Antennas & Wave Propagation (PET6J012)

Text Book

1. Antennas & Wave Propagation, A.R. Harish, M. Sachidananda, Oxford University Press, 2007.
2. Antenna Theory Analysis and Design, C.A. Ballanis, John Wiley Publications, Second Edition, 2005.

Reference Books:-

1. Antennas for all Applications, J.D. Kraus, Ronald J. Marhefka, and Ahmad S Khan, Tata McGraw-Hill Book Company, Third Edition, 2008.
2. Electromagnetic Waves and Radiating Systems, E.C. Jordan and K.G. Balmain, Pearson Education Publications, 1968.
3. Antenna & Wave Propagation, K.D. Prasad, Satya Prakashan, New Delhi.

(Antenna)

Defn:- A means for radiating or receiving radio waves.

In other words the antenna is the transitional structure between free space and a guiding device.

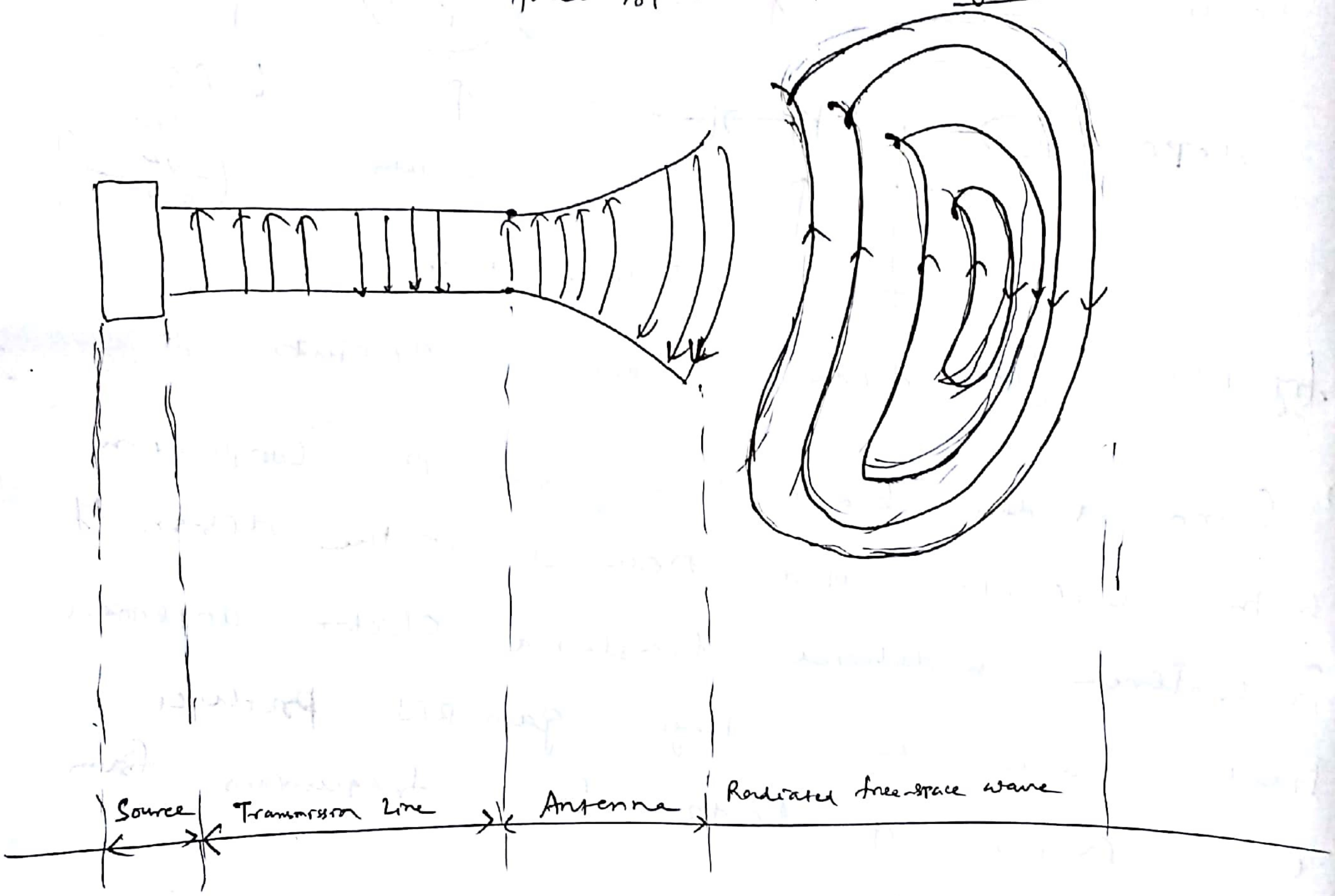


fig 1:- Antenna as a transition device.

Types of antennas

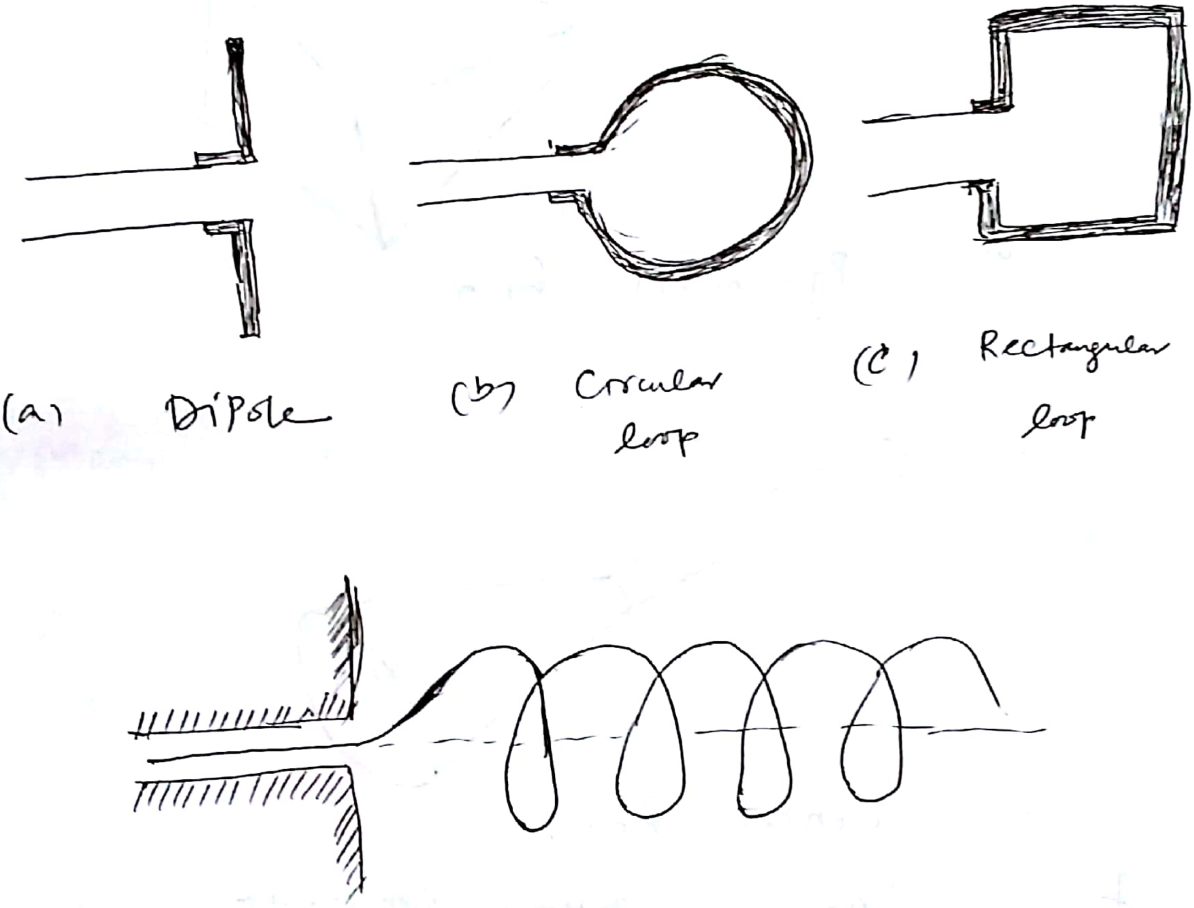
1) Wire Antenna

→ There are different shapes of wire antennas such as straight wire (dipole), loop and helix

→ loop antenna may be circular, rectangular, square etc.

→ The circular loop is most common because of its simplicity in construction.

→ Wire antennas used in automobiles, buildings, ships, aircrafts, space crafts etc.



(a) Dipole (b) Circular loop (c) Rectangular loop

(d) helix

fig 2:- Wire antenna configuration.

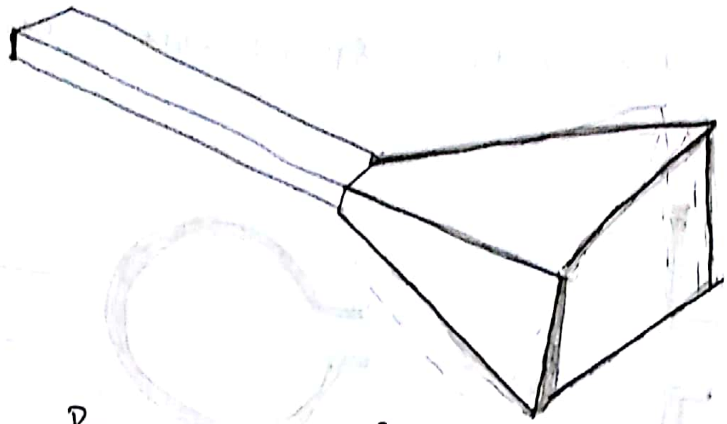
2) Aperture Antenna:-

→ Sophisticated form of antenna and used for high frequencies

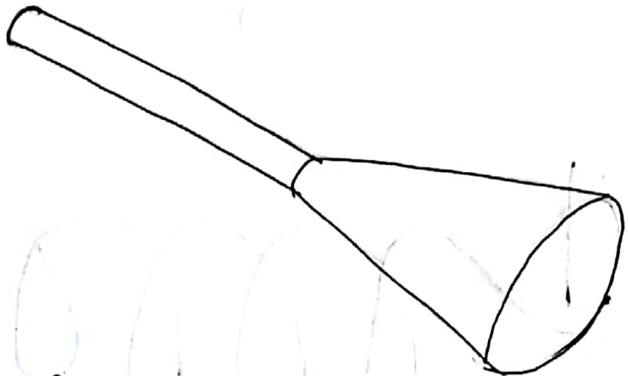
→ Useful for aircraft, spacecraft application, because they can be very conveniently flush-mounted on the skin of aircraft and space craft.

→ In addition, they can be covered with

dielectric material to protect them from hazardous conditions of environment.



(a) Pyramidal horn



(b) Conical horn.

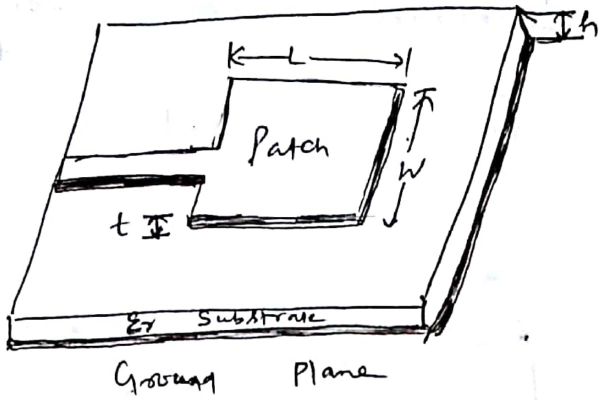
fig 3: Aperture antenna Configuration.

3) Microstrip Antennas:-

→ ~~Met~~ These antennas consist of a metallic patch on a grounded substrate.

→ Metallic patch may take different configurations ex:- Circular, Rectangular etc.

→ These antennas can be mounted on the surface of high-performance aircraft, space craft, satellites, missiles, Cars and even hand held mobile Phones.



(a) Rectangular microstrip (Patch) antenna.

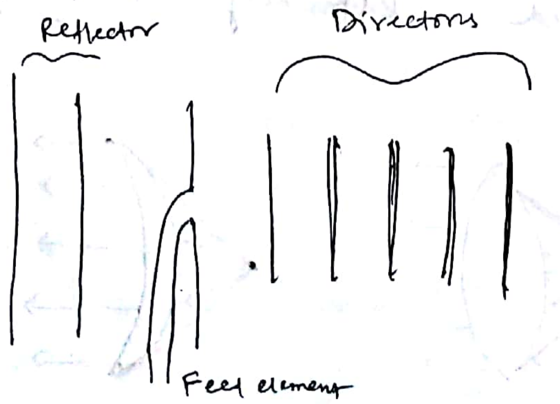
4) Array Antennas :-

→ Many applications require radiation characteristics that may not be achievable by a single element.

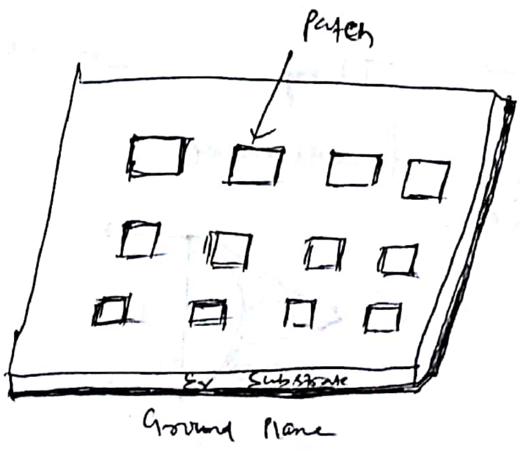
→ It may, however, be possible that an aggregate of radiating elements in an electrical and geometrical arrangement (an array) will result in the desired radiation characteristics.

→ The arrangement of the array may be such that the radiation from the elements adds up to give a radiation maximum in a particular direction or directions, minima in others, or otherwise as desired.

ex :-



(a) Yagi-Uda Array



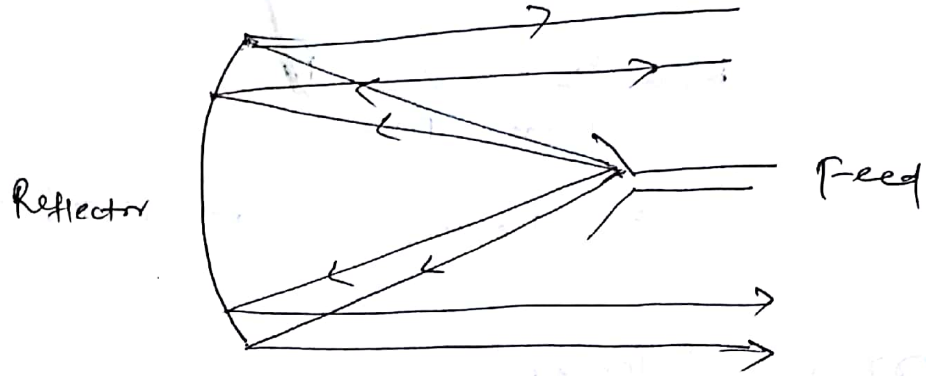
(b) Microstrip Patch array

5) Reflector Antennas:-

→ To Communicate over long distance

→ ex:- Parabolic reflector Antenna

→



(a) Parabolic Reflector with front feed.

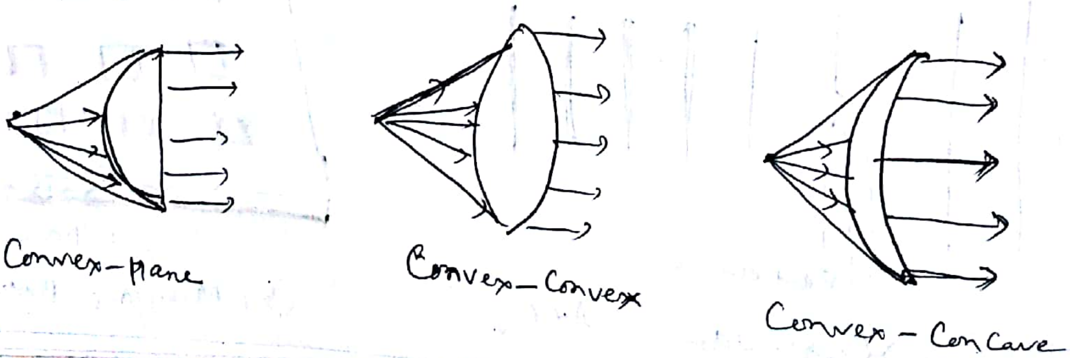
6) Lens Antennas:-

→ Lenses are primarily used to Collimate incident divergent energy to prevent it from spreading in undesired direction.

→ By properly shaping the geometrical Configuration and Choosing the appropriate material for lenses, they can transform various forms of divergent energy into plane waves.

→ They Can be used, especially at higher frequencies.

ex:-



~~Radiation Mechanism :-~~

Note :- An ideal antenna is one that will radiate all the power delivered to it from the transmitter in a desired direction or directions. In practice, however, such ideal performances can't be achieved but may be closed approached. Various types of antennas are available and each type can take different forms in order to achieve the desired radiation characteristics for the particular application.

Radiation Mechanism :-

(i) Single wire

Conducting wires are materials whose prominent characteristic is the motion of electric charges and creation of current flow. If the wire is very thin (ideally zero radius), then the current in the wire can be represented by

$$\boxed{I_z = q_e \cdot v_z} \quad \text{—— (1)}$$

where q_e = ~~Coul~~ Coulombs/meter. (Charge per unit length)

v_z = Charge q moving ~~moving~~ in Z-direction with uniform velocity v_z (m/sec)

[Note :- $I = \frac{dq}{dt} = \frac{\text{Coul}}{\text{sec}}$, Here, $I_z = \frac{\text{Coul}}{\cancel{m}} \times \frac{\cancel{m}}{\text{sec}}$]

→ If the current is time varying, then the derivative of the current (eq. 1) can be written as,

$$\frac{dI_z}{dt} = q_e \cdot \frac{dv_z}{dt} = q_e \cdot a_z \quad \text{--- (2)}$$

Where $\frac{dv_z}{dt} = a_z \left(\frac{m}{sec^2}\right)$ is the acceleration.

→ If the wire is of length 'l', then Eqⁿ (2), Can be written as

$$l \cdot \frac{dI_z}{dt} = l q_e \frac{dv_z}{dt} = l \cdot q_e \cdot a_z \quad \text{--- (3)}$$

→ Eqⁿ (3) is the basic relation between Current and Charge, and it also serves as the fundamental relation of electromagnetic radiation.

→ It states that, to create radiation, there must be a time-varying current or an acceleration (or deceleration) of charge.

→ So to create charge acceleration or (deceleration) the wire must be curved, bent, discontinuous or terminated.

Note:- $\frac{dI_z}{dt} = \frac{d}{dt}\left(\frac{dq}{dt}\right) = \frac{d^2q}{dt^2}$

Summary:-

1) If a charge is not moving, current is not created and there is no radiation.

2) If a charge is moving with a uniform velocity:

a. There is no radiation if the wire is straight and infinite in extent.

2. There is radiation if the wire ~~the~~ is curved, bent, discontinuous, terminated or truncated as shown in figure 1.

3. If charge is oscillating in a time-motion, it radiates even if the wire is straight.

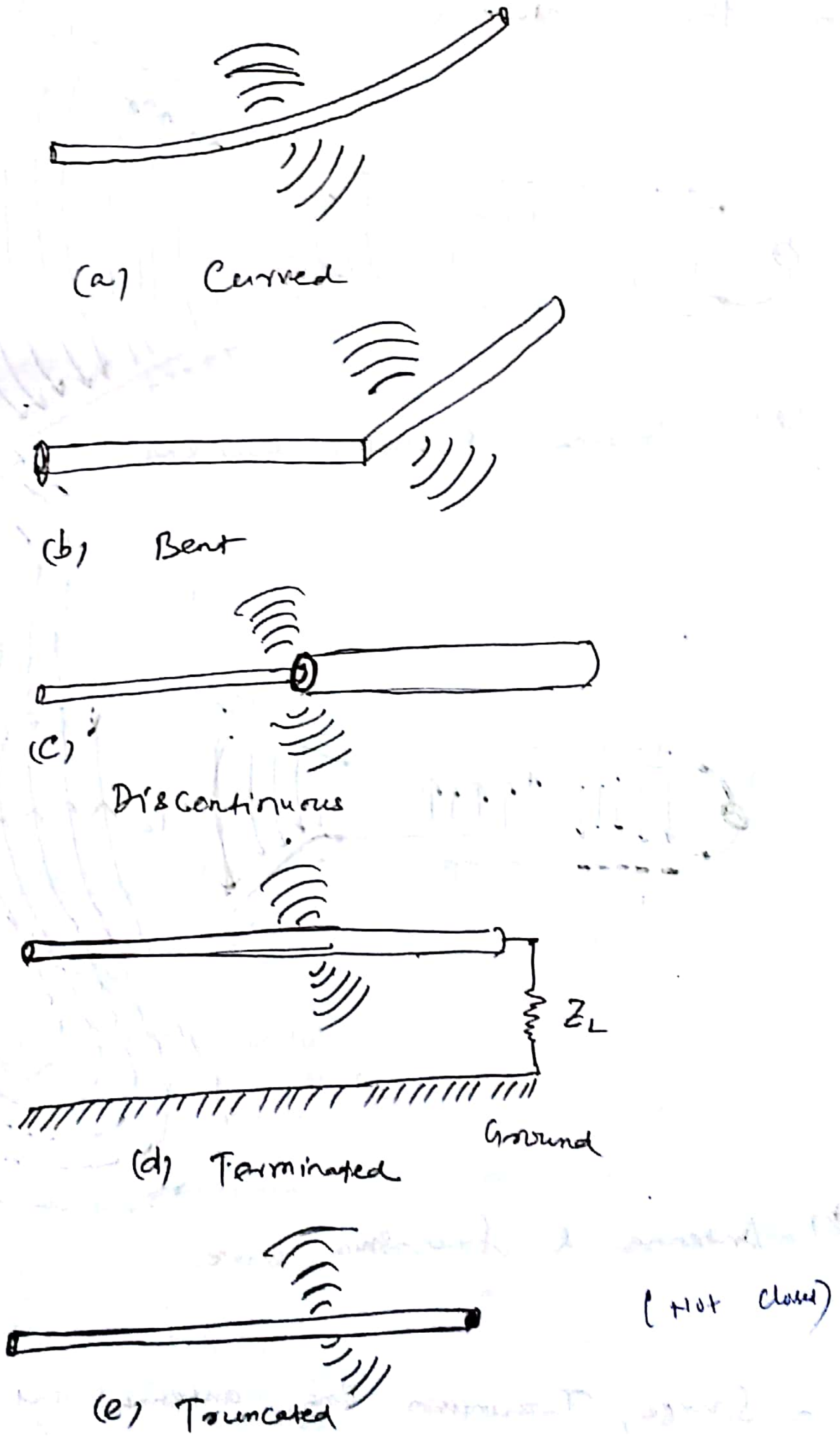
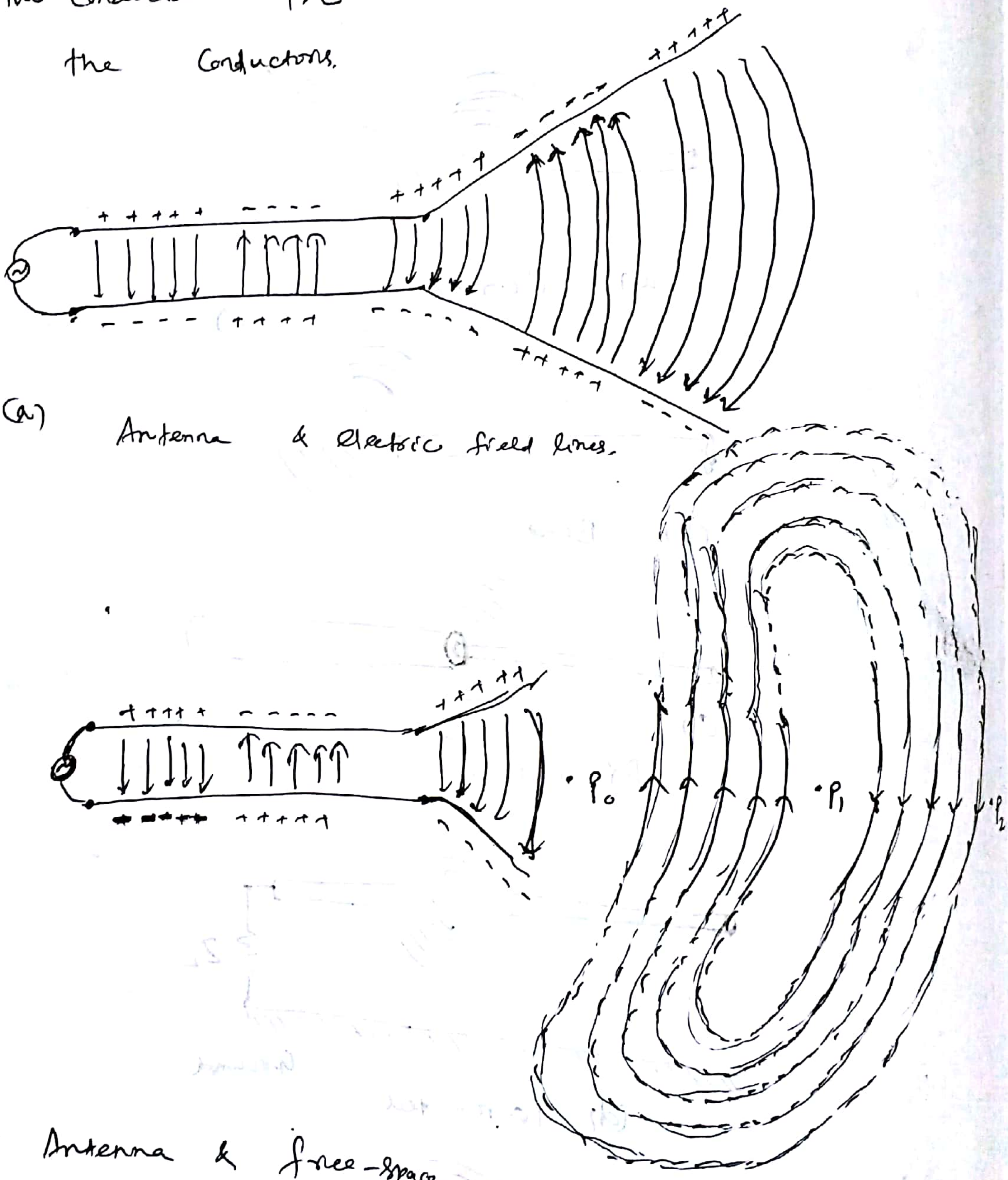


fig 1:- Wire Configuration for radiation.

2. Two-wire

→ Let us consider a voltage source connected to a two-conductor transmission line which is connected to an antenna, shown in fig 2(a). Applying a voltage across the two-conductor T.L creates an electric field between the conductors.



(a) Antenna & electric field lines.

(b) Antenna & free-space wave

fig 2:- Source, Transmission line, antenna and detachment of electric field lines.

→ The electric field has associated with it electric lines of force which are tangent to the electric field at each point and their strength is proportional to the electric field intensity.

→ The electric lines of force have a tendency to act on the free electrons associated with each conductor and force them to be displaced.

→ The movement of the charges creates a current that in turn creates a magnetic field intensity. Associated with the magnetic field intensity are magnetic lines of forces which are tangent to the magnetic field.

→ We have accepted that electric field lines start on +ve charges and end on -ve charges. Magnetic field lines always form closed loops encircling current-carrying conductors because physically there are no magnetic charges.

→ The electric field lines drawn between the conductors help to exhibit the distribution of charge. If we assume source voltage to be sinusoidal, we expect the electric field between the conductors to also be sinusoidal with a period equal to that of the applied source.

→ The relative magnitude of the electric field intensity is indicated by density of lines of force with the arrows showing the relative direction.

✓The creation of time-varying electric and magnetic fields between the conductors forms electromagnetic waves which travel along the T.L as shown in fig 2(a).

→ The e.m waves enter the antenna and have ✓associated with them electric charges and corresponding currents. If we remove part of the antenna structure ✓as shown in fig 2(b), free-space waves can be formed by "connecting" the open ends of electric lines. (shown dashed)

→ The free space waves are also periodic but ✓a constant phase point P_0 moves outwardly with the speed of light and travels a distance of $\frac{\lambda}{2}$ (to P_1) in the time of one half of a period.

→ It has been shown that close to the antenna the constant phase point 'P_0' moves faster than the speed of light but approaches the speed of light at points faraway from the antenna (analogous to phase velocity inside a rectangular wave guide).

How the guided waves are detached from the antenna to create free-space waves?

Suppose a pebble is dropped in a calm body of water. Once the disturbance on the water has been initiated, water waves are created which begin to travel outwardly.

→ If disturbance has been removed, the waves don't stop or extinguish themselves but continue their course of travel. If disturbances persists, new waves are continuously created which lag in their travel behind the others.

→ The same is true with e.m waves created by electric disturbance. If the initial electric disturbance by the source is of short duration, the created e.m waves travel inside the transmission lines, then into the antenna, and finally are radiated as free-space waves, even if the electric source has ceased to exists.

→ If the electric disturbance is of a continuous nature, e.m waves exist continuously and follow on their travel behind the others.

→ When the em waves are within T.L and antenna, their existance is associated with the presence of charges inside the conductors. However, when the waves are radiated, they form closed loops and there are no charges to sustain their existance.

→ Thus, electric charges are required to excite the fields but are not needed to sustain them and may exist in their absence.

