ଜୟ ଜଗନ୍ନାଥ

Equation of motion for uniformly accelerated motion of bodies moving in a straight line.

Formulae :

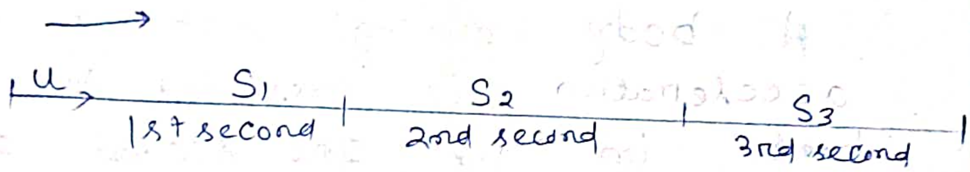
1. $V = u + at$
2. $V^2 - u^2 = 2as$
3. $S = ut + \frac{1}{2}at^2$
4. $\frac{S}{t} = \bar{V}$ that is average speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

$$= \frac{S_1 + S_2 + S_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$$

5. $S_n = u + \frac{a}{2}(2n-1)$

(where S_n = distance covered during the nth second)

Ex :-



$$S_3 = (S_1 + S_2 + S_3) - (S_1 + S_2)$$

= (Total distance covered in 3 seconds) − (Total distance covered in 2 seconds)

$$= \left[u \cdot 3 + \frac{1}{2} \cdot a \cdot (3)^2\right] - \left[u \cdot 2 + \frac{1}{2} \cdot a (2)^2\right]$$

$$= \left[3u + \frac{1}{2} a \cdot 9\right] - \left[2u + \frac{1}{2} a \cdot 4\right]$$

$$= 3u + \frac{9a}{2} - 2u - \frac{4a}{2}$$

$$= u + 5\frac{a}{2}$$

Derivation of the formula

$$S_n = u + \frac{a}{2}(2n-1)$$

S_n = [Total distance covered in n second] − [Total distance covered in (n−1) second]

$$= \left[u \cdot n + \frac{1}{2} \cdot a n^2\right] - \left[u \cdot (n-1) + \frac{1}{2} a \cdot (n-1)^2\right]$$

$$= \left(un + \frac{1}{2} an^2\right) - \left[un - u + \frac{1}{2} a (n^2 + 1 - 2n)\right]$$

$$= \left(un + \frac{1}{2} an^2\right) - \left(un - u + \frac{1}{2} an^2 + \frac{1}{2} a - \frac{2an}{2}\right)$$

$$= \left(\cancel{un} + \frac{1}{2} \cancel{an^2} - \cancel{un} + u - \frac{1}{2} \cancel{an^2} - \frac{1}{2} a + an\right)$$

$$= u - \frac{1}{2} a + \frac{an}{2}$$

$$= u + \frac{a}{2}(2n-1)$$

Problem :1

A body moving with a uniform acceleration is observed to travel 24 feet in the 2nd second and 100 feet in the 4th second of its motion. How far will it go in the fifth second? (Ans: 138 feet)

Ans: We know the formula

$$S_n = u + \frac{a}{2}(2n-1)$$

$$\Rightarrow S_2 = u + \frac{a}{2}\{2 \cdot (2) - 1\}$$

$$\Rightarrow S_2 = u + \frac{a}{2}(3)$$

$$\Rightarrow 24 = u + \frac{3a}{2} \quad \text{——— (i)}$$

$$S_4 = u + \frac{a}{2}\{2 \cdot (4) - 1\}$$

$$\Rightarrow S_4 = u + \frac{a}{2}(7)$$

$\Rightarrow \; 100 = u + \frac{7a}{2}$ ——— (ii)

Subtracting from equation (2) ~~ao~~ to equation (1)

$$u + \frac{7a}{2} = 100$$
$$u + \frac{3a}{2} = 24$$
$$(-) \; (-) \qquad (-)$$
$$\frac{4a}{2} = 76$$
$$\Rightarrow 4a = 152$$
$$\Rightarrow a = \frac{152}{4} = 38 \text{ feet}/s^2$$

Putting the value in eq^n (1) we get

$$u + \frac{3a}{2} = 24$$
$$\Rightarrow u + \frac{3.(38)}{2} = 24$$
$$\Rightarrow u + \frac{114}{2} = 24$$
$$\Rightarrow u + 57 = 24$$
$$\Rightarrow u = 24 - 57 = -33 \text{ feet/sec}$$

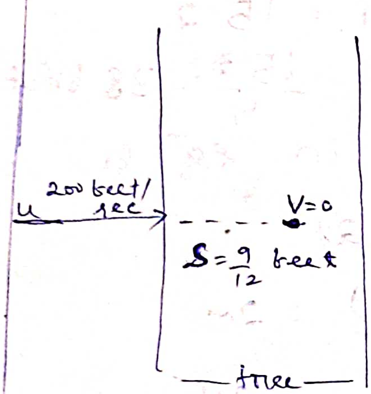
$$S_5 = u + \frac{a}{2}\{(2).(5) - 1\}$$
$$= u + \frac{a}{2}(9)$$
$$= u + \frac{9a}{2}$$
$$= (-33) + \frac{9.(\cancel{38}^{19})}{\cancel{2}}$$
$$= -33 + 171$$
$$= 138 \text{ feet/sec}.$$

∴ It will go 138 feet in fifth second.

2.) A bullet moving at the rate of 200 feet/sec is fired into trunk of a tree in which it penetrates 9 inches. If the bullet moving with the same velocity were fired into a

Similar piece of wood 5 inches thick, with what velocity would it emerge, supposing the resistance to be uniform.
(Ans: 400/3 = 133.33 feet/sec)

Ans:
12 inch = 1 feet
1 inch = $\frac{1}{12}$ feet
9 inch = $\frac{9}{12}$ feet
5 inch = $\frac{5}{12}$ feet.



We know from the 3rd equation of motion that $v^2-u^2=2as$
After moving 9 inches the bullet must be stopped.
So final speed = V = 0,
u = 200 feet/sec.
$s = \frac{9}{12}$ feet.

$$v^2 - u^2 = 2as$$
$$\Rightarrow 0-(200)^2 = \cancel{2}(a).\frac{9}{\cancel{12}_6}$$
$$\Rightarrow -40000 = \frac{9a}{6}$$
$$\Rightarrow 9a = -240000$$
$$\Rightarrow a = \frac{-240000}{9} = \frac{-80000}{3} \text{ feet/second}^2$$

In the second time the bullet was fired into 5 inches.
u = 200 feet/sec.
$S = \frac{5}{12}$ feet.
$a = -\frac{80,000}{3}$ feet/sec^2

$$v^2-u^2 = 2as$$
$$\Rightarrow v^2-(200)^2 = \cancel{2}.\left(-\frac{80,000}{3}\right).\left(\frac{5}{\cancel{12}_6}\right)$$

$\Rightarrow v^2 - 40,000 = \frac{-400000}{18}$

$\Rightarrow 18v^2 - 720000 = -400000$

$\Rightarrow 18v^2 = 720000 - 400000$
$= 320000$

$\Rightarrow v^2 = \frac{320000}{18} = \frac{160000}{9}$

$\Rightarrow v = \frac{400}{3} = 133.33$ feet/sec.

$\therefore$ It would emerge in 133 feet/sec.

3. A body projected upwards with a velocity 32 feet/sec.
(a) How high the body will rise?
(b) what is the time taken by the body to reach the highest point?
(c) what is the time taken by the body to touch the ground?
(d) with what velocity the body will strike the ground?

Ans: (a) At the highest point the final speed $v = 0$,
$u = 32$ feet/sec.
$a = -32$ feet/sec

The third eqn of motion
$v^2 - u^2 = 2as$
$\Rightarrow 0 - 1024 = 2.(-32).s$
$\Rightarrow 0 - 1024 = -64s$
$\Rightarrow 64s = 1024$
$\Rightarrow s = \frac{1024}{64} = 16$ feet upwords.

So the body will ~~rise~~ rise 16 feet.

(b) The first eqn of motion

$$V = u + at$$

$$\Rightarrow 0 = 32 + (-32).t$$

$$\Rightarrow -32 = -32t$$

$$\Rightarrow t = \frac{-32}{-32} = 1 sec,$$

(c.) At the time of falling

$u = 0$, $g = 32$ feet/sec^2

$\Rightarrow a = 32$ feet/sec^2

$s = 16$ feet

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 16 = 0.t + \frac{1}{2}.(32).t^2$$

$$= 16t^2$$

$$\Rightarrow t^2 = 1$$

$$\Rightarrow t = 1 sec$$

$\therefore$ It takes 1 sec to fall to the ground.

(d) We know that $V = u + at$

$$\Rightarrow V = 0 + 32(1)$$

$$= 32 \text{ feet/sec}$$

$\therefore$ The body will strike the ground with velocity 32 feet/sec

4. A ball thrown up is caught the thrower 3 seconds after its starts.

(a) How high did it go?

(b) With what speed was it thrown?

(c) How far below its highest point was it 2 seconds after its start ?

(Ans: 36 feet, 48 feet/sec, 4 feet)

Ans :- (b) Time taken to rise = Time taken to fall.

Here $t = \frac{3}{2}$ seconds.

$a = -32$ feet/sec.

$v = 0$

$v = u + at$

~~(a) $s = ut + \frac{1}{2}at^2$~~

$V = u + at$

$\Rightarrow 0 = u + (\cancel{-32}^{16}) \cdot (\frac{3}{\cancel{2}} sec)$

$= u - 48 \; sec$

$\Rightarrow u = 48$ feet/sec.

(a) $S = ut + \frac{1}{2}at^2$

$= (\cancel{48}^{24})(\frac{3}{\cancel{2}}) + \frac{1}{\cancel{2}} \cdot (\cancel{-32}^{-16}) \cdot (\frac{9}{4})$

$= 72 + \frac{-144}{4}$

$= 72 - 36$

$= 36$ feet.

(c) Out of two seconds the body takes 1.5 sec to reach the highest point and then it falls down freely in .5 second.

So $u = 0$, $t = \frac{1}{2}$ sec.

$S = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \cdot (\cancel{32}^{16 \; 4}) \cdot (\frac{1}{4})$

$= 4$ feet.

5. Calculate the Average speed of a Car moving from A to B with a speed 40 K.m / hour and returning from B to A with a speed of 60 K.m / hour.

Ans: Let the distance ~~into~~ between A and B $= x$

The	Car	runs	40 km	in	1	hour
"	"	"	1 K.m	in	$\frac{1}{40}$	hour
"	"	"	x K.m	in	$\frac{x}{40}$	hour.

On returning journey it ~~feet~~ runs 60 K.m. in 1 hour.

1 K.m in $= \frac{1}{60}$ hour.

x K.m in $= \frac{x}{60}$ hour

Average speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

$$= \frac{x+x}{\frac{x}{40}+\frac{x}{60}} = \frac{2x}{\frac{3x+2x}{120}} = \frac{2\cancel{x} \times \cancel{120}^{24}}{\cancel{5}\cancel{x}} = 48 \text{ K.m/hour.}$$

∴ So the Average speed of the Car is 48 K.m / hour.

6. A stone falls from a cliff and travels 144 feet in last second before it reaches the ground at the foot of a cliff. Find the height of the cliff. (A: 400 feet)

According to the question $S_n = 144$ feet

beginning speed: $u = 0$

$a = 32$ feet / sec^2.

$$S_n = u + \frac{a}{2}(2n-1)$$

$$\Rightarrow 144 = 0 + \frac{32}{2}(2n-1)$$

$$= 0 + 16(2n-1)$$

$$= 32n - 16$$

$$\Rightarrow 32n = 144 + 16$$

$$\Rightarrow n = \frac{160}{32} = 5$$

Total distance covered in ~~the~~ 5 seconds $= S = ut + \frac{1}{2}at^2$

$$= 0.t + \frac{1}{2}.(32).(5)^2$$

$$= \frac{1}{\cancel{2}} . \cancel{32}^{16} . 25$$

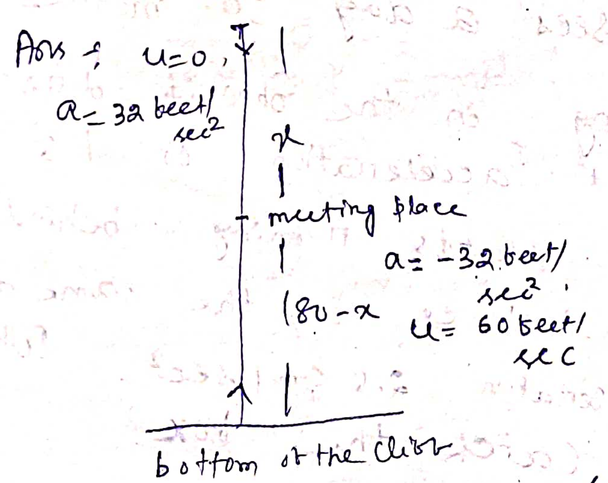
$$= 400 \text{ feet.}$$

$\therefore$ The height of the cliff = 400 feet

7. A stone is dropped from the top of a cliff 180 feet height. At the same instantance another stone is thrown upwards with a velocity of 60 feet/second from the bottom of the cliff. When and where will the stones meet?

(Ans: 3 sec, 144 feet from the top)

Ans: $u = 0$, $a = 32$ feet/sec^2



Let the stones meet ~~two~~ at a point x feet below the highest point.

For the downward stone $u = 0$, $a = 32$ feet/sec, $S = x$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow x = 0.t + \frac{1}{2}.(32)t^2$$

$\Rightarrow x = 16t^2$ ——— (i)

For the upward stone $u = 60$ feet / second
$a = -32$ feet / sec^2
$S = 180 - x$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 180 - x = 60.t + \frac{1}{2}.(-32).t^2$$

$$\Rightarrow 180 - x = 60t - 16t^2$$

$$\Rightarrow 180 - (16t^2) = 60t - 16t^2$$

{Putting the value of x in eq^n (i)}

$$\Rightarrow 180 - \cancel{16t^2} = 60t - \cancel{16t^2}$$

$$\Rightarrow 60t = 180$$

$$\Rightarrow t = \frac{180}{60} = 3 \text{ sec}$$

From equation (i) $x = 16t^2$

$$\Rightarrow x = 16.(3)^2$$

$$\Rightarrow x = 16.9$$

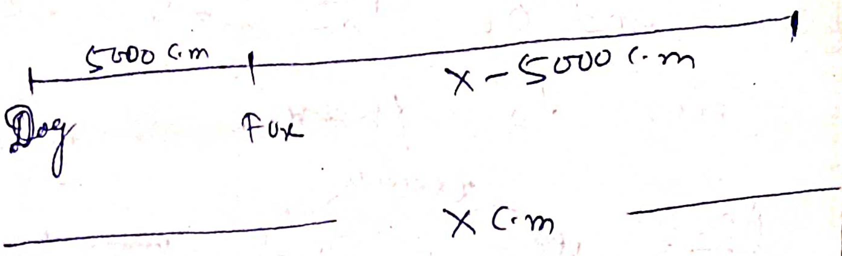
$$\Rightarrow x = 144 \text{ feet}$$

$\therefore$ The stones meet at a point which is 144 feet from the highest point after 3 seconds of their start.

15. A fox sees a dog 50 metres away and starts running in the opposite direction with a constant acceleration of 1.5 cm/sec^2. The dog also starts running behind it in the same direction at the same instance with an acceleration 2.5 cm/sec^2. When will the dog catch the fox?

(Ans : 100 sec)

Ans : Let the dog catch the fox ~~distance~~ after a time t second from start. After travelling through a distance of x c.m.



Distance covered by the fox $= (x - 5000$ c.m$)$

Here the acceleration of fox $= 1.5$ c.m/sec^2.

" " " of dog $= 2.5$ c.m/sec^2.

The distance covered by the fox

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow x - 5000 = 0.t + \frac{1}{2}.(1.5).t^2$$

$$\Rightarrow x - 5000 = .75\,t^2$$

$$\Rightarrow x = 5000 + .75\,t^2 \quad \text{--- (i)}$$

The distance covered by the dog

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow x = 0.t + \frac{1}{2}.(2.5).\,t^2$$

$$\Rightarrow x = 1.25\,t^2 \quad \text{--- (ii)}$$

From eq^n (i) and (ii)

$$5000 + .75t^2 = 1.25t^2$$

$$\Rightarrow 5000 = 1.25\,t^2 - .75t^2$$

$$\Rightarrow 5000 = .50\,t^2 = \frac{1}{2}t^2$$

$$\Rightarrow t^2 = 10,000$$

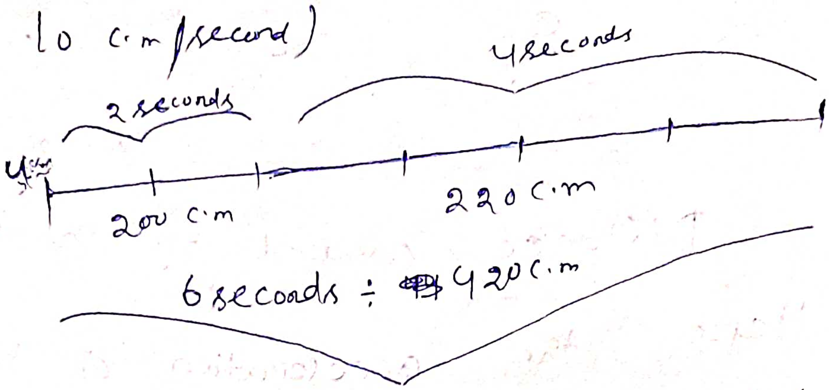
$$\Rightarrow t = \sqrt{10,000} = 100 \text{ sec.}$$

$\therefore$ After 100 seconds the dog will catch the fox.

16. A body travels 200 cm in the first 2 seconds and 220 cm in the next 4 seconds. What will be the velocity at the end of the 7th second from start.

(Ans ÷ 10 cm/second)

Ans ÷



In the first 2 seconds the body travels the distance 200 c.m.

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow S = u.2 + \frac{1}{2}a.(2)^2$$

$$\Rightarrow 200 = 2u + 2a$$

$$\Rightarrow 200 = 2(u+a)$$

$$\Rightarrow u + a = 100 \quad \text{——— (i)}$$

In the ~~last~~ next 4 seconds the body travels 220 c.m.

So the total distance = 420 c.m.
Total time = 6 second

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 420 = u(6) + \frac{1}{2}.a.(6)^2$$

$$\Rightarrow 420 = 6u + \frac{1}{2}a.36$$

$$420 = 6u + 18a$$

$$\Rightarrow 420 = 6(u+3a)$$

$$\Rightarrow 70 = u + 3a \quad \text{——— (ii)}$$

Subtracting eqn (ii) from eqn (i)

$$= \begin{array}{r} u + a = 100 \\ \underset{(-)}{u} \underset{(-)}{+3a} = \underset{(-)}{70} \\ \hline -2a = 30 \end{array}$$

$$\Rightarrow a = -15,$$

putting the value of a in eqn (i)

$$= u + a = 100$$

$\Rightarrow u - 15 = 100$

$\Rightarrow u = 115$

$V = u + at$ The velocity at end of 7th second

$\Rightarrow V = 115 + (-15) \cdot 7$

$= 115 - 105$

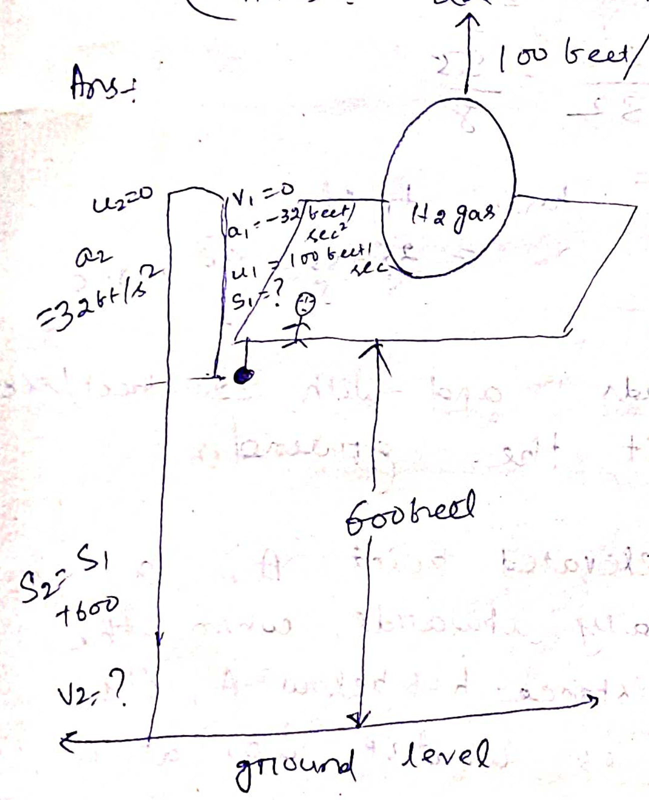
$= 10$ cm/sec.

