klystron amplifier has the following parameters

$V_0 = 1000\,V$, $R_0 = 40\,k\Omega$,

$I_0 = 25\,mA$, $f = 3\,GHz$

Gap spacing in either cavity (d) = 1 mm

Spacing between the two cavities (L) = 4 cm

Effective shunt impedance, $R_{sh} = 30\,k\Omega$

(a) Find the i/p gap voltage to give max^m voltage V_2.

(b) Find the voltage gain,

(c) Find the efficiency of the amplifier

~~(d)~~ Answer :- (a) For max^m V_2, $J_1(x)$ must be maximum. This means $J_1(x) = 0.582$ at $x = 1.841$. The electron velocity just leaving the cathode is

$$v_0 = 0.593 \times 10^6 \sqrt{V_0}$$

$$v_0 = 0.593 \times 10^6 \sqrt{1000}$$

$$\Rightarrow \boxed{v_0 = 1.88 \times 10^7 \ \frac{m}{sec}}$$

From eq^n (1) of Reflex klystron

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

$e = 1.6 \times 10^{-19}$ coulombs

$m = 9.1 \times 10^{-31}$ kg

The gap transit angle,

$$\theta_g = \frac{\omega d}{v_0} = \frac{2 \times \pi \times (3 \times 10^9) \times 10^{-3}}{1.88 \times 10^7} = 1 \text{ rad.}$$

Beam coupling coefficient (β_i)

$$\beta_i = \beta_0 = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\frac{\theta_g}{2}\right)} = \frac{\sin\left(\frac{1}{2}\right)}{(1/2)} = 0.952$$

The dc transit angle betn the cavities is

$$\theta_0 = \omega T_0 = \frac{\omega \cdot L}{v_0} = \frac{2\pi \times 3 \times 10^9 \times 4 \times 10^{-2}}{1.88 \times 10^7} = 40 \text{ rad.}$$

The max^m i/p voltage $(V_1) = \dfrac{2V_0 X}{\beta_i \theta_0}$ (From eq^n (13))

$$\therefore V_1 = \frac{2 \times 10^3 \times 1.841}{0.952 \times 40} = 96.5\ V$$

($\because$ For max^m (V_1) $x = 1.841$, as per Bessel's function)

(b) The Voltage gain is found as,

$$A_v = \frac{\beta_0^2 G_0}{R_0} \frac{J_1(x)}{x} R_{sh}$$

$$= \frac{(0.952)^2 \times 40 \times (0.582) \times (30 \times 10^3)}{(4 \times 10^4) \times (1.841)}$$

$\because$ $R_0 = \frac{V_0}{I_0} = \frac{1000}{25 \times 10^{-3}} = 4 \times 10^4$

$$A_v = 8.595$$

(c) Efficiency

$$\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} \quad \text{[From eq}^n \text{ (11)]}$$

$$= \frac{(0.952) \times [2 I_0 J_1(x)] \times [\beta_0 I_2 R_{sh}]}{2 I_0 V_0}$$

~~$= (0.952) \times [2 \times 25 \times 10^{-3} \times 0.582] [0.952 \times 2 \times$~~

$$\therefore I_2 = 2 I_0 J_1(x) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1 \times 10^{-3}\ A$$

$$\therefore \eta = \frac{(0.952) \times [29.1 \times 10^{-3}] \times [0.952 \times 29.1 \times 10^{-3} \times 30 \times 10^3]}{2 \times 25 \times 10^{-3} \times 10^3}$$

$$\therefore \boxed{\eta = 46.2\ \%}$$

Ch-6 - Multicavity Magnetron

Magnetron belongs to Cross-field tube type or M-type device. In Cross-field devices, the d.c magnetic field and d.c electric fields are perpendicular to each other, and the d.c magnetic field plays a direct role in the RF interaction process.

The electrons emitted by Cathode are accelerated by the electric field and gain velocity, but greater their velocity, the more their path is bent by the magnetic field.

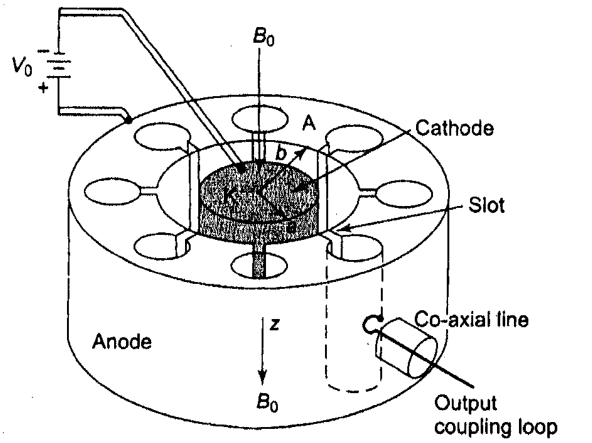
Multicavity Magnetron :-

Magnetron is the first high power microwave source or oscillator. The multicavity magnetron (more generally known as multicavity travelling wave magnetrons) are superior in performance and technically highly developed devices. The examples of different types of travelling-wave magnetrons are cylindrical magnetron, linear magnetron, Planar magnetron, Co-axial magnetron, voltage-tunable magnetron, inverted Co-axial magnetron etc.

→ Used to generate high MW power required in radar & Communication system.

→ Magnetron are Cross field tubes (M-type) in which the d.c magnetic field and d.c electric field are perpendicular to each other.

→ A high power microwave oscillator uses a traveling wave ~~ckt~~ cylindrical magnetron tube as shown in fig 59 & 60.



59
Fig. 9.24 *Basic magnetron oscillator*

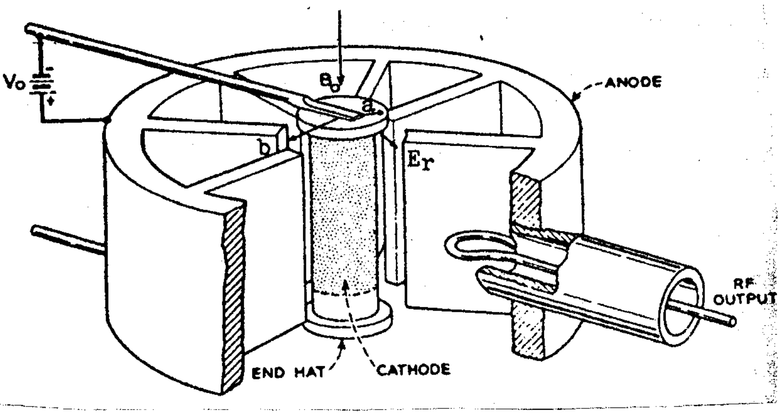


fig 60:- Interior view of magnetron

→ Magnetron consists of a cylindrical cathode 'K' [fig 59] of finite - length - radius 'a' at the center surrounded by a cylindrical anode 'A' of radius 'b'.

The anode is a slow-wave structure consisting of several re-entrant cavities equispaced around the circumference and coupled together through the anode - cathode space by means of slots.

Radial electric field is established by d.c voltage

'V_0' in between the Cathode and the anode and an axial d.c magnetic flux density 'B_0' is maintained in the +ve Z- direction by means of a permanent magnet or an electromagnet.

Principle of operation:-

Magnetron theory of operation is based on the motion of electrons under the influence of combined electric and magnetic fields. In Fig 61, trajectories a', b', c', d' of the electrons are shown for different magnetic field strengths.