Radiation Pattern :-

→ An antenna radiation pattern or antenna pattern is defined as " mathematical function or a graphical representation of the radiation properties of the antenna as a function of space co-ordinates".

→ The radiation property is the two or three dimensional spatial distribution of radiated energy as a function of the observer's position along a path or surface of constant radius.

→ A trace of received electric field at a constant radius is called the amplitude field pattern. On the other hand, a graph of the spatial variation of the power density along ~~the~~ a constant radius is called ~~the~~ an amplitude power pattern.

→ Often the field and Power patterns are normalized w.r.t their maxm value, yielding normalized field and power patterns.

→ Also, the power pattern is usually plotted on a logarithmic scale or more commonly in decibel (dB). This scale is usually desirable because a logarithmic scale can accentuate in more details those parts of the pattern that have very low values (minor lobes)

→ Thus,

(a) Field pattern (in linear scale): typically represents a plot of magnitude of the electric or magnetic field as a function of the angular space.

(b) Power pattern (in linear scale) :- typically represents a plot of the square of the magnitude of the electric or magnetic fields as a f^n of angular space.

(c) ~~Power~~ Field pattern (in dB) represents the magnitude of the electric or magnetic field in dB as function of angular space.

→ The 2D normalized field pattern, power pattern and power pattern in dB, are shown in fig (3).

The Half Power (−3dB) points can be found out relative to the max^m value of the patterns as follows. i.e

(a) field pattern at 0.707 ($\frac{1}{\sqrt{2}}$) value of its max^m [fig 3(a)]

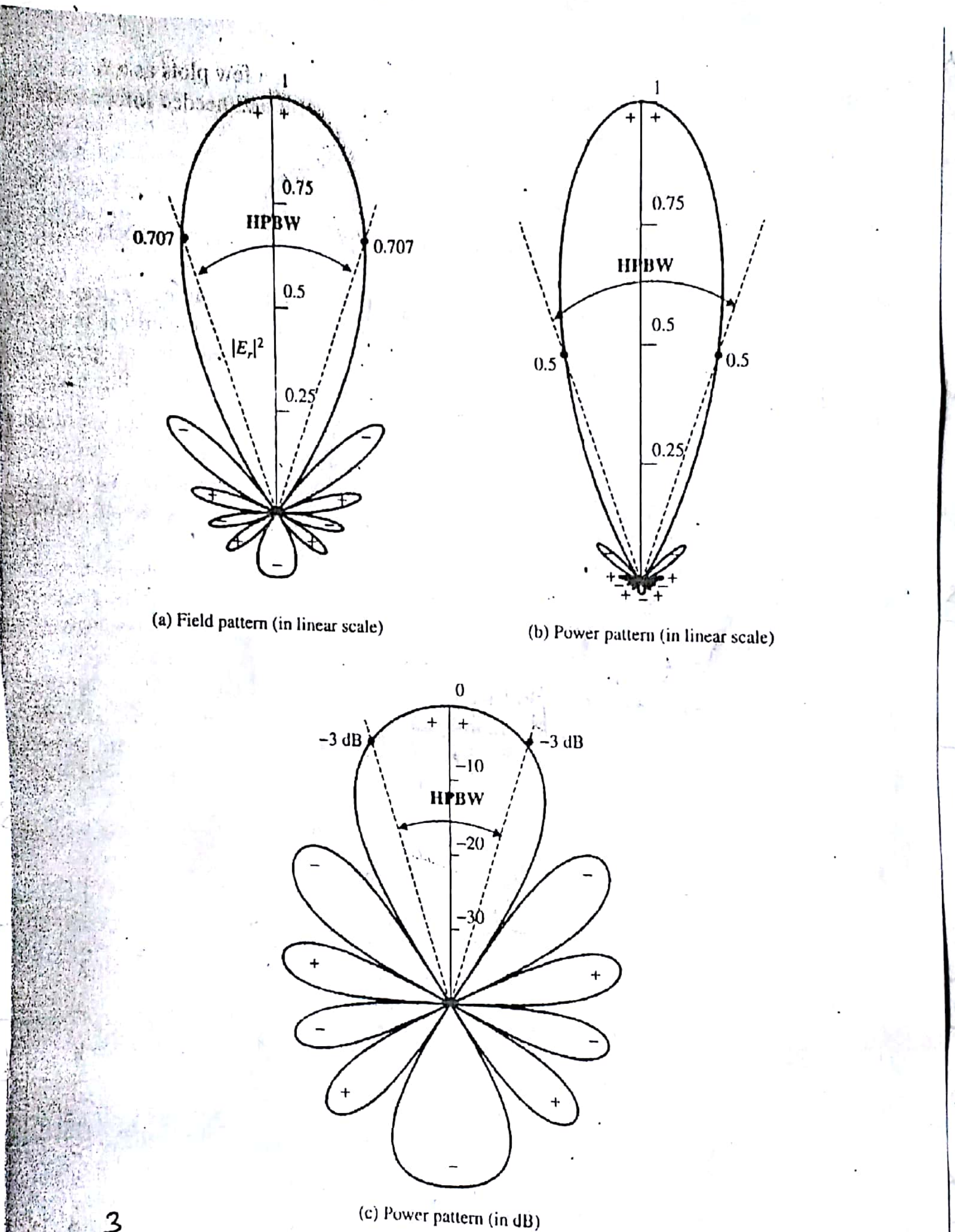


Figure 3 Two-dimensional normalized *field* pattern (*linear scale*), *power* pattern (*linear scale*), and *power* pattern (*in dB*) of a 10-element linear array with a spacing of $d = 0.25\lambda$.

(b). Power pattern (in a linear scale) at its 0.5 value of its maximum [fig 3(b)]

(c) Power pattern (in dB) at −3dB value of its max^m [fig 3(c)]

→ The angular separation between the two half-power points is called, Half Power Beam width (HPBW).

→ In all three patterns here it is 38.64° [fig 3]

Radiation Pattern Lobes

→ A major lobe (also called main beam) is defined as "the radiation lobe containing the direction of maxm radiation."

→ In some antennas, there may exist more than one major lobe.

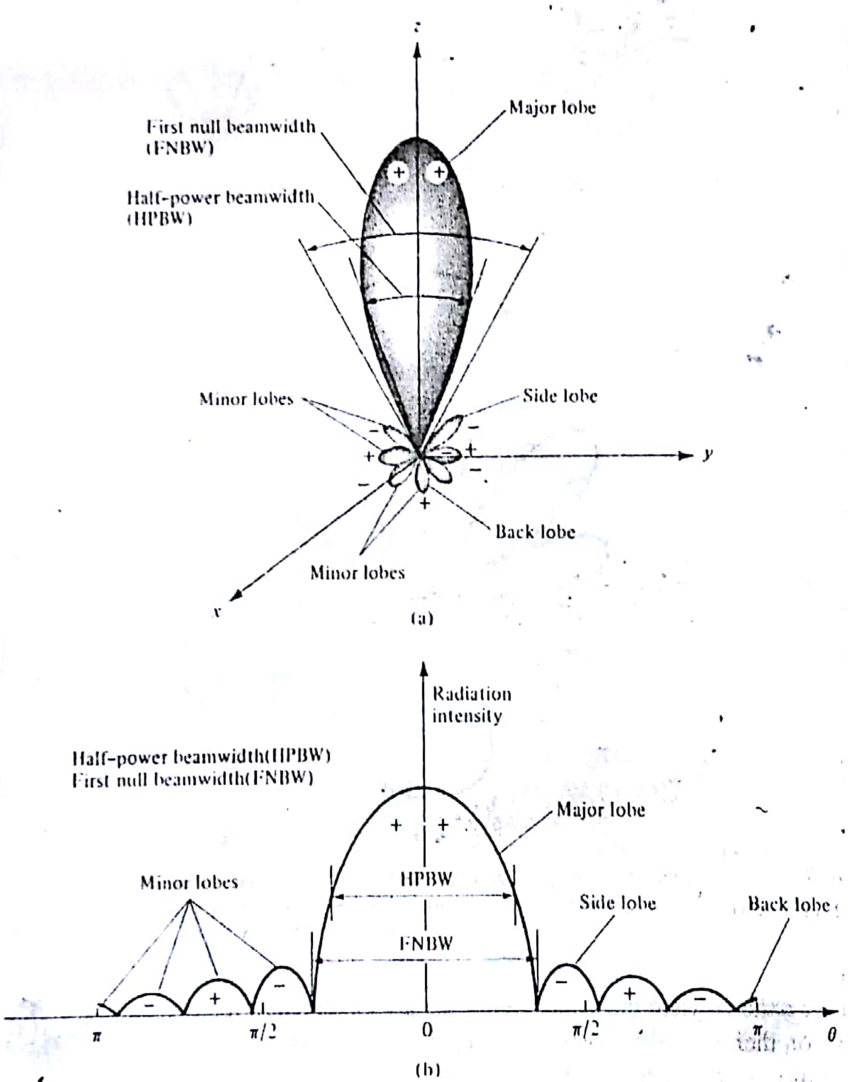


Figure 4 (a) Radiation lobes and beamwidths of an antenna pattern. (b) Linear plot of power pattern and its associated lobes and beamwidths.

→ A minor lobe is any lobe except a major lobe.

→ In fig 4 (a) & (b) all the lobes with the exception of the major can be classified as minor lobes.

→ A side lobe is "a radiation lobe in any direction other than the intended lobe". (Usually a sidelobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam).

→ A backlobe is "a radiation lobe whose axis makes an angle of approximately 180° w.r.to ~~to~~ the beam of an antenna."

→ Minor lobes usually represent radiation in undesired directions, and they should be minimized.

→ Side lobes are normally the largest of the minor lobes.

Isotropic, Directional & Omnidirectional Patterns

→ An isotropic radiator is defined as "a hypothetical lossless antenna having equal radiation in all directions."

→ Although it is ideal and not physically realizable, it is often taken as a reference for expressing the directive properties of actual antennas.

→ A directional antenna is one "having the property of radiating or receiving e.m waves more effectively in some directions than in other."

→ Azimuthal Plane [$\theta = $ const (say $\frac{\pi}{2}$), varying ϕ i.e $f(\phi)$]

→ Elevation plane [ϕ = constant, varying θ i.e $g(\theta)$].

→ A omnidirectional pattern is defined as one "having essential nondirectional pattern in a given plane (say azimuthal) and directional pattern in any orthogonal plane (say elevation). ~~An~~

→ An omnidirectional pattern is then a special type of directional patterns.

Principal Pattern :-

→ For a linearly polarized antena, performance is often described in terms of its principal E- and H-plane patterns.

→ The E-plane is defined as "the plane containing the electric field vector and the direction of maximum radiation".

→ The H-plane is defined as" the plane containing the magnetic-field vector and the direction of max^m radiation".

→ It is usual practice to orient most antennas so that at least one of the principal plane patterns coincide with one of the geometrical principal planes.

→ As shown in fig (5) & (6), for example the x-z plane (~~azimuthal plane;~~ ~~$\theta = \frac{\pi}{2}$~~) (elevation plane; $\phi = 0$) is the principal E-plane & x-y plane (azimuthal plane, $\theta = \frac{\pi}{2}$) is the principal H-plane

→

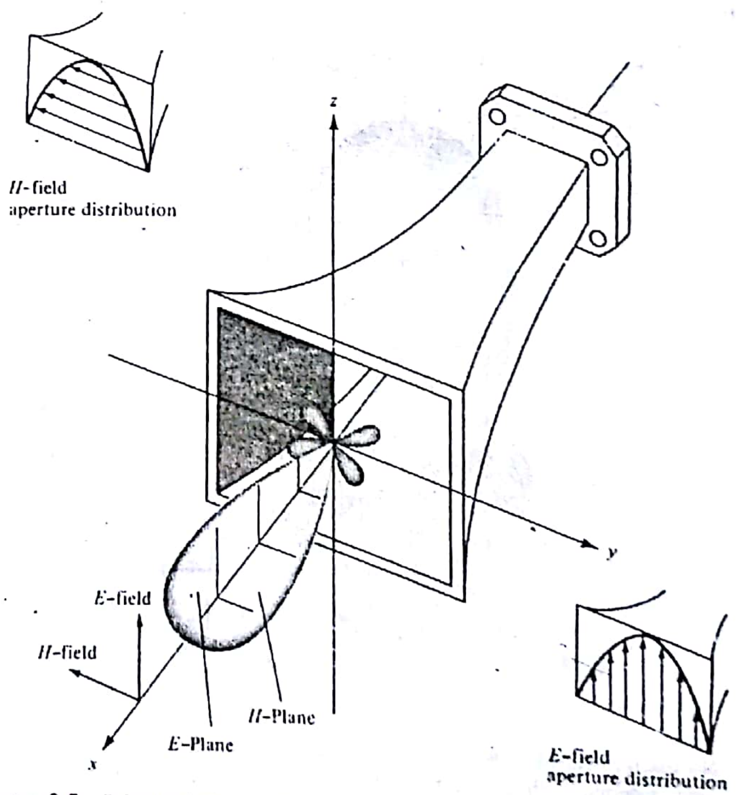


Figure 5 Principal *E*- and *H*-plane patterns for a pyramidal horn antenna.

→ The omni-directional pattern of fig (6) has an infinite number of principal E-planes (elevation planes, $\phi = \phi_c$) and one principal H-plane (azimuthal plane $\theta = 90°$) - [∵ Max^m of azimuthal planes is at $\theta = 90°$]

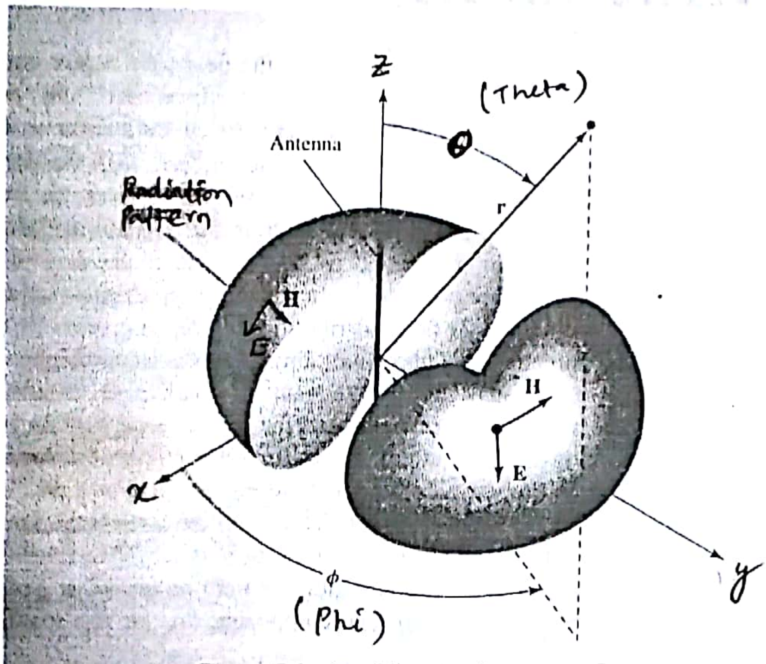


Figure 8.6 Omnidirectional antenna pattern

Field Regions :-

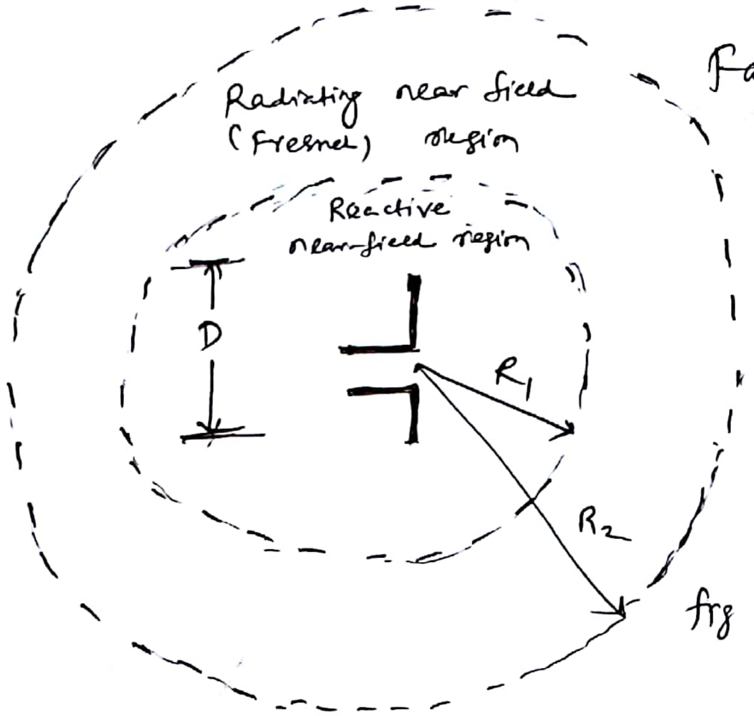
✓ The space surrounding an antenna is usually subdivided into three regions (a) reactive near-field (b) radiating near-field (Fresnel) and (c) far field (Fraunhofer)

(a) Reactive near-field

Reactive near-field region is defined as " that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates."

→ For most of the antennas, the outer boundary of this region is commonly taken to exist at a distance $0.62\sqrt{\frac{D^3}{\lambda}}$ from the antenna surface,

where λ is the wavelength and D is the largest dimension of the antenna.



$R_1 = 0.62\sqrt{\frac{D^3}{\lambda}}$

$R_2 = \frac{2D^2}{\lambda}$

fig 7:- Field regions of an antenna

→ Radiating near-field (Fresnel) region is defined as "that region of the field of an antenna between the reactive near-field region and the far-field region wherein ~~Radiating~~ radiation fields predominates. and wherein the angular field distribution is dependent upon the distance from the antenna.

→ If $D << \lambda$, this region may not exist.

→ For an antenna focused at infinity the radiating near-field region is sometimes referred to as the Fresnel region on the basis of analogy to optical terminology.

→ So $0.62\sqrt{\frac{D^3}{\lambda}} \leq$ Radiating near field region $< \frac{2D^2}{\lambda}$.

→ Far field (Fraunhofer) region is defined as "that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna."

→ The far-field region is commonly taken to exist at distance greater than $\frac{2D^2}{\lambda}$ from the antenna,

→ For an antenna focused at infinity, the far-field region is sometimes referred to as the Fraunhofer region on the basis of analogy to optical terminology".

→ The amplitude pattern of an antenna, as the observation distance is varied from the reactive near field to the far field, changes in shape because of variations of the fields, both magnitude and phase.

→ A typical progression of the shape of an antenna, with the largest dimension D, is shown in figure 8.

→ It is apparent that in the reactive near-field region the pattern is more spread out and nearly uniform, with slight variations.

→ As the observation is moved to the radiating near-field region (Fresnel), the pattern begins to smooth and form lobes.

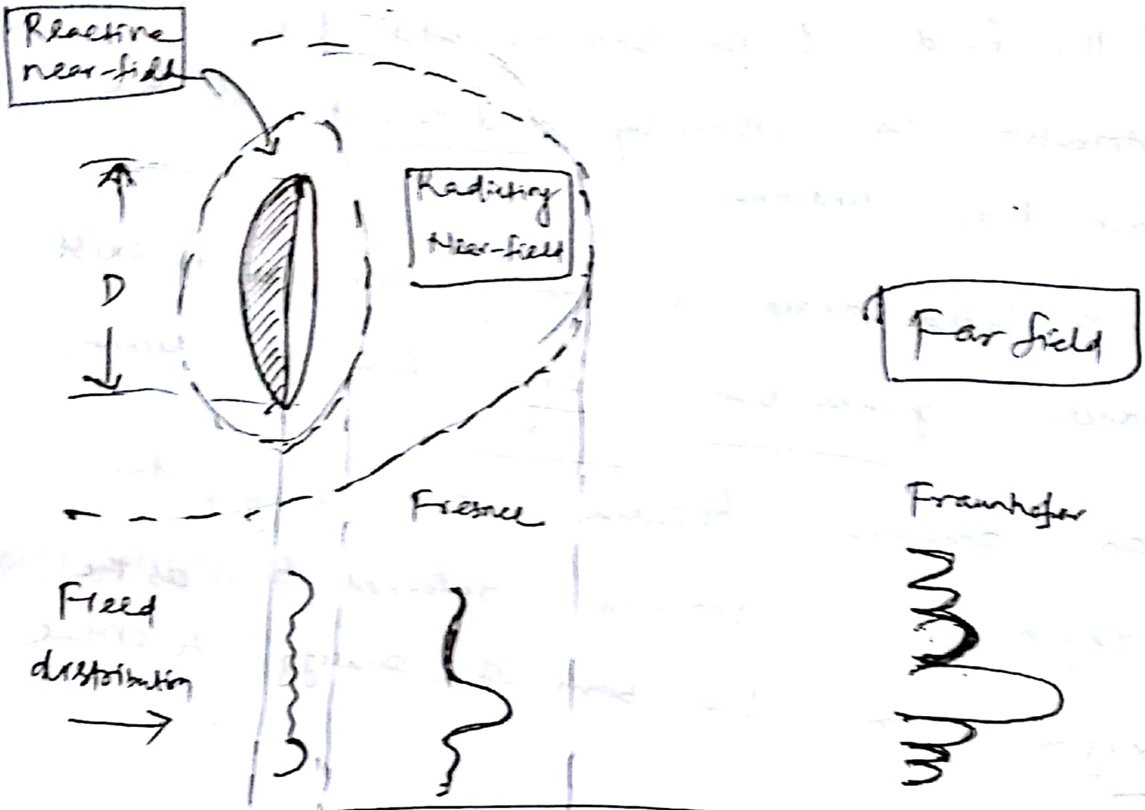


fig 8 :- Typical changes of antenna amplitude pattern shapes from reactive near-field toward the far field.

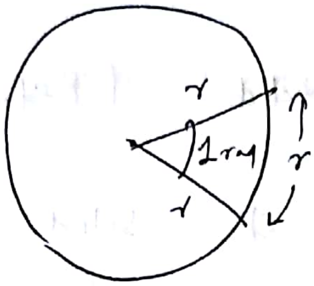
→ In the far-field region (Fraunhofer), the pattern is well formed, usually consisting of few minor lobes and one or more major lobes.

Radian and Steradian :-

The measure of plane angle is radian. One radian is defined as the plane angle with its vertex at the center of a circle of radius 'r' that is subtended by an arc whose length is 'r'.

$\left(\because \quad \theta = \frac{l}{r} \quad \text{If} \quad l = r, \quad \theta = 1 \text{ radian} \right)$

i.e.



Since the circumference of circle = $2\pi r$. There are 2π $\left(\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi \right)$ rad in a full circle.

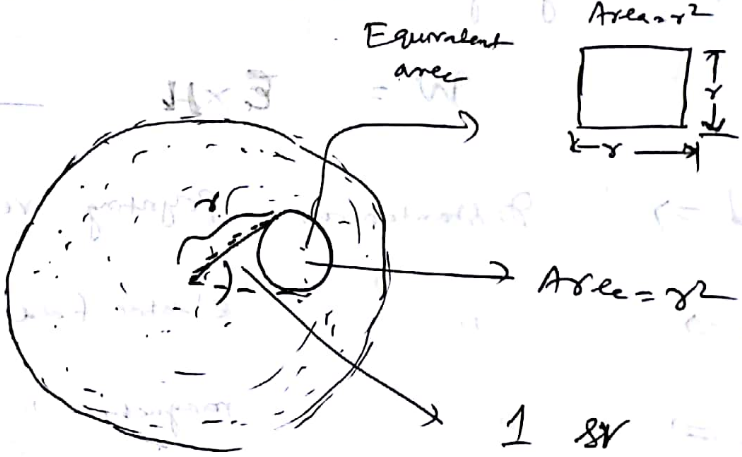
Steradian (~~Beam Solid Angle~~)

The measure of solid angle is a steradian.

One steradian is defined as the solid angle with its vertex at the center of a sphere of radius 'r' that is subtended by a spherical surface area equal to that of a square with each side of length 'r'.

$\frac{\text{Area (steradian)}}{r^2}$

$1\ Sr = \frac{r^2}{r^2} = \frac{\text{Area of square}}{r^2}$



→ Since the area of a sphere of radius 'r' is $A = 4\pi r^2$, there are 4π Sr $\left(\frac{4\pi r^2}{r^2}\right)$ in a closed sphere.

For $r^2 \rightarrow 1$ Steradian,
for $4\pi r^2 \rightarrow 4\pi$ Steradians

Note: -

→ The infinitesimal area 'dA' on the surface of a sphere of radius 'r' is given by,

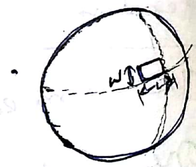
$dA = r^2 \sin\theta \, d\theta \, d\phi$ (Refer EMT book)

Therefore, the element of solid angle $\underline{d\Omega}$ of a sphere can be written as

$$d\Omega = \frac{dA}{r^2} = \frac{\cancel{r^2} \sin\theta \, d\theta \, d\phi}{\cancel{r^2}} = \sin\theta \, d\theta \, d\phi \, (Sr)$$

[1 Solid angle refers to angle by subtended area r^2.
$1 Sr \rightarrow r^2$]

$$\boxed{d\Omega = \sin\theta \, d\theta \, d\phi \, (Sr)}$$



$L = r \sin\theta \, d\phi$
$W = r \, d\theta$
$dA = L \times W = r^2 \sin\theta \, d\theta \, d\phi$

Beam Area (or Beam Solid Angle) $[\Omega_A]$

In Polar two-dimensional Co-ordinates an incremental area dA on the surface of a sphere is the product of length $r\,d\theta$ in the 'θ' direction (latitude) and $r\sin\theta\,d\phi$ in the ϕ direction (longitude) as shown in figure below.

Thus

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) \quad \text{—— (1)}$$

$$dA = r^2 \sin\theta\, d\theta\, d\phi \quad \text{—— (2)}$$

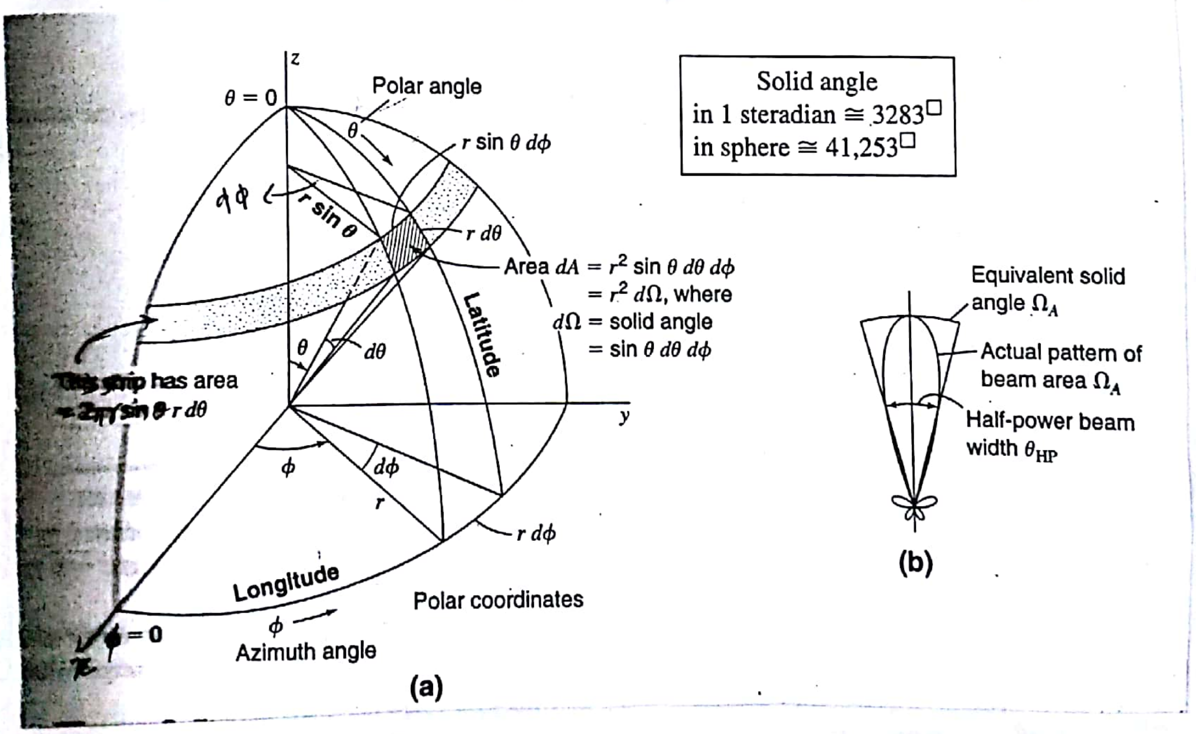


Fig:- Polar Coordinates showing incremental solid angle $dA = r^2 d\Omega$ on the surface of sphere of radius r where $d\Omega$ = solid angle subtended by area dA (b) Antenna Power pattern & its equivalent solid angle or beam area (Ω_A)

$\therefore \quad dA = r^2 \sin\theta \, d\theta \, d\phi$

$$dA = r^2 \, d\Omega \quad \text{——— (3)}$$

<u>Where</u> $\quad \therefore d\Omega = \sin\theta \, d\theta \, d\phi$ ——— (4)

$d\Omega$ = Solid angle expressed in steradians (sr) or square degrees. ($\square$)

$d\Omega$ = Solid angle subtended by area dA

The area of the strip of width <u>$r\,d\theta$</u> extending around the sphere at a constant angle θ is given by

$$(2\pi r \sin\theta) \cdot (r\,d\theta),$$

($\because$ Perimeter of a circle having radius '$r\sin\theta$' $= (2\pi)(r\sin\theta)$)

Integrating this for θ value from 0 to π yields the area of the sphere.

$$\text{Area of sphere} = \int_0^{\pi} (2\pi r \sin\theta) \cdot (r\,d\theta)$$

$$= 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta$$

$$= 2\pi r^2 \left[-\cos\theta\right]_0^{\pi}$$

$$= 2\pi r^2 \left[\cos\theta\right]_{\pi}^{0}$$

$\therefore$ Area of the sphere

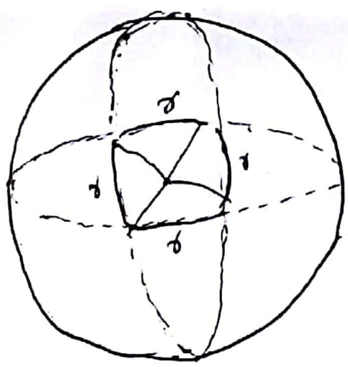
$= 2\pi r^2 [1 - (-1)]$

$= 2\pi r^2 \times 2$

Area of sphere $= 4\pi r^2$ ——— (5)

where 4π = Solid angle subtended by a sphere, Sr

$\therefore$ 1 Steradian = 1 Sr = Solid angle subtended by a square of side 'r' with center of the sphere



= Angle due to Horizontal side of square × Angle due to Vertical side of square

= 1 rad × 1 rad

$\boxed{1 Sr = 1 \text{ rad}^2}$

(∵ Angle due to a arc of length 'r' = 1 rad)

$= 1 \times \left(\frac{180}{\pi}\right)^2 \deg^2$

$\boxed{1 Sr = 3282.8064 \deg^2} \simeq 3283^{\square}$ → indicate square degree

$\therefore$ 4π Steradian = $4\pi \times 3282.8064 \simeq 41,253 \deg^2$

$= 41,253^{\square}$ = Solid angle in a sphere.