∴ The velocity at the end of the 7the second = 10 cm / sec.

17. A ballon is going up at the rate of 100 feet/sec and a stone is dropped when it is at a height of 600 feet. When and with what velocity does it hit the ground?

(Ans: 220 feet / second, 10 second)

Ans:



If the stone is dropped due to inertial motion it goes upwards till its velocity is 0.

In the rising part

$u_1 = 100$ feet/sec.

$V_1 = 0$,

$a_1 = -32$ feet/second.

$v_1^2 - u_1^2 = 2a_1 s_1$

$\Rightarrow 0 - (-100)^2 = 2 \cdot (-32) \cdot S_1$

$\Rightarrow -10{,}000 = -64 S_1$

$\Rightarrow S_1 = \frac{-10000}{-64} = \frac{625}{4}$

$V_1 = u_1 + a_1 t_1$

$\Rightarrow 0 = 100 + (-32) \cdot (t_1)$

$\Rightarrow 32 t_1 = 100 \Rightarrow t_1 = \frac{100}{32} = \frac{25}{8}$, In the falling part

$u_2 = 0, \quad V_2 = ?, \quad a_2 = 32 \text{ ft}/s^2$

$$S_2 = S_1 + 600 = \frac{625}{4} + 600 = \frac{625 + 2400}{4}$$

$$= \frac{3025}{4}.$$

$$V_2^2 - u_2^2 = 2 a_2 . S_2$$

$$\Rightarrow V_2^2 - 0 = 2 . \cancel{(32)}^{8} . \left(\frac{3025}{\cancel{4}}\right)$$

$$= 16 (3025)$$

$$\Rightarrow V_2^2 = 48400$$

$$\Rightarrow V_2^2 = \sqrt{48400} = 220 \text{ feet/sec.}$$

$$V_2 = u_2 + a t_2$$

$$\Rightarrow 220 = 0 + 32 . t_2$$

$$\Rightarrow t_2 = \frac{220}{32} = \frac{55}{8}$$

The total time taken $= t_1 + t_2$

$$= \frac{25}{8} + \frac{55}{8} = \frac{80}{8} = 10 \text{ sec.}$$

$\therefore$ After 10 seconds and with 220 feet/sec velocity it will hit the ground.

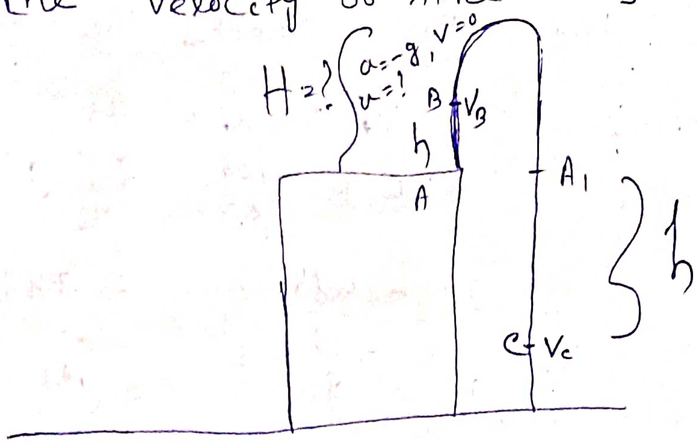
18. From an elevated point A, a stone is projected vertically upwards, when the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Show that the greatest height attained by the stone above A is $\frac{5h}{3}$.

Ans:- When the stone reaches a distance 'h' below that point $= C$.

There its velocity $= V_c$

The same height 'h' Above 'A' = 'B'.

There the velocity of stone = V_B,



Let the greatest point attained by the stone = 'H' distance from 'A'.

According to question = $V_c = 2V_B$

When the stone is projected upwards to the point B there

$$V_B^2 = u^2 + 2as$$

$$\Rightarrow V_B^2 = u^2 + 2.(-g).h$$

$$\Rightarrow V_B^2 = u^2 - 2gh$$

$$\Rightarrow V_B = \sqrt{u^2 - 2gh}$$

When the stone reaches at point c there

$$V_c^2 = u^2 + 2as$$

$$\Rightarrow V_c^2 = u^2 + 2.(g).h$$

$$\Rightarrow V_c = \sqrt{u^2 + 2gh},$$

According to the question

$$V_c = 2V_B$$

$$\Rightarrow \sqrt{u^2 + 2gh} = 2.\sqrt{u^2 - 2gh}$$

$$\Rightarrow \left(\sqrt{u^2 + 2gh}\right)^2 = \left(2\sqrt{u^2 - 2gh}\right)^2$$

(squaring both side)

$$\Rightarrow u^2 + 2gh = 4(u^2 - 2gh)$$

$$\Rightarrow u^2 + 2gh = 4u^2 - 8gh$$

$$\Rightarrow 3u^2 = 10gh$$

$\Rightarrow u^2 = \frac{10gh}{3}$

When the stone reaches the highest point

there $v^2 - u^2 = 2as$

$\Rightarrow 0 - \frac{10gh}{3} = 2.(-g).(H)$

$\Rightarrow \frac{-10gh}{3} = -2gH$

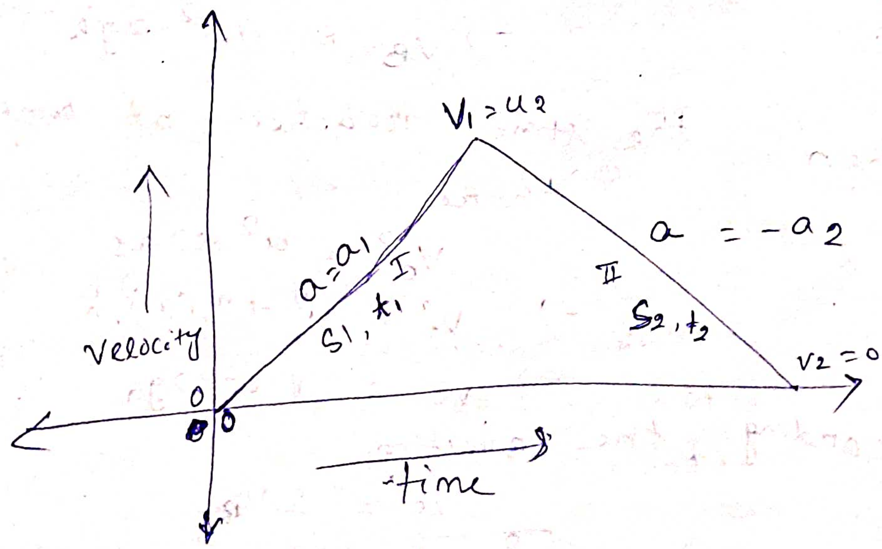
$\Rightarrow \cancel{+}10gh = \cancel{+}6gH$

$\Rightarrow H = \frac{5\cancel{10gh}}{3\cancel{6g}} = \frac{5h}{3}$ (proved)

19. A train starts from rest with a constant acceleration a_1 for some time and then moves with constant retardation a_2 till it comes to rest. If the total time taken is $= t$, calculate

(a) The maximum velocity.

(b) The total distance travelled.



Given $t_1 + t_2 = t$

$S_1 + S_2 = ?$

During acceleration the velocity changes from 0 to V_1. Time taken be $= t_1$ sec

Using the formula: $V_1 = u_1 + a_1 t_1$

we get $\Rightarrow V_1 = 0 + a_1 t_1$

$\Rightarrow V_1 = a_1 t_1$

and $V_1 = u_2 = a_1 t_1$

During retardation the velocity decreases from u_2 to 0 in t_2 second.

Using the formula $V_2 = u_2 + (-a_2) t_2$

we get $\Rightarrow 0 = a_1 t_1 - a_2 t_2$

$\Rightarrow a_1 t_1 = a_2 t_2$

$\Rightarrow t_2 = \dfrac{a_1 t_1}{a_2}$

But $t_1 + t_2 = t$

$\Rightarrow t_1 + \left(\dfrac{a_1 t_1}{a_2}\right) = t$

$\Rightarrow t_1 \left(1 + \dfrac{a_1}{a_2}\right) = t$

$\Rightarrow t_1 \left(\dfrac{a_2 + a_1}{a_2}\right) = t$

$\Rightarrow t_1 = \dfrac{a_2 t}{a_1 + a_2}$

Maximum ~~value~~ velocity $V_1 = a_1 t_1$

$= a_1 \cdot \left(\dfrac{a_2 t}{a_1 + a_2}\right)$

$= \dfrac{a_1 a_2 t}{a_1 + a_2}$

Distance covered in the first part

$S_1 = ut + \frac{1}{2} at^2$

$\Rightarrow S_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$

$= 0 \cdot t_1 + \frac{1}{2} a_1 t_1^2$

$= \frac{1}{2} a_1 t_1^2$

Distance covered in the second part

$S_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2$

$= (a_1 t_1)\left(\dfrac{a_1 t_1}{a_2}\right) + \dfrac{1}{2} (-a_2) \cdot \left(\dfrac{a_1 t_1}{a_2}\right)^2$

$= \dfrac{a_1^2 t_1^2}{a_2} + \dfrac{1}{2} (-\cancel{a_2}) \cdot \dfrac{(a_1 t_1)^2}{a_2^{\cancel{2}}}$

$= \dfrac{a_1^2 t_1^2}{a_2} - \dfrac{1}{2} \dfrac{(a_1 t_1)^2}{a_2}$

Total distance covered

$$= S_1 + S_2$$

$$= \left(\frac{1}{2} a_1 t_1^2\right) + \left(\frac{a_1^2 t_1^2}{a_2} - \frac{1}{2}\frac{(a_1 t_1)^2}{a_2}\right)$$

$$= a_1 t_1^2 \left\{\frac{1}{2} + \frac{a_1}{a_2} - \frac{1}{2}\frac{a_1}{a_2}\right\}$$

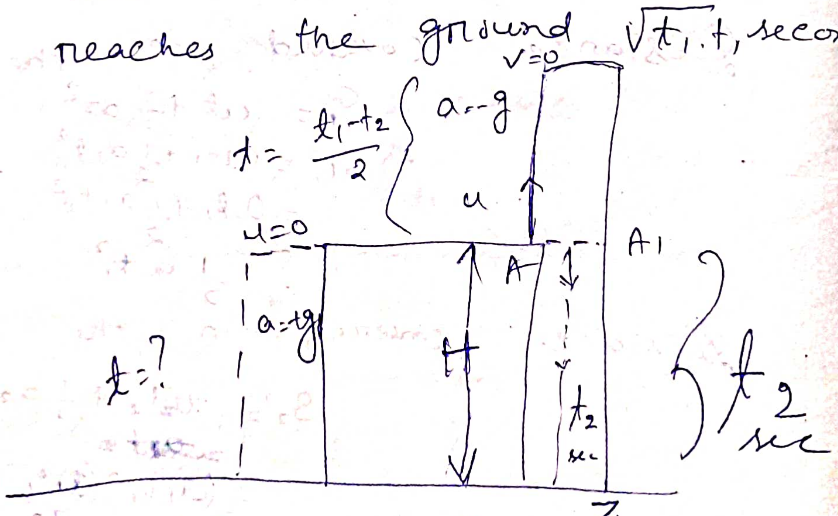
$$= a_1 t_1^2 \left(\frac{1}{2} + \frac{1}{2}\frac{a_1}{a_2}\right)$$

$$= a_1 \left(\frac{a_2 t}{a_2 + a_2}\right)^2 \left\{\frac{1}{2}\left(1 + \frac{a_1}{a_2}\right)\right\}$$

$$= a_1 \frac{(a_2^2 t^2)}{(a_1+a_2)^{\cancel{2}}} \cdot \frac{1}{2} \cdot \frac{\cancel{(a_1+a_2)}}{\cancel{a_2}}$$

$$= \frac{1}{2} \frac{a_1 a_2 t^2}{(a_1+a_2)} \quad \text{(ans)}$$

20. A body is thrown up vertically from a top of a tower and reaches the ground in t_1 second. It reaches the ground in t_2 second when thrown down from the same height with same velocity if it falls freely from rest at the same height. Show that it reaches the ground $\sqrt{t_1 \cdot t_2}$ second.



When the body is projected downwards with a velocity u, time taken is t_1 second and S

$= ut + \frac{1}{2}at^2$ gives

$\Rightarrow H = u \cdot t_2 + \frac{1}{2} \cdot (g) \cdot (t_2)^2$ ——— (i)

For the rising part of the body upto the highest point

$v = 0, \quad a = -g$

The body takes t_1 second to reach the ground 'Z'.

and it takes t_2 second from A_1 to Z.

So it takes $(t_1 - t_2)$ second to reach A to A_1

So it takes $\frac{t_1 - t_2}{2}$ for its rising part.

So for the rising part

$v = u + at$ gives

$\Rightarrow 0 = u + (-g) \cdot \left(\frac{t_1 - t_2}{2}\right)$

$\Rightarrow u = \frac{g}{2}\left(t_1 - t_2\right)$ ——— (ii)

using eqn. (ii) in eq (i)

$H = ut_2 + \frac{1}{2} g t_2^2$

$\Rightarrow H = \frac{g}{2}(t_1 - t_2) t_2 + \frac{1}{2} g t_2^2$

$= \left(\frac{gt_1 - gt_2}{2}\right) t_2 + \frac{1}{2} g t_2^2$

$= \frac{g t_1 t_2 - g t_2^2}{2} + \frac{g t_2^2}{2}$

$= \frac{g t_1 t_2 - \cancel{g t_2^2} + \cancel{g t_2^2}}{2}$

$\Rightarrow H = \frac{1}{2} g t_1 t_2$ ——— (iii)

For free fall from height H we have $u = 0$,

$a = g$, $s = H$

Using formula $S = ut + \frac{1}{2}at^2$

$$\Rightarrow H = 0.t + \frac{1}{2}.(g).(t^2)$$

$$\Rightarrow H = \frac{1}{2}gt^2 \quad \text{————— (iv)}$$

Comparing eqn (ii) and (iv) we get

$$\frac{\cancel{8}gt_1t_2}{\cancel{2}} = \frac{\cancel{8}gt^2}{\cancel{2}}$$

$$\Rightarrow \cancel{g}t_1t_2 = \cancel{g}t^2$$

$$\Rightarrow t_1t_2 = t^2$$

$$\Rightarrow t = \sqrt{t_1t_2}$$ (proved)

(33) If a body travels half of its total path in the last second of its fall from rest, find the time and height of its fall?
(Ans: 3.5414 second, 187 feet)

34. A car moving with constant acceleration covers the distance between two points 180 feet apart in 6 second. Its speed as it passes the second point is 45 feet/sec.
(a) What is its speed at the first point?
(b) What is its acceleration?
(c) At what prior distance from the first point was the car at rest?
(Ans: 15 feet/sec, 5 feet/sec^2, 23 feet)

35. Two trains one travelling at 60 mile/hour and other at 80 miles/hour are ~~headed~~ headed towards

One another along straight lable track when they are two miles apart, Both engineers simultaneously (একই সাথে) of the strain and apply their ~~breaks~~ brakes. If the brakes decelerates each train at the rate of 3 feet/sec². Determine wheather there is a ~~collision~~ collision. (Ans= No)

36. The engineer of a train moving at a speed $\underline{v_1}$ sights, a freight train at a distance 'd' ahead of him on the same track moving in the same direction with a slower speed v_2. He puts on the brakes and gives ~~his~~ his strain a constant deceleration 'a'. Show that if

$$d > \frac{(v_1 - v_2)^2}{2a}$$ there will be no collision.

If $d < \frac{(v_1 - v_2)^2}{2a}$ there will be a collision.

37. (Last page) A lead ball is dropped into a lake from a ~~diving~~ diving board 16 feet above the water. It hits the water with a certain velocity and then sinks to the bottom with the same constant velocity. It reaches the bottom five second after it is dropped.

(a) How deep is the lake?
(b) What is the avarage velocity

of the ball?

(c) Suppose all the water is drained from the lake. The ball is thrown from the diving board so that it again reaches the bottom in five seconds. What is the initial velocity of the ball?

(Ans: 128 feet, 28.8 feet/sec, $\frac{256}{5}$ feet/sec upwards)

38. A rocket is fired vertically and ascends with a constant vertical acceleration of 64 feet/sec^2 for one minute. Its fuel is all used and it continue as a free particle.

(a) What is the maximum altitude (height) reached?

(b) What is the total time elapsed from take-off until the rocket strikes the earth?

(Ans: 330 second, 3.5×10^5 feet

39. Two trains, each having a speed of 30 miles/hour, are headed at each other on the same straight track. A bird that can fly 60 miles/hour flies off one train when there 60 miles apart and heads directly for the other train. On reaching the other train it flies directly back to the first train and so forth.

(a) How many trips can be the bird make from one train to the other before they crash ?

(b) What is the total distance the bird travels ?

(Ans : Infinite. 60 miles.)

40. A train is 150 meter long. While in uniformly accelerated motion, It cross a lamp post in 1.5 seconds, 2.5 seconds later, it crosses another lamp post in 1 seconds. Calculate the acceleration of the train.

