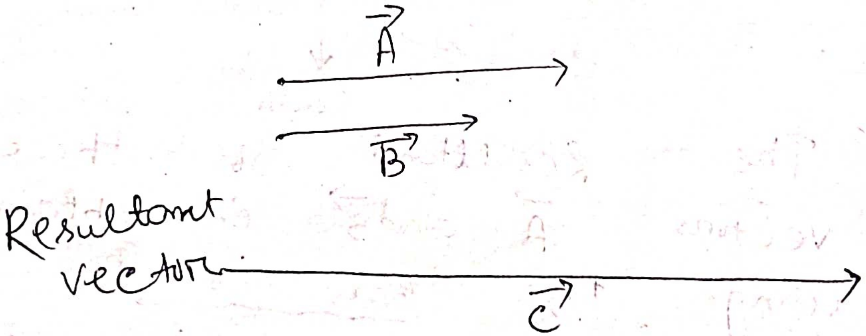
Scalar quantities : are those which have only magnitude.

Ex: Length, mass, time, speed, volume, density, temperature, pressure, work, energy, power, electric potential, Magnetic flux, Electric flux, Specific heat, Thermal conductivity.

Vector quantities are those which have magnitude as well as direction.

Ex: Displacement, Velocity, acceleration, Force, linear momentum, Angular momentum, Angular velocity, Angular acceleration, torque, electric field intensity, Magnetic induction.

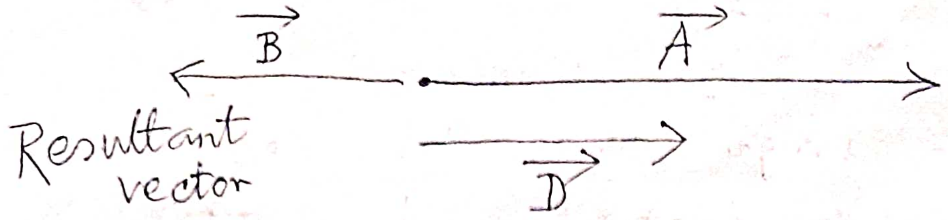
Addition of vectors - Vectors acting in the same direction



C = A+B

(i) The magnitude of the Resultant vector is equal to the sum of the magnitudes of the individual vectors and its direction is along any of the vectors.

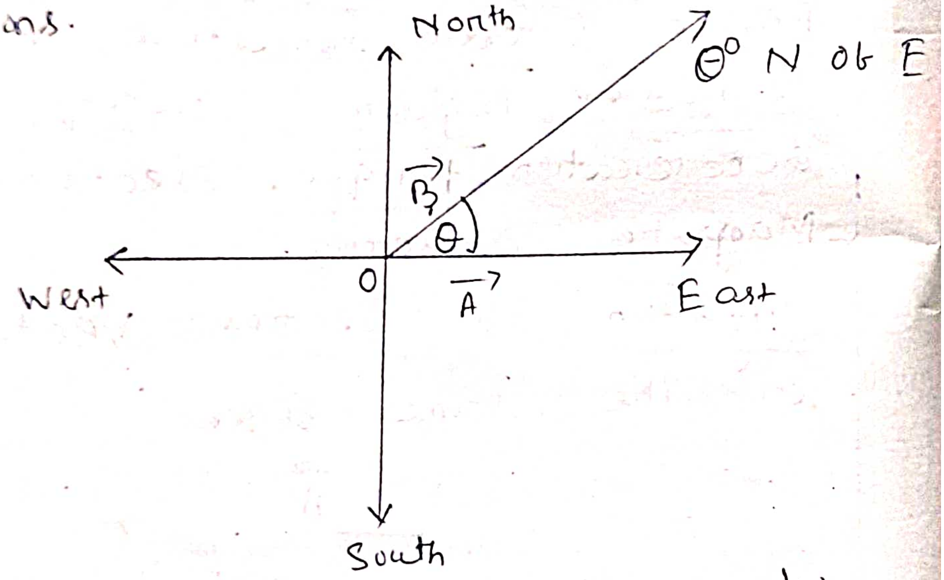
(ii) vectors acting in opposite directions-



$$D = A - B$$

The magnitude of the Resultant vector is equal to the difference of the magnitudes of the two vectors and acts along the bigger vector.

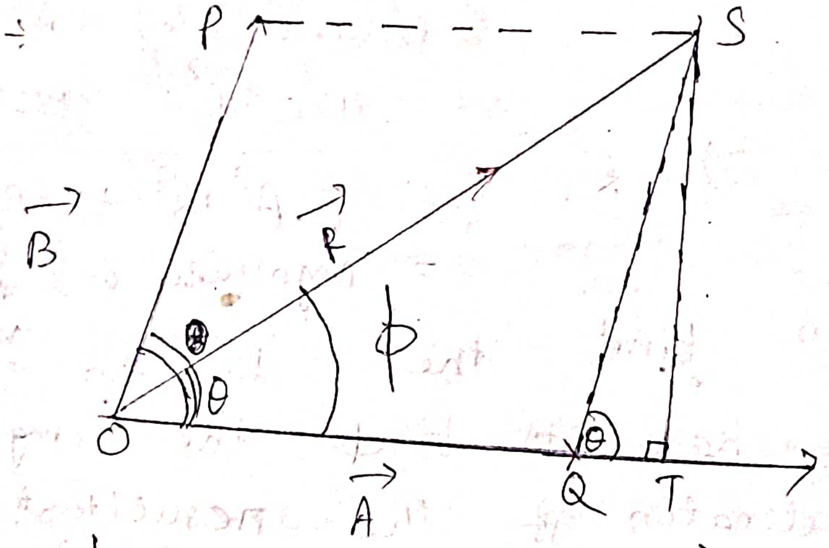
(iii) Vectors acting in arbitrary directions.



The Resulttant of these two vectors $\vec{A}$ and $\vec{B}$ is obtained by using the law of parallelogram of vectors.

Statement: If two vectors acting at a point be represented in magnitude ([illegible]) and direction by the two adjacent sides of a parallelogra two adjacent sides then the Resulttand is represented in ([illegible]) magnitude and direction by the diagonal of the parallelogram passing through that point.

Proof :-



The two vectors $\vec{A}$ and $\vec{B}$ acting at the point O having ~~Re~~ represented by the two ~~adjient~~ adjacent side $\overline{OQ}$ and $\vec{OP}$ respectively. The parallelogram is completed and a perpendicular $\overline{ST}$ be dropped from S on the extention of $\overline{OQ}$ SQT is a right angled triangle.

$$\therefore \quad \sin\theta = \frac{ST}{SQ} = \frac{ST}{B}$$

$$\Rightarrow \quad ST = B\sin\theta \quad \text{———} (i)$$

$$\cos\theta = \frac{QT}{SQ} = \frac{QT}{B}$$

$$\Rightarrow \quad QT = B\cos\theta \quad \text{———} (ii)$$

Now SOT is another right angled triangle. Using pythagoras theorem we get $OS^2 = ST^2 + OT^2$

$$\Rightarrow R^2 = (B\sin\theta)^2 + (OQ+QT)^2$$

$$\Rightarrow R^2 = B^2\sin^2\theta + (A+B\cos\theta)^2$$
$$= B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$
$$= B^2(\sin^2\theta + \cos^2\theta) + A^2 + 2AB\cos\theta$$
$$= B^2 + A^2 + 2AB\cos\theta \text{ —— (iii)}$$

$$\Rightarrow R = \sqrt{A^2+B^2+2AB\cos\theta} \text{ —— (iv)}$$

= Magnitude of Resultant vector.

To find the direction of $\vec{R}$ we have to find the angle of inclination of the resultant vector with the base vector ($\vec{A}$).

From the right angled triangle SOT we have $\tan\phi = \frac{ST}{OT} = \frac{ST}{OQ+QT}$
$$= \frac{B\sin\theta}{A+B\cos\theta}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right) \text{ —— ⑤}$$

Special cases.

(i) $\theta = 0$.

That is two vectors are acting along the same direction.

(ii) From the law of parallelogram vectors we can write

$$R = \sqrt{A^2+B^2+2AB\cos 0}$$
$$= \sqrt{A^2+B^2+2AB}$$
$$= \sqrt{(A+B)^2}$$
$$= A+B$$

= Maximum (because $\cos 0 = 1$ = maximum)

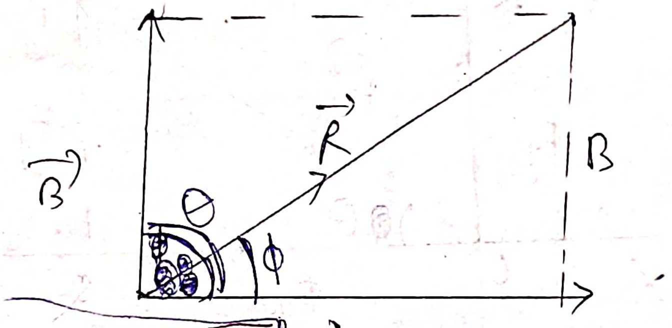
$$\tan\phi = \frac{B\sin 0^\circ}{A+B\cos 0^\circ} = \frac{B.0}{A+B} = \frac{0}{A+B} = 0$$

$$\therefore \phi = \tan^{-1}(0) = 0^\circ$$

Thus the resultant has the maximum magnitude A+B and its direction is along the base vector $\vec{A}$.

(ii) $\theta = 90^\circ$

That is the two vectors are at right angles to each other



Here $R = \sqrt{A^2+B^2}$ $\vec{A}$

$$\tan\phi = \frac{B}{A}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{B}{A}\right)$$

(iii) $\theta = 180^\circ$

That is the two vectors are acting at a point in opposite directions. From the law of parallelogram vectors we can write

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$= \sqrt{A^2+B^2+2AB.\cos 180^\circ}$$
$$= \sqrt{A^2+B^2+2AB(-1)}$$
$$= \sqrt{A^2+B^2-2AB}$$
$$= \sqrt{(A-B)^2}$$
$$= A-B$$

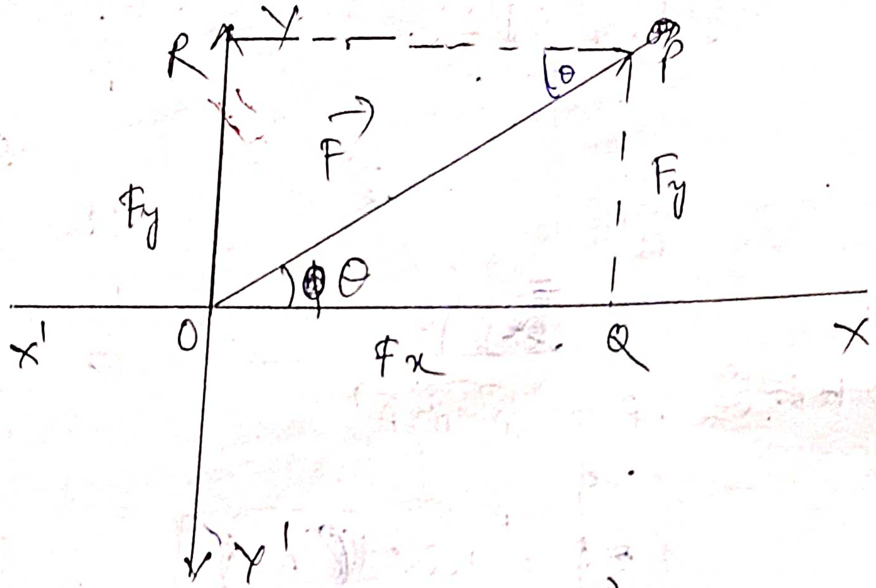
= Minimum (because cos 180° = -1 = minimum)

$$\tan\phi = \frac{B\sin 180^\circ}{A+B\cos 180^\circ} = \frac{B\cdot 0}{A+B(-1)} = \frac{0}{A-B} = 0$$

$$\therefore \phi = 0$$

Thus the resultant has minimum magnitude A−B and its direction is along the base vector ($\vec{A}$))

Resolution of a vector into two rectangular components :



Let us represent $\vec{F}$ in the $\overline{XY}$ plane start from the origin O. Let us dropp two perpendiculars, one on the X-axis and other on the Y axis. The projection on the X-Axis is called X-components (F_x). From the figure we see that $\cos\theta = \frac{OQ}{OP} = \frac{F_x}{F}$

$$\Rightarrow \boxed{F_x = F\cos\theta}$$

The projection on the Y axis is called Y-component of the vector (F_y).

⑨. From the figure we see that,

$$\sin\theta = \frac{PQ}{OP} = \frac{F_y}{F}$$

$$\Rightarrow \boxed{F_y = F\sin\theta}$$

Squaring and adding these two expressions we get

$$(F_x)^2 + (F_y)^2 = F^2$$

$$\Rightarrow F = \sqrt{(F_x)^2 + (F_y)^2}$$

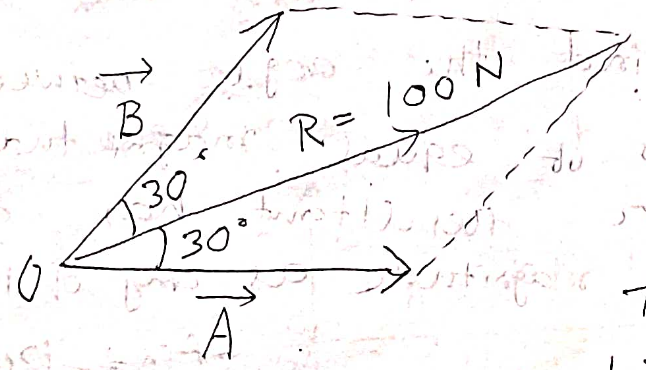
$$\tan\theta = \frac{PQ}{OQ} = \frac{F_y}{F_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Problems

④.

p.36



In the case of a rhombus, the diagonal bisects the angle

Hence the two components $\vec{B}$ & $\vec{A}$ will have the same magnitude

ie $B = A$.

Using the law of parallelogram of vectors, we have

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow (100N)^2 = A^2 + A^2 + 2AA\cos 60^\circ$$

$$= 2A^2 + \cancel{2}A^2 \cdot \frac{1}{\cancel{2}}$$

$$= 3A^2$$

$$\Rightarrow 100\,N = \sqrt{3}.A$$

$$\Rightarrow A = \frac{100\,N}{\sqrt{3}} = \frac{100\sqrt{3}}{3} = \frac{100\times 1.732}{3}$$

$$= \frac{173.2}{3} = 57.73\,N$$

Thus, each component has a magnitude 57.73 N.

4. Question Resolve a force of 100 N into two component which lie on opposite sides of the force and each of which makes an angle 30° with the force.

Imp Q : Find the angle between two vectors of equal magnitude when their resultant has also the same magnitude as any of them.
(Ans : 120°)

Soln Given $|\vec{R}| = |\vec{A}| = |\vec{B}| = A$ (Say)

From the law of parallelogram of vector we ~~get~~ have

$$R^2 = A^2 + B^2 + 2AB\cos\alpha$$

$$= A^2 + A^2 + 2A.A^2\cos\alpha$$

$$\Rightarrow A^2 = 2A^2 + 2A^2\cos\alpha$$

$\Rightarrow 2A^2 - A^2 = -2A^2\cos\theta$

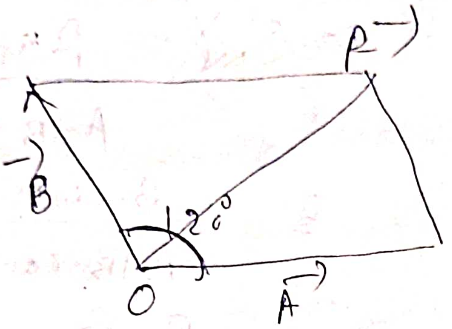
$\Rightarrow \cancel{A^2} = -2\cancel{A^2}\cos\theta$

$\Rightarrow -2\cos\theta = 1$

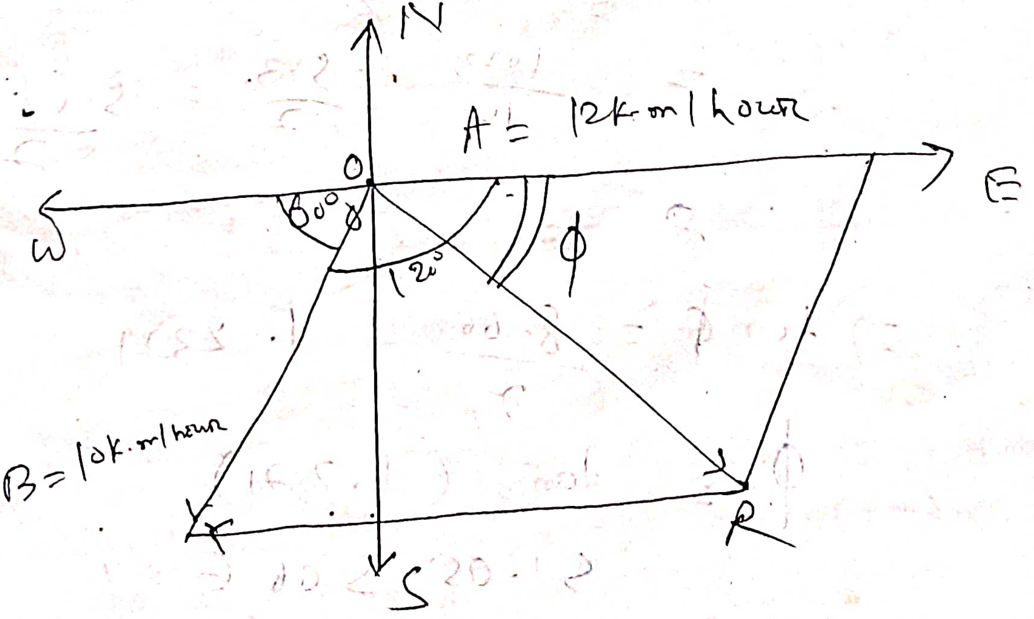
$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\theta = \cos 120^\circ$

$\Rightarrow \theta = 120^\circ$ (Ans)



20. A boat travels 10 k.m/hour in still water. It it is headed 60° S of W in a current that moves ~~east~~ at 12.00 km/hour due east. What is the resultant velocity of the ~~boa~~ boat? (Ans 11.1 km h at 51° S of E)



From the law of parallelogram of vector we have

$R^2 = A^2 + B^2 + 2AB\cos\theta$

⑫ ⇒ $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{(12)^2 + (10)^2 + 2(12).(10).(-\frac{1}{2})}$

$= \sqrt{144 + 100 + \overset{120}{\cancel{240}} . -\frac{1}{\cancel{2}}}$

$= \sqrt{244 - 120}$

$= \sqrt{124}$

$= 11.1$ k.m / hour.

$\tan\phi = \cancel{\tan^{-1}} \frac{B\sin\theta}{A + B\cos\theta}$

$= \frac{B.\sin 120^\circ}{A + B\cos 120^\circ}$

$= \frac{B.\frac{\sqrt3}{2}}{A + B(-\frac{1}{2})} = \cancel{\frac{\frac{B\sqrt3}{2}}{-\frac{A}{2}}}$

$= \cancel{\frac{10\sqrt3}{(+12) - (20)}} = \cancel{\frac{10\sqrt3}{-20}}$

$=$

$= \frac{\frac{B\sqrt3}{2}}{A - \frac{B}{2}} = \frac{\frac{B\sqrt3}{\cancel{2}}}{\frac{2A - B}{\cancel{2}}} = \frac{10\sqrt3}{24 - 10}$

$= \frac{10\sqrt3}{14} = \frac{5\sqrt3}{7} = \frac{5.(1.732)}{7}$

Average

1.2349 } diff .0044

tan 51° 1.2393

1.2321

1.2349

.0022

$\Rightarrow \tan\phi = \frac{8.66}{7} = 1.2371$

$\phi = \tan^{-1}(1.2371)$

$= 51.05°$ S of E

next One of the the rectangular components of force of 65 gwt is 25 gwt

⑬ Find the other component.

(Ans: 60 g wt)

Let us $F_x = 25$ g wt

$F = 65$ g wt

$$\Rightarrow \begin{bmatrix} F_x \\ 25 \end{bmatrix} = \begin{bmatrix} F \\ 65 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \cos\theta \end{bmatrix}$$

we know that $F^2 = (F_x)^2 + (F_y)^2$

$\Rightarrow (65\text{ gwt})^2 = (25)^2 + (F_y)^2$

$\Rightarrow (65)^2\text{gwt}^2 - (25)^2\text{gwt} = F_y^2$

$\Rightarrow (65+25)(65-25) = F_y^2$

$\Rightarrow 90 \cdot 40 = F_y^2$

$\Rightarrow 3600 = (F_y)^2$

$\Rightarrow F_y = 60$ gwt.

* Task

What is the angle between two vectors $(P+Q)$ & $(P-Q)$ so that their resultant is $\sqrt{3P^2+Q^2}$

(Ans $- 60^\circ$)

From the law of parallelogram of vector we have

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow 3P^2+Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q).(P-Q)$

$\Rightarrow 3P^2+Q^2 = 2(P^2+Q^2) + 2(P^2-Q^2)\cos\theta\ \cos\theta$

$= 2P^2+2Q^2+2P^2\cos\theta - 2Q^2\cos\theta$

$\Rightarrow 3P^2+Q^2 - 2P^2 - 2Q^2 = 2\cos\theta(P^2-Q^2)$

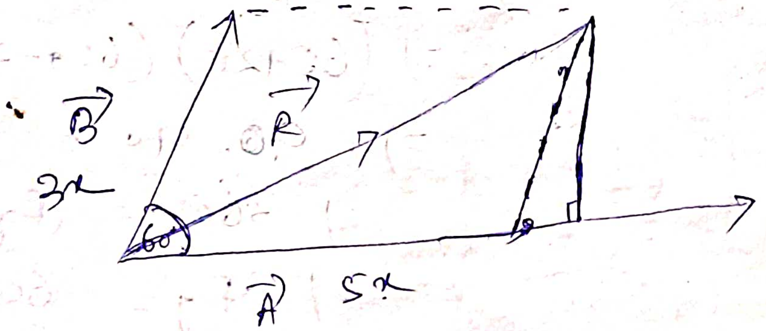
$$\Rightarrow \cancel{P^2 - Q^2} = 2\cos\theta(\cancel{P^2 - Q^2})$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

Tue → 7.00 P.M

8. Two forces whose magnitudes are in the ratio 3:5 give a resultant equal to 35N. If the angle between them is 60°. Find the magnitude of each force (Ans:- 15N, 25N)



Let The forces whose magnitudes are in the ration 3:5 be $\vec{B}$ and $\vec{A}$ and $\vec{A} = 5x$, $\vec{B} = 3x$

According to ~~the~~ parallelogram law of vectors

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow (35)^2 = (5x)^2 + (3x)^2 + 2\cdot(5x)\cdot(3x)\cdot\cos 60^\circ$$

$$\Rightarrow 1225 = 25x^2 + 9x^2 + \cancel{30}^{15} x^2 \cdot \frac{1}{\cancel{2}}$$

$$= 49x^2$$

$$\Rightarrow x^2 = \frac{1225}{49} = 25$$

$$\Rightarrow x = 5$$

$$\Rightarrow 3x = 3.5 = 15N$$

$$5x = 5.5 = 25N$$

∴ So the magnitude of each force = 15N and 25N,

9, The greatest and least resulttant of two forces acting at a point are 29 k.g wt and 5 kg wt respectively If each force is increased by 3 k.g wt. find the resulttant of two forces when acting at angle 90° with each other. (Ans: 25 k.g wt)

A→ The resulttant of two force acting at a point is maximum when angle between them 0°,

Two forces are $\vec{A}$ and $\vec{B}$,

According to parallelogram law of vectors

$$R^2 = A^2 + B^2 + 2AB.\cos\theta$$

$$\Rightarrow R^2 = A^2 + B^2 + 2.AB.\cos 0^\circ$$

$$= A^2 + B^2 + 2AB.1$$

$$= A^2 + B^2 + 2AB$$

$$\Rightarrow R = \sqrt{(A+B)^2}$$

$$= A+B,$$

$$\Rightarrow 29 = A+B \longrightarrow (i)$$

The resulttant of two forces acting at point is minium when angle between them 180°. According to parallelogram law of vectors

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos 180^\circ}$$

⑯

$$\Rightarrow R = \sqrt{A^2+B^2+2AB\cos(180^\circ)}$$

$$= \sqrt{A^2+B^2-2AB}$$

$$\Rightarrow R = \sqrt{(A-B)^2} = A-B$$

$$\Rightarrow 5 = A-B \text{ ——— } (ii)$$

$$\begin{array}{r} A+B = 29 \\ A-B = 5 \\ (-)\;(+)\quad(-) \\ \hline 2B = 24 \end{array} \quad \Rightarrow \vec{B} = \frac{24}{2} = 12$$

Subtracting eqⁿ (ii) from (i)

$$A+B = 29$$

$$\Rightarrow A+12 = 29$$

$$\Rightarrow A = 17$$

A 17 k.g weight, and B = 12k.g wet. There resultant is increased 3k.g wet. So $\vec{A}$ = 20k.g, B = 15k.g, When $\vec{A}$ and $\vec{B}$ forces are acting 90° with each other then according the parallelogram law of vector

$$R = \sqrt{A^2+B^2}$$

$$= \sqrt{(20)^2+(15)^2}$$

$$= \sqrt{400+225}$$

$$= \sqrt{625}$$

$$= 25 \text{ kg wt.}$$

∴ The resultant of two forces when acting at an angle 90° is equal to 25 k.g wt.

— o —

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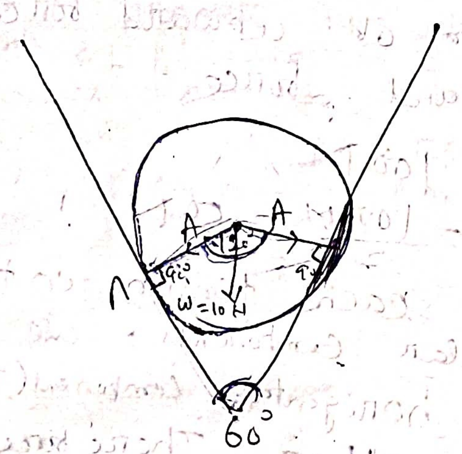
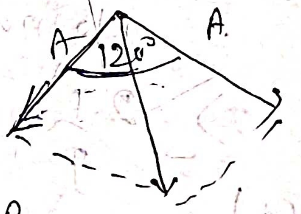
18. An iron sphere weighs 10 N and rests in a V shaked through whose sides from an angle of 60°. What is the normal force exerted by ~~by~~ sphere on each side of the ~~tro~~ trough?

(Ans: 10 N)

Ans:



Here A = B due to semetry.



R = W = 10 N

From the law of parallelogram vectors we have

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow (10N)^2 = A^2 + A^2 + 2AA \cdot \cos 120^\circ$$

$$\Rightarrow 100N^2 = A^2 + A^2 + \cancel{2}A^2 \cdot -\frac{1}{\cancel{2}}$$

$$= A^2 + \cancel{A^2} - \cancel{A^2}$$

$$= A^2$$

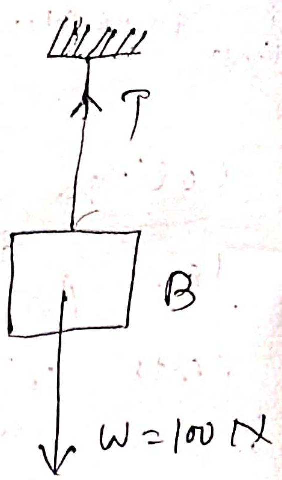
$$\Rightarrow 100N^2 = A^2$$

$$\Rightarrow A = 10N \quad \text{(Ans)}$$

∴ Force is 10 N.

19.

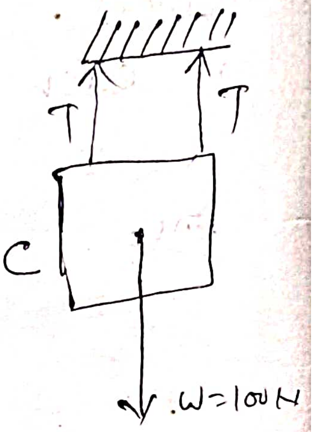
1.12.(B) Tension is a force that acts towards support through the string.



(সাম্য)
For equilibrium, net force on the body must be zero. Hence, upward force = downward force

$\Rightarrow T = W = 100\,N$

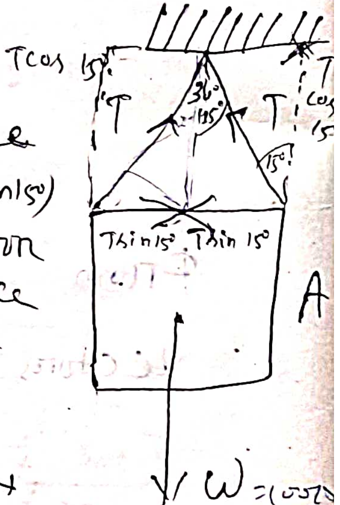
(C) For equilibrium, net force on the body must be zero. Hence sum of upward force = downward force



$\Rightarrow 2T = 100\,N$

$\Rightarrow T = \frac{100\,N}{2} = 50\,N$

(A) Resolving each tension into two rectangular components, we see that the horizontal component (T sin15°) cancel each other. Therefore for equilibrium, the net upward force must be equal to the net downward force.

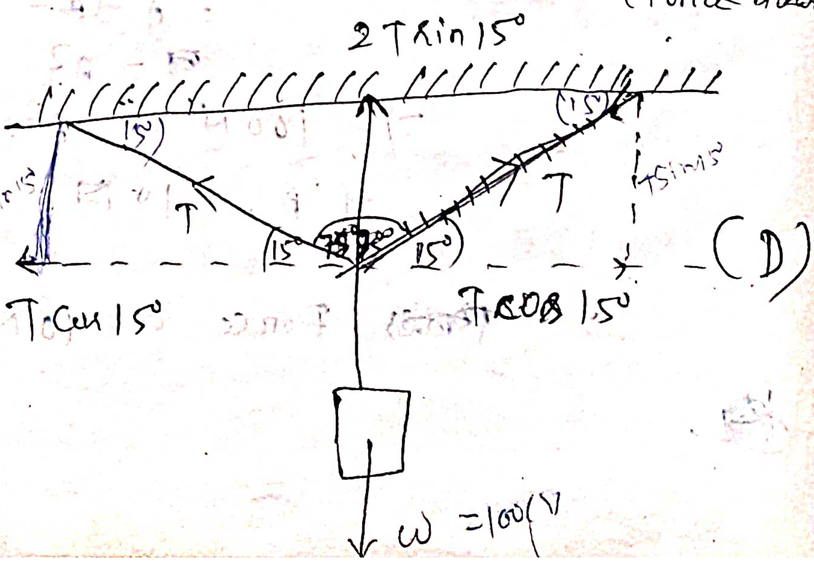


Then $2T \cos 15° = W$

$\Rightarrow 2T \cdot (0.9659) = 100\,N$

$\Rightarrow T = \frac{50}{.9659} = 51.8\,N$

Tension
(Force towards



(D) Resolving each tension into two rectangular components, we see that the horizontal component $(T\cos 15^\circ)$ cancel each other. Therefore for equilibrium the net upward force must be equal to the net downward force

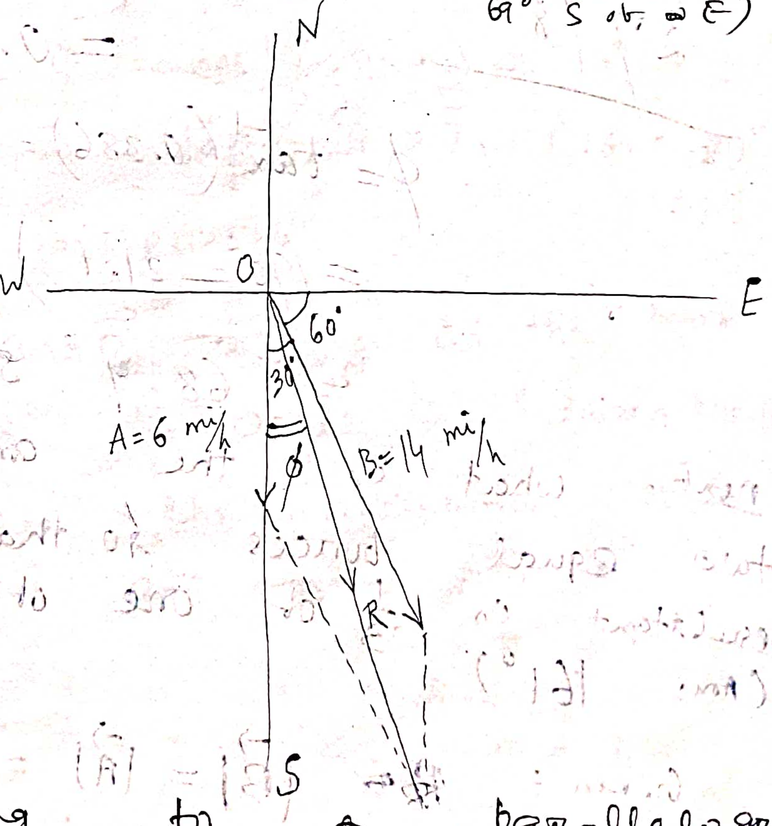
Then $2T\sin 15^\circ = 100\ N$

$\Rightarrow T\sin 15^\circ = 50\ N$

$\Rightarrow T(0.2588) = 50\ N$

$\Rightarrow T = \dfrac{50\ N}{.2588} = 193\ N$

Q:- A boat travels 14 mile/hour in still water. If it is headed 60° S of E in a current which is directed due south at 6 mile/hour, what is the resultant velocity of the boat? (Ans: 19.4 mile/hour, at 69° S of E)



According to parallelogram law of vectors

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(6)^2 + (14)^2 + 2(6)(14)\cos 30^\circ}$$

$$= \sqrt{36 + 196 + \cancel{2}(84)\sqrt{3}}$$

$$= \sqrt{232 + 84(1.732)}$$

= 19.4 mile/hour.