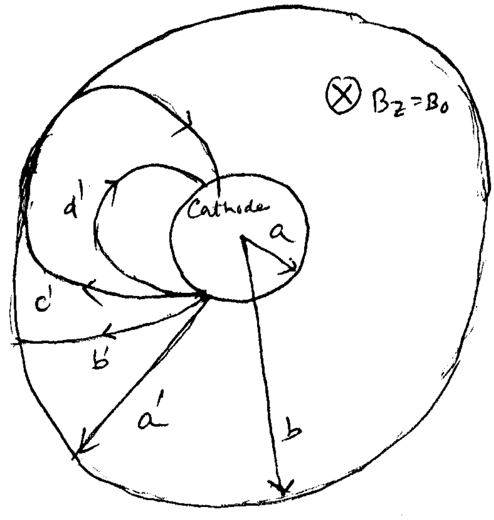


fig 61:- Electron trajectory

→ At Zero magnetic field, the electron take the straight path a', by the influence of electric field only and collected by anode

→ For a given 'V_0' if the magnetic field is increased, the electron take the curved path b' due to the force 'F' to reach the anode. [given by eqn (1)]

After emergence from the Cathode with Zero velocity (say), the electrons will acquire velocity 'v' having a tangential as well as radial components due to Force 'F' exerted by Cross fields E and H. $(B = \mu H)$

$$F = -eE - e(v \times B) \qquad (1)$$

At a critical value of magnetic field Bc, (say), the electrons just graze the anode surface at radius 'b' and take the path c' to return to the cathode for a given voltage Vo.

The value Bc is called cutoff magnetic flux density. If the magnetic field is greater than Bc, all the electrons return to the cathode as shown by a typical path d' without reaching the anode.

Due to excitation of the anode cavities by RF noise voltage in the biasing circuit, the RF field lines are fringed out of the cavity slot to the space between the anode and cathode. The accelerated electrons in the trajectory, when retarded by this RF field, transfer energy from the electron to the cavities to grow RF oscillations. When the system RF losses balance the RF oscillation energy, a stable oscillation is achieved. Output power is extracted through an external line coupled to the cavity.

Equations of Electron trajectory :-

The equations of motion for electrons in a cylindrical magnetron can be written with and of eqn ② & ③ as [In cylindrical co-ordinates (r, ϕ, z)]

$$\frac{d^2 r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m} E_r - \frac{e}{m} r B_z \cdot \frac{d\phi}{dt} \quad \text{—} (2)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \frac{e}{m} B_z \frac{dr}{dt} \quad \text{—} (3)$$

where $\frac{e}{m} = 1.759 \times 10^{11} \frac{C}{kg}$ is the charge-to-mass ratio of the electron. and $B_0 = B_z$ is assumed in the +ve Z- direction.

Rearranging eqn ③,

$$\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \frac{e}{m} B_z \cdot r \frac{dr}{dt} = \frac{1}{2} w_c \frac{dr^2}{dt} \quad \text{—} (4)$$

— (4A)

where $\boxed{w_c = \frac{e}{m} B_z}$, is the cyclotron angular frequency. $\left(\because \frac{d}{dt} r^2 = 2r \frac{dr}{dt} \right)$

Integrating eqn (5), we have

$$\int d\left(r^2 \frac{d\phi}{dt}\right) = \frac{1}{2} w_c \int d(r^2)$$

$$\Rightarrow \quad r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 + \text{constant} \quad \text{—} (5)$$

At $r = a$, where 'a' is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$.

Putting this condⁿ in eqⁿ (5), we have

$$\Rightarrow \quad 0 = \frac{1}{2} \omega_c a^2 + \text{Constant}$$

$$\Rightarrow \quad \text{Constant} = -\frac{1}{2} \omega_c a^2$$

∴ Eqⁿ (5) becomes,

$$\Rightarrow \quad r^2 \frac{d\phi}{dt} = \frac{1}{2} \omega_c r^2 - \frac{1}{2} \omega_c a^2$$

$$\Rightarrow \quad \frac{d\phi}{dt} = \frac{1}{2} \omega_c - \frac{1}{2} \omega_c \frac{a^2}{r^2}$$

∴ Angular velocity is expressed as,

$$\boxed{\frac{d\phi}{dt} = \frac{1}{2} \omega_c \left(1 - \frac{a^2}{r^2}\right)} \quad \text{———} (6)$$

The kinetic energy of the electron is given by,

$$\frac{1}{2} m v^2 = e V_0 \quad \text{———} (7)$$

However, the electron velocity has r and ϕ components such as

$$v^2 = v_r^2 + v_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r \frac{d\phi}{dt}\right)^2 = \frac{2 e V_0}{m} \quad \text{—} (8)$$

At $r = b$, where 'b' is the radius from the centre of the Cathode to the edge of the anode,

(Radial velocity = 0)

$\frac{dr}{dt} = 0$, when the electrons just graze the anode.

$\therefore$ Eqn ⑥ & ⑧ becomes,

$$\frac{d\phi}{dt} = \frac{1}{2} w_c \left(1 - \frac{a^2}{b^2}\right) \quad \text{—— (9)}$$

$$b^2 \left(\frac{d\phi}{dt}\right)^2 = \frac{2eV_0}{m} \quad \text{—— (10)}$$

Substituting eqn ⑨ into eqn ⑩, we have

$$b^2 \left[\frac{1}{2} w_c \left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2eV_0}{m}$$

$$\Rightarrow \left[\frac{1}{2} W_c \left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2eV_0}{b^2 m}$$

$$\Rightarrow \frac{1}{2} W_c \left(1 - \frac{a^2}{b^2}\right) = \sqrt{\frac{2eV_0}{b^2 m}}$$

$$\Rightarrow W_c = \frac{2\sqrt{\frac{2eV_0}{b^2 m}}}{\left(1 - \frac{a^2}{b^2}\right)}$$

$$\Rightarrow \sqrt{\frac{e}{m}} \cancel{\frac{e}{m}} B_z = \frac{2\sqrt{\frac{2eV_0}{b^2 m}}}{\left(1 - \frac{a^2}{b^2}\right)} = \frac{\sqrt{8} \cdot \cancel{\sqrt{\frac{e}{m}}} \cdot \frac{\sqrt{V_0}}{b}}{\left(1 - \frac{a^2}{b^2}\right)}$$

$\left(\because W_c = \frac{e}{m} B_z \text{ from eqn } 4(A)\right)$

$$\Rightarrow B_z = \frac{\sqrt{8 \frac{m}{e} V_0}}{b\left(1 - \frac{a^2}{b^2}\right)} \quad \text{—— (11)}$$

The electron will acquire a tangential as well as radial velocity. For a given Vo, The magnetic flux density for which electron will just graze the anode and return towards the cathode [Curve c'' on fig 61] is known as cutoff magnetic flux density. The Hull cutoff magnetic eqn obtained from eqn (11) as

$$\boxed{B_{oc} = \frac{\left(8 V_o \frac{m}{e}\right)^{\frac{1}{2}}}{b\left(1-\frac{a^2}{b^2}\right)}} \quad \text{—— (12)}$$

Conversely, for a given magnetic flux density Bo, the d.c voltage for which electron will just graze the anode and return towards the cathode is known as cutoff voltage. The Hull cutoff voltage eqn obtained from eqn (11), as

$$\boxed{V_{oc} = \frac{e}{8m} B_0^2 b^2 \left(1-\frac{a^2}{b^2}\right)^2} \quad \text{—— (13)}$$

→ From eqn (12), it is clear that iff $B_o > B_{oc}$, for a given V_o, the electron will not reach the anode and return to the cathode [Curve d' on fig 61].

→ From eqn (13), it is clear that if $V_o < V_{oc}$ for a given Bo, the electron will not reach the anode.