(Ans : $\frac{40}{3}$ meter/sec^2)

—o—

40. Let the engine of the train reaches the lamp post with a speed u meter/second. The speed of the last compartment at the lamp post is v meter/sec = u+at

$= u+a.(1.5)$

Average speed of the train while crossing the first lamp post

$$= \frac{u+v}{2}$$

$$= \frac{u + u+1.5a}{2}$$

$$= \frac{2u+1.5a}{2} = \frac{150 \text{ metre}}{1.5 \text{ sec}}$$

$$\Rightarrow \frac{2u+1.5a}{2} = 100$$

$$\Rightarrow 2u+1.5a = 200$$

The speed of the engine when it reaches the second lamp post is

$V + 2.5a = u + 1.5a + 2.5a = u + 4a$

Speed of the last compartment when it crosses the second lamp post

$= (u + 4a) + 1a = u + 5a$

Average speed of the train when it crosses the second lamp post

$= \frac{u + 4a + u + 5a}{2} = \frac{2u + 9a}{2} = \frac{150}{1}$

$\Rightarrow 2u + 9a = 300$ ——— (ii)

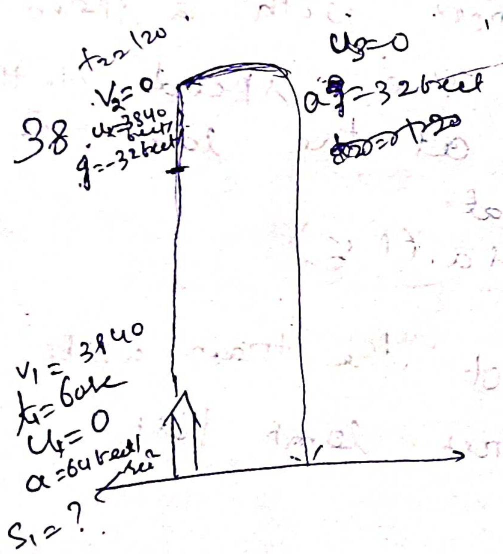
Subtracting eq^n (i) from (ii) we get

32
31
192
96
1152

$\cancel{2u} + 9a - \cancel{2u} - 1.5a = 100$

$\Rightarrow 7.5a = 100$

$\Rightarrow a = \frac{100}{7.5} = \frac{1000}{75} = \frac{40}{3}$ m/sec^2



For the first part

$t = 60$ sec,
$u_1 = 0$,
$a = 64$ feet,

$\Rightarrow V_2 = u + at$
$\Rightarrow V = 0 + 64 \cdot 60$
$= 3840$ feet/sec

$S_1 = ut + \frac{1}{2}at^2$
$= 0 \cdot t + \frac{1}{2} \cdot \cancel{(64)}^{32} \cdot (3600)$
$= 0 + 115200$
$= 115200$

For the second part

$u_2 = 3840$
$V_2 = 0$,
$a = -32$ feet,

~~$S_2 = ut + \frac{1}{2}at^2 = 3840 \cdot t + \frac{1}{2} \cdot (-32) \cdot t^2$~~

$V_2^2 - u_2^2 = 2aS_2$

$\Rightarrow 0 - (3840)^2 = 2 \cdot (-32) \cdot S_2$

$\Rightarrow 3840 \times 3840 = 64 S_2$

$\Rightarrow S_2 = \dfrac{\cancel{3840}^{\,60} \times 3840}{\cancel{64}} = 230400$ feet

180
140
320.6

$S_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2$

$\Rightarrow 230400 = 3840 \cdot t_2 + \frac{1}{2} \cdot (-32) \cdot t_2^2$

$\Rightarrow 230400 = 3840\, t_2 - 16\, t_2^2$

$V_2 = u_2 + a_2 t_2$

$\Rightarrow 0 = (3840) + (-32) \cdot t_2$

$\Rightarrow -3840 = -32\, t_2$

$\Rightarrow \dfrac{3840}{32} = t_2$

$\Rightarrow 120$ sec $= t_2$

For the falling part

$u_3 = 0$, $a_3 = 32$ feet

$S_3 = S_1 + S_2 = 115200 + 230400$
$= 345600$

$S_3 = u_3 t_3 + \frac{1}{2} a_3 t_3^2$

$\Rightarrow 345600 = 0 \cdot t_3 + \frac{1}{2} \cdot (32) \cdot t_3^2$

$= 16\, t_3^2$

$\Rightarrow t_3^2 = \dfrac{345600}{16} = 21600$

Total time ta $\Rightarrow t_3 = \sqrt{21600}$

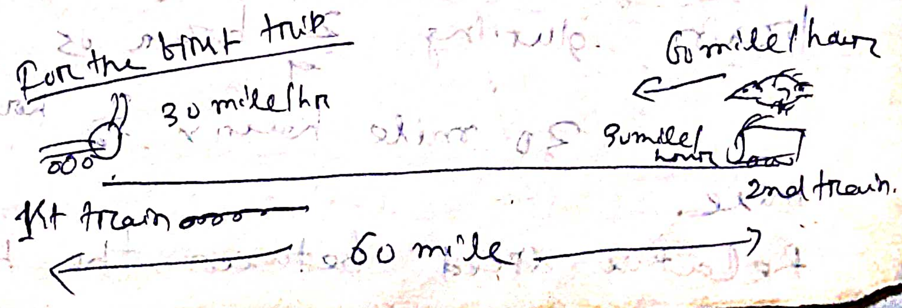
$= 146.9$

So the total time $= t_1 + t_2 + t_3$

$= 60 + 120 + 146.9$

$= 326.9$

39.

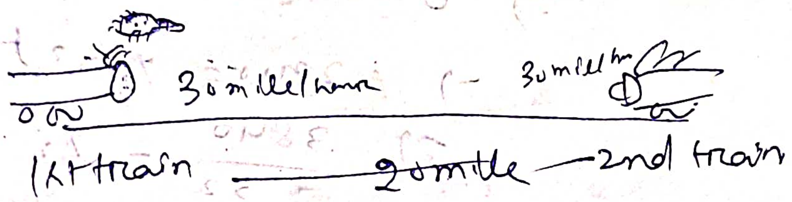


Relative speed between the bird and the 1st train = 60 mile / hour + 30 mile/hour
$= 90$ mile/hour

time required $= \dfrac{\text{Distance covered}}{\text{Relative speed}}$

$t_1 = \dfrac{60\text{ mile}}{90\text{ mile/hour}} = \dfrac{2}{3}$ hour

For the second trip

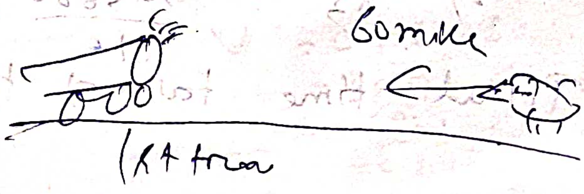


Distance covered by each train during $\frac{2}{3}$ hour is $S = 30$ mile/hr $\times \frac{2}{3}$ hour
$= 20$ mile

Relative speed between the bird and the 2nd train $60+30 = 90$ mile/hour

$t_2 = \dfrac{20\text{ mile}}{90\text{ mile}} = \dfrac{2}{9}$ hour

For the third part



$\longrightarrow \dfrac{20}{3}$ mile.

Distance covered by each train during $\frac{2}{9}$ hour is
30 mile/hour $\times \frac{2}{9}$ hour $= \frac{20}{3}$ mile.

Relative speed between the bird &

The first train = 90 mile/hour.

$$t_3 = \frac{20/3 \text{ mille}}{90 \text{ mille hour}} = \frac{2}{27} \text{ hour.}$$

Time for each trip is $\frac{1}{3}$ of ~~for~~ the time taken for the previous trip. Thus time decreases very fast and tends to zero.

Therefore number of trip = infinite.

Total time taken for all the trips $= t_1 + t_2 + t_3 + - - - -.$

$$= \frac{2}{3}\text{ hour} + \frac{2}{9}\text{ hour} + \frac{2}{27}\text{ hour}$$

$$= \frac{2}{3}\text{hr}\left(1 + \frac{1}{3} + \frac{1}{(3)^2} + \frac{1}{(3)^3} + \cdots\right)$$

$$= \frac{2}{3}\text{ hour}\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$= 1 \text{ hour}$$

Distance covered by the bird during 1 hour is 60 miles.

35. 15 mile/hour = 22 feet/hour.

$$1 \text{ mile/hour} = \frac{22}{15}$$

$$60 \text{ mile/hour} = \frac{22}{15} \times 60^4 = 88 \text{ feet/s.}$$

$$80 \text{ mille hour} = \frac{22}{15_3} \times \cancel{80}^{16} = \frac{352}{3} \text{ feet/s.}$$

For the 1st train

$u_1 = 88$ feet/sec.

~~$80 = 2$ mille~~.

$a_1 = -3$ feet/sec,

$V_1 = 0$

$$V_1^2 - u_1^2 = 2as_1$$

$$\Rightarrow 0 - (88)^2 = 2.(-3).S \qquad \Rightarrow \frac{7744}{6} = S_1$$

$$\Rightarrow (88)^2 = 6S$$

29

For the second train

$u_2 = \frac{352}{3}$ feet/sec

$V_2 = 0$

$a_2 = -3$ feet/sec^2

$$V_2^2 - u_2^2 = 2a_2 S_2$$

$$\Rightarrow 0 + \left(\frac{352}{3}\right)^2 = 2 \cdot (+3) \cdot S_2$$

$$\Rightarrow \frac{352 \times 352}{9} = 6 S_2$$

$$\Rightarrow \frac{352 \times 352}{54} = S_2$$

$$S_1 + S_2 = \frac{7744}{6} + \frac{123904}{54}$$

$$= \frac{7744 \times 9 + 123904}{54}$$

$$= \frac{69696 + 123904}{54}$$

$$= \frac{193600}{54} = 3585.18 \text{ feet}$$

22 feet/sec = 15 mile/hour

$1 = \frac{15}{22}$

$\cancel{3585} \text{ feet} = \frac{15}{22} \times 3585$

$= \frac{53775}{22}$

$=$

36. Let the time taken by the first train to decrease its speed from v_1 to v_2 be t sec.

During this time the distance covered by the first train be S_1 and that by the second train be $V_2 t$.

For no collision, $S_1 < d + v_2 t$

$$\Rightarrow \frac{v_2^2 - v_1^2}{2\cdot(-a)} < d + v_2 \cdot \left(\frac{v_2 - v_1}{-a}\right)$$

$(\because v_2^2 - v_1^2 = 2\cdot(-a) S_1,\ v_2 = v_1 + (-a)t)$

$$\Rightarrow \frac{v_1^2 - v_2^2}{2a} < d + \frac{v_2 (v_1 - v_2)}{a}$$

$$\Rightarrow \frac{v_1^2 - v_2^2}{2a} - \frac{v_2 \cdot (v_1 - v_2)}{a} < d$$

$$\Rightarrow \frac{v_1^2 - v_2^2 - 2v_2 (v_1 - v_2)}{2a} < d$$

$$\Rightarrow d > \frac{(v_1 - v_2)^2}{2}$$

(For no collision)

$V_2, V_1 \Rightarrow \frac{v_1^2 - v_2^2}{2a} - v_2 \left(\frac{v_1 - v_2}{a}\right) < d$

$$\Rightarrow \frac{v_1 - v_2}{a} \left[\frac{v_1 + v_2}{2} - v_2\right] < d$$

$$\Rightarrow \frac{v_1 - v_2}{a} \left(\frac{v_1 + v_2 - 2v_2}{2}\right) < d$$

$$\Rightarrow \left(\frac{v_1 - v_2}{a}\right)\left(\frac{v_1 - v_2}{2}\right) < d$$

$$\Rightarrow \frac{(v_1 - v_2)^2}{2a} < d$$

$$\Rightarrow d > \frac{(v_1 - v_2)^2}{2a},$$

(b) For collision

$$S_1 > d + V_2 t$$

$$\Rightarrow \frac{V_2^2 - V_1^2}{2.(-a)} > d + V_2 \left(\frac{V_2 - V_1}{-a}\right)$$

$$\left\{ \because V_2^2 - V_1^2 = 2.(-a).S_1 , \quad V_2 = \cancel{V_1}(-a)t \right\}$$

$$\Rightarrow \frac{V_1^2 - V_2^2}{2a} > d + \frac{V_2 (V_1 - V_2)}{a}$$

$$\Rightarrow \frac{V_1^2 - V_2^2}{2a} - \frac{V_2 (V_1 - V_2)}{a} > d$$

$$\Rightarrow \frac{V_1 - V_2}{a} \left[\frac{V_1 + V_2}{2} - V_2\right] > d$$

$$\Rightarrow \frac{V_1 - V_2}{a} \left(\frac{V_1 + V_2 - 2V_2}{2}\right) > d$$

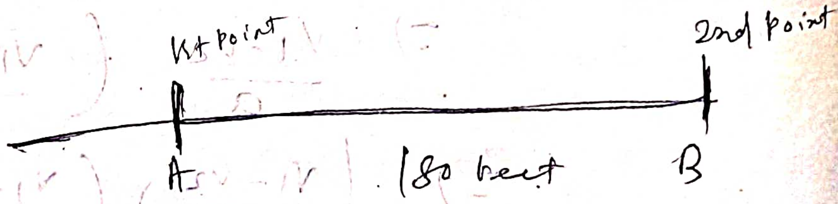
$$\Rightarrow \frac{V_1 - V_2}{a} \left(\frac{V_1 - V_2}{2}\right) > d$$

$$\Rightarrow \frac{(V_1 - V_2)^2}{2a} > d$$

$$\cancel{\Rightarrow d > \frac{(V_1 - V_2)^2}{2a}}$$

$$\Rightarrow d < \frac{(V_1 - V_2)^2}{2a}$$

34.



A car moving with a constant acceleration $= a$

The distance between two point $S = 180$ feet

The total time taken $t = 6$ sec

~~The~~ The final speed of the car at the second point is 45 feet/sec

$\Rightarrow V = 45$ feet/sec,

We know that $V = u + at \Rightarrow V = u + a.6$
$= u + 6a$
$\Rightarrow v = u + 6a$

The average speed

$$= \frac{u+v}{2} = \frac{u + u + 6a}{2}$$

$$\Rightarrow \frac{2u + 6a}{2} = \frac{180}{6} = 30$$

$$\Rightarrow 2u + 6a = 60 \quad \text{——(i)}$$

Again using the formula

$= V = u + at$

$\Rightarrow 45 = u + a.(6)$ ——(ii)

$\Rightarrow 45 = u + 6a$

To equal u in two eq^n we multiply (2) with eq^n (ii)

$$2u + 12a = 90$$
$$2u + 6a = 60$$
$$(-) \quad (-) \quad (-)$$

Substracting

$$6a = 30$$

(ii) $\Rightarrow a = \frac{30}{6} = 5$ feet/sec^2

Putting the value of a in eq^n (i)

$$2u + 6a = 60$$
$$\Rightarrow 2(u) + 6(5) = 60$$
$$\Rightarrow 2u + 30 = 60$$
$$\Rightarrow 2u = 30$$
$$\Rightarrow u = \frac{30}{2} = 15 \text{ feet/sec.}$$

(i) So the speed at the first point 15 feet/sec.

(iii) At what time or distance from the first point the car was at rest

So initial speed $u = 0$,

$V = 15$ feet/sec

constant acceleration $a = 5$ feet/sec^2

$s = ?$

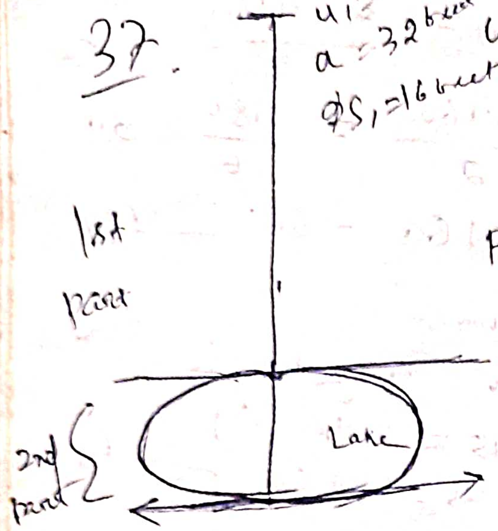
$v^2 - u^2 = 2as$

$\Rightarrow (15)^2 - (0)^2 = 2.(5)s$

$\Rightarrow 225 = 10s$

$\Rightarrow s = \frac{225}{10} = 22.5$ feet (Ans)

37.



When the ball is thrown its initial velocity

For the first part $u_1 = 0$,
$S_1 = 16$ feet
$a = 32$ feet

$v_1^2 - u_1^2 = 2aS_1$

$\Rightarrow v_1^2 - 0 = 2.(32).(16)$

$\Rightarrow v_1^2 = 1024$

$\Rightarrow v_1 = 32$ feet/sec,

We know

$v_1 = u_1 + at_1$

$\Rightarrow 32 = 0 + 32t_1$

$\Rightarrow t_1 = 1$ sec

Total time $= 5$ sec

For the second part time $= 5 - 1 = 4$ sec;

For the second part the ball sinks with a constant velocity

So $a = 0$,

The initial velocity $= u_2 = v_1 = 32$,

$t_2 = 4$ sec,

$S_2 = u_2t_2 + \frac{1}{2}at_2^2$

$\Rightarrow S_2 = (32).(4) + \frac{1}{2}.0\,t_2^2$

$= 128 + 0$

(i) $\longrightarrow$ ans $= 128$ feet

So the lake is 128 feet deep.

(ii) The average velocity of ball

$= \frac{\text{Total distance covered}}{\text{Total time taken}}$

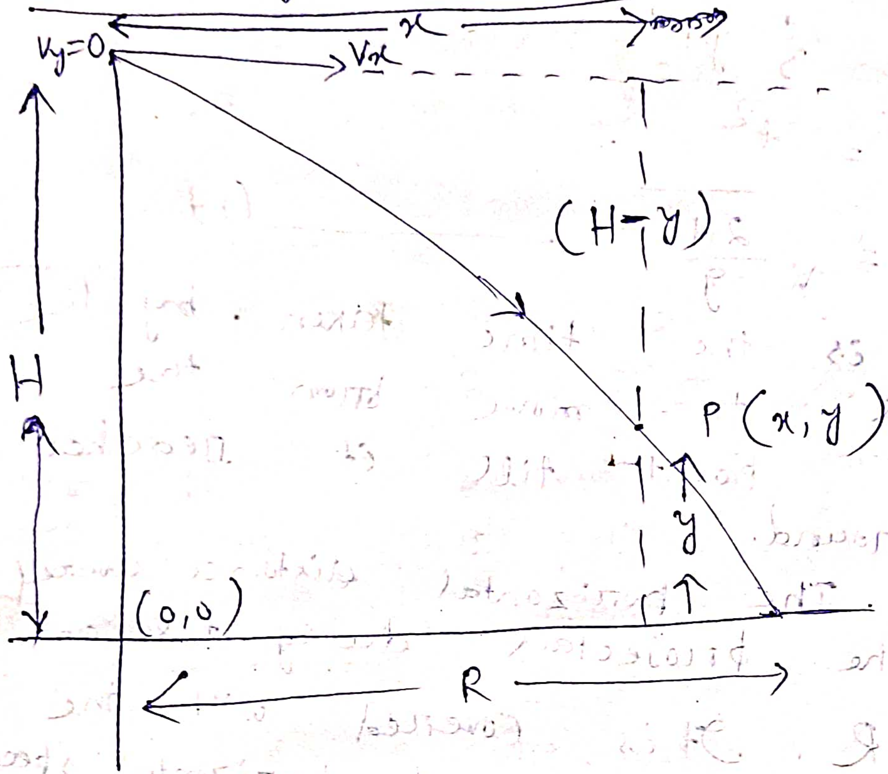
$= \frac{S_1 + S_2}{t_1 + t_2}$

$= \frac{16+128}{1+4} = \frac{144}{5} = 28.8$ feet/second

(c)

~~doubts 33 and 36 (c)~~

Motion of a particle throw on horizontaly from an elevated point.



Let a particle be thrown from a height H with a velocity V_x in the horizontal direction. The initial Y component of the velocity is 0.

As usual the projectial motion can be regarded as consisting of three kinds of motions, one due to the horizontal component of the velociy (V_x) which remains constant through out the motion.

And the other component which is affected by gravity. The Y component gradually increases in the downward direction. Using the y component of the velocity and using the formula

$S = ut + \frac{1}{2}at^2$, we get

$\Rightarrow H = 0.t + \frac{1}{2}.(g).t^2$

$\Rightarrow H = \frac{1}{2}gt^2$

$\Rightarrow \frac{2H}{g} = t^2$

$\Rightarrow t = \sqrt{\frac{2H}{g}}$ ——— (i)

This is the time taken by the projectaile to move from the highest point till it reaches the ground.

The horizontal distance covered by the projectail during t second is R. It is covered with the help of the constant horizontal speed V_x

$\therefore R = V_x \cdot t = V_x \cdot \sqrt{\frac{2H}{g}}$ ——— (ii)

<u>To prove that the path of this kind of projectaile is also a parabola</u>

Let's consider a point P having Co-ordinates x,y on the path of the projectail reached by it after t second from start.

The horizontal distance covered by the projectail is x during

The time t second. It is covered with the constant horizontal speed V_x.

$$\therefore x = V_x . t \text{ ———— (i)}$$

The vertical distance covered by the projectile during t second is $H - y$ which is covered with the help of vertical component of the velocity.

$$\therefore S = ut + \frac{1}{2}at^2 \text{ gives}$$

$$\Rightarrow H - y = 0.t + \frac{1}{2}.(g).t^2$$

$$\Rightarrow H - \frac{1}{2}gt^2 = y \text{ ———— (ii)}$$

If we can eliminate t from eqns (i) and (ii), we can get a relation between x and y.