$= 41,253^{\square}$

The beam area or beam solid angle or Ω_A of an antenna is given by integrating the normalized power pattern over a sphere (4π Sr)

$$\boxed{\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi} \quad (7) \to (6)$$

$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta, \phi) \, d\Omega} \quad (S_r) \to (7)$$

where $\boxed{d\Omega = \sin\theta \, d\theta \, d\phi, \ S_r}$

The beam area Ω_A is the solid angle through which all of the Power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its max^m value over $\underline{\Omega_A}$ and $\underline{\text{Zero}}$ elsewhere

$\therefore$ Thus power radiated $= P(\theta, \phi) \, \Omega_A$ Watts.

<u>Note</u> :- The beam area of an antenna can often be described approximately in terms of the angle subtended by half-power two points of the main lobe in the principal planes. [E - plane & H - plane]

Thus,

$$\text{Beam Area} \cong \Omega_A = \theta_{HP} \cdot \phi_{HP} \quad (S_r) \quad (8)$$

Where

θ_{HP} & ϕ_{HP} are the half power beamwidth (HPBW) in the two principal planes, minor lobes being neglected.

Radiation Power density:-

✓ Power associated with an e.m wave is the instantaneous Poynting vector defined as

$$\mathcal{W} = E \times H \quad \text{--- (1)}$$

$\mathcal{W} \Rightarrow$ Instantaneous Poynting vector $\left(\frac{\text{Watt}}{m^2}\right)$

$E \Rightarrow$ " electric-field intensity $\left(\frac{\text{Volt}}{m}\right)$

$H \Rightarrow$ " magnetic - " , $\left(\frac{\text{Amp}}{m}\right)$

→ Since the Poynting vector is power density, the total power crossing a closed surface can be obtained by integrating the normal component of the poynting vector over the entire surface.

$$P = \oiint_S \mathcal{W} \cdot ds = \oiint_S \mathcal{W} \cdot \hat{n} \, da \quad \text{--- (2)}$$

$\oint \mathcal{P}$ = Instantaneous total power (W)

$\hat{n}$ = Unit vector normal to the surface.

da = Infinitesimal area of the closed surface (m^2)

Note :- The script letters are used to denote instantaneous fields and quantities, where as roman letters are used to represent their complex counterparts.

∴

$$\mathcal{E}(x,y,z;t) = Re\left[E(x,y,z).e^{j\omega t}\right] \quad -③$$

$$\mathcal{H}(x,y,z;t) = Re\left[H(x,y,z)\,e^{j\omega t}\right] \quad -④$$

We know that

$$Re\left[E\,e^{j\omega t}\right] = \frac{1}{2}\left[E.e^{j\omega t} + E^{*}\,e^{-j\omega t}\right] \quad -⑤$$

Using eqn ⑤ in eqn ①

$$\mathcal{W} = \frac{1}{2}\left[E.\,e^{j\omega t} + E^{*}\,e^{-j\omega t}\right] \times \frac{1}{2}\left[H.e^{j\omega t} + H^{*}\,e^{-j\omega t}\right] \quad -⑥$$

Simplifying, the time avg Poynting vector (Avg power density) can be written as

$$W_{av}(x;y,z) = \left[\mathcal{W}(x,y,z;t)\right]_{av} = \frac{1}{2}Re\left[E \times H^{*}\right] -⑦$$

Note :- Real part represent avg (real) power density,

Img. part " reactive (stored) power density associated with e.m fields.

From eqn (7), the avg power radiated by an antenna can be written as,

$$P_{rad} = P_{av} = \oint\!\!\oint_S W_{rad} \cdot ds$$

$$= \oint\!\!\oint_S W_{av} \cdot \hat{n}\, da$$

$$P_{rad} = \frac{1}{2} \oint\!\!\oint_S Re[E \times H^*] \cdot ds \quad \text{—— (8)}$$

Note:- 1) The power pattern (As discussed before) of the antenna, is just a measure, as a function of direction, of the avg power density radiated by the antenna.

2) The observation are usually made on a large sphere of constant radius extending into the far field.

→ For an isotropic antena, (radiates equally in all direction), its Poynting vector will n't be a fn of the ~~Spher~~ spherical Co-ordinates angle θ and ϕ. It will have ~~be~~ only a radial component.

Thus, total power radiated by it is given by

$$P_{rad} = \oint\!\!\oint_S W_0 \cdot ds = \int_0^{2\pi}\int_0^{\pi} [\hat{a}_r W_0(r)] \cdot [\hat{a}_r r^2 \sin\theta\, d\theta\, d\phi] \quad \text{—— (9)}$$

$[\because ds = r^2 \sin\theta\, d\theta\, d\phi$, as discussed earlier$]$

$$\boxed{P_{rad} = 4\pi r^2 W_0} \quad \text{--- (10)}$$

$\therefore \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi \cdot r^2$

$2\pi \times \cos\theta]_{\pi}^{0} = 2\pi \times [2] \times r^2 = 4\pi r^2$

$\therefore$ Power density is given by,

$$\vec{W_0} = \hat{a}_r W_0 = \hat{a}_r \left(\frac{P_{rad}}{4\pi r^2}\right) \frac{Watt}{m^2} \quad \text{--- (11)},$$

(vector)

which is uniformly distributed over the surface of a sphere of radius 'r'.

Radiation Intensity:-

→ Radiation Intensity in a given direction is defined as "the power radiated from an antenna per unit solid angle."

→ Radiation intensity is a far-field parameter, and it can be obtained by simply multiplying the radiation density by the square of the distance..

Mathematically,

$$\boxed{U = r^2 W_{rad}}$$

Note:- over $4\pi r^2$ → P_{rad} power radiated

over 1 → $\frac{P_{rad}}{4\pi r^2}$

over r^2 → $\left(\frac{P_{rad}}{4\pi r^2}\right) \times r^2 = W_{rad} \cdot r^2$

$= P_{rad}/4\pi$. (12)

4π steradian corresponds to $4\pi r^2$ area.

$1\ Sr = \frac{4\pi r^2}{4\pi}$

$1\ Sr = r^2$

Unit solid angle corresponds to r^2.

where U = radiation intensity $\left(\frac{Watt}{unit\ solid\ angle}\right)$

W_{rad} = Radiation density $\left(\frac{Watt}{m^2}\right)$.

$\therefore U = r^2 \times \frac{Watt}{r^2}$

= Watt per unit solid angle

The total power is obtained by integrating the radiation intensity, as given in eq^n (12), over the entire solid angle of 4π.

Thus,

$$P_{rad} = \oiint_{\Omega} U \, d\Omega$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin\theta \, d\theta \, d\phi \quad \text{--- (13)}$$

where $d\Omega$ = element of solid angle $= \sin\theta \, d\theta \, d\phi$

→ For an isotropic source U will be independent of the angles 'θ and ϕ', as was the case of W_{rad}. Thus eqn (13) can be written as

$$P_{rad} = \oiint_{\Omega} U_0 \, d\Omega = U_0 \oiint d\Omega = 4\pi U_0 \quad \text{--- (14)}$$

$$\Rightarrow \quad U_0 = \frac{P_{rad}}{4\pi} \quad \text{--- (15)}$$

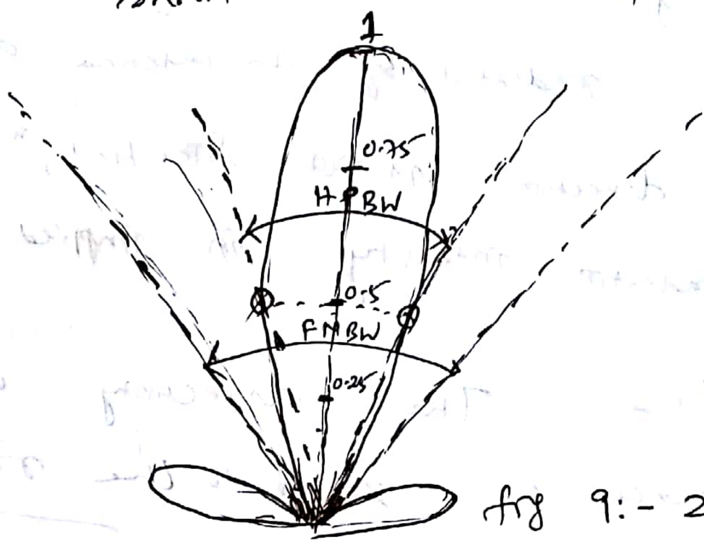
→ Radiation intensity of an isotropic source is given by $\boxed{U_0 = \frac{P_{rad}}{4\pi}}$,

Beam width:-

The beamwidth of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum.

One of the most widely used beamwidths is Half-power Beamwidth (HPBW), defined as " In a plane containing the direction of the maximum of a beam, the angle between ~~two~~ the two directions in which the radiation intensity is one-half value of the beam!"

→ This is shown in figure 9:-



exc -

HPBW = 28.79°

FNBW = 60°

fig 9:- 2D Power Pattern (in linear scale)

→ Another important Beamwidth is the angular separation between the first nulls of the pattern. and it is referred to as First-null Beamwidth (FNBW) (shown in fig 9)

→ Often there is Trade off between beamwidth & side lobe level; i.e as the beamwidth decreases the side lobe increases and vice versa.

→ Note:-

$$HPBW \simeq \frac{FNBW}{2}$$

Directivity:-

→ ~~Def~~ Directivity of an antenna is defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions."

→ The average radiation intensity is equal to the total power radiated by an antenna divided by 4π.

→ If the direction is not specified, the direction of maxm radiation intensity is implied.

→ Another Defn:- The directivity of a non-isotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

$$\therefore D = \frac{U}{U_0} = \frac{U}{\left(\frac{P_{rad}}{4\pi}\right)} \qquad \left[\because U_0 = \frac{P_{rad}}{4\pi} \quad \text{eq}^n \ (15)\right]$$

$$\Rightarrow \boxed{D = \frac{4\pi U}{P_{rad}}} \quad -(16) \quad \& \quad \boxed{D = \frac{U}{U_0}} \quad -(17)$$

If direction is not specified, it implies the direction of maxm radiation intensity (maxm directivity) expressed as

$$D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{U_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} = \frac{4\pi U_{max}}{P_{rad}} \quad -(18)$$

where

$D =$ Directivity (dimensionless)

$D_0 =$ Max^m directivity

$U =$ Radiation Intensity $\left(\frac{W}{\text{Unit Solid angle}}\right)$

$U_{max} =$ Max^m " " (")

$U_0 =$ radiation intensity of isotropic source (")

$P_{rad} =$ total radiated power (watt)

→ Note :-

1) For isotropic Source $U = U_{max} = U_0$

so $\boxed{D = D_{max} = D_0 = 1}$

2) $0 \leq D \leq D_0$

→

Gain of Antenna :-

Any Physical antenna has losses associated with it. Depending on the antenna structure, both ohmic and dielectric losses can be present in the antenna. Let P_{loss} be the power dissipated in the antenna due to the losses in the structure and

$$\boxed{P_{in} = P_{rad} + P_{loss}}$$; be the power i/p to the antenna,

(Radiation efficiency, η), of an antenna is the ratio of the total power radiated to the net power i/p to the antenna.

$$\eta = \frac{P_{rad}}{P_{in}} \quad \text{——— (1)}$$

Taking losses into account, the gain of the antenna is defined as

$$G(\theta, \phi) = \eta \, . \, D(\theta, \phi) \quad \text{——— (2)}$$

i.e

$$\text{Gain} = \left(\begin{matrix}\text{Radiation} \\ \text{efficiency}\end{matrix}\right) \times (\text{Directivity})$$

$$\therefore \quad G(\theta, \phi) = \eta \, . \, \frac{4\pi U(\theta, \phi)}{P_{rad}} \quad \text{——— (3)}$$

$$G(\theta,\phi) = \frac{\cancel{P_{rad}}}{P_{in}} \times \frac{4\pi U(\theta,\phi)}{\cancel{P_{rad}}}$$

$$\boxed{G = \frac{4\pi U}{P_{in}}} \quad \text{——} \quad (4)$$

we know $\boxed{D = \dfrac{4\pi U}{P_{rad}}}$ —— (5)

$$\therefore \frac{D}{G} = \frac{\cancel{4\pi U}}{P_{rad}} \times \frac{P_{in}}{\cancel{4\pi U}} = \frac{P_{in}}{P_{rad}} = \frac{1}{\eta}$$

$$\Rightarrow \boxed{G = \eta D}$$

Since $\eta < 1$

$$\boxed{G < D}$$

Gain of Antenna < Directivity of antenna.

Since $\boxed{G = \dfrac{U(\theta,\phi)}{(P_{in}/4\pi)}}$, we can define gain of the antenna as follows.

Gain of an antenna

Although the gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities.

→ ✓ Absolute gain of an antenna (in a given direction) is defined as "the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (i/p) by the antenna divided by 4π"

$$\therefore \text{Gain} = \frac{\text{Radiation intensity}}{\left(\frac{\text{Total input (accepted power)}}{4\pi}\right)} = \frac{4\pi \cdot U(\theta, \phi)}{P_{in}}$$

(dimensionless)

→ The Relative gain, is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced directions". The power i/p must be the same for both antennas. In most cases, the reference antenna is a lossless isotropic source.

Thus,

$$G = \frac{4\pi U(\theta,\phi)}{P_{in}\ (\text{lossless isotropic source})} \quad \text{—} (24)$$

Note:-

When the direction is not stated, the power gain is usually taken in the direction of max^m radiation.

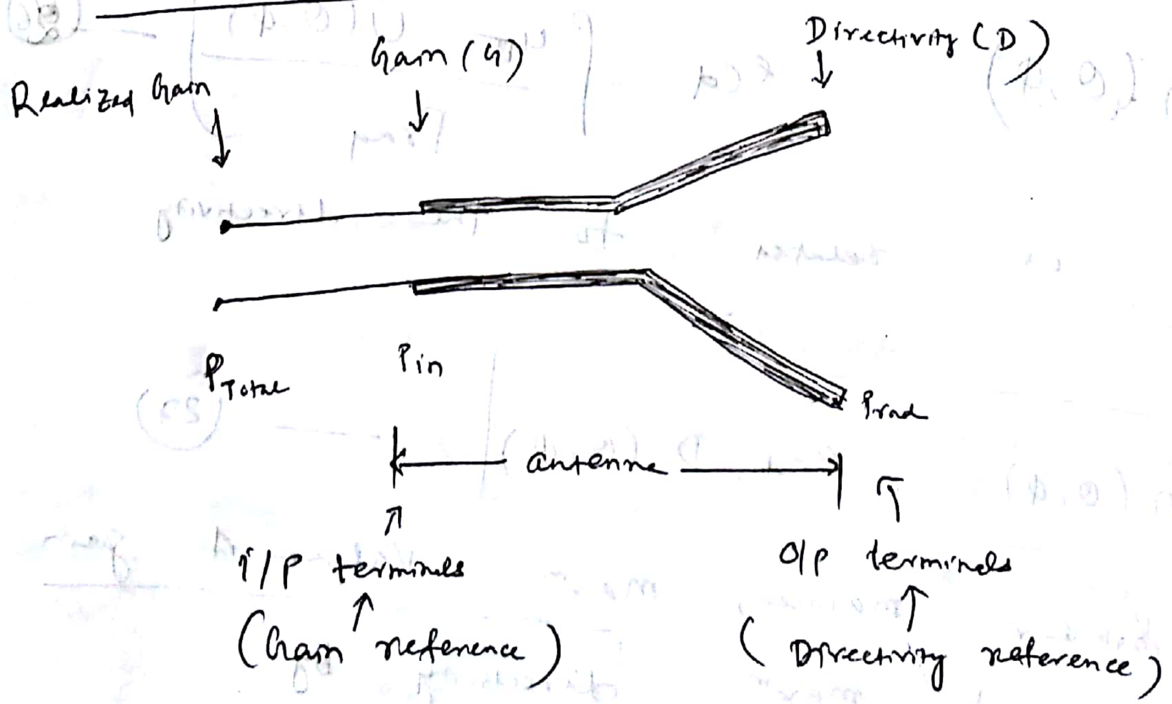


Fig 10:- Antenna reference terminals.

Referring to fig (10), we can write that radiated power (P_{rad}) is related to the total i/p power (P_{in}) by,

$$P_{rad} = e_{cd} . P_{in} \quad \text{——} \quad (25)$$

where e_{cd} is the antenna radiation efficiency <page-132> (will be discussed later) of the eqn)

(e_{cd} = conduction dielectric efficiency)

Using eqn (25) in eqn (24)

$$G(\theta,\phi) = \frac{4\pi . U(\theta,\phi)}{\left(\frac{P_{rad}}{e_{cd}}\right)}$$

$$G(\theta,\phi) = e_{cd}\left[\frac{4\pi \quad U(\theta,\phi)}{P_{rad}}\right] \quad \text{——} \quad (26)$$

which is related to the directivity

$$\boxed{G(\theta,\phi) = e_{cd} . D(\theta,\phi)} \quad \text{——} \quad (27)$$

In similar manner, maxm value of gain is related to maxm directivity by

$$G_0 = G(\theta,\phi)\Big|_{max} = e_{cd} . D(\theta,\phi)_{max} = e_{cd} . D_0 \quad \text{—} \quad (28)$$

Antenna Efficiency :-

The total antenna efficiency 'e_0' is used to take into account losses at the i/p terminals and within the structure of the antenna. Such losses may be due to, referring to fig 11.

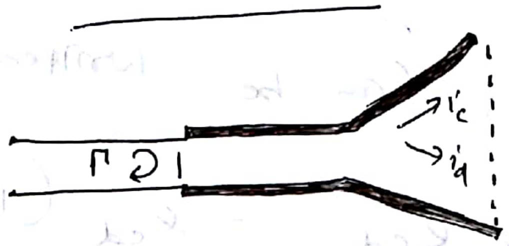


fig 11:- Reflection, conduction, and dielectric losses of an antenna

1. Reflections because of the mismatch between the transmission line and the antenna. (Γ)

2. I^2R losses (conduction & dielectric)

In general, the overall efficiency can be written as

$$e_0 = e_r \cdot e_c \cdot e_d \quad \text{——— (29)}$$

where e_0 = total efficiency (Dimensionless)

e_r = reflection (mismatch) efficiency
$= (1 - |\Gamma|^2)$ (Dimensionless)

e_c = Conduction efficiency (dimensionless)

e_d = dielectric efficiency (dimensionless)

Γ = Voltage reflection coefficient $= \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0}$

Where

Z_{in} = antenna I/P impedance

Z_0 = Characteristic impedance of the Transmission line.

Usually, e_c and e_d are very difficult to compute, they can be determined experimentally.

So eqn (29), can be written as

$$e_0 = e_r . e_{cd} = e_{cd}(1-|\Gamma|^2) \quad (30)$$

$$(\because e_r = (1-|\Gamma|^2))$$

Where $e_{cd} = e_c . e_d$ = antena radiation efficiency (31)

Which used to relate gain and directivity.

(Refer eqn (27))

Bandwidth :-

The bandwidth can be considered to be range of frequencies, on either side of a center frequency where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency.

→ For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation.

ex:- A 10:1 BW indicates the upper freq is 10 times greater than the lower.

→ For narrowband antennas, the BW is expressed as % of freq difference (upper minus lower) over the center freq of the BW.

ex:- a 5% BW indicates that the freq difference of acceptable operation is 5% of the center freq of the BW.

BW can be classified as

1) Pattern BW
2) Impedance BW

Associated with Pattern BW are gain, sidelobe level, beamwidth, polarization and beam direction.

while associated with impedance BW are I/P impedance and radiation efficiency.

Bandwidth

The performance parameters of an antenna, such as, the i/p reflection coefficient, the pattern gain, etc., are function of frequency.

The range of frequencies over which the performance of an antenna is within some specified limits is known as Bandwidth (BW) of the antenna.

It is called the i/p bandwidth if the performance parameter corresponds to the i/p characteristics.

If the performance parameter refers to the pattern characteristics, it is called pattern bandwidth.

For Example:- Over a frequency band covering f_L to f_U ($f_U > f_L$), if the antenna gain is within the specified limits, the bandwidth of the antenna expressed as a percentage of the center frequency, f_0, is

f_U = Upper freq.

f_L = Lower freq.

$$\boxed{BW = \frac{f_U - f_L}{f_0} \times 100\%} \quad \text{—— (1)}$$

where $f_0 = \frac{f_L + f_U}{2}$. Sometimes the bandwidth is expressed as a ratio of the two frequencies.

$$\boxed{BW = \frac{f_U}{f_L}} \quad \text{—— (2)}$$

The second defination is used for antennas having a very large bandwidth, known as broadband antennas.

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Polarization :-

→ Polarization of an antenna in a given direction is defined as " the Polarization of the wave transmitted (radiated) by the antenna, Note: when the direction is not stated, the Polarization is taken to be polarization in the direction of maxm gain ".

→ ✓ Polarization of a radiated wave is defined as " that property of an electromagnetic wave describing the time varying direction and relative ~~at a fixed~~ magnitude of the electric-field vector; ~~location in space~~ specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space and observed along the direction of propagation.

✓→ Polarization ~~a~~ then is the curve traced by the end point of the arrow representing the instantaneous electric field.

→ Polarization may be classified as linear, circular or elliptical.

✓→ If the vector that describes the electric field at a point in space as function of time is always directed along a line, the field is said to be linearly polarized.

→ In general, however, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized.

→ Linear and circular polarizations are special cases of elliptical.

→ The figure of the electric field is traced in a clockwise (CW) or counterclockwise (CCW) sense.

→ Clockwise rotation of the electric field vector is designated as right-hand polarization and counterclockwise as lefthand polarization.

Mathematical Analysis:-

The instantaneous field of a plane wave, traveling in the -ve Z-direction, can be written as

$$\mathcal{E}(z;t) = \hat{a}_x \mathcal{E}(z;t) + \hat{a}_y \mathcal{E}(z;t) \quad (1)$$

The instantaneous components are related to their complex counterparts by,

$$\mathcal{E}_x(z;t) = Re\left[E_x^- e^{j(\omega t + kz)}\right] = Re\left[E_{xo} e^{j(\omega t + kz + \phi_x)}\right]$$

$$\therefore \mathcal{E}_x(z;t) = E_{xo} \cos(\omega t + kz + \phi_x) \quad (2)$$

Similarly

$$\mathcal{E}_y(z;t) = E_{yo} \cos(\omega t + kz + \phi_y) \quad (3)$$

Where E_{xo} and E_{yo} are, respectively, the maximum magnitudes of the x and y components.

A. Linear Polarization

For the wave to have linear polarization, the time-phase difference between two components must be

$$\Delta\phi = \phi_y - \phi_x = n\pi \quad , \quad n = 0,1,2,3,\ldots$$

B. Circular Polarization

Circular polarization can be achieved only when the magnitude of the two components are the same and the time-phase difference between them is odd multiples of $\frac{\pi}{2}$.

i.e $$|E_x| = |E_y| \Rightarrow E_{x0} = E_{y0} \quad \text{——— (4)}$$

$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +\left(\frac{1}{2}+2n\right)\pi, & n = 0,1,2\ldots \text{ for CW} \quad (5) \\ -\left(\frac{1}{2}+2n\right)\pi, & n = 0,1,2\ldots \text{ for CCW} \quad (6) \end{cases}$$

CW → clockwise
CCW → counter clockwise

Note :- If the direction of wave propagation is reversed (i.e +Z direction), the phases in eqn (5) & (6) for CW and CCW rotation must be interchanged.

C. Elliptical Polarization

Elliptical polarization can be attained only when the time-phase difference between the two components is odd multiple of $\pi/2$ and their magnitudes are not the same.

~~or~~ when the time-phase difference between two components is not equal to multiple of $\frac{\pi}{2}$ (irrespective of their magnitude)

i.e. ~~when~~

$$|E_x| \neq |E_y| \Rightarrow E_{x0} \neq E_{y0} \quad — (7)$$

$$\text{when} \quad \Delta\phi = \phi_y - \phi_x = \begin{cases} +(\frac{1}{2}+2n)\pi & \text{for CW} \\ -(\frac{1}{2}+2n)\pi & \text{for CCW} \end{cases} \quad — (8)$$

where $n = 0, 1, 2, \ldots$

or

$$\Delta\phi = \phi_y - \phi_x \neq \pm\frac{n\pi}{2} = \begin{cases} >0, & \text{for CW} \\ <0, & \text{for CCW} \end{cases} \quad — (9)$$

where $n = 0, 1, 2, 3 = \ldots$

(Irrespective of ~~......~~ their magnitude in this case)

Antenna Radiation efficiency :- (e_{cd})

The conduction-dielectric efficiency (e_{cd}) is defined as the ratio of the power delivered to the radiation resistance R_r to the power delivered to R_r and R_L.

$$\boxed{e_{cd} = \frac{R_r}{R_r + R_L}} \quad — (10)$$

$$\left(\because e_{cd} = \frac{P_{rad}}{P_{Total}} = \frac{I^2 R_r}{P_{Loss} + P_{rad}} = \frac{I^2 R_r}{I^2 R_L + I^2 R_r} = \frac{R_r}{R_L + R_r}\right)$$

Radiation Resistance (R_r)

The antenna is a radiating device, in which the power (i.e energy per time) is radiated into space in the form of e.m waves. Hence, there must be power dissipation which may be expressed in usual manner as,

$$W' = I^2 R \quad \text{——— (11)}$$

→ If it is assumed that all this power appears as e.m waves, then the power (W') can be divided by square of the current (I^2)

i.e $R = \frac{W'}{I^2}$, at the point where it is fed to the antenna and obtain a fictitious ~~fictious~~ resistance (not actually imaginary) called as radiation resistance

→ Thus, the radiation resistance (R_r) is defined as, that ~~fictious~~ fictitious resistance, which, when substituted in series with the antenna will consume the same power as is actually radiated.

→ As a matter of fact, the energy supplied to an antenna is dissipated

(a) In, the form of e.m waves

(b) As ohmic losses in the antenna wire and nearby dielectrics (e.g insulator, gnd, surrounding object)

Thus, Total power loss = Ohmic loss + Radiation loss

$$\Rightarrow W = W'' + W'$$

$$\Rightarrow W = I^2 R_e + I^2 R_r$$

$$\Rightarrow W = I^2 (R_e + R_r)$$

$$\Rightarrow W = I^2 R,$$

where $R = R_e + R_r$.

Radiation Resistance (R_r) depends on

(i) Configuration of antenna

(ii) The point ~~at~~ where radiation resistance is considered.

(iii) Location of the antenna w.r.to ground and other objects.

(iv) Ratio of length and diameter of the conductor used.

Effective Area or Effective Aperture or Capture Area of an Antenna (A_e)

A transmitting antenna transmits e.m waves and receiving antenna receives a fraction of the same.

The concept of effective area or

aperture is best understood by considering an antenna to have an effective area or aperture over which it extracts the e.m energy from the travelling e.m waves.

→ It may be defined as the ratio of power received at the antenna load terminal to the Poynting vector (or power density) in Watts/m² of the incident wave.

Thus,

$$A_e = \frac{\text{Power received}}{\text{Poynting vector of incident wave}} \quad \text{--- (1)}$$

$$= \frac{W}{P}$$

$$\Rightarrow \boxed{W = P A_e} \quad \text{--- (1A)}$$

where, W = Power received in watts.

P = Poynting vector of the incident waves in watts/m², w power flow per sq. meter for the incident wave.

A_e = Effective or Capture area or Effective aperture in m².

→ Let a receiving antenna be placed in the field of plane polarised travelling wave as shown in fig. having effective area (A_e) and the receiving antenna (dipole) is terminated at a

Load impedance ($Z_L = R_L + jX_L$).