Ex -1 Liao Book

An X-band pulsed cylindrical magnetron has the following operating parameters:

Anode voltage = 26 kV (V_o)

Beam current (I_o) = 27 A

Magnetic flux density (B_o) = 0.336 $\frac{Wb}{m^2}$

Radius of cathode cylinder (a) = 5 cm

Radius of vane edge to the center (b) = 10 cm

Compute

(a) Cyclotron angular freq

(b) The cutoff voltage for a fixed B_o

(c) The cutoff magnetic flux density for a fixed V_o

Ans :- From eqn 4(A), cyclotron angular freq

$$W_c = \frac{e}{m} B_o$$

$$= (1.759 \times 10^{11}) \times 0.336$$

(a) $$W_c = 5.91 \times 10^{10} \text{ rad}$$

(b) The cutoff voltage for a fixed B_o is

$$V_{oc} = \frac{1}{8} \times \frac{e}{m} \times B_o^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$= \frac{1}{8} \times 1.759 \times 10^{11} \times (0.336)^2 \times (10 \times 10^{-2})^2 \times \left(1 - \left(\frac{5}{10}\right)^2\right)^2$$

$$= \frac{1}{8} \times 1.759 \times (.336)^2 \times \frac{9}{16} \times 10^9$$

$\Rightarrow \quad V_0 = 139.62 \times 10^{-4} \times 10^{9}$

$\Rightarrow \quad \boxed{V_0 = 139.62 \times 10^{5}\ V}$

[Check the ans But in Box = 139.62 kV]

(c) The cutoff magnetic flux density for a fixed V_0 is

$$B_0 = \frac{\left(8 V_0 \frac{m}{e}\right)^{\frac{1}{2}}}{b\left(1 - \frac{a^2}{b^2}\right)}$$

$$\Rightarrow \quad B_0 = \frac{\sqrt{8 \times 26 \times 10^{3} \times \frac{1}{1.759 \times 10^{11}}}}{10 \times 10^{-2} \left(1 - \left(\frac{5}{10}\right)^2\right)}$$

$$= \frac{\sqrt{118.24 \times 10^{-8}}}{10^{-1} \times \left(\frac{3}{4}\right)}$$

$$= \frac{10.87 \times 10^{-4} \times 4}{10^{-1} \times 3}$$

$$B_0 = 14.498 \times 10^{-3}$$

$$\Rightarrow \quad \boxed{B_0 = 14.498\ m\,\frac{Wb}{m^2}}$$

Cyclotron Angular frequency:-

Since the magnetic field is normal to the motion of electrons that travel in a cycloidal path, the outward centrifugal force is equal to the pulling force. Hence

$$\frac{mv^2}{R} = evB$$

where R = radius of the cycloidal path,

v = tangential velocity of the electron.

The cyclotron angular freq of the circular motion of the electron is then given by,

$$W_c = \frac{v}{R} = \frac{eB}{m} \quad \text{—(15)} \qquad \left[\because \frac{mv^{\cancel{2}}}{R} = e\cancel{v}B \Rightarrow \frac{v}{R} = \frac{eB}{m} \right]$$

Resonant Modes in a Magnetron:-

The nature of field distribution in the magnetron cavities is such that the alternating RF magnetic flux lines pass through the cavities parallel to the cathode axis, and the alternating RF electric fields are concentrated across the slot and fringe out to the interaction space between the anode & cathode, in the transverse direction.

Since the slow wave structure is closed on it self, the total phase shift around the internal periphery must be an integral multiple of '2π' for possible oscillations. The phase shift between two adjacent cavities is given by

$$\phi_n = \frac{2\pi}{N} \cdot n \quad — (16)$$

| N = No of cavities.

where $n = \pm 1, \pm 2, \pm 3, ---, \pm \frac{N}{2}$

indicates the nth mode of oscillation.

Imp → Magnetrons are ordinarily operated on the π-mode. i.e

$$\phi_n = \pi \quad (\pi - \text{mode})$$

This occurs when $n = \frac{N}{2}$.

e.g If a magnetron having 8-cavities.

$$N = 8, \quad n = \frac{N}{2} = \frac{8}{2} = 4.$$

$$\phi_n = \frac{2\pi \times 4}{8_2} = \pi$$

Since the phase difference between two successive cavities is 'π', excitation is maxm in the cavities.

Note:- Net phase shift of 8 cavities $= 8\pi$, which is multiple of 2π.

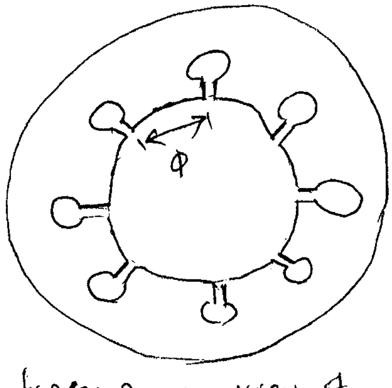
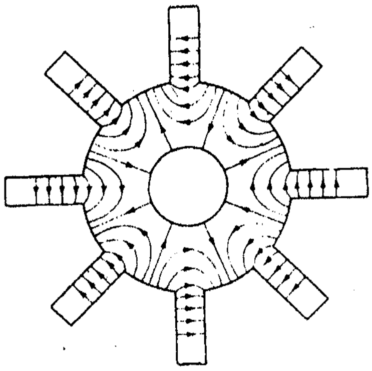


fig 62: Inside view of 8-cavity magnetron.

Fig 63:- Shows the lines of force in the π-mode of an eight-cavity magnetron. It is evident that in the π-mode the excitation is large in the cavities, having opposite phase in successive cavities.



63
Figure ~~10-1-3~~ Lines of force in π mode of eight-cavity magnetron.

→ The successive rise and fall of adjacent anode-cavity fields may be regarded as a travelling wave along the surface of the slow-wave structure. For the energy to be transferred from the moving electrons to the travelling field, the electrons must be deaccelerated by ~~the~~ a retarding field when they pass through each anode cavity.

→ If 'L' is the mean-separation between cavities, the phase constant of the fundamental mode field is given by

$$\beta_0 = \frac{2\pi n}{NL} \quad -(17)$$

Note:- In Transmission line we have $\tan\beta l$

$\therefore \beta l = \text{Angle}$

$\Rightarrow \beta = \frac{\text{Angle}}{l} = \frac{\phi}{L}$

Here $\phi = \frac{2\pi n}{N}$

...e traveling field of the fundamental mode ...avels around the structure with angular velocity $\frac{d\phi}{dt} = \frac{\omega}{\beta_0}$ —— (18)

Note:—
In Rectangular wave guide
Phase velocity
$u = \frac{\omega}{\beta}$

When the cyclotron frequency of ...e electrons is equals to the angular frequency of the field, the interaction between the field and electron occures and energy is transferred.

$$\omega_c = \omega$$

$$\Rightarrow \boxed{\omega_c = \beta_0 \frac{d\phi}{dt}} \text{ —— (19)}$$

[From eq^n 18]

Where

$\omega_c = \frac{eB}{m}$ from eq^n (15)

...ower o/p & efficiency of Magnetron

A magnetron can deliver a peak power o/p up to 40 MW with d.c voltage of 50 KV ... 10 GHZ. The average power o/p is of ...e order of 800 KW.

The magnetron possesses a very high ~~frequency~~ efficiency ranging from 40 to 70%.

Magnetrons are commercially available for peak power output from 3KW and higher.

Traveling - Wave Tubes (TWT) (Liao Book)

→ Kompfner invented helix TWT in 1944.

→ Broadband appl^n → Helix TWT used.

→ High-avg-power purpose such as radar transmitters Coupled-Cavity TWT are used.