⑳ The direction of the resulttant vector is obtained from the angle ϕ, $\tan\phi = \frac{B\sin\theta}{A+B\cos\theta}$

$$\Rightarrow \tan\phi = \frac{14\sin 30^\circ}{6+14\cos 30^\circ}$$

$$= \frac{14\cdot\frac{1}{2}}{6+\overset{7}{\cancel{14}}\cdot\frac{\sqrt{3}}{\cancel{2}}} = \frac{7}{6+7\cdot\sqrt{3}}$$

$$= \frac{7}{6+7\cdot(1.732)}$$

$$\Rightarrow \tan\phi = \frac{7}{18.124} = \frac{\cancel{7000}}{\cancel{18124}}$$

1750
~~3500~~
~~7000~~
~~18124~~
~~9062~~
4531

4531) 17500 (0.386
13593
39070
36248
28220
27186

$$= 0.386$$

$$\phi = \tan^{-1}(0.386) = 21.1^\circ \text{ E of S}$$

$$= (90^\circ - 21.1^\circ) \text{ S of E}$$

$$= 68.9^\circ \text{ S of E}$$

next: what is the angle between two equal forces so that the resulttant is $\frac{1}{3}$ of one of the forces (Ans 161°)

Given: $|\vec{B}| = |\vec{A}| = A$ (Say)

$$|\vec{R}| = \frac{A}{3}$$

Using the parallelogram law of vector we get

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$

㉑ $\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \left(\frac{A}{3}\right)^2 = A^2 + A^2 + 2A\cdot A\cdot\cos\theta$

$= 2A^2 + 2A^2\cos\theta$

$\Rightarrow \frac{A^2}{9} - 2A^2 = 2A^2\cos\theta$

$\Rightarrow \frac{A^2 - 18A^2}{9} = 2A^2\cos\theta$

$\Rightarrow -\frac{17A^2}{9} = 2A^2\cos\theta$

$\Rightarrow -17\cancel{A^2} = 18\cancel{A^2}\cos\theta$

$\Rightarrow \cos\theta = \frac{-17}{18} = -0.9444$

Let $\theta = 180° - \phi$.

$\Rightarrow \cos\theta = \cos(180° - \phi)$

$\Rightarrow -0.9444 = -\cos\phi$

$\Rightarrow \cos\phi = 0.9444$

$\therefore \phi = \cos^{-1}(.9444) = 19°.2°$

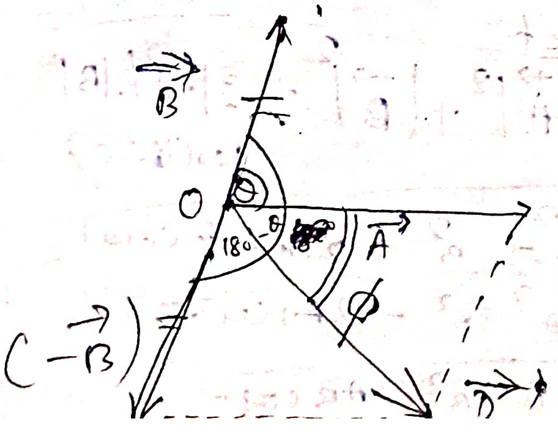
$\therefore \theta = (180° - 19.2°) = (160°.8)°$

Vector difference

Let $\vec{A}$ and $\vec{B}$ acting at the point O making an angle θ between them. Let's call the difference of these two vectors as $\vec{D} = \vec{A} - \vec{B}$

$= \vec{A} + (-\vec{B})$

= Addition of $\vec{A}$ and reversed $\vec{B}$.



projectile
g. m. [illegible]

(22) Using the law of parallelogram of vectors, we get

$$|\vec{B}| = \sqrt{|\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}|.|-\vec{B}|\cos(180^\circ-\theta)}$$

$$\Rightarrow D = \sqrt{A^2 + B^2 + 2A.B.(-\cos\theta)}$$

$$\Rightarrow D = \sqrt{A^2 + B^2 - 2AB\cos\theta} \quad \text{—(1)}$$

$$\tan\phi = \frac{|-\vec{B}|\sin(180^\circ-\theta)}{|\vec{A}| + |-\vec{B}|\cos(180^\circ-\theta)}$$

$$= \frac{B.\sin\theta}{A+B(-\cos\theta)}$$

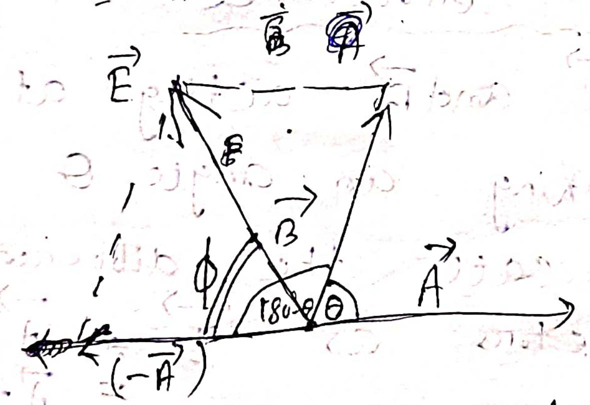
$$= \frac{B\sin\theta}{A-B\cos\theta}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{B.\sin\theta}{A-B\cos\theta}\right) \quad \text{—(2)}$$

If $\vec{E} = \vec{B} - \vec{A}$

$= \vec{B} + (-\vec{A})$

$=$ Addition of $\vec{B}$ and reversed $\vec{A}$



From the law of parallelogram vectors we get

$$|\vec{E}| = \sqrt{|-\vec{A}|^2 + |\vec{B}|^2 + 2|-\vec{A}|.|\vec{B}|\cos(180^\circ-\theta)}$$

$$= \sqrt{A^2 + B^2 + 2AB(-\cos\theta)}$$

$$= \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\Rightarrow E = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\phi = \frac{|\vec{B}|\sin(180-\theta)}{|\vec{A}|+|\vec{B}|\cos(180-\theta)}$$

$$= \frac{B\sin\theta}{A+B(-\cos\theta)}$$

$$= \frac{B\sin\theta}{A-B\cos\theta} = \frac{B\sin\theta}{A-B\cos\theta}$$

~~$\phi = \tan^{-1}(0) = 0$~~

$$\phi = \tan^{-1}\left(\frac{B\sin\theta}{A-B\cos\theta}\right) \text{ ——— } ④$$

Now $\vec{D} = \vec{A} - \vec{B}$

$= -(\vec{B} - \vec{A})$

$= -(\vec{E})$

This shows that $\vec{D}$ and $\vec{E}$ have the same magnitude but the directions are opposite.

Special Cases ~~No 1. θ = 0°~~

① $\theta = 0°$

That is $\vec{A}$ and $\vec{B}$ are acting the same direction.

$$D = \sqrt{A^2+B^2-2AB\cos\theta}$$

$$= \sqrt{A^2+B^2-2A\cdot B(\cos 0°)}$$

$$= \sqrt{A^2+B^2-2AB(1)}$$

$$= \sqrt{A^2+B^2-2AB}$$

$$= \sqrt{(A-B)^2}$$

$$= A-B,$$

= Difference of the magnitudes of two vectors.

$$\tan\phi = \frac{B\sin\theta}{A-B\cos\theta}$$

$$= \frac{B \sin 0^\circ}{A - B \cos 0^\circ} = \frac{0}{A-B} = 0$$

$$\Rightarrow \phi = 0^\circ$$

(ii) $\theta = 90^\circ$

That is $\vec{A}$ and $\vec{B}$ are acting at ~~same~~ right angles ~~direction~~ to each other

$$D = \sqrt{A^2 + B^2 - 2AB\cos\alpha}$$
$$= \sqrt{A^2 + B^2 - 2 \cdot AB \cdot \cos 90^\circ}$$
$$= \sqrt{A^2 + B^2 - 2A \cdot B \cdot 0}$$
$$= \sqrt{A^2 + B^2}$$

$$\tan\phi = \frac{B \sin\alpha}{A - B\cos\alpha}$$
$$= \frac{B \sin 90^\circ}{A - B\cos 90^\circ} = \frac{B \cdot 1}{A - B \cdot 0}$$
$$= \frac{B}{A}$$

$$\Rightarrow \phi = \tan^{-1}\frac{B}{A}$$

(iii) $\theta = 180^\circ$,

That is $\vec{A}$ and $\vec{B}$ are acting opposite directions.

$$D = \sqrt{A^2 + B^2 - 2AB\cos\alpha}$$
$$= \sqrt{A^2 + B^2 - 2AB\cos(180^\circ)}$$
$$= \sqrt{A^2 + B^2 - 2AB(-1)}$$
$$= \sqrt{A^2 + B^2 + 2AB}$$
$$= \sqrt{(A+B)^2}$$
$= A + B$ (Sum of the magnitude of two vectors)

$$\tan\phi = \frac{B \sin\alpha}{A - B\cos\alpha}$$

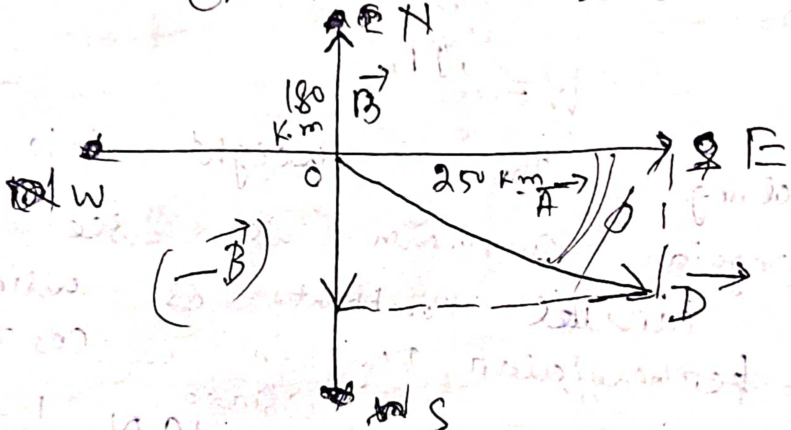
$$= \frac{B \sin(180^\circ)}{A - B\cos(180^\circ)} = \frac{B \cdot 0^\circ}{A - B\cancel{\cos}} = 0^\circ$$

$$\Rightarrow \quad \phi = 0^\circ$$

3 | 9 4 | 9 , (3.7
9
61) 4 49
4 27

Page. 36

No. 5. Two aeroplanes starts from the point, one travelling 250 k.m E and other 180 k.m N. What is the vector difference of these two placements? what does it signify?
(Ans: 308 km, 35°.8 S of E)



18
1·8
144
18
324

$$D = \sqrt{A^2 + B^2}$$
$$= \sqrt{(250)^2 + (180)^2}$$
$$= \sqrt{62500 + 32400}$$
$$= \sqrt{94900}$$
$$\Rightarrow D = 308 \text{ km}$$

$$\tan\phi = \frac{B}{A} = \frac{180}{250} = \frac{18}{25} = \frac{72}{100} = 0.72$$

$$\Rightarrow \phi = \tan^{-1}(.72)$$
$$= 35^\circ.8^\circ \text{ S of E}$$

Thu : 7

College physics

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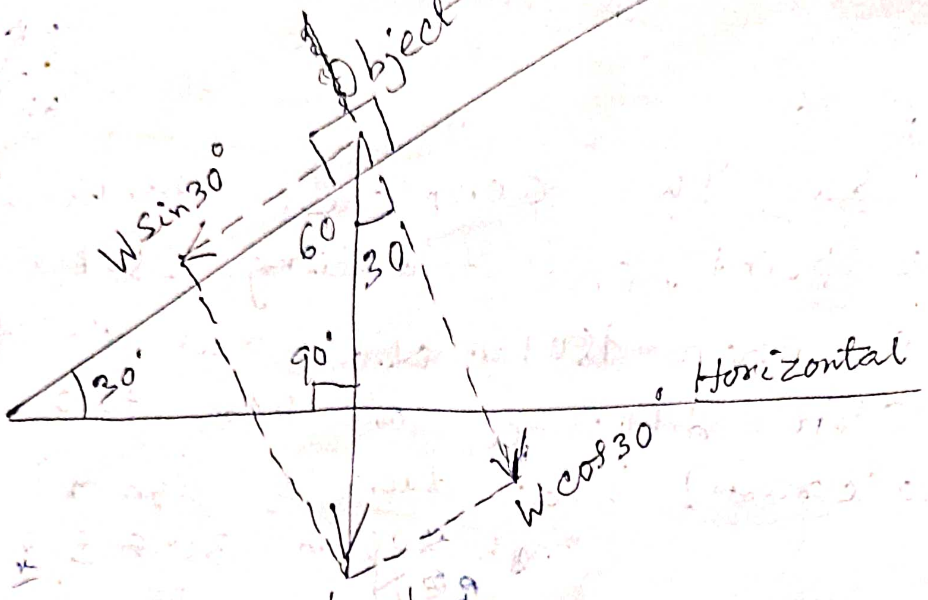
(26)



$W = 40N$

Resolving the weight into two rectangular components, we see that the parallel component is $w\sin 30^\circ$ and the perpendicular " is $w \cos 30^\circ$

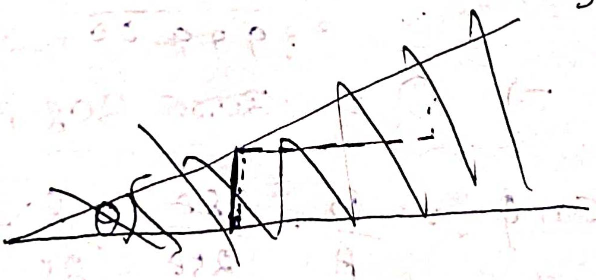
∴ parallel component $= w\sin30^\circ = 40N \cdot \frac{1}{2} = 20 N$

perpendicular " $= 40 N \cdot \frac{\sqrt{3}}{\cancel{2}} = 20\sqrt{3}$

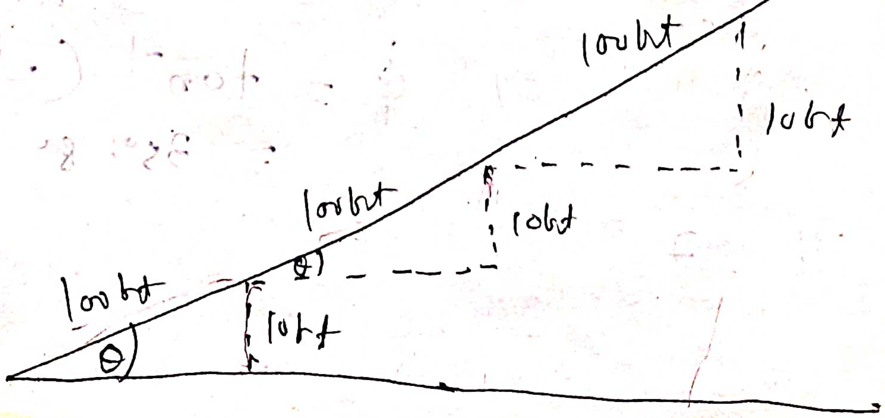
$= 20(1.732)$

$= 34.64 N$

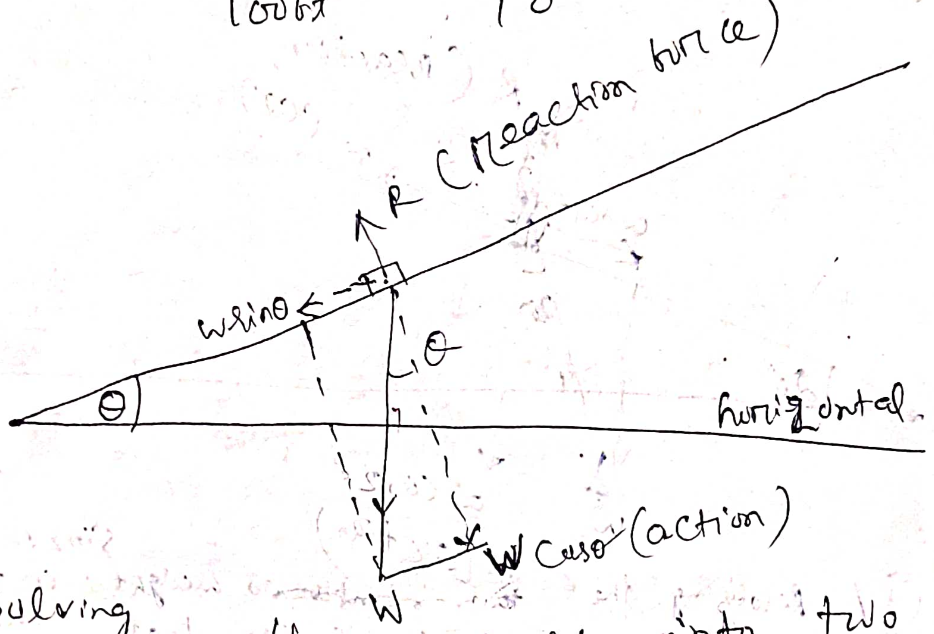
II.



4.



$$\sin\theta = \frac{10ft}{100ft} = \frac{1}{10}$$



Resolving the weight into two ~~rea~~ rectangular components, One being parallel to the inclined plane and the other perpendicular to the incline plane; we see that w cos θ acts like action which produces an equal and opposite reaction (R), from Newtons third law. we know that action and reaction are equal and opposite.

Therefore the only effective force acting on the car is w sin θ, down the incline plane = 3,200 lb × $\frac{1}{10}$

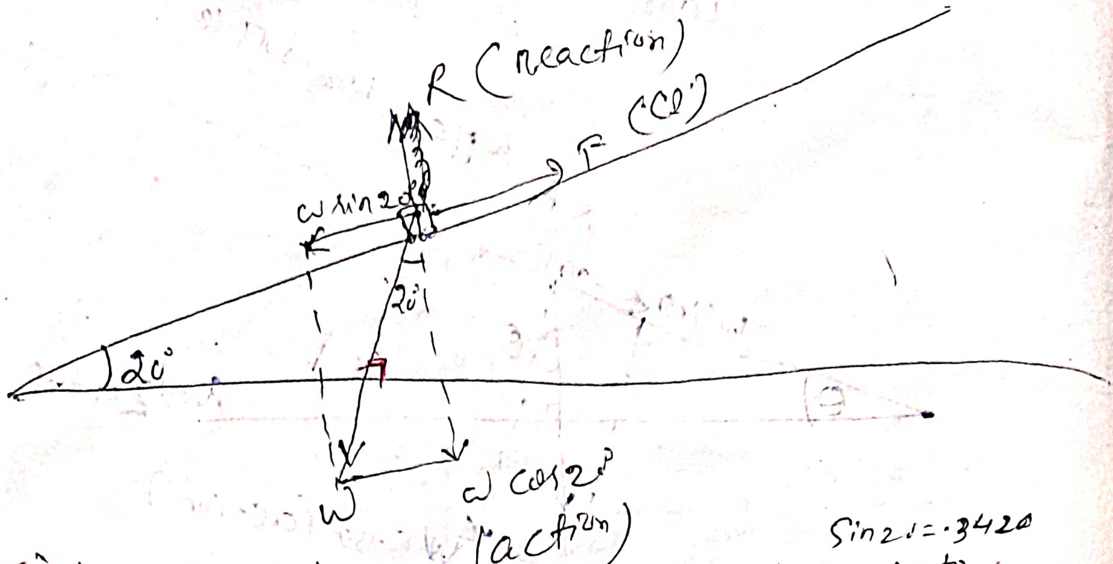
= 320lb.

2: Find the force in a cable necessary just to keep in equilibrium a three thousand lb car on a 20° incline. (a) when the cable is parallel to the incline

(b) when the cable makes an angle of 50° above the horizontal.

(Ans : 1026 lb , 1184·7 lb)

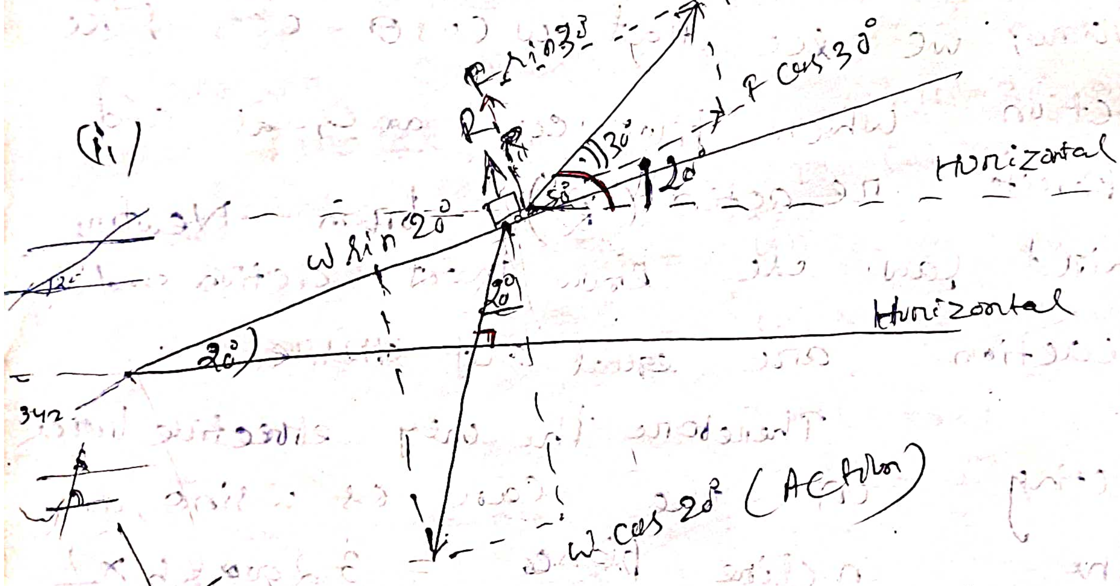
(1) ㉘



Sin 20 = .3420

(i/ Resolving the ~~factor components~~ weight in to two rectangular components, we see that the vertical component acts like action which produces an equal and opposite reaction. Hence the force with which the car goes down the inclined plance = w sin 20° ± 3,000 × .342 = 1026 lb

(ii/



(ii/ Resolving the applied force into two rectangular components, we see that $w \sin 20° = F \cos 30°$

$\Rightarrow 1026\ lb = F \cdot \frac{\sqrt{3}}{2}$

$\Rightarrow F = \frac{1026 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$ $= \frac{\overset{342}{\cancel{1026}} \times 2 \times \sqrt{3}}{\cancel{3}}$

$= 684 \times (1.732)$

$= 1184.688$

$= 1184.7 lb$

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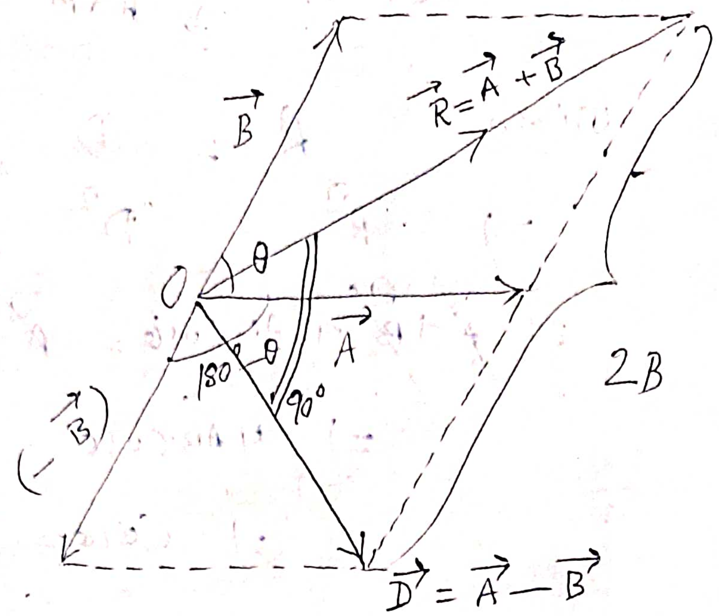
5.



From pythagoras therem, we have

$$R^2 + D^2 = (2B)^2$$

where R = Magnitude of the sum of the two vectors $= \sqrt{A^2+B^2+2AB\cos\theta}$

D = Magnitude of the difference of two vectors

$$= \sqrt{A^2+B^2-2AB\cos\theta}$$

Thus

$$A^2+B^2+\cancel{2AB\cos\theta} + A^2+B^2-\cancel{2AB\cos\theta} = 4B^2$$

$$\Rightarrow 2A^2+2B^2 = 4B^2$$

$$\Rightarrow \cancel{2}A^2 = \cancel{2}B^2$$

$$\Rightarrow A^2 = B^2$$

$$\Rightarrow A = B \text{ (proved)}$$

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6. Given

$$R = D$$

where R = Magnitude of the sum of the two vectors $= \sqrt{A^2+B^2+2AB\cos\theta}$

(30) D = Magnitude of difference of two vectors = $\sqrt{A^2+B^2-2AB\cos\alpha}$

Given : $R = D$

$\Rightarrow R^2 = D^2$

$\Rightarrow \cancel{A^2}+\cancel{B^2}+2AB\cos\theta = \cancel{A^2}+\cancel{B^2}-2AB\cos\alpha$

$\Rightarrow 4AB\cos\theta = \cancel{0}\ 0$

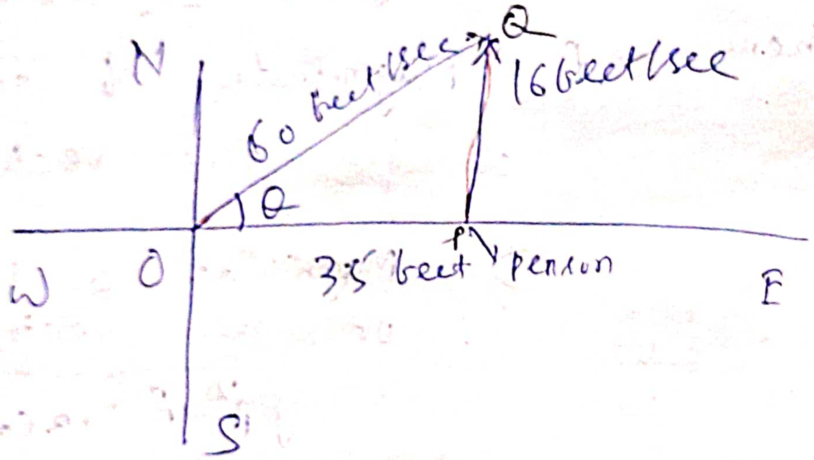
$\Rightarrow \cos\theta = \frac{0}{4AB} = 0$

$\Rightarrow \cos\theta = \cos 90°$

$\Rightarrow \theta = 90°$

38 page

22.



Let the person run for t second when he is hit by the ball

∴ PQ = distance covered by the person in t second = ~~16~~ speed. time

= 16 feet/sec . t sec

= 16 t feet

OQ = distance covered by the ball during t second = speed. time

= 60 ft/sec. t sec

= ~~60tft~~

= 60 t feet

Now $\sin\theta = \frac{PQ}{OQ}$

$= \frac{16t \text{ feet}}{60t \text{ feet}}$

$= \frac{4}{15}$

$= 0{\cdot}2666$

$\therefore \theta = \sin^{-1}({\cdot}2666)$

$= 15.4^\circ$ N of E

$\tan\theta = \frac{PQ}{OP} = \frac{16t \text{ feet}}{35 \text{ feet}} = \frac{16t}{35}$

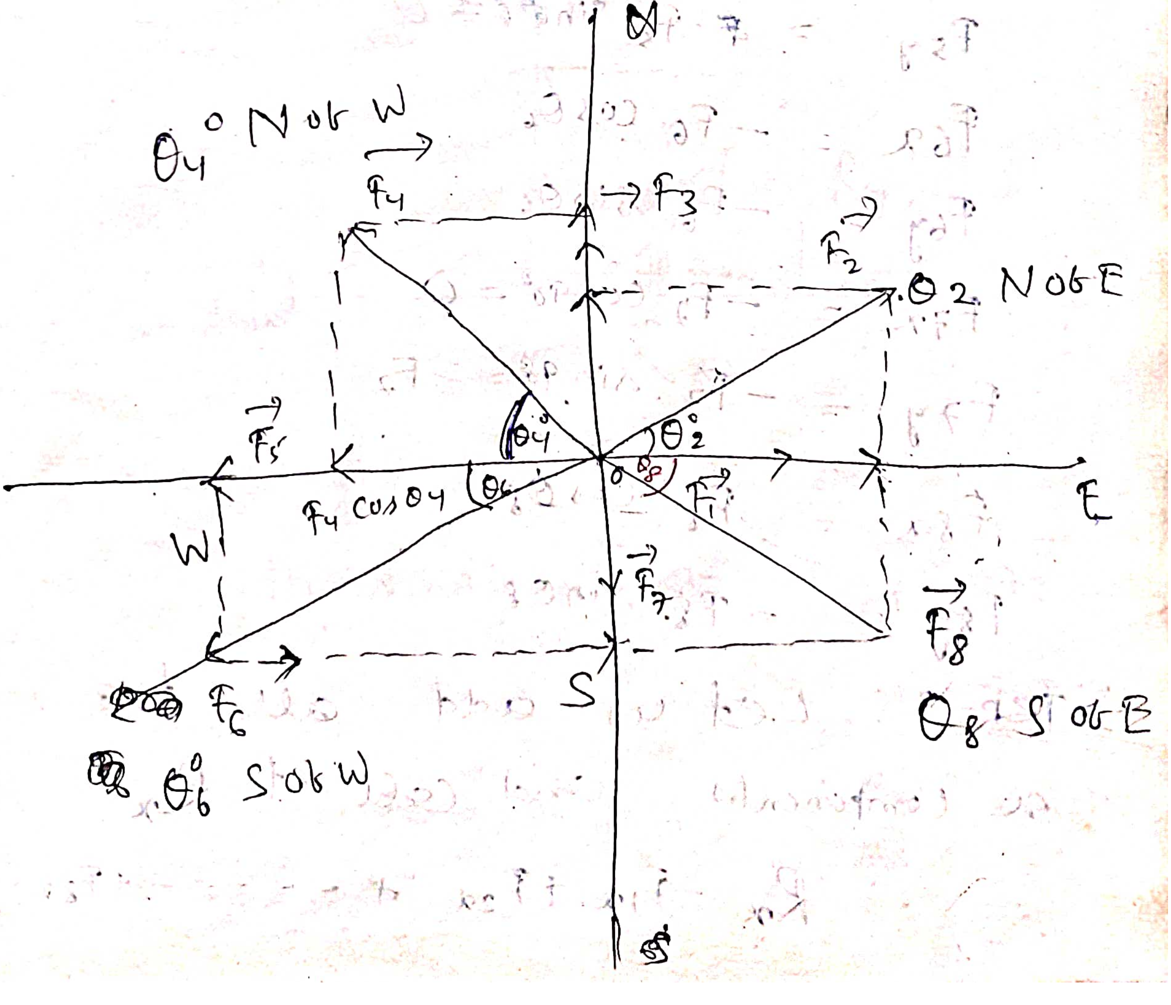
$\tan 15^\circ.4^\circ = 0.2754$ $= \frac{16t}{35}$

$\Rightarrow t = \frac{35 \times .2754}{16}$

$= .600372$ sec

$= .6$ sec

Algebraic method of adding many vectors acting at a point in a plane (Component method of adding vectors)



(32) **Step (1)** All the vectors be represented in a plane on a graph paper.