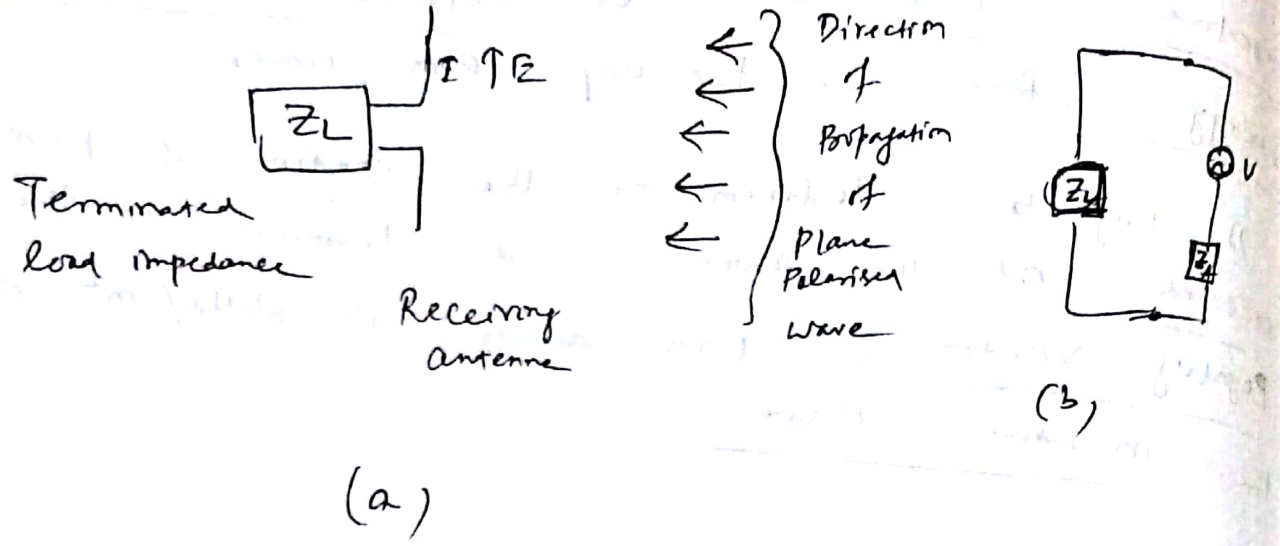


fig:- 12 (a) Receiving antenne in the field of plane polarised wave

(b) Equivalent ckt of fig (a).

→ If I be the terminal current, then received power,

$$W = I_{rms}^2 R_L \quad \text{——— (2)}$$

where R_L = Load resistance, in Ω

I_{rms} = Terminal rms current

$$A_e = \frac{W}{P} = \frac{I_{rms}^2 R_L}{P} \quad \text{——— (3)}$$

Since the antenne extracts energy from incident e.m waves, delivers the same to terminated load impedance Z_L and the power flowing per square meter in Poynting vector is P (W/m²).

This entire system can be replaced by an equivalent ckt [fig 12 (b)], according to Thevenin's th^m.

In fig 12(b),

$V =$ Equivalent Thevenin's Voltage

$Z_A =$ Equivalent " Impedance

The voltage 'V' is induced by passing e.m. waves which produces current I_{rms} through terminal load impedance Z_L.

$$I_{rms} = \frac{\text{Equivalent Voltage}}{\text{Equivalent Impedance}}$$

$$I_{rms} = \frac{V}{Z_L + Z_A} \text{ Amp} \quad \text{— (4)}$$

Where $Z_A = R_A + jX_A =$ Complex Antenna impedance

and $R_A = R_r + R_\ell =$ Radiation Resistance + Loss resistance.

$\therefore R_A = R_r$, if $R_\ell = 0$ is assumed.

Now putting Z_L and Z_A, we have

$$I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)} \quad \text{— (5)}$$

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}} \quad \text{— (6)}$$

or

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L + R_r + R_e)^2 + (X_L + X_A)^2}} \quad -(7)$$

where

X_L = Load reactance, in Ω

X_A = Antenna reactance, in Ω

The power received by terminal load impedance is given by eqn (2),

i.e $W = I_{rms}^2 R_L$

So

Putting eqn (6), in eqn (2)

$$W = \frac{V^2 \cdot R_L}{(R_L + R_A)^2 + (X_L + X_A)^2} \quad \text{—— } (8)$$

This is the power delivered by antenna at the terminating load impedance Z_L. Now by definition of effective aperture (A_e), eqn (1),

we have,

$$A_e = \frac{\text{Power received } (W)}{\text{Poynting vector } (P)} \quad (m^2) \text{ or } (\lambda^2)$$

$$A_e = \frac{V^2 R_L}{[(R_L + R_A)^2 + (X_L + X_A)^2] \cdot P} \quad (m^2) \text{ or } (\lambda^2) - (9)$$

$$A_e = \frac{V^2 R_L}{[(R_L + R_r + R_e)^2 + (X_L + X_A)^2] \cdot P} \quad -(10)$$

The induced voltage 'V' is max^m when antenna is oriented for max. response and the antenna and the incident wave both have same polarisation.

→ ~~According~~ In eq^n (10), $R_A = R_r + R_\ell$ where R_ℓ accounts for any mismatch between antenna & Z_L and antenna losses.

→ According to max^m power transfer theo^m, max^m power will be transferred from antenna to the antenna terminating load, if

$$\left.\begin{aligned} X_L &= -X_A \\ \text{and} \quad R_L &= R_A = R_r + R_\ell \\ \text{or} \quad R_L &= R_r \quad \text{if } R_\ell = 0 \end{aligned}\right] \quad (11).$$

Thus, max^m power received ~~for~~ in antenna terminating load impedance Z_L can be obtained if cond^n of eq^n (11), is substituted in eq^n (8),

$$W_{max} = \frac{V^2 R_L}{4R_L^2} = \frac{V^2}{4R_L} = \frac{V^2}{4R_r}$$

$$\boxed{W_{max} = \frac{V^2}{4R_r}} \quad \text{———} \quad (12)$$

$$(\because R_L = R_r \text{ eq}^n \ (11))$$

This is the max^m power received in antenna terminating load impedance Z_L under the condition

If max^m power transfer and without ~~loss~~
antenna loss and corresponding effective aperture is known as max^m effective aperture.

∴ Max^m effective Aperture

$$(A_e)_{max} = \frac{\text{Max}^{n} \text{ Received power}}{\text{Power density of incident wave}}$$

$$= \frac{\left(\frac{V^2}{4R_r}\right)}{P}$$

$$(A_e)_{max} = \frac{V^2}{4R_r \cdot P}$$

$$\boxed{(A_e)_{max} = \frac{V^2}{4PR_r} \ (m^2) \text{ or } (\lambda^2)} \quad (13)$$

Note:- Gain of an antenna can be expressed as $\boxed{G = \frac{4\pi A_e}{\lambda^2}}$ (For problems)

Effective length:-

The term 'effective length' of an antenna represents the effectiveness of an antenna as radiator or collector of e.m wave energy. In other words, effective length indicates how-far an antenna is effective in transmitting or receiving e.m wave energy.

For a receiving antenna, the effective length may be defined in terms of induced voltage

'V' and incident field.

(2) Effective length is nothing but the ratio of induced voltage at the terminal of the receiving antenna under open circuited condition to the incident Electric field intensity (or strength E). Thus

$$\text{Effective length} = \frac{\text{open circuited voltage}}{\text{Incident field strength (electric)}}$$

$$\boxed{l_e = \frac{V}{E} \text{ meter} \sim \text{wavelength}} \quad (14)$$

Since induced voltage V also depends on the effective aperture and hence effective length and effective aperture of an antenna are related to each other as shown below.

From eq^n (9),

$$A_e = \frac{V^2 R_L}{\left[(R_L+R_A)^2 + (X_L+X_A)^2\right] . P}$$

$$\Rightarrow V^2 = \frac{A_e . \left[(R_L+R_A)^2 + (X_L+X_A)^2\right] . P}{R_L}$$

$$\Rightarrow V^2 = \frac{A_e . \left[(R_L+R_A)^2 + (X_L+X_A)^2\right] . \frac{E^2}{Z}}{R_L} \quad \left[\because P = \frac{E^2}{Z}\right]$$

$$\Rightarrow V = \frac{\sqrt{A_e \left[(R_L+R_A)^2 + (X_L+X_A)^2\right] E^2}}{\sqrt{Z R_L}}$$

$$l_e = \frac{V}{E} = \frac{\sqrt{A_e\left[(R_A+R_L)^2+(X_A+X_L)^2\right]}}{\sqrt{Z R_L}} \quad \text{——} (15)$$

under the condition of max^m aperture, when

$$X_A = -X_L$$

$$R_A = R_r + R_l = R_L$$

$$R_A = R_r = R_L \quad \text{when } R_l = 0.$$

$$l_e = \frac{\sqrt{A_e \cdot 4R_r^2}}{\sqrt{Z R_r}} = \frac{\sqrt{A_e} \cdot 2\cancel{R_r}^{\sqrt{R_r}}}{\sqrt{Z}\,\cancel{\sqrt{R_r}}}$$

$$\boxed{l_e = \frac{2\sqrt{(A_e)_m R_r}}{\sqrt{Z}} \quad m \text{ or } \lambda} \quad (16)$$

or

$$\boxed{(A_e)_{max} = \frac{l_e^2 Z}{4 R_r}} \quad \text{——} (17)$$

This is the relation between max^m effective aperture and effective length.

→ Now for the transmitting antenna, the effective length is that length of an equivalent linear antenna that has the same current I(C) (as at the terminal of the actual antenna) at ~~the~~ all the point along its length and that radiates the same field intensity E as the actual antenna.

$I(C)$ = Current at the terminals of actual antenna.

$I(Z)$ = Current at any point Z of an antenna.

l_e = effective length.

l = Actual length

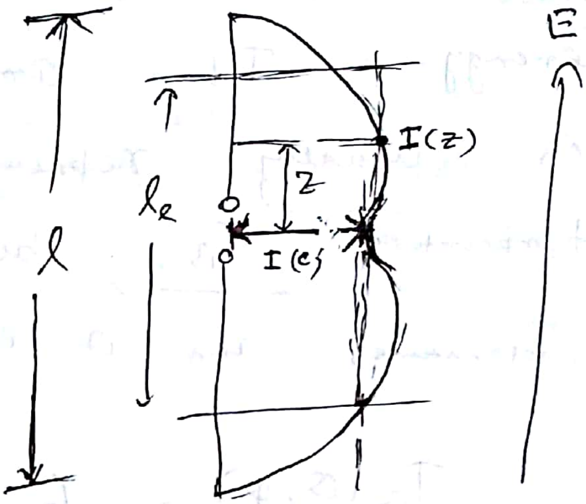
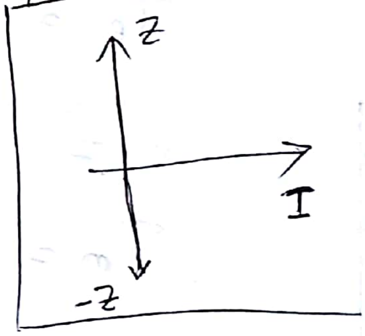


Fig:- Illustration of effective length for transmitting antenna.

Note:-



Hence for transmitting antenna,

$$I(C)\cdot l_{ef} = \int_{-l/2}^{+l/2} I(Z)\,dZ$$

$$l_{ef} = \frac{1}{I(C)} \cdot \int_{-l/2}^{l/2} I(Z)\,dZ$$

$$\boxed{l_{ef} = \frac{2}{I(C)} \int_{0}^{l/2} I(Z)\,dZ} \quad — (18)$$

Note:- $I(C) \times l_{eff}$ = Area of Rectangle.

$\int_{-l/2}^{l/2} I(Z)\,dZ$ = Area of current distribution 

Because integration indicates area under the curve.

& Both the areas should be same. (Equated)

Antenna temperature:-

Every object with a physical temperature above absolute zero (0K = −273°C) radiates energy. The amount of energy radiated is usually represented by an equivalent temperature T_B, better known as brightness temperature, and it is defined as

$$T_B(\theta,\phi) = \epsilon(\theta,\phi)\, T_m = (1-|\Gamma|^2)\, T_m \quad (19)$$

where

T_B = brightness temperature (equivalent temperature; K)

ϵ = emmissivity (dimensionless)

T_m = molecular (physical) temperature (K)

$\Gamma(\theta,\phi)$ = Reflection coefficient of the surface for the polarization of the wave

→ Since the values of emmissivity are $0 \le \epsilon \le 1$, the max^m value the brightness temperature can achieve is equal to the molecular temperature.

→ Usually the emissivity is a function of the frequency of operation, polarization of the emitted energy and molecular structure of the object.

The brightness temperature emitted by the different sources (ground, sky) is intercepted by antennas, and it appears at their terminals as an antenna temperature.

The temperature appearing at the terminals of an antenna is that given by (19), after it is weighted by the gain pattern of the antenna. In equation form, this can be written as

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\,.\,G(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi} \quad (20)$$

Where T_A = antenna temperature (effective noise temperature of the antenna radiation resistance; K) (in Kelvin)

~~G(θ,φ)~~ $G(\theta,\phi)$ = Gain (Power) pattern of the antenna.

Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by,

$$P_r = K T_A \Delta f \quad (21)$$

Where

P_{r} = antenna noise power (Watt)

K = Boltzmann's constant ($1.38 \times 10^{-23} \frac{J}{K}$)

T_A = Antenna temperature (K)

Δf = Bandwidth (Hz)

→ If the antenna and transmission line are maintained at certain physical temperature and the transmission line between the antenna and receiver is lossy, the antenna temperature T_A as seen by the receiver through (21), must be modified to include the other contributions and the line losses.

→ If the antenna itself is maintained at a certain physical temperature T_P and a transmission line of length 'l', constant physical temperature T_0 throughout its length, and uniform attenuation of 'α' (Np/unit length) is used to connect an antenna to a receiver, as shown in fig.

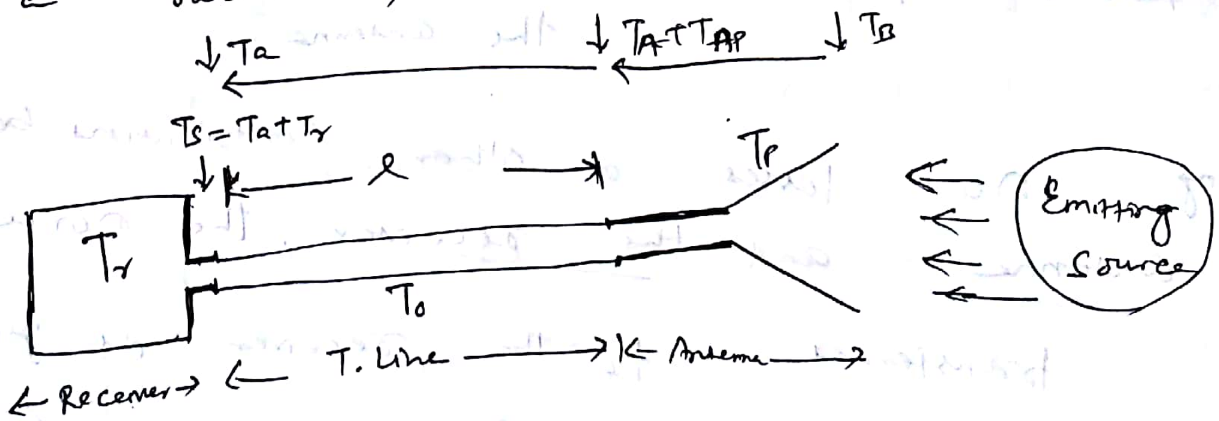


fig:- Antenna, transmission line and receiver arrangement for system noise power calculation.

The effective antenna temperature at the receiver terminal is given by,

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0 (1 - e^{-2\alpha l}) \quad \text{(22)}$$

where

$$T_{AP} = \left(\frac{1}{e_A} - 1 \right) T_p \quad \text{(23)}$$

T_a = antenna temperature at the receiver terminal (K)

T_A = antenna noise temperature at the antenna terminals [eq^n. 20]

T_{AP} = antenna temperature at the antenna terminals due to physical temperature.

T_p = antenna physical temperature

α = attenuation coefficient of Tr Line (Np/m)

e_A = thermal efficiency of antenna (dimensionless)

l = length of transmission line (m)

T_0 = Physical temperature of the Tr Line

So the antenna noise power of (21) must also be modified and written as

$$P_r = K T_a \Delta f \quad \text{(24)}$$

where T_a is the antenna temperature at the receiver i/p as given in eq^n (22).

If the receiver itself has a certain noise temperature (T_r) [due to thermal noise in the receiver components], the system noise power at the receiver terminals is given by

$$P_s = K(T_a + T_r)\Delta f = K T_s \Delta f \quad \text{—— (25)}$$

where

P_s = System noise power (at receiver terminals)

T_a = Antenna noise temperature (at receiver terminals)

T_r = receiver noise temperature (at receiver terminals)

$T_s = T_a + T_r$ = Effective system noise temperature (at receiver terminals)

→ The effective system noise temperature T_s of radio astronomy antennas and receivers varies from very few degrees (typically $\simeq 10K$) to thousand kelvins depending upon the type of antenna, receiver and frequency of operation.

→ Antenna temperature changes at the antenna terminals, due to variations in the target emissions, may be as small as a fraction of one degree. To detect such changes, the receiver must be very sensitive and be able to differentiate changes of a fraction of a degree.

~~Arrays~~:-

Input Impedance:-

In a practical antenna connected to a transmitter via transmission line, the applied radio frequency voltage establishes a current distribution on the antenna structure. This, in turn, radiates power into free space.

A small part of I/P power is dissipated due to ohmic/dielectric losses in the antenna. Further, the applied voltage also establishes a reactive field in the vicinity of the antenna.

Imp — One can think of the antenna as an equivalent complex impedance Z_a, which draws exactly the same amount of complex power from the transmission line as the antenna.

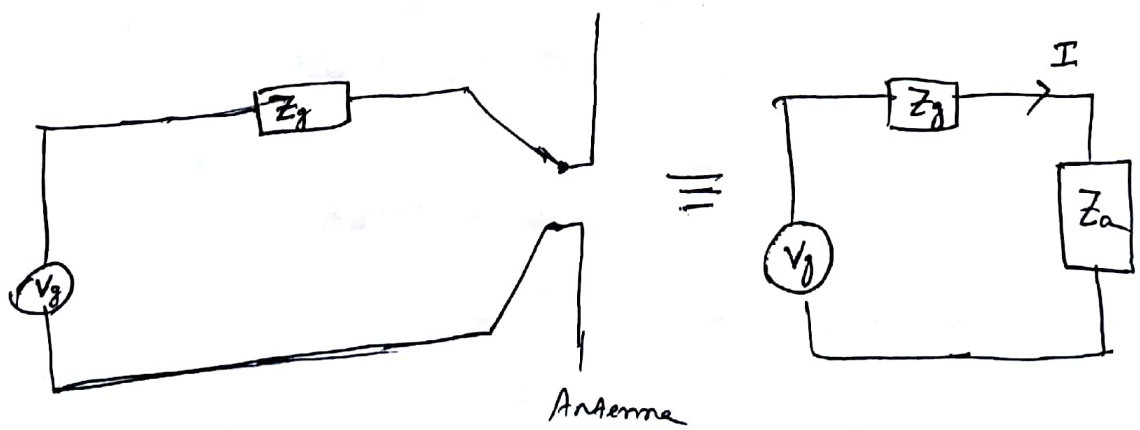


Fig:- An antenna connected to a source and its equivalent circuit.

This is known as antenna i/p impedance (Z_a). The real part accounts for the radiated power and power dissipated in the antenna.

The reactive part accounts for the reactive power stored in the near field of the antenna.

Note :- 'Z_a' is the ratio of the i/p voltage to i/p current at the Antenna terminals.

Consider an antenna in the transmit mode, having i/p impedance of $\boxed{Z_a = R_a + jX_a}$, — (1)

where R_a and X_a are the resistive and reactive parts respectively, connected directly to a source having equivalent Thevenin voltage, V_g and an internal impedance $\boxed{Z_g = R_g + jX_g}$ as — (2)

shown in Figure.

The Max^m power transfer takes place when the antenna's conjugate match to the source, i.e

$$R_a = R_g \quad \text{and} \quad X_a = -X_g \qquad (3)$$

Under the complex conjugate-match condition, the antenna i/p current is,

$$I = \frac{V_g}{2R_g} = \frac{V_g}{2R_a} \qquad (4)$$

and the real power supplied by the source is

$$P_g = \frac{1}{2} \, Re \left\{ V_g . I^* \right\}$$

$$= \frac{1}{2} \, Re. \left\{ V_g . \frac{V_g}{2R_a} \right\} \qquad \left[\text{Using eqn } (4) \right]$$

$$= \frac{1}{2} . \frac{V_g^2}{2R_a}$$

$$= \frac{|V_g|^2}{4R_a}$$

$$\therefore \boxed{P_g = \frac{|V_g|^2}{4R_a}} \qquad \text{——— } (5)$$

Half the power supplied by the source is lost in the source resistance, R_g, and the other half gets dissipated in the antenna resistance, R_a. Power i/p to the antenna is

$$P_{in} = \frac{P_g}{2} = \frac{|V_g|^2}{8R_a} = \frac{1}{2} |I|^2 R_a \qquad \text{——— } (6)$$

The antenna resistance, R_a, is comprised of two components, namely, the radiation resistance and the loss resistance.

$$R_a = R_{rad} + R_{loss} \qquad \text{——— } (7)$$

The total power dissipated in the antenna resistance R_a, can be split into two parts

$$P_{rad} = \frac{1}{2} |I|^2 R_{rad} \quad \text{—— (8)}$$

$$P_{loss} = \frac{1}{2} |I|^2 R_{loss} \quad \text{—— (9)}$$

Where P_{rad} is the power dissipated in R_{rad} which is actually radiated power. P_{loss} represents the ohmic losses in the antennas.

For a matched antenna these are given by

$$P_{rad} = \frac{1}{2} |I|^2 \cdot R_{rad}$$

$$\cancel{2 \cdot \frac{1}{2} \times |I|^2 \times}$$

[From eqn (6) $|I|^2 = \frac{|V_g|^2}{4 R_a^2}$]

$$P_{rad} = \frac{1}{2} \times \frac{|V_g|^2}{4 \cdot R_a^2} \times R_{rad}$$

$$P_{rad} = \frac{1}{8} \times \frac{V_g^2}{R_a^2} \times R_{rad} \quad \text{—— (10)}$$

Similarly

$$P_{loss} = \frac{1}{8} \times \frac{V_g^2}{R_a^2} \times R_{loss} \quad \text{—— (11)}$$

We know radiation efficiency

$$\eta' = \frac{P_{rad}}{P_{in}} \quad \text{—— (12)}$$

Putting eqⁿ 6 & 10 in eqⁿ 12

$$\eta' = \frac{\frac{|V_g|^2}{8R_a^2} \times R_{rad}}{\frac{|V_g|^2}{8R_a}}$$

$$= \frac{\cancel{|V_g|^2} \times R_{rad}}{\cancel{8} R_a^{\cancel{2}} R_a} \times \frac{8\,\cancel{R_a}}{\cancel{|V_g|^2}}$$

$$\eta' = \frac{R_{rad}}{R_a}$$

$$\boxed{\eta' = \frac{R_{rad}}{R_{rad} + R_{loss}}}$$ [Using eqⁿ 7] — 13

For an Hertzian dipole

(Total power radiated) → $P_{rad} = \eta \cdot \frac{\pi}{3} \left(|I_0| \frac{dl}{\lambda} \right)^2$ | η = Intrinsic Impedance

But

$P_{rad} = \frac{1}{2} |I_0^2| R_{rad}$ [Eqⁿ 8]

Equating Both P_{rad},

$$\eta \cdot \frac{\pi}{3} \left(|I_0| \cdot \frac{dl}{\lambda} \right)^2 = \frac{1}{2} I_0^2 R_{rad}$$

$$\Rightarrow \eta \cdot \frac{\pi}{3} \cdot \cancel{|I_0|^2} \cdot \left(\frac{dl}{\lambda}\right)^2 = \frac{1}{2} \cdot \cancel{|I_0|^2} \cdot R_{rad}$$

$$\Rightarrow \boxed{R_{rad} = \frac{2}{3}\pi\eta\left(\frac{dl}{\lambda}\right)^2}$$

$\therefore$ The Radiation resistance of Hertzian dipole is given by

$$\boxed{R_{rad} = \frac{2}{3}\pi\eta \cdot \left(\frac{dl}{\lambda}\right)^2} \quad \text{--- (14)}$$

Ex :- 1) A Voltage source of amplitude $V_g = (50 + j40)$ volt and a source impedance $Z_g = 50\Omega$ is connected to an antenna having radiation resistance $R_{rad} = 70\Omega$, loss resistance $R_{loss} = 1\Omega$ and a reactance $jX = j25\Omega$. Calculate (i) the radiation efficiency, (ii) the real power delivered by the source, (iii) the real power i/p to the antenna, (iv) Power radiated by the antenna and (v) power dissipated in the antenna.

Ans :- (i) Radiation efficiency (η')

$$\eta' = \frac{R_{rad}}{R_a} = \frac{70}{R_{rad} + R_{loss}} = \frac{70}{70+1} = \frac{70}{71} = 0.986$$

∴ Radiation efficiency = 98.6 %.

(ii) The current through the circuit is

$$I = \frac{V_g}{R_g + R_{rad} + R_{loss} + jX} = \frac{50+j40}{50+70+1+j25}$$

$$= \frac{50+j40}{121+j25} = \frac{64.03 \angle 38.66^\circ}{123.56 \angle 11.67^\circ}$$

[Converting rectangular to polar using Calculator]

$$\boxed{I = 0.518 \angle 26.99 \text{ Ampere}}$$

[Magnitude divided, Angle subtracted]

The real power delivered by the source

$$P_g = \frac{1}{2} Re \left[V_g \cdot I^* \right]$$

$$= \frac{1}{2} Re \left[(50+j40) \cdot 0.518 \angle -26.99 \right]$$

$$\downarrow$$

$$= \frac{1}{2} Re \left[(64.03 \angle 38.66)(0.518 \angle -26.99) \right]$$

$$= \frac{1}{2} Re \left[33.16754 \angle 11.67 \right]$$

[Angle subtracted Magnitude multiplied]

$$= \frac{1}{2} Re \left[32.48 + j\, 6.7089 \right]$$

$P_g = \frac{1}{2} \times 32.48$ [Real part is taken]

$$\boxed{P_g = 16.24 \text{ Watt}}$$

(iii) The real power i/p to the antenna

$$P_{in} = \frac{1}{2} |I|^2 [R_{rad} + R_{loss}]$$

$$= \frac{1}{2} \times 0.518^2 [70 + 1]$$

$$\Rightarrow \boxed{P_{in} = 9.53 \text{ Watt}}$$

(iv) Power radiated by the antenna is

$$P_{rad} = \frac{1}{2} |I|^2 \times R_{rad} = \frac{1}{2} \times 0.518^2 \times 70 = 9.39 \text{ Watt}$$

(v) Power dissipated in the antenna is

$$P_{loss} = \frac{1}{2} |I|^2 \times R_{loss} = \frac{1}{2} \times 0.518^2 \times 1 = 0.134 \text{ watt}.$$

2) Calculate the radiation resistance and efficiency of a Hertzian dipole of length $\underline{dl = 0.05\lambda}$, having loss resistance of 1Ω, a reactance of $-j100\Omega$, and radiating into free space.

If the dipole is connected to 100V (peak voltage) source having source impedance 50Ω, Calculate the real power radiated by the antenna and the power generated by the source. 20

Ans:-

(i) The radiation resistance of a Hertzian dipole is given by

$$R_{rad} = \frac{2}{3}\pi\eta\left(\frac{dl}{\lambda}\right)^2$$

$$= \frac{2}{\cancel{3}} \times \pi \times \cancel{120}^{40}\pi \times \left(\frac{0.05\lambda}{\lambda}\right)^2$$

$\left[\eta = \text{Intrinsic impedance of freespace} = 120\pi\right]$

$$= 80\pi^2 \times (0.05)^2$$

$$\boxed{R_{rad} = 1.97\Omega}$$

(ii) $$\eta' = \frac{R_{rad}}{R_{rad} + R_{loss}} = \frac{1.97}{1.97+1} = 0.66$$

∴ The radiation efficiency is 66 %.