→ Comparision of operating principle of TWT & Klystron

→ In case of TWT, the MW ckt is non resonant and wave propagates with same speed as the electrons in the beam. The initial effect on the beam is a small amount of velocity modulation caused by weak electric fields associated with traveling wave.

But in case of Klystron, this velocity modulation later translates to current modulation, which then induces RF current in the ckt, causing amplification.

Major difference bet^n Klystron & TWT

1. The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit, but the interaction in Klystron occures only at the gap of a few resonant cavities.
2. The wave in the TWT is a propagating wave; the wave in the Klystron is not propagating.
3. In the Coupled-Cavity TWT there is a coupling

effect betⁿ the cavities, where as each cavity on the klystron operates independently.

4. In case of TWT, the microwave circuit is non resonant; but in case klystron it is resonant.

Principle of operation :-

→ A helix TWT consist of an electron beam and a slow-wave-structure. The electron beam is focused by a constant magnetic field along the electron beam & the slow-wave structure. [see fig 64]

→ This is termed as O-type travelling-wave tube. The slow-wave structure is either helical type or folded-back line. The applied signal propagates around the turns of the helix and produces an electric field at the center of the helix, directed along the helix axis.

→ The axial electric field progresses with a velocity that is very close to the velocity of light multiplied by the ratio of helix pitch to helix circumference $\left(\frac{v_p}{\pi d}\right)$. When the electrons enter the helix tube, an interaction takes place between the moving axial electric field and the moving electrons.

→ On the average, the electrons transfer energy to the wave on the helix. This interaction

Causes the signal wave on the helix to become larger. The electrons entering the helix at zero field are not affected by the signal wave; those electrons entering the helix

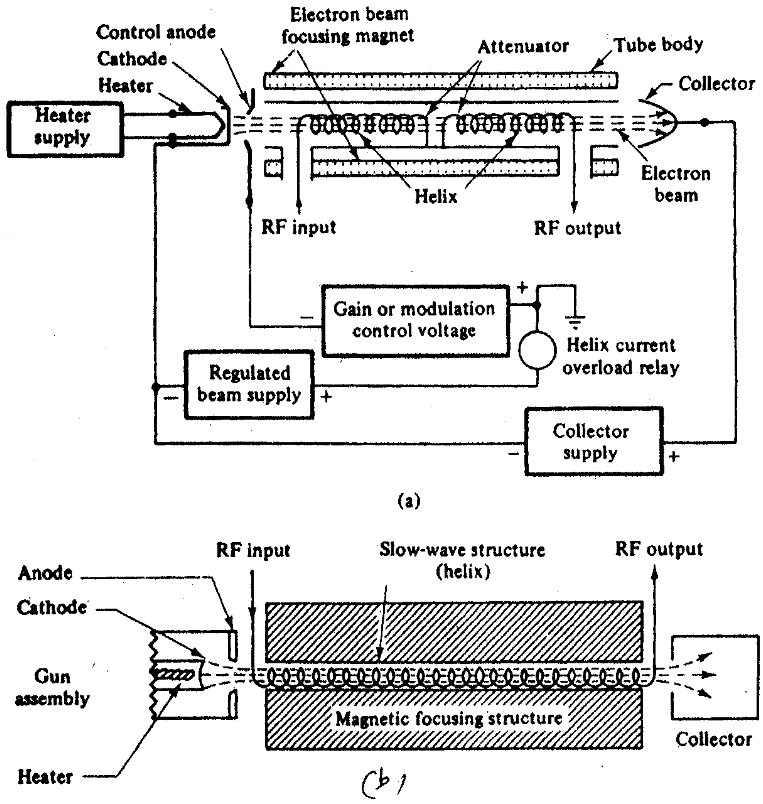


fig 64:- Diagram of helix TWT : (a) Schematic diagram of helix TWT
(b) simplified ckt.

→ at the accelerating field are accelerated [fig 68]
and those at the retarding field are deaccelerated
As the electrons travel further along the helix, they bunch at the collector end. [fig 64]
→ The bunching shift the phase by $\frac{\pi}{2}$.
Each electrons in the bunch encounters a stronger retarding field. Then the MW energy of the electrons is delivered ~~to~~ by the electron bunch to the wave on the helix. The amplification of the

Signal wave is accomplished.

Characteristics of TWT

Freq Range - 3 GHz and higher

Band width - about 0.8 GHz.

Efficiency - 20 to 40 %.

Power o/p - up to 10 kW average

Power gain - up to 60 dB.

Slow-wave Structures

Slow-wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact. Different types of slow-wave structure is shown below.

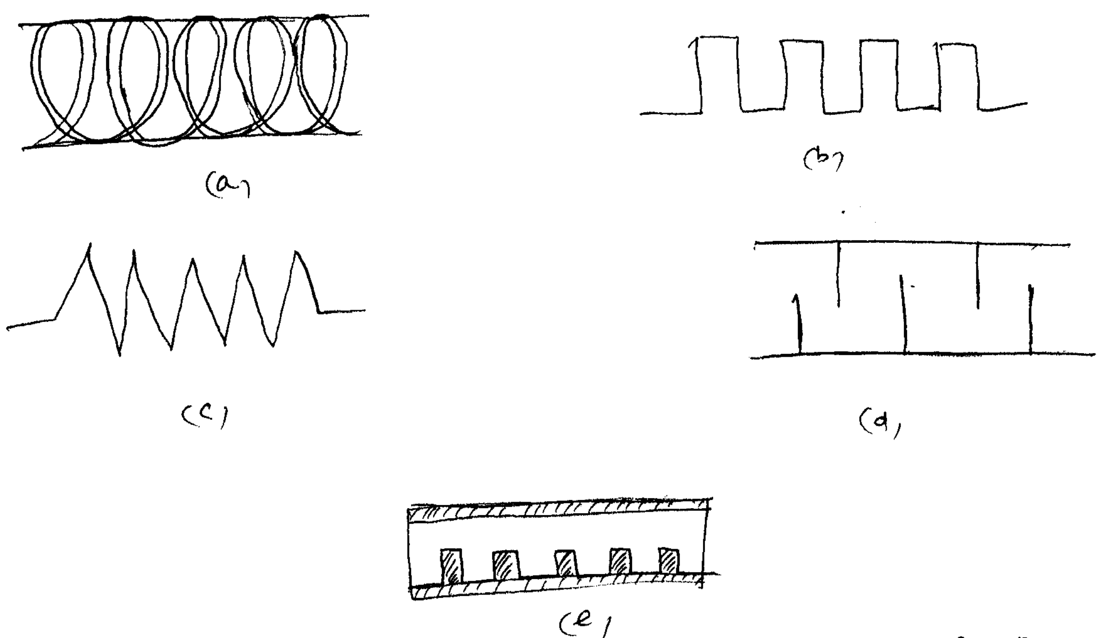
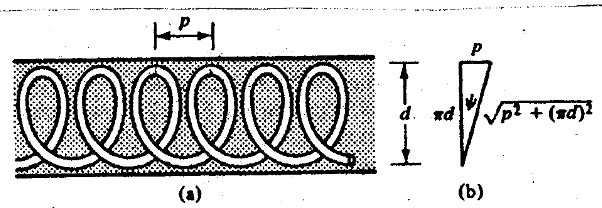


fig 65: Slow-wave structures (a) Helical type (b) Folded-back line (c) Zig-zag line (d) Interdigital line (e) Corrugated waveguide.

The Phase velocity of the wave in ordinary waveguide is greater than the velocity of light in a vacuum. [$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$]

In the operation of traveling-wave and magnetron-type devices, the electron beam must keep in step with microwave signal. Since the electron beam can be accelerated only to to velocities that are about a fractions of the velocity of light, a slow-wave structure must be incorporated in the microwave devices so that the phase velocity of the MW signal ~~must~~ can keep pace with that electron beam for effective interactions.