Step (2): Each vector be resolved into two rectangular components.

$$F_{1x} = F_1 \cos 0^\circ = F_1$$

$$F_{1y} = F_1 \sin 0^\circ = 0$$

$$F_{2x} = F_2 \cos \theta_2$$

$$F_{2y} = F_2 \sin \theta_2$$

$$F_{3x} = F_3 \cos 90^\circ = 0$$

$$F_{3y} = F_3 \sin 90^\circ = F_3$$

$$F_{4x} = -F_4 \cos \theta_4$$

$$F_{4y} = F_4 \sin \theta_4$$

$$F_{5x} = -F_5 \cos 0^\circ = -F_5$$

$$F_{5y} = F_5 \sin 0^\circ = 0$$

$$F_{6x} = -F_6 \cos \theta_6$$

$$F_{6y} = -F_6 \sin \theta_6$$

$$F_{7x} = -F_7 \cos 90^\circ = 0$$

$$F_{7y} = -F_7 \sin 90^\circ = -F_7$$

$$F_{8x} = F_8 \cos \theta_8$$

$$F_{8y} = -F_8 \sin \theta_8$$

Step-3: Let us add all the x components and call it R_x

$$\therefore R_x = F_{1x} + F_{2x} + \text{------} + F_{8x}$$

(33)

$$R_x = \sum_{i=1}^{8} F_{ix}$$

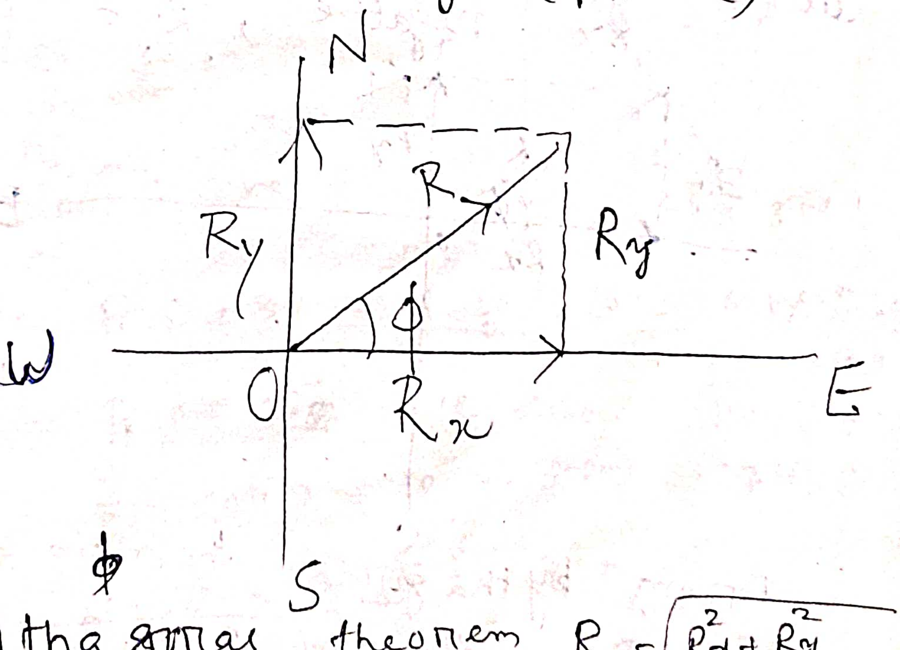
Let us add all the y- components and call it R_y

$$\therefore R_y = F_{1y} + F_{2y} + F_{3y} - - - - + F_{8y}$$

$$= \sum_{i=1}^{8} F_{iy}$$

STEP(4) :- Let us combine R_x and R_y vectorially to find the resultant ~~ϕ~~ vector. The following four cases arise

Case I ([illegible]) [illegible] = R_x (positive), R_y (positive)



From ~~ϕ~~

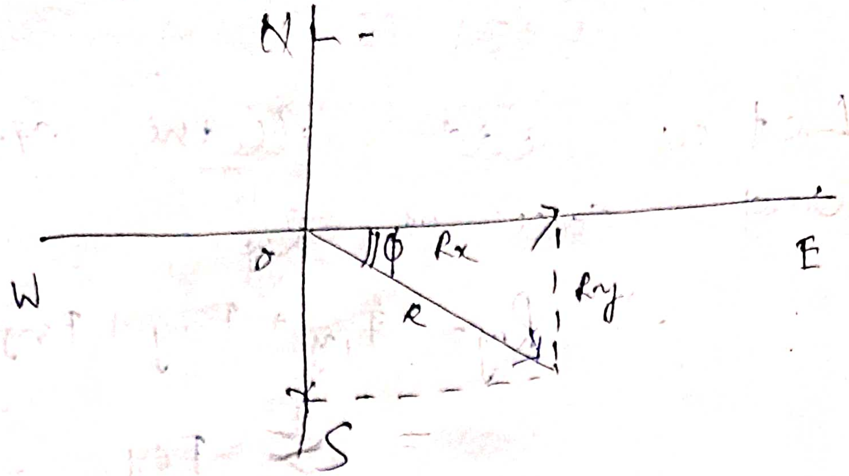
Pythagoras theorem $R = \sqrt{R_x^2 + R_y^2}$

$$\tan\phi = \frac{R_y}{R_x}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)^{\circ} \text{N of E}$$

Case (2) R_x = positive , R_y (Negative)

(34)



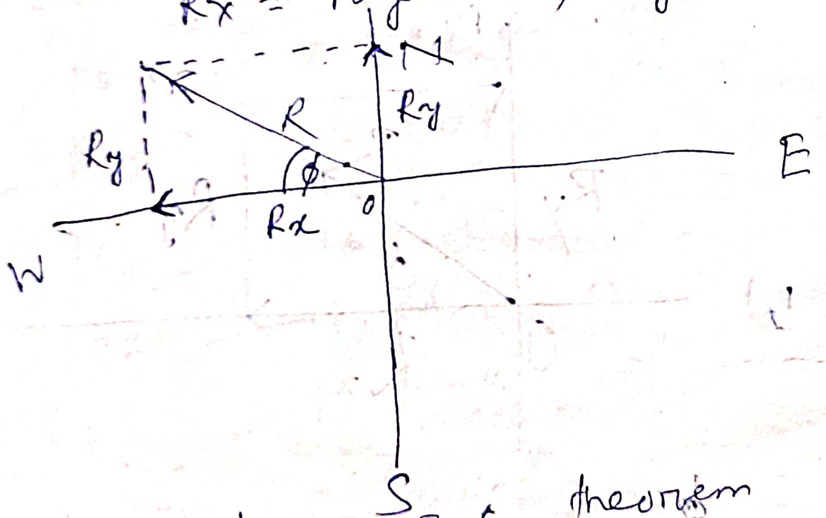
From pythagoras theorm

$$\therefore R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\phi = \frac{R_y}{R_x}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) \text{ S of E}$$

Case (3) R_x = negative , R_y = positive



From pythagoras theorem

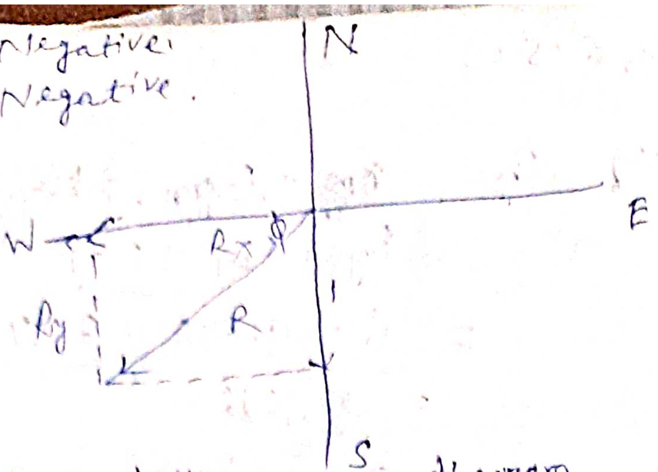
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\phi = \frac{R_y}{R_x}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) \text{. N of W}$$

Case-(4) R_x = Negative
R_y = Negative.

(35)



From the pythagoras theorem

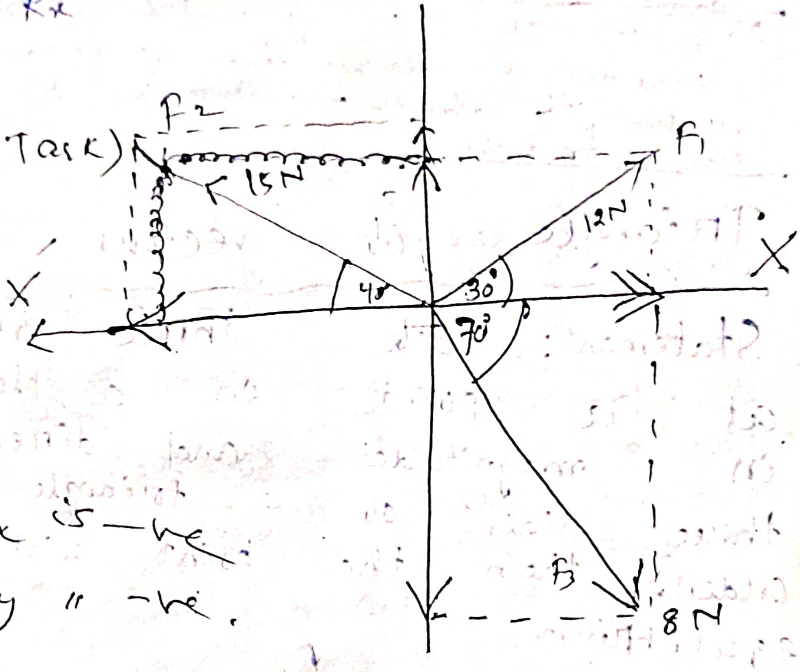
$$R = \sqrt{R_x^2 + R_y^2}$$

&

$$\tan\phi = \frac{R_y}{R_x} \Rightarrow \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$ S of W

Page 37 : 17 (Task)

horizontal :
Resolve :
Component :
Morning batch.



F_{3x} is −ve.
F_{3y} " −ve.

$F_1x = F_1 \cos 30^\circ = 12 . \frac{\sqrt{3}}{2} = 6\sqrt{3} = 6.(1.732)$
$= 10.392$ N

$F_2x = F_2 \cancel{\sin} \cos 40^\circ = 15.(.7660) = 11.49000$ N

$F_3x = F_3 . \cos 70^\circ = 8.(.3420) = 2.7360$ N

$F_1y = F_1 . \sin 30^\circ = 12 . \frac{1}{2} = 6$ N

$F_2y = F_2 . \sin 40^\circ = 15 . (.6428) = 9.6420$

$F_3y = F_3 . \sin 70^\circ = 8 . (.9397) = 7.5176$

Let us add all the x components and call it R_x

$R_x = F_1x + F_2x + F_2x = 10.392 + 11.49 + 2.736$

$= 24.618$

Similarly $R_y =$ ~~[illegible]~~

$F_{1y} + F_{2y} + F_{3y}$

$= 6 + 9.6420 + 7.5176$

$= 23.1596$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(24.618)^2 + (23.1596)^2}$$

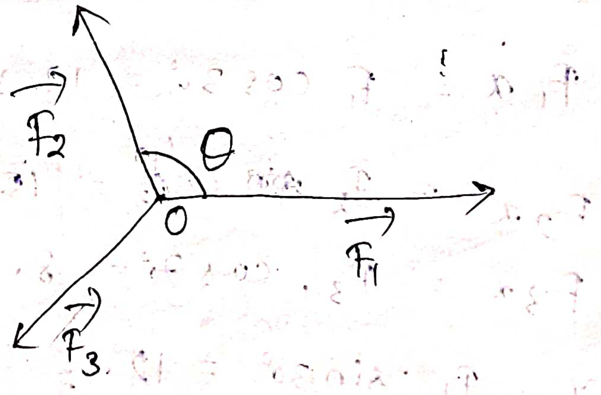
$=$

OR Triangle Law

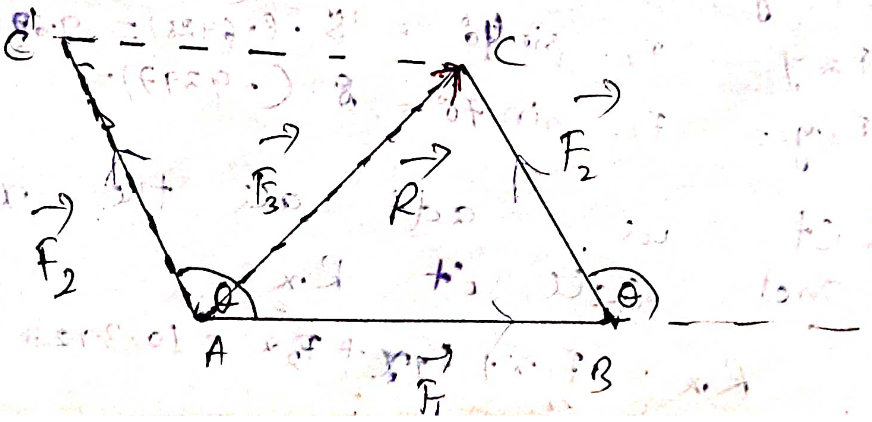
If two vectors are represented (in magnitude and direction) by the two sides of a triangle, taken in the same order, then their resultant is represented (in magnitude and direction) by the third side of the triangle

Triangle law of vectors

Statement: If three vectors acting at a point in a plane be represented in magnitude and direction by the three sides of a triangle taken in order, then the point will be in equilibrium



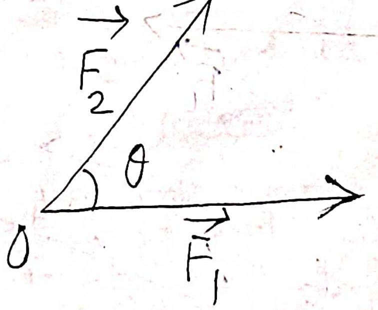
Proof:



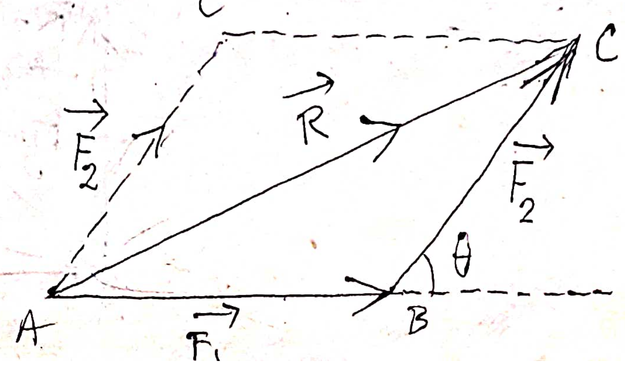
(37) The three vectors $\vec{F_1}, \vec{F_2}, \vec{F_3}$ acting at the point O have been represented by the three sides of the triangle AB, BC and CA of the triangle ABC. ~~by~~

By shifting $\vec{BC}$ into the position $\vec{AC'}$, we see that the resultant of $\vec{F_1}$ and $\vec{F_2}$ is $\vec{R}(\vec{AC'})$ which cancels $\vec{F_3}$. Therefore, the net resultant of the three vectors becomes zero and the point O will be in equilibrium.

Corollary (ଅନୁସିଦ୍ଧାନ୍ତ): If two vectors acting at a point in a plane be represented in such a manner that the tip of the ~~second~~ first vector touches the tail of the second vector, then the resultant of these two vectors is obtained by joining the tail of the first vector with the tip of the second vector.



Proof



38

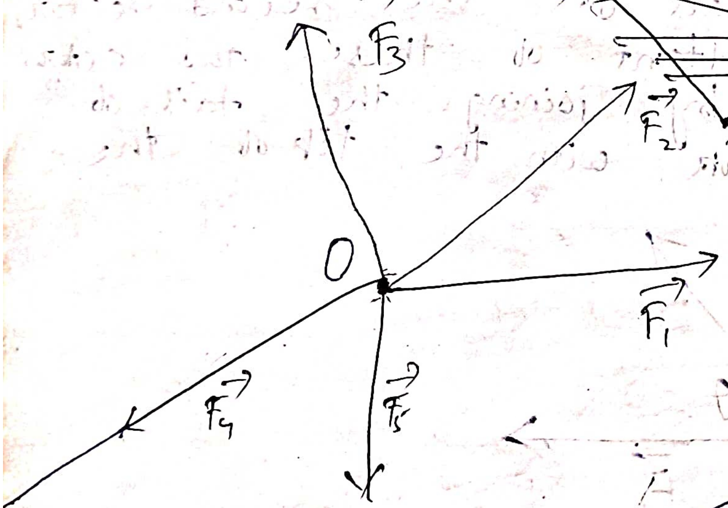
If we will represent $\overrightarrow{F_1}$ (AB) and $\overrightarrow{F_2}$ (BC) and complete the parallelogram then the resulttant vector is $\vec{R}$. If we think $\overrightarrow{BC} = \overrightarrow{F_2}$ then

$$\vec{R} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$= \overrightarrow{F_1} + \overrightarrow{F_2}$$

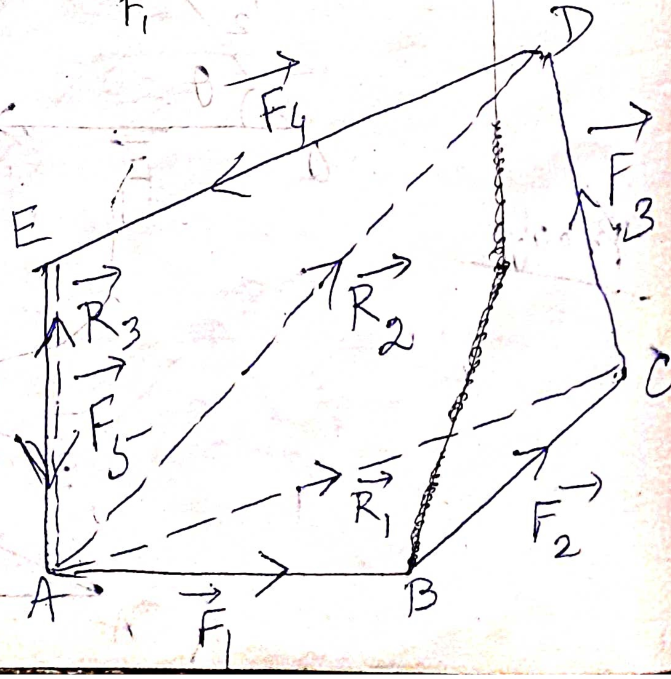
(Proved)

Polygon law of vectors.

Statement: If many vectors acting at a point in a plane be represented by the sides of a polygon taken in order, then the point will be in equilibrium.



Proof



(39) The five vectors $\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$ and $\vec{F_5}$ acting at the point O in the plane of the paper have been represented by the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, EA of the pentagon respectively.

From the corollary of the triangle of vectors we can write the following equations

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$$
$$\overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$$

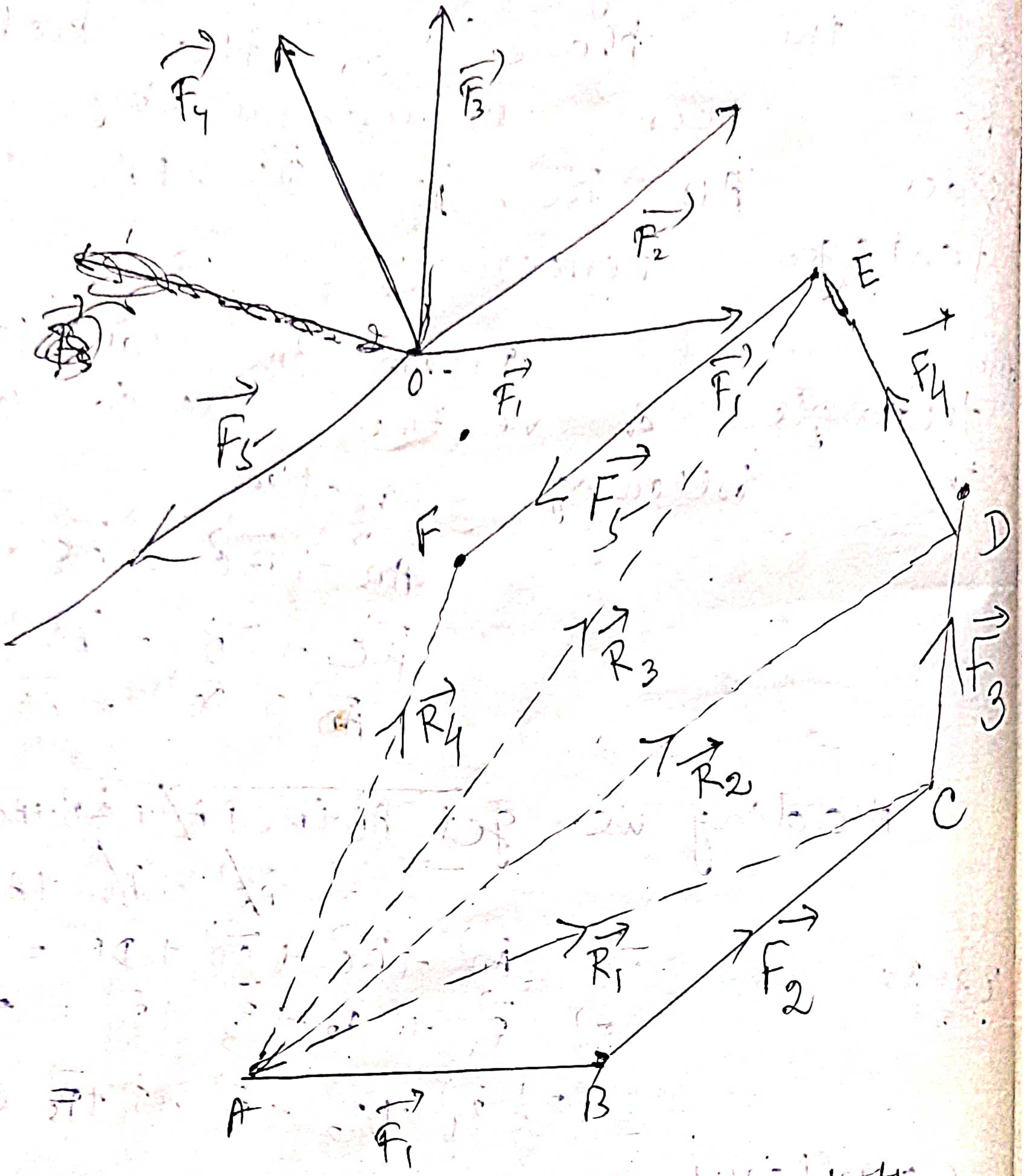
Adding we get $\overrightarrow{AB} + \overrightarrow{BC} + \cancel{\overrightarrow{AC}} + \overrightarrow{CD} + \cancel{\overrightarrow{AD}} + \overrightarrow{DE}$

$$= \cancel{\overrightarrow{AC}} + \cancel{\overrightarrow{AD}} + \overrightarrow{AE}$$

$\overrightarrow{EA} = \vec{F_5}$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$$
$$\Rightarrow \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = (-\vec{F_5})$$
$$\Rightarrow \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = 0$$

Corollary :- If many vectors acting at a point in a plane can not be represented by the sides of a closed polygon, then the point will not be in equilibrium and the resultant of these vectors can be obtained by joining the tail of the first vector with the tip of the last vector.



The five vectors acting at the point O have been represented by the sides $\vec{AB}$, $\vec{BC}$, $\vec{CD}$, $\vec{DE}$, and EF of the incomplete polygon.

From the corollary triangle of vectors one can write

$$\vec{AB} + \vec{BC} = \vec{AC}$$
$$\vec{AC} + \vec{CD} = \vec{AD}$$
$$\vec{AD} + \vec{DE} = \vec{AE}$$
$$\vec{AE} + \vec{EF} = \vec{AF}$$

Adding we get: $\overline{\vec{AB}+\vec{BC}+\cancel{\vec{AC}}+\vec{CD}+\cancel{\vec{AD}}+\vec{DE}+\cancel{\vec{AE}}+\vec{EF}}$

$= \cancel{\vec{AC}} + \cancel{\vec{AD}} + \cancel{\vec{AE}} + \vec{AF}$

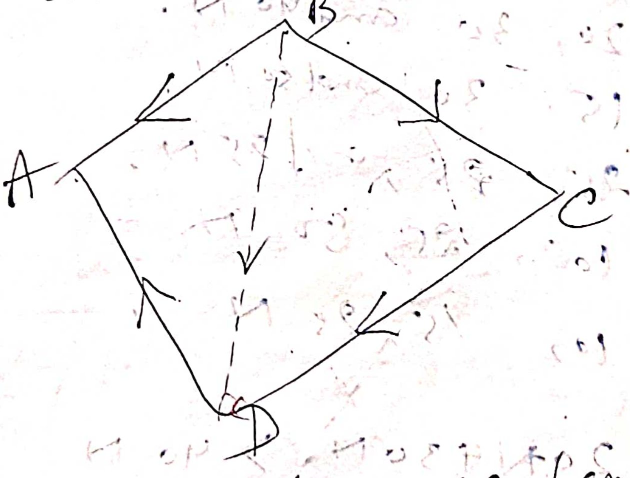
(41) $\Rightarrow \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} \overset{+F_5}{\cancel{3}} = \overrightarrow{AF} = \vec{R_4}$ (proved)

1. Pro.

which of the following groups of ~~concurrent~~ ~~fo~~ forces could be in equilibrium?

(a) 20, 30, and 40 N ✓
(b) 15, 30, and 50 N ✗
(c) 25, 25, 25 N ✓
(d) 100, 50, 25 N ✗
(e) 10, 15, 25 N ✓

~~(E)~~ 2. ABCD is a quadrilateral. Show that the resultant of the four vectors ~~be~~ $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ equals $2\overrightarrow{BA}$ vector.



From triangle law, we have,

$$\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$$
$$\overrightarrow{BD} + \overrightarrow{DA} = \overrightarrow{BA}$$

Adding we get,

$$\overrightarrow{BC} + \overrightarrow{CD} + \cancel{\overrightarrow{BD}} + \overrightarrow{DA} = \cancel{\overrightarrow{BD}} + \overrightarrow{BA}$$

Adding $\overrightarrow{BA}$ to both the sides, we get.

$$\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 2\overrightarrow{BA}$$ (proved)

(i) From the triangle of vectors

(42) We know that ~~[illegible]~~ if three vectors are acting at a point in a plane ~~by~~ be represented in magnitude and direction by the three sides of of a triangle taken in order. then the point will be in equilibrium.

(ii) We know that a triangle is completed if the sum of two ~~angles~~ side of triangle is greater than the other ~~angle~~ side

(iii) So these following groups of forces could be in equilibrium if ~~[illegible]~~ the sum of two sides is great than the third side...

(a) 20, 30 and 40 N.

(b) 15, 30 and 50 N.

(c) 25, 25, and 25 N.

(d) 100, 25, 50 N

(e) 10, 15, 25 N

(a)
$20N + 30N > 40\ N$
$30N + 40\ N > 20\ N$
$20N + 40\ N > 30\ N$

So the triangle is possible. ~~[illegible]~~ So this groups of forces could be in equilibrium.

(b)
$15 N + 30 N < 50 N$

So the triangle is not completed.

So this groups of forces could not be in equilibrium

How $\vec{A}$namic $\vec{R}$

㊸ (c) 25N+25N > 25N
So this group of forces could be in equlibrium

(d) 25N+50N < 100N. So this group of force could not be in equlibrium

~~(e) 10N+15 ≮ 25N. So this group of forces could not be in equlibrium.~~

(e) ?   ~~How $\vec{R} = \vec{AB} + \vec{BC}$~~
$\overline{2\vec{F_1} + \vec{F_2}}$

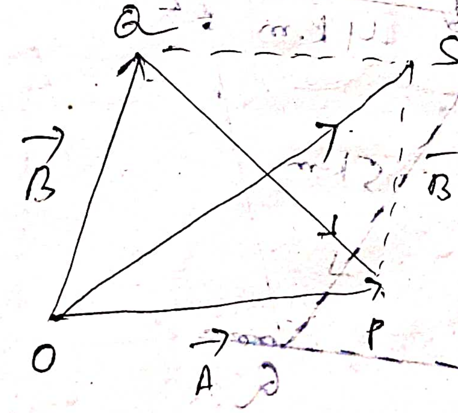
If 10N and 15N will act in one direction and 25N will act in the opposite direction, then equlibrium is possible.

— u —

6.06.2K

Page-36

13.



Using the corollary of triangle law of vectors, we can write

$$\vec{OS} = \vec{OP} + \vec{PS}$$
$$= \vec{A} + \vec{OQ}$$
$$= \vec{A} + \vec{B}$$

Thus the digonal $\vec{OS}$, represents sum of two vectors.

Using triangle law in the triangle OPQ

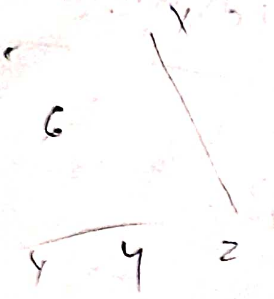
㊹ we get $\overrightarrow{OQ} + \overrightarrow{QP} = \overrightarrow{OP}$

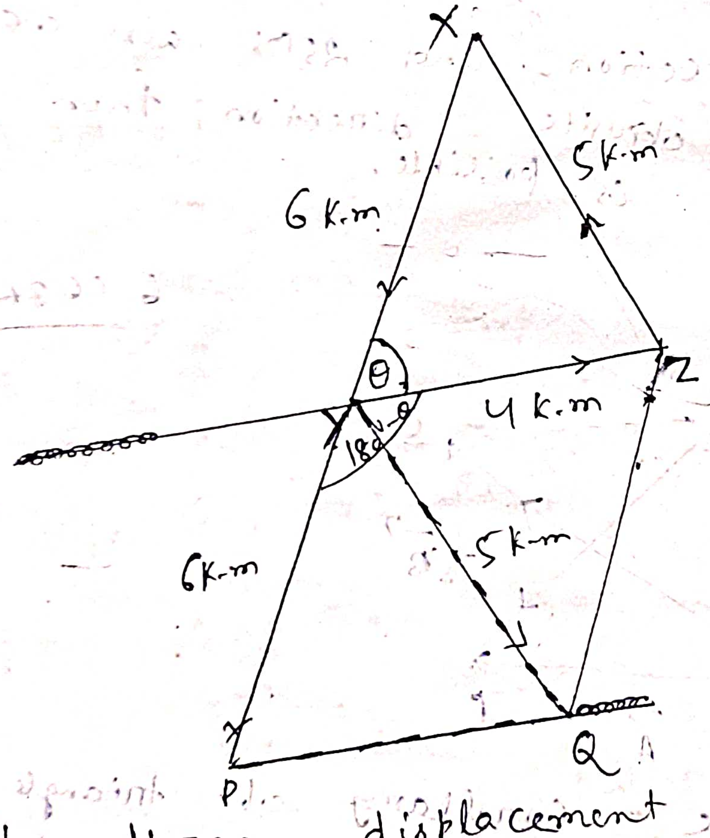
$\Rightarrow \vec{B} + \overrightarrow{QP} = \vec{A}$

$\Rightarrow \overrightarrow{QP} = \vec{A} - \vec{B}$

Thus the digonal $\overrightarrow{QP}$ represents the difference of the two vectors.

6.





The three displacement vectors are represented as the sides $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$ of the triangle XYZ. They produce equlibrium. that is the resultant of any two displacement vectors must be equal and opposite to the third displacement vector.