(iii) The real power radiated by the antenna

$$P_{rad} = \frac{1}{2}|I|^2 \times R_{rad}$$

$$= \frac{1}{2} \times \left|\frac{100}{50+1.97+1-j100}\right|^2 \times 1.97$$

$$= \frac{1}{2} \times \frac{100^2}{|52.97-j100|^2} \times 1.97$$

$$\Rightarrow P_{rad} = \frac{1}{2} \times \frac{100^2}{\left|\sqrt{52.97^2 + 100^2}\right|^2} \times 1.97$$

$$\Rightarrow P_{rad} = \frac{1}{2} \times \frac{100^2}{113.16^2} \times 1.97$$

$$\Rightarrow \boxed{P_{rad} = 0.77 \text{ Watt}}$$

(iv) The real Power generated by source

$$P_g = \frac{1}{2} Re\left[V_g \cdot I^*\right]$$

$$= \frac{1}{2} Re\left[V \times \left(\frac{V}{Z}\right)^*\right]$$

$$= \frac{1}{2} Re\left[\frac{V^2}{Z^*}\right]$$

$$= \frac{1}{2} Re\left[\frac{100^2}{52.97 + j100}\right]$$

or

$$P_g = \frac{1}{2} Re\left[V_g \cdot I^*\right]$$

$$= \frac{1}{2} Re\left[100 \times \left(\frac{100}{52.97 - j100}\right)^*\right]$$

$$= \frac{1}{2} Re\left[\frac{100^2}{52.97 + j100}\right]$$

$\rightarrow$ ($\because$ $Z = 52.97 - j100$

$Z^* = 52.97 + j100$)

$$= \frac{1}{2} \times Re\left[\frac{100^2}{113.16 \angle 62.08}\right]$$

$$= \frac{100^2}{2} \times Re\left[\frac{1}{113.16 \angle 62.08}\right]$$

$$= \frac{100^2}{2} \times Re\left[8.837 \times 10^{-3} \angle -62.08\right]$$

$$= \frac{100^2}{2} \times Re\left[4.137 \times 10^{-3} - j\, 7.808\right]$$

$$= \frac{100^2}{2} \times 4.137 \times 10^{-3} = 20.68 \text{ Watt}.$$

$$\boxed{P_g = 20.68 \text{ watt}}$$

(V) We know,

Radiation efficiency $= \dfrac{P_{rad}}{P_{in}}$

$\Rightarrow 0.66 = \dfrac{P_{rad}}{P_{in}}$

$\Rightarrow 0.66 = \dfrac{0.77 \text{ Watt}}{P_{in}}$

$\Rightarrow P_{in} = \dfrac{0.77}{0.66} = 1.167$ Watt

$\boxed{P_{in} = 1.167 \text{ Watt}} \simeq 1.17$ watt

∴ Total power i/p to the antenne is 1.167 Watt.

or $\simeq 1.17$ Watt.

(V) Power dissipated in the antenne due to loss resistance is $1.17 - 0.77 = 0.4$ watt

(V) Power dissipated by the source resistance

$(R_g) = P_g - P_{in}$

$= 20.68 - 1.17 = 19.51$ Watt

∴ 19.51 Watt power is dissipated by R_g.

Equivalence of Radiation & Receive Patterns

The radiation pattern (power pattern) of an antenna is the distribution of the radiated power in the far-field region as a function of the angle. This can be determined by measuring the power density distribution as a function of the angle at a constant distance from the antenna and normalizing it with respect to the peak measured power-density.

Similarly, when an antenna is used as receiver, the power delivered into a matched load is a function of the direction of the incident plane wave with a constant power density and a given polarization. This is known as the receive pattern of the antenna. The receive pattern is generally normalized w.r.to the max^m received power. From the reciprocity th^m, we can establish that ~~two~~ these two patterns are the same.

Consider a two-dipole situation as

Shown in figure 1.

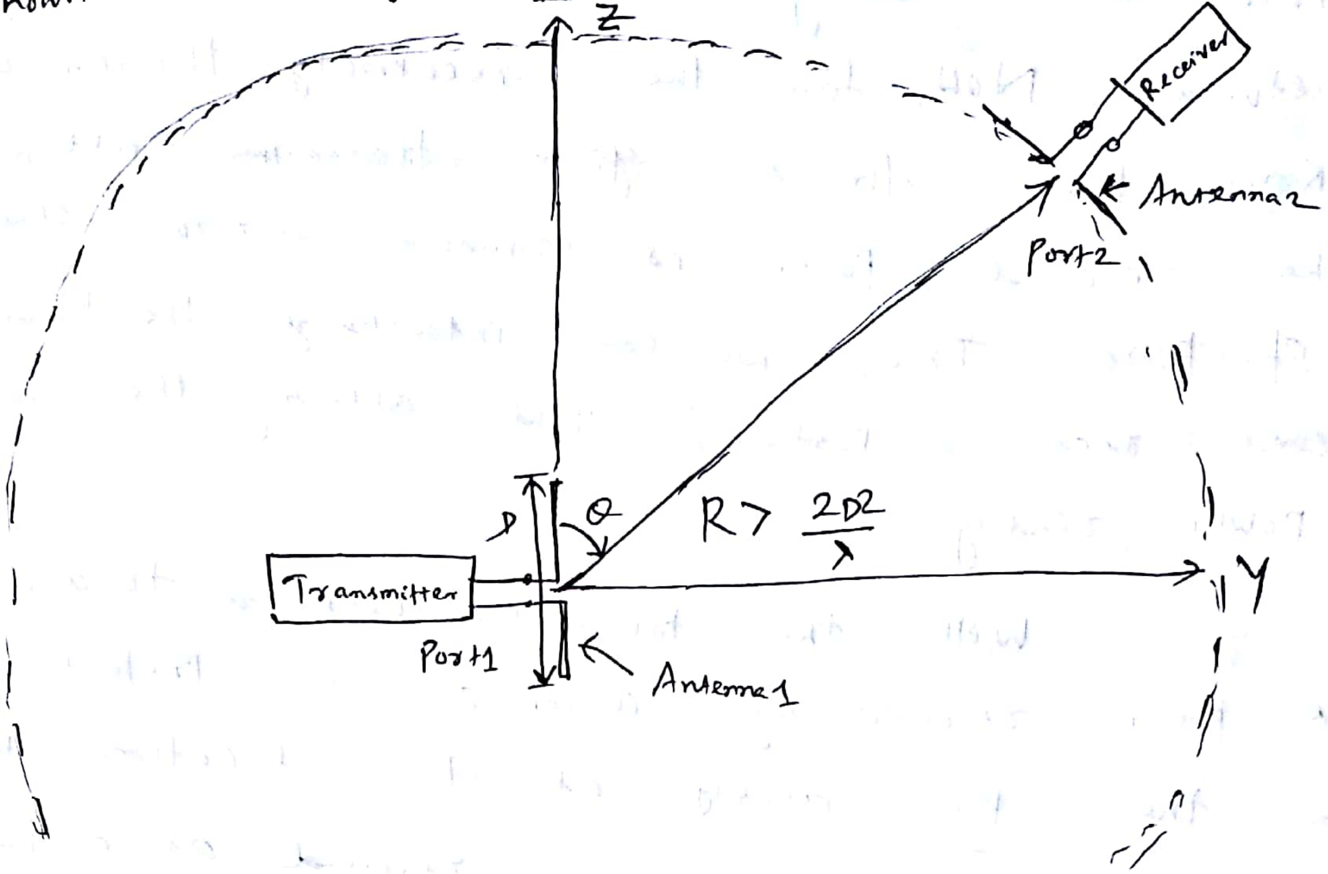


Fig 1:- Measurement of radiation Pattern.

Select a sufficiently large distance, R, between the antennas so that the antennas are in the far field of each other. We connect a transmitter to antenna 1 and a receiver to antenna 2 to measure the received Power.

It is also assumed that the transmitter source impedance and the receiver i/p impedance are ~~matching~~ matched to the respective antennas. Typically all RF systems are matched to the transmission line impedances, either 50Ω or 75Ω.

The two antennas terminals can be treated as ports of a two-port network, with the entire free space being inside the two-port network. Now, from the reciprocity theorem, we know that for a given transmitter voltage, the received power is invariant w.r.to a change of ports. Thus, we can interchange the transmitter and receiver positions, without affecting the received power reading.

With the transmitter connected to antenna 1, the power received by antenna 2 is proportional to the power density at its location. Hence, the plot of the power received as a function of θ gives the radiation pattern of antenna 1, provided the orientation of antenna 2 w.r.to the radial vector from antenna 1 is kept constant.

Now, if we interchange the positions of the transmitter & receiver, antenna 2 produces a plane wave of constant power density at the location of antenna 1, incident from angle θ, with the E-field oriented along the a_θ - direction.

By definition, a plot of the received power at antenna 1 as antenna 2 is moved around at a constant R, keeping the orientation of antenna 2 along a_θ, we get the receive pattern of antenna 1. Since, the received power is independent of the position of transmitter & receiver, both plots will be the same. Hence, we conclude that the transmit & receive patterns are the same for an antenna.

In a practical antenna pattern measurement system, both the antenna locations are generally fixed at a convenient distance. To measure the pattern of antenna 1, instead of moving antenna 2, antenna 1 is rotated about an axis to obtain the same effect as moving antenna 2 around.

By selecting the rotation axis, different pattern cuts can be plotted. Since the transmit and receive patterns are identical, it is common practice to connect the receiver to antenna 1 and transmitter to antenna 2.

Equivalence of Impedance :-

Consider a transmit - receive system using two antennas separated by a distance R. [Refer to figure for : Equivalence of Radiation & Receive patterns]. The terminals of the two antennas can be treated as the ports of a two - port n/w with the entire free space being inside the two-port n/w.

A two port n/w can be characterized by its Z. Matrix. Let the transmitter be connected to antenna 1 and receiver to antenna 2. In the transmit mode, the i/p impedance of the antenna 1 is the (V/I) ratio at its terminals

$$Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_R + Z_{22}} \quad \text{——— (1)}$$

Where Z_R is the receiver impedance.

[<u>Derivation</u> :- For a two port system with receiver impedance (Z_R)

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{——— (2)}$$

$$V_2 = Z_{21} I_1 + (Z_{22} + Z_R) I_2 \quad \text{— (3)}$$

To get the I/P impedance,

$Z_{in} = \frac{V_1}{I_1}$, we make o/p voltage $V_2 = 0$.

When $V_2 = 0$, Eq^n (3) becomes

$$0 = Z_{21} I_1 + (Z_{22} + Z_R) I_2$$

$$\Rightarrow I_2 = -\frac{Z_{21} I_1}{(Z_{22} + Z_R)} \quad \text{——— (4)}$$

Putting eq^n (4), in eq^n (2), we have

$$V_1 = Z_{11} I_1 + Z_{12} \cdot \left(\frac{-Z_{21} I_1}{Z_{22} + Z_R} \right)$$

$$\Rightarrow \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_R}$$

$$\Rightarrow \boxed{Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_R}} \quad \text{——— (5)}$$

The i/p impedance depends on the entire structure seen by the terminals, which includes the two antennas, the entire free space, and the load connected to antenna 2.

In the receive mode, antenna 1 driving a receiver can be modelled as a Thevenin's equivalent source.

The Thevenin's equivalent source impedance is the antenna impedance. This impedance can be measured by connecting a transmitter with a source impedance, Z_R (must be same as the receiver impedance), to antenna 2.

The ratio of the open-circuit voltage & the short-ckt current, measured at the antenna 1 terminals, gives Thevenin's equivalent impedance. For a two-port network, this is the same as the I/P impedance given by eqn (5).

The impedance of an antenna radiating into infinite free space is known as the self-impedance. As the distance between the two antennas tends to infinity, Z_{12} and Z_{21} tends to zero.

Therefore, as the antenna separation tends to infinity, $(R \to \infty)$, the second term in eqn (5), tends to zero and Z_{11} tends to self-impedance of the antenna.

Although we can't measure the impedance of the antenna as a receive antenna if we remove the second antenna to infinity, we can infer from previous result that as a receiver, the antenna's impedance also tends towards the self-impedance for large value of R.

Module-I : Electromagnetic radiation and antenna fundamentals:

Review of Electromagnetic Theory :-

Maxwell's Equations

Differential form

1\. (a) $\nabla \cdot D = \rho_v$

i.e The Divergence of electric flux density (D) is always equals to the volume charge density (ρ_v)

Integral form

(b) $\oint_S D \cdot ds = \int_V \rho_v \, dv = Q$

i.e The net electric flux through any closed surface S is always equal to the total charge enclosed. (Gauss's law for electric fields)

2\. (a) $\nabla \cdot B = 0$

The divergence of magnetic flux density is always zero.

Since

(i) The magnetic field lines are always closed in nature

(ii) The magnetic charges in isolated form don't exist in nature

(b) $\oint_S B \cdot ds = 0$

The net magnetic flux through any closed surface 'S' is always equal to zero.

[Gauss's law for magnetic fields]

3\. (a) $\nabla \times E = -\frac{\partial B}{\partial t}$

The curl of electric field 'E' is always equal to the rate of decrease of magnetic flux density $\vec{B}$ w.r.to time.

(b) $\oint_L E \cdot dl = -\frac{\partial}{\partial t} \int_S B \cdot ds$

The net E.M.f produced is always equal to the surface integral of the rate of decrease of magnetic flux density 'B' w.r.to time 't'. [Faraday's law of electromagnetic induction]

4) (a) $\nabla \times H = J + \frac{\partial D}{\partial t}$

$J = J_c$ = Conduction current density

$\frac{\partial D}{\partial t} = J_D$ = Displacement Current density

The curl of magnetic field intensity 'H' is always equals to the sum of conduction current density (J_c) & displacement current density (J_d)

In summary

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

(b) $\oint_L H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$ (Integration) ③

The MMF (Magnetic Motive Force) produced is always equal to the surface integral of sum of conduction current density (J_c) and displacement current density ($J_d = \frac{\partial D}{\partial t}$).

$$\oint_S D \cdot ds = \int_V \rho_v \, dv$$

$$\oint_S B \cdot ds = 0$$

$$\oint_L E \cdot dl = -\frac{\partial}{\partial t} \int_S B \cdot ds$$

$$\oint_L H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$$

Constitutive Relations

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

Where

E = Electric field intensity $\left(\frac{\text{volts}}{\text{meter}}\right)$ $\left(\frac{V}{m}\right)$

H = Magnetic field intensity $\left(\frac{\text{Ampere}}{\text{meter}}\right)$ $\left(\frac{A}{m}\right)$

D = Electric flux density $\left(\frac{\text{Coulomb}}{\text{meter}}\right)$ $\left(\frac{C}{m}\right)$

B = Magnetic flux density $\left(\frac{\text{Weber}}{\text{meter}}\right)$ $\left(\frac{Wb}{m}\right)$

J = Current density $\left(\frac{\text{Ampere}}{\text{meter}^2}\right)$ $\left(\frac{A}{m^2}\right)$

ρ = Charge density $\left(\frac{\text{Coulomb}}{\text{meter}^3}\right)$ $\left(\frac{C}{m^3}\right)$

Why Vector Potential Approach? (11)

One of the problems in antenna analysis is that of finding the $\underline{E}$ and $\underline{H}$ fields in the space surrounding the antenna. An antenna operating in the transmit mode is normally excited at a particular i/p point in the antenna structure. (The same point is connected to the receiver in the receive mode)

Given an antenna structure and an i/p excitation, the current distribution on the antenna structure is established in such a manner that Maxwell's equations are satisfied everywhere and at all times.

The antenna analysis can be splitted into 2 parts

(a) Determination of the current distribution on the structure due to the excitation

(b) Evaluation of the field due to this current distribution in the space surrounding the antenna

This first part generally leads to an integral equation, the treatment of which is beyond the scope of the syllabus.

So we will be mainly concerned with the

Second part, i.e establishing the antenna fields, given the current distribution.

Maxwell's eqn

$$\nabla \times E = -j\omega\mu H = -\frac{\partial B}{\partial t} \quad \text{—— } (1)$$

$$\nabla \times H = J + j\omega\epsilon E = J + \frac{\partial D}{\partial t} \quad \text{—— } (2)$$

$$\nabla \cdot D = \rho \quad \text{—— } (3)$$

$$\nabla \cdot B = 0 \quad \text{—— } (4)$$

Maxwell's eqns [1 to 4] are time-independent, first order differential equations to be solved simultaneously. It is common practice to reduce these equations to two second order differential equations called wave equations.

E.g. in a source-free region ($\rho = 0$, $J = 0$) we can take curl of first equation, substitute it in the second equation to eliminate H and get the wave equation to

$$\nabla^2 E + k^2 E = 0 \quad \text{—— } (A)$$, satisfied equation by E-field.

Note: (Derivation)

—— From eqn (1)

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times (\nabla \times E) = -j\omega\mu\,(\nabla \times H)$$ [Taking curl both the sides]

$$\Rightarrow \nabla \times H = -\frac{1}{j\omega\mu}\,\nabla \times (\nabla \times E) \quad \text{—— } (5)$$

Putting eqn (5) in eqn (2)

$$-\frac{1}{j\omega\mu}\,\nabla \times (\nabla \times E) = 0 + j\omega\epsilon E$$

$$\nabla \times (\nabla \times E) = \omega^2 \mu \epsilon E$$

$$\Rightarrow \nabla(\nabla \cdot E) - \nabla^2 E = k^2 E$$

For source free region $\nabla \cdot E = 0$

$$\Rightarrow \nabla^2 E = -k^2 E$$

$$\Rightarrow \nabla^2 E + k^2 E = 0 \quad \text{——— (6)}$$

Similarly, we can also derive the wave equation satisfied by the H field. Thus, it is sufficient to solve one equation to find both E and H fields, since they satisfy the same wave equation.

To simplify the problem of finding E and H fields due to a current distribution, we can split it into two parts by defining intermediate potential functions which are related to the $\boxed{E}$ & $\boxed{H}$ fields. This is known as vector potential approach and is discussed below.

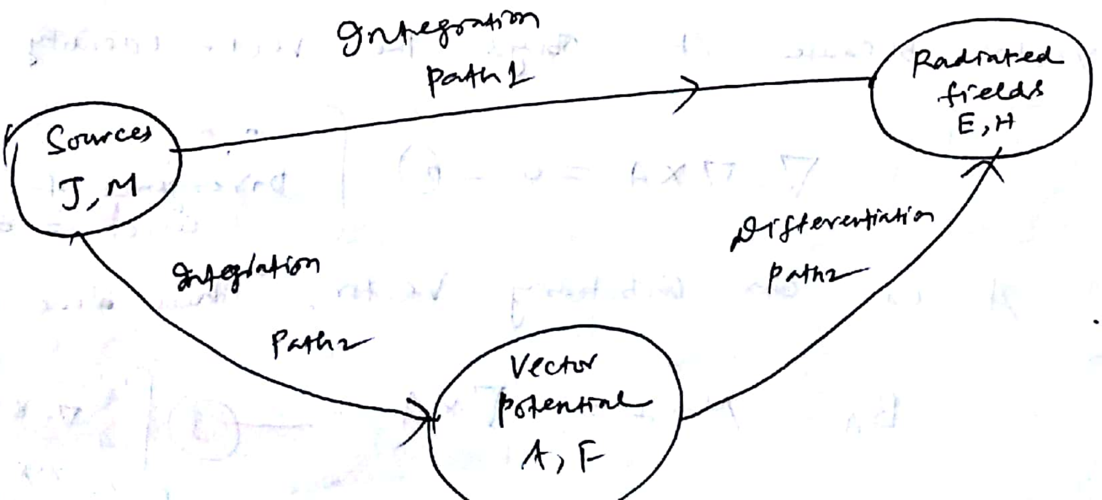
Vector Potential :—

→ In the analysis of radiation problems, the usual procedure is to specify the sources and then require the fields radiated by sources. This is in contrast to the synthesis problem where the radiated fields are specified, and we are required to find the sources.

→ It is very common practice in the analysis procedure to introduce auxiliary function, known as vector potential, which will aid in the solution of the problems.

→ The most common vector potential functions are the magnetic vector potential (A) and electric vector potential (F).

→ It is possible to calculate the E and H fields directly from the source-current densities J and M as shown below.



It is usually much simpler to calculate the auxiliary potential functions first and then determine ~~the~~ the $\underline{E}$ and $\underline{H}$. This 2 step procedure is shown in previous figure.

Although the 2-step procedure requires both integration and differentiation, where path 1 requires only integration, the integrands in the 2-step procedure are much simpler. So we go for 2-step procedure.

The Vector potential 'A' for an electric current source 'J':-

→ The vector potential 'A' is useful in solving for the EM field generated by a given harmonic electric current 'J'. The magnetic flux B is always solenoidal; i.e $\boxed{\nabla . B = 0}$ — ①

Therefore, it can be represented as the curl of another vector because it obeys the vector identity

$$\nabla . \nabla \times A = 0 \quad - ② \qquad \left[\because \text{Divergence of curl} = 0 \right]$$

where A is an arbitrary vector. Thus we define

$$B_A = \mu H_A = \nabla \times A \quad — ③$$

$\left[\because \nabla . B = 0 \text{ and } \nabla.(\nabla \times A) = 0 \Rightarrow B = \nabla \times A \right.$

or $H_A = \frac{1}{\mu} \nabla \times A$ ——— (4)

where 'A' subscript indicates the field due to the 'A' potential. Substituting eqn (4), into Maxwell's curl eqn, we have

$\nabla \times E_A = -j\omega\mu H_A$ ——— (5)

∵ From Maxwell eqn
$\nabla \times E = -\frac{\partial B}{\partial t}$
$= -j\omega\mu H$
∵ $\frac{\partial}{\partial t} = j\omega$, $B = \mu H$

Putting eqn (4) in eqn (5),

$\nabla \times E_A = -j\omega \cancel{\mu} \left(\frac{1}{\cancel{\mu}} \nabla \times A \right)$

$\nabla \times E_A = -j\omega (\nabla \times A)$ ——— (6)

which can also be written as

$\nabla \times [E_A + j\omega A] = 0$ ——— (7)

From the vector identity

(Curl of gradient = 0
ex:- $\nabla \times (\nabla F) = 0$)

$\nabla \times (-\nabla \phi_e) = 0$ ——— (8)

So eqn (7) can be written as using eqn (8).

$\nabla \times (E_A + j\omega A) = \nabla \times (-\nabla \phi_e)$

$\Rightarrow E_A + j\omega A = -\nabla \phi_e$

$\Rightarrow \boxed{E_A = -\nabla \phi_e - j\omega A}$ ——— (9)

The scalar function ϕ_e represents an arbitrary electric scalar potential which is a function of position.

Taking the Curl of eqⁿ 3

i.e ~~$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$~~ [using vector identity]

and ~~$\nabla \times (\mu H_A) =$~~

$$\nabla \times (\nabla \times A) = \nabla \times (\mu H_A)$$

$$\Rightarrow \nabla(\nabla \cdot A) - \nabla^2 A = \nabla \times (\mu H_A) \quad \text{--- (10)}$$

from vector identity

$$\left[\because \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \right]$$

For a homogeneous medⁿ, ~~From~~ eqⁿ (10) can be reduced to

$$\mu(\nabla \times H_A) = \nabla(\nabla \cdot A) - \nabla^2 A \quad \text{--- (11)}$$

Using Maxwell's eqⁿ.

$$\nabla \times H_A = J + j\omega\epsilon E_A \quad \text{--- (12)}$$

$$\left[\because \text{Maxwell eqⁿ} \quad \nabla \times H = J + \frac{\partial D}{\partial t} = J + j\omega\epsilon E \right]$$

Using eqⁿ (12) in eqⁿ (11), we have

$$\mu(J + j\omega\epsilon E_A) = \nabla(\nabla \cdot A) - \nabla^2 A \quad \text{--- (13)}$$

Putting eqⁿ (9) in eqⁿ (13), we have

$$\mu\left[J + j\omega\epsilon(-\nabla\phi_e - j\omega A)\right] = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\mu J - \nabla(j\omega\epsilon\mu\phi_e) + \omega^2\mu\epsilon A = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\Rightarrow \quad \nabla^2 A + K^2 A = -\mu J + \nabla(\nabla \cdot A) + \nabla(j\omega\mu\epsilon\phi_e)$$

where $K^2 = \omega^2 \mu \epsilon$

$$\Rightarrow \quad \nabla^2 A + K^2 A = -\mu J + \nabla(\nabla \cdot A + j\omega\mu\epsilon\phi_e) \quad \text{(14)}$$

Let

$$\nabla \cdot A = -j\omega\mu\epsilon\phi_e$$

$$\Rightarrow \quad \phi_e = -\frac{1}{j\omega\mu\epsilon}(\nabla \cdot A) \quad \text{(15)}$$

(which is known as Lorentz Condition)

Putting eqn (15), in eqn (14), we have

$$\nabla^2 A + K^2 A = -\mu J + \nabla\left[\nabla \cdot A + \cancel{j\omega\mu\epsilon} \times \frac{-1}{\cancel{j\omega\mu\epsilon}}(\nabla \cdot A)\right]$$

$$\Rightarrow \quad \boxed{\nabla^2 A + K^2 A = -\mu J} \quad \text{(16)}$$

In addition, eqn (9) Can be reduced to

$$E_A = -\nabla\phi_e - j\omega A$$

$$= -j\omega A - \nabla\left[-\frac{1}{j\omega\mu\epsilon}(\nabla \cdot A)\right] \quad \text{[Using eqn (15)]}$$

$$\boxed{E_A = -j\omega A - \frac{j}{\omega\mu\epsilon}\nabla(\nabla \cdot A)} \quad (\because j^2 = -1) \quad \text{(17)}$$

Once 'A' is known 'HA' Can be found from eqn (4) and 'E_A' can be found from eqn (17).

The vector potential 'F' for a magnetic current source 'M'

Although magnetic current appears to be physically unrealizable, equivalent magnetic currents arise when we use the volume or surface equivalence theorems.

→ The fields created by a harmonic magnetic current in a homogeneous region, with $J = 0$ but $M \neq 0$, must satisfy $\nabla \cdot D = 0$

$$\nabla \cdot D = 0 \quad \text{———} \quad (1)$$

But we know that Divergence of Curl $= 0$

$$\nabla \cdot (-\nabla \times F) = 0 \quad \text{——} \quad (2)$$

where F is vector potential.

Equating eqn (1) & (2), we have

$$D = -\nabla \times F$$

$$\Rightarrow \epsilon E_F = -\nabla \times F$$

$$\Rightarrow E_F = -\frac{1}{\epsilon}(\nabla \times F) \quad \text{————} \quad (3)$$

Substituting (3), into Maxwell's curl eqn

$$\nabla \times H_F = \cancel{J} \; j\omega \epsilon E_F \quad \left(\because \nabla \times H = J + \frac{\partial D}{\partial t} = 0 + j\omega \cdot \epsilon E \right)$$

reduces to

$$\nabla \times H_F = \cancel{j\omega\epsilon} \times \left(\frac{1}{\cancel{\epsilon}}\right)(\nabla \times F)$$

$$\Rightarrow \nabla \times (H_F + j\omega F) = 0 \quad \text{————} \quad (4)$$

From the vector identity,

Curl of gradient $= 0$,

$$\nabla \times (-\nabla \phi_m) = 0 \quad \text{--- (5)}$$

Equating eqn (4) & (5), we have.

$$H_f + jwF = -\nabla \phi_m$$

$$\Rightarrow \boxed{H_f = -\nabla \phi_m - jwF} \quad \text{--- (6)}$$

where ϕ_m represents an arbitrary magnetic scalar potential which is a function of position.

Taking curl of eqn (3), we have

$$\nabla \times E_F = -\frac{1}{\epsilon} \nabla \times (\nabla \times F) = -\frac{1}{\epsilon}(\nabla \nabla . F - \nabla^2 F) \quad \text{--- (7)}$$

$$(\because \nabla \times \nabla \times A = \nabla(\nabla . A) - \nabla^2 A)$$

~~Equating (7) to~~ From maxwell's eqn i.e

$$\nabla \times E_f = -M - jw\mu H_F \quad \text{--- (8)}$$

$$\left(\nabla \times E = -M - \frac{\partial B}{\partial t} = -M - jw.\mu H \right)$$

$$\because \frac{\partial}{\partial t} = jw)$$

Comparing eqn (7) & (8) we have,

$$-\frac{1}{\epsilon}(\nabla \nabla . F - \nabla^2 F) = -M - jw\mu H_F$$

$$-(\nabla \nabla . F - \nabla^2 F) = -\epsilon M - jw\mu\epsilon H_F$$

$$\Rightarrow \nabla^2 F + jw\mu\epsilon H_f = \nabla \nabla . F - \epsilon M \quad \text{--- (9)}$$

Substituting eqn (6) into eqn (9), we have

$$\nabla^2 F + jw\mu\epsilon(-\nabla \phi_m - jwF) = \nabla \nabla . F - \epsilon M$$

$$\nabla^2 F - \nabla(jw\mu\epsilon \phi_m) + w^2\mu\epsilon F = \nabla \nabla . F - \epsilon M$$

$$\Rightarrow \nabla^2 F + K^2 F = -\epsilon M + \nabla \nabla \cdot F + \nabla(j\omega\mu\epsilon\phi_m)$$
$$(\because K = \omega^2\mu\epsilon) \quad \text{--- (10)}$$

<u>Letting</u>

$$\nabla \cdot F = -j\omega\mu\epsilon\phi_m \quad \text{[Lorentz's condn]}.$$

$$\Rightarrow \phi_m = -\frac{1}{j\omega\mu\epsilon}(\nabla \cdot F) \quad \text{--- (11)}$$

Putting eqn (11), in eqn (10),

$$\nabla^2 F + K^2 F = -\epsilon M + \nabla \nabla \cdot F + \nabla\left(\cancel{j\omega\mu\epsilon}\left(\frac{-1}{\cancel{j\omega\mu\epsilon}}\nabla \cdot F\right)\right)$$

$$\nabla^2 F + K^2 F = -\epsilon M + 0$$

$$\boxed{\nabla^2 F + K^2 F = -\epsilon M} \quad \text{--- (12)}$$

and eqn (6) becomes,

$$H_F = -\nabla\phi_m - j\omega F$$
$$= -\nabla\left[\frac{-1}{j\omega\mu\epsilon}(\nabla \cdot F)\right] - j\omega F$$

$$\Rightarrow \boxed{H_F = -j\omega F - \frac{j}{\omega\mu\epsilon}\nabla(\nabla \cdot F)} \quad (\because j^2 = -1)$$
$$\text{--- (13)}$$

Once F is known, E_F can be found from eqn (3) and H_F from eqn (13) or (8) with $M = 0$.

Solution of the inhomogeneous Vector Potential wave equation

→ Let us assume that a source with current density J_z, which in the limit is an infinitesimal source, is placed at the origin of a x, y, z co-ordinate system.