→ The commonly used slow-wave structure is a helical coil with a concentric conducting cylinder. [fig 6.6]



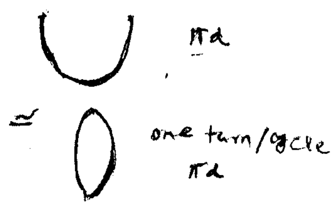


Figure ~~9-6-3~~ 6.6 Helical slow-wave structure. (a) Helical coil. (b) One turn of helix.

→ It can be shown that the ratio of phase velocity v_p along the pitch to the phase velocity along the coil [Along the coil signal travels with velocity of light] is given by

$$\frac{v_p}{c} = \frac{p}{\sqrt{p^2+(\pi d)^2}} = \sin\psi \quad -(1)$$

$\because \sin\psi = \frac{v_p}{c}$
and $\sin\psi = \frac{p}{\sqrt{p^2+(\pi d)^2}}$

where

$c = 3\times10^8$ m/sec

p = helix pitch

d = diameter of the helix

ψ = pitch angle

→ In general, the helical core maybe within a dielectric - filled cylinder. The phase velocity in the axial direction is expressed as,

$$v_{pe} = \frac{p}{\sqrt{\mu\epsilon\left[p^2+(\pi d)^2\right]}} \quad -(2)$$

$\because$ From eq^n (1)
$v_p = \frac{p.c}{\sqrt{p^2+(\pi d)^2}}$
and $c = \frac{1}{\sqrt{\mu\epsilon}}$

→ If pitch angle is very small,

$$p << \pi d$$

$$p^2 << (\pi d)^2$$

the phase velocity along the ~~coil~~ pitch in free space is approximately represented by,

$$\frac{v_p}{c} \simeq \frac{p}{\pi d}$$

{ from eq^n (1)
$\because p^2 << (\pi d)^2$

$$\Rightarrow V_p \simeq \frac{pc}{\pi d} = \frac{\omega}{\beta} \quad -(3)$$

($\because V_p = \frac{\omega}{\beta}$ As studied in rectangular wave guide)

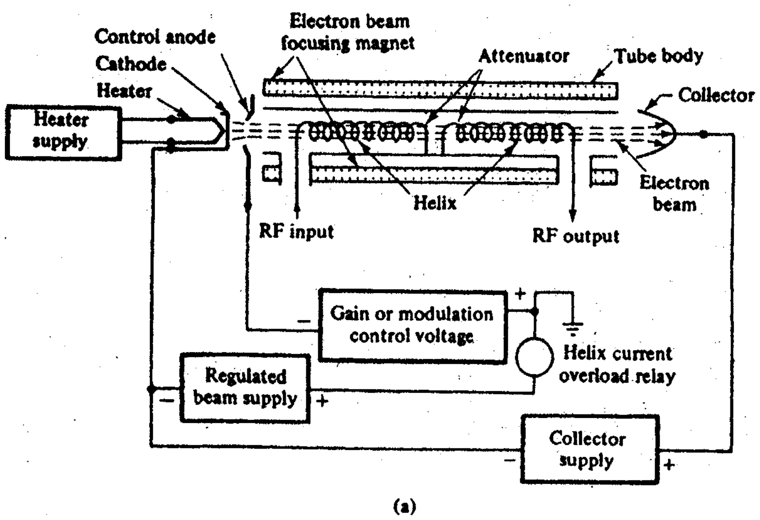
V. Imp Amplification Process:— (BPUT 2010, 2012)

A Schematic diagram of a helix-type traveling-wave tube is shown in fig 67.

→ Write principle of operation in page 307.

Write principle of operation in page 307.

→ The motion of electrons in the helix type TWT can



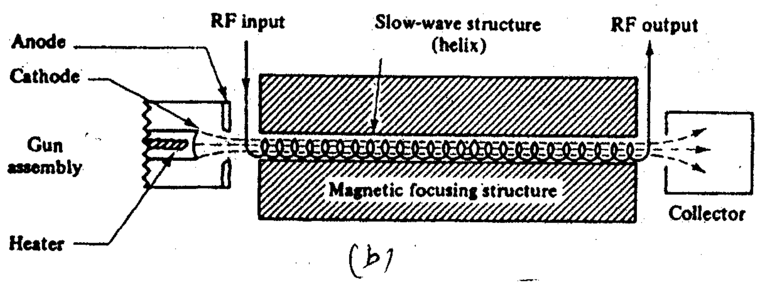


fig 67:- Diagram of helix TWT : (a) Schematic diagram of helix TWT (b) Simplified ckt.

be quantitatively analyzed in terms of the axial electric field. If the traveling wave is propagating in the Z- direction, the Z- Component of the electric field can be expressed

$$E_z = E_1 \sin(\omega t - \beta_p z) \quad (4)$$

...re E_1 is the magnitude of the electric ...d in the Z-direction. If $t = t_0$, at $z = 0$, the electric field is assumed maximum.

$\beta_p = \frac{\omega}{v_p}$ = is the axial phase constant of the microwave

v_p = axial phase velocity of the wave.

The axial electric field exerts a force on the electrons

$$F = (-e).E_z = (-e)\left[E_1 \sin(\omega t - \beta_p z)\right] \quad \text{—} (5)$$

But

$$F = ma = m.\frac{dv}{dt} \quad \text{———} (6)$$

Equating (5) & (6)

$$m\frac{dv}{dt} = -eE_1 \sin(\omega t - \beta_p z) \quad \text{———} (7)$$

Assume that velocity of electron is

$$v = v_0 + v_e \cos(\omega_e t + \theta_e) \quad \text{———} (8)$$

...ere

v_0 = d.c electron velocity

v_e = magnitude of velocity fluctuation in the velocity-modulated electron beam

ω_e = angular freq of velocity fluctuation.

θ_e = phase angle of the fluctuations.

Then

$$\frac{dv}{dt} = -v_e\omega_e \sin(\omega_e t + \theta_e) \quad \text{———} (9)$$

...ting eqn (9) in eqn (7), we have

$$m.\left[-v_e\omega_e \sin(\omega_e t + \theta_e)\right] = -eE_1 \sin(\omega t - \beta_p z)$$

$$\Rightarrow m v_e \omega_e \sin(\omega_e t + \theta_e) = eE_1 \sin(\omega t - \beta_p z) \quad \text{———} (10)$$

For interaction between electrons and the electric field, the velocity of the ~~electron~~ velocity-modulated electron beam must be approximately equal to the d.c electron velocity.

$$v \simeq v_0 . \qquad \text{——— (11)}$$

Hence the distance z travelled by the electron is

$$z = v_0 (t - t_0) \qquad \text{—— (12)}$$

and eq^n (10) becomes,

$$m v_e w_e \sin(w_e t + \theta_e) = e E_1 \sin[wt - \beta_p (v_0(t-t_0))]$$

$$\Rightarrow m v_e w_e \sin(w_e t + \theta_e) = e E_1 \sin[(w - \beta_p v_0)t + \beta_p v_0 t_0] \qquad \text{—— (13)}$$

Comparing left and right hand side of eq^n (13), we have

$$m v_e w_e = e E_1$$

$$\Rightarrow v_e = \frac{e E_1}{m w_e} \qquad \text{——— (14)}$$

$$w_e = w - \beta_p v_0 \qquad \text{——— (15)}$$

$$= \cancel{\beta_p v_p - \beta_p v_0} \qquad \left[\because \beta_p = \frac{w}{v_p} \right]$$

$$= \beta_p v_p - \beta_p v_0$$

$$\Rightarrow w_e = \beta_p (v_p - v_0) \qquad \text{—— (16)}$$

and $\theta_e = \beta_p v_0 t_0$ —— (17)

[illegible] can be seen that the magnitude of velocity fluctuation (v_e) of the electron beam is directly proportional to the magnitude of the [illegible] electric field.