(45) In our diagram we have extended $\overrightarrow{XY}$ in to the position $\overrightarrow{YP}$. Completing the parallelo gram we see that $\overrightarrow{YQ}$ ~~BQ~~ has~~the~~ the magnitude of 5 k.m. Using the formula

$$R^2 = A^2+B^2+2AB\cos\theta,$$

we get $\Rightarrow \cancel{4Q}(5)^2 = (6)^2+(4)^2+2.(6).(4)\cos(180^\circ-\theta)$

$$\Rightarrow 25 = 36+\cancel{24}^{16}+48.\cancel{\cos 180}\overset{(-\cos\theta)}{}$$

$$\cancel{60}52\cancel{-24} - 48\cos\theta$$

$$\Rightarrow \cancel{60}^{52}-25 = 48\cos\theta$$

$$\Rightarrow \cancel{35}^{27} = 48\cos\theta$$

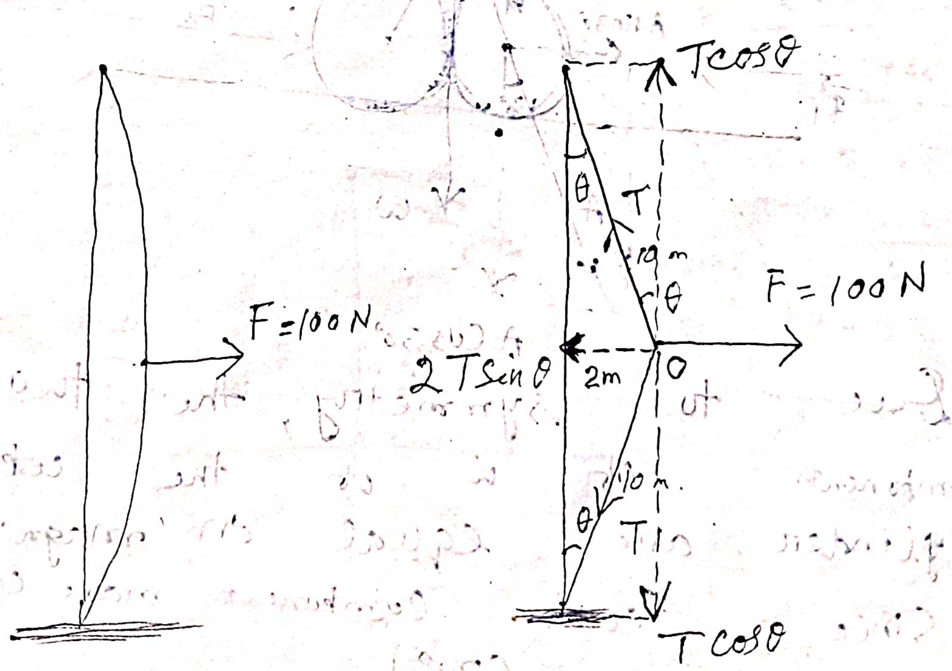
~~$\Rightarrow \cos\theta = \frac{35}{48}$~~

$$\Rightarrow \cos\theta = \frac{27}{48} = \frac{9}{16} = .5625$$

$$\Rightarrow \cancel{\cos}\theta = \cos^{-1}(.5625)$$

$$\Rightarrow \theta = 55^\circ.7^\circ$$

(10)



$$\sin\theta = \frac{2\,m}{10\,m} = \frac{1}{5}$$

(46) Resolving each tension into two rectangular components, we see that the vertical components are each T cosθ and they (equal, opposite (because)) cancel each other.

The horizontal compents are in the same directions and add up. Since there is equlibrium (zero resultant is 0). The net resultant

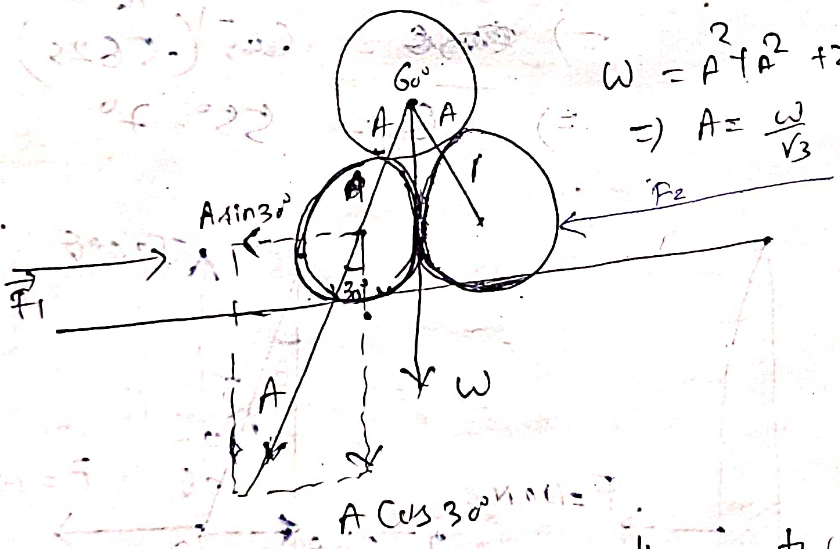
Hence $2T\sin\theta = 100\text{ N}$

$\Rightarrow T\sin\theta = 50\text{ N}$

$\Rightarrow T\cdot\frac{1}{5} = 50\text{ N}$

$\Rightarrow T = 250\text{ N}$

14.



$W^2 = A^2 + A^2 + 2A\cdot A\cos 60°$

$\Rightarrow A = \frac{W}{\sqrt{3}}$

Due to symmetry, the two components of W of the upper cylinder are equal in magnitude (Since each component make equal angle with W (30°))

Using the law of parallelogram of vectors we can write

$W^2 = A^2 + A^2 + 2A\cdot A\cos 60°$

(47) $\Rightarrow (5)^2 = A^2 + A^2 + \cancel{2}A \cdot A \cdot \cos\frac{1}{\cancel{2}}$

$= 3A^2$

$\Rightarrow 25 = 3A^2$

$\Rightarrow A^2 = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt3}$

Let us ~~[illegible]~~ again resolved each two rectangular component vector in to components as shown in the figure

The vertical component is A Cos 30° which presses the lower cylinder.

The horizontal component is A sin30° and it tries to separate the cylinders This has to be prevented by the application of an extrional force $\vec{F_1}$.

$\therefore F_1 = A \sin 30° = \frac{5}{\sqrt3} \cdot \frac{1}{2}$

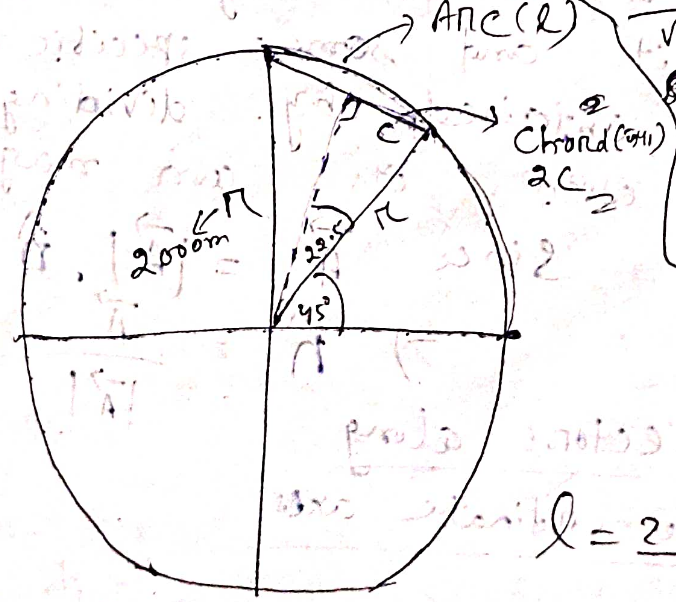
$= \frac{5 \cdot \sqrt3}{\sqrt3 \cdot 2}$ .866

$= \frac{5 \cdot \cancel{(1.732)}}{\sqrt3 \cdot 2 \times \sqrt3}$

$= \frac{4.33}{3}$

1.443

25.



$l = \frac{2\pi r}{8}$

$\sin 22.5° = \frac{c}{r}$,

$\Rightarrow c = ?, 2c = ?$

$l - 2c$

(48) $\sin 22.5^\circ = \frac{c}{r} \Rightarrow .3827 = \frac{c}{2000}$

$\Rightarrow \quad c = 7654000 = 765.4$

$\Rightarrow 2c = 1530.8$

$l = \frac{2\pi r}{8} = \frac{2 \cdot \frac{22}{7} \cdot (2000)}{8} = \frac{44 \times \overset{\overset{35.7}{\cancel{250}}}{\cancel{2000}}}{\cancel{7}} \times \frac{1}{\cancel{8}}$

$= \underline{1570.8}$

$l - 2c = 1570.8 - 1530.8$

$= \underline{40\ m}$

Unit vector

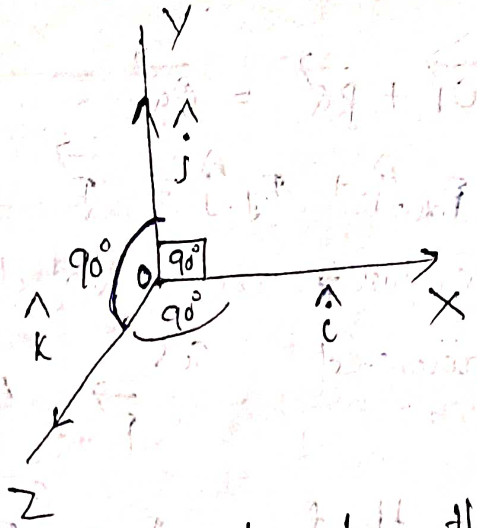
It is a vector having magnitude one unit and some specific direction. It is obtained by dividing a vector with its own magnitude

$$\text{Since } \vec{A} = |\vec{A}| \cdot \hat{n}$$

$$\Rightarrow \hat{n} = \frac{\vec{A}}{|\vec{A}|}$$

Unit vectors along the co-ordinate axes

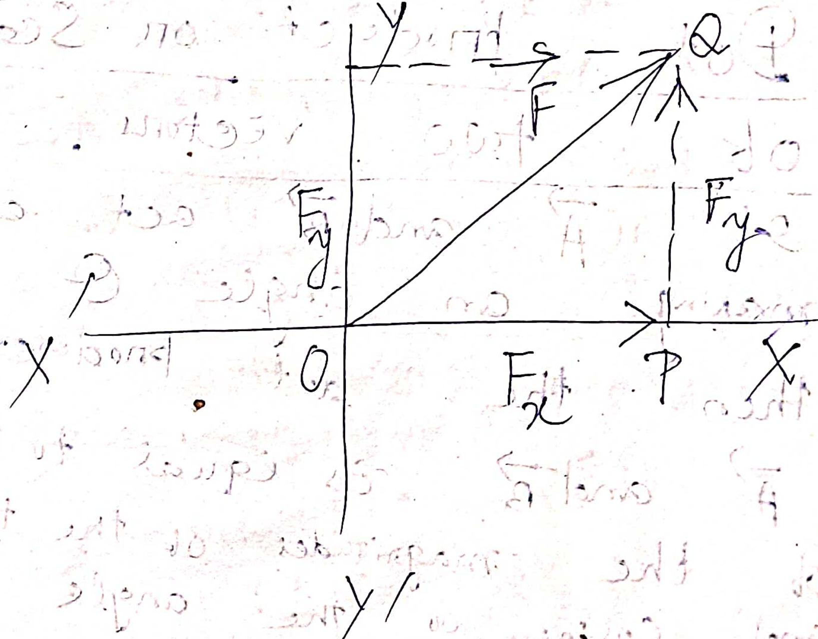
7) 25 (35.71
21
40
35
50
49
10

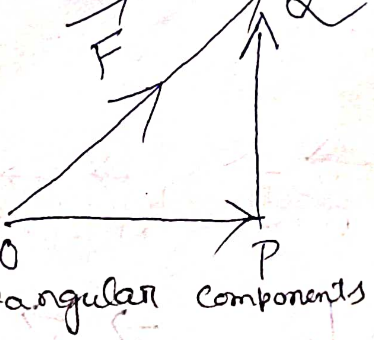


The symbols used for the unit vectors along X, Y, and Z axes are $\hat{i}$, $\hat{j}$, $\hat{k}$ respectively.

$$\therefore |\hat{i}| = 1 = |\hat{j}| = |\hat{k}|$$

Representation of a vector in terms of its components





If the two rectangular components of $\vec{F}$ be F_x and F_y along the X and Y directions, then triangle law can be used.

$$\therefore \vec{OP} + \vec{PQ} = \vec{OQ}$$

$$\Rightarrow F_x.\hat{i} + F_y.\hat{j} = \vec{F}$$

Thus, a vector in two dimension can be represented as $\vec{F} = \hat{i} F_x + \hat{j} F_y$

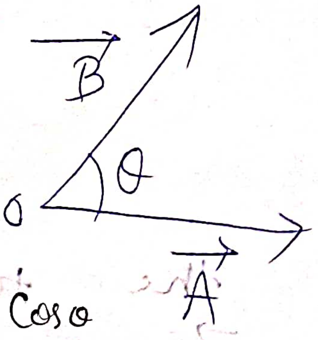
Such that $F = \sqrt{F_x^2 + F_y^2}$

If a vector is in ~~the~~ three dimension then it will have three components like F_x, F_y, F_z along x, y, and z axes. Thus $\vec{F} = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z$

$$\Rightarrow F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Dot product or Scalar product of two vectors

If $\vec{A}$ and $\vec{B}$ act at a point making an angle θ between them, then the dot product between $\vec{A}$ and $\vec{B}$ is equal to the product of the magnitudes of the two vectors and cosine of the angle between them.



$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Now $\vec{B}.\vec{A} = |\vec{B}||\vec{A}| \cos\theta$

$= |\vec{A}||\vec{B}| \cos\theta$

$= \vec{A}.\vec{B}$

(51) Dot product between unit vectors.

$$\hat{i}.\hat{i} = |\hat{i}||\hat{i}| \cos 0^\circ$$
$$= (1)(1)(1)$$
$$= 1$$

Similarly $\hat{j}.\hat{j} = 1 = \hat{k}.\hat{k}$

Now $\hat{i}.\hat{j} = |\hat{i}||\hat{j}| \cos 90^\circ$
$$= 1.1.0$$
$$= 0$$

Similarly $\hat{j}.\hat{k} = 0$ and $\hat{k}.\hat{i} = 0$.

Dot product between two vectors in component form

Let $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$

and $\vec{B} = \hat{i}B_x + \hat{j}B_y + \hat{k}B_z$

Hence $\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z)$

$$= (\hat{i}.\hat{i})A_xB_x + (\hat{i}.\hat{j})A_xB_y + (\hat{j}.\hat{i})A_yB_x + (\hat{i}.\hat{k})A_xB_z + (\hat{j}.\hat{k})A_yB_z + (\hat{j}.\hat{j})A_yB_y + (\hat{k}.\hat{i})A_z.B_x + (\hat{k}.\hat{j})A_z.B_y + (\hat{k}.\hat{k})A_z.B_z$$

$$= 1A_xB_x + 0 + 0 + 0 + A_yB_y + 0 + 0 + 0 + A_zB_z$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Applications

(i) To find the angle between two vectors

(52) $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z$

$$\Rightarrow \quad \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{|\vec{A}||\vec{B}|}$$

(where $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
and $|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

(ii) Condition of perpendicularity of two vectors.

If $\theta = 90°$, then $\cos\theta = 0$

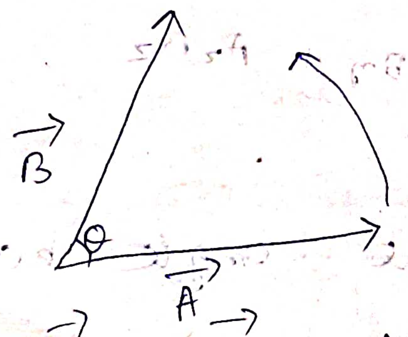
$$\text{Thus} \quad 0 = \frac{A_xB_x + A_yB_y + A_zB_z}{|\vec{A}|.|\vec{B}|}$$

$$\boxed{\Rightarrow A_xB_x + A_yB_y + A_zB_z = 0}$$

Cross product between two vectors (vector product of two vectors)

It is denoted by giving a cross in between the two vectors and the magnitude is AB sinθ with a direction indicated by a unit vector $\hat{n}$.

The direction of $\hat{n}$ is decided by the right hand screw rule.



$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

Here $\hat{n}$ is upward and perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ (plane of the paper)

Hence $\vec{B} \times \vec{A} = BA \sin\theta (-\hat{n})$

$= -(\vec{A} \times \vec{B})$

Cross products between unit vectors

$\hat{i} \times \hat{i} = |\hat{i}| |\hat{i}| \sin 0 \hat{n}$

$= 1 . 1 . 0 . \hat{n}$

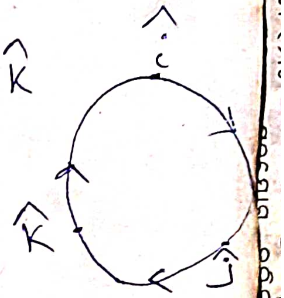
$= 0$

Similarly $\hat{j} \times \hat{j} = 0 = \hat{k} \times \hat{k}$

$\hat{i} \times \hat{j} = |\hat{i}| |\hat{j}| \sin 90^\circ \hat{n}$

$= 1 . 1 . 1 . \hat{n}$

$= \hat{k}$



Similarly $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

$\hat{k} \times \hat{j} = -(\hat{j} \times \hat{k}) = -\hat{i}$ etc

Cross product between two vectors in component form

~~Cross product~~

$\vec{A} \times \vec{B} = (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z) \times (\hat{i} B_x + \hat{j} B_y + \hat{k} B_z)$

$= (\hat{i} \times \hat{i}) A_x B_x + (\hat{i} \times \hat{j}) A_x B_y + (\hat{i} \times \hat{k}) A_x B_z$

$+ (\hat{j} \times \hat{i}) A_y B_x + (\hat{j} \times \hat{j}) A_y B_y + (\hat{j} \times \hat{k}) A_y B_z + (\hat{k} \times \hat{i}) A_z B_x + (\hat{k} \times \hat{j}) A_z B_y + (\hat{k} \times \hat{k}) A_z B_z$

$= 0 + (\hat{k}) A_x B_y + (-\hat{j}) A_x B_z + (-\hat{k}) A_y B_x + 0$

$+ (\hat{i}) A_y B_z + (\hat{j}) A_z B_x + (-\hat{i}) A_z B_y + 0$

$= (\hat{k}) A_x B_y + (-\hat{j}) A_x B_z + (-\hat{k}) A_y B_x$

$+ (\hat{i}) A_y B_z + (\hat{j}) A_z B_x + (-\hat{i}) A_z B_y$

$= (\hat{k})(A_x B_y - A_y B_x) + \hat{j}(A_z B_x - A_x B_z)$

$+ \hat{i}(A_y B_z - A_z B_y)$

$= \hat{i}(A_y B_z - A_z B_y) + \hat{j}(A_z B_x - A_x B_z)$

$+ \hat{k}(A_x B_y - A_y B_x)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Problems

1. Find the angle between the two vectors $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$

(ans - 79°)

2. Prove that the vectors $\vec{A} = 3\hat{i} - 3\hat{j} + \hat{k}$ are and $\vec{B} = -6\hat{i} + 6\hat{j} - 2\hat{k}$ anti parallel = ~~180~~ . (Ans $\theta = 180°$)

3. For what values of n, the two vectors $\vec{A} = n\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2n\hat{i} + n\hat{j} - 4\hat{k}$ will be perpendicular? (Ans $n = 2, -1$

4. If $\vec{A} = 5\hat{i} + 3\hat{j} - 3\hat{k}$ and $\vec{B}$

(55)

$= 3\hat{i} - 2\hat{j} + 4\hat{k}$

Find $|\vec{A} + \vec{B}|$ and $|\vec{A} - \vec{B}|$

Ans - 8.12, 8.83

⑤ A force $\vec{F} = \hat{i} + 2\hat{j} + \hat{k}$ displaces a particle along a vector $\vec{S} = 4\hat{i} - \hat{j} + 7\hat{k}$ Find the work done.

Ans ÷ 9 units of work.

Hints $W = \vec{F} . \vec{S}$

6. A force of $7\hat{i} + 6\hat{k}$ Newton displaces a body on a rough plane with a velocity of $3\hat{j} + 4\hat{k}$ m/s. Calculate the power in watts.

Hints $P = \vec{F} . \vec{V}$

Ans ÷ 24 watts,

⑦ Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the co-ordinate axes.

Ans ÷ 64.6°, 149°, 73.4°

Mod → ~~[illegible]~~ $AB\sin\theta = |\vec{A} . \vec{B}|$?

⑧ Find a unit vector which is perpendicular to both the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$

Ans:

(56)
$$\frac{3\hat{i} - \hat{j} + 5\hat{k}}{\sqrt{35}}$$

Hints: $\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$$\therefore \hat{n} = \frac{\vec{A} \times \vec{B}}{AB \sin\theta} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

⑨ Prove that $\vec{P} \cdot (\vec{Q} + \vec{R})$
$= \vec{P} \cdot \vec{Q} + \vec{P} \cdot \vec{R}$

and $\vec{P} \times (\vec{Q} + \vec{R}) = \vec{P} \times \vec{Q} + \vec{P} \times \vec{R}$

⑩ Prove that
$$\vec{A} \cdot \vec{B} = \frac{1}{4} \left[|\vec{A} + \vec{B}|^2 - |\vec{A} - \vec{B}|^2 \right]$$

11. A particle is in equilibrium under the action of 3 vectors acting simultaneously (Bu 17odd) prove that that $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$

Ans: Since the point is in equilibrium under the action of 3 vectors, the vector sum must be zero.

$$\therefore \vec{A} + \vec{B} + \vec{C} = 0$$

$$\therefore \vec{A} + \vec{B} = -\vec{C} \quad \text{———} (i)$$

Taking cross product of Both

the sides of equation (1) by $\vec{A}$

(57) we get $(\vec{A}+\vec{B})\times\vec{A} = -\vec{C}\times\vec{A}$

or $\vec{A}\times\vec{A} + \vec{B}\times\vec{A} = -(\vec{C}\times\vec{A})$

or $0 + \{-(\vec{A}\times\vec{B})\} = -(\vec{C}\times\vec{A})$

$\therefore \vec{A}\times\vec{B} = \vec{C}\times\vec{A}$ —(2)

Taking cross product of both the sides of equation (1) by $\vec{B}$ we get $(\vec{A}+\vec{B})\times\vec{B} = -\vec{C}\times\vec{B}$

or $\vec{A}\times\vec{B} + \vec{B}\times\vec{B} = -(\vec{C}\times\vec{B})$

or $\vec{A}\times\vec{B} + 0 = \vec{B}\times\vec{C}$ —(3))

From (2) and (3) we get

$\vec{A}\times\vec{B} = \vec{B}\times\vec{C} = \vec{C}\times\vec{A}$

(proved)

12. The condition of co-planarity of 3 vector is

$$(\vec{A}\times\vec{B})\cdot\vec{C} = 0$$

Find the value of 'a' such that the three vectors $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{C} = 3\hat{i} + a\hat{j} + 5\hat{k}$ are co-planar.

Ans: $a = -4$

Determinant - Short form of writing a big eqn

(58)

$$A_1(B_2C_3 - B_3C_2) + A_2(B_3C_1 - B_1C_3) + A_3(B_1C_2 - B_2C_1) = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$$

(3x

deter

mina

Ex ÷ To find $(3\hat{i} + 2\hat{j} - \hat{k}) \times (2\hat{i} - 3\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix}$$

$$= \hat{i}(2\times 2 - (-1)(-3)) + \hat{j}\{2(-1) - (3\times 2)) + \hat{k}\{3(-3) - 2\times 2\}$$

$$= \hat{i}(4-3) + \hat{j}(-2-6) + \hat{k}(-9-4)$$

$$= \hat{i} - 8\hat{j} - 13\hat{k}$$

OR $\vec{A} \times \vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$

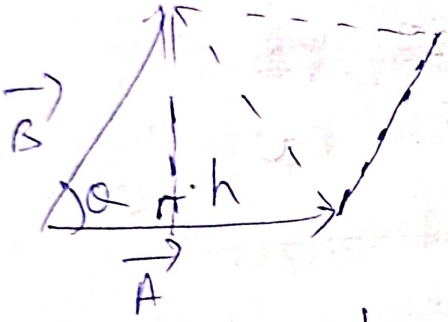
$$= \hat{i}\{2(2) - (-1).(-3)\} + \hat{j}\{(-1).(2) - 3.2\} + \hat{k}\{(3.(-3) - 2.2\}$$

$$= \hat{i}(4-3) + \hat{j}(-2-6) + \hat{k}(-9-4)$$

$$= \hat{i} - 8\hat{j} - 13\hat{k}$$

59

13. To find the area of a triangle and parallelogram form out of two vectors as its two adjacent ([illegible]) sides.



Area of the triangle $= \frac{1}{2} \cdot \text{base} \times \text{height}$

$$= \frac{1}{2} \times |\vec{A}| \times h$$

But $\sin\theta = \dfrac{h}{|\vec{B}|}$

$$\Rightarrow h = |\vec{B}| \sin\theta$$

$\therefore$ Area of the triangle

$$= \frac{1}{2} \cdot |\vec{A}| \cdot |\vec{B}| \sin\theta$$

$$= \frac{1}{2} \left(\vec{A} \times \vec{B}\right)$$

Area of the parallelogram

= 2 × area of triangle

$$= |\vec{A} \times \vec{B}|$$

Problem

~~Q.~~ ⑬ Calculate the area of the parallelogram and triangle whose adjacent sides are represented by vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$

Ans : 17.32 , 8.66 square units

Pr. 17. At what angles two ~~for~~ forces P + Q and P − Q should be inclined so as to have a resultant $\sqrt{3P^2 + Q^2}$

Ans: 60°.

~~Problem~~ Solutions

Q. $\vec{A} = 3\hat{i} - 3\hat{j} + \hat{k}$

$\vec{B} = -6\hat{i} + 6\hat{j} - 2\hat{k}$

Here $A_x = 3$, $A_y = -3$, $A_z = 1$

$B_x = -6$, $B_y = 6$, $B_z = -2$

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\theta$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$\Rightarrow \cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$= \frac{(3)(-6) + (-3).(6) + (1).(-2)}{\sqrt{9+9+1} + \sqrt{36+36+4}}$$

$$= \frac{-18 - 18 - 2}{\sqrt{19} \cdot \sqrt{76}} = \frac{-38}{\sqrt{19} \cdot 2\sqrt{19}}$$

$$= \frac{-\cancel{38}\,{-19}}{\cancel{2} \cdot 19} = -1 \text{ (ans)}$$

$\Rightarrow \quad \cos\theta = -1 = \cos 180^\circ$

$\Rightarrow \quad \theta = 180^\circ$ | (ans)

Hence $\vec{A}$ and $\vec{B}$ are antiparallel.

9. $\vec{A} = n\hat{i} - 2\hat{j} + \hat{k}$

$\vec{B} = 2n\hat{i} + n\hat{j} - 4\hat{k}$

Here $A_x = n$, $A_y = -2$, $A_z = 1$

$B_x = 2n$, $B_y = n$, $B_z = -4$

~~$\vec{A} \cdot \vec{B}$~~

~~$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| \cdot |\vec{B}|}$~~

~~$\Rightarrow \cos 90^\circ = "$~~

~~Condition~~

$\Rightarrow \quad 0$ ~~$= A_x B_x + A_y B_y$~~

Condition of perpendicularity of two vectors is

$$A_x B_x + A_y B_y + A_z B_z = 0$$

$\Rightarrow \quad n \cdot 2n + (-2) \cdot (n) + (1) \cdot (-4) = 0$

$\Rightarrow \quad 2n^2 - 2n - 4 = 0$

$\Rightarrow \quad n^2 - n - 2 = 0$

$\Rightarrow \quad n^2 - 2n + n - 2 = 0$

~~$\Rightarrow n(n+2) -1(n+2) = 0$~~

~~$\Rightarrow (n+2)(n-1) = 0$~~

~~$\Rightarrow n = -2$ or $n = 1$~~

$\Rightarrow n(n-2) +1(n-2) = 0$

$\Rightarrow (n-2)(n+1) = 0$

$\Rightarrow n = 2$, or $n = -1$ (Ans)

12. Given

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{C} = 3\hat{i} + a\hat{j} + 5\hat{k}$

~~Given that~~

~~$|\vec{A}|.B(\vec{A}\times\vec{B})$~~

Here $A_x = 2$, $A_y = -1$, $A_z = 1$

$B_x = 1$, $B_y = 2$, $B_z = -3$

$C_x = 3$, $C_y = a$, $C_z = 5$

$=$ ~~$AB\sin\theta\,\hat{n}$~~

$= \vec{A} \times \vec{B}$

$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$

$= \hat{i}\{(-1).(-3) - (1).(2)\} + \hat{j}\{(1).(1) - (2).(-3)\} + \hat{k}\{(2).(2) - (-1)(1)\}$

$= \hat{i}\{3 - 2\} + \hat{j}(1+6) + \hat{k}(4+1)$

$= \hat{i}(1) + \hat{j}(7) + \hat{k}(5)$

$= \hat{i} + 7\hat{j} + 5\hat{k}$

Now $(\vec{A}\times\vec{B}).\vec{C}=0$ provided the three vectors are in one plane

~~$\Rightarrow(\hat{i}+7\hat{j}+5\hat{k})$~~ $\cdot\vec{C}$

$\Rightarrow(\hat{i}+7\hat{j}+5\hat{k}).(3\hat{i}+a\hat{j}+5\hat{k})=0$

$\Rightarrow(1).(3)+7a+25=0$

$\Rightarrow 3+7a+25=0$

$\Rightarrow 7a+28=0$

$\Rightarrow 7a=-28$

$\Rightarrow a=\frac{-28}{7}=-4$ (Ans)

4. Given $\vec{A}=5\hat{i}+3\hat{j}-3\hat{k}$

$\vec{B}=3\hat{i}-2\hat{j}+4\hat{k}$

$\vec{A}+\vec{B}=5\hat{i}+3\hat{j}-3\hat{k}+3\hat{i}-2\hat{j}+4\hat{k}$

$=8\hat{i}+\hat{j}+\hat{k}$

$|\vec{A}+\vec{B}|=\sqrt{(8^2)+(1)^2+(1)^2}$

$=\sqrt{64+1+1}$

$=\sqrt{66}$

$=8.12$

| ~~A×B~~ +

$\vec{A}-\vec{B}=(5\hat{i}+3\hat{j}-3\hat{k})-(3\hat{i}-2\hat{j}+4\hat{k})$

$=5\hat{i}-3\hat{i}+3\hat{j}+2\hat{j}-3\hat{k}-4\hat{k}$

$=2\hat{i}+5\hat{j}-7\hat{k}$

04)

$$|\vec{A} - \vec{B}| = \sqrt{(2)^2 + (5)^2 + (-7)^2}$$

$$= \sqrt{4 + 25 + 49}$$

$$= \sqrt{29 + 49}$$

$$= \cancel{80} = \sqrt{78}$$

$$= 8.83 \quad \text{(ans)}$$

1.

$$\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$$ Here $A_x = 2, A_y = 2$, $A_z = -1$

$$\vec{B} = 6\hat{i} + -3\hat{j} + 2\hat{k}$$ $B_x = 6,\ B_y = -3,\ B_z = 2$

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\Rightarrow \cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| \cdot |\vec{B}|}$$

$$\Rightarrow \cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$\Rightarrow \cos\theta = \frac{(2).(6) + (2).(-3) + (-1).(2)}{\sqrt{4+4+1} \cdot \sqrt{36+9+4}}$$

$$= \frac{12 + -6 - 2}{3 \cdot 7}$$

$$= \frac{4}{\cancel{7\sqrt{9}}} = \frac{4}{21} = .1904$$

$$\Rightarrow \cos\theta = \cos 79^\circ$$

$$\Rightarrow \theta = 79^\circ$$

$\therefore$ The angle between two vectors is 79°

5. A force $\vec{F} = \hat{i} + 2\hat{j} + \hat{k}$ ⑥⑤

here $A_x = 1$, $A_y = 2$ $A_z = 1$

It displace a particle along a vector $\vec{S} = 4\hat{i} - \hat{j} + 7\hat{k}$

Here $B_x = 4$, $B_y = -1$, $B_z = 7$

Work $= \vec{F} \cdot \vec{S}$

$= A_xB_x + A_yB_y + A_zB_z$

$= 4 - 2 + 7$

$= 2 + 7$

$= 9$ units of work. (ans)

⑤ → $W = \vec{F} \cdot \vec{S}$ ← because work is scalar

or ~~$\vec{F} \times \vec{S}$~~

6. A force (F) of $7\hat{i} + 6\hat{k}$ Newton displaces a body on a rough plane with a velocity (v) of $3\hat{j} + 4\hat{k}$ m/s

Power $= \frac{W}{t} = \frac{FS}{t} = F \cdot V$

$= (7\hat{i} + 6\hat{k})(3\hat{j} + 4\hat{k})$

$= (7\times3)(\hat{i}\cdot\hat{j}) + (7\times4)(\hat{i}\cdot\hat{k})$

$+ (6\times3)(\hat{k}\cdot\hat{j}) + (6\cdot4)(\hat{k}\cdot\hat{k})$

$= (7\times3).0 + (28).0 + (6\times3).0$

$+ 24.(1)$

$= 0 + 24$

$= 24$ Watt (ans)

8 A unit vector is perpendicular

b)) to Both vectors

$\vec{A} = 2\hat{i} + \hat{j} - \hat{K}$, Here $A_X = 2$, $A_y = 1$ $A_Z = -1$

$\vec{B} = 3\hat{i} + 4\hat{j} - \hat{K}$ $\quad B_x = 3$ $\quad B_y = 4$ $\quad B_z = -1$

We know that $\vec{A} \times \vec{B} = AB \sin\theta \, . \hat{n}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{AB \sin\theta}$$

$$= \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$= \frac{(2\hat{i} + \hat{j} - \hat{K}) \times (3\hat{i} + 4\hat{j} - \hat{K})}{|\vec{A} \times \vec{B}|}$$

$$= \frac{i(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{j}(A_xB_y - A_yB_x)}{|\vec{A} \times \vec{B}|}$$

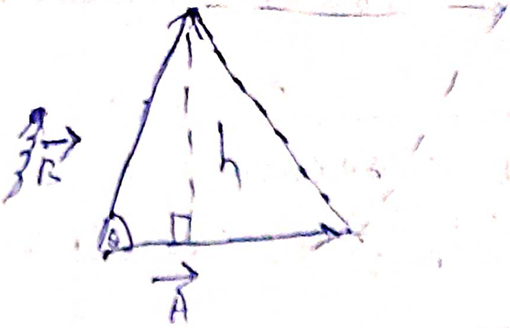
$$= \frac{\hat{i}(-1 + \overset{4}{3}) + \hat{j}(-3+2) + \hat{K}(8-3)}{|\vec{A} \times \vec{B}|}$$

$$= \frac{2\hat{i} - \hat{j} + 5\hat{K}}{|\vec{A} \times \vec{B}|}$$

$$= \frac{2\hat{i} - \hat{j} + 5\hat{K}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \frac{3\hat{i} - \hat{j} + 5\hat{k}}{\sqrt{9+1+25}}$$

$$= \frac{3\hat{i} - \hat{j} + 5\hat{K}}{\sqrt{35}}$$ (ans)

12.