→ Since the current density is directed along z-axis (J_z), only A_z component will exist.

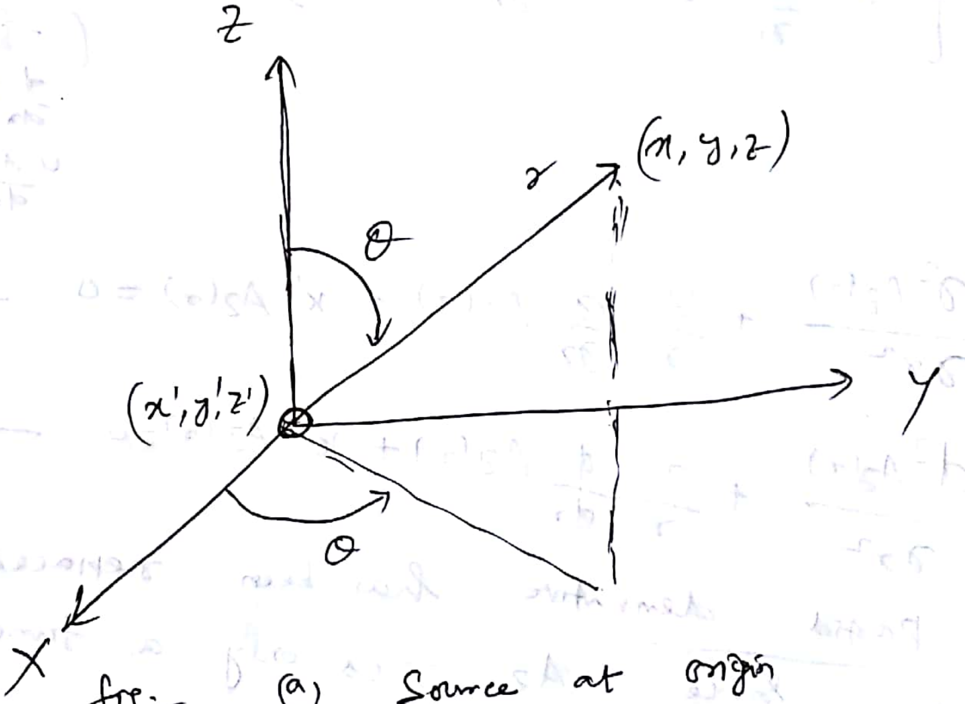


fig :- (a) Source at origin

→ Thus, we can write eqn (16) as

$$\nabla^2 A_z + k^2 A_z = -\mu J \quad \text{—— (1)}$$

At points removed from the source ($J_z = 0$), the wave equation reduces to

$$\nabla^2 A_z + k^2 A_z = 0 \quad \text{—— (2)}$$

Since ~~to~~ in the limit the source is a point, it requires that A_z is not a function of

direction (θ and ϕ) ; in a spherical Co-ordinate 158
system , $A_z = A_z(r)$ where r is the ~~radial~~
radial distance.

Thus eqn (2) Can be written as

$$\nabla^2 A_z(r) + K^2 A_z(r) = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r} A_z(r)\right] + K^2 A_z(r) = 0 \quad \text{—— (3)}$$

$$\Rightarrow \frac{1}{r^2}\left[r^2 \cdot \frac{\partial^2}{\partial r^2} A_z(r) + \frac{\partial}{\partial r} A_z(r) \cdot 2r\right] + K^2 A_z(r) = 0$$

$$\left(\because \frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}\right)$$

$$\Rightarrow \frac{\partial^2 A_z(r)}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} A_z(r) + K^2 A_z(r) = 0 \quad \text{—— } \cancel{(4)}$$

$$\Rightarrow \frac{d^2 A_z(r)}{dr^2} + \frac{2}{r}\frac{d}{dr} A_z(r) + K^2 A_z(r) = 0 \quad \text{—— (4)}$$

The partial derivative has been replaced by ordinary derivatives since A_z is only a function of the radial co-ordinates.

The differential eqn (4) has 2 independent sol's

$$A_{z1} = C_1 \frac{e^{-jKr}}{r} \quad \text{—— (5)}$$

$$A_{z2} = C_2 \frac{e^{+jKr}}{r} \quad \text{—— (6)}$$

Equation (5) represents an outwardly (in the radial direction) travelling wave and eqn (6) describes an inwardly travelling wave. For this problem, the source is placed at the origin with the radiated fields travelling in the outward radial direction. Therefore, we choose the solution of (5), thus

$$A_z = A_{z1} = C_1 \frac{e^{-jkr}}{r} \quad \text{—— (7)}$$

In the static case $(\omega = 0,\ k = \omega\sqrt{\mu\epsilon} = 0)$

k → wave number

So eqn (7) simplifies to,

$$A_z = \frac{C_1}{r} \quad \text{—— (8)}$$

which is a solution to the wave equation to eqn (2), (3) or (4), when $k=0$

Thus at points removed from the source, the time-varying and the static solution of (7) & (8) differ only by e^{-jkr} factor or the time-varying solution of (7) can be obtained by multiplying the static solution of (8) by e^{-jkr}.

→ In the presence of the source $(J_z \neq 0)$ and $(k=0)$ the wave equation of (1) reduces to

$$\nabla^2 A_z = -\mu J_z \quad \text{—— (9)}$$

This equation is recognized to be Poisson's equation whose solution is widely ~~solution~~ documented.

The must familiar equation with Poisson's form is that relating to the scalar electric potential (ϕ) to the electric charge density ρ. This is given by

$$\nabla^2 \phi = -\frac{\rho}{\epsilon} \quad \text{—— (10)}$$

whose solution is

$$\phi = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho}{r} \, dv' \quad \text{—— (11)}$$

where r is the distance from any point on the charge density to the observation point.

Since (9) is similar in form to (10), its solution is similar to (11), or

$$A_z = \frac{\mu}{4\pi} \iiint_V \frac{J_z}{r} \, dv' \quad \text{—— (12)}$$

Equation (12) represents the solution to (1) when $K = 0$ (static case). Using the comparative analogy between (7) & (8), the time varying solution of eq^n (1) can be obtained by multiplying the static solution of eq^n (12) by e^{-jkr}. Thus

$$\boxed{A_z = \frac{\mu}{4\pi} \iiint_V J_z \cdot \frac{e^{-jkr}}{r} \cdot dv'} \quad \text{—— (13)}$$

Which is a solution to eq^n (1).

If the current densities were in x- and y- directions (J_x and J_y), the wave equation for each would reduce to

$$\nabla^2 A_x + k^2 A_x = -\mu J_x \quad \text{—— (14)}$$

$$\nabla^2 A_y + k^2 A_y = -\mu J_y \quad \text{—— (15)}$$

with corresponding solution similar in form to (13) or

$$\boxed{A_x = \frac{\mu}{4\pi} \iiint_V J_x \cdot \frac{e^{-jkr}}{r} \cdot dv'} \quad \text{—— (16)}$$

$$\boxed{A_y = \frac{\mu}{4\pi} \iiint_V J_y \cdot \frac{e^{-jkr}}{r} \cdot dv'} \quad \text{—— (17)}$$

The solutions of (13), (16) and (17) allows us to write the solution to the vector wave equation of [$\nabla^2 A + k^2 A = -\mu J$], as (In general)

$$\boxed{A = \frac{\mu}{4\pi} \iiint_V J \cdot \frac{e^{-jkr}}{r} \cdot dv'} \quad \text{—— (18)}$$

If the source is removed from the origin and placed at a position represented by the prime coordinates (x', y', z') as shown below,

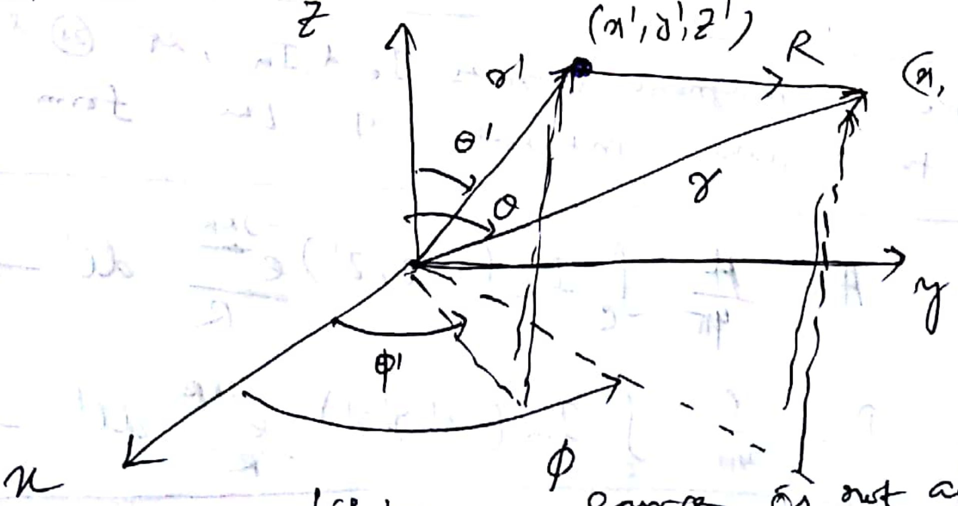


fig 1 - source is not at origin.

Thus eqⁿ (18), Can be written as

$$\boxed{A(x,y,z) = \frac{\mu}{4\pi} \iiint J(x',y',z') \frac{e^{-jkR}}{R} dv'} \quad (19)$$

Where the prime Co-ordinates represents the source, the unprimed to the observation point, and R the distance from any point on the source to the observation point. In a similar fashion we Can show the solution of $[\nabla^2 F + K^2 F = -\epsilon M]$ is given by

$$\boxed{F(x,y,z) = \frac{\epsilon}{4\pi} \iiint_V M(x',y',z') \frac{e^{-jkR}}{R} dv'} \quad (20)$$

If J and M represent linear densities (m^{-1}), eqn (19) & (20) reduces to surface integral.

$$A = \frac{\mu}{4\pi} \iint_S J_s(x',y',z') \frac{e^{-jkR}}{R} ds' \quad (21)$$

$$F = \frac{\epsilon}{4\pi} \iint_S M_s(x',y',z') \frac{e^{-jkR}}{R} ds' \quad (22)$$

For electric & magnetic Currents I_e & I_m, eqn (21) & (22) Can be reduce to line integral of the form

$$A = \frac{\mu}{4\pi} \int_C I_e(x',y',z') \frac{e^{-jkR}}{R} dl' \quad (23)$$

$$F = \frac{\epsilon}{4\pi} \int_C I_m(x',y',z') \frac{e^{-jkR}}{R} dl' \quad (24)$$

Duality Theorem :-

→ When two equations that describe the behavior of two different variables are of the same mathematical form, their solutions will also be identical.

→ The variables in the two equations that occupy identical positions are known as dual quantities and a solution of one can be formed by a systematic ~~exchange~~ interchange of symbols to the other. This concept is known as the duality theorem.

Dual quantities for Electric (J) and Magnetic (M) Current Sources

Electric Sources ($J \neq 0, M = 0$)		Magnetic Sources ($J = 0, M \neq 0$)
E_A	$\leftrightarrow$	H_F
H_A	$\leftrightarrow$	$-E_F$
J	$\leftrightarrow$	M
A	$\leftrightarrow$	F
ϵ	$\leftrightarrow$	μ
μ	$\leftrightarrow$	ϵ
k	$\leftrightarrow$	k
η	$\leftrightarrow$	$\frac{1}{\eta}$
$1/\eta$	$\leftrightarrow$	η

Dual equations for Electric (J) and Magnetic (M) Current Sources

Electric Sources ($J \neq 0, M = 0$)	Magnetic Sources ($J = 0, M \neq 0$)
$\nabla \times E_A = -j\omega\mu H_A$	$\nabla \times H_F = j\omega\epsilon E_F$
$\nabla \times H_A = J + j\omega\epsilon E_A$	$-\nabla \times E_F = M + j\omega\mu H_F$
$\nabla^2 A + k^2 A = -\mu J$	$\nabla^2 F + k^2 F = -\epsilon M$
$A = \frac{\mu}{4\pi}\iiint_V J \frac{e^{-jkR}}{R} dv'$	$F = \frac{\epsilon}{4\pi}\iiint_V M \frac{e^{-jkR}}{R} dv'$
$H_A = \frac{1}{\mu}\nabla \times A$	$E_F = -\frac{1}{\epsilon}\nabla \times F$
$E_A = -j\omega A - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla . A)$	$H_F = -j\omega F - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla . F)$

Thus knowing the solution to one set (i.e $J \neq 0, M = 0$) the solution to other set ($J = 0, M \neq 0$) can be formed by a proper ~~set~~ interchange of quantities.

Reciprocity thm for antennas

→ Reciprocity thm states that :- If an emf is applied to the terminals of an antenna 'A' and the current measured at the terminals of another antenna 'B', then an equal current (in both amplitude and phase) will be obtained at the terminals of antenna 'A' if the same emf is applied to the terminals of antenna 'B'.

→ It is assumed that the emfs are of the same frequency and that the media are linear, passive and also isotropic.

Case I :-

→

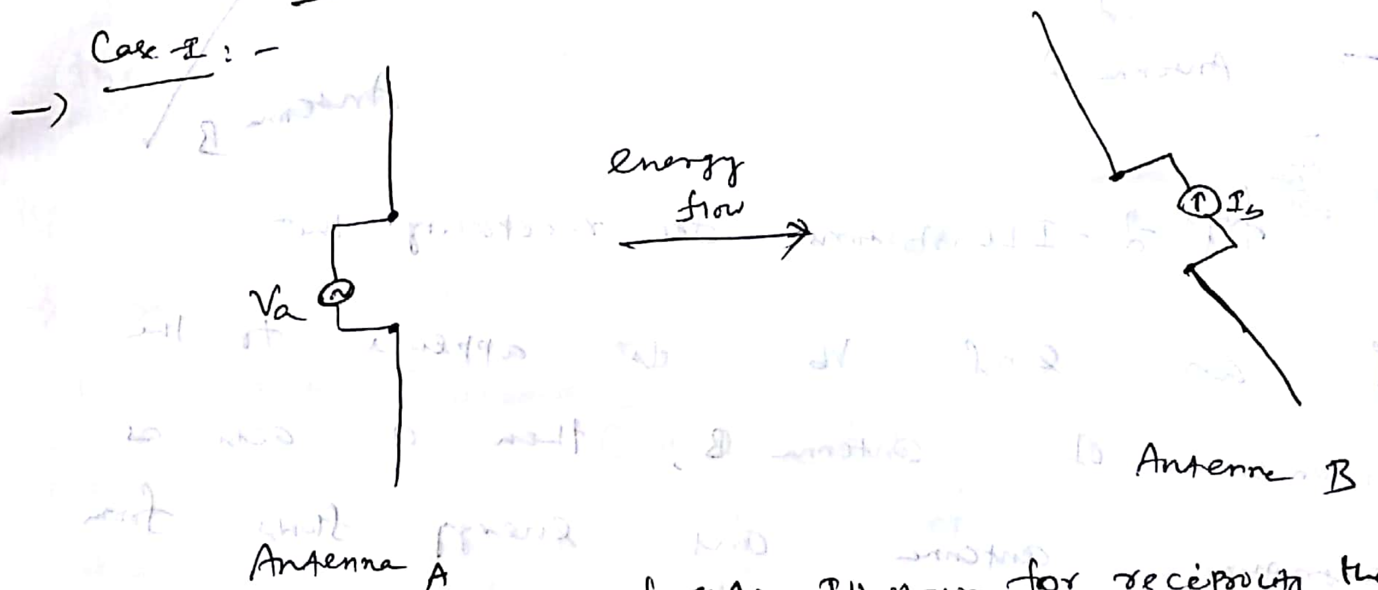


fig 1:- Illustration for reciprocity thm

Let an emf 'Va' be applied to the terminals of antenna A. This antenna acts as transmitting antenna and energy flows from it to antenna 'B', which may be considered as a receiving antenna, producing a current I_b at the terminals. It is assumed that the generator supplying the emf and the ammeter for measuring the current have zero impedance, or if not zero, that the generator and ammeter impedances are equal.

<u>Case 2:</u>

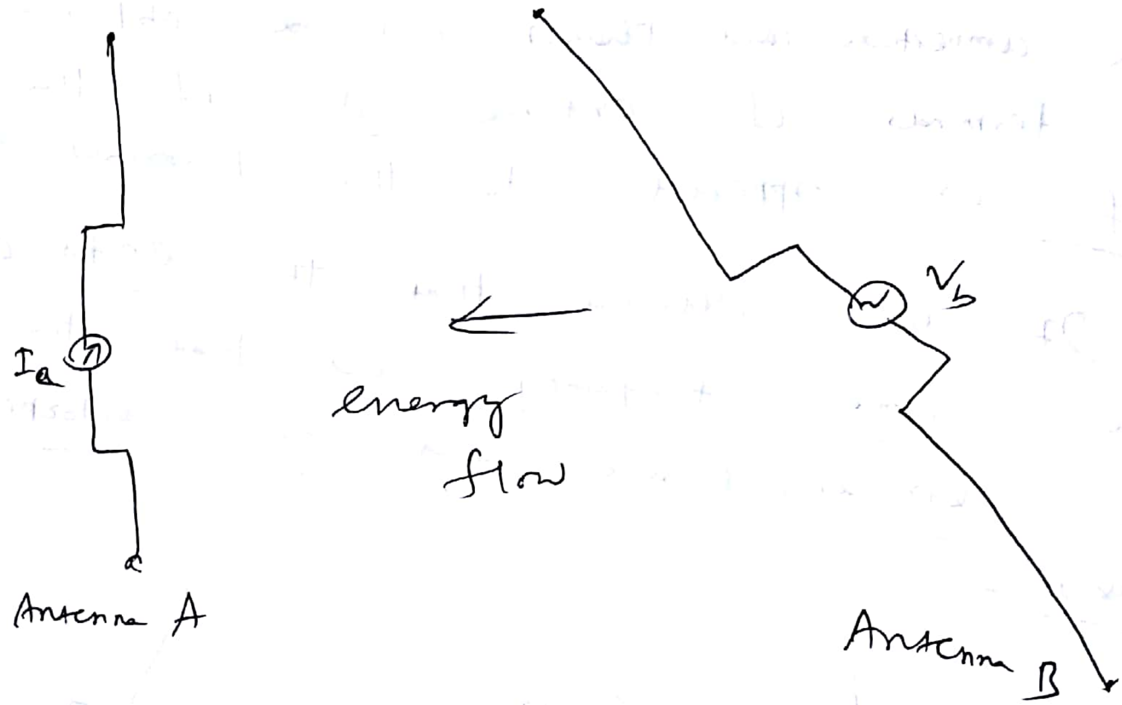


fig 2:- Illustrations for reciprocity them

If an emf V_b is applied to the terminals of antenna B, then it acts as transmitting antenna and energy flows from it to antenna A (as in fig), producing a current I_a' at its terminals.

Now if $V_b = V_a$ then by reciprocity theorem $I_a = I_b$.

The ratio of an emf to current is an impedance. In case 1, the ratio of 'V_a' to 'I_b' may be called the transfer impedance Z_{ab}, ~~Then~~ and in case 2 the ratio V_b to I_a may be called the transfer impedance Z_{ba}. Then, by reciprocity theorem it follows that these impedances are equal, thus,

$$\frac{V_a}{I_b} = Z_{ab} = Z_{ba} = \frac{V_b}{I_a} \quad \text{——— (1)}$$

Proof :-

→ In order to prove the reciprocity thm for antennas, let the antennas and the space between them be replaced by a n/w of linear, passive, bilateral impedances ~~→ [fig 1]~~

→ Since any 4-terminal n/w can be reduced to equivalent T-section, the antenna arrangement of case 1 (fig 1) can be replaced by the n/w fig (3)

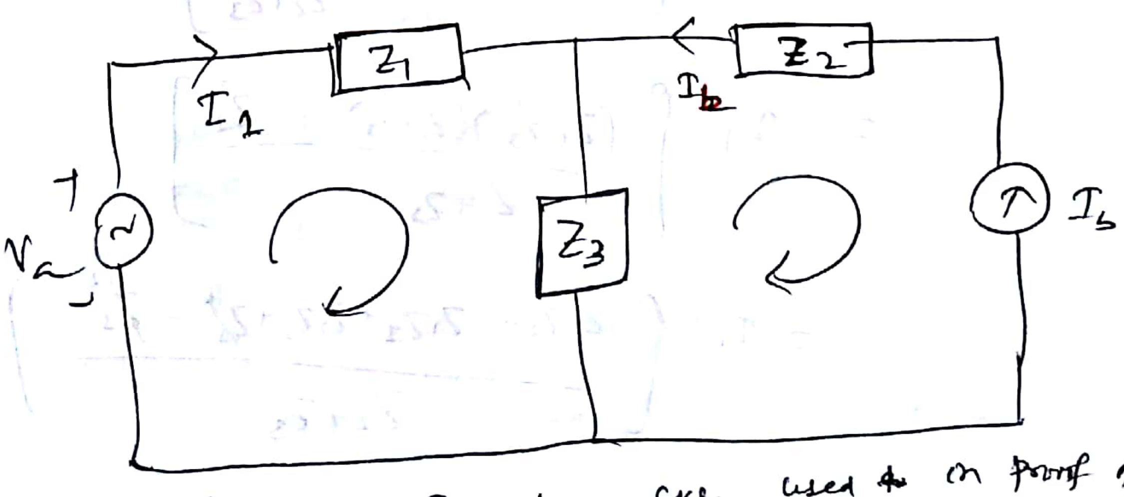


fig 3:- Equivalent ckts used in proof of reciprocity thm.

Putting KVL in the 1st loop,

$$V_a - I_1 Z_1 - (I_1+I_b) Z_3 = 0$$

$$\Rightarrow V_a = I_1 (Z_1+Z_3) + I_b Z_3 \quad \text{—(1)}$$

Putting KVL in the 2nd loop,

$$I_b Z_2 + (I_1+I_b) Z_3 = 0$$

$$\Rightarrow I_b (Z_2+Z_3) = -I_1 Z_3$$

$$\Rightarrow I_b = -\frac{I_1 Z_3}{Z_2+Z_3} \quad \text{—(2)}$$

Putting eqn (2) in eqn (1), we have

$$V_a = I_1 (Z_1+Z_3) + \left(\frac{-I_1 Z_3}{Z_2+Z_3}\right) Z_3$$

$$= I_1 \left[(Z_1+Z_3) - \frac{Z_3^2}{Z_2+Z_3} \right]$$

$$= I_1 \left[\frac{(Z_1+Z_3)(Z_2+Z_3) - Z_3^2}{Z_2+Z_3} \right]$$

$$= I_1 \left(\frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + \cancel{Z_3^2} - \cancel{Z_3^2}}{Z_2+Z_3} \right)$$

$$\Rightarrow \quad I_1 = \frac{V_a\,(Z_2+Z_3)}{(Z_1Z_2+Z_2Z_3+Z_3Z_1)} \quad \text{—— (4)}$$

Putting eqn (4) in eqn (2), yields the current through the meter in terms of emf V_a and the n/w impedances. Thus

$$I_b = \cancel{I_1 Z_3} - \frac{V_a\,\cancel{(Z_2+Z_3)}}{(Z_1Z_2+Z_2Z_3+Z_3Z_1)} \cdot \frac{Z_3}{\cancel{(Z_2+Z_3)}}$$

$$\Rightarrow \quad I_b = \frac{-V_a Z_3}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \quad \text{—— (5)}$$

→ If location of the emf and current meter are interchanged as in fig (4), we have

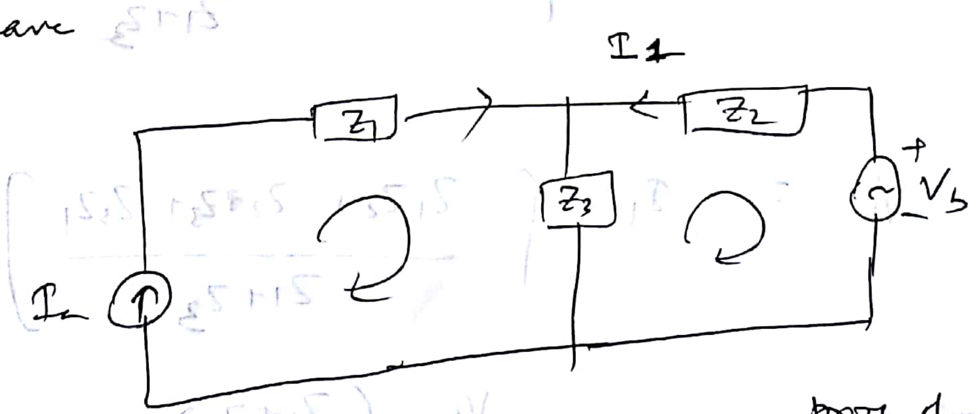


fig 4:- Equivalent circuits used in proof of reciprocity thm.

$$-I_a Z_1 - (I_1+I_a)Z_3 = 0$$

$$\Rightarrow \quad I_a(Z_1+Z_3) = -I_1 Z_3$$

$$\Rightarrow \quad I_a = -\frac{I_1 Z_3}{Z_1+Z_3} \quad \text{—— (6)}$$

KVL on the 2nd loop

$$I_1 Z_2 - V_b + (I_a + I_1) Z_3 = 0$$

$$\Rightarrow \quad V_b = I_1(Z_2+Z_3) + I_a Z_3 \quad \text{--- (7)}$$

Putting eqn (6) in eqn (7), we have

$$V_b = I_1(Z_2+Z_3) + \left(\frac{-I_1 Z_3}{Z_1+Z_3}\right) Z_3$$

$$= \frac{I_1(Z_2+Z_3)(Z_1+Z_3) - I_1 Z_3^2}{(Z_1+Z_3)}$$

$$\Rightarrow = I_1 \left[\frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1 + \cancel{Z_3^2} - \cancel{Z_3^2}}{Z_1+Z_3}\right]$$

$$V_b = I_1 \left[\frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1+Z_3}\right]$$

$$\Rightarrow I_1 = \frac{V_b(Z_1+Z_3)}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \quad \text{--- (8)}$$

Putting eqn (8) in eqn (6), we have

$$I_a = -\frac{I_1 Z_3}{(Z_1+Z_3)}$$

$$\Rightarrow \quad I_a = -\frac{V_b \cdot \cancel{(Z_1+Z_3)}}{(Z_1Z_2+Z_2Z_3+Z_3Z_1)} \times \frac{\cancel{(Z_1+Z_3)}\, Z_3}{\cancel{(Z_1+Z_3)}}$$

$$\Rightarrow \quad I_a = -\frac{V_b \cdot Z_3}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \quad \text{---} (9)$$

Comparing eqn. (5) & (9), it follows that if $V_a = V_b$ then $I_a = I_b$, proving the theorem.

Note :- Relation between Beam solid Angle (Ω_A) & Directivity (D) :-

The radiated power can be expressed in terms of beam solid angle and maximum radiation intensity as

$$P_{rad} = \Omega_A \cdot U_{max} \quad \text{(watt)} \quad \text{--- (1)}$$

The Maxm directivity can be defined as

$$D = \frac{4\pi U_{max}}{P_{rad}} \quad \text{--- (2)}$$

Putting eqn (1) in eqn (2), we have

$$D = \frac{4\pi \cancel{U_{max}}}{\Omega_A \cdot \cancel{U_{max}}} = \frac{4\pi}{\Omega_A}$$

$$\therefore \boxed{D = \frac{4\pi}{\Omega_A}} \quad \text{--- (3)}$$

We know

$$\Omega_A \simeq \theta_{HP} \cdot \phi_{HP}$$

(HP = Half-power beamwidth)

$$\therefore \boxed{D \simeq \frac{4\pi}{\theta_{HP} \cdot \phi_{HP}} = \frac{41253}{\theta_{HP} \cdot \phi_{HP}}} \quad \text{--- (4)}$$

→ degree²

Hertzian Dipole (Infinitesimal Dipole)

A Hertzian dipole is 'an elementary source consisting of time-harmonic electric current element of a specified direction and infinitesimal length'. ($l << \lambda$)

Although a single current element can't be supported in free space, because of the linearity of Maxwell's equations, one can always represent any arbitrary current distribution in terms of the current elements of the type that a Hertzian dipole is made of.

If the field of a current element is known, the field due to any current distribution can be computed using a superposition integral or summing the contributions due to all the current elements comprising the current distribution. Thus, the Hertzian dipole is the most basic antenna element and the starting point of antenna analysis.

Consider an infinitesimal time-harmonic current element, $I = a_z I_0 dl$, kept at the origin with the current flow directed along the z-direction indicated by the unit vector (a_z) as shown in figure. I_0 is the current and dl is elemental length of the current element. Time variation of the type $e^{j\omega t}$ is implied in saying current is time-harmonic. Consider the relationship between the current distribution I and vector potential A, as shown in eqn (1).