From eqn (i) $x = V_x . t$

$$\Rightarrow t = \frac{x}{V_x}.$$

Putting this value of time in eqn (ii) we get

$$H - \frac{1}{2}gt^2 = y$$

$$\Rightarrow H - \frac{1}{2}.g.\left(\frac{x}{V_x}\right)^2 = y$$

$$\Rightarrow H - \frac{1}{2}g\frac{x^2}{V_x^2} = y$$

$$\Rightarrow H - \left(\frac{1}{2}\frac{g}{V^2x}\right)x^2 = y$$

$$\boxed{\Rightarrow H - bx^2 = y} \text{ ———— (iii)}$$

where b is a constant given by the expression $b = \left(\frac{1}{2}\frac{g}{V_x^2}\right)$

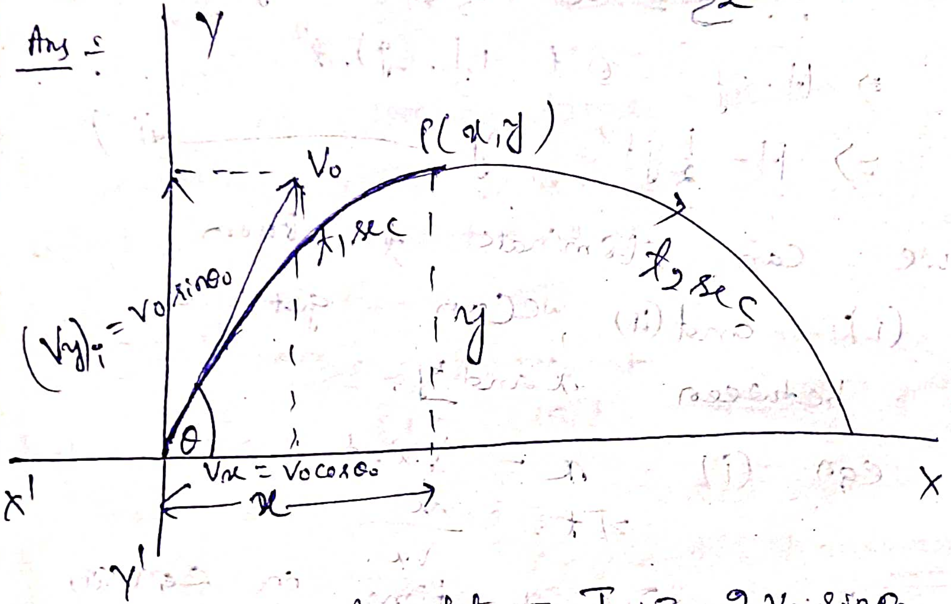
This eqn (iii) contains x^2 as well as

y. Therefore it is a eqn of a parabola.

Problem

1. A projectile reaches a particular point of its path after t_1 second from start and takes t_2 second further to reach the ground again. Prove that the height reached after t_1 second is $\frac{1}{2} \cdot g\, t_1 . t_2$

Ans :



Time of flight $= T = \dfrac{2 V_0 \sin\theta_0}{g}$

$= t_1 + t_2$

$\Rightarrow V_0 \sin\theta_0 = \dfrac{g(t_1+t_2)}{2}$

The vertical distance covered by the projectile during t_1 second is y. Using the formula

$S = ut + \frac{1}{2}at^2$, we get

$\Rightarrow y = V_0 \sin\theta_0 . t_1 + \frac{1}{2}(-g).(t_1)^2$

Putting the value $V_0 \sin\theta_0 = \frac{g}{2}(t_1+t_2)$ in above eqn

$\Rightarrow y = \frac{g}{2}(t_1+t_2)t_1 - \frac{g t_1^2}{2}$

$\Rightarrow y = \frac{g}{2} t_1 (\cancel{t_1} + t_2 - \cancel{t_1})$

$= \frac{g}{2} t_1 . t_2$ (Proved)

2. Prove that the height of projectaile is $\frac{1}{4}$: Range x $\tan\alpha$ where α = angle of projection

3. Prove that the maxm horizontal Range $2H + \frac{R^2}{8H}$

Hints : ~~R = Rmax~~ $R_{max} = \frac{V_0^2}{g}$

or Explanation of black body radiation spectrum

4 The horizontal range of a ball for two different directions of projections is the same and equal to R. If H and H' are the greatest height in the two paths. prove that

$R^2 = 16 H . H'$

5. A body is projected twice from a point with a same velocity but in different directions. If in each case the rang is R, the time of flight are t and t' then prove that

$R = \frac{1}{2} g . t . t'$

6. Prove that $g^2 . T^4 - 4 T^2 . u^2 + 4 R^2 = 0$

and $16 g H^2 - 8 u^2 . H + g R^2 = 0$

where (u = velocity of projection $= V_0$)

7. From the point on the ground at a distance x from a vertical ~~one~~ wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance y on the otherside. Show that the height of the wall is $\frac{xy}{x+y}$

99 page → 12, 8. Cofficient of friction → μ

8. Frictional force $= f = \mu N$
$= \mu W$
$= \mu mg$

It opposes motion. Hence, eqⁿ of motion for the sled on the plane surface is

$$ma = -f = -\mu mg$$

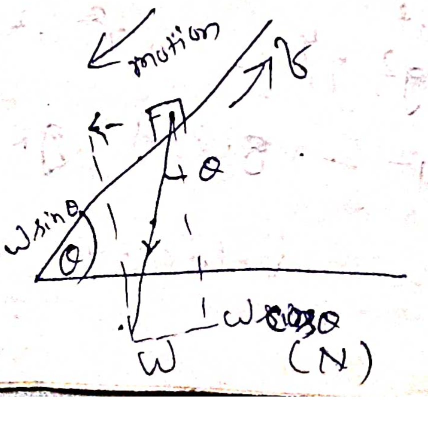
$$\Rightarrow a = -\mu g$$
$$= -(0.03).(32)\ ft/s^2$$
$$= -0.96\ ft/s^2$$

using the formula $v^2 - u^2 = 2as$ we get

$$\Rightarrow 0^2 - (40)^2 = 2(-0.96).s$$
$$= -1.92s$$

$$\Rightarrow s = \frac{1600}{1.92} = 833.33\ ft$$

10/ 22.



Frictional force $f = \mu N$

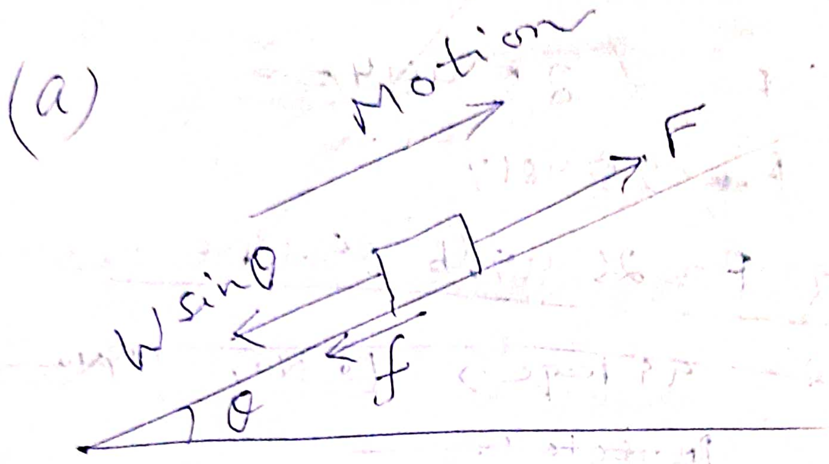
$= \mu W \cos\theta$

$= (0.18).(300).\cos 15^\circ$

$= (.18)(300).(.9659)$

$= 52.1586$ lb.

(a)



eqn of motion for the body is

$F - (W\sin\theta + f) = ma$

$F - W\sin\theta - f = ma$

But, $a = 0$ (∵ body is moving with constant speed)

$\Rightarrow F - W\sin\theta - f = 0$

$\Rightarrow F - (300)\sin 15^\circ - 52.1586 = 0$

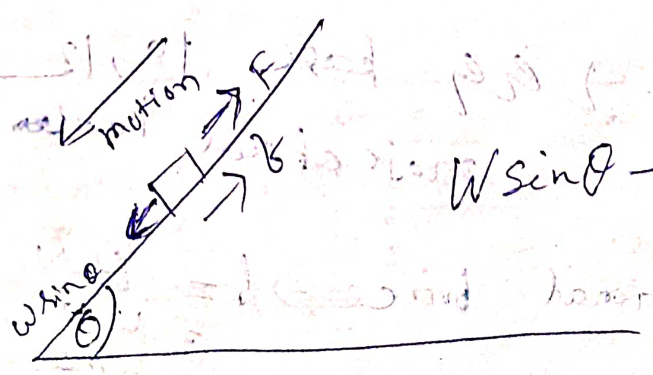
$\Rightarrow F - (300)(.2588) - 52.1586 = 0$

$\Rightarrow F - 77.64 - 52.1586 = 0$

$\Rightarrow F = 129.79$

$\Rightarrow F = 130$ lb.

(b)



$W\sin\theta - F - f = ma$

eqn for the body is

$F - \cancel{W\sin\theta} \mp f = ma$

But ~~a=0~~

$\cancel{\Rightarrow (F - \mu\sin\theta + b = 0}$

$\cancel{\Rightarrow (F - (300)(.2588) + 52.1586 = 0}$

$\cancel{\Rightarrow (F - 77.64 + 52.1586 = 0}$

$\cancel{\Rightarrow (F - 25.4814 = 0}$

$\cancel{\Rightarrow F = 25.4814}$

$\cancel{\Rightarrow F = 25.49\ lb}$ ~~(ans)~~

Task $\Rightarrow$ 29 page $\rightarrow$ 10 No, 12 No

~~Projectail.~~

But $a=0$

$w\sin\theta - (F + b) = ma$

$\Rightarrow w\sin\theta - F - b = 0 \quad (\because a=0)$

$\Rightarrow (300).(.2588) - F - (52.1586) = 0$

$\Rightarrow 77.64 - 52.1586 - F = 0$

$\Rightarrow 25.4814 = F$

$\Rightarrow F = 25.4814$

$\Rightarrow F = 25.49\ lb$ (ans)

Task $\rightarrow$ 29 page 10, 12
projectail problem.

10. Frictional force $\rightarrow b = \mu N$
$= \mu w$
$= \mu m a$

12. 15 mile/hour = 22 feet/sec
$\Rightarrow$ 1 mile/hour = $\frac{22}{15}$

50 mile/hour = $\frac{22}{15} \times 50 = \frac{1100}{\cancel{15}} = \frac{220}{3}$

97 opposes the motion, hence eqn of motion for the automobile on the level road $ma = -f = -\mu mg$

$\Rightarrow a = -\mu g$

$= -(.75).(32)$

$= -23.40$

$= -23.5$

Using the formula $v^2 - u^2 = 2as$ we get.

$\Rightarrow 0 - \left(\frac{220}{3}\right)^2 = 2.(-23.5)s$

$\Rightarrow \frac{48400}{9} = 47 s$

$\Rightarrow s = \frac{48400}{47 \times 9}$

$= \frac{48400}{423}$

$= 114$ feet

Projection

Q. 2 To prove that

$H = \frac{1}{4}$. Range $\times \tan\alpha$.

(where α = angle of projection)

~~We know that~~

R.H.S

$= \frac{1}{4}$. Range . $\tan\alpha$

$= \frac{1}{4} \cdot \frac{V_0^2 \sin 2\alpha}{g} \cdot \frac{\sin\alpha}{\cos\alpha}$

$= \frac{1}{\cancel{4}_2} \frac{v_0^2 \cancel{2}\sin\alpha . \cancel{\cos\alpha}}{g} \cdot \frac{\sin\alpha}{\cancel{\cos\alpha}}$

$= \frac{V_0^2 \sin^2\alpha}{2g}$

We know $H = \frac{v_0^2 \sin^2\alpha}{2g}$

$\therefore H = \frac{1}{4}$ Range. $\tan\alpha$. (Proved)

12. The potential energy of the Car at the top of the hill

$= mgh$, Kinetic energy $= 0$,

total energy $= mgh$

The Kinetic energy of Car at the bottom of hill $= \frac{1}{2}mv^2$, potential energy 0

total energy $= \frac{1}{2}mv^2$

according to law of conservation of energy. we have total energy at the top most point = total energy at the bottom

$$\Rightarrow mgh = \frac{1}{2}mv^2$$

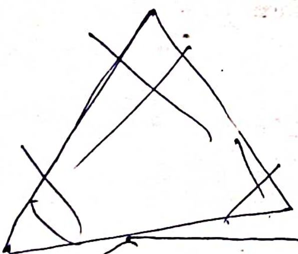
$$\Rightarrow gh = \frac{v^2}{2}$$

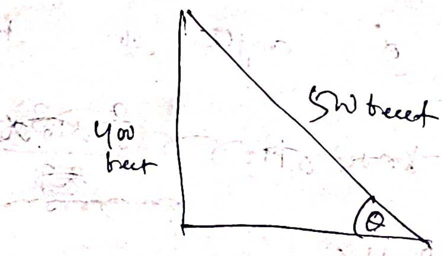
$$\Rightarrow v^2 = 2gh$$

$$\Rightarrow v^2 = 2.(32).(400)$$

$$\Rightarrow v^2 = 25600$$

$$\Rightarrow v = 160 \text{ feet/sec.}$$





$w \sin\theta = ma$

$\Rightarrow mg \sin\theta = ma$

$\Rightarrow a = g\sin\theta$ $\Rightarrow a = g \sin\theta$

$\sin\theta = \frac{400}{500} = \frac{4}{5}$

$u = 0$, $s = 500$

using the formula $v^2 - u^2 = 2as$ we get

$$\Rightarrow v^2 - 0 = 2.g\sin\theta.500$$

$$\Rightarrow v^2 = 2(32)(\cancel{500}^{100}).\frac{4}{\cancel{5}}$$

$$= 25600$$

$$\Rightarrow v = 160 \text{ ft/sec}$$

3. $2H + \frac{R^2}{8H}$

$$= \cancel{2 \cdot \frac{v_0^2 \sin^2\theta}{2g} + \frac{v_0^2 \sin 2\theta}{g}}$$

$$= \cancel{\frac{v_0^2 \sin^2\theta_0}{g} + \frac{v_0^2 \cdot 2\sin\theta \cos\theta}{g}}$$

$$= \cancel{\frac{v_0^2 \sin^2\theta_0 + v_0^2 \, 2\sin\theta \cdot \cos\theta}{g}}$$

$$= 2 \cdot \frac{v_0^2 \sin^2\theta_0}{2g} + \frac{\left(\frac{v_0^2 \sin 2\theta_0}{g}\right)^2}{8 \cdot \left(\frac{v_0^2 \cdot \sin^2\theta_0}{g}\right)}$$

$$= \frac{v_0^2 \cdot \sin^2\theta_0}{g} + \frac{v_0^4 \cdot \sin^2 2\theta_0}{g^2} \times \frac{g}{8 v_0^2 \cdot \sin^2\theta_0}$$

$$= \frac{g \, 8 \, v_0^4 \sin^4\theta_0 + v_0^4 \, 4\sin^2\theta_0 \cdot \cos^2\alpha}{g \, 8 v_0^2 \sin^2\theta_0}$$

3. $2H + \frac{R^2}{8H}$

$$= 2 \cdot \frac{v_0^2 \cdot \sin^2\theta_0}{2g} + \frac{\left(\frac{v_0^2 \cdot \sin 2\theta_0}{g}\right)^2}{8 \cdot 4\left(\frac{v_0^2 \cdot \sin^2\theta_0}{2g}\right)}$$

$$= \frac{v_0^2 \cdot \sin^2\theta_0}{g} + \frac{v_0^4 \cdot 4\sin^2\theta_0 \cdot \cos^2\alpha}{g^2} \times \frac{g}{4 v_0^2 \cdot \sin^2\theta_0}$$

$$= \frac{v_0^2 \sin^2\theta_0}{g} + \frac{v_0^2 \cos^2\theta_0}{g}$$

$$= \frac{v_0^2 \sin^2\alpha + v_0^2 \cos^2\alpha}{g} = \frac{v_0^2 (\sin^2\theta_0 + \cos^2\theta_0)}{g}$$

$$= \frac{v_0^2}{g} = R_{max} \quad (\text{Proved})$$

(because $R = \frac{v_0^2 \cdot \sin 2\alpha}{g} = \frac{v_0^2 \cdot 1}{g} = \frac{v^0}{g}$
($\because$ Maximum value of $\sin 2\alpha = 1$))

Unit-2

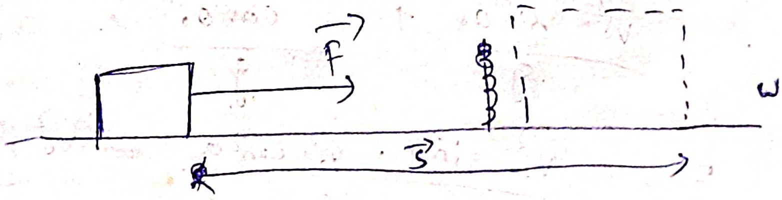
Work:- Work is said to be done when a force acting on a body displaces it through a certain distance along the direction of the force.

It is ~~measuser~~ measured by the product of magnitude of the force and projection of the displacement along the direction of the force. Or it is measured by the product of the component of the force along the displacement of the direction and magnitude of the ~~dis~~ displacement vector.

If the displacement is along the same direction as the of the force then
$W = FS$

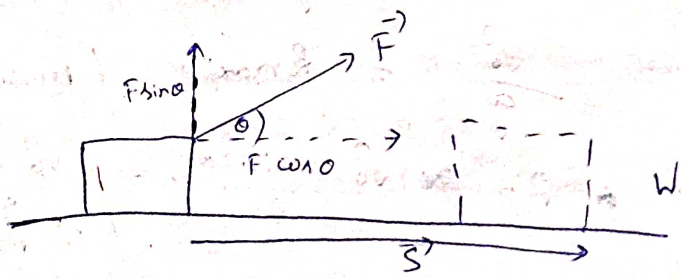
If the angle between force and displacement is θ, then $w = F\cos\theta \cdot s$
$W = FS\cos\theta$
$= \vec{F} \cdot \vec{S}$

Figure :- 1



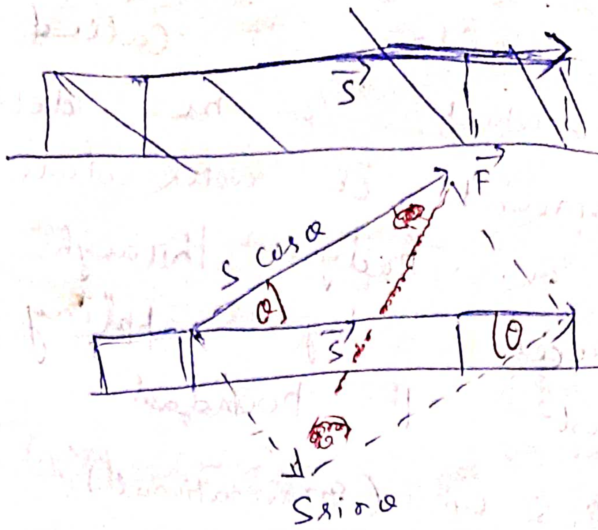
$W = \vec{F} \cdot \vec{S}$

Figure :- 2



$W = (F\cos\theta)S$

Figure - 3



$W = F \cdot S\cos\theta$

$= \vec{F} \cdot \vec{S}$

Dimension

Work = Force x displacement

$[W] = [F] \cdot [S]$

$= [ma] \cdot [L]$

$= [M \cdot LT^{-2}] \cdot [L]$

$= [ML^2T^{-2}]$

Units of work

(a). C.G.S unit = dyne x c.m = erg

The amount of work is said to be one erg when a force of dyne acting on a body displaces it through a distance of 1 c.m along the direction of the force.