Area of triangle $= \frac{1}{2}$. base height

$= \frac{1}{2} . |\vec{A}| . h$

But $\sin\theta = \frac{h}{|\vec{B}|}$

$\Rightarrow h = |\vec{B}| \sin\theta$

So Area of triangle $= \frac{1}{2} |\vec{A}| . |\vec{B}| \sin\theta$

$= \frac{1}{2} | \vec{A} \times \vec{B} |$

Here $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{B} = \hat{i} - 3\hat{j} + 4\hat{k}$

Here $A_x = 3$, $A_y = 1$, $A_z = 2$

$B_x = 1$, $B_y = -3$, $B_z = 4$

$\vec{A} \times \vec{B} =$

$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z)$

$+ \hat{k}(A_xB_y - A_yB_x)$

$= \hat{i}(4+6) + \hat{j}(2-12) + \hat{k}(-9-1)$

$= 10\hat{i} - 10j - 10\hat{k}$

$= \frac{1}{2}\sqrt{(2)^2 + (10)^2 + (10)^2}$

$= \frac{1}{2}\sqrt{100 + 100 + 100}$

$= \frac{1}{2} \times \sqrt{300}$

$= \frac{1}{2} . (17.32)$

$= \frac{1}{2} = ~~8.995~~$

$= 8.66$ sq units

Area of parallelogram

$\frac{1}{2} \times 2 |\vec{A} \times \vec{B}|$

$= |\vec{A} \times \vec{B}| = \sqrt{(10)^2 + (10)^2 + (10)^2}$

$= \sqrt{300}$ $\cdot 2$ ~~17.91~~ $= 10\sqrt{3} = 10(1.732)$

$= 17.32$ sq units

14. According to the law of parallelogram

(68) of vectors

$$R^2 = \sqrt{A^2+B^2+2AB\cos\alpha}$$

$\Rightarrow 3p^2+q^2 = (p+q)^2 + (p-q)^2 + 2(p+q).(p-q)\cos\alpha$

$= \cancel{p^2+q^2+2pq}$

$= 2(p^2+q^2) + 2(p^2-q^2)\cos\alpha$

$\Rightarrow 3p^2+q^2-2p^2-2q^2 = 2(p^2-q^2)\cos\alpha$

$\Rightarrow (\cancel{p^2-q^2}) = 2(\cancel{p^2-q^2})\cos\alpha$

$\Rightarrow \cos\alpha = \frac{1}{2} = \cos 60$

$\theta = 60^\circ$

7.(a) Let's take $\vec{B} = \hat{i}$ when angle made by $\vec{A}$ with the ~~x~~ X axis ~~is~~to to be bound out. Let the angle be θ_1

$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta_1$

$\Rightarrow (3\hat{i}-6\hat{j}+2\hat{k}).\hat{i} = \sqrt{(3)^2+(-6)^2+(2)^2}.|\hat{i}|.\cos\theta_1$

$\Rightarrow 3(\hat{i}.\hat{i}) - 6(\hat{j}.\hat{i}) + 2(\hat{k}.\hat{i}) = \sqrt{49}.|\hat{i}|.\cos\theta_1$

$\Rightarrow 3-6(0)+2(0) = 7\cos\theta_1$

$\Rightarrow 3 = 7\cos\alpha$

$\Rightarrow \cos\theta_1 = \frac{3}{7} = 0.4285$

$\Rightarrow \theta_1 = \cos^{-1}(0.4285) = 64.6^\circ$

(b) Let us take $\vec{B} = \hat{j}$ when angle made ~~[illegible]~~ by $\vec{A}$ with Y axis is to be bound out, let the angle be θ_2

$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta_2$

$\Rightarrow (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \hat{j} = \sqrt{9+36+4} \cdot |\hat{j}| \cos\theta_2$

$\Rightarrow 3(\hat{i}\cdot\hat{j}) - 6(\hat{j}\cdot\hat{j}) + 2(\hat{k}\cdot\hat{j}) = 7\,\hat{j}\cdot|\hat{j}| \cos\theta_2$

$\Rightarrow 3(0) - 6(1) + 2(0) = 7 \cos\theta_2$

$\Rightarrow -6 = 7 \cos\theta_2$

$\Rightarrow \cos\theta_2 = \frac{-6}{7} = -.8521$

Let $\theta_2 = (180° - \phi)$

$\Rightarrow \cos\theta_2 = \cos(180° - \phi)$

$\Rightarrow -.8571 = -\cos\phi$

$\Rightarrow$ ~~cos θ 180°~~

$\cos\phi = .8571$

$\phi = \cos^{-1}(.8571) = 31°$

$\theta = 180° - \phi = 180° - 31° = 149°$

(C) Lets take $\vec{B} = \hat{k}$ when angle made by the x axis is to found

Let the angle be θ_3

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\theta_3$

$\Rightarrow (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \hat{k} = \sqrt{9+36+4} \cdot (\hat{k}) \cos\theta_3$

$= 7\,\hat{k} \cos\theta_3$

$\Rightarrow 3(\hat{i}\cdot\hat{k}) - 6(\hat{j}\cdot\hat{k}) + 2(\hat{k}\cdot\hat{k}) = 7\hat{k} \cos\theta_3$

$\Rightarrow 3(0) - 6(0) + 2$

$2 = 7\cos\theta_3$

$\Rightarrow \cos\theta_3 = \frac{2}{7} = .2852$

$\Rightarrow \theta_3 = \cos^{-1}(.2857) = 73.4°$

10. $\vec{A} \cdot \vec{B} = \frac{1}{4}\left[|\vec{A}+\vec{B}|^2 - |\vec{A}-\vec{B}|^2\right]$

R.H.S

$$\frac{1}{4}\left[|\vec{A}+\vec{B}|^2 - |\vec{A}-\vec{B}|^2\right]$$

$$= \frac{1}{4}\left[\{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}|.|\vec{B}|\} - \{|\vec{A}|^2 + |\vec{B}|^2 - 2.|\vec{A}||\vec{B}|\}\right]$$

$$\cancel{= \frac{1}{4}\{4|\vec{A}||\vec{B}|\}} - \frac{1}{4}\left[4.\vec{A}\cdot\vec{B}\right]$$

$$= \vec{A}\cdot\vec{B}$$

$\therefore$ L.H.S = R.H.S (proved)

9. Prove that
(a) $\vec{P} \cdot (\vec{Q} + \vec{R}) = \vec{P}\cdot\vec{Q} + \vec{P}\cdot\vec{R}$
(b) $\vec{P} \times (\vec{Q} + \vec{R}) = \vec{P}\times\vec{Q} + \vec{P}\times\vec{R}$

(a) R.H.S $\vec{P}\cdot\vec{Q} + \vec{P}\cdot\vec{R}$

$$= |\vec{P}|.|\vec{Q}| \cos\theta + |\vec{P}|.|\vec{R}| \cos\theta$$

$$= |\vec{P}| \cos\theta \{|\vec{Q}| + |\vec{R}|\}$$

(b) R.H.S $\vec{P}\times\vec{Q} + \vec{P}\times\vec{R}$

$$= |\vec{P}|.|\vec{Q}| \sin\theta\, \hat{n} + |\vec{P}|.|\vec{R}| \sin\theta\, \hat{n}$$

$$= |\vec{P}|.|\vec{Q}| \sin\theta . 1 + |\vec{P}|.|\vec{R}| \sin\theta . 1$$

$$= |\vec{P}|.\cancel{|\vec{Q}|} \sin\theta \{|\vec{Q}| + |\vec{R}|\}$$

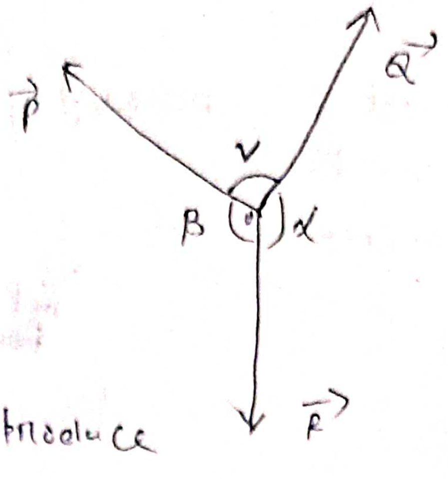
$$= |\vec{P}|. \sin 90^\circ \{|\vec{Q}| + |\vec{R}|\}$$

$$= |\vec{P}|. 1 \{|\vec{Q}| + |\vec{R}|\}$$

$$= |\vec{P}|.\{|\vec{Q}| + |\vec{R}|\}$$

LAMI'S theorem

Statement· If 3 vectors acting at a point in a plane produce equilibrium then each force [illegible] bears constant ratio with the sine of the angle between the other two



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Proof :

Since the 3 vectors produce equilibrium, the vector sum of of the 3 forces must be zero

$$\vec{P} + \vec{Q} + \vec{R} = \vec{0}$$

It can be shown that

prove it

$$\vec{P}\times\vec{Q} = \vec{Q}\times\vec{R} = \vec{R}\times\vec{P}$$

Taking magnitude of each cross product we get

$$= |\vec{P}||\vec{Q}|\sin\gamma$$

$$= |\vec{Q}|.|\vec{R}|\sin\alpha$$

$$= |\vec{R}||\vec{P}|\sin\beta$$

Dividing each quantity with $|\vec{P}||\vec{Q}||\vec{R}|$

we get $$\frac{\sin\gamma}{|\vec{R}|} = \frac{\sin\alpha}{|\vec{P}|} = \frac{\sin\beta}{|\vec{Q}|}$$

$$\Rightarrow \frac{|\vec{R}|}{\sin\gamma} = \frac{|\vec{P}|}{\sin\alpha} = \frac{|\vec{Q}|}{\sin\beta}$$ (proved)

Last topic doubt

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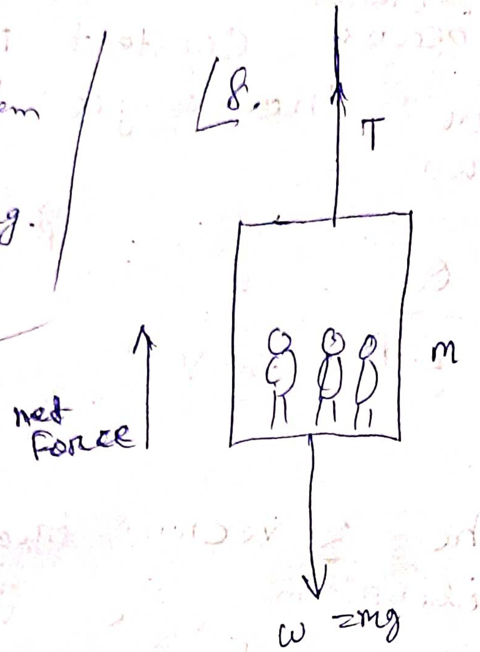
F.P.S System

Foot. m=slag.
pound.
second.



The tension must be always greater than the weight of the elevator.

Net force = T−w, upwards

$T - w = ma$

$\Rightarrow a = \frac{T-w}{m}$ upwards

$= \frac{T-1600}{50}$, upwards

(where $m = \frac{w}{g} = \frac{1600}{32}$ slug
= 50 slug)

Since the elevator was moving downwards at 20 feet/sec and brought to rest in ~~[illegible]~~ to 50 feet, we have u=20 feet/sec, V=0, S= 50 feet

Using the formula

$v^2 - u^2 = 2as$ we get

$\Rightarrow 0 - 400 = 2.\ a.(50)$

$\Rightarrow -400 = 100\ a$

7) $\Rightarrow$ $a = -4$ feet/sec^2

The Negative sine indicates that the acceleration is 4 feet/sec^2 upwards.

Hence

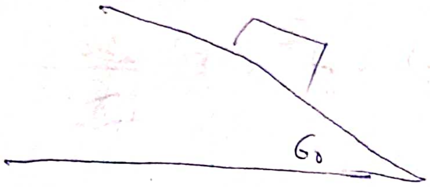
$$\frac{T-1600}{50} = 4$$

$$\Rightarrow T - 1600 = 200$$

$$\Rightarrow T = 1800 \text{ lb.}$$

25 . 1 ton = 2000lb .

(60)



Resolving the weight into two rectangular components we see that the component normal to the plane is the component w cosθ that acts like action which produces an equal and opposite reaction force and they make each other in effective. Hence the net force on the box is the component w sinθ acting parallel to the inclined plane downwards

~~force~~ $100\text{lb.} \sin 60°$

$= 100\text{lb.} \frac{\sqrt{3}}{2}$

$= 50\sqrt{3}$ lb

$= 50(1.732)$ lb

$= 86.6$ lb

(b) The box starts from rest

So $u = 0$, $S = 20$ feet

$a = g \sin\theta$, acting down the inclined plane.

Using the formula $S = ut + \frac{1}{2}at^2$ we get

$$\Rightarrow 20 = 0.t + \frac{1}{2}\cdot\overset{32}{\cancel{32}}\sin 60^\circ \cdot t^2$$

$$\Rightarrow 20 = \frac{1}{\cancel{2}}\cdot\overset{8\ 16}{\cancel{32}}\cdot\frac{\sqrt{3}}{\cancel{2}}\cdot t^2$$

$$\Rightarrow 20 = 8\sqrt{3}\, t^2$$

$$\Rightarrow t^2 = \frac{\overset{10\ 5}{\cancel{20}}}{\underset{4\ 2}{\cancel{8}}\sqrt{3}} = \frac{\cancel{10}\,5}{2\sqrt{3}}$$

$$= \frac{5}{2(1.732)}$$

$$= \frac{5}{3.464}$$

$$= 1.443$$

$$\Rightarrow t = 1.2 \text{ sec}$$

(c) Let us find the time for 40 feet

$S = 40$ feet

$a = g \sin\theta$

$u = 0$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 40 = 0 + \frac{1}{\cancel{2}}\cdot(\overset{16}{\cancel{32}})\sin 60^\circ \cdot t^2$$

$$\Rightarrow 40 = \frac{\overset{8}{\cancel{16}}\cdot\sqrt{3}}{\cancel{2}}\, t^2$$

$$\Rightarrow \frac{40}{8\sqrt{3}} = t^2$$

$\Rightarrow \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = t^2$

~~$\Rightarrow \frac{5}{1.732} = t^2$~~

$\Rightarrow \frac{5\sqrt{3}}{6} = t^2$ (1.732)

$\Rightarrow \frac{8.66}{3} = t^2$

$\Rightarrow 2.886 = t^2$

$\Rightarrow t = \sqrt{2.886} = 1.69$ sec,

The time taken to the next 20 feet = time taken to 40 feet − time taken to 20 feet

$= 1.69 - 1.2$

$= 0.49$ sec (ans)

22. $W = mg$

$\Rightarrow 20\,lb = m.32$

$\Rightarrow m = \frac{20\,lb}{32} = \frac{100}{16} = \frac{50}{8} = \frac{25}{4}$

$= 6.25$ slug.

If reaction of the floor be R and weight of the ~~body~~ man = W then eqn of motion is

$R - W = m.a$

$\Rightarrow \vec{a} = \frac{R-W}{m}$ upwards

(a) $R = W = 20\,lb$

(b) $R - w = ma$

$\Rightarrow R - 200 =$

(a) Stationary means displacement=0, velocity=0 acceleration=0.

$R - w = ma$

$\Rightarrow R - w = m.(0)$

$\Rightarrow R - w = 0$

$\Rightarrow R = w$

$\Rightarrow R = 200$lb (ans)

(b) Here $a = 16$ feet/sec², upwards

$R - w = ma$

$\Rightarrow R - 200 = (6.25) \cdot 16$

$\Rightarrow R - 200 = 100.$

$\Rightarrow R = 200 + 100 = 300$lb

(c) Here $a = 0$, $R = w = 200$ lb

(d) Here elevelator is moving upwards but decelerating 12ft/s²

So $a = -12$ feet/s²

$R - w = ma$

$\Rightarrow R - 200 = (6.25).(-12)$

$\Rightarrow R - 200 = -75$

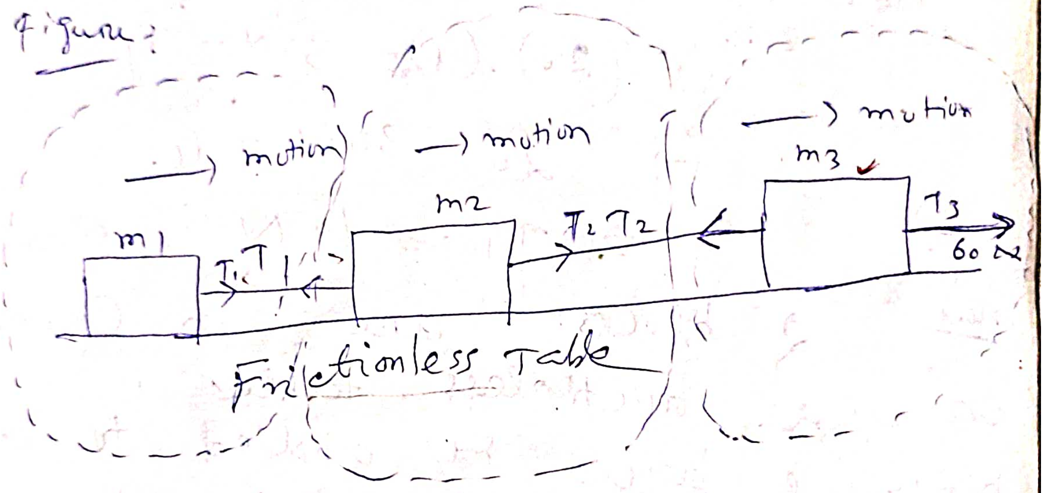
$\Rightarrow R = 200 - 75 = 125$ lab.

pblm

1. Three blocks are connected as shown in the figure on a horizontal frictionless table and pulled to the right with a force 60 Newton. If $m_1 = 10$ K.G
$m_2 = 20$ K.G
$m_3 = 30$ K.G,
and the tensions T_1 and T_2?

(ans. a = 1 meter/sec^2, 60N, 30N)

figure:



Net force on the body m_3

$$m_3 a = 60N - T_2 \text{ —(i)}$$

Net force on the body m_2

$$m_2 a = \cancel{8} T_2 - T_1$$

Net force on the body m_1

$$m_1 a = T_1$$

$$\Rightarrow m_1 a + m_2 a + m_3 a = 60N - \cancel{T_2} + \cancel{T_2} - \cancel{T_1} + \cancel{T_1}$$

$$= 60N$$

$$\Rightarrow a(m_1 + m_2 + m_3) = 60N$$

$$\Rightarrow a(60 \text{ K.G}) = 60N$$

$$\Rightarrow a = 1 \text{ m/s}^2$$

Putting the value of (a) in equation (i).

$m_3 a = 60N - T_2$

$\Rightarrow \overset{30}{\cancel{m_3}}.(1) = 60N - T_2$

~~$\Rightarrow m_3 = 60N$~~

$\Rightarrow 30N = 60N - T_2$

$\Rightarrow T_2 = 30N$

equ (ii) $m_2 a = T_2 - T_1$

$\Rightarrow (20).(1) = 30N - T_1$

$\Rightarrow 20 = 30N - T_1$

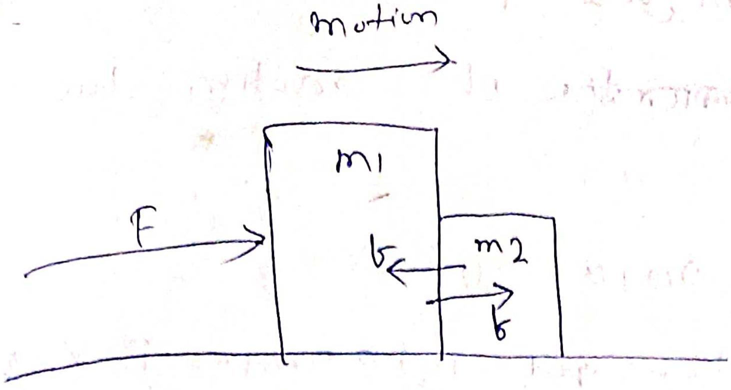
$\Rightarrow T_1 = 10N$,

next 2 blocks are in contaned on a frictionless table. A horizontal force is applied to one block as shown in the figure (1).

(a) If $m_1 = 2k.g$
$m_2 = 1k.g$
and $F = 3N$
Find the force of contact between the 2 blocks.

(b) Show that if the same force is applied to m_2 rather than to m_1 the force of contact between the block is 2N which is not the same as the value derived in a.

a)



Frictionless. Table

f = Force of contact.

equation of motion for the mass m_1

$m_1 a = F - f$ ——(i)

equation of motion for mass m_2

$m_2 a = f$ ——(ii)

Adding we get $m_1 a + m_2 a = F - \cancel{f} + \cancel{f}$

$\Rightarrow a(m_1 + m_2) = F$

$\Rightarrow a(3) = 3$ N

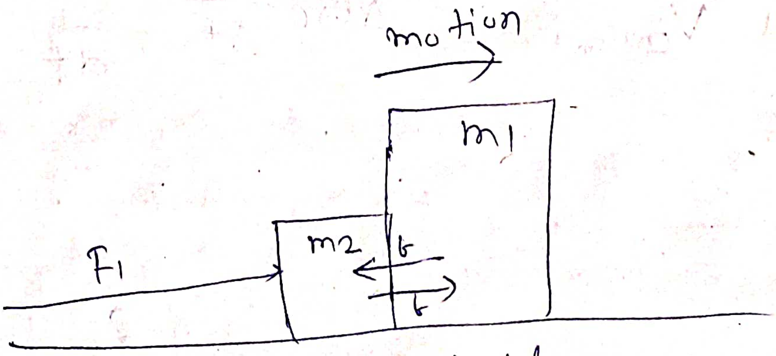
$\Rightarrow a = 1$ meter/sec²

$m_1 a = F - f$

$\Rightarrow 2 \cdot 1 = 3 - f$

$\Rightarrow f = 3 - 2 = 1$ N

(b)



Frictionless table.

equation of motion for the mass m_2.

$m_2 a = F - f$

Equation ~~for the~~ of motion for mass m_1

$m_1 a = f$

adding eqⁿs we get $m_2 a + m_1 a = F - \cancel{f} + \cancel{f}$

$\Rightarrow a(m_1 + m_2) = F$

$\Rightarrow a(3) = 3N$

$\Rightarrow a = 1\,m/s^2$

$m_2 a = F - f$

$\Rightarrow (1).(1) = 3N - f$

$\Rightarrow 1N = 3N - f$

$\Rightarrow f = 2N$

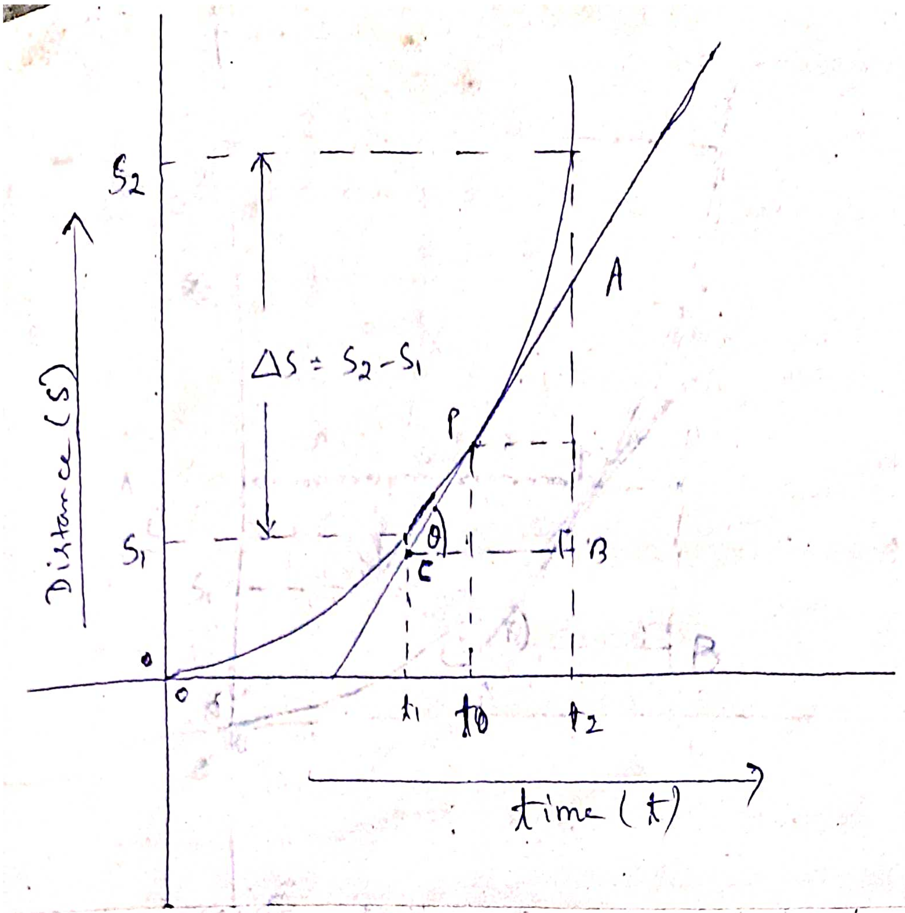
So the force of contact between the block is 2N.

Instantaneous velocity

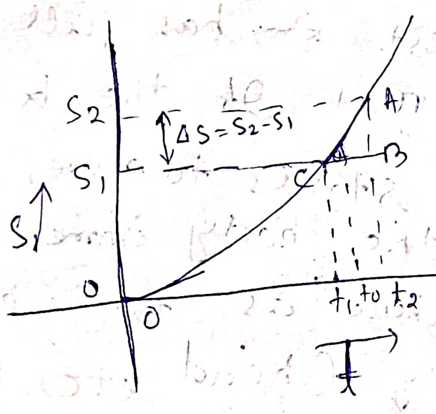
The velocity of a body at any instant of time defined as instantaneous velocity given by the expression

$$\vec{V}_{instant} = \lim_{\Delta t \to 0} \frac{\vec{\Delta s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

(P. T. O)



Slope $\tan\theta = \frac{AB}{BC} \neq \frac{\Delta S}{\Delta t} \neq V$ instant (because $AB \neq \Delta S$)



(Graph - 2)

Slope $= \tan\theta$
$= \frac{AB}{BC}$
$= \frac{\Delta S}{\Delta t}$
because $\Delta t = 0$ $=$ V instant.

~~because $\Delta t = 0$~~

<u>Instantaneous</u> acceleration : The acceleration of a body at any instant or a time is called instantaneous acceleration and is given by the expression

$$\vec{a}_{instant} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Newton's laws of motion:-

Newton has given three laws regarding the definition and measurement of force.

1st law :- Everybody continues in its state of rest or uniform motion in a straight line unless it is acted upon by a force.

→ So Force is a agent which when applied externally to a body changes or tends to change the state of rest or uniform motion along the straight line.

Thus the first law gives a qualitative definition of force.

(a) Inertia of rest

If a body is at rest, then it has a tendency to remain at rest.

Example:- A man sitting in a bus or train at rest ~~as~~ has all parts of his body at rest. If the bus or train suddenly starts to move, the lower side of the body immediately starts to move, where as the upper side of the body (head etc) remain as it is.

As a result he leans in the backward direction.

(b) Inertia of motion

When a man is inside a running ~~bus~~ bus or train, all parts of his body are moving with the same speed as that of the bus or train. If he will get down, the legs immediately comes to rest being in contact with the ground, where as the upper part of the body continues to move in the original direction. Hence he will fall forward

2nd law

This law gives a quantitative expression for measuring force. It can be stated in two different ways.

Method-1 The rate of change of momentum of a body is directly proportional to the net external unbalanced force acting on the body.

Let the initial velocity of the body be $\vec{u}$ and the final velocity be $\vec{V}$ after a time Δt sec.

$$\vec{V} = \vec{u} + \vec{a}\,\Delta t$$

or $$\vec{V} - \vec{u} = \vec{a}\,\Delta t$$

Multiplying by mass of the body to both the sides, we have

$$m\vec{V} - m\vec{u} = m\vec{a}\,\Delta t$$

or $$\vec{P_f} - \vec{p_i} = m\vec{a}\,\Delta t$$

or $$\Delta\vec{p} = m\vec{a}\,\Delta t$$

or $$\frac{\Delta\vec{p}}{\Delta t} = m\vec{a} =$$ rate of change of linear momentum

According to Newton's 2nd law,

$$\frac{\Delta\vec{p}}{\Delta t} \propto \vec{F}$$ where $\vec{F}$ = Net external unbalanced force

or $m\vec{a} \propto \vec{F}$

or $m\vec{a} = K\vec{F}$

where K is constant

$$\therefore \vec{F} = \frac{1}{K} \cdot m\vec{a} = K_1 \cdot m\vec{a}$$

where K_1 = Someother constant $= \frac{1}{K}$

The unit of force is defined in such a way that K_1 becomes equal to unity. Example:- 1 dyne is that a amount of force which when acts on a body of mass 1 gram produces an acceleration 1 cm/s^2

Thus 1 dyne = $K_1 \times 1\,gm \times 1\,cm/s^2$

This implies $K_1 = 1$

$$\boxed{\therefore \vec{F} = m\vec{a}}$$

$\therefore$ Net unbalanced external force = mass of the body × acceleration produced in the body. ⊛

Method - 2

Experimentally it is found that acceleration produced in a body is directly proportional to the force acting on the body, when mass of the body is kept constant.

That is $\vec{a}$

$\vec{a} \propto \vec{F}$ (when m is kept constant)

The acceleration is found to be inversely proportional to the mass of the body when magnitude of the force is kept constant. i.e

$$|\vec{a}| \propto \frac{1}{m} \text{ (when } \vec{F} \text{ is kept constant)}$$

Combining these two variations by the theorem of joint variation,

Statement

If A varies as B, when C is kept constant and A varies as C, when B is kept constant, then A will vary as BC when Both B and C vary.
([illegible])

We get

$$\vec{a} \propto \frac{\vec{F}}{m} \text{ (when } \vec{F} \text{ and } m \text{ vary)}$$

$$\text{or } \vec{a} = K\frac{\vec{F}}{m}$$

where K is constant

$$\text{or } \vec{F} = \frac{1}{K} m\vec{a} = K_1 m\vec{a}$$

where K_1 = some other constant $= \frac{1}{K}$

The units of force are defined in such a way that K_1 becomes equal to unity.

Example :- 1 Newton is that of amount of force which when acts on a body of mass 1 kilogram produces an acceleration of 1 m/s².

When a force acts on a body of mass 1kg produces an accel^n of 1 m/s² then it is called 1 Newton

$1 \text{ Newton} = K_1 \cdot 1 \text{ K.g} \cdot 1 \text{ m/s}^2$

This implies $K_1 = 1$

$$\therefore \boxed{\vec{F} = m\vec{a}}$$

3rd law

To every action, there is an equal and opposite reaction.