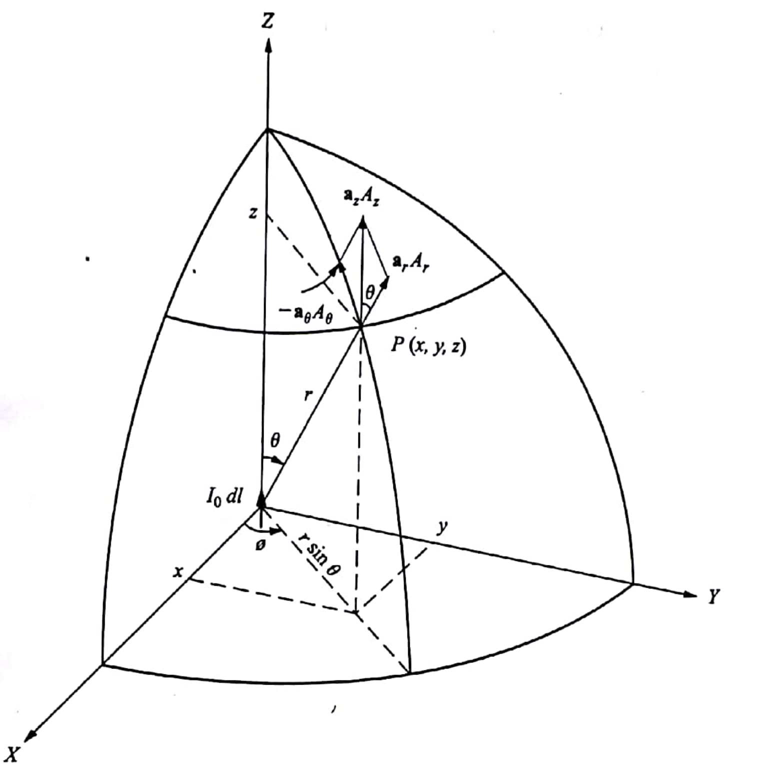


Fig. 1.5 Components of the vector potential on the surface of a sphere of radius r, due to a z-directed current element kept at the origin

$$A(r,\theta,\phi) = \frac{\mu}{4\pi}\int_{c'} I(x',y',z')\,\frac{e^{-jkR}}{R}\,dl' \quad (1)$$

Since we have an infinitesimal current element kept at the origin

Note
c' → indicate line integral

$$x' = y' = z' = 0$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2+y^2+z^2} = r$$

Now the vector potential due to a current element can be written as

(b)

$$A(r,\theta,\phi) = a_z \frac{\mu}{4\pi} I_0\, dl \frac{e^{-jkr}}{r} = a_z A_z \quad (2)$$

$\left(\because A = \frac{\mu}{4\pi} I_0 \cdot \frac{e^{-jkr}}{r} \int_{-dl/2}^{dl/2} dz' = \frac{\mu}{4\pi} I_0 \frac{e^{-jkr}}{r} \cdot dl \right)$ const, due to small length

Note that the vector potential has the same vector direction as the current element. In this case, the a_z - directed current element produces only the A_z - component of the vector potential.

The H and E fields of a Hertzian dipole are computed using the relationship,

$\because \nabla \times H = J + \frac{\partial D}{\partial t}$
$= 0 + j\omega D$
$= 0 + j\omega \epsilon E$
$\Rightarrow E = \frac{1}{j\omega\epsilon}(\nabla \times H)$

$$H = \frac{1}{\mu}(\nabla \times A) \quad (3)$$

&

$$E = \frac{1}{j\omega\epsilon}(\nabla \times H) \quad (4)$$

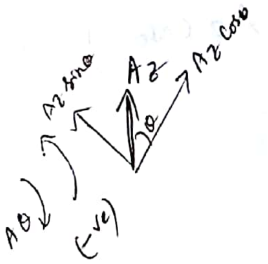
[See the derivations of vector potential]

The E and H fields are generally computed in spherical coordinates for the following reasons

(a) the $\frac{e^{-jkr}}{r}$ term indicates that fields consists of outgoing spherical waves which are simple to represent mathematically in spherical coordinates.

(b) the spherical co-ordinates system allows easy visualization of the behaviour of the fields as a function of direction and simplifies the mathematical representation of the radiated fields.

From figure 1.5, we can relate the z-component of the vector potential, A_z, to the components in spherical co-ordinates A_r, A_θ and A_ϕ as,



$$A_r = A_z \cos\theta \quad \text{——— (5)}$$

$$A_\theta = -A_z \sin\theta \quad \text{——— (6)}$$

$$A_\phi = 0 \quad \text{——— (7)}$$

Taking curl of 'A' in spherical co-ordinates,

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r\,a_\theta & r\sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix} \quad \text{——— (8)}$$

Substituting eqn (5), (6) & (7) in to eqn (8)

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r\,a_\theta & r\sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_z\cos\theta & -r A_z \sin\theta & 0 \end{vmatrix}$$

Since A_z is a function of 'r' alone, its derivative w.r.to θ & ϕ are zero.

(9)

$$\therefore \nabla \times A = \frac{1}{r^2 \sin\theta}\left[a_r \left(0 - \cancelto{0}{\frac{\partial}{\partial \phi} r A_z \sin\theta}\right) - r a_\theta \left(0 - \cancelto{0}{\frac{\partial}{\partial \phi} A_z \cos\theta}\right) + r\sin\theta\, a_\phi \left(\frac{\partial}{\partial r}\cdot(-r A_z \sin\theta) - \frac{\partial}{\partial \theta} A_z \cos\theta\right)\right]$$

$$\nabla \times A = \frac{1}{\cancel{r^2}\cancel{\sin\theta}}\, a_\phi \left[\cancel{r}\cancel{\sin\theta}\left(\frac{\partial}{\partial r}(-r A_z \sin\theta) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right)\right]$$

$$\nabla \times A = a_\phi \times \frac{1}{r}\left[\frac{\partial}{\partial r}(-r A_z \sin\theta) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right] \quad \text{—— (9)}$$

Substituting the expression for A_z and

[From eqn (2), $A_z = \frac{\mu}{4\pi} I_0\, dl\, \frac{e^{-jkr}}{r}$]

performing the indicated differentiation in eqn (9)

$$\nabla \times A = a_\phi \frac{1}{r} A_z \sin\theta\, (jkr + 1) \quad \text{—— (10)}$$

[Proof:- From eqn (9),

$$\nabla \times A = a_\phi \times \frac{1}{r}\left[\frac{\partial}{\partial r}(-r A_z \sin\theta) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right]$$

Putting $A_z = \frac{\mu}{4\pi} I_0 \, dl \frac{e^{-jkr}}{r}$ (20)

$$\nabla \times A = a_\phi \times \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\cancel{r} \times \frac{\mu}{4\pi} I_0 \, dl \cdot \frac{e^{-jkr}}{\cancel{r}}\right)\sin\theta - \frac{\partial}{\partial \theta}\left(\frac{\mu}{4\pi} I_0 \, dl \frac{e^{-jkr}}{r} \cdot \cos\theta\right)\right]$$

$$= a_\phi \times \frac{1}{r}\left[-\frac{\mu}{4\pi} I_0 \, dl \cdot e^{-jkr}(-jk)\sin\theta + \frac{\mu}{4\pi} \cdot I_0 \cdot dl \cdot \frac{e^{-jkr}}{r} \cdot \sin\theta\right]$$

$$= \frac{a_\phi}{r} \frac{\mu}{4\pi} I_0 \, dl \cdot \frac{e^{-jkr}}{r} \sin\theta \left[1 + jkr\right]$$

$$= \frac{a_\phi}{r} \cdot A_z \left(1 + jkr\right)$$

Substituting eqn (10), in eqn (3) and simplifying, we get the expression for the Horizontal component of H field of a dipole in spherical Coordinates as,

From eqn (3) $H = \frac{1}{\mu}(\nabla \times A)$

$$\therefore H = \frac{1}{\mu} \times a_\phi \frac{1}{r} A_z \sin\theta (jkr + 1)$$

$$= 0 \cdot a_r + 0 \cdot a_\theta + \frac{1}{\cancel{\mu}} a_\phi \cdot \frac{1}{r} \times \frac{\cancel{\mu}}{4\pi} I_0 \, dl \cdot \frac{e^{-jkr}}{r} \sin\theta \cdot (jkr + 1)$$

$$= 0 \cdot a_r + 0 \cdot a_\theta + \frac{1}{r^2} \frac{I_0 \, dl \sin\theta \, e^{-jkr}}{4\pi} (1 + jkr)$$

$$= 0 \cdot a_r + 0 \cdot a_\theta + \frac{1}{r^2} \frac{I_0 \, dl \sin\theta \, e^{-jkr}}{4\pi} \left(\frac{1 + jkr}{jkr}\right) \times jkr$$

$$\therefore H = 0 \cdot a_r + 0 \cdot a_\theta + \frac{1}{r\cancel{\mu}} \cdot \frac{I_o\, dl \sin\theta \cdot e^{-jkr}}{4\pi} \left[1 + \frac{1}{jkr}\right] \cdot jk_{\mu}$$

$$\Rightarrow H = 0 \cdot a_r + 0 \cdot a_\theta + jk \frac{I_o\, dl \sin\theta\, e^{-jkr}}{4\pi r} \cdot \left(1 + \frac{1}{jkr}\right)$$

$$\therefore \quad H_r = 0 \quad \text{——— (11)}$$

$$H_\theta = 0 \quad \text{——— (12)}$$

$$H_\phi = jk \frac{I_o\, dl \sin\theta}{4\pi} \frac{e^{-jkr}}{r} \left(1 + \frac{1}{jkr}\right) \quad \text{— (13)}$$

The electric field can be obtained from Maxwell's curl equation. Substituting the expression for H in eqn (4)

$$E = \frac{1}{j\omega\epsilon} (\nabla \times H)$$

$$= \frac{1}{j\omega\epsilon} \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r \sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r \sin\theta\, H_\phi \end{vmatrix} \quad \text{— (14)}$$

Expanding the determinant & substituting eqn (13) into eqn (14), we have

$$E = \frac{1}{jw\epsilon} \times \frac{1}{r^2 \sin\theta} \left[a_r \frac{\partial}{\partial\theta}(r\sin\theta H_\phi) - r a_\theta \frac{\partial}{\partial r}(r\sin\theta H_\phi) \right]$$

Consider

$$\frac{\partial}{\partial\theta}(r\sin\theta H_\phi)$$

$$= \frac{\partial}{\partial\theta}\left\{ \cancel{r}\sin\theta \cdot \frac{jK I_0 dl \sin\theta}{4\pi} \cdot \frac{e^{-jkr}}{\cancel{r}} \left(1+\frac{1}{jkr}\right)\right\}$$

$$= \frac{jK I_0 dl}{4\pi} e^{-jkr}\left(1+\frac{1}{jkr}\right) \cdot \frac{\partial}{\partial\theta}\sin^2\theta$$

$$= \frac{jK I_0 dl}{4\pi} \cdot e^{-jkr}\left(1+\frac{1}{jkr}\right) 2\sin\theta \cdot \cos\theta$$

$$\vec{E}_r = \frac{1}{\cancel{j}w\epsilon} \times \frac{1}{r^2 \cancel{\sin\theta}} \cdot a_r \cdot \frac{\cancel{j}K I_0 dl}{\cancel{2}\cancel{4}\pi} \cdot e^{-jkr}\left(1+\frac{1}{jkr}\right) \cancel{2}\cancel{\sin\theta}\cos\theta$$

$$= \frac{K}{w\epsilon} \frac{I_0 dl \cos\theta \, e^{-jkr}}{2\pi r} \cdot \frac{e^{-jkr}}{r} \cdot \left(1+\frac{1}{jkr}\right)\hat{a}_r$$

$$E_r = \eta \frac{I_0 dl \cos\theta}{2\pi r} \cdot \frac{e^{-jkr}}{r}\left(1+\frac{1}{jkr}\right) \text{ --- } (15)$$

$\because \frac{K}{w\epsilon} = \eta$

= Intrinsic Impedance of the medium

Similarly

Consider $r \cdot \frac{\partial}{\partial r}(r\sin\theta H_\phi)$

$$r \cdot \frac{\partial}{\partial r}\left(r \sin\theta \, H_\phi\right)$$

$$= r \frac{\partial}{\partial r}\left(\cancel{r}\sin\theta \cdot \frac{jk I_0 dl \sin\theta}{4\pi} \frac{e^{-jkr}}{\cancel{r}}\left(1+\frac{1}{jkr}\right)\right)$$

$$= r \cdot \frac{jk I_0 dl \sin^2\theta}{4\pi} \frac{\partial}{\partial r}\left(e^{-jkr}\right)\left(1+\frac{1}{jkr}\right)$$

$$= r \cdot \frac{jk I_0 dl \sin^2\theta}{4\pi}\left[e^{-jkr}\left(0+\frac{1}{jk}\cdot\frac{-1}{r^2}\right) + \left(1+\frac{1}{jkr}\right) \cdot e^{-jkr} \cdot (-jk)\right]$$

$$= r \cdot \frac{jk I_0 dl \sin^2\theta}{4\pi}\left[-\frac{e^{-jkr}}{jkr^2} - e^{-jkr}(jk)\left(1+\frac{1}{jkr}\right)\right]$$

$$= r \cdot \frac{jk I_0 dl \sin^2\theta}{4\pi} \times \left(-e^{-jkr} \cdot jk\right)\left[1+\frac{1}{jkr} - \frac{1}{k^2 r^2}\right]$$

$$\vec{E}_\theta = \frac{-1}{\cancel{j}\omega\epsilon} \times \frac{1}{r^2 \sin\theta} \times r \times \frac{\cancel{j}k I_0 dl \sin^2\theta}{4\pi} \times -e^{-jkr}(jk)\left[1+\frac{1}{jkr} - \frac{1}{(kr)^2}\right]\hat{a}_\theta$$

$$E_\theta = \frac{k}{\omega\epsilon} \cdot \frac{1}{r^2 \sin\theta} \cdot r \cdot \frac{I_0 dl \sin^2\theta}{4\pi} \cdot e^{-jkr} \cdot jk\left[1+\frac{1}{jkr} - \frac{1}{(kr)^2}\right]$$

$$2)\; E_\theta = \eta \cdot \frac{1}{r^2 \sin\theta} \cdot r \cdot \frac{I_0 dl \sin^2\theta}{4\pi} \cdot e^{-jkr} jk\left[1+\frac{1}{jkr} - \frac{1}{(kr)^2}\right]$$

$$\Rightarrow E_\theta = \cancel{r}\, j \cdot \frac{\eta \cdot K I_0 dl \sin^{\cancel{2}}\theta}{\cancel{(r\sin\theta)}\, 4\pi} \cdot \frac{e^{-jkr}}{r}\left(1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right)$$

$$\therefore E_\theta = j\eta \frac{K I_0 dl \sin\theta}{4\pi} \cdot \frac{e^{-jkr}}{r}\left(1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right)$$

and $E_\phi = 0$ (Since there is no $\hat{a}_\phi$ term).

$\therefore$ Summary

$$\boxed{\begin{aligned} E_r &= \eta \frac{I_0 dl \cos\theta}{2\pi r} \cdot \frac{e^{-jkr}}{r}\left(1+\frac{1}{jkr}\right) \quad &(15)\\ E_\theta &= j\eta \frac{K I_0 dl \sin\theta}{4\pi} \cdot \frac{e^{-jkr}}{r}\left(1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right) \quad &(16)\\ E_\phi &= 0 \quad &(17)\end{aligned}}$$

where $\eta = \frac{K}{\omega\epsilon} = \frac{\cancel{\omega}\sqrt{\mu\epsilon}}{\cancel{\omega}\epsilon} = \frac{\sqrt{\mu}}{\sqrt{\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$ — (18)

= Intrinsic Impedance of the medium

Vector Effective Length

The effective length of an antenna, whether it be a linear or an aperture antenna, is a quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it.

Let I_{in} be the input current at the terminals of a transmit antenna producing an electric field, E_a, in the far-field region. The vector effective length, l_{eff}, is related to E_a by the relation

$$E_a = a_\theta E_\theta + a_\phi E_\phi = j\eta \frac{k I_{in} e^{-jkr}}{4\pi r} \cdot l_{eff} \quad — (1)$$

The vector effective length can be written in terms of its components l_θ and l_ϕ along the θ and ϕ directions, respectively, as

$$l_{eff} = a_\theta l_\theta + a_\phi l_\phi \quad — (2)$$

It should be noted that it is also referred to as effective height. In general, l_θ and l_ϕ can be complex quantities. For an ideal current element of length dl carrying a current of I_0, the electric field in the far-field region is given by

$$E = a_\theta \, j\eta \, \frac{K I_o dl \sin\theta}{4\pi} \cdot \frac{e^{-jKr}}{r} \quad \text{—(3)}$$

Proof:- For an Hertzian dipole,

$$E_\theta = j\eta \cdot \frac{K I_o dl \sin\theta}{4\pi} \cdot \frac{e^{-jKr}}{r}\left(1 + \frac{1}{jKr} - \frac{1}{(Kr)^2}\right)$$

$$E_\phi = 0$$

Multiplying $r \rightarrow$ into the denominator of bracket terms.

$$E_\theta = j\eta \frac{K I_o dl \sin\theta}{4\pi} \cdot e^{-jKr}\left[\frac{1}{r} + \frac{1}{jKr^2} - \frac{1}{K^2 r^3}\right]$$

Since $\frac{1}{r^2}$ and $\frac{1}{r^3}$ terms decay much faster than $\frac{1}{r}$ term, at large distance from the dipole $(r >> \lambda)$, we can neglect these terms

$$\therefore E_\theta = j\eta \frac{K I_o dl \sin\theta}{4\pi} \frac{e^{-jKr}}{r}$$

and $E_\phi = 0$.

Comparing eqn 1 & 3, we have

$$\cancel{j\eta} \frac{\cancel{K} \cancel{I_o} \cancel{e^{-jKr}}}{\cancel{4\pi r}} \cdot l_{eff} = \cancel{j\eta} \frac{\cancel{K}\cancel{I_o} dl \sin\theta}{\cancel{4\pi}} \frac{\cancel{e^{-jKr}}}{\cancel{r}} a_\theta$$

$$\Rightarrow l_{eff} = dl \sin\theta \, a_\theta \quad \text{—(4)}$$

'l_{eff}' is maximum when $\underline{\theta = 90^\circ}$.

Thus vector effective length of a Hertzian dipole is maximum along the direction orthogonal to its axis and zero along its axis.

In the receiving mode, the o/p voltage developed at the terminals of an antenna due to an incident e.m. wave having an electric field $\underline{E}^i$ is given by

$$V_{oc} = E^i \cdot l_{eff} \qquad \text{—— (5)}$$

where

V_{oc} = open-ckt voltage at antenna terminals

E^i = Incident Electric field

l_{eff} = Vector effective length.

In eqn (5), V_{oc} can be thought of as the voltage induced in a linear antenne of length 'l_{eff}', when 'l_{eff}' and 'E_i' are linearly polarized. From eqn (5), the effective length of a linearly polarized antenna receiving a plane wave in a given direction is defined as "the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization.

Alternatively, the effective length is

the length of a thin straight conductor oriented perpendicular to the given direction and parallel to the antenna polarization, having a uniform current equal to that at the antenna terminals and producing the same far-field strength as the antenna in that direction."

Problem

1) The farfield zone field radiated by a small dipole of length $\left(l < \frac{\lambda}{10}\right)$ and with a triangular current distribution is given by

$$E_\theta = \hat{a}_\theta \frac{j\eta\, k\, I_m\, l\, e^{-jkr}}{8\pi r} \sin\theta.$$

Determine the vector effective length of the antenna.

Ans: - Comparing eqn ① with the field given in the example, we have

$$\frac{\cancel{j\eta}\, \cancel{k}\, \cancel{I_m}\, \cancel{e^{-jkr}}}{\cancel{4\pi r}}\, l_{eff} = \frac{\cancel{j\eta}\, \cancel{k}\, \cancel{I_m}\, l\, \cancel{e^{-jkr}} \sin\theta}{2\,\cancel{8\pi r}}\, \hat{a}_\theta$$

$$\boxed{l_{eff} = \frac{l}{2} \sin\theta\, \hat{a}_\theta}$$

Relation between Maximum Directivity (D_o) and Maximum Effective Area (A_{em})

To derive the relationship between directivity and maximum effective area, the geometrical arrangement of figure 1 is chosen. Antenna 1 is used as a transmitter and 2 as a receiver. The effective areas and directivities of each are designated as A_t, A_r and D_t, D_r. If antenna 1 were isotropic, its radiated power density at distance R would be

$$W_o = \frac{P_t}{4\pi R^2} \quad \text{——— (1)}$$

Where P_t is the total radiated power.

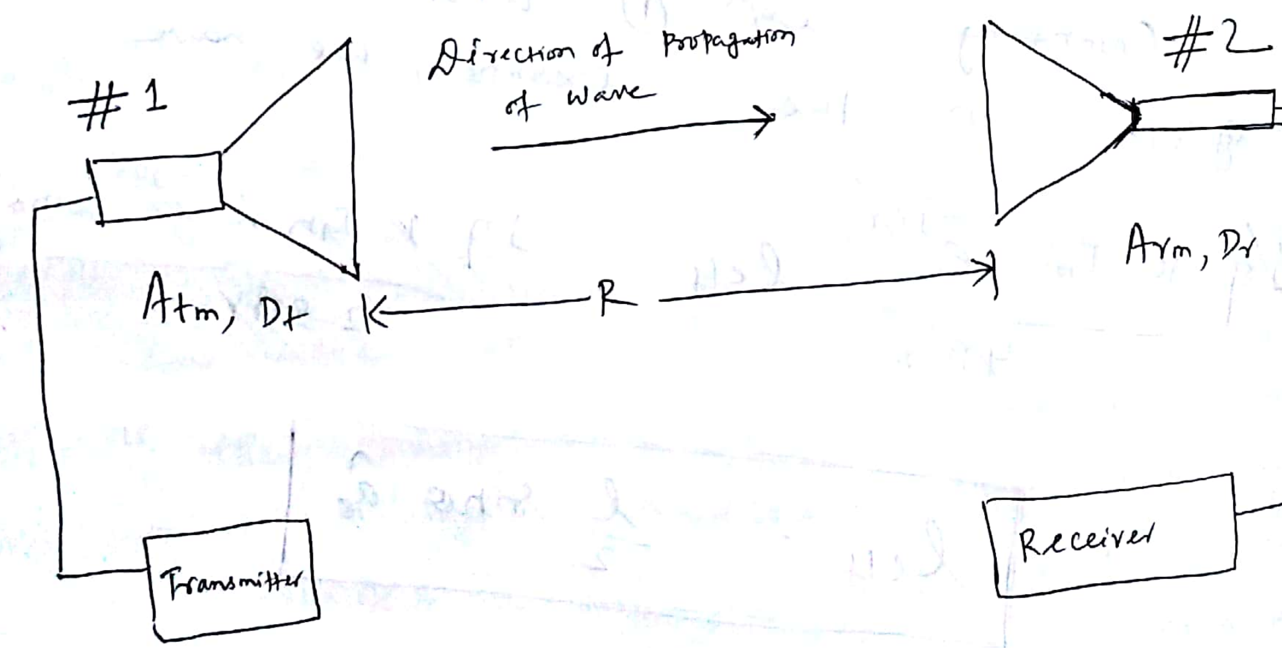


Fig 1:- Two antennas separated by distance R

Because of directive Properties of the antenna, its actual density is

$$W_t = W_0 \cdot D_t = \frac{P_t}{4\pi R^2} \cdot D_t \quad \text{——} (2)$$

(∵ For a directive antenne, W_t is D_t time of isotropic)

The power Collected (received) by the antenne and transferred to the load would be

$$P_r = W_t . A_r = \frac{P_t D_t}{4\pi R^2} . A_r \quad \text{——} (3)$$

or $$D_t A_r = \frac{P_r (4\pi R^2)}{P_t} \quad \text{——} 3(a)$$

If antenne 2 is used as transmitter, 1 as receiver, and the intervening medium is linear, passive and isotropic, we can write

$$D_r A_t = \frac{P_r}{P_t} (4\pi R^2) \quad \text{——} (4)$$

Equating 3(a) & (4), we have

$$D_t A_r = D_r A_t$$

$$\Rightarrow \frac{D_t}{A_t} = \frac{D_r}{A_r} \quad \text{——} (5)$$

Increasing the Directivity of an antenna increases its effective area in direct proportion.

Thus eqn (5), Can be written as

$$\frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}} \quad \text{——— (6)}$$

where A_{tm}, and A_{rm} (D_{ot} and D_{or}) are the maximum effective areas (directivities) of antennas 1 and 2, respectively.

If antenna 1 is isotropic, then $D_{ot} = 1$ and its maximum effective area can be expressed as

$$\frac{1}{A_{tm}} = \frac{D_{or}}{A_{rm}}$$

$$\Rightarrow A_{tm} = \frac{A_{rm}}{D_{or}} \quad \text{——— (7)}$$

Equation (7), states that the maximum effective area of an isotropic source is equal to the ratio of the maximum effective area to the maximum directivity of any other source.

For example:- Let the other antenna be a very short dipole ($l << \lambda$) whose effective area $\left(\frac{3\lambda^2}{8\pi}\right)$ and maximum directivity (1.5) are known.

The maxm effective area of the

isotropic source is then equal to

$$A_{tm} = \frac{A_{rm}}{D_{or}} = \frac{\cancel{3}\lambda^2}{\cancel{8}\pi_{4}} \times \frac{1}{1.\cancel{5}} = \frac{\lambda^2}{4\pi} \quad \text{--- (8)}$$

Using eq^n (8), we can write eq^n (7) as

$$A_{rm} = A_{tm} \cdot D_{or} = \frac{\lambda^2}{4\pi} \cdot D_{or} \quad \text{--- (9)}$$

In general, the max^m effective aperture (A_{em}) of any antenna is related to its maximum directivity (D_o) by

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_o} \quad \text{--- (10)}$$

or

$$\boxed{D_o = \frac{4\pi A_{em}}{\lambda^2}} \quad \text{--- (11)}$$

Eq^n (10), is multiplied by the power density of the incident wave it leads to the max^m power that can be delivered to the load. This assumes that there are no conduction-dielectric losses (radiation efficiency ($e_{cd} = 1$)), the antenna is matched to the load (reflection efficiency $e_r = 1$). If there are losses associated with an antenna, its max^m effective aperture of eqn (10) must be

modified to account for Conduction- dielectric losses (radiation efficiency). Thus,

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_0$$

$$A_{em} = (D_0 \times e_{cd}) \times \frac{\lambda^2}{4\pi}$$

$$A_{em} = G \times \frac{\lambda^2}{4\pi}$$

where

Gain = efficiency × Directivity

$\lambda \rightarrow$ Wavelength

$$\Rightarrow \boxed{G = \frac{4\pi A_{em}}{\lambda^2}} \quad (12)$$

(Proved)

Problems

1) Calculate the effective length of an $\lambda/2$ antenna. Given $R_r = 73\Omega$, $(A_e)_{max} = 0.13\lambda^2$ and $\eta = 120\pi$. (Intrinsic Impedance)

Ans :-

$$l_e = \frac{\sqrt{4 A_{em} R_r}}{\sqrt{Z}} = \frac{\sqrt{4 \times 0.13\lambda^2 \times 73}}{\sqrt{120\pi}}$$

$$\boxed{l_e = 0.3174\lambda}$$

2) Calculate the max^m effective aperture of an antenna which is operating at a wavelength of 2 meters and has a directivity of 100.

Ans:- $A_{em} = ?$, $\lambda = 2m$, $D = 100$.

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$\Rightarrow 100 = \frac{\cancel{4}\pi \times A_{em}}{\cancel{2^2}}$$

$$\Rightarrow \boxed{A_{em} = \frac{100}{\pi} = 31.84\, m^2} \quad \text{(Ans)}.$$

3) What is the BW of a resonant circuit whose $Q = 100$ (Quality factor) and resonant freq is 5 MHZ. BW → Bandwidth.

Ans:- We know,

$$BW = \frac{f_r}{Q}$$

$$\therefore BW = \frac{5 \times 10^6}{100}$$

$$= 5 \times 10^4 \text{ HZ} = 50 \times 10^3 \text{ HZ}$$

$$\boxed{BW = 50 \text{ KHZ}}$$

4) Calculate the max^m effective aperture of a microwave antenna which has directivity of 900.

Ans :-
$$D = \frac{4\pi A_{em}}{\lambda^2}$$

$$\Rightarrow A_{em} = \frac{D \times \lambda^2}{4\pi}$$

$$A_{em} = \frac{900 \times \lambda^2}{4\pi} = 71.61\lambda^2$$ (Ans)

5) Determine the maximum effective aperture of a beam antenna having HPBW of 30° & 35° in perpendicular planes intersecting in the beam axis. Assume small side lobes.

Ans :- We know,

$$\text{Directivity } (D) = \frac{4\pi}{\theta_{HP} \cdot \phi_{HP}} = \frac{41253 \cancel{\text{degree}}^2}{30 \cancel{\text{deg}} \times 35 \cancel{\text{degree}}}$$

$$D = 39.28 \quad \text{--- (1)}$$

$$D = \frac{4\pi A_{em}}{\lambda^2}$$

$$\Rightarrow 39.28 = \frac{4\pi \times A_{em}}{\lambda^2}$$

$$\Rightarrow \frac{39.28}{4\pi}\lambda^2 = A_{em}$$

$$\Rightarrow \boxed{A_{em} \simeq 3.13\lambda^2}$$

6) Calculate the gain of an antenna with circular aperture of diameter 3 meter at a frequency of 5 GHz.

Ans ÷ Assume no losses present,

$D = G$ (Directivity = Gain)

$$G = \frac{4\pi A_e}{\lambda^2}$$

$f = 5\,GHz$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^9} = \frac{3}{50}$

$A_e = \pi r^2 = \frac{\pi \times d^2}{4} = \frac{\pi \times 3^2}{4} = \frac{9\pi}{4}$

$$G = \frac{4\pi A_e}{\lambda^2}$$

$$G = \frac{\cancel{4}\pi \times \cancel{9}\pi \times 50^2}{\cancel{4} \times \cancel{3^2}}$$

$$G = 2500 \times \pi^2$$

$$\boxed{G \simeq 24674}$$ (Ans).