(b) M.K.S unit = Newton x metre = Joule

The amount of work is said to be one Joule when a force of ~~Nen~~ Newton acting on a body displaces it through a distance of 1 metre along the direction of force.

(2) F.P.S unit (absolute) -

The units is called Feet, poundal which can be defined as the amount of work done to displace a body through distance of 1 feet by appling a force of 1 poundal

(3) F.P.S unit (gravitational)

The unit is called feet pound which can be defined as the amount of work done to displace a body through a distance of 1 feet by appling a force of 1 pound.

(4) Relation between different units of work

(1) Between Joule and erg.

1 Jolue = 1 Newton x 1 metre

$= 10^5$ dyne x 10^2 c.m

$= 10^7$ (dyne x c.m)

$= 10^7$ erg.

(2) Between feet poundal and erg

1 feet poundal = 1 poundal x 1 feet

= 13825.728 dyne x 12 x 2.54 c.m

= 421408.17 erg

$= 4214 \times 10^5$ erg

(3) between ~~foot poundal~~ pound and feet poundal

$$1 \text{ foot pound} = 1 \text{ pound} \times 1 \text{ feet}$$
$$= 32.17 \text{ poundal} \times 1 \text{ foot}$$
$$= 32.17 \text{ foot poundal}$$

(4) Foot pound and Joule (task)

$\boxed{\text{ENERGY}}$

$$1 \text{ foot pound} = 1 \text{ pound} \times 1 \text{ feet}$$
$$= 32.17 \text{ poundal} \times 12 \times 2.54 \text{ cm}$$
$$= (32.17) \times (13825.728) \times 12 \times 2.54 \text{ erg}$$
$$= (32.17) \times (13825.728) \times 12 \times 2.54 \times 10^{-7} \text{ Joule}$$
$$= 13556701 \times 10^{-7} \text{ Joule}$$
$$= 1.3556701 \text{ Joule}$$

ENERGY : The ability or capacity to do work is called energy.

Types of energy :

(1) Mechanical (a) Potential
(b) Kinetic

(2) Heat
(3) Light
(4) Sound
(5) Electrical
(6) Atomic
(7) Nuclear
(8) Chemical
(9) Magnetic
(10) Solar
(11) wind
(12) Tidal
(13) Geo-thermal.

Principle of Conservation of energy

Statement: Energy can neither be created nor it can be destroyed, but can always be transformed from one type into the other type. The sum total of energy, in the universe, is always same.

Ex: (1) When we rub our hands, kinetic energy gets converted into heat energy.

(2) In a battery chemical energy is converted into electrical energy.

(3) In a dynamo, mechanical energy is converted to electrical energy.

(4) In a motor electrical energy is converted to kinetic energy.

(5) In a nuclear reactor nuclear energy is converted to electrical energy.

(6) In an electric calling bell electrical energy converted to sound energy.

(7) In a telephone sound energy is converted to electrical energy.

(8) In solar cells light energy is converted to electrical energy or heat energy.

Note: Potential energy 2 expression by ∫ see Back P.S.

Potential energy:

The energy possessed by body on account of its position or configuration is called potential energy.

Ex: (1) A body of mass (m) lying on the

ground presses the ground with its weight (mg), acting downwards. In order to raise the body a minimum force of amount (mg) is necessary. The amount of work done to raise it up to a height (h) = Force x displacement

$$\cancel{mgh} = mg \times h$$
$$= mgh$$

This much of work done on the body is stored in it. as potential energy.

Ex: (2) A watch requires winding everyday by this the spring inside the watch are compressed. There by, energy is stored in the springs which is utilized during the next 24 hours in rotating the three hands of watch.

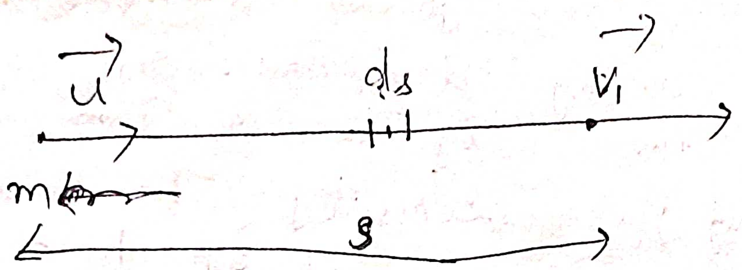
<u>Kinetic energy :-</u>

$\boxed{\text{Expression K.E 24 page Interpretation alternative method. Here Ans in Derivation } \Delta W = \Delta E_K}$

The energy possessed by a body on account of its motion is called kinetic energy.

An expression for the kinetic energy can be derived by the method of calculus.

Let there be a body of mass (m) moving with initial velocity $(\vec{u})$ which changes $\vec{v_1}$ when an external force $\vec{F} = m\vec{a}$, acts on it through a distance of S units.



Let's divide the distances into small segments of thickness ds. One such segment has been shown in the figure where the instamtanious velocity is given by $v = \frac{ds}{dt}$ $\Rightarrow ds = v . dt$

dw = small amount of work done to displace the body by a small amount ~~ds~~ ds

$$= F . ds$$

$$= ma . ds$$

$$= m . \frac{dv}{dt} . \cancel{dt} v\, dt$$

$$= mv . dv$$

Integrating both the sides with proper limits, we get

$$\int_0^w dw = \int_u^{v_1} mv . dv$$

$$(w)\Big|_0^w = m \int_u^{v_1} v . dv$$

$$\Rightarrow w - 0 = m \left(\frac{v^2}{2}\right)\Big|_u^{v_1}$$

$$\Rightarrow \Delta W = m\left(\frac{v_1^2}{2} - \frac{u^2}{2}\right)$$

$$\Rightarrow \Delta w = \frac{1}{2} m (v_1^2 - u^2)$$

$$\Rightarrow \Delta w = \frac{1}{2} m v_1^2 - \frac{1}{2} m u^2$$

$$= (E_K)_f - (E_K)_i$$ $= \Delta E_K$

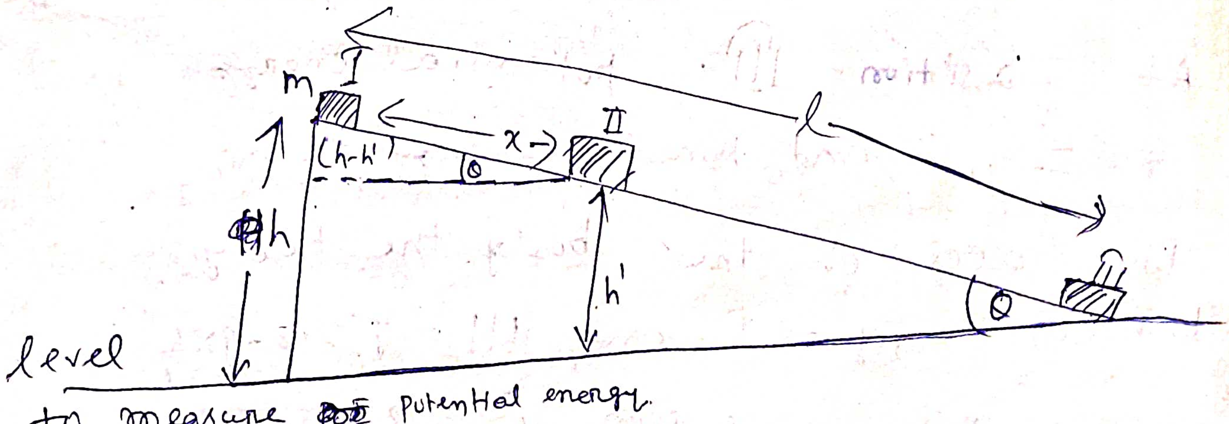
where we have defined kinetic energy as $E_K = \frac{1}{2}$ mass . Velocity^2

Thus $\boxed{\Delta W = \Delta E_K}$ and $\boxed{E_K = \frac{1}{2} m v^2}$

That is Change in work done = gain of Kinetic energy.

Inter conversion between potential energy and Kinetic energy.

Ex : (1) To prove that the total energy remains as mgh when a body slides on a frictionless inclined plane



At position I when the body just starts to slide, the initial velocity is zero Hence kinetic energy

$= \frac{1}{2} m v^2 = 0$

Potential energy = mgh

$\therefore$ Total energy at position I = mgh ——(1)

At position II potential energy mgh'

and kinetic energy = $\frac{1}{2} m v^2$

The accl. of the body between I and II $= g \sin\theta = g \frac{(h-h')}{x}$

From the relation $v^2 - u^2 = 2as$ we have $v^2 - 0^2 = 2 . g . \left(\frac{h-h'}{x}\right) . x$

$\Rightarrow v^2 = 2g(h-h')$

$\therefore$ Kinetic energy $= \frac{1}{2} m v^2$

$= \frac{1}{2} . m . 2g(h-h')$

$= mgh - mgh'$

Total energy at position II

$= mgh' + mgh - mgh'$

$= mgh$ ——(2)

At position III potential energy $= 0$, and kinetic energy $= \frac{1}{2} m v_1^2$

The accl of the body the body between position I and III $= g \sin\theta = g \times \frac{h}{x}$

From the relation $v_1^2 - u^2 = 2as$

we get $v_1^2 - 0^2 = 2 . g . \frac{h}{x} . x$

$\Rightarrow v_1^2 = 2gh$

$\therefore$ Kinetic energy $= \frac{1}{2} m v_1^2$

$= \frac{1}{2} \cdot m \cdot 2gh$

$= mgh$

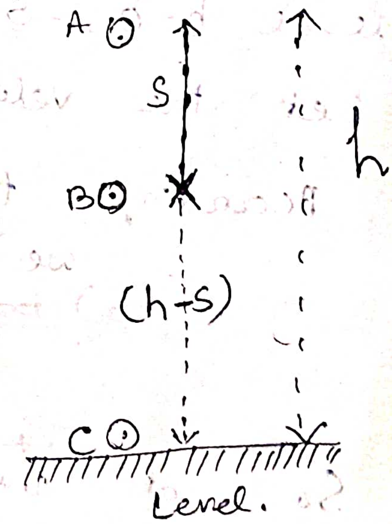
$\therefore$ Total ~~position~~ energy at position III $= mgh$ —— (3)

From these three eqⁿs we see that the total energy remains the same although there is conversion of potential into kinetic.

Task: Freely falling body:

To prove that the total energy remains same as mgh when a body falls freely.

Let the mass of the body $= m$.



(Conservation of energy on a freely falling body)

(a) At the position 'A' the height from the level is 'h'

(a) At the position A when the body just starts to fall the initial velocity is zero.

Hence Kinetic energy, $E_K = 0$

Potential energy, $E_P = mgh$.

$\therefore$ Total energy at position "A" $E_A = E_P + E_K = mgh + 0 = mgh$ —— (i)

(b) After falling S distance the body reaches the 'B' position. The kinetic energy must be increased and potential energy must be decreased.

b) At the position 'B' the the distance from level (h-s). So its potential energy.

$E_P = mg(h-s)$

From the 3rd ~~[illegible]~~ equation of motion we know

$$v^2 - u^2 = 2gs$$
$$\Rightarrow v^2 - 0 = 2.gs$$
$$\Rightarrow v^2 = 2gs$$

$\therefore$ Its kinetic energy, $E_K = \frac{1}{2} . m . v^2$

$$= \frac{1}{2} . m . 2gs$$
$$= mgs$$

Total energy at the position B

$$E_B = E_P + E_K = mg(h-s) + mgs$$
$$= mgh \quad \text{------} (ii)$$

(c) ~~[illegible]~~ At the position C the height from level $h = 0$. So potential energy $E_P = 0$

Let the velocity ~~at~~ in this time $= V_1$

According to the 3rd equation of motion

we know $v_1^2 - u^2 = 2gh$

$$\Rightarrow v_1^2 = 2gh \quad (\because u_1 = 0)$$

(because it travels distance h)

So at the position C the kinetic energy $E_K = \frac{1}{2} m . v_1^2 = \frac{1}{2} . m . 2gh = mgh$

So the total energy at the position C

$$E_C = E_P + E_K = 0 + mgh = mgh \quad \text{------} (iii)$$

From these three equations we see that the total energy remains same although there is conversion of potential to kinetic.

Elements of Integration :

Integration is a process of summation. It is indicated by the symbol $\int$.

Formulae on ~~Indefinite~~ indefinite Integration

(i) $$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

where c is the constant of integration.

Ex-1

$$\int dx = \int x^0\,dx$$
$$= \frac{x^{0+1}}{0+1} + c_1$$
$$= x + c_1$$

Ex-2

$$\int x.\,dx = \int x^1.\,dx$$
$$= \frac{x^{1+1}}{1+1} + c_2$$
$$= \frac{x^2}{2} + c_2$$

Ex-3

$$\int x^2\,dx = \frac{x^{2+1}}{2+1} + C_3$$
$$= \frac{x^3}{3} + C_3$$

Ex-4

$$\int \sqrt{x}.\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C_4$$
$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C_4$$
$$= \frac{2x^{\frac{3}{2}}}{3} + C_4$$

Ex-5

$$\int \frac{dx}{\sqrt{x}} = \int \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C_5$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C_5$$

$$= 2x^{\frac{1}{2}} + C_5$$

In the above formula ~~for~~ is applicable for all values of n except $n = -1$.

(2) $$\int \frac{dx}{x} = \log_e x + c$$

$$= 2.3026 \log_{10} x + c$$

(3) $$\int k x^n \, dx$$

where k = a constant

$$= k \int x^n dx$$

$$= \cancel{k \int x^{n+1}} = k\left(\frac{x^{n+1}}{n+1} + c\right)$$

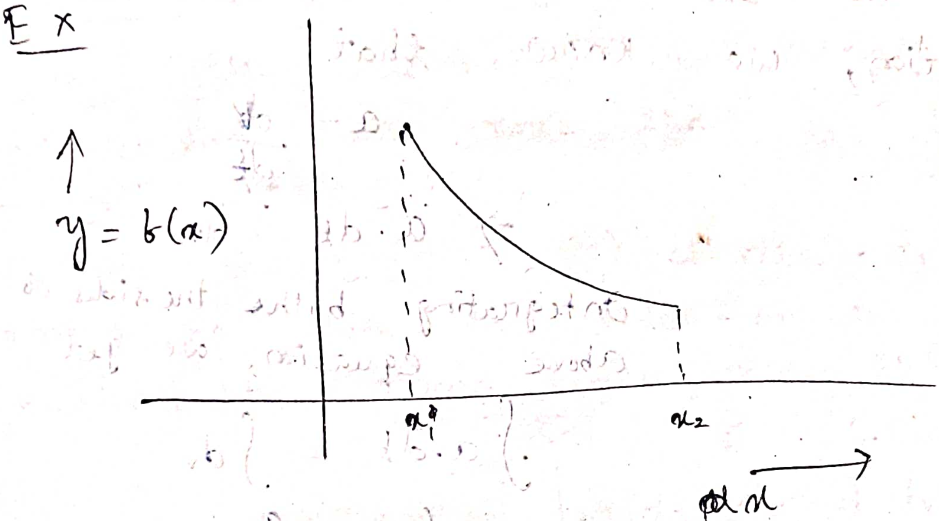
$$= \frac{k}{n+1} \cdot x^{n+1} + kc$$

$$= a \cdot x^{n+1} + b$$, where a and b are constants given by $\frac{k}{n+1}$, kc

Definite integration:

If the integration is combined to a particular region so that there will be a lower limit and upper limit, then the constant of integration will be inaffective. This is ~~explaned~~ explained by the following example.

Ex



Suppose, $y = f(x) = x^2$

$$\therefore \int x^2\,dx = \left[\frac{x^{2+1}}{2+1} + C\right]\Bigg|_{x_1}^{x_2} = \left[\frac{x^3}{3} + C\right]\Bigg|_{x_1}^{x_2}$$

$$= \left[\frac{x_2^3}{3} + C\right] - \left[\frac{x_1^3}{3} + C\right]$$

$$= \frac{x_2^3}{3} - \frac{x_1^3}{3}$$

$$= \frac{x^3}{3}\Bigg|_{x_1}^{x_2}$$

Derivation of eq^ns of motion for uniformly accelerated bodies by Calculus method

(i) To derive $v = u + at$

Method (1) (by indefinite integration)

From the definition of instantaneous acceleration, we know that

$$a = \frac{dv}{dt}$$

$$\Rightarrow a \cdot dt = dv$$

Integrating both the sides of the above equation, we get

$$\int a \cdot dt = \int dv$$

$$\Rightarrow a \int dt = \int dv$$

(where 'a' could be taken out of the integration since it is a constant quantity for uniformly accelerated body.)

$$\therefore a(t + C_1) = v + C_2$$

where C_1 and C_2 are the constants of integration.

$$\Rightarrow at + aC_1 = v + C_2$$

$$\Rightarrow v = at + (aC_1 - C_2)$$

$$\Rightarrow v = at + C \quad \text{—— (i)}$$

where C = another constant $= aC_1 - C_2$

To evaluate C, let's use the initial condition at time $t = 0$ when velocity of the moving body $= u$

Using this condition in eqn -(i), we get

$$u = a.(0) + c$$

$$\Rightarrow u = c \quad \text{———— (ii)}$$

Using eqn (ii) in eqn (i) we get

$$v = at + u$$

$$\Rightarrow v = u + at$$

Method-2 (by definite integration)

From the definition of instantaneous acceleration, we know that $a = \frac{dv}{dt}$

$$\Rightarrow a.dt = dv$$

Integrating both the sides of the above equation, with proper limits, we get

$$\int_0^t a\,dt = \int_u^v dv$$

$$\Rightarrow a\int_0^t dt = \int_u^v dv$$

Where a could be taken out of the integration because it is a constant quantity for uniformly accelerated bodies.

$$\therefore a.(t)\Big|_0^t = (v)\Big|_u^v$$

$$\Rightarrow a(t-0) = v-u$$

$$\Rightarrow at = v-u$$

$$\Rightarrow v = u + at$$

(ii) To derive $S = ut + \frac{1}{2}at^2$

Method-1 (by indefinite integration)

From the definations of instantaneous velocity, we know that $V = \frac{ds}{dt}$

$\Rightarrow V \cdot dt = ds$

$\Rightarrow (u + at) \cdot dt = ds$

$\Rightarrow u\,dt + at \cdot dt = ds$

Integrating both the sides of the above equation, we get $\int u\,dt + \int at\,dt = \int ds$

Since u and a are constants, they can be taken out of the integrations $\Rightarrow u \int dt + a \int t \cdot dt = \int ds$

$\Rightarrow u(t + C_1) + a\left(\frac{t^2}{2} + C_2\right) = (S + C_3)$

When C_1, C_2 and C_3 are the constant of integration. $\Rightarrow ut + uC_1 + \frac{at^2}{2} + aC_2 = S + C_3$

$\Rightarrow S = ut + \frac{1}{2}at^2 + (uC_1 + aC_2 - C_3)$

$\Rightarrow S = ut + \frac{1}{2}at^2 + C$ ——— (iii)

To evaluate C, let's use the initial condition at time $t = 0$, when $S = 0$.