Example: If we strike a table with our hand, then the table reacts, which is easily realized (felt).

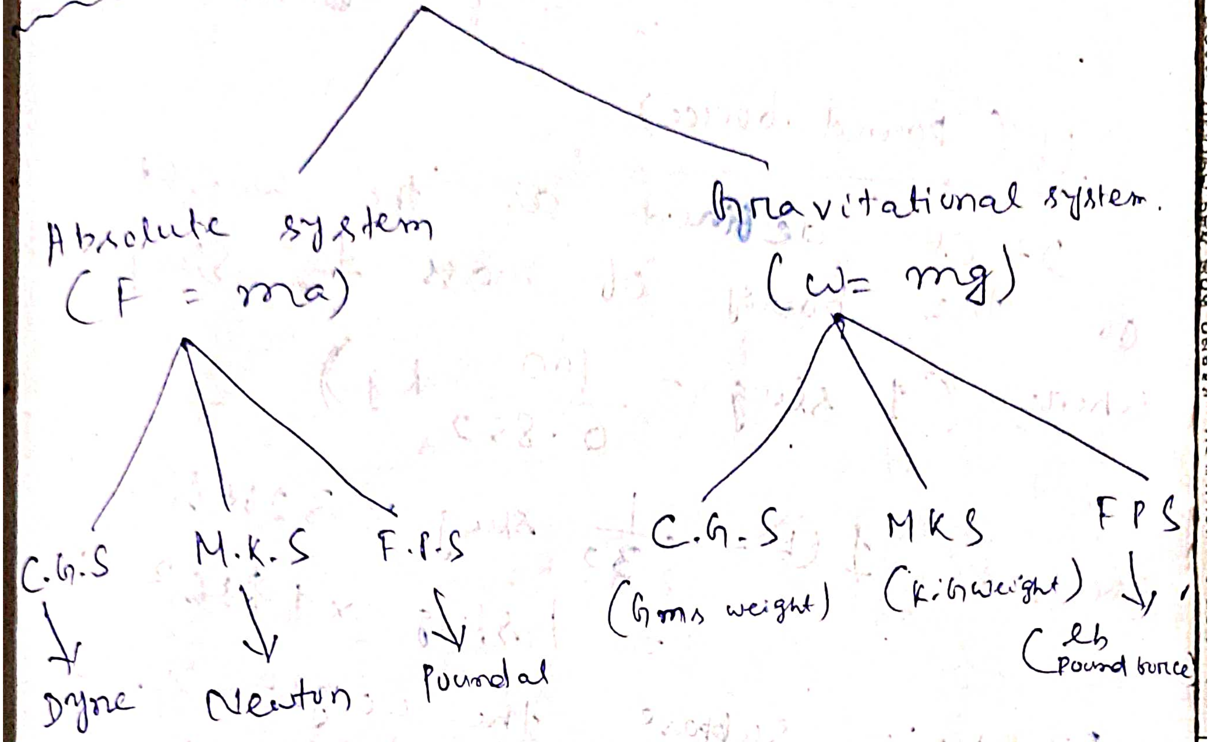
(*) → See (71) Page 124 for Third Law,

Difference between mass and weight

Mass	weight
1. It is a scalar quantity.	1. It is a vector quantity.
2. It is the quantity of matter present in a body.	2. It is the force with which a body is attracted towards the center of the earth. $\therefore \vec{W} = m\vec{g}$
3. The dimension of mass $[M]$	3. Dimension of weight is $[M]\left[\frac{L}{T^2}\right] = [MLT^{(-2)}]$ $= [MLT]^{-2}$
4. Mass is a Constant quantity unless the body moves with very high velocity.	4. weight is a variable quantity because it depends on 'g' which changes from place to place on the surface of the earth.
5. The units of mass are gm, Kg, pound mass, slug.	5. The units of weight are dyne, newton, poundal, gms weight, Kg weight, lbf (pound force).

6. Mass is measured by a physical balance.	6. Weight is measured by a spring balance.

Units of force



Poundal

It is that amount of force which when acts on a body of mass one pound (453.6 gm) produces an acceleration of $1 \, ft/sec^2$.

Grams weight :

It is defined as the weight of a body of mass 1 gm

$$\therefore \; 1 \text{ gms weight} = 1 \, gm \times 980 \, cm/s^2$$
$$= 980 \text{ dyne.}$$

The value of 1 gms wt varies from place to place due to variation of 'g'.

K.g wt

It is defined as the weight of a body of mass 1 K.g

$$\therefore \quad 1 \text{ K.g wt} = 1 \text{ K.g} \times 9.8 \text{ m/s}^2$$
$$= 9.8 \text{ Newton.}$$

lb (pound force)

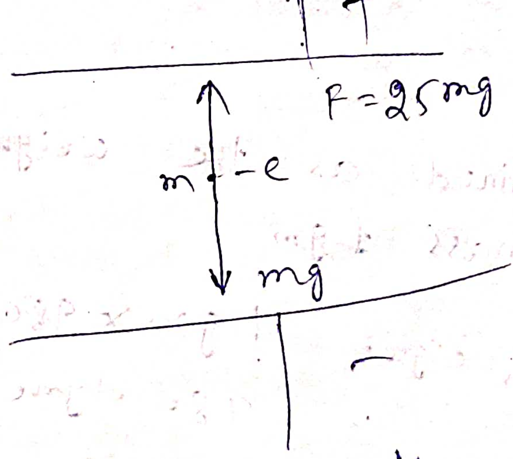
It is defined as the weight of the body of mass $\frac{1}{32}$ slug, where $\left(1 \text{ slug} = \frac{100}{6.852} \text{ Kg}\right)$)

$$\therefore \quad 1 \text{ lb} = \frac{1}{32} \text{ slug} \times 32 \text{ ft/s}^2$$
$$= 1 \text{ slug} \times 1 \text{ ft/s}^2$$

Example:- suppose the weight of a body is given in 64 lb then its mass is $\frac{64}{32} = 2$ slug (because $w = mg$, so that $m = \frac{w}{g}$)

page-77

7.



Net force on the electron F =

$$= (25mg - mg), \text{ upwards}$$
$$= 24 \text{ mg upwards}$$

$24 mg = ma$

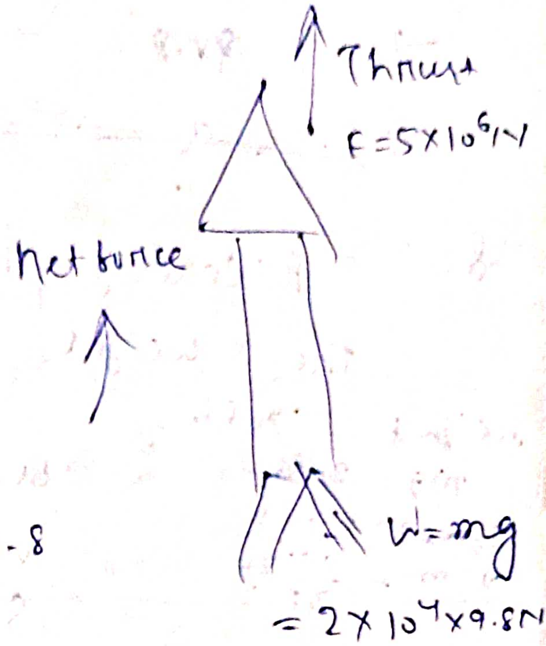
$\therefore a = \frac{24mg}{m} = 24 \times 9.8$ m/s², upwards

= 235 2 m/s² upwards)

1. 1 sled, which with its load

weighs

13.



Net force

$= 5 \times 10^6 N - 2\times10^4 \times 9.8$

$= 10^4 (500N - 19.6)$

$= 10^4 (480.4)$

$\therefore ma = 10^4 (480.4)$

$\Rightarrow 2 \times \cancel{10^4} \times ~~9.8~~ \; a = \cancel{10^4} (480.4)$

$\Rightarrow ~~19.6~~ \; 2a = 480.4$

$\Rightarrow a = \cancel{\frac{480.4}{19.6}} \; \frac{480.4}{2} = 240.2$

$$5\times10^6 - 2\times10^4 \times 9.8 = (2\times10^4)\times a$$

$$\Rightarrow \cancel{10^4}(500 - 19.6) = 2\times\cancel{10^4} \times a$$

$$\Rightarrow a = \frac{480.4}{2} = 240.2$$

m/s² upwards.

(b) $m = 1 \times 10^4 \times 9.8$

$ma = 5 \times 10^6 - 2 \times 10^4 \times 9.8$

$\Rightarrow (2 \times \cancel{10^4})\, a = \cancel{10^4}(500 - \overset{19.6}{\cancel{9.8}})$

$\Rightarrow a = \dfrac{480.4}{2}$ m/s² upwards

$= 240.2$ m/s²

~~$\Rightarrow a = \dfrac{480.6}{9.8}$~~

8. According the question the weight of the object = 20lb.

weight = 20lb

$mg = 20$lb.

$\Rightarrow m \cdot 32 = 20$lb

$\Rightarrow m = \dfrac{20}{32}$ lb

$u = 18$ feet/s

$v = 50$ feet/s.

$s = 40$ft

$v^2 - u^2 = 2as.$

$\Rightarrow 2500 - 324 = 2 \cdot (a) \cdot (40ft)$

$\Rightarrow 2176 = 80\, a$

$\Rightarrow a = \dfrac{2176}{80} = 27.2$ ft/sec

$$\begin{array}{r} 18 \\ 18 \\ \hline 144 \\ 18 \\ \hline 324 \end{array} \qquad \begin{array}{r} 2500 \\ 324 \\ \hline 2176 \end{array}$$

$$80)\,2176\,(27.2$$
$$\begin{array}{r} 160 \\ \hline 576 \\ 560 \\ \hline 160 \\ 160 \end{array}$$

$$8)\,27.2\,(3.4$$
$$\begin{array}{r} 24 \\ \hline 32 \\ 32 \end{array}$$

accelerating force

$F = ma$

$= \dfrac{\overset{5}{\cancel{20}}}{\underset{\underset{8}{\cancel{16}}}{\cancel{32}}} \cdot (\overset{3.4}{\cancel{27.2}})$

$= 5 \times 3.4$

$= 17.0 = 17$lb

6. The mass of rifle bullet

$m = 10g.$

$u = 0, \quad V = 400$ m/s $= 40000$ cm/s.

$S = 50$ cm.

$v^2 - u^2 = 2as$

$\Rightarrow (40000)^2 - (0) = 2.(a).50$

$\Rightarrow 16\,0000,00\cancel{00} = \cancel{2}\,1\cancel{00}\,a$

$\Rightarrow 1.6 \times 10^7 \ c.m/s^2 = a$

$\Rightarrow a = 1.6 \times 10^7 \ c.m/s^2.$

barrel, → বন্দুক নল

accelerating force

$= m a$

$= 10.g \times 1.6 \times 10^7 \ c.m/s^2$

$= 1.6 \times 10^8 \ \frac{gm.cm}{s^2}$

$= 1.6 \times 10^8$ dyne

Relation between different units of force

(1) Newton & dyne

$1\,N = 1\,K.G \,.\, 1\,m/s^2$

$= 10^3\,gm.\ 10^2\ c.m/s^2$

$= 10^5$ dyne

(2) Poundal and dyne

1 poundal $= 1$ pound (mass) $\times 1\,ft/s^2$

$= 453.6\,gm \times 12 \times 2.54\ c.m/s^2$

$= 453.6 \times 30.48$ dyne

$= 13825.728$ dyne

(3) Pound and poundal

$1\,lb = 1$ slug $\times 1\,ft/s^2$

Relation
Pound and Newton

1 lb
$= 1\ \text{slug} \times 1\ \text{ft/s}^2$
$= \frac{100}{6.852} \times \frac{12\ \text{m/s}^2}{100}$
$= 1.75\ \text{N}$

$$= \frac{100}{6.852}\ \text{Kg} \times 1\ \text{ft/s}^2$$

$$= \frac{100 \times 1000\ \text{gm}}{6.852} \times 1\ \text{ft/s}^2$$

$$= \frac{10^5\ \text{gm}}{6.852 \times 453.6}\ \text{pound mass} \times 1\ \text{ft/s}^2$$

$$= 32.17\ \text{poundal}$$

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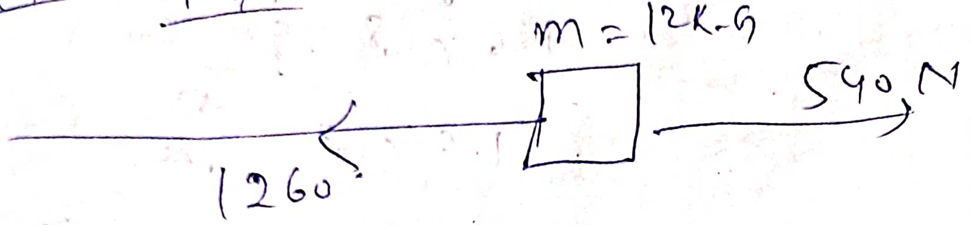


Net force on the body

$= (1260 - 540)\ \text{N}$

$= 720\ \text{N}$

$= ma$

$= 12a$ (From Newton's 2nd law)

$$\Rightarrow \quad a = \frac{720}{12} = 60\ \text{m/s}^2$$

Direction of the accl. is along the bigger vector.

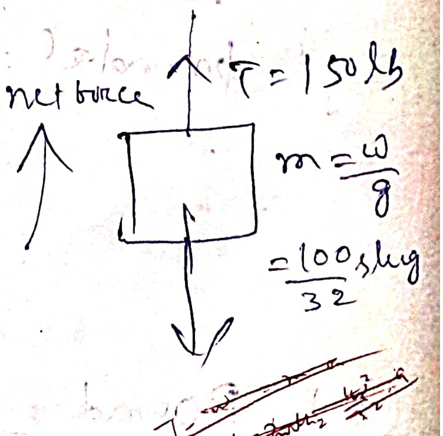
(4) ~~From Newtons 2nd law~~

~~Net force~~ ~~= F~~

$T - W = ma$

$$\Rightarrow 50\ \text{lb} = \frac{100}{32}\ \text{slug} \times a$$

$$\Rightarrow \vec{a} = \frac{50 \times 32}{100}\ \text{ft/s}^2$$



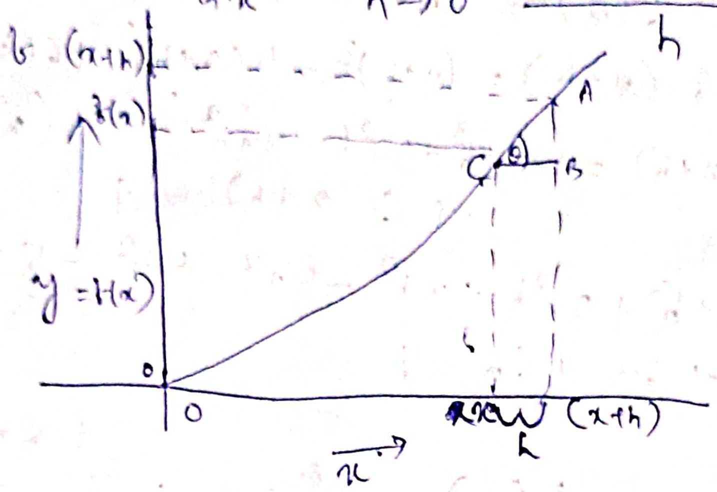
= 16 feet/sec upward

— 0 —

Derivative of a function

Let a function be denoted by f(x).
The derivative of f(x) is defined
as

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$$\text{Slope} = \tan\theta = \frac{AB}{BC} = \frac{f(x+h) - f(x)}{h}$$

Example-1 $y = x^2 + 2x + 3$

To find $\frac{dy}{dx}$

$$f(x) = x^2 + 2x + 3$$

$$f(x+h) = (x+h)^2 + 2(x+h) + 3$$

$$f(x+h) - f(x) = (x+h)^2 + 2(x+h) + 3 - (x^2+2x+3)$$

$$= \cancel{x^2} + h^2 + 2hx + \cancel{2x} + 2h + \cancel{3} - \cancel{x^2} - \cancel{2x} - \cancel{3}$$

$$= h^2 + 2hx + 2h$$

$$= h(h + 2x + 2)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(h+2x+2)}{h} = h + 2x + 2$$

Now using the ~~formula~~ $\lim_{h \to 0}$ we get

$$\frac{dy}{dx} = h + 2x + 2 = 0 + 2x + 2 = 2x + 2$$

Example - 2

If $y = x^3 - 4x^2 + 6x - 9$

Since $f(x) = x^3 - 4x^2 + 6x - 9$

$$f(x+h) = (x+h)^3 - 4(x+h)^2 + 6(x+h) - 9$$

$$\Rightarrow f(x+h) = (x^3 + h^3 + 3x^2h + 3xh^2) - 4(x^2 + h^2 + 2xh) + 6(x+h) - 9$$

$$= x^3 + h^3 + 3x^2h + 3xh^2 - 4x^2 - 4h^2 - 8xh + 6x + 6h - 9$$

$f(x+h) - f(x)$

$$= (x^3 + h^3 + 3x^2h + 3xh^2 - 4x^2 - 4h^2 - 8xh + 6x + 6h - 9) - (x^3 - 4x^2 + 6x - 9)$$

$$= \cancel{x^3} + h^3 + 3x^2h + 3xh^2 - \cancel{4x^2} - 4h^2 - 8xh + \cancel{6x} + 6h - \cancel{9} - \cancel{x^3} + \cancel{4x^2} - \cancel{6x} + \cancel{9}$$

$$= h^3 + 3x^2h + 3xh^2 - 4h^2 - 8xh + 6h$$

$$= h(h^2 + 3x^2 + 3xh - 4h - 8x + 6)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cancel{h}(h^2 + 3x^2 + 3xh - 4h - 8x + 6)}{\cancel{h}}$$

$$= h^2 + 3x^2 + 3xh - 4h - 8x + 6$$

Now using the lim $h \to 0$

we get $0 + 3x^2 + 0 - 0 - 8x + 6$

$= 3x^2 - 8x + 6$ (ans)

Formulae on differentiation (to find derivative)

1. If $y = x^n$, then $\frac{dy}{dx} = \frac{dx^n}{dx} = n.x^{n-1}$

2. If $y = c$ = (constant) then $\frac{dy}{dx} = 0$

3. If $y = c.x^n$. then $\frac{dy}{dx} = c.\frac{d(x^n)}{dx}$

$= c.n.x^{n-1}$

Example-1

$y = x^2 + 2x + 3$

To find $\frac{dy}{dx}$ now $\frac{dy}{dx} = \frac{d(x^2+2x+3)}{dx}$

$= \frac{d(x^2)}{dx} + \frac{d(2x)}{dx} + \frac{d(3)}{dx}$

$= 2.x^{2-1} + 2.\frac{d(x^1)}{dx} + 0$

$= 2x^1 + 2 \times 1 * x^{1-1}$

$= 2x + 2 * x^0$

$= 2x + 2 * 1$

$= 2x + 2$

Example-2

$y = x^3 - 4x^2 + 6x - 9$

To find $\frac{dy}{dx}$

now $\frac{dy}{dx} = \frac{d(x^3 - 4x^2 + 6x - 9)}{dx}$

$$= \frac{d(x^3)}{dx} - \frac{d(4x^2)}{dx} + \frac{d(6x)}{dx} - \frac{d(9)}{dx}$$

$$= 3.x^{3-1} - 4.2.x^{2-1} + 6\times1(x^0) + 0$$

$$= 3.x^2 - 8x + 6$$

(3) Four particles A, B, C and D in such a way that their distances at any instant of time from a fixed point given by the eqns

$$x_A = 3t^2 + 5t - 9$$

$$x_B = 5t^3 + 9t^2 - 3t + 6$$

$$x_C = 5\sin 60^\circ - 3\cos 30^\circ +$$

$$x_D = 2t + 5$$

Which of them moves with

D (a) uniform velocity ?

A (b) uniformly accelerated motion?

(c) Non-uniformly accelerated motion. B

C (d) 0 velocity (at rest) ?

Ans :- Let's find the velocity of the four particles at any instant of time.

$\therefore v_A = \frac{dx_A}{dt}$

$= \frac{d(3t^2+5t-9)}{dt}$

$= \frac{d\,3t^2}{dt} + \frac{d(5t)}{dt} - \frac{d(9)}{dt}$

~~$= 3.t^{2-1} + 5.t^{1-1} - 0$~~

~~$= 6t^2 + 5$~~

$= 3.\frac{2\,d t^2}{dt} + 5.\frac{dt}{dt} - 0$

$= 3.2\,t^{(1-1)} + 5.t^0$

$= 6t + 5$

$V_B = \frac{dx_B}{dt} = \frac{d(5t^3+9t^2-3t+6)}{dt}$

$= \frac{d\,5t^3}{dt} + \frac{d(9t^2)}{dt} - \frac{d(3t)}{dt} + \frac{d(6)}{dt}$

$= 5.\frac{d(t^3)}{dt} + 9\frac{dt^2}{dt} - 3\frac{dt}{dt} + 0$

$= 5\times 3t^{(3-1)} + 9\,2t^{(2-1)} - 3.\frac{dt}{dt}$

$= 5.3t^2 + 9\,2t^1 - 3.1.t^0$

$= 15t^2 + 18t^1 - 3$

$= 15t^2 + 18t - 3.$

$V_C = \frac{dx_C}{dt} = \frac{d(8\sin 60^\circ - 3\cos 30^\circ + 7)}{dt}$

$= 0$

$V_D = \frac{dx_D}{dt} = \frac{d(2t+5)}{dt} = \frac{d(2t)}{dt} + \frac{d(5)}{2t}$

$= 2\times 1\times t^0$

$= 2$

(a) Particle D moves with uniform velocity

(d) Particle C reach at rest

(b) Let us find the accl of A

$$a_A = \frac{dv_A}{dt} = \frac{d}{dt}(6t-15)$$

$$= \frac{d(6t)}{dt}$$

$$= 6\times1\times t^0 = 6\ m/s^2$$

$$a_B = \frac{dv_B}{dt} = \frac{d(15t^2+18t-3)}{dt}$$

$$= 15\frac{d(t^2)}{dt} + \frac{d\,18t}{dt} - \frac{d(3)}{dt}$$

$$= 15\times2.t^{(2-1)} + 18.t^{(1-1)} - 0$$

$$= 30t + 18$$

Thus the particle A is moving with uniform accl and B is moving with non-uniform accl.

Q: A body moves in a straight line along x axis. Its distance from the origin is given by the eqn $x = 8t - 3t^2$ where x is measured in metre and t in sec. Find the average velocity in the interval t=0 to t=1 and from t=0 to t=4 sec.

Ans: Lets find instantinious velocity $\frac{ds}{dt}$

$$= \frac{d(8t - 3t^2)}{dt}$$

$$= \frac{d(8t)}{dt} - \frac{d(3t^2)}{dt}$$

$$= 8 \cdot \frac{d(t)}{dt} - 3 \cdot \frac{dt^2}{dt}$$

$$= 8(1) t^0 - 3 \cdot 2 \cdot t^{2-1}$$

$$= 8 - 6t$$

taking t=0, $8 - 6.0 = 8 \text{ m/s}^2$,

taking t=1, $8 - 6(1) = 2 \text{ m/s}$

taking t=4, $8 - 6(4) = -16 \text{ m/s}$

Avarage velocity (b) $\frac{t_0 + t_4}{2} = \frac{8-16}{2} = -4 \text{ m/s}$

(a) $\frac{t_0 + t_1}{2} = \frac{2+8}{2} = 5 \text{ m/s}$

3. The motion of a particle along x axis is given by the equation: $x = 9 + 5t^2$ where x is the distance in c.m and t is the time in seconds. Find

(a) the displacement after 3 seconds and 5 seconds

(100) (b) Avarage ([illegible]) velocity during the interval from t = 3 sec to t = 5 sec.

(C) Instantanious velocity after t = 3 secs.

(Ans: 54 ~~sec~~ c.m, 134 c.m, 40 c.m/sec 30 c.m/sec)

Ans: Let the instantaneous velocity

$$V_x = \frac{dx}{dt}$$

$$= \frac{d(9+5t^2)}{dt}$$

$$= \frac{d(9)}{dt} + \frac{d(5t^2)}{dt}$$

$$= \cancel{9\,0\,dt} = 0 + 5 \cdot \frac{dt^2}{dt}$$

$$= 5 \cdot 2 \cdot t^{2-1}$$

$$= 10t$$

velocity ~~at~~ when t = 3sec, and t = 5sec

10t = 10×3 = 30 ~~sec~~ c.m / ~~t~~ sec

= 10×5 = ~~50 sec/t~~ = 50 c.m/sec

(a) Displacement

t = 3sec, x = 9 + 5t²

= 9 + 5(9)

= 9 + 45

= 54 c.m

t = 5sec, x = 9 + 5t²

= 9 + 5(5)²

= 9 + 125

= 134 c.m

(i) Avairage ~~speed~~ velocity $= \frac{30+50}{2} = \frac{80}{2} = 40$ m/sec.

(ii) Instantinous velocity after 3 secs

$= 10t$

$= 10 \cdot (3 \text{ sec})$

$= 30$ m/sec

4. A particle is moving in a straight line and its distance in metre from a given point along a line after t sec from start is given by $x = t^3 - 2t - 16$

(a) find the velocity at the ~~end~~ of three sec.

(b) find the accl at the end of 4 sec.

(c) what is the accl when it is at a distance 5 metre from the given point

(Ans 25 m/s, 24 m/s², 18 m/s²)

Ans : $x = t^3 - 2t - 16$

$$V = \frac{dx}{dt} = \frac{d(t^3 - 2t - 16)}{dt}$$

$$= \frac{dt^3}{dt} - \frac{d(2t)}{dt} - \frac{d(16)}{dt}$$

$$= 3t^2 - 2 \cdot \frac{dt}{dt} - 0$$

$$= 3t^2 - 2$$

(a) after 3secs

$3t^2 - 2$

$= 3(9) - 2$

$= 27 - 2$

$= 25$ m/s,

~~(b) after 4secs~~

~~$3t^2 - 2$~~

~~$= 3(16) - 2$~~

$=$

(b) Lets find the accl

$$a_A = \frac{dv_A}{dt} = \frac{d(3t^2 - 2)}{dt}$$

$$= \frac{d(3t^2)}{dt} - \frac{d(2)}{dt}$$

$$= 3.2.t^1 - 0$$

$$= 6t$$

$a = 6t = 6.4 = 24$ m/s²

(c) If x is 5 meter then,

$5 = \cancel{3t^2} \; t^3 - 2t - 16$

$\Rightarrow t^3 - 2t = 21$

$\Rightarrow t^3 - 2t - 21 = 0$

by trial and error method, Let's check whheather t = 1 sec satisfy the eqn

$1^3 - 2.(1) - 21 = 22 \neq 0$

If t = 2 sec satisfy the eqn

$2^3 - 2(2) - 21 = 17 \neq 0$

If t = 3 sec satisfy the eqn

∴ $(3)^3 - 2(3) - 21 = 0$

Thus $(t-3)$ must be a factor of the cubic eqn to find the other two factors lets proceed as follows.

$$t^3 - 2t - 21 = 0$$

$$\Rightarrow t^3 - 3t^2 + 3t^2 - 2t - 21 = 0$$

$$\Rightarrow t^2(t-3) + 3t^2 - 9t + 9t - 2t - 21 = 0$$

$$\Rightarrow t^2(t-3) + 3t(t-3) + 7t - 21 = 0$$

$$\Rightarrow t^2(t-3) + 3t(t-3) + 7(t-3) = 0$$

$$\Rightarrow (t-3)(t^2 + 3t + 7) = 0$$

If $t^2 + 3t + 7 = 0$ then

$$t = \frac{-3 \pm \sqrt{(3)^2 - 4.(1).(7)}}{2.(1)}$$

$$= \frac{-3 \pm \sqrt{9 - 28}}{2}$$

$$= \frac{-3 \pm \sqrt{-19}}{2} = \frac{-3 \pm \sqrt{-1 \times 19}}{2}$$

$$= \frac{-3 \pm i\sqrt{19}}{2}$$

= imaginary which is not acceptable.

Thus $t = 3$ sec is the only physically acceptable time

$a = 6t$

$= 6 \times 3$

$= 18 m/s^2$

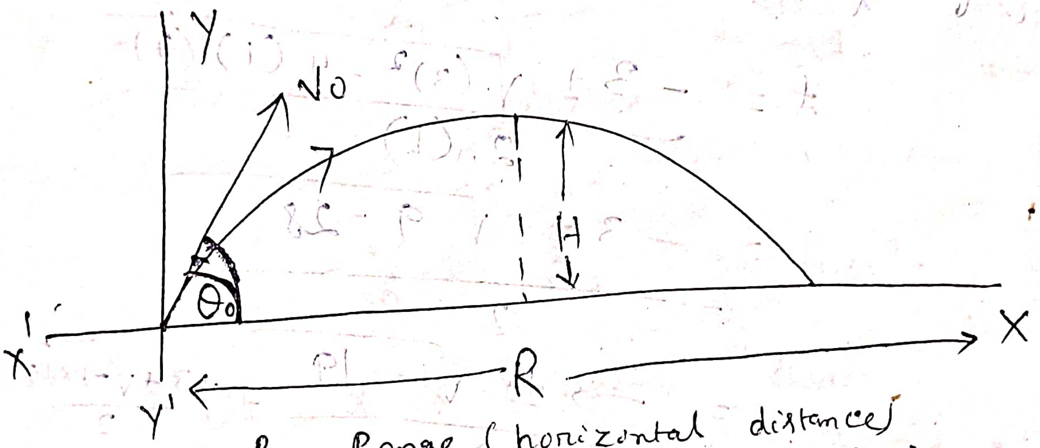
Unit four B

Projectile motion:

Defn of a ~~projectile~~ projectile

A Projectile is a body having no motive power of its own and which is projected into space making some angle with the horizontal such that after projection its motion is controlled by gravity and air resitance.

Definations associated with projectile motion.



R= Range (horizontal distance)

(1) Time of ~~fligt~~ flight (T)

It is defined as the total time taken by a projectaill to move from the starting point till it reaches the horizontal surface.

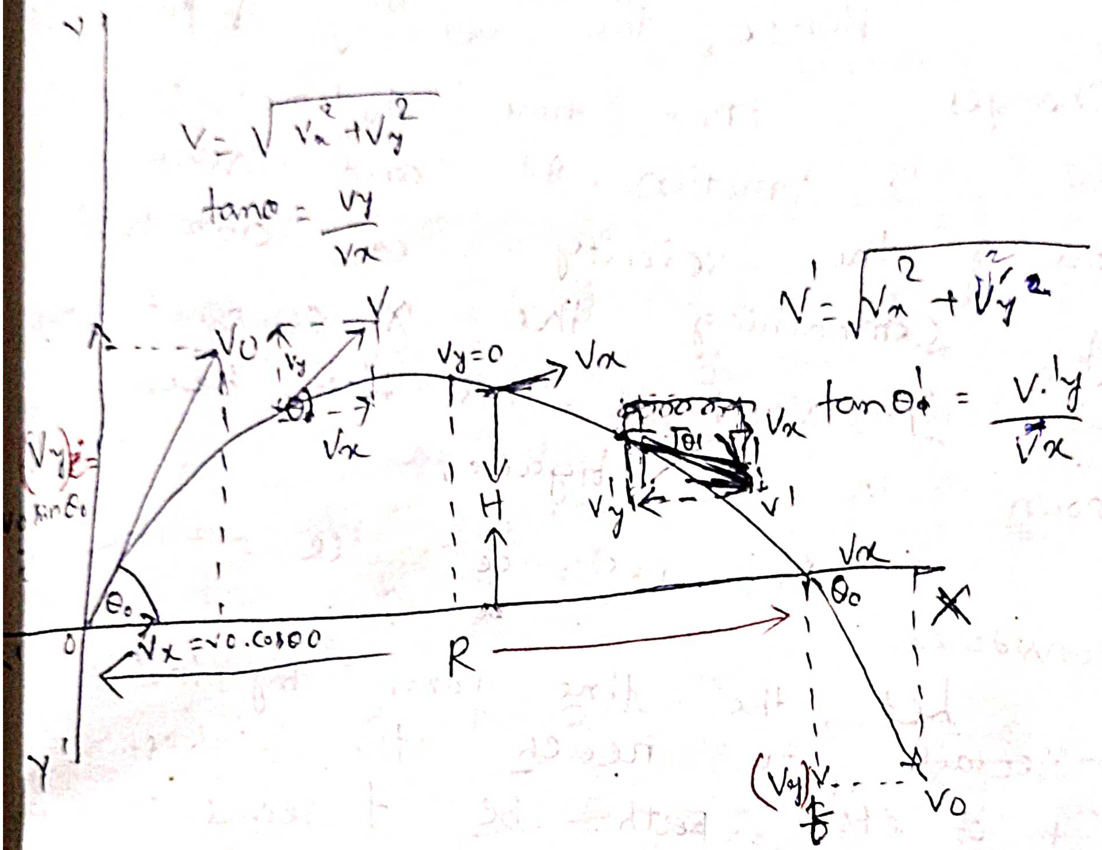
(2) height (H)

It is the maximum vertical distance covered by the projectail during its motion

(e) Range (R)

It is defined as the maximum horizontal distance covered by the projectial during its motion.