[illegible]te :- The electrons entering the retarding field are deaccelerated and those in the accelerating field are accelerated. They begin forming a bunch centered about those electrons that enter the helix during the zero field.

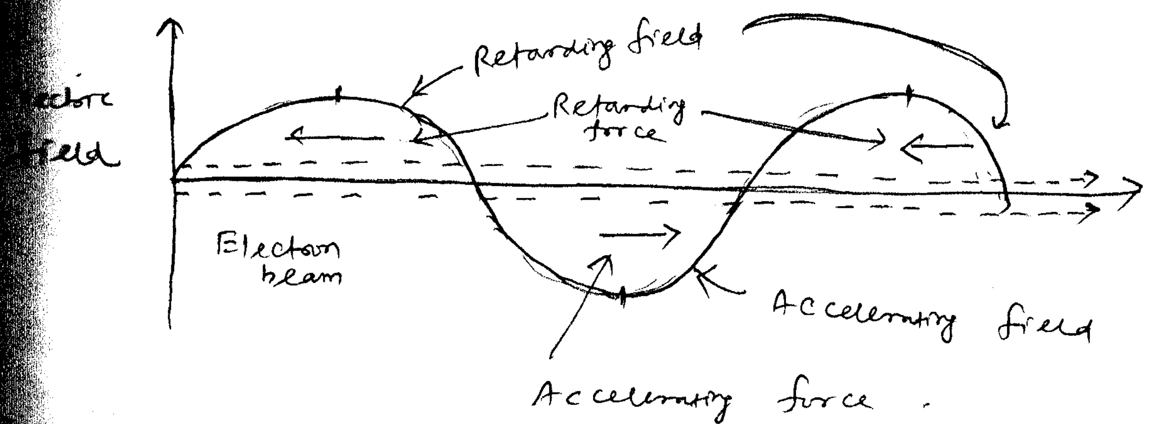


Fig 68:- Interaction between electron beam & electric field

Since the d.c velocity of the electrons is slightly greater than the axial wave velocity, more electrons are in the retarding field than in the accelerating field and a great amount of energy is transferred from the beam to the e.m field.

The mw signal voltage is, in turn amplified by the amplified field. The bunch continues to become more compact and a larger amplification of the signal voltage occures at the end of the helix. The magnet produces an axial magnetic field to prevent spreading of the electron beam as it travels down the tube.

An attenuator placed near the center of the helix, to attenuate any reflected wave generated ~~during~~ due to impedance mismatch that could ~~cause~~ be fed back to the i/p to cause oscillations.

Klystron Amplifier	TWT Amplifier
1) Linear beam or 'O' type device	1) Linear beam or 'O' type device
2) Electric field is stationary only e-beam travels.	2) Electric field travels along the electron beam
3) Interaction of e-field & e-beam ~~is conti~~ takes place only at the gap of few resonant cavities.	3) The interaction of e-field & e-beam takes place over the entire length of the ckt.
4) The wave (i/p) is not propagating.	4) The wave (i/p) in TWT is propagating
5) Use resonant MW ckts	5) Use non-resonant MW ckts.
6) Narrowband device due to resonant cavities.	6) Wideband device because of non-resonant wave circuits.

(Appl^n) of TWT :-

1) (TWT) amplifiers are used in medium-power (Satellite) and high-power satellite transponder o/p.

2) Repeater amplifier in wideband Comm^n line.

3) ~~Ex~~ Used in Radar transmitter

(For) Problems :-

The o/p Power gain in dB is defined as

$$\boxed{A_p} = 10 \log \left| \frac{\text{o/p voltage}}{\text{i/p voltage}} \right|^2 = \boxed{-9.54 + 47.3\,NC}$$

first term -9.54 dB represents a loss. ~~due to fact that~~

$$N = \frac{l}{\lambda_e}$$

$$\lambda_e = \frac{2\pi}{\beta_e}$$

l = length of slow-wave structure in meter.

N = circuit length in electronic wavelength.

[N = How many multiple of λ_e]

$\beta_e = \frac{\omega}{u_0}$, $u_0 = \sqrt{\frac{2eV_0}{m}}$

The factor 'C' is the gain parameter of the circuit defined by

$$\boxed{C = \left(\frac{I_0 Z_0}{4 V_0}\right)^{\frac{1}{3}}}$$

I_0 = d.c beam current

V_0 = ~~$~~ d.c beam voltage

Z_0 = Characteristic impedance of the helix.

BPUT-2012 / Liao ex: 9.5-1 **Ex-1 :-** A TWT has the following characteristics. Beam voltage $V_0 = 3KV$, Beam current $I_0 = 30mA$, $Z_0 = 10\Omega$, Circuit length $N = 50$, freq $(f) = 10\,GHz$.

Determine (a) The gain parameter (C)

(b) Power gain in dB.

Ans → (a) $C = \left(\dfrac{I_0 Z_0}{4V_0}\right)^{\frac{1}{3}}$

$$= \left(\frac{\cancel{30}^{10} \times 10^{-3} \times 10}{4 \times \cancel{3} \times 10^{3}}\right)^{\frac{1}{3}}$$

$$= (25)^{\frac{1}{3}} \times 10^{-2}$$

$$\boxed{C = 2.92 \times 10^{-2}}$$

(b) $A_p = -9.54 + 47.3\, N.C$

$$= -9.54 + (47.3)(50)(2.92) \times 10^{-2}$$

$$= -9.54 + 69.058$$

$$\boxed{A_p = 59.518\ dB}$$

be

e

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$$\boxed{\eta = 46.2\ \%}$$

Forward-Wave Cross-field Amplifier (FWCFA/ CFA)

The Crossed-field amplifier (CFA) is an outgrowth of the magnetron. CFAs can be grouped by their mode of operation as forward-wave or backward-wave types and by their electron stream source as emitting-sole or injected-beam types.

The first group concerns the direction of the phase and group velocity of the energy on the microwave circuit.

The second group emphasizes the method by which electrons reach the interaction region and how they are controlled. This can be shown in Fig 16.

In the forward-wave mode, the helix-type slow-wave structure is often selected as the microwave circuit for the cross-field amplifier; in the backward-wave mode, the strapped bar line represents a satisfactory choice. The structure of strapped cross-field amplifier is shown in fig. 17.

Principle of Operation:-

In the emitting-sole tube, the current emanated from the cathode is in response to the electric field forces in the space between the cathode & anode.

The amount of current is a function of the dimension, the applied voltage, and the emission properties of the cathode.

In the injected-beam tube the electron beam is produced in a separate gun assembly and is injected into the interaction region.

The beam-circuit interaction features are similar in both emitting-sole & the injected-beam tubes. Favorably phased electrons continue towards the +vely polarized anode & are ultimately collected, where as the unfavourably phased electrons are directed toward the -vely polarized electrode.

In the CFA, the electrons is exposed to the dc electric field force, magnetic field force, and the electric field force of the RF field.

Under the influence of the three forces, the electrons travel in spiral trajectories on a direction tending along equipotentials. Fig 18. shows the patterns of electron flow in the CFA by computerised techniques.