Putting this condition in eqn (iii) we get $\Rightarrow 0 = 0 + 0 + C$

$\Rightarrow C = 0$ ——— (iv)

Using eqn (iv) in eqn (iii) we get

$\Rightarrow S = ut + \frac{1}{2}at^2 + 0$

$= ut + \frac{1}{2}at^2$

Method :- 2 (By definite integration)

From the defination of instantanious velocity, we know that $V = \frac{ds}{dt}$

$$\Rightarrow V \cdot dt = ds$$
$$\Rightarrow (u+at)\,dt = ds$$
$$\Rightarrow u\,dt + at\,dt = ds$$

Integrating both the sides with proben limits, we get

$$\int_0^t u\,dt + \int_0^t at\,dt = \int_0^s ds$$

Since u and a are constents they can be taken out of the integrations

$$\Rightarrow u\int_0^t dt + a\int_0^t t\,dt = \int_0^s ds$$

$$\cancel{\Rightarrow u\int_0^t t\Big|_0^t + a\int_0^t \frac{t^2}{2} =}$$

$$\Rightarrow u(t)\Big|_0^t + a\left(\frac{t^2}{2}\right)\Big|_0^t = (s)\Big|_0^s$$

$$\Rightarrow u(t-0) + a\left(\frac{t^2}{2} - 0\right) = s - 0$$

$$\Rightarrow ut + \frac{at^2}{2} = s$$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

(iii) To derive $v^2 - u^2 = 2as$

Method -(1) (by indefinite integration)

From the definations of instantaneous velocity and accleration we know that

~~$V = u + at$~~ $\quad V = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$

$$\Rightarrow \quad dv = a.dt$$

Now $\quad V.\,dV = \frac{ds}{dt}.(a.dt)$

$$= a.ds$$

Integrating both the sides, we get

$$\int V.dv = \int a.ds$$

Since a = Constant for uniformly accelerated body it can be taken out of the integration.

$$\therefore \int V.dv = a\int ds$$

$$\Rightarrow \frac{V^2}{2} + C_1 = a(S + C_2)$$

where C_1 and C_2 are the constants of integration

$$\Rightarrow \frac{V^2}{2} = aS + (aC_2 - C_1)$$

$$\Rightarrow \frac{V^2}{2} = aS + C \quad \text{———} \quad (v)$$

Where C is a another constant $= aC_2 - C_1$

To evaluate C we can use the initial condition at time $t = 0$, when $S = 0$ and $V = u$

Putting this condition in eq^n (iv) we get

$$\Rightarrow \frac{u^2}{2} = a.0 + C$$

$$\Rightarrow C = \frac{u^2}{2} \quad \text{———} \quad (vi)$$

using eq^n (vi) in eq^n(v), we get

$\Rightarrow \frac{v^2}{2} = as + \frac{u^2}{2}$

$\Rightarrow \frac{v^2}{2} = \frac{2as+u^2}{2}$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow v^2 - u^2 = 2as$

Method-2 (by definite integration)

From the definations of instantanious velocity and acceleration we know that

$v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$

$\Rightarrow dv = a.dt$

Now $v.dv = \frac{ds}{dt}(a.dt)$

$= a\,ds$

integrating both the sides with proper limits, we get

$$\int_u^v v.dv = \int_0^s a\,ds$$

Since a = constant for uniformly accelerated body it can be taken out of the integration $\int_u^v v.dv = a\int_0^s ds$

$\Rightarrow \left(\frac{v^2}{2}\right)\Big|_u^v = a(s)\Big|_0^s$

$\Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a(s-0)$

$\Rightarrow \frac{v^2-u^2}{2} = as$

$\Rightarrow v^2 - u^2 = 2as$

$\boxed{11.30 \text{ to } 1}$ Tuesday

In the exam Case

(1.) Derivation of $S = ut + \frac{1}{2}at^2$

Method-1

From the definitions of instantanious velocity, we know that $V = \frac{ds}{dt}$

$\Rightarrow V.dt = ds$ ——— (i)

Derivation of $V = u + at$

From the ~~derivation~~ defination of instantanious ~~velocity~~ acceleration, we know that,

$a = \frac{dv}{dt}$

$\Rightarrow a.dt = dv$

Integrating both the sides of the eqn with proper limits, we get

$\int_0^t a.dt = \int_u^v dv$

$= a \int_0^t dt = \int_u^v dv$

where, a could be taken out of the integration because it is a constant quantity for uniformly accelerated bodies

$a.(t)\Big|_0^t = v\Big|_u^v$

$\Rightarrow a(t-0) = \cancel{v} \; v - u$

$\Rightarrow at = v - u$

$\Rightarrow v = u + at$ ——— (ii)

Putting the value of v in eqn (i), we get ~~~~ $V.dt = ds$

$\Rightarrow (u+at).dt = ds$

$\Rightarrow udt + at.dt = ds$

Integrating both the sides with proper limit, we get $\int_0^t u\,dt + \int_0^t at.dt = \int_0^s ds$

~~proved~~

Since u and a are constants they can be taken out of the integration.

$$u\int_0^t dt + a\int_0^t t.dt = \int_0^s ds$$

$$\Rightarrow u(t)\Big|_0^t + a\left(\frac{t^2}{2}\right)\Big|_0^t = s\Big|_0^s$$

$$\Rightarrow u(t-0) + a\left(\frac{t^2}{2}-0\right) = s-0$$

$$\Rightarrow ut + \frac{at^2}{2} = s$$

$$\Rightarrow s = ut + \frac{1}{2}at^2 \quad \text{(proved)}$$

(2) Derivation of $v^2 - u^2 = 2as$

Method: 1

From the definitions of instantaneous velocity, we know that $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$

$$\Rightarrow dv = a.dt$$

Now $v.dv = \frac{ds}{dt}.(a.dt)$

$$= a\,ds$$

Integrating both the sides with proper limit we get

$$\int_u^v v.dv = \int_0^s a.ds$$

Since a = constant for uniformly accelerated body it can be taken out of the integration.

$$\int_u^v v.dv = a\int_0^s ds$$

$$\Rightarrow \left(\frac{v^2}{2}\right)\Big|_u^v = a(s)\Big|_0^s$$

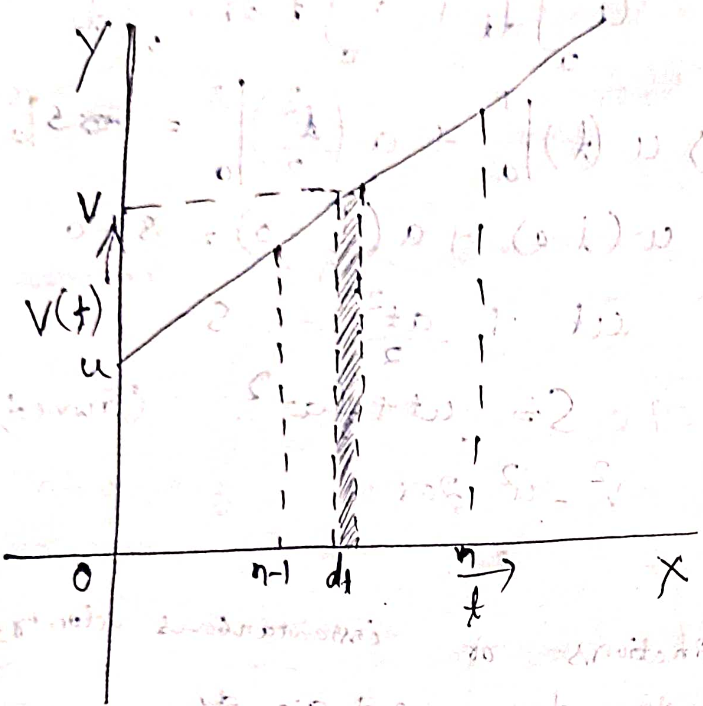
$$\Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a(s-0)$$

$$\Rightarrow \frac{v^2-u^2}{2} = as$$

$$\Rightarrow v^2-u^2=2as \quad \text{(Proved)}$$

<u>4th</u> :- To derive the formula

$$S_n = u+\frac{a}{2}(2n-1)$$



Area of the strip $\boxed{V = \text{length},\ dt = \text{breadth}}$

$= V.\cancel{\ }dt$

$= \frac{ds}{dt} \times dt$

$= ds$

= small distance or displacement

From the definition of instantaneous velocity, we know that $V = \frac{ds}{dt}$

$\Rightarrow ds = V.dt$

If a graph be plotted between velocity along the Y axis and time along the X axis then a straight line is obtained. Area of a thin strip represents the displacement in a small time dt second.

Integrating the expression for ds with proper limits like $t=n-1$ to $t=n$, we can get the distance covered during n^{th} second. Let's write S as a function of t

$$\therefore \; ds(t) = v(t).dt$$
$$= (u+at)\,dt$$
$$= u.dt + at.dt \quad \text{(where } a \text{ is = constant for uniformly accelerated body)}$$

Integrating both the sides of the above eqn with proper limits, we get

$$\int_{t=n-1}^{t=n} ds(t) = \int_{n-1}^{n} u\,dt + \int_{n-1}^{n} at\,dt$$

$$\Rightarrow [S(t)]\Big|_{t=n-1}^{t=n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt$$

$$\Rightarrow S(n) - S(n-1) = u\,(t)\Big|_{n-1}^{n} + a\left(\frac{t^2}{2}\right)\Big|_{n-1}^{n}$$

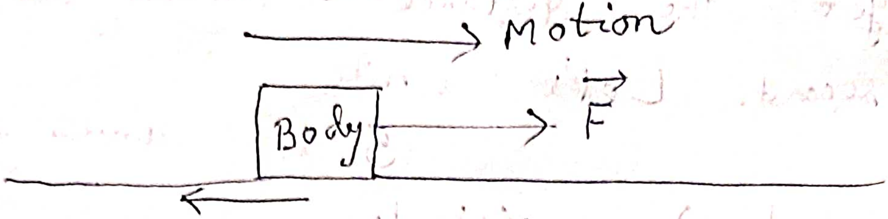
$$\Rightarrow S_n = u\,[n-(n-1)] + \frac{a}{2}\left[n^2-(n-1)^2\right]$$

When u and a could be taken out of the integration because they are constants for uniformly accelerated bodies.

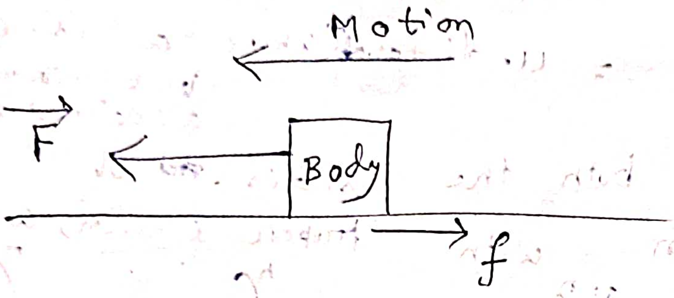
$$\therefore S_n = u(\cancel{n}-\cancel{n}+1) + \frac{a}{2}(\cancel{n^2}-\cancel{n^2}-1+2n)$$
$$= u(1) + \frac{a}{2}(2n-1)$$
$$= u + \frac{a}{2}(2n-1)$$

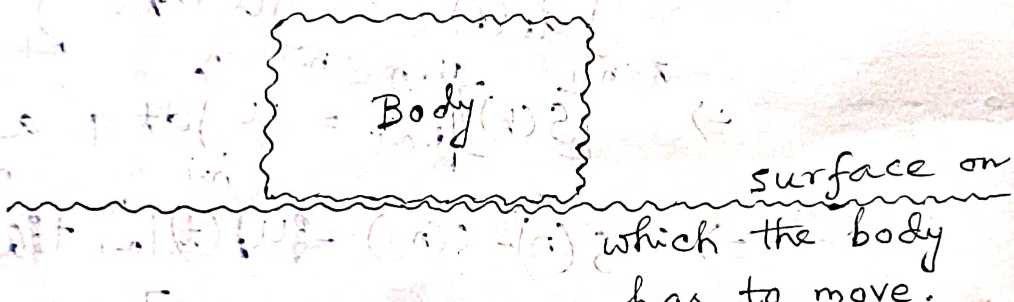
Friction

It opposes motion. It arises out of the interlocking of the two surfaces in contact.



f = frictional force





Laws of friction

From large number of experiments the following three laws have been established. (1) The direction of force of friction is always opposite to the direction of motion.

(2) (1) The frictional force depends on the nature & surface polish of the two surfaces in contact and acts tangentially to the interface between two surfaces.

(3) (2) The magnitude of the frictional force is directly proportional to the normal force pressing (N) $\therefore f \propto N$

$$\Rightarrow \boxed{f = \mu N}$$

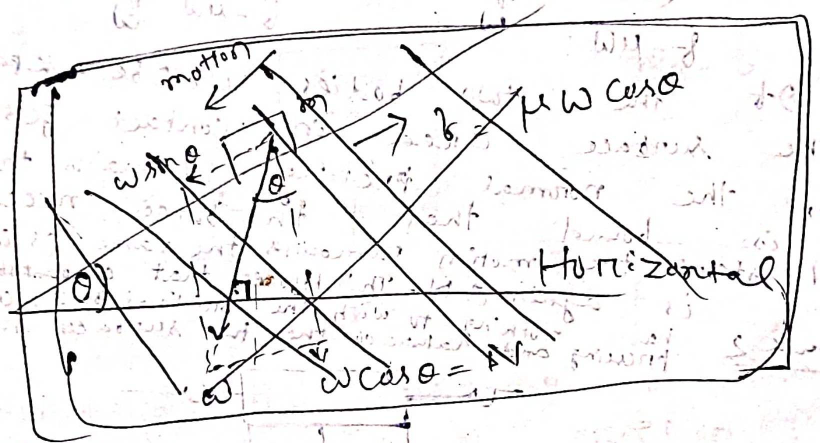
where, μ is the co-efficient of friction which depends on the nature of the two surfaces in contact.

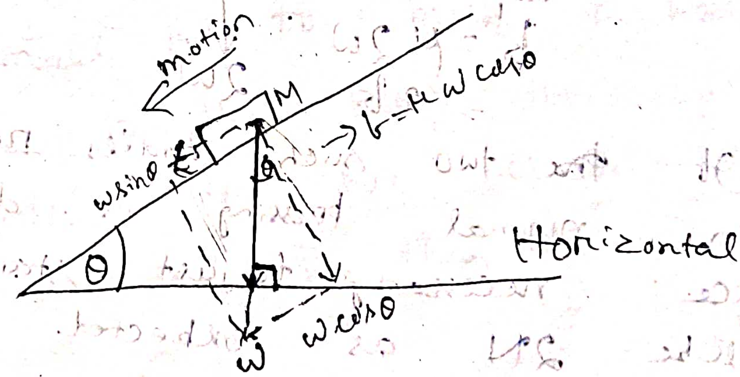
Ex:1

If a body slides on a plane surface then $N = W$ (W = weight)

$\therefore f = \mu W$

Ex:2 If a body slides on an inclined plane, then $N = W\cos\theta$





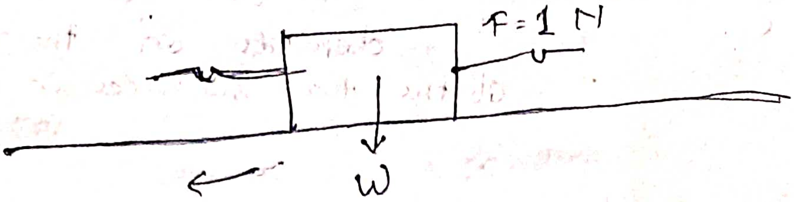
(4)

(3) The frictional force is roughly independent of the area & shape ~~constant~~ of surface ~~area~~ of the body that ~~or~~ is contact with the surface so long as the normal reaction remains the same.

~~Figure-1~~ Illustration

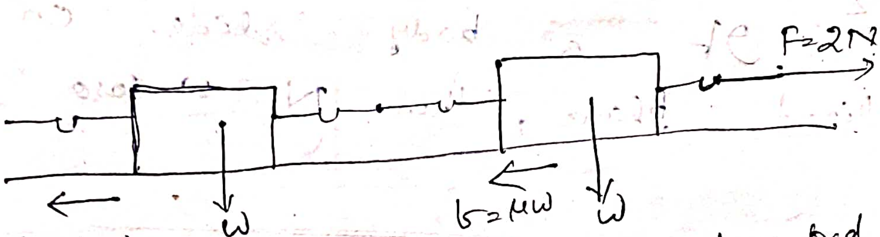
Let's apply a force just sufficient to make the body move. If this force be 1 N then f = 1 N

Figure-1



$f = \mu W$

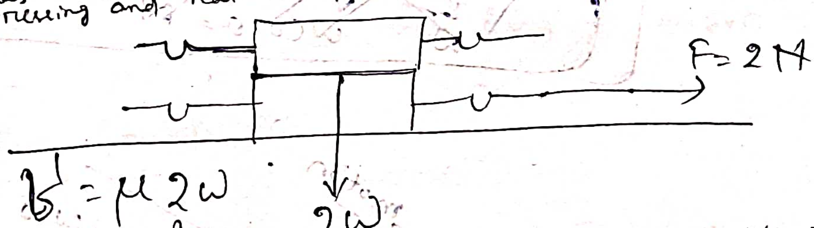
Figure-3



$f = \mu W$

If the two bodies will be separated, the surface area in contact gets doubled. The normal pressing remains the same. But, the force necessary to just start the motion remains the same as in case(2). That it is bound that the force is again 2 N. This proves that amount of surface area has nothing to with the frictional force, only normal pressing and nature of the two surfaces in contact are important.

Figure-2



$f' = \mu 2W$
$= 2f$

If ~~the~~ two such bodies replace one over the another, then normal pressing is doubled. The force necessary for just start the motion will be 2N as expected.

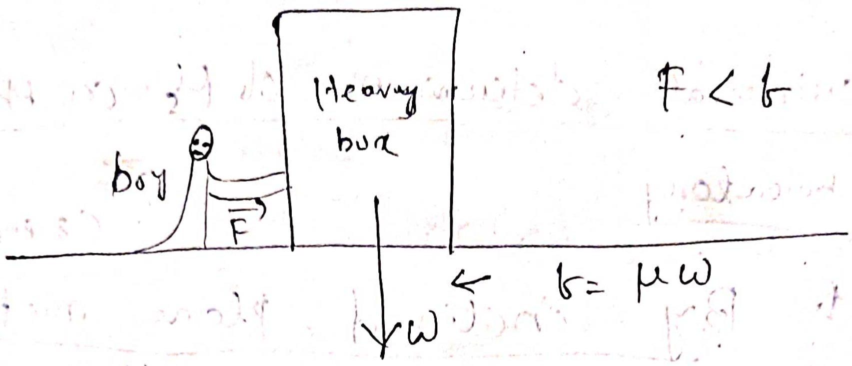
Types of Co-efficient of friction

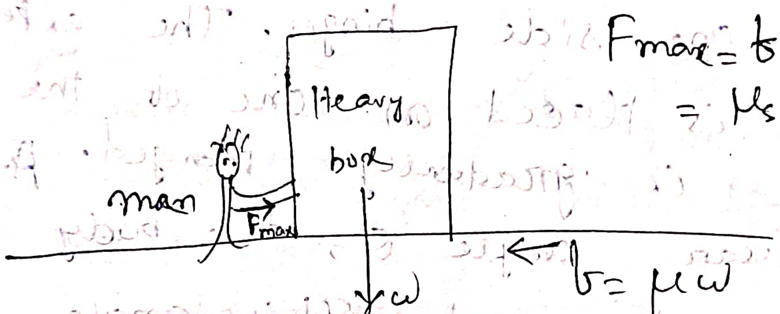
3 types of co-efficient of friction are observed in nature
(1) ~~Static~~ static Co-efficent of ~~floating~~ friction (μ_s)

(2) Coefficient of starting kinetic friction (μ_k)

(3.) Coefficient of rolling friction (μ_r)

~~Coeb~~ Coefficient of Static friction





To understand the meaning of this kind of coefficient of friction, let's ask a small boy to push a heavy (ক্ষুদ্র) box lying on the floor. Obviously he cannot displace the box because his applied force is less than the frictional force.

Let's ask a man to apply a force just sufficient to make the box move ~~a bit~~ a little. This force be called F_{max}.

