Noise in Frequency Modin System

The ~~[illegible]~~ Superheterodyne receiver used in AM can be used for FM. Only change is AM demodulator is replaced by FM demodulator, such as limitor - discriminator, shown below.

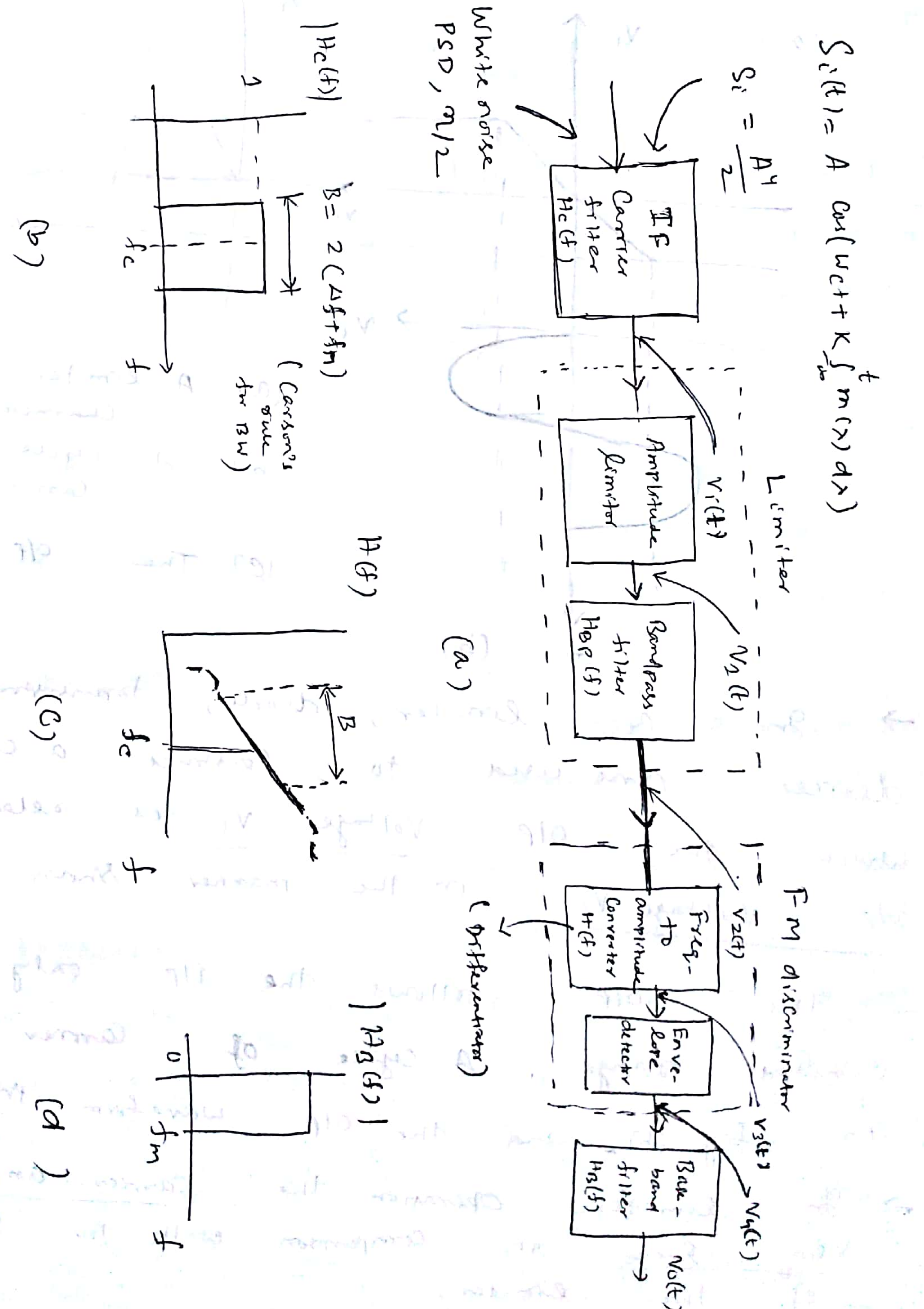