It can be seen that when the spoke is +vely polarized or the RF field is in the +ve half cycle, the electron speed up ~~forward~~ toward the anode; while the spoke is -vely polarized or the RF field is in the -ve half cycle, the electrons are returned towards the cathode. Consequently, the electrons beam moves in a spiral path in the interaction

Forward-Wave Crossed-Field Amplifier (FWCFA or CFA)

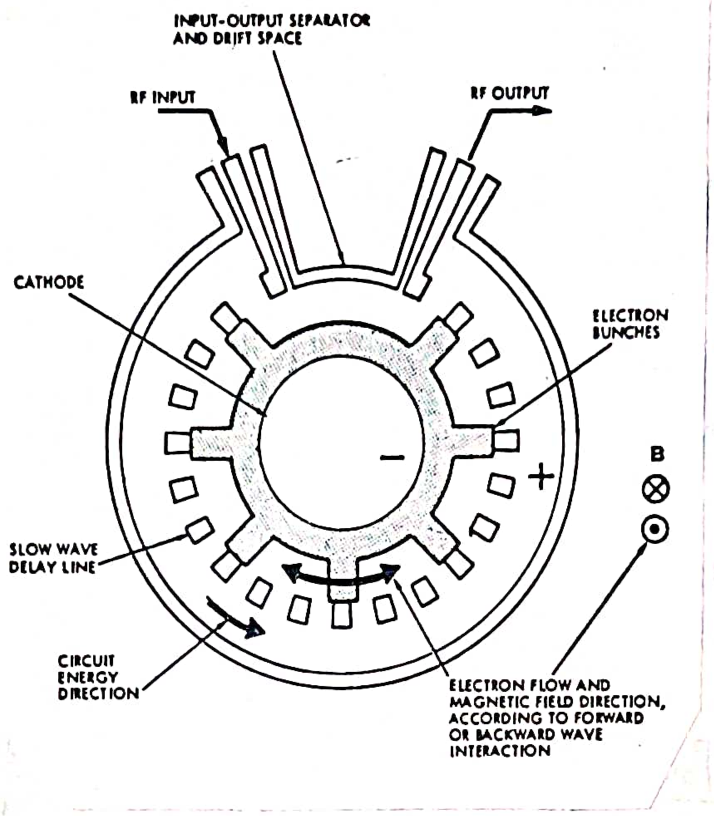

Fig 16. Schematic diagram of CFA

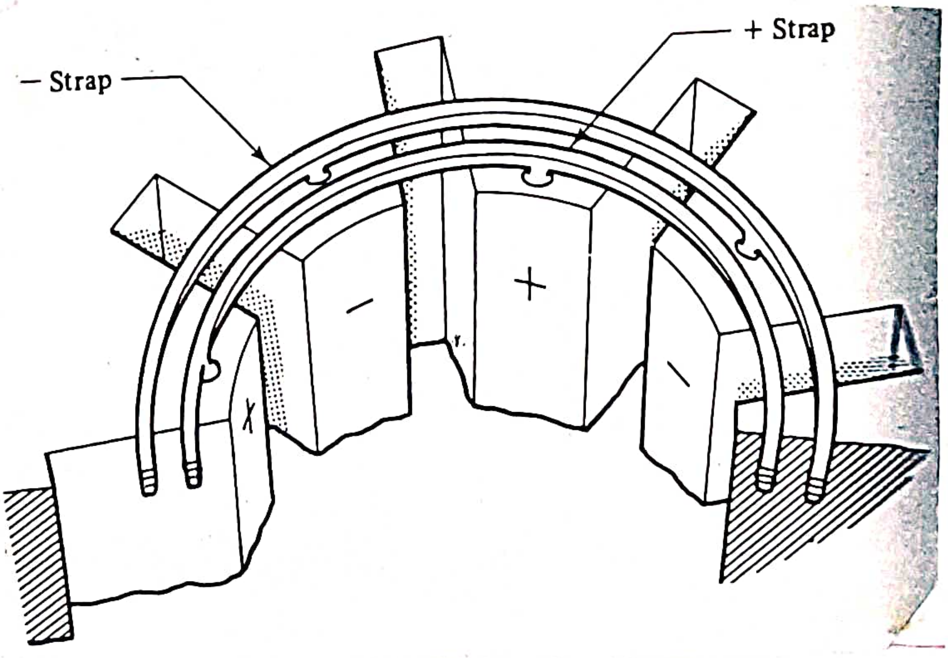

Fig 17. Diagram of a Strapped CFA

region,

The total power generated in a given CFA is independent of RF i/p power, as long as the i/p power exceeds the threshold value for space stability at the i/p.

The power generated can be increased only by increasing the anode voltage & current. Neglecting the circuit attenuation, the o/p power of CFA is equal to the sum of i/p power & power generated in the interaction region, i.e the power gain of a CFA is given by

$$g = \frac{P_{out}}{P_{in}} = \frac{P_{in} + P_{gen}}{P_{in}} = 1 + \frac{P_{gen}}{P_{in}} \quad \text{--- (1)}$$

where $P_{out} = P_{in} + P_{gen}$ = RF o/p power

P_{in} = RF i/p power

P_{gen} = RF power induced onto the anode circuit by electrons.

Therefore, the CFA is not a linear amplifier but rather is termed a saturated amplifier.

→ The efficiency of a CFA is defined as the product of the electronic efficiency (η_e) and circuit efficiency (η_c). The electronic efficiency (η_e) is defined as

$$\eta_e = \frac{P_{gen}}{P_{dc}} \quad \text{--- (2)}$$

&

$$\eta_c = \frac{P_{out} - P_{in}}{P_{gen}} \quad \text{--- (3)}$$

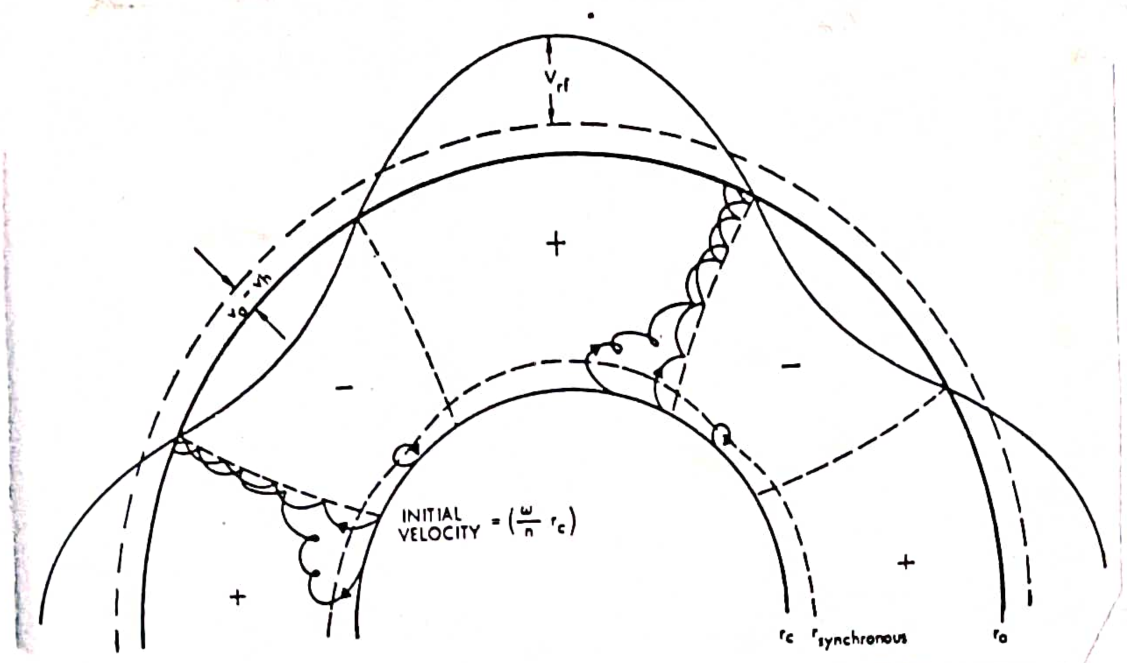

Fig 18. Motion of electrons in CFA.