Naturally $F_{max} = \mu_s N$

$$\Rightarrow \boxed{\mu_s = \frac{F_{max}}{N}} \quad \text{where } N = \omega$$

When the motion is just started, the type of coefficient of friction encountered is called coefficient of static friction

Experimental determination of μ_s in the laboratory

(2 page আগে figure)

No 1. By inclined plane method.

There are two wooden planks joined on one side hinges. The experimental body is placed on one of the planks which is gradually ranged. At a particular angle θ, the body just starts to slide. This angle is called limiting angle of friction or angle of repose.

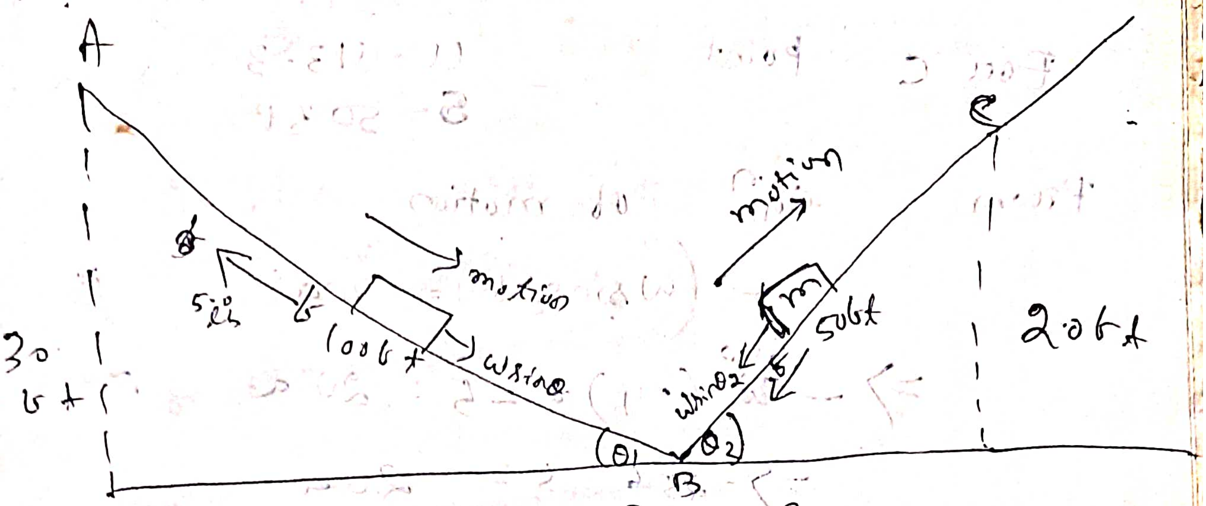
$$\therefore \omega \sin\theta = f = \mu_s \, \omega \cos\theta$$

$$\Rightarrow \boxed{\mu_s = \tan\theta = \frac{p}{b}}$$

Other observations are taken by placing additional mass on the body and repeating the above procedure

Page 99 → 16 no

$-(w\sin\theta_2 + b) = ma$



$\sin\theta_1 = \frac{30}{100} = .3$

$\sin\theta = \frac{20}{50} = .4$

The roller ~~coas~~ coaster weighs $= 640$ lb

$ma = 640$ lb

$\Rightarrow m = \frac{640}{32}$

$= 20$ slug

The frictional force (b_1) $= 5$ lb

~~we know that F - b = m~~

eqn of motion for the roller coaster is

$w\sin\theta_1 - b = ma$

$\Rightarrow 640(.3) - (5) = 20 \cdot a$

$\Rightarrow 192 + 5 = 20a$

~~$\Rightarrow 197 = 20a$~~

~~$\Rightarrow a = \frac{195}{20}$~~

$\Rightarrow 187 = 20a$

$\Rightarrow a = \frac{187}{20} = 9.35$

From 3rd eqn of motion we know that $v^2 - u^2 = 2as$

$\Rightarrow v^2 - 0 = 2 \cdot (9.35)(100\ ft)$

21.3 ? 2-1-4

$\Rightarrow v^2 = 1870$

$v = 43.243$

$= 43.3$ ft/s at point B

For C point $u = 43.3$

$S = 50$ ft

From eqn of motion

$-(w\sin\theta_2 + F) = ma$

$\Rightarrow -640(.4) + -5 = 20a$

$\Rightarrow -256 - 5 = 20a$

$\Rightarrow -261 = 20a$

$\Rightarrow a = -13.05$

We know that $v^2 - u^2 = 2as$

$\Rightarrow v^2 - (43.3)^2 = 2.(-13.05).(50)$

$\Rightarrow v^2 - 1874.89 = -1350$

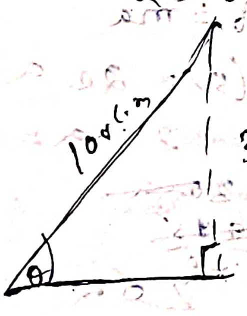
$\Rightarrow v^2 = 1874.89 - 1305$

$\Rightarrow v^2 = 569.89$

$\Rightarrow v = 23.8$ ft/s at point C.

- velocity is 43.3 ft/s at B and 23.8 at C

24\. A coin lying on a meterstick is raised to slide starts 36.4 cm above the horizontal



$\sin\theta = \frac{p}{h} = \frac{36.4}{100} = .364$

$\theta = \sin^{-1}(.3640)$

$= 21.3^\circ$

$\tan(21.3^\circ) = .3899$

We know that

$\mu_s = \tan\theta$

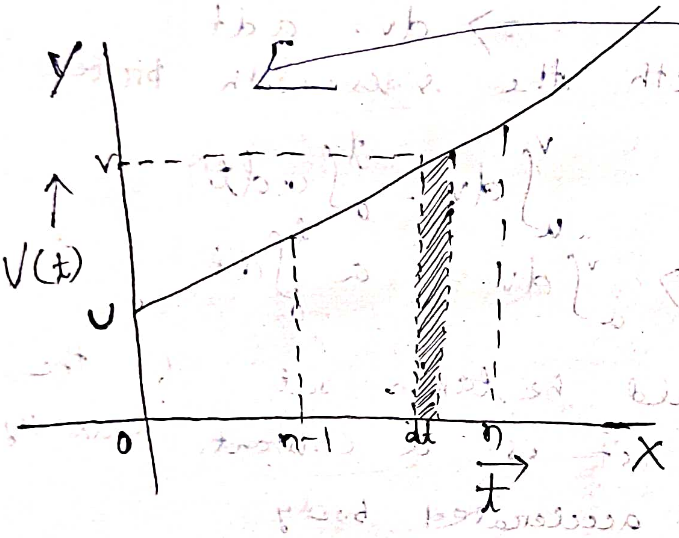
$\Rightarrow \mu_s = ~~.389~~ .3899$

∴ The limiting angle $\theta = 21.3°$ and the coefficient of the starting friction .3899.

— ◡ —

exam case ~~Q.~~

Derrivation of $S_n = ~~\frac{a}{2}+2a~~ u + \frac{a}{2}(2n-1)$



Area of the strip

$= v \cdot dt$

$= \frac{ds}{dt} \cdot dt$

$= ds$

From the defination of instantaneous velocity $v = \frac{ds}{dt}$

$\Rightarrow ds = v \cdot dt$

If a graph is plotted between velocity along Y axis and time along X axis then a straight line is obtained.

Area of a thin strip represent the displacement in a small time dt second.

Integrating the expression for ds with proper limits like $t = n-1$ to $t = n$, we can get the distance covered during n^{th} second.

Let's write S as function of t

$$\therefore ds(t) = v(t)dt \quad \text{——— (i)}$$

Let's derive $v = ?$

From definition of instantaneous acceleration, we know that, $a = \frac{dv}{dt}$

$$\Rightarrow dv = a\,dt$$

Integrating both the sides with proper limits, we get $\int_u^v dv = \int_0^t a\,dt$

$$\Rightarrow \int_u^v dv = a\int_0^t dt$$

where a could be taken out of the integration since it is a constant quantity for uniformly accelerated body

$$\Rightarrow (v)\big|_u^v = a(t)\big|_0^t$$

$$\Rightarrow v - u = a(t - 0)$$

$$\Rightarrow v = u + at \quad \text{——— (ii)}$$

Putting the value of (v) in eqn (i) we get

$$ds(t) = (u + at)\,dt$$

$$= u\,dt + at.dt$$

(where a is ~~taken~~ constant for uniformly accelerated body)

Integrating both the sides with proper limits we get

$$\int_{t=n-1}^{t=n} ds(t) = \int_{n-1}^{n} u\,dt + \int_{n-1}^{n} at\cdot dt$$

$$\Rightarrow \left[S(t)\right]\Big|_{t=n-1}^{t=n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\cdot dt$$

where u and a could be taken out of the integration because they are constants for uniformly accelerated bodies

$$\Rightarrow S(n) - S(n-1) = u\cdot(t)\Big|_{n-1}^{n} + a\left(\frac{t^2}{2}\right)\Big|_{n-1}^{n}$$

$$= u[(n)-(n-1)] + \frac{a}{2}[n^2-(n-1)^2]$$

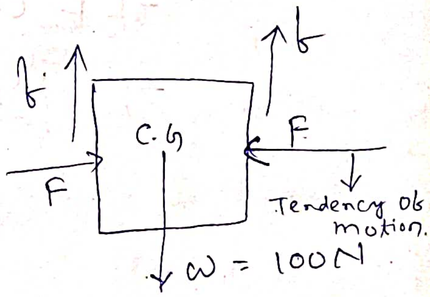
$$\Rightarrow S_n = u[\cancel{n} - \cancel{n}+1] + \frac{a}{2}[\cancel{n^2} - \cancel{n^2} - 1 + 2n]$$

$$= u + \frac{a}{2}(2n-1)$$ (proved)

— o —

Static friction:? 99 page College Physics

13. F = Pressing force = ?



$f = \mu_s N$

$= \mu_s F$

$= 0.3F$

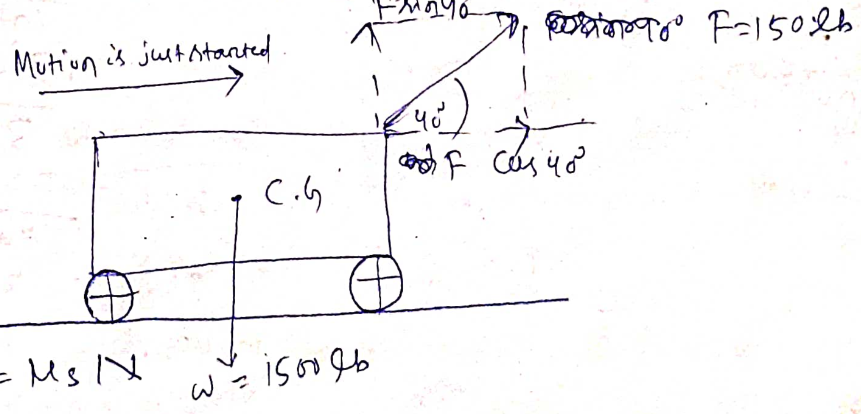
For equilibrium, total upward force = total downward force.

$$2f = w$$

$$\Rightarrow 2\mu_s F = 100$$

$$\Rightarrow F = \frac{100}{2\cdot(0.3)} = \frac{50}{.3} = \frac{500}{3} = 166.66\ N$$

14.



$f = \mu_s N$

Here $N = w - F\sin 40°$ ——— (i)

To just start the motion

$F\cos 40° = \sigma$ ——— (ii)

From eqn (i)

$N = w - F\sin 40°$

$\Rightarrow N = 1500 - (150)(.6428)$

$= 1500 - 96.42$

$= 1403.58$

From eqn (ii)

$F\cos 40° = \sigma$

$\Rightarrow F\cos 40° = \mu_s N$

$\Rightarrow \mu_s = \dfrac{F\cos 40°}{N}$

$= \dfrac{150 \times .7660}{1403.58}$

$= \dfrac{114.9}{1403.58}$

$= .082$

college physics 1st part

34. Efficiency of a machine

$= \dfrac{\text{output}}{\text{input}} \times 100$

$= \dfrac{(300-75)}{300} \times 100$

$= \dfrac{225}{300} \times 100$

$= 75\%$

$\boxed{\text{Output} = \text{input} - \text{friction}}$

$.6428$
1.50
96.4200
1500.00
96.42
1403.58

1. What is the efficiency of a $\frac{1}{4}$ h.p motor rated at 300 watts power ~~consumption~~ consumption? (Ans) - 62%.

Consume - inpute.

Ans : The efficiency of machine is $\frac{\text{out put} \times 100}{\text{input}}$

$$= \frac{\frac{1}{4} \cdot 746}{300} \times 100$$

$$= \frac{186.5}{300} \times 100$$

$$= 62\%.$$

2. A power station conveyer (ચઢાવનાર યંત્ર) lifts 500 tones of coal per hour to a height of 80 feet. what avarage h.p is required? (Ans: 40.4 h.p)

Ans :

1 ton $= 2000$ lb

500 " $= 10^5$ lb

Work done to raise the coal upto a height (h) $= mgh$

$$= 10^5 \times 80 \text{ foot.lb}$$

$$\text{power} = \frac{\text{work done}}{\text{time}} = \frac{10^5 \times 80 \text{ foot lb}}{3600 \text{ sec}}$$

but 550 foot lb / sec $= 1$ h.p

1 foot lb/sec $= \frac{1}{550}$ h.p

$$\frac{10^5 \times 80}{3600} \text{ footlb/sec} = \frac{1}{550} \times \frac{10^5 \times 80}{3600}$$

$$= \frac{8 \times 10^{6} 10^{2}}{198 \times 10^{4}} = \frac{800}{198} = 40.4 \text{ h.p}$$

$\boxed{\text{weight is a force.}}$

Task

3 What is the avarage power extended to accelerate a bullet of 15 gm mass to a speed of 900 mt/sec in 1.3 millisecond? (Ans: 4.7 megawatt)

4 If a 3000lb car were to accelerated uniformly at 10 ft/sec^2 for eight second, what avarage power would be needed for the first second and for the eighth second, {frictional effects being neglected}

(8.5 h.p, 127.5 h.p)

5 what avarage power is needed during the first, second, third seconds to accelerate a 5 Kg object at a uniform acceleration of 5 m/s^2

Power

The rate of doing work is called power.

Avarage power: $= \frac{\text{Total work done}}{\text{time taken}} = \frac{W}{t}$

$$= \frac{\vec{F} \cdot \vec{S}}{t}$$

If the force is constant, then

$\overline{P}$ (Average power) $= \vec{F} \cdot \left(\frac{\vec{S}}{t}\right) = \vec{F} \cdot \vec{V}_{(Avg)}$

Instantaneous power :

It is defined as the power at any instant of time and given by the expression :

$$P_{instant} = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt}$$

$$= \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta\vec{S}}{\Delta t}$$

$$= \vec{F} \lim_{\Delta t \to 0} \frac{\Delta \vec{S}}{\Delta t} \quad (\text{if } \vec{F} = \text{a Constant})$$

$$\therefore P_{instant} = \vec{F} \cdot \vec{V}_{instantaneous}$$

Dimension of power $= \frac{[Work]}{[time]} = \frac{ML^2T^{-2}}{T} = ML^2T^{-3}$

Units of power

(a) C.g.S unit : $\frac{erg}{sec}$

(b) M.K.S unit = $\frac{Joule}{sec}$ or watt

(c) F.P.S unit (absolute) = $\frac{\text{Foot poundal}}{sec}$

(d) F.P.S unit (gravitional) = $\frac{\text{Foot lb}}{sec}$

The practical unit for power is called horse power (h.p)

$$1 \text{ h.p} = 550 \text{ foot lb/sec}$$
$$= 746 \text{ watt}$$

To prove that 1 h.p = 746 Watt

$1\ h.p = 550$ foot lb/sec

$= 550 \times 32.17$ foot poundal/sec

$= 550 \times 32.17 \times 1$ poundal $\times 1$ foot/sec

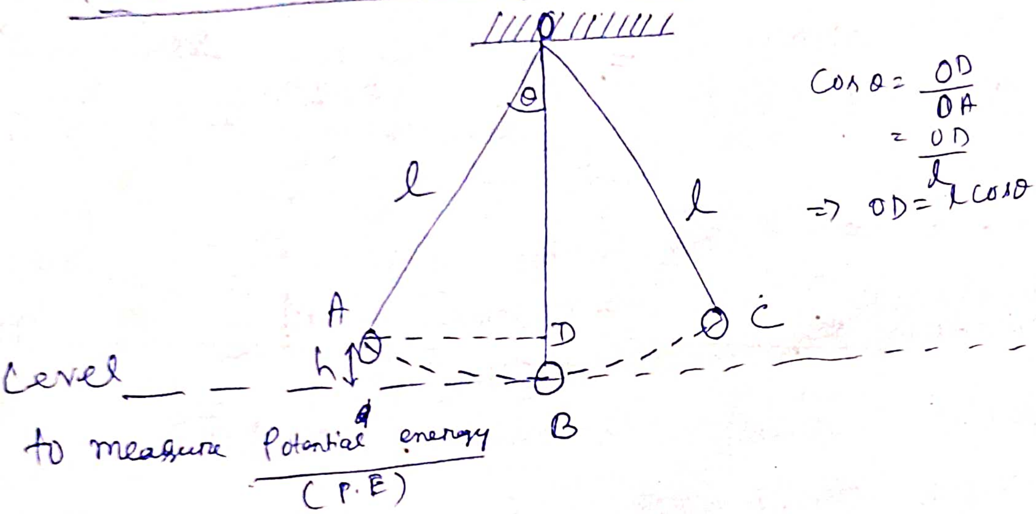
$= 550 \times 32.17 \times (13825.728\ \text{dyne}) \times \dfrac{(12 \times 2.54\ \text{cm})}{\text{sec}}$

$= \dfrac{550 \times 32.17}{1} \times \left(\dfrac{13825.728}{10^5}\right) \text{Newton} \times \dfrac{30.48}{100} \text{m/sec}$

$= 745.6$ watt.

(1). Inclined plane
(2) Freely falling

Example (3) for interconversion between potential energy and kinetic energy



$\cos\theta = \dfrac{OD}{OA} = \dfrac{OD}{l}$

$\Rightarrow OD = l\cos\theta$

In order to find the velocity with which the pendulum bob passes the equilibrium position, we can use the principle of conservation of energy at position A and B.

At position A, the particle is ~~momentary~~ momentaryly at rest. So that its kinetic energy $= 0$,

but, potential energy $= mgh$.

$\therefore$ Total energy of the pendulum bob at position $A = mgh$

At position B

Potential energy of the ball

$= mg.(0)$

$= 0$

Kinetic energy $= \frac{1}{2} mv^2$

Total energy at position B

$= \frac{1}{2} mv^2$

Thus principle of conservation of energy

gives Total energy at position B

= total energy at position A

$\therefore \frac{1}{2} \cancel{m} v^2 = \cancel{m} gh$

$\Rightarrow v^2 = 2gh$

$\Rightarrow v = \cancel{2gh} \sqrt{2gh}$

We can express V as a function

of θ, $OB - OD = h$

$\Rightarrow l - l\cos\theta = h$

$\cancel{\Rightarrow} \therefore h = \cancel{\quad} l(1-\cos\theta)$

$\boxed{\text{92 page example}}$

$\therefore v = \sqrt{2gl(1-\cos\theta)}$

Task: 3. when the bullet starts

its initial velocity $u = 0$

$v = 900 \, m/sec$

~~....~~ And the time taken

$t = 1.3$ milisec

$= \frac{1.3}{1000}$ sec

$= \frac{13 \times 1}{10 \times 1000} = \frac{13}{10000}$ sec.