Expressions for T, H, R



The project~~al~~ile motion is characterised by the initial speed with which it is projected (Vo) and the angle made with the horizontal (θo). The project~~al~~ile motion is confined to a two dimensional plane, say, XY plane.

Therefore, the initial velocity of projection can be resolved into two rectangular components, one along the horizontal of magnitude of Vo cosθo which remains constant through out the motion. The other component is $V_0 \sin \theta_0$ along the vertical direction which is controlled by the gravity. Therefore its value decreases till the maximum height is reached and there after its value increases.

Hence, the velocity of projectaile changes from time to time through out its motion. At any instant of time the velocity is obtained by combaining the X component and the Y component. This has been shown in ~~the~~ figure.

Air resitance will not be considered.

Let the time taken by the projectaile to reach the highest point of its path be t second. considering the y component of ~~the~~ velocity and using the formula $V = u + at$

we get $u = (V_y)_i = V_0 \sin \theta_0$

$V = 0$, $a = -g$

$$V = u + at$$

$$\Rightarrow 0 = V_0 \sin \theta_0 - gt$$

$$\Rightarrow t = \frac{V_0 \sin \theta_0}{g} \quad \text{—— (i)}$$

Time of flight = Total time taken by the projectile to move from the starting point till it reaches the horizontal surface.

$$\therefore \quad T = 2t = \frac{2 v_0 \sin\theta_0}{g} \quad \text{—— (ii)}$$

(Because time of ascent = time of descent)

Using the formula $v^2 - u^2 = 2as$, we get

$$\Rightarrow 0^2 - (v_0 \sin\theta_0)^2 = 2.(-g)H$$

$$\Rightarrow H = \frac{v_0^2 \sin^2\theta_0}{2g} \quad \text{—— (iii)}$$

The horizontal distance covered by the projectaile during T second is the Range of the projectaile. It will be covered with the help of the horizontal component of the velocity.

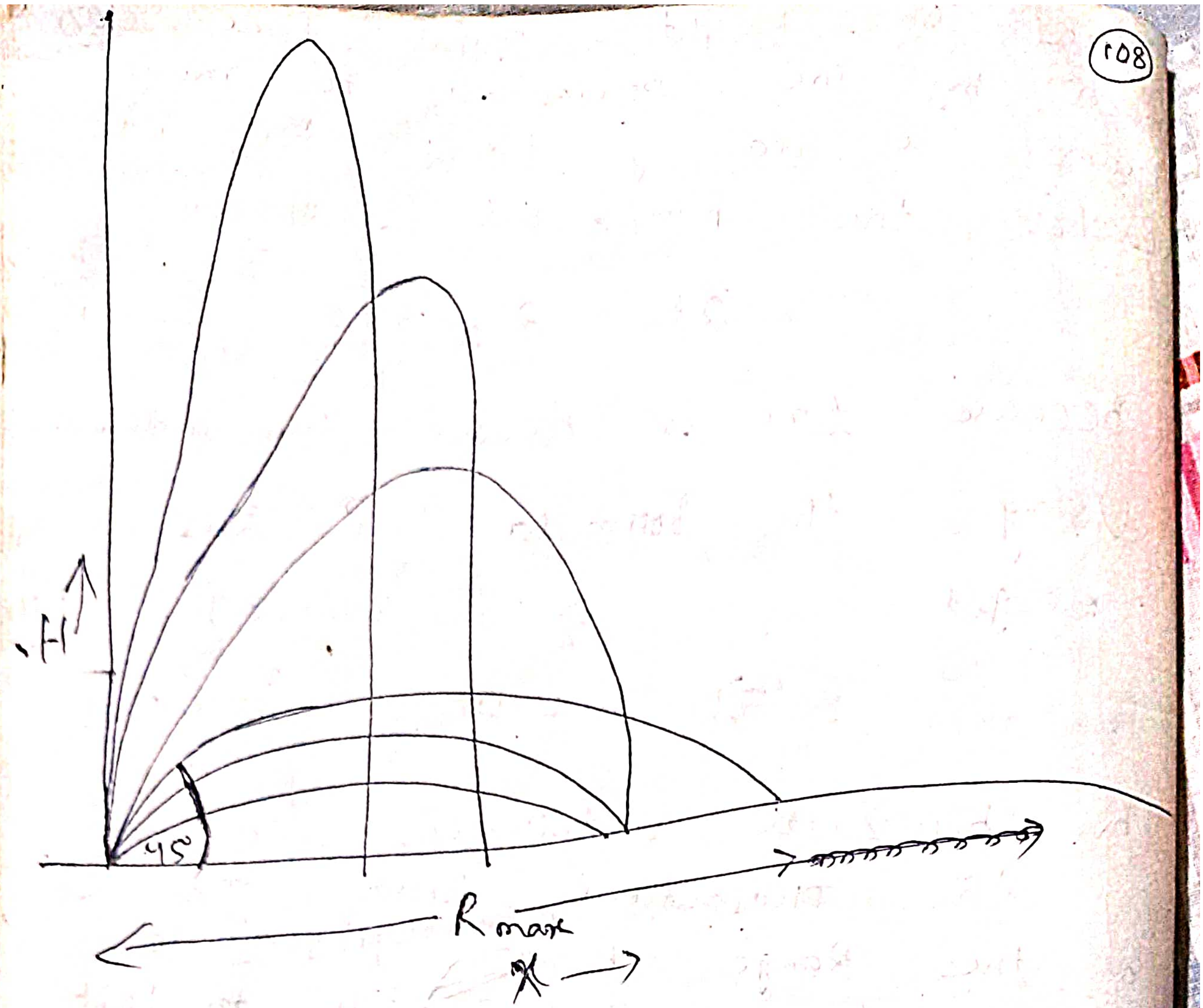
$$\therefore R = v_x . T$$

$$= v_0 \cos\theta_0 . \frac{2 v_0 \sin\theta_0}{g}$$

$$\boxed{= \frac{v_0^2 \sin 2\theta_0}{g}} \quad \text{—— (iv)}$$

To prove that Range of a projectaile is maximum when the angle of projection is 45°

P.T.O



~~Form the~~ From the expression of the projectail we see that R will be maximum only when

$\sin 2\theta_0 = \text{maximum} = 1$

Thus $\sin 2\theta_0 = \sin 90^\circ$

$\Rightarrow 2\theta_0 = 90^\circ$

$\Rightarrow \theta_0 = 45^\circ$

To prove that range of a projectail remains the same for two different angles of projection θ_1 and θ_2 which satisfy the relation $\theta_1 + \theta_2 = 90^\circ$

Proof :- Let the ranges be R_1 and R_2 when angles of projection are θ_1 and θ_2 respectively while speed of projection (v_0) remains the same.

$$\therefore R_1 = \frac{v_0^2 \sin 2\theta_1}{g}$$

$$R_2 = \frac{v_0^2 \sin 2\theta_2}{g}$$

For $R_1 = R_2$

$$V_0^2 \sin 2\theta_1 = V_0^2 \sin 2\theta_2$$

$$\Rightarrow \sin 2\theta_1 = \sin(180^\circ - 2\theta_2)$$

$$\Rightarrow 2\theta_1 = 180^\circ - 2\theta_2$$

$$\Rightarrow 2(\theta_1 + \theta_2) = 180^\circ$$

$$\Rightarrow \theta_1 + \theta_2 = 90^\circ$$

Problem:

Prove that the heights reached by two projectails having the same Range, but projected at angles 30° and 60° is 1:3

Ans: Since $R_1 = R_2$

$$\Rightarrow \frac{V_1^2 \sin(2\cdot 30^\circ)}{g} = \frac{V_2^2 \sin(2\times 60)}{g}$$

(Note sin 60° and sin 120° cancel)

$$\Rightarrow V_1^2 = V_2^2 \Rightarrow V_1 = V_2$$

Thus the speed of projection are the same.

$$\therefore \frac{H_1}{H_2} = \frac{\frac{V_1^2 \sin^2 30^\circ}{2g}}{\frac{V_2^2 \sin^2 60^\circ}{2g}}$$

$$= \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{\frac{1}{4}}{\frac{3}{4}} = 1:3$$

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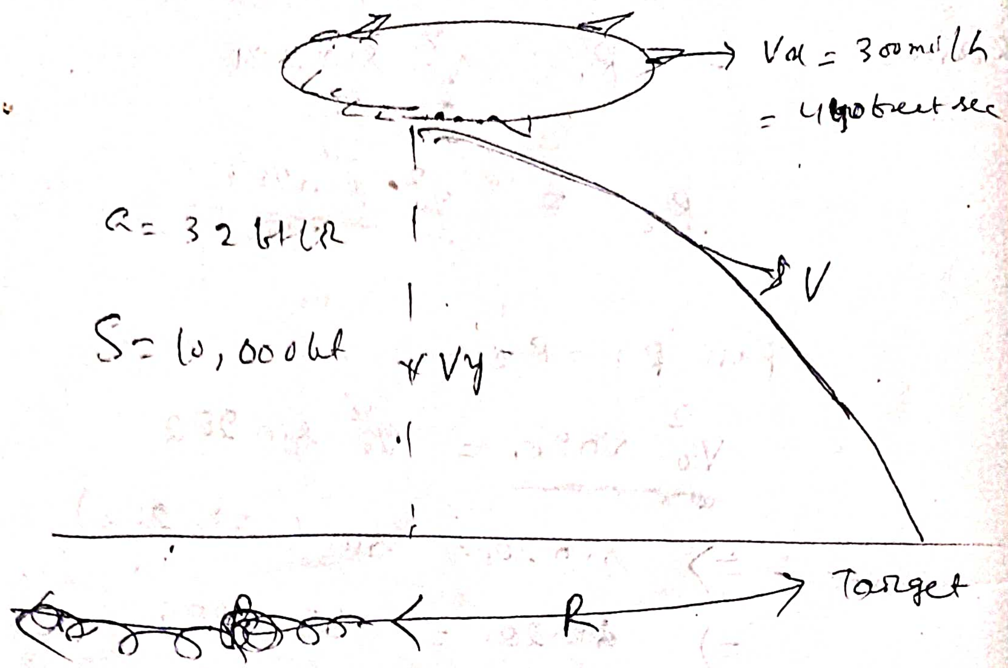
24.



15 mile/h = 22 feet / sec

$\Rightarrow$ 1 mile/h = $\frac{22}{15}$

$\Rightarrow$ 300 = $\frac{22 \times \cancel{300}^{20}}{\cancel{15}}$ = 440 feet/sec

Considering the y component of the velocity and using the formula $S = ut + \frac{1}{2}at^2$

$\Rightarrow$ 10,000 ft = $0 . t + \frac{1}{\cancel{2}} \cancel{(32)}^{16} . t^2$

$\Rightarrow$ $t^2 = \frac{10,000}{16} = 625$

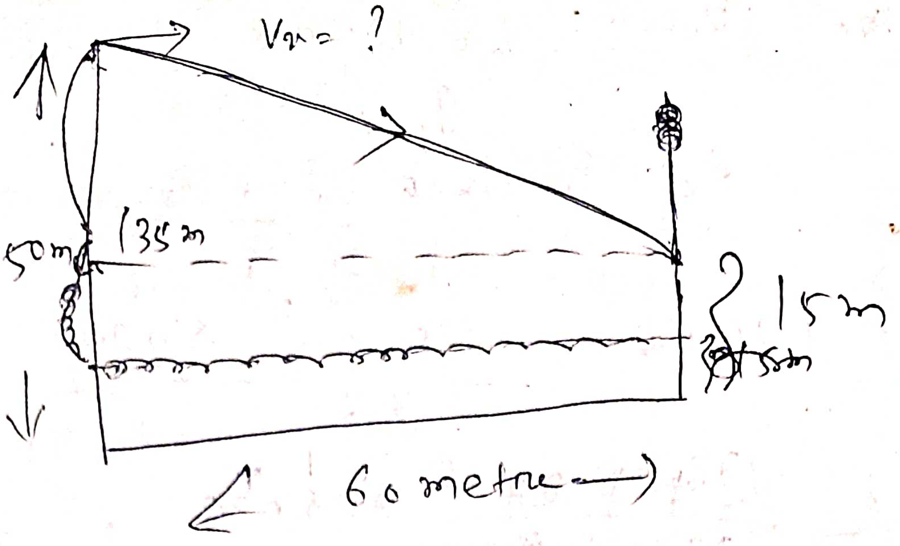
$\Rightarrow$ t = 25 second

V = u + at
$\Rightarrow$ ~~3~~440 = 32 . t
$\Rightarrow$ t = $\frac{440}{32}$ = 1

R = Range = horizontal component of velocity × time := 440 feet/sec × 25 sec
= ~~11~~,000 feet

= $\frac{\cancel{11,000}^{\cancel{100}\,25}}{\cancel{1760} \times 3}$ = $\frac{25}{12}$ $\frac{\cancel{12,000}^{\cancel{1400}\ \cancel{100}\ 50}}{\cancel{1760} \times \cancel{3}}$ = $\frac{25}{12}$ = 2.083 miles
$\cancel{16}$ 4 = 2.083 mile/hour.
~~44~~
~~24~~ 11

17.



$S = 135$ m

$g = 9.8$m

$S = ut + \frac{1}{2}at^2$

$\Rightarrow 135 = 0 + \frac{1}{2}\cdot\overset{4.9}{\cancel{(9.8)}}\cdot t^2$

$\Rightarrow t^2 = \dfrac{135}{4.9} = \sqrt{\dfrac{1350}{49}} = \dfrac{36.742}{7}$

$\Rightarrow t = 5.248$

$R = V_x t$

$\Rightarrow V_x = \dfrac{R}{t} = \dfrac{60}{5.248} = 11.43$ m/s.

28. $V_x = 15,000$ ft/s,

$R = 1,200$ mile

15 mile/h = 22 feets

$\therefore 1 = \dfrac{22}{15}$

1200 mile/h $= \dfrac{22}{15} \times \dfrac{\overset{80}{1200}}{3600}$

$= 1760$ feet

~~$R = \dfrac{V_x^2 \cdot \sin 2\theta}{g}$~~

$S = ut + \frac{1}{2}at^2$

$\Rightarrow 1760 = 15000 + \frac{1}{2}\cdot(0)t^2$

$= 15,000 + 0$

$\Rightarrow t = \dfrac{1760}{15,000}$

$V_x = 15{,}000$, $R = 1200$ mile $= 1760 \times 3$ feet

$g = 32$

$$R = \frac{V_0^2 . \sin 2\theta}{g}$$

$$\Rightarrow 1760 = \frac{(15{,}000)^2 . \sin 2\theta}{32}$$

$$\Rightarrow \frac{17\cancel{6}0 \times \cancel{3} \times 32}{\underset{75}{\cancel{225}} \times 10^5} = \sin 2\theta$$

200

$$\Rightarrow \frac{176 \times 32}{75 \times 10^5} = \sin 2\theta$$

$$\Rightarrow \sin 2\theta_0 = 0.9$$

$$\Rightarrow 2\theta_0 = \sin^{-1}(0.9)$$

$$= 64^\circ . 3'$$

$$\Rightarrow \theta_d = 32.15^\circ$$

$$T = \frac{2 V_0 \sin\theta_0}{g} = \frac{2 . (15{,}000) . \sin 32.15^\circ}{32}$$

$$= \frac{2 (15{,}000) 0.5321}{32}$$

$$= 498.4 \text{ sec}$$

---

1 mile = 1760 × 3 ft

$$15 \text{ mi}/\text{hr} = \frac{\cancel{15} \times \overset{22}{\cancel{1760}} \times \cancel{3}}{\underset{\cancel{120}/\cancel{8}}{\cancel{3600}} \text{ sec}} \text{ ft}$$

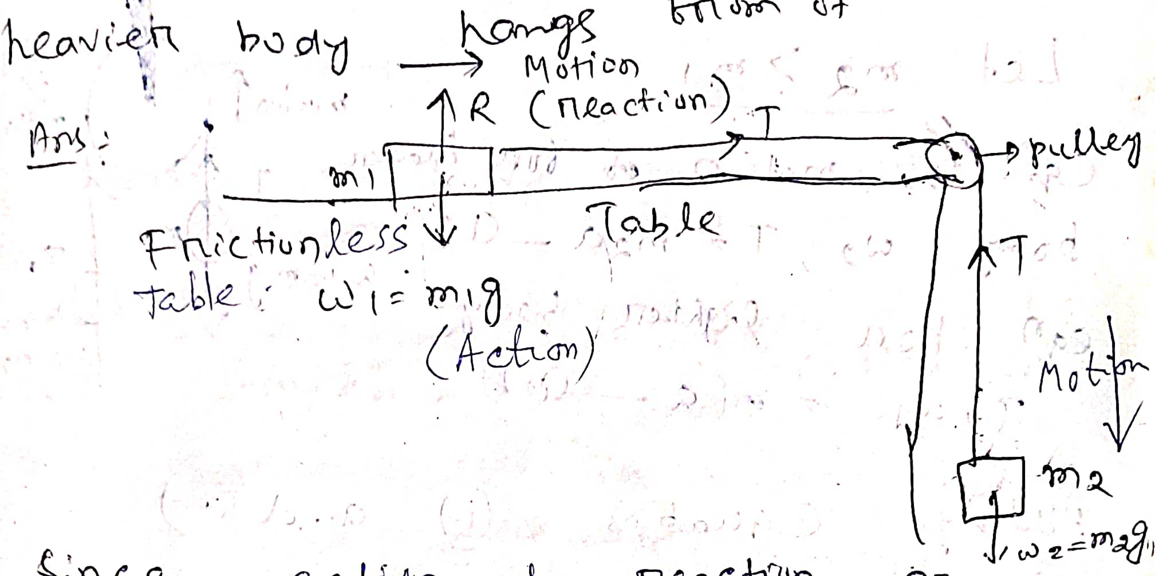
$$= 22 \text{ ft}/\text{sec}$$

$$300 \text{ mi}/\text{hr} = 22 \times 20 = 440 \text{ ft}/\text{sec}$$

20.6.21

1. A body placed on a frictionless horizontal table and draged by string that pass over a pulley at the end of the table a heavier body hangs from it

Ans:



Since action & reaction are equal and opposite they make each other in abbective so that the net force on the body of mass m1 is the tension (T)

Newtons 2nd law gives

$$T = m_1 a \quad \text{——(i)}$$

Since the 2nd body of mass m_2 moves down net force on it is

$$w_2 - T = m_2 a \quad \text{——(ii)}$$

(accⁿ is same because connected together)

Adding (i) and (ii) we get

$$\cancel{T} + w_2 - \cancel{T} = m_1 a + m_2 a$$

$$\Rightarrow w_2 = a(m_1 + m_2)$$

$$\Rightarrow \underline{m_2 g} = a(m_1 + m_2)$$

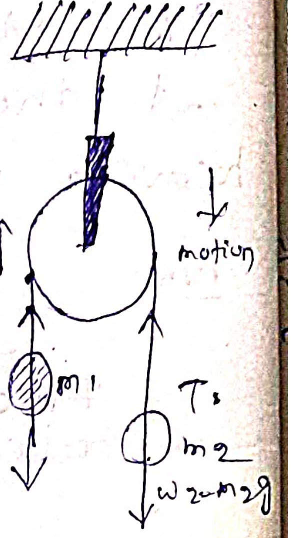
$$\Rightarrow a = \frac{m_2 g}{m_1 + m_2}$$

Putting the value of a in eqn (i)

$$T = m_1 . \left(\frac{m_2}{m_1+m_2} \times g \right)$$

$$= \frac{m_1 . m_2 . g}{m_1+m_2}$$

2. Atwood's machine



Let $m_2 > m_1$

eqn of motion ~~of~~ for heavier body $w_2 - T = m_2 a$ — (i)

eqn for lighter body

$T - w_1 = m_1 a$ — (ii)

Adding equations (i) and (ii)

$$w_2 - \cancel{T} + \cancel{T} - w_1 = m_2 a + m_1 a$$

$$\Rightarrow w_2 - w_1 = a(m_1 + m_2)$$

$$\Rightarrow a = \frac{W_2 - W_1}{m_1+m_2}$$

$$= \frac{m_2 g - m_1 g}{m_1+m_2}$$

$$= \frac{g(m_2 - m_1)}{m_1+m_2}$$

Putting this value in equation (i)

$$w_2 - T = m_2 a$$

$$\Rightarrow T = w_2 - m_2 a$$

$$= m_2 g - m_2 . \left\{ \frac{g(m_2 - m_1)}{m_1+m_2} \right)$$

$$= m_2 g . \cancel{g m_2} \left\{ \overset{1}{\cancel{m_2}} - \frac{\cancel{m_2}(m_2 - m_1)}{(m_1+m_2)} \right\}$$

(115)

$$= \cancel{g\left\{\frac{m_2 m_1 + m_2^2 + -m_2^2 + m_2}{m_1 + m_2}\right\}} = m_2 g\left(\frac{m_1 + m_2 - m_2 + m_1}{m_1 + m_2}\right)$$

$$\cancel{= g\left(\frac{m_1 m_2 + m_2}{m_1 + m_2}\right)} \qquad = m_2 g.\frac{(2m_1)}{m_1 + m_2}$$

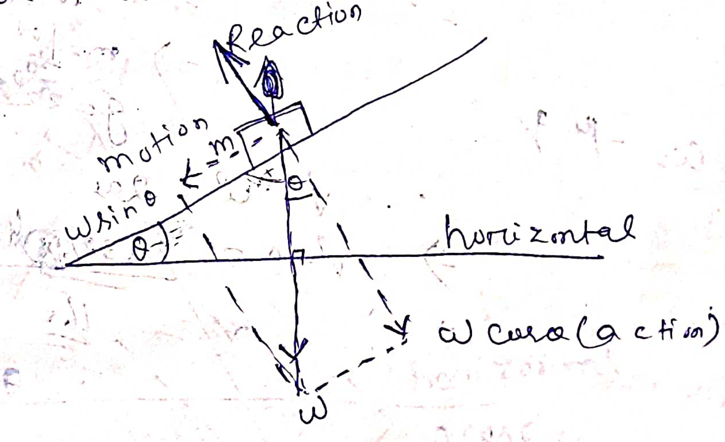
$$\cancel{= g\,\frac{m_2(m_1 + 1)}{(m_1 + m_2)}} \qquad = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

$$\cancel{m_2 g}$$

3. Motion of body on a frictionless inclined plane.

Case -1

No - external force is acting on the body.



Resolving the weight ^{w} in to two rectangular components, we see that w cosθ is perpendicular to the inclined plane and acts like action which produces an equal and opposite reaction force and then make each other ineffective. Hence the other component w sin θ acting parallel to the inclined plane is the net force acting on the body. This produces acceleration.

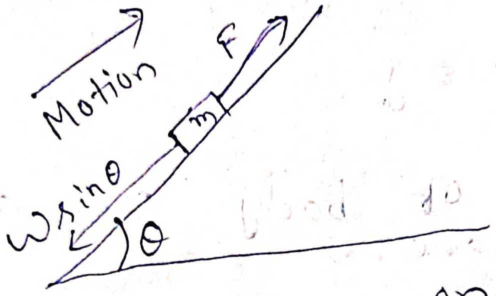
$ma = w\sin\theta$, down the inclined plane

$\Rightarrow a = \frac{w\sin\theta}{m}$, " " " "

$\Rightarrow a = g \sin\theta$ down the inclined

IInd Case

A body dragged up an inclined plane with same acceleration.



Net force acting on the body

$= F - w\sin\theta = ma$

$\Rightarrow a = \dfrac{F - w\sin\theta}{m}$ directed up the inclined plane.

IIIrd Case

A body dragged upon inclined plane with constant velocity.

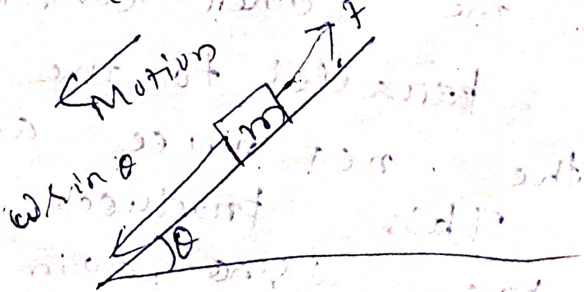
Here $a = 0$ [∵ Constant velocity)

Thus $F - w\sin\theta = m.a = m.0 = 0$

$\Rightarrow F = w\sin\theta$

Case-4

A body dragged up an inclined plane, yet allowed to move down the inclined plane.



Net force $= w\sin\theta - F = ma$

(17) => $\vec{a} = \frac{w\sin\theta - F}{m}$, down the inclined plane.

$= g\sin\theta - \frac{F}{m}$

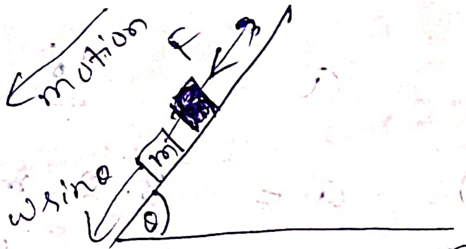
Case 5 : A body dragged up an inclined plane yet allowed to move down the inclined plane with constant velocity.

Here also $a = 0$

Thus $w\sin\theta - F = m.(0)$

$\Rightarrow w\sin\theta = F$

Case 6 : A body being pushed down an inclined plane with some force

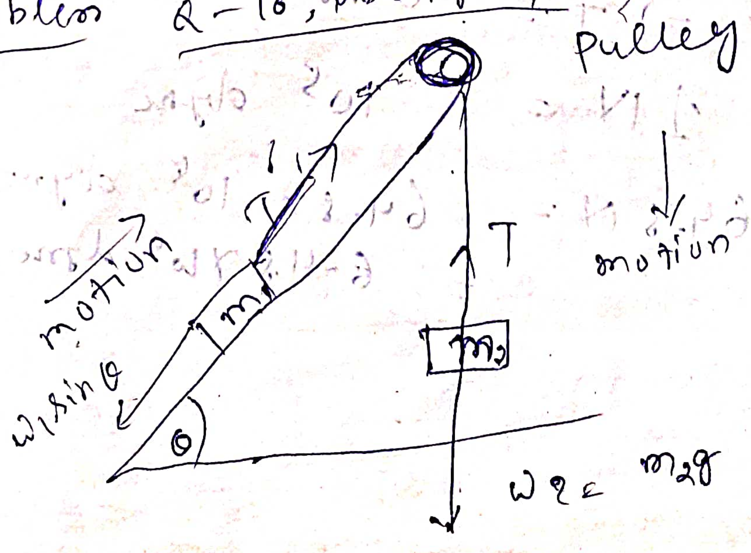


Thus $w\sin\theta + F = ma$.

$\Rightarrow a = \frac{w\sin\theta + F}{m}$ directed along down the plane

$= g\sin\theta + \frac{F}{m}$ "

Problem Q-16, ~~p.no~~ Pg-77



Net force, ~~[illegible]~~ $T - w_1 \sin\theta = m_1 a$ —(i)

Net force $w_2 - T = ma$ —(ii)

Adding (i) and (ii)

$\cancel{T} - w_1 \sin\theta + w_2 - \cancel{T} = m_1 a + m_2 a$

$\Rightarrow w_2 - w_1 \sin\theta = a(m_1 + m_2)$

$\Rightarrow a = \frac{w_2 - w_1 \sin\theta}{m_1 + m_2}$

$\Rightarrow a = \frac{m_2 g - m_1 g \sin\theta}{8 + 10}$

$\Rightarrow a = \frac{10 \times 9.8 - 8 \times (9.8) \times \frac{1}{2}}{18}$

$\Rightarrow a = \frac{98 - 39.2}{18} = \frac{58.8}{18}$

$\Rightarrow a = 3.2 \ m/s^2$

Putting the value of (a) in eq (i)

$T - w_1 \sin\theta = m_1 a$

$\Rightarrow T - 9.8 \times 8 \times \frac{1}{2} = 8 . \cancel{a}(3.2)$

$T - 39.2 = 25.6$

$\Rightarrow T$

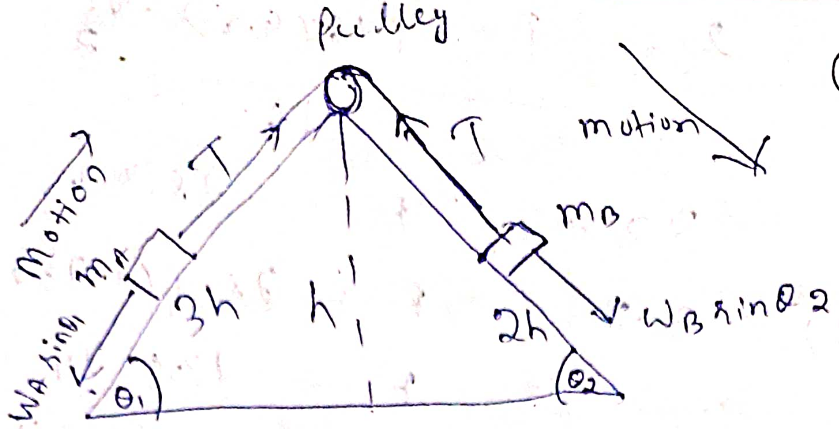
$\Rightarrow T = 64.8 \ N$

$1 N = 10^5$ dyne

$64.8 N = 64.8 \times 10^5$ dyne

$= 6.48 \times 10^6$ dyne.

24.



$\sin\theta_1 = \frac{h}{3h} = \frac{1}{3}$

$\sin\theta_2 = \frac{h}{2h} = \frac{1}{2}$

Net force $T - W_A \sin\theta_1 = m_A a$ —(i)

Net force $W_B \sin\theta_2 - T = m_B a$ —(ii)

Adding we get $\cancel{T} - W_A \sin\theta_1 + W_B \sin\theta_2 - \cancel{T}$

$= a(m_A + m_B)$

$\Rightarrow -W_A \frac{1}{3} + W_B \frac{1}{2} = a(m_A + m_B)$

$\Rightarrow \frac{-2W_A + 3W_B}{6} = a(2M_B + m_B)$

$a(3M_B)$

$\Rightarrow \frac{-2W_A + 3W_B}{6} =$

$-2W_A + 3W_B = 18 M_B a$

$\Rightarrow -2W_A + 3W_B \qquad = a(18 m_B)$

$\Rightarrow 3M_B g - 2 W_A g = a(18 m_B)$

$\Rightarrow g(3M_B -$ [illegible]$) = a(18 m_B)$

$\Rightarrow g(-m_B) = a(18 m_B)$

$\Rightarrow a = \frac{g(-m_B)}{18 m_B} = \frac{-g}{18} = \frac{-9.8}{18}$

$= -0.54 m/s^2$

upwards

21. $a = \frac{F - W\sin\theta}{m}$

$\Rightarrow 2 = \frac{F - W\sin 60^\circ}{16}$

$$\Rightarrow \quad 2 = \frac{F - (16\times 9.8)\cdot\frac{\sqrt{3}}{2}}{16}$$

$$\Rightarrow \quad 32 = F - 78.4\sqrt{3}$$

$$\Rightarrow \quad 32 = F - 78.4(1.732)$$

$$= F - 135.79$$

$$\Rightarrow \quad F = 32 + 135.79 = 167.79\text{N} (\text{ans})$$

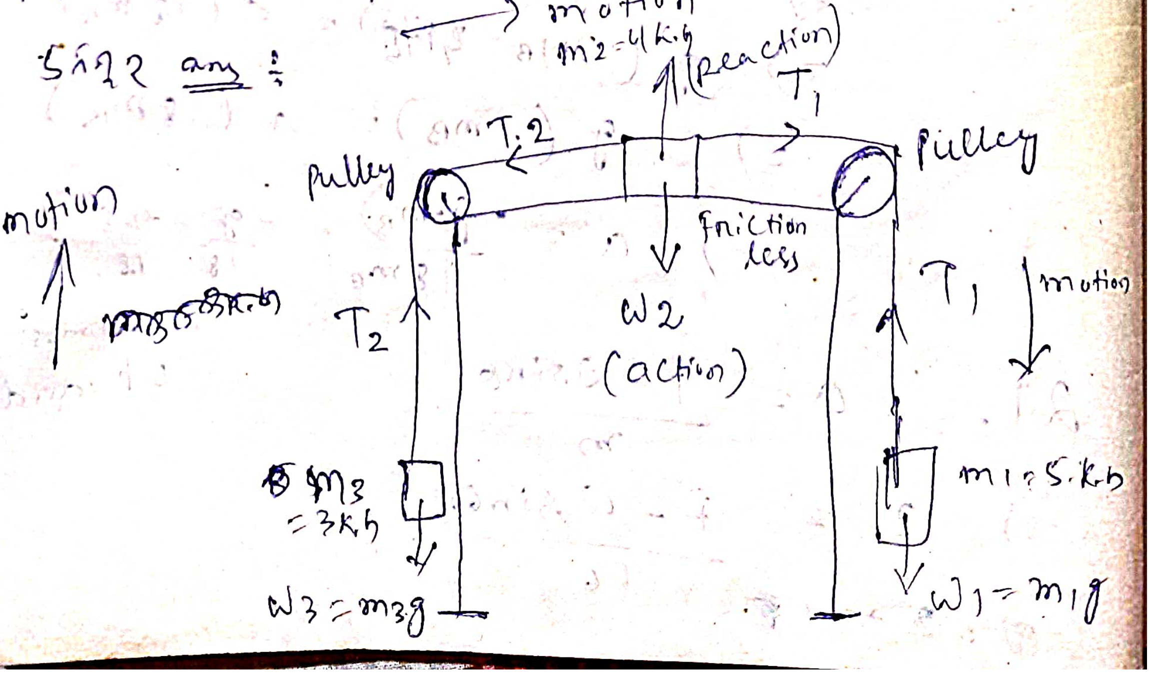
5. A four K.G body rests on a smooth horizontal table ~~top~~ top Cords that are attached to the body pass over pulleys at opposite ends of the table and are attached to a 5K.G body at 1 end & to a 3.K.G body at the other end.