∴ The overall efficiency is then expressed as

$$\eta = \eta_c \times \eta_e = \frac{P_{out} - P_{in}}{\cancel{P_{gen}}} \times \frac{\cancel{P_{gen}}}{P_{dc}}$$

$$\eta = \frac{P_{out} - P_{in}}{P_{dc}} \quad \text{—— (4)}$$

Where $P_{dc} = V_{ao} \cdot I_{ao}$ is d.c power

V_{ao} = Anode d.c Voltage

I_{ao} = Anode d.c Current

From eqn (2),

$$P_{gen} = \eta_e \cdot P_{dc} = \eta_e \times V_{ao} I_{ao} \quad \text{—— (5)}$$

Since the power generated per unit length is constant, the o/p power is given by

$$P_{out} = P_{in}\, e^{-2\alpha l} + \int_0^l \frac{P_{gen}}{l}\, e^{-2\alpha(l-\phi)}\, d\phi \quad \text{—— (6)}$$

$$P_{out} = P_{in}\, e^{-2\alpha l} + \frac{P_{gen}}{l} \times e^{-2\alpha l} \int_0^l e^{+2\alpha\phi}\, d\phi$$

$$= P_{in}\, e^{-2\alpha l} + \frac{P_{gen} \times e^{-2\alpha l}}{l} \times \left. \frac{e^{2\alpha\phi}}{2\alpha} \right]_0^l$$

$$P_{out} = P_{in}\, e^{-2\alpha l} + \frac{P_{gen} \times e^{-2\alpha l}}{2\alpha . l} \times \left[e^{2\alpha l} - 1 \right]$$

$$P_{out} = P_{in}\, e^{-2\alpha l} + \frac{P_{gen}}{2\alpha l} \left[1 - e^{-2\alpha l} \right] \quad \text{——— (7)}$$

Where

$\alpha =$ Circuit Attenuation Constant

$l =$ Circuit length in ϕ direction,

Putting eqn (7) in eqn eqn (3), we have

$$\eta_c = \frac{P_{out} - P_{in}}{P_{gen}} = \frac{P_{in}.e^{-2\alpha l} + \frac{P_{gen}}{2\alpha l}\left[1 - e^{-2\alpha l}\right] - P_{in}}{P_{gen}}$$

$$= \frac{-P_{in}\left[1 - e^{-2\alpha l}\right] + \frac{P_{gen}}{2\alpha l}\left[1 - e^{-2\alpha l}\right]}{P_{gen}}$$

$$= \frac{\left[1 - e^{-2\alpha l}\right]\left[\frac{P_{gen}}{2\alpha l} - P_{in}\right]}{P_{gen}}$$

$$= \left[1 - e^{-2\alpha l}\right]\left[\frac{1}{2\alpha l} - \frac{P_{in}}{P_{gen}}\right]$$

$$\Rightarrow \eta_c = \left[\frac{1}{2\alpha l} - \frac{P_{in}}{P_{gen}}\right]\left[1 - e^{-2\alpha l}\right] \quad \text{--- (8)}$$

The term $\frac{P_{in}}{P_{gen}}$ becomes negligible for high-gain CFA.

Prob

1) A CFA operates under the following parameters

Anode d.c voltage $V_{ao} = 2\,KV$

Anode d.c current $I_{ao} = 1.5\,A$

Electronic efficiency $(\eta_e) = 20\%$.

RF i/p power $P_{in} = 80W$

Calculate

(a) The induced RF power

(b) The total RF o/p power

(c) The power gain in dB.

Ans: (a) The induced RF power

$P_{gen} = \eta_e \times V_{ao} \times I_{ao}$ [From eq^n (5)]

$= 0.2 \times 2\times10^3 \times 1.5$

$= 2\times10^{-1} \times 2\times10^3 \times 1.5$

$= 6\times10^2$

$P_{gen} = 600$ watt

(b) The total RF o/p power

$$P_{out} = P_{in} + P_{gen} = 80 + 600 = 680 \text{ watt}$$

(c) The power gain (in dB)

$$g = \frac{P_{out}}{P_{in}} = \frac{680}{80} = 8.50$$

$$(g)_{dB} = 10\log(8.5) = 9.294 \text{ dB}$$

Backward-wave Cross-field Oscillator (Carcinotron)

Linear M-Carcinotron

The M-Carcinotron oscillator is an M-type backward-wave oscillator. The interaction between the electrons and the slow wave structure takes place in a space of cross field. A linear model of the M-Carcinotron oscillator is shown below.

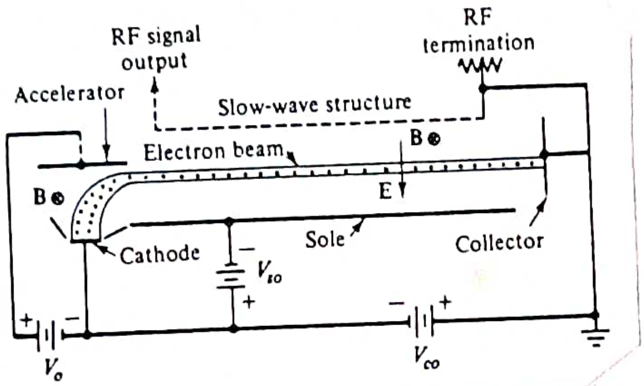

Fig 19:. Linear Model of an M-Carcinotron Oscillator

The slow wave structure is in parallel with an electrode known as sole. A d.c electric field is maintained between the grounded slow-wave structure and the -ve sole. A dc magnetic field is directed into the page. The electrons emitted from the cathode are bent through a 90° angle by the magnetic field. The electrons interact with a backward-wave space harmonic of the circuit, and the energy on the circuit flows opposite to the direction of the electron motion. The slow-wave structure is terminated at the collector end, and the RF signal o/p is removed at the electron-gun end.

Since the M-Carcinotron is a Crossed-field device, its efficiency is very high, ranging from 30% to 60%.

The Perturbed electrons moving in synchronism with the wave in a linear M-Carcinotron are shown in fig 20.

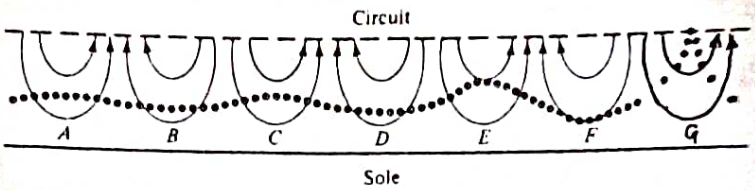

Fig 20:- Beam Electrons & electric field lines in an M-Carcinotron

Electrons at Position 'A' near the beginning of the circuit are moving toward the circuit, whereas electrons at position 'B' are moving toward the sole. Farther down the circuit, electrons at position 'C' are closer to the circuit, and electrons at position 'D' are closer to the sole.

However, electrons at position 'C' have departed a greater distance from the unperturbed path than have electrons at position D. Thus, the electrons have lost a net amount of Potential energy, this energy having been transferred to the RF field.

The reason for the greater displacement of the electrons moving toward the circuit is that these electrons are in stronger RF fields, since they are closer to the circuit.

Electrons at position 'G' have moved so far from the unperturbed position that some of them are being intercepted on the circuit.

The length from the position 'A' through position 'G' is a half cycle of the electron motion.