$v = u + at$

$\Rightarrow 900 = 0 + a \cdot \left(\frac{13}{10000}\right)$

$\Rightarrow 13a = 900 \times 10000 = 9 \times 10^6$

$\Rightarrow a = \frac{90}{13} \times 10^5 = 6.92 \times 10^5 \, m/s^2$

From 2nd eqn of motion

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow S = 0 + \frac{1}{2}\cdot(6.92\times10^{8})\times\frac{169}{\cancel{10^8}\,10^3}$$

$$= \frac{3.46\times169}{10^3}$$

$$= \frac{584.74}{1000}$$

$$= .58474\ m$$

Avarage power = $\frac{\text{Total work done}}{\text{total time taken}} = \frac{mgh}{.0013}$

Mass = 15 gm

$= \cancel{.0015}$ k.g

$= .015$

$$= \frac{m.a.S}{.0013} = \frac{.015\times(6.92\times10^{5})\times.58474}{.0013}$$

$$= \frac{\cancel{.060069}\ .060696012 \times 10^5}{.0013}$$

$$= \cancel{46689061\times10^5}$$

$$= \cancel{4668.461\times10^6\ \text{watt}}$$

$$= \frac{\cancel{.60696012\times10^6}}{.0013}$$

$$= \frac{.0060696012\times10^6}{.0013}$$

$$= 4.66\times10^6\ \text{watt}$$

10^6 watt = 1 megawatt

$$4.66\times10^6\ \text{watt} = \frac{1}{10^6}\times4.66\times10^6$$

$$= 4.66\ \text{megawatt}$$

$\therefore$ The avarage power is $\approx$ 4.7 megawatt.

4. The weight of car = 3000 lb

$\Rightarrow mg = 3000$ lb

$\Rightarrow m = \frac{3000}{32}$

$= 93.75$ slug

and acceleration $a = 10\ ft/s^2$

Total distance taken in 1st second

$$S_1 = u + \frac{a}{2}(2n-1)$$
$$= 0 + \frac{10}{2}(1)$$
$$= 5 \text{ mitre}$$

Total distance taken is 8^{th} second

$$S_8 = u + \frac{a}{2}(2.(8)-1)$$
$$= 0 + \frac{10}{2} \cdot 15$$
$$= 75 \text{ mitre}.$$

Avarage power in 1st second

$$\frac{\text{Total work done}}{\text{time taken}} = \frac{mgh}{t} = \frac{mas}{t} = \frac{(93.75).(10).(5)}{1}$$
$$= 4687.5\ ~~\text{pound}~~\ ft\frac{lb}{sec}$$

$550\ ft lb/sec = 1\ h.p$

$1\ ft lb/sec = \frac{1}{550}\ h.p$

$4687.5\ ft lb/sec = \frac{4687}{550} = 8.5\ h.p$

Average power in 8^{th} second

$$= \frac{mas}{t} = \frac{(93.75)(10).(75)}{1} = \left(\frac{93.75 \times 10 \times 5}{1}\right) \times 15$$
$$= 4687.5 \times 15\ ft lb/sec$$

$550\ ft lb/sec = 1\ h.p$

4687.5×15 " " $= \frac{1}{550} \times 4687.5 \times 15$

$$= \frac{4687.5}{550} \times 15$$
$$= 8.5 \times 15$$
$$= 127.5\ h.p$$

∴ Avarage power in 1st second 8.5 h.p and in 8^{th} sec is 127.5 h.p

5. Similarly to Q-(4)

$m = 5 \cdot kg$

$a = 5\ m/s^2$

$S_1 = u + \frac{a}{2}(2n-1) = 0 + \frac{5}{2}(1) = \frac{5}{2}$ mitre $= 2.5 m$

$S_2 = u + \frac{a}{2}(2n-1) = 0 + \frac{5}{2}(3) = \frac{15}{2}$ mitre $= 7.5 m$

$S_3 = u + \frac{a}{2}(2n-1) = 0 + \frac{5}{2}(5) = \frac{25}{2}$ mitre $= 12.5 m$,

Avarage power in 1^{st} second $= \frac{\text{Total work done}}{\text{time taken}}$

$= \frac{m.(a).S}{t} = \frac{5 \times 5 \times 2.5}{1}$

$= 62.5$ watt

2nd second $= \frac{m.a.S}{t} = \frac{5 \times 5 \times (7.5)}{1}$

$= 187.5$ watt

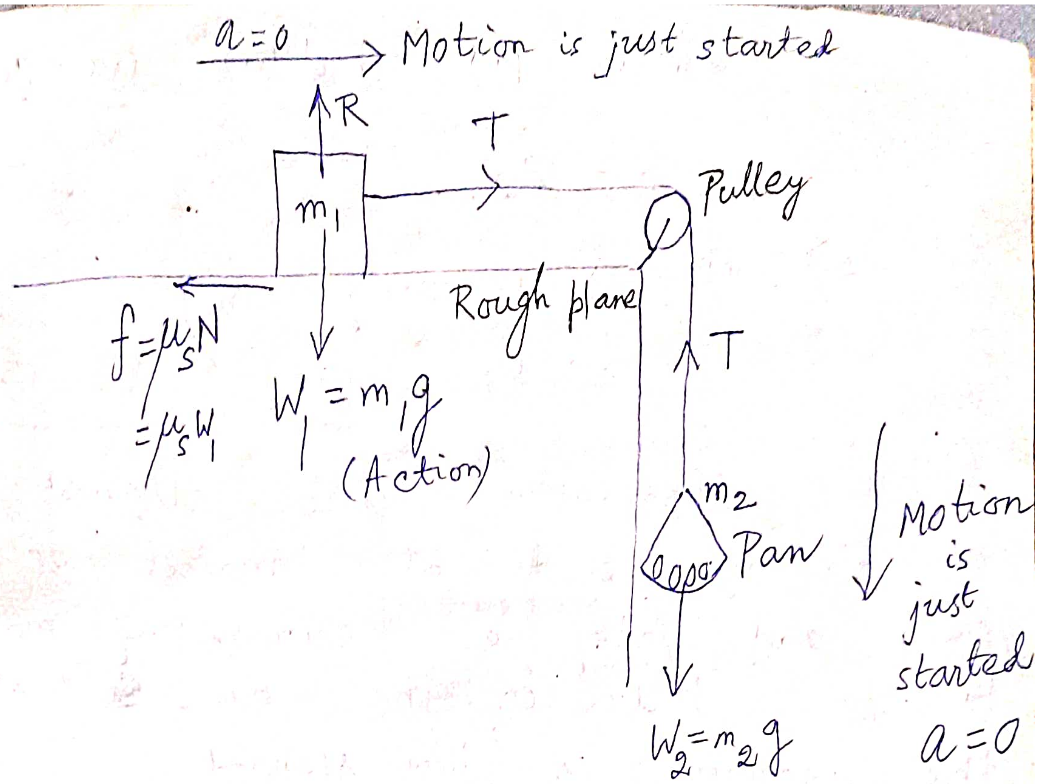
3^{rd} second $= \frac{mas}{t} = \frac{5 \times 5 \times (12.5)}{1}$

$= 312.5$ watt

— o —

~~Physics~~ ? Experimental determination of ~~coeffi~~ Co-efficient of static friction in the ~~labority~~ laboratory by horizontal plane method.

The experimental body is placed on the horizontal wooden plank which is connected with a string into a pan. The string passes over the pulley as shown in the figure. Gradually



masses are added to the pan till the body just starts to slide.

Under this condition, the acceleration of the body and pan are Zero. Equations of motion can be obtained from Newton's second law.

For the body of mass (m_1)

net force $F = T - f$

$\Rightarrow m_1 a = T - f$

$\Rightarrow m_1 . 0 = T - f$

$\Rightarrow 0 = T - f$

$\Rightarrow T = f$ ——— (i)

Because action and reaction make each other in-effective.

Net force on the pan

$= W_2 - T$

$= M_2 a$

$= m_2 . 0$

$= 0$

$\Rightarrow W_2 = T$ ——— (ii)

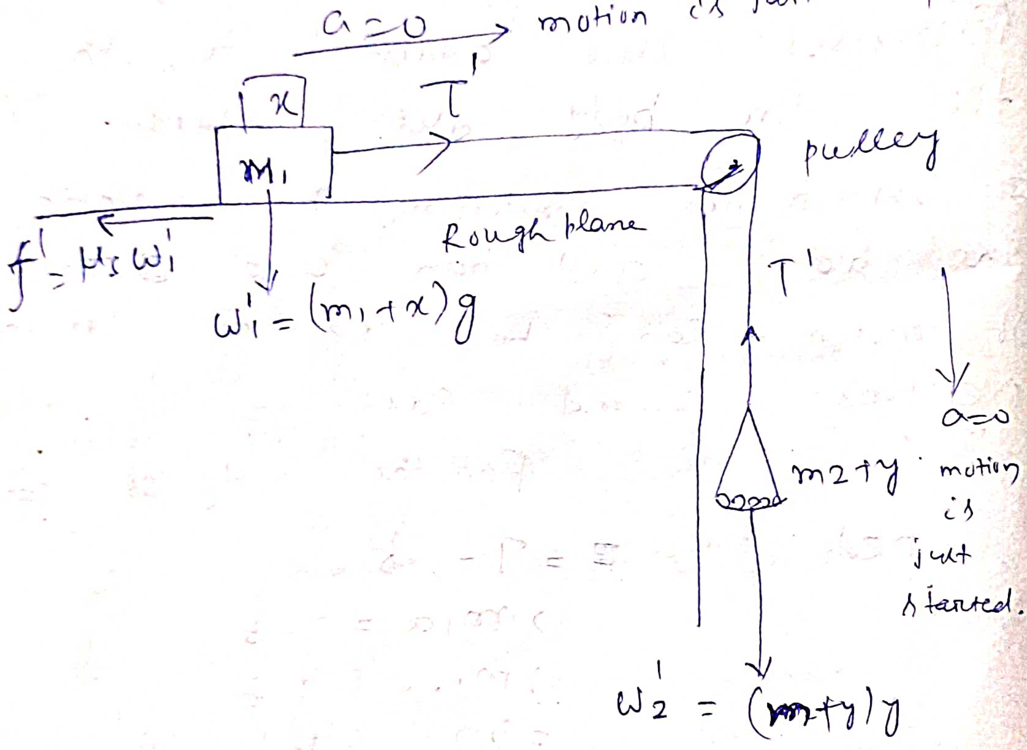
From eqⁿs (i) and (ii) we get

$$f = w_2$$

$$\Rightarrow \mu_s \cdot m_1 g = m_2 g$$

$$\Rightarrow \mu_s = \frac{m_2}{m_1} \quad \text{———— (iii)}$$

Other observations are obtained by putting additional masses on the body and finding the corresponding masses to be put on the pan. so that the motion is just started



Let x = mass put on the body
and y = mass put on the pan to just start the motion.

For the body (m_1+x) net force

$$F = T' - f'$$

$$\Rightarrow (m_1+x)\,a = T' - f'$$

$$\Rightarrow (m_1+x)\cdot 0 = T' - f'$$

$\Rightarrow T_d' = f'$ ———— (iv)

Because action and reaction make each other in-effective.

Net force on the pan

$$\cancel{(m_2+y)}$$
$$= W_2' - T'$$
$$= (m_2 y)\, a$$
$$= (m_2 y)\, 0$$
$$= 0$$

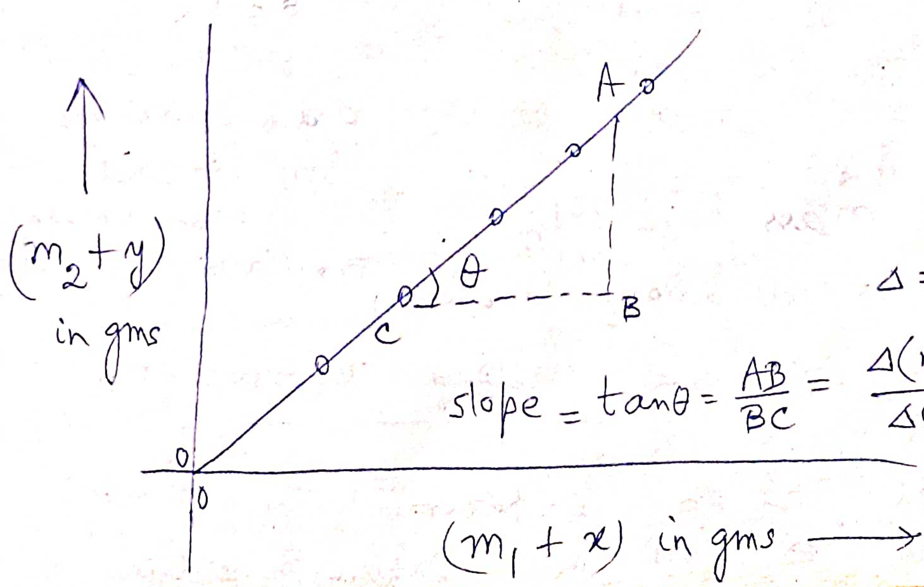
$\Rightarrow W_2' = T'$ ———— (v)

From eq^n (i) and (ii) we get

$$W_2' = f'$$

$$\cancel{\Rightarrow \mu_s (m}$$
$$\cancel{\Rightarrow (m_1+x) g = \mu_s W_1'}$$
$$\Rightarrow (m_2+y)\, g = \mu_s W_1' = \mu_s (m_1+x) g$$

$$\Rightarrow \mu_s = \frac{m_2+y}{m_1+x} \quad \text{———— (vi)}$$



Δ = change

$$\text{slope} = \tan\theta = \frac{AB}{BC} = \frac{\Delta(m_2+y)}{\Delta(m_1+x)} = \mu_s$$

A graph can be plotted between m_2+y along the Y axis and m_1+x along the X axis which comes out to be a straight line pass through the origin. Slope of the graph also gives the value of μ_s

Co-efficient of kinetic friction (μ_k)

When a body moves on a surface then the interlocking is usually small compared to the situation when a body was at rest.

Therefore
$$\mu_k < \mu_s$$

Equation of motion of a moving body is $F - f = ma$ (where F = applied external force on the body, f = frictional force produced $= \mu_k . W$ for plane surface and $f = \mu_k W \cos\theta$ for inclined plane surface

m = mass of the body

a = acceleration produced.)

Co-efficient of rolling friction (μ_r)

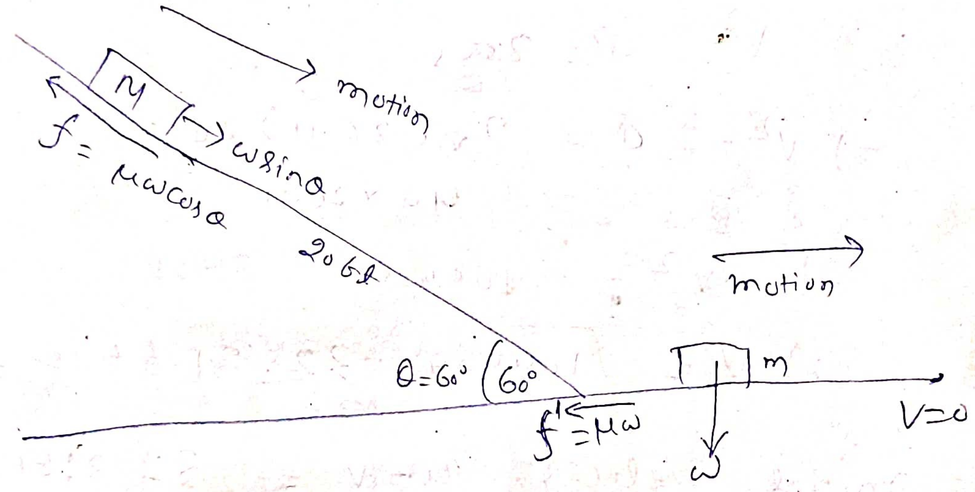
It has been experimentaly found that it is always easier to roll a body ~~and~~ than to slide it.

Therefore $\boxed{\mu_r < \mu_k < \mu_s}$

This properiety has been used to make all kinds of vehicles used by human beings

$$\begin{array}{r} 1.732 \\ 16 \\ \hline 27.712 \end{array}$$

Q3. page (100)



weight = 100 lb

=) ma = 100 lb

=) m = $\frac{100}{32}$ ~~= ...~~.

Coefficient of kinetic friction μ_k = .001.

$w\sin\theta - f = F$

Net force $w\sin\theta - f = ma$

$f = \mu w\cos\theta$

$= \cancel{.001}\,.100 \times 100 \times \cos 60^\circ$

$= \frac{\cancel{100}\,10}{\cancel{1000}} \times \cancel{100} \times \frac{1}{2}$

$= 5$ lb

$=) \; 100 \cdot \sin 60^\circ - (.001) = \frac{100}{32} \cdot a$

$=) \; \cancel{100}^{50} \cdot \frac{\sqrt{3}}{\cancel{2}} - \frac{1}{1000} = \frac{100a}{32}$

$=) \; \frac{1000 \times 50\sqrt{3} - 1}{1000} = \frac{100a}{32}$

$=) \; a = \frac{32 \times (50000\sqrt{3} - 1)}{100000}$

$= \frac{\cancel{32}^{16} \times \cancel{50000}\sqrt{3}}{\cancel{100000}_{2}} - 32$

$= 16(1.732) - 32$

$\Rightarrow 100 \cdot \sin 60^\circ - \cancel{8}5 = \frac{100}{32} a$

$\Rightarrow \frac{\overset{50}{\cancel{100}} \sqrt{3}}{\cancel{2}} - \cancel{8}5 = \frac{100}{32} a$

$\Rightarrow \frac{\cancel{500\sqrt{3}} - 1}{10} = \frac{100 a}{32}$

$\Rightarrow \cancel{10000 a} = \frac{\cancel{32 \times 50 \sqrt{3} - 32}}{\cancel{1000}}$

$\Rightarrow 32(50\sqrt{3} - 5) = 100 a \Rightarrow a = \frac{32 \times 5(10\sqrt{3} - 1)}{100} = 26.11$ ft/sec²

$a = 26.11$ ft/sec²

$v^2 - u^2 = 2as$

$\Rightarrow v^2 - 0 = 2 \times (26.11)\, 20$

$= 40 \times 26.11$

$\Rightarrow v^2 = 1044.4$

$\Rightarrow v = \sqrt{1044.4} = 32.31$ ft/sec

Initial velocity $u = v = 32.31$ ft/sec,

$f' = \mu_k w$ $\quad V = 0$

$= .100 \times 100$

$= \frac{100}{1000} \times 100$

$= 10$ lb

$F = -f' = -10$ lb

$ma = f'$

$\Rightarrow a = \frac{m}{f'} = \frac{100}{32} \times \frac{-10}{1000} = \frac{-1000}{32} = -3.2$ ft/sec

$\Rightarrow a' = \frac{f'}{m} = \frac{-10}{\frac{100}{32}} = \frac{-10 \times 32}{100} = -3.2$ ft/sec

$V = u + at$

$\Rightarrow 0 = 32.31 + (-3.2) t$

$\Rightarrow -32.31 = -3.2t$

$\Rightarrow t = \dfrac{32.31}{3.20} = \dfrac{3231}{320} = 10.09\ sec$

$\cancel{S = ut + \frac{1}{2}at^2}$

$\cancel{= (32.31).(10.09) + \frac{1}{2}.(32.31).(10.09)^2}$

Body moves on the body

$\cancel{S = ut + \frac{1}{2}at^2}$

$\Rightarrow \cancel{S = (32.31)(10.09) + \frac{1}{2}.(-3.2).(10}$

$v^2 - u^2 = 2as$

$\Rightarrow 0 - (32.31)^2 = 2.(-3.2).S$

$\Rightarrow \cancel{-}1043.9361 = \cancel{-}6.4\ S$

$\Rightarrow \dfrac{1043.9361}{6.4} = S$

$\Rightarrow S = 163\ ft$

2. A body sliding on a rough inclined plane takes twice the time to cover the same type of frictionless inclined plane. If the angle of inclination of the inclined plane be 45° in both of the cases, calculate μ_k