(a) Find acc^n of 4K.G body when the system is released also find the tensions in each cord while the bodies are accelerated.

(Ans: 1.63 m/sec, 40.8 N, 34.3 N

Page 772 Q:- 12} 15, 21.2 ans (25lb, 167N

5.22 ans :



Motion equation for 1st case

Net force $w_1 - T_1 = m_1 a$ —— (i)

Motion eqn for 2nd case

$m_2 = 4$ k.g.

Net force $T_1 - T_2 = m_2 a$ — (ii)

Motion eqn for 3rd case

$T_2 - W_3 = m_3 a$ — (iii)

Adding eqn (i), (ii) and (iii) we have

$w_1 - \cancel{T_1} + \cancel{T_1} - \cancel{T_2} + \cancel{T_2} - w_3 = m_1 a + m_2 a + m_3 a$

$\Rightarrow w_1 - w_3 = a(m_1 + m_2 + m_3)$

$\Rightarrow m_1 g - m_3 g = a(5+4+3)$

$\Rightarrow g(m_1 - m_3) = 12a$

$\Rightarrow g(5-3) = 12a$

$\Rightarrow 2(9.8) = 12a$

$\Rightarrow 19.6 = 12a$

$\Rightarrow a = \frac{19.6}{12} = 1.63$ m/sec

Putting the value of (a) in eqn (i)

$W_1 - T_1 = m_1 a$

$\Rightarrow m_1 g - T_1 = (5)(1.63)$

$\Rightarrow 5.(9.8) - T_1 = 8.15$

$\Rightarrow 49 - T_1 = 8.15$

$\Rightarrow T_1 = 49 - 8.15 = 40.85$

Putting the value of (a) in eqn (iii) we have

$T_2 - W_3 = m_3 a$

$\Rightarrow T_2 - m_3 g = 3(1.63)$

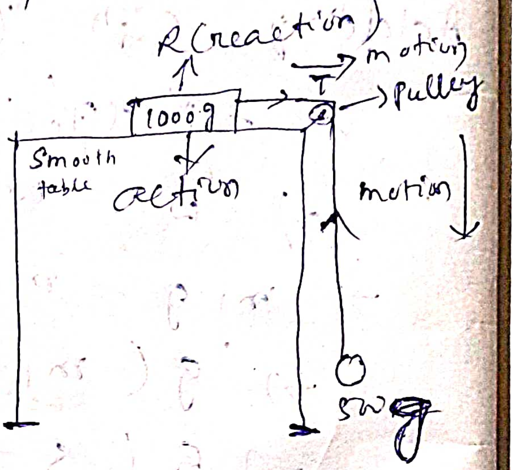
$\Rightarrow T_2 - (3).(9.8) = 4.89$

$\Rightarrow T_2 - 29.4 = 4.89$

$\Rightarrow T_2 = 29.4 + 4.89 = 34.29$

$\therefore T_1 = 40.85$

$T_2 = 34.29 = 34.3$ } Ans



12. $T = m_1 a$ —(i)

Net force

$W_2 - T = m_2 a$ —(ii).

Adding eqn (i) and (ii) we get

$\cancel{T} + W_2 - \cancel{T} = a(m_1 + m_2)$

$\Rightarrow m_2 g = a(1 + \frac{1}{2}\ k.g.)$

$\Rightarrow \frac{1}{2}.(9.8) = (\frac{3}{2} a)$

$\Rightarrow 4.9 = \frac{3}{2} a$

$\Rightarrow a = \frac{4.9 \times 2}{3} = \frac{9.8}{3} = 3.27$ m/sec

$= 327$ cm/sec²

Putting the value of (a) in eqn (i) we get

$T = m_1 a$

$= 1000 gm \times (327$ cm/sec²

$= 327000$

$= 3.27 \times 10^5$ dyne

(123)

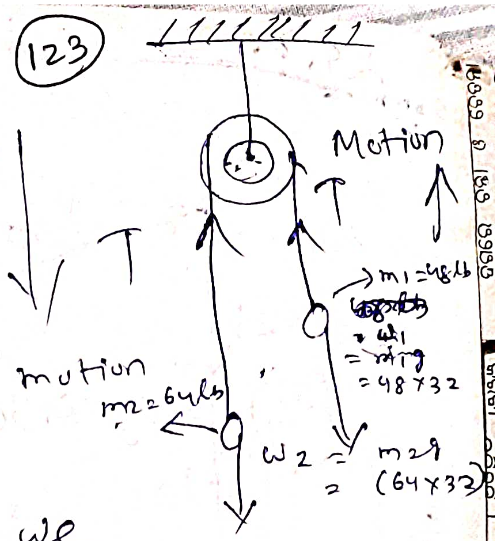
as eqn of motion for heavier body

$W_2 - T = m_2a$ —(i)

eqn of motion for lighter body

~~[illegible]~~

$T - W_1 = m_1a$ —(ii)



Adding eqn (i) and (ii) we get

$W_2 - \cancel{T} + \cancel{T} - W_1 = a(m_1 + m_2)$

$\Rightarrow W_2 - W_1 = a(m_1 + m_2)$

$\Rightarrow (64 \times 32) - (48 \times 32) = a(48 + 64)$

$\Rightarrow 2048 - 1536 = 112a$

$\Rightarrow \frac{512}{112} = a$

$\Rightarrow 4.57 = a$

putting the value of (a) in eqn (i) we get

$W_2 - T = m_2a$

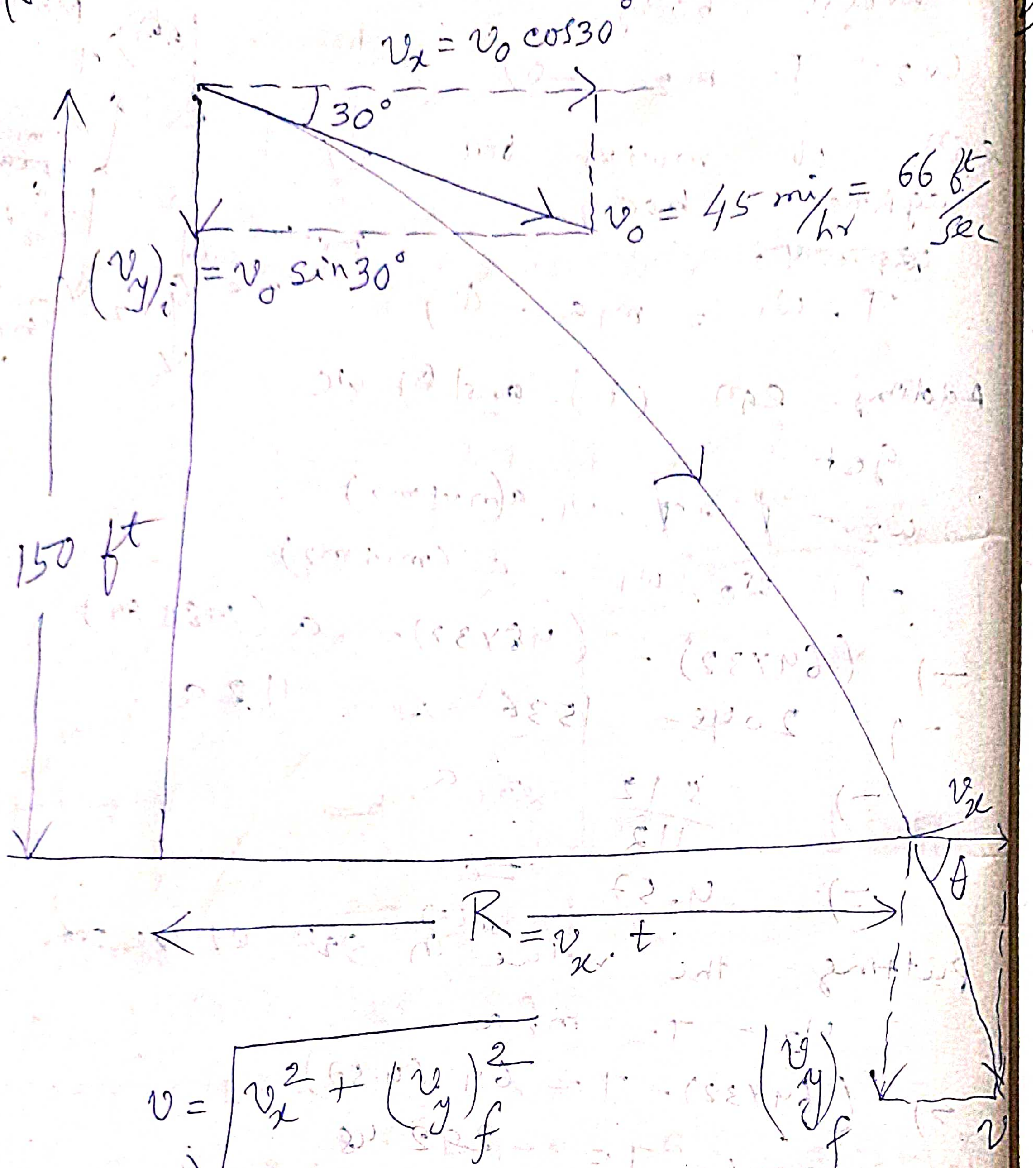
$\Rightarrow (64 \times 32) - T = 64 \cdot (4.57)$

$\Rightarrow 2048 - T = 292.48$

29.06.22

Sstask

10.



$$v = \sqrt{v_x^2 + (v_y)_f^2}$$

$$\tan\theta = \frac{(v_y)_f}{v_x}$$

Resolving the initial velocity of projection in to two rectangular components, we see that the horizontal component is $V_x = V_0 \cos 30°$

$= 66 \cdot \frac{\sqrt{3}}{2}$

$= 33\sqrt{3}\ ft/s^2$

(12.5) The vertically component acting downwards is given by $(V_y)(i) = V_0 . \sin 30$

$= 66 \times \frac{1}{2}$

$= 33$ ft/s

The vertical distance of 150 ft is covered with the help of the vertical component of the velocity.

Using the formula $S = ut + \frac{1}{2}at^2$, we get

$$150 = 33.t + \frac{1}{2}.(32).t^2$$

$$\Rightarrow 150 = 33t + 16t^2$$

$$\Rightarrow 16t^2 + 33t - 150 = 0.$$

$$t = \frac{-33 \pm \sqrt{1089 - 4.(16).(-150)}}{2(16)}$$

$$= \frac{-33 \pm \sqrt{10689}}{32}$$

$$= \frac{-33 + 103.4}{32}$$

= ~~103.4~~ ~~[illegible]76875~~

= 2.2 sec,

$R = V_x . t$

$= 33\sqrt{3} \times 2.2$

$= 33(1.732) \times 2.2$

$= 125.7432$ feet

$$(V_y)_f = (V_y)_i + gt$$

$$= (33) + 32.(2.2)$$

$$= 33 + 70.4$$

$$= 103.4$$

$$V = \sqrt{(V_x)^2 + (V_y)_f^2}$$

$$= \sqrt{(33\sqrt{3})^2 + (103.4)^2}$$

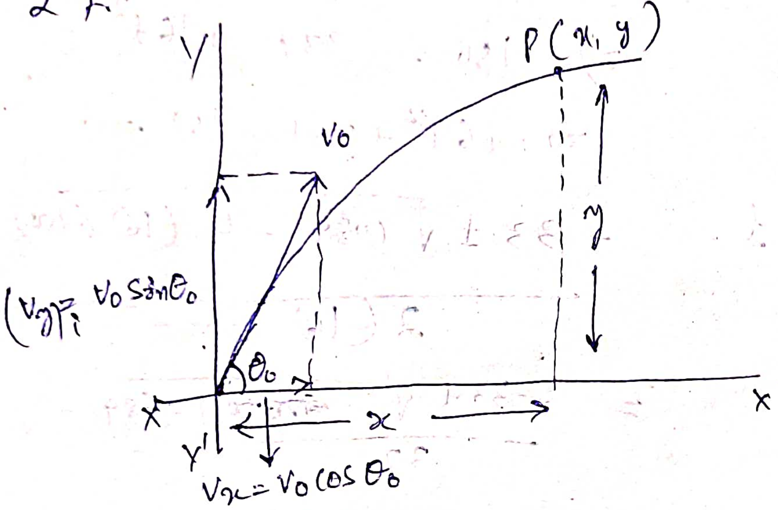
$$= \cancel{33}\sqrt{1089.3 + (103.4)^2}$$

$$= \sqrt{3267 + 10691.56}$$

$$= \sqrt{13958.56}$$

$$= 118.14 \text{ ft/sec}$$

27.



Given:- $t = 2$ sec

$x = 15$ m

$y = 5$ m

The horizontal distance of 15 metre is covered with the help of the horizontal component of the velocity.

$$\therefore x = V_x \cdot t$$

$$\Rightarrow 15\text{m} = V_0 \cos\theta_0 \cdot (2)$$

$$\Rightarrow V_0 \cos\theta_0 = 7.5 \text{ m/s} \quad \text{——(i)}$$

(127) The vertical distance of 5 metre is covered with the help of the vertical component of velocity.

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow 5 = (v_0 \sin\theta_0) \cdot 2 + \frac{1}{2} \cdot (-9.8)\, 4$$

$$\Rightarrow 5 = v_0 \sin\theta \cdot 2 - 19.6$$

$$\Rightarrow v_0 \sin\theta_0 = \frac{5 + 19.6}{2}$$

$$= \frac{24.6}{2}$$

$$= 12.3 \text{ m/s} \quad \text{---(ii)}$$

Squaring and adding eqns (i) and (ii) gives

~~$v_0^2 \cos^2\theta_0 = 56.25$~~

~~$v_0^2 \sin^2\theta_0 = 151.29$~~

Adding $v_0^2 \cos^2\theta + v_0^2 \sin^2\theta = 56.25 + 151.29$

$$\Rightarrow v_0^2 (\sin^2\theta + \cos^2\theta) = 207.54$$

$$\Rightarrow v_0 = \sqrt{207.54}$$

$$= 14.40 \text{ m/s}.$$

$$\tan\theta_0 = \frac{v_0 \sin\theta_0}{v_0 \cos\theta_0} = \frac{(12.3)}{7.5} = 1.64$$

$$\Rightarrow \theta = \tan^{-1}(1.64)$$

$$= 58.65$$

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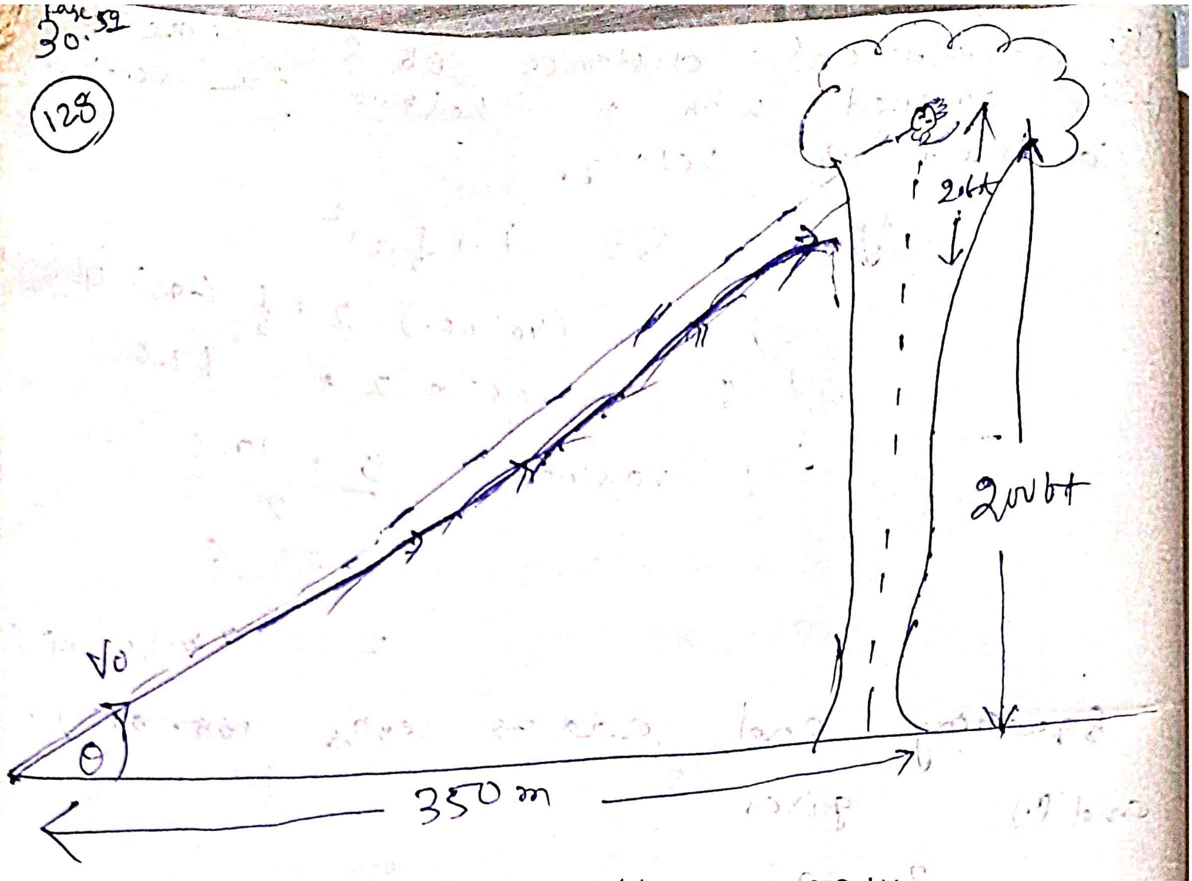
(128)



$$\tan\theta_0 = \frac{200}{350} = \frac{4}{7} = 0.5714$$

~~tan~~ $\theta_0 = \tan^{-1}(.5714)$

$= \cancel{30.08^\circ}\ 29.7^\circ$

$$V_x = V_0 \cos\theta_0 = V_0 \cos 29.7^\circ$$
$$= V_0 .\ (.8686)$$

$$R = V_x t$$
$$\Rightarrow \cancel{R}\,350 = V_0\,(.8686).\,t \text{ ——— (i)}$$

To find t, we see that the time
taken by the bullet to move 350ft
is equal to the time taken by the
to ball freely through
monkey
20 ft

$\therefore\ u=0,\ S=20,\ a=32$

$$S = ut + \tfrac{1}{2}at^2$$
$$\Rightarrow 20 = 0 + \tfrac{1}{2}\,32.\,t^2$$

$\Rightarrow 20 = 16t^2$

$\Rightarrow t^2 = \frac{20}{16} = 1.25$

$\Rightarrow t = \sqrt{1.25} = 1.118$

taking the value of t in eqn(1)

we get

$350 = v_0 (.8686).(1.118)$

$\Rightarrow \frac{350}{(.8686)\times(1.118)} = v_0$

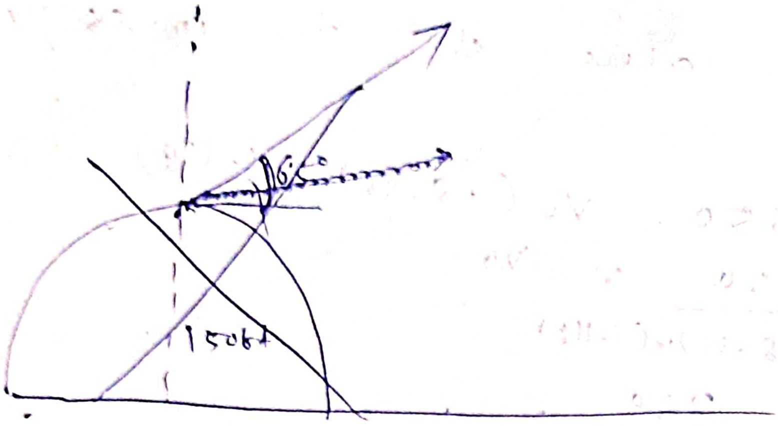
$\Rightarrow \frac{350}{.971} = v_0$

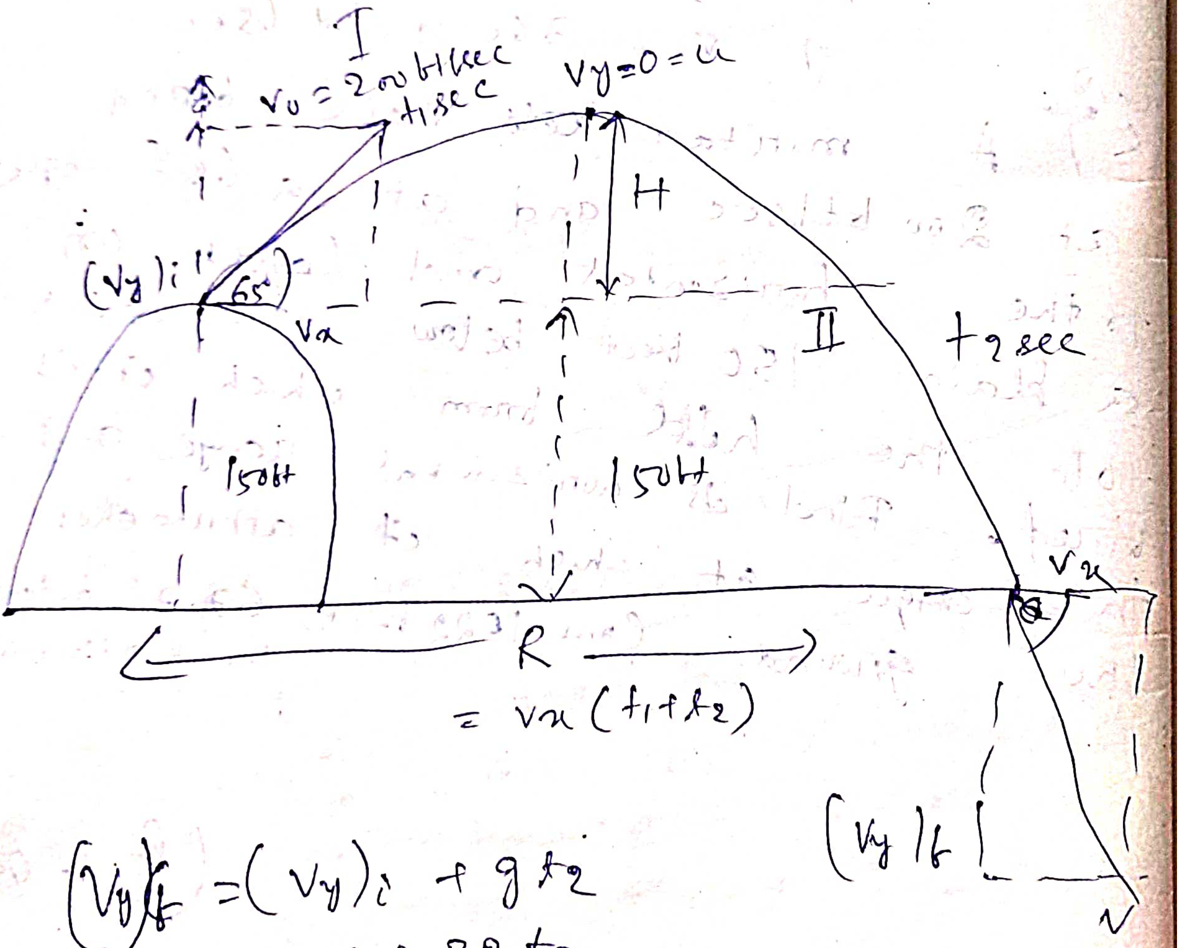
$\Rightarrow 360.4 = v_0$

$\Rightarrow v_0 = 360.4$ ft/sec

(5) A mortar shell fired at 200 ft/sec and at a 65° angle to the horizontal and lands on a plain 150 feet below the level of the hill from which it is fired. Find its horizontal range and the angle at which it approaches the ground. (Ans 1022 feet, 67.7° below the horizontal

(P.T.O)





$$(V_y)_f = (V_y)_i + g t_2$$

$$= 0 + 32 t_2$$

$$\tan\theta = \frac{(V_y)_f}{V_x}$$

Ans:

$V_x = V_0 . \cos 65°$

$= (200)(.4226)$

$= 84.52 \, ft$

$(V_y)_i = V_0 . \sin 65°$

$= 200(.9063)$

$= 181.26$

For first point

$$H = \frac{V_0 . \sin^2 65°}{2g} = \frac{(200)^2 . (.9063)^2}{2(32)}$$

$$= \frac{(200)^2 (.8213)}{64}$$

$$= \frac{32852}{64}$$

$$= 513.3125$$

~~For the 2nd~~ $t_1 = \frac{V_0 . \sin\theta_0}{g} = \frac{V_0 . \sin 65°}{32}$

$= \frac{181.26}{32}$

$= 5.66$

~~Total d h~~

For the second point

$S = 150 + H = 150 + 513$

$= 663$

$S = ut_2 + \frac{1}{2} a t_2^2$

$\Rightarrow 663 = 0 t_2 + \frac{1}{2} . (32) . t^2$

$= 16 t^2$

$\Rightarrow t_2^2 = \frac{663}{16}$ $\Rightarrow t_2 = \sqrt{41.43} = 6.43$

$R = V_x (t_1 + t_2)$

$= (84.52)(5.66 + 6.43)$

$= (84.52)(12.09)$

$= 1021.84$

$= 1022$ ft

~~$\tan\theta = \frac{(V_y)f}{V_x} =$~~

$(V_y)_f = (V_y)_i + g t_2$

$= 0_i + (32).(6.43)$

$= 205.76$

$\tan\theta = \frac{(V_y)f}{V_x} = \frac{205.76}{84.52} = 2.437$

$\Rightarrow \theta = \tan^{-1}(2.434)$

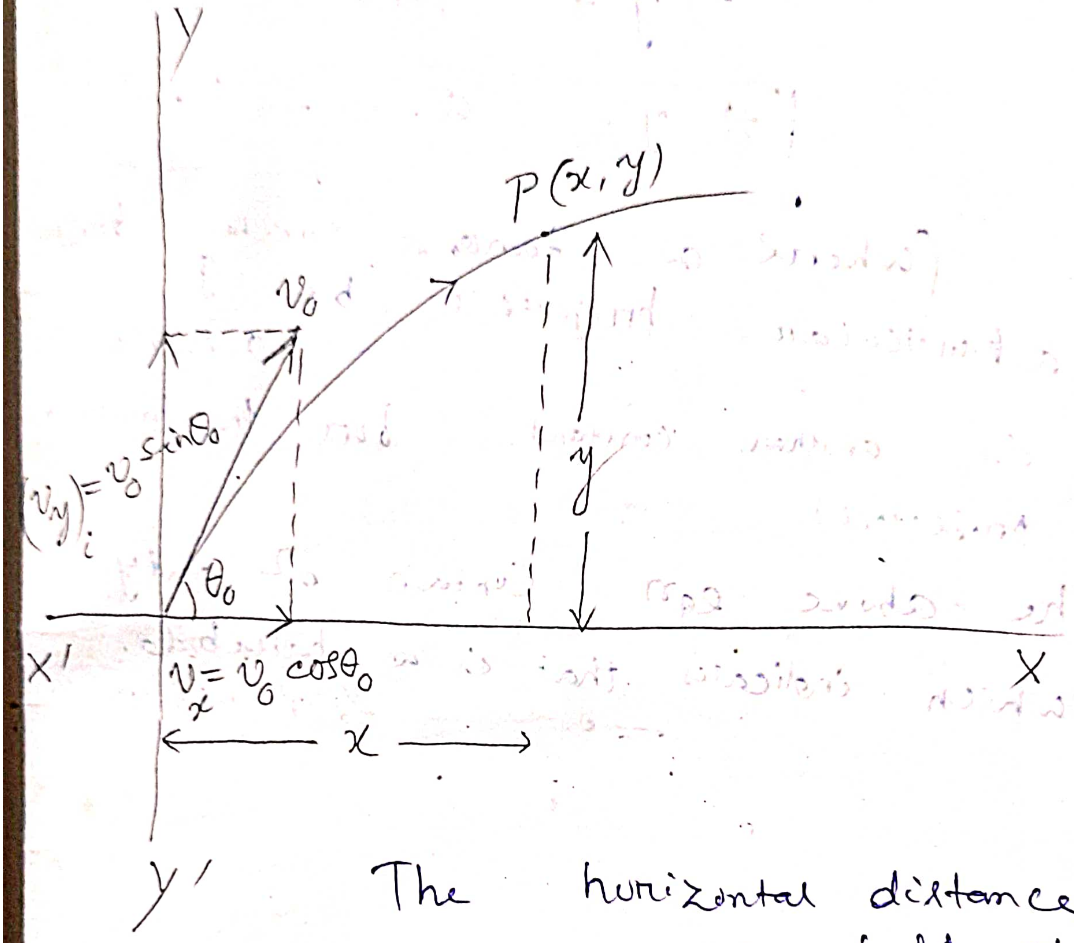
$= 67.7°$ below the horizontal

To prove that the trajectory or path of a projectile is a parabola (curve)

Let's consider a projectile in the xy plane. The initial velocity of projection is V_0 and intial angle of projection θ_0. We can resolve the initial velocity into two rectangular components. The horizontal component is $V_x = V_0 \cos\theta_0$ which is unaffected by gravity

The vertical component is Vo sin θo which is affected by gravity.

P is a point on the path of the projectile reached after t second from start. The co-ordinate of the point P are (x, y).



The horizontal distance x is covered with the help of horizontal component of the velocity in a time t second.

$$\therefore x = V_x . t = V_0 \cos\theta_0 . t \quad \text{—(i)}$$

The vertical distance (y) is covered with the help of the vertical component of the velocity during t second.

Using the formula $S = ut + \frac{1}{2}at^2$ we get

$$\Rightarrow y = V_0 \sin\theta_0 . t + \frac{1}{2}(-g).t^2 \quad \text{—(ii)}$$

(9.20241

By eliminating t between eqns (i) and (ii), We can get the relation between x and y. From eqn (i) $t = \frac{x}{V_0 \cdot \cos\theta_0}$ which can be place in eqn (ii) $\therefore y = V_0 . \sin\theta_0 \cdot \frac{x}{V_0 \cos\theta_0} - \frac{1}{2}(g)\left(\frac{x^2}{V_0^2 . \cos^2\theta_0}\right)$

$$\Rightarrow y = \tan\theta_0 . x - \left(\frac{g}{2 V_0^2 \cos^2\theta_0} . x^2\right)$$

$$\boxed{\Rightarrow y = a . x - b . x^2}$$

(where $a = \tan\theta_0$ a constant for a particular projectile, $b = \frac{g}{2 V_0^2 . \cos^2\theta_0}$ is another constant for the particular projectile)

The above eqn contain x^2 and y. which indicates that is a parabola.

[illegible] y^2, x [illegible] (parabola)