7) The radiation resistance of an antenna is $72\,\Omega$ and loss-resistance is $8\,\Omega$. What is the directivity, if the power gain is 16.

Ans ÷ Given

$R_r = 72\,\Omega$

$R_e = 8\,\Omega$

Radiation efficiency $(\eta) = \frac{R_r}{R_r + R_L}$

$= \frac{72}{72+8}$

$= \frac{72}{80} = \frac{9}{10}$

$\eta = 0.9$

~~$D = \eta \times \text{Gain} = 0.9 \times 16$~~

$\text{Gain} = \eta \times D = 0.9 \times D$

$\Rightarrow D = \frac{\text{Gain}}{0.9} = \frac{16}{0.9} = 17.78$ (Ans)

8) A thin dipole antenna is $\frac{\lambda}{15}$ long. If its loss resistance is $1.5\,\Omega$. Find the radiation resistance & the efficiency.

Ans: For a small dipole, $(l << \lambda)$

$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$

$= 80\pi^2 \left(\frac{\cancel{\lambda}}{15\cancel{\lambda}}\right)^2$

$R_r = 3.5\,\Omega$

$\eta = \frac{R_r}{R_r + R_L} = \frac{3.5}{3.5+1.5} = \frac{3.5}{5} = 0.7$

$\therefore$ $\boxed{\eta = 70\%}$

9) Calculate the radiation resistance of an antenna which is drawing 15 Ampere current and radiating 5 KW.

Ans :
$$W = I_{rms}^2 \cdot R_r$$

$$\Rightarrow 5\times10^3 = 15^2 \times R_r$$

$$\Rightarrow R_r = \frac{5\times10^3}{225} = 22.22 \text{ ohm}$$

(Ans).

10) An antenne having a gain of 6 dB over a reference antenne is radiating 700 watts. Calculate the power that ~~reference~~ antenne must radiate in order to be equally effective in the must prefered direction.

Ans :- Given $P_2 = 700$ watt, $G = 6\text{dB}$

$P_1 = ?$

$$G_{dB} = 10 \log \frac{P_1}{P_2}$$

$$\Rightarrow 6 = 10 \log_{10} \frac{P_1}{700}$$

$$\Rightarrow \log_{10}\left(\frac{P_1}{700}\right) = \frac{6}{10} = 0.6$$

$$\Rightarrow \frac{P_1}{700} = 10^{0.6}$$

$$\Rightarrow \boxed{P_1 = 2786.75 \text{ Watt}}$$

(Ans)

11) Calculate the gain of receiving antenna in dB which delivers a 50 μV signal to a transmission line over that of an antenna which delivers 25 μV signal under identical conditions.

Ans :-
$$G_{dB} = 20 \log \frac{V_1}{V_2}$$

$$= 20 \log \left(\frac{50}{25}\right)$$

$$G_{dB} = 20 \log (2)$$

$$= 20 \times 0.3010$$

$$\boxed{G_{dB} = 6.02 \text{ dB}}$$

12) The radiation resistance of an antenna is 72 Ω and loss resistance is 8 Ω. What is the directivity in dB if the power gain is 16.

Ans :-
$$\eta = \frac{72}{72+8} = 0.9 \qquad \left(\eta = \frac{R_r}{R_r + R_L}\right)$$

$$\cancel{D = \eta \times G = 0.9 \times 16}$$

$$G = \eta \times D = 0.9 \times D$$

$$\Rightarrow D = \frac{G}{0.9} = \frac{16}{0.9} = 17.77$$

$$D|_{(\text{in dB})} = 10 \log_{10} 17.77 = 12.498 \text{ dB}$$

(Ans)

13) An antenna has a loss resistance 10 ohms, power gain of 20 and directivity 22. Calculate Radiation resistance. (40)

Ans :- Given

$$R_l = 10\Omega, \quad G = 20, \quad D = 22.$$

$$\eta = \frac{G}{D} = \frac{20}{22} \quad \text{--- (1)}$$

$$\eta = \frac{R_r}{R_r + R_l}$$

$$\Rightarrow \frac{20}{22} = \frac{R_r}{R_r + 10}$$

$$\Rightarrow 20(R_r + 10) = 22 R_r$$

$$\Rightarrow 20 R_r + 200 = 22 R_r$$

$$\Rightarrow 2 R_r = 200$$

$$\Rightarrow \boxed{R_r = 100\,\Omega} \quad \text{(Ans)}.$$

14) Calculate the antenna efficiency & directivity.

Given :- $R_r = 72\,\Omega$, $R_l = 8\,\Omega$. $G = 12\,dB$

Ans : $10 \log_{10} G = 12 \Rightarrow \log_{10} G = 1.2$

$$\Rightarrow G = 10^{1.2} = 15.84$$

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72+8} = \frac{72}{80} = 0.9$$

$$D = \frac{G}{\eta} = \frac{15.84}{0.9} = 17.61$$

In dB

$$D_{dB} = 10 \log 17.61 = 12.458 \text{ dB}$$

(Ans).

15) An antenna having an effective temperature of 15°K is fed into a microwave amplifier has an effective noise temperature of 20°K. (a) Calculate the available noise power per unit Bandwidth, at i/p for this particular antenna temperature. → (b) Calculate the available noise power for a noise bandwidth of 4 MHZ.

Ans: - Given

$$T_a = 15°K, \quad T_r = 20°K.$$

$$P_s = K(T_a + T_r) \cdot \Delta f$$

$$= 1.38 \times 10^{-23} \times (15+20) \cdot \Delta f$$

$\Delta f = BW$

K = Boltzmann's Constant

(a)

$$\Rightarrow \frac{P_s}{\Delta f} = 1.38 \times 10^{-23} \times 35 = 48.3 \times 10^{-23} \frac{\text{watt}}{\text{Hz}}.$$

(b)

$$P_s = (48.3 \times 10^{-23}) \frac{\text{watt}}{\text{Hz}} \times 4 \times 10^{6}$$

$$\boxed{P_s = 193.2 \times 10^{-17} \text{ watt}}$$

→ Total noise power for a noise BW of 4 MHZ.

(Extra)

* Equivalent Noise temperature of Antenna

The noise introduced by a n/w may also be expressed as effective noise temperature, **Te**, is defined as "that fictional temperature at the i/p of the n/w which would account for noise ΔN at the o/p". where ΔN is the additional noise introduced by the n/w itself.

The noise figure 'F', is related to effective noise temperature 'Te' as

$$F = 1 + \frac{T_e}{T_o}$$

$$\Rightarrow (F-1) = \frac{T_e}{T_o}$$

$$\Rightarrow \boxed{T_e = (F-1)\,T_o} \quad \text{——— (1)}$$

where T_e = Effective Noise Temperature. in °K.

T_o = 290°K (273+17)

F = Noise figure (Dimensionless)

The noise figure 'F' in dB is given by

$$\boxed{(F)_{dB} = 10 \log_{10} F} \quad \text{——— (2)}$$

16) The noise figure of an amplifier at room temperature ($T_0 = 290^\circ K$) is 0.2 dB. Find the equivalent temperature.

Ans :-

$$(F)_{dB} = 10 \log_{10} F$$

$$\Rightarrow 0.2 = 10 \log_{10} F$$

$$\Rightarrow \log_{10} F = 0.02$$

$$\Rightarrow F = 10^{0.02} = 1.047$$

$$T_e = (F-1) T_0 = (1.047 - 1) \times 290 = 13.63^\circ K$$

17) The equivalent noise temperature of cooled parametric amplifier is $20^\circ K$. Find the noise figure

Ans :- Given

$$T_e = 20^\circ K.$$

$$T_e = (F-1) T_0$$

$$\Rightarrow 20^\circ = (F-1) \times 290$$

$$\Rightarrow F-1 = \frac{2\cancel{0}}{29\cancel{0}} = 0.0689 \simeq 0.069$$

$$\Rightarrow F = 1 + 0.069$$

$$\Rightarrow \boxed{F \simeq 1.069}$$ (Ans)

Radiation Pattern of Hertzian dipole

The normalized power pattern, P_n, is obtained by the normalizing the radiation intensity, U, or the time-averaged Poynting vector, S, w.r.to their maximum values.

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U_{max}} = \frac{\cancel{r^2} \times S(r,\theta,\phi)}{\cancel{r^2} \times S_{max}(r)} = \frac{S(r,\theta,\phi)}{S_{max}(r)} \quad \text{--- (1)}$$

The normalized power is a dimensionless quantity and it is expressed in dB as

$$P_{n\,dB}(\theta,\phi) = 10 \log_{10}(P_n(\theta,\phi)) \quad \text{--- (2)}$$

For a Hertzian dipole is given by

$$P_n(\theta,\phi) = \frac{\cancel{\frac{\eta}{2}} \cdot \cancel{\left|\frac{KI_0 dl}{4\pi}\right|^2} \sin^2\theta}{\cancel{\frac{\eta}{2}} \cancel{\left|\frac{KI_0 dl}{4\pi}\right|^2}} = \sin^2\theta \quad \text{--- (3)}$$

$$P_{n\,dB}(\theta,\phi) = 10 \log_{10}(\sin^2\theta) \quad \text{--- (4)}$$

A plot of the far-field electric or magnetic field intensity as a function of direction at a constant distance from antenna is known as the electric field pattern or magnetic field pattern.

Dividing the field quantities by their respective maximum values we get the normalized field patterns. For a Hertzian dipole, the normalized field pattern is given by

$$E_{\theta n}(\theta,\phi) = \frac{E_\theta(r,\theta,\phi)}{E_{\theta max}(r)}$$

$$= \frac{\cancel{\dfrac{j\eta k I_0 dl}{4\pi}} \sin\theta \cdot \cancel{\dfrac{e^{-jkr}}{r}}}{\cancel{\dfrac{j\eta k I_0 dl}{4\pi}} \cdot \cancel{\dfrac{e^{-jkr}}{r}}}$$

$$E_{\theta n}(\theta,\phi) = \sin\theta \qquad \text{—— (5)}$$

Similarly

$$H_{\phi n}(\theta,\phi) = \sin\theta \qquad \text{—— (6)}$$

A normalized field (Maximum amplitude equal to unity) pattern (3D) is shown below for Hertzian dipole.

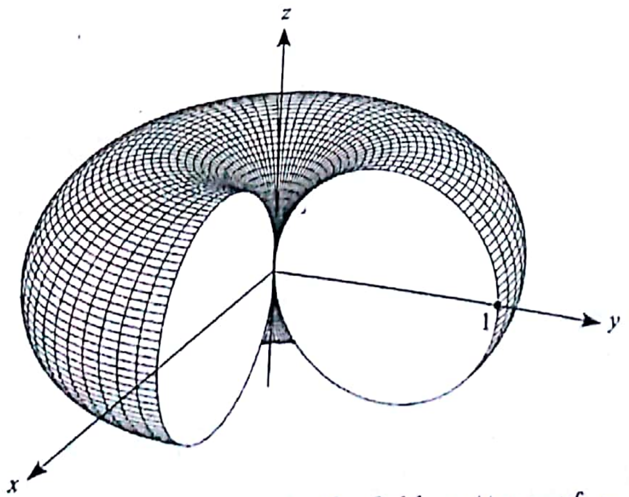


Fig. 2 Normalized E_θ field pattern of a Hertzian dipole

It is generally convenient to present the pattern in two-dimensional (2D) plots by considering the two orthogonal principal plane cuts of the 3D pattern. Principal plane implies that the cut is through the pattern maximum. For a linearly polarized antenna, the pattern cuts in the principal planes parrall to the E and H field vectors are chosen.

These patterns are called the E-plane and H-plane patterns, respectively. For a Hertzian dipole, the X-Z and X-Y plane cuts of the pattern shown in figure 3. The E-plane pattern (X-Z plane cut) and H-plane pattern (XY plane cut) resembles the shape of figure-of-eight and a circle respectively.

A 3D plot of the normalized power pattern expressed in dB [Eqn (4)] is shown in figure 4(a) and E-plane pattern is shown in fig 4(b). The pattern has maximum along $\theta = 90^\circ$ which is known as broadside direction of the dipole. The plot indicates the relative level of radiation intensity w.r.to maximum. For example, along $\theta = 30^\circ$ the radiation intensity is 6dB below the maximum. The pattern has nulls along $\theta = 0^\circ$ and $\theta = 180^\circ$, which are directions along the axis of the dipole.

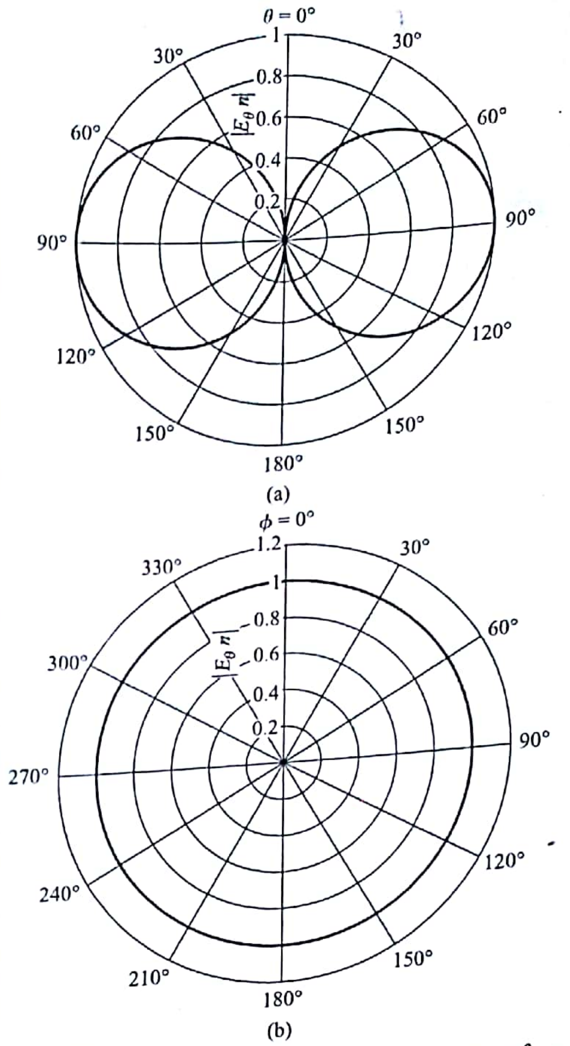


Fig. 3 Normalized E_θ field pattern of a Hertzian dipole: (a) **E**-plane pattern (x-z plane); (b) **H**-plane pattern (x-y plane)

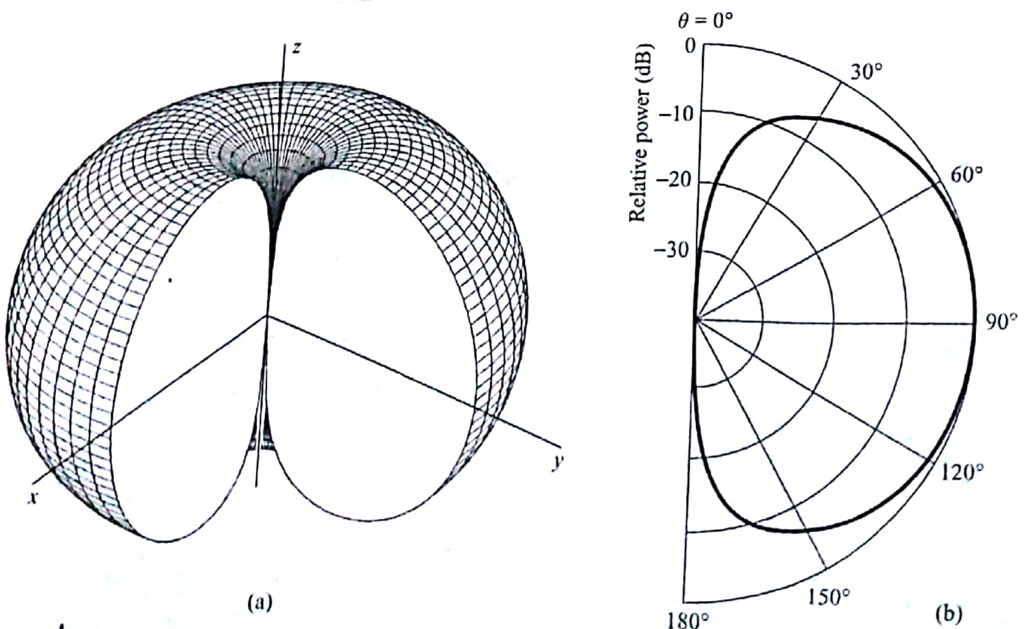


Fig. 4 Normalized power pattern of a Hertzian dipole (a) 3D view; (b) **E**-plane pattern ($\phi = 0°$ plane) in dB units

Directivity & Radiation Resistance Calculation of a Hertzian Dipole

Consider the fields of an infinitesimal Hertzian electric dipole of length dl kept at the origin of the co-ordinate system. Let I_0 be the Z-directed current in the Hertzian dipole radiating into free space. In spherical co-ordinates, the expressions for the E and H fields are given by [As derived earlier]

$$E_r = \frac{\eta I_0 \, dl \cos\theta}{2\pi} e^{-jkr} \left[\frac{1}{r^2} + \frac{1}{jkr^3} \right] \quad \text{—(1)}$$

$$E_\theta = j\eta \frac{k I_0 \, dl \sin\theta}{4\pi} e^{-jkr} \left[\frac{1}{r} + \frac{1}{jkr^2} - \frac{1}{k^2 r^3} \right] \quad \text{—(2)}$$

$$\text{—(3)}$$

$$E_\phi = 0 \quad \text{—(4)}$$

$$H_r = 0 \quad \text{—(5)}$$

$$H_\theta = 0$$

$$H_\phi = jk \frac{I_0 \, dl \sin\theta}{4\pi} e^{-jkr} \left[\frac{1}{r} + \frac{1}{jkr^2} \right] \quad \text{—(6)}$$

Where the impedance of the medium, η is given by $\eta = \frac{k}{\omega\epsilon} = \frac{\omega\sqrt{\mu\epsilon}}{\omega\epsilon} = \sqrt{\frac{\mu}{\epsilon}}$ = Intrinsic Impedance of the medium.

$$\text{—(7)}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

In eqn (7), μ & ϵ are the permeability and permittivity of the medium respectively.

For free space,

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 376.99 \simeq 377\,\Omega$$

$$\therefore \boxed{\eta = 120\pi = 377\,\Omega} \quad \text{———} (8)$$

The eqn (1) to (6) are valid everywhere except on the dipole itself. Since the $\frac{1}{r^2}$ & $\frac{1}{r^3}$ terms decay much faster than $\frac{1}{r}$ term, at large distances from the dipole $(r >> \lambda)$ we can neglect these terms and simplify the field expression by taking only the $\frac{1}{r}$ terms. This simplified expression is known as far-field expression.

In the far-field region, only E_θ and H_ϕ exist and form a spherical wavefront emanating from the antenna.

[Note:- Eqn (1) contains $\frac{1}{r^2}$ & $\frac{1}{r^3}$ terms, they vanish for far field region]

Thus, the far-field expressions for E and H reduces to

$$E = a_\theta E_\theta = a_\theta\, j\eta \frac{k I_0 dl \sin\theta}{4\pi} \frac{e^{-jkr}}{r} \quad \text{———} (9)$$

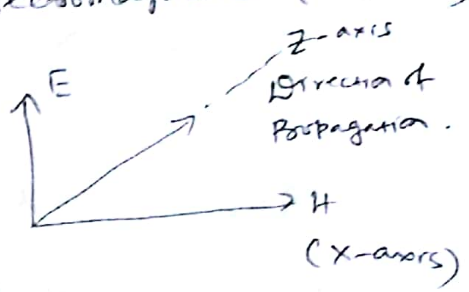
$$H = a_\phi H_\phi = a_\phi \frac{jk I_0 dl \sin\theta}{4\pi} \frac{e^{-jkr}}{r} \quad \text{———} (10)$$

In the far-field, E_θ and H_ϕ are $\perp$ to each other and transverse to the direction of propagation. The ratio of the two field components is the same as the intrinsic impedance, η, of the medium.

$$\frac{E_\theta}{H_\phi} = \eta \quad \text{———— (11)} \qquad \left[\frac{|E q^n (9)|}{|E q^n (10)|} = \eta \right]$$

Thus, the radiated EM wave satisfies all the properties of a transverse electromagnetic (TEM) wave.

E $\perp$ H $\perp$ Z-axis (Direction of Propagation)



$\therefore$ The time-averaged power density vector of the wave is given by

$$S = \frac{1}{2} \operatorname{Re}[E \times H^*] = a_r \frac{1}{2} E_\theta \cdot H_\phi^*$$

$$= a_r \frac{1}{2} E_\theta \cdot \frac{E_\theta^*}{\eta} \qquad [\text{From eq}^n \text{ (11)}]$$

$$S = a_r \frac{1}{2\eta} |E_\theta|^2 \ \frac{\text{Watt}}{m^2} \quad \text{———— (12)}$$

Substituting the value of E_θ from eq^n (9), the time-averaged power density or the Poynting vector for a Hertzian dipole reduces to

$$S = a_r \times \frac{1}{2\eta} \times \eta^2 \times \left| \frac{\kappa I_0 dl}{4\pi} \right|^2 \times \frac{\sin^2\theta}{r^2} \quad \text{——}$$

$$\therefore S(r,\theta,\phi) = a_r \frac{1}{2}\eta \left|\frac{k I_0 dl}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2} \quad \text{—(13)}$$

This shows that in the far-field of the antenna, the power flows radially outward from the antenna, but the power density is not the same in all directions.

We know radiation intensity

$$U(\theta,\phi) = r^2 . S(r,\theta,\phi)$$

As discussed earlier
$U = r^2 . W_{rad}$
Here,
$W_{rad} = S(r,\theta,\phi)$
$U \rightarrow$ Radiation Intensity
$W_{rad} = S =$ Power density
= Poynting vector

$\therefore$ The radiation intensity of a Hertzian dipole is

$$U(\theta,\phi) = r^2 S(r,\theta,\phi)$$

$$= \cancel{r^2} \times \frac{1}{2} \times \eta \left|\frac{K I_0 dl}{4\pi}\right|^2 \frac{\sin^2\theta}{\cancel{r^2}}$$

$$U(\theta,\phi) = \frac{1}{2}\times\eta \left|\frac{K I_0 dl}{4\pi}\right|^2 \sin^2\theta \quad \frac{\text{Watt}}{\text{Sr}} \quad \text{—(14)}$$

The total power radiated by a Hertzian dipole is given by

$$P_{rad} = \oint_\Omega U(\theta,\phi)\, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi) \sin\theta\, d\theta\, d\phi \quad \text{—(15)}$$

Putting eqn (14) in eqn (15), we have (48)

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{0}^{\pi} \frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2 \sin^2\theta . \sin\theta \, d\theta \, d\phi$$

$$= \int_{\theta=0}^{\pi}\left(\frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2 . \sin^3\theta \, d\theta\right)\times 2\pi$$

$\because \int_0^{2\pi} d\phi = 2\pi$

$$= \frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2\times 2\pi\int_0^{\pi}\sin^3\theta\, d\theta$$

$$= \frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2\times\left[\frac{\cos^3\theta}{3} - \cos\theta\right]_0^{\pi}\times 2\pi$$

$$= \frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2\times\left[\left(\frac{-1}{3}+1\right) - \left(\frac{1}{3}-1\right)\right]\times 2\pi$$

$$P_{rad} = \frac{1}{2}\times\eta\times\left|\frac{K I_o dl}{4\pi}\right|^2\times\left[\frac{4}{3}\right]\times 2\pi \quad \text{--- (16)}$$

$$= \frac{1}{2}\times\eta\times\left|\frac{\cancel{2\pi}}{\lambda} . \frac{I_o dl}{\cancel{4\pi}_{2}}\right|^2\times\frac{4}{3}\times 2\pi$$

$$= \frac{1}{\cancel{2}}\times\eta\times\left(\frac{I_o dl}{\lambda}\right)^2\times\frac{1}{\cancel{4}}\times\frac{\cancel{4}}{3}\times\cancel{2}\pi$$

$$P_{rad} = \eta\,\frac{\pi}{3}\left(\frac{I_o dl}{\lambda}\right)^2 \text{ watt} \quad \text{--- (17)}$$

$\because \int \sin^3\theta \, d\theta$

$= \int \sin\theta . \sin^2\theta \, d\theta$

$= \int \sin\theta\,(1-\cos^2\theta)\, d\theta$

Put $\cos\theta = t$

$dt = -\sin\theta\, d\theta$

$= -\int (1-t^2)\, dt$

$= -\left[t - \frac{t^3}{3}\right]$

$= \frac{t^3}{3} - t$

$= \frac{\cos^3\theta}{3} - \cos\theta$

$K = \omega\sqrt{\mu\epsilon} = \frac{\omega}{\frac{1}{\sqrt{\mu\epsilon}}} = \frac{\omega}{c}$

$= \frac{2\pi f}{c} = \frac{2\pi}{c/f} = \frac{2\pi}{\lambda}$

$=$ Phase constant

Directivity :-

The directivity of a Hertzian dipole is calculated by dividing the radiation intensity, $U(\theta, \phi)$, by the average radiation intensity $\left(U_{avg} = \frac{P_{rad}}{4\pi} \right)$

$$D = \frac{U(\theta,\phi)}{\left(\frac{P_{rad}}{4\pi}\right)} \qquad \text{[By def}^n\text{]}$$

$$D = \frac{4\pi \; U(\theta,\phi)}{P_{rad}} \quad \text{--- (18)}$$

Putting eqn (14) & (16) in eqn (18), we have

$$D = \frac{4\pi \times \cancel{\frac{1}{2} \times \eta} \cancel{\left| \frac{K I_0 dl}{4\pi} \right|^2} \cdot \sin^2\theta}{\cancel{\frac{1}{2} \times \eta} \cancel{\left| \frac{K I_0 dl}{4\pi} \right|^2} \times \frac{8\pi}{3}}$$

$$= \frac{\cancel{4\pi} \times \sin^2\theta \times 3}{\cancel{8\pi}_{2}}$$

$$\boxed{D = \frac{3}{2} \sin^2\theta} \quad \text{--- (19)}$$

The max^m value of directivity of a Hertzian dipole is $\boxed{D_0 = \frac{3}{2} = 1.5}$ — (20)

$$\boxed{(D_0)_{dB} = 10 \log 1.5 = 1.76 \text{ dB}} \quad \text{— (21)}$$

It occurs at $\underline{\theta = 90^\circ}$, $\underline{\sin\theta = 1}$.

Effective Aperture :-

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$\Rightarrow \frac{3}{2} = \frac{4\pi \times A_e}{\lambda^2}$$

$$\Rightarrow A_e = \frac{3\lambda^2}{8\pi}$$

$$\Rightarrow \boxed{A_{em} = \frac{3\lambda^2}{8\pi}} \quad \text{— (22)}$$

Radiation Resistance :-

$$P_{rad} = \frac{1}{2} |I_0|^2 \times R_r \quad \text{— (23)}$$

Also $P_{rad} = \eta \times \frac{\pi}{3} \times \left(\frac{I_0 dl}{\lambda}\right)^2$ — (24)

$\because$ Power $= I_{rms}^2 \cdot R$

$= \left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot R_r$

$= \frac{I_0^2}{2} R_r$

Eq^n (24) is from eq^n (17)

Equating (23) & (24)

$$\frac{1}{2} \cancel{|I_0|^2} \times R_r = \eta \times \frac{\pi}{3} \times \cancel{|I_0|^2} \cdot \left(\frac{dl}{\lambda}\right)^2$$

$$\Rightarrow \boxed{R_r = \frac{2\pi}{3} \times \eta \left(\frac{dl}{\lambda}\right)^2} \quad \text{— (25)}$$

Putting $\eta = 120\pi$, $R_r = \frac{2\pi}{\cancel{3}} \times \cancel{120}^{40}\pi \left(\frac{dl}{\lambda}\right)^2 = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

$$\boxed{R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \text{ ohm}} \quad -(26)$$

Ex:- 18) A Hertzian dipole of length $dl = 0.5$ meter is radiating into free space. If the dipole current is 4 A and frequency is 10 MHz, Calculate the highest power density at a distance of 2 KM from the antenna.

Ans:- The power density of a Hertzian dipole is given by eqn (13)

$$S(r,\theta,\phi) = a_r \frac{1}{2}\eta \left|\frac{K I_0 dl}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2} \quad —(1)$$

For maxm power, the dipole should be oriented along Z-axis and at $\theta = 90°$, maxm radiation occurs.

$$S(r,\theta,\phi)_{max} = \frac{1}{2}\eta \times \left|\frac{K I_0 dl}{4\pi}\right|^2 \times \frac{1}{r^2}$$

$$K = \frac{2\pi}{\lambda} = \text{Phase Constant} = \frac{2\pi}{\left(\frac{c}{f}\right)} = \frac{2\pi f}{c} = \frac{2\pi \times 1\cancel{0} \times 10^6}{3 \times 10^{\cancel{8}}} = \frac{2\pi}{30}$$

$$K = \frac{2\pi}{30} \frac{\text{rad}}{\text{meter}}.$$

$$S = \frac{1}{2} \times 120\pi \times \left|\frac{\frac{2\pi}{30} \times 4 \times 0.5}{4\pi}\right|^2 \times \frac{1}{2000^2} = 5.24 \times 10^{-8} \frac{W}{m^2}$$

$$\boxed{S_{max} = 5.24 \times 10^{-8} \frac{\text{watt}}{m^2}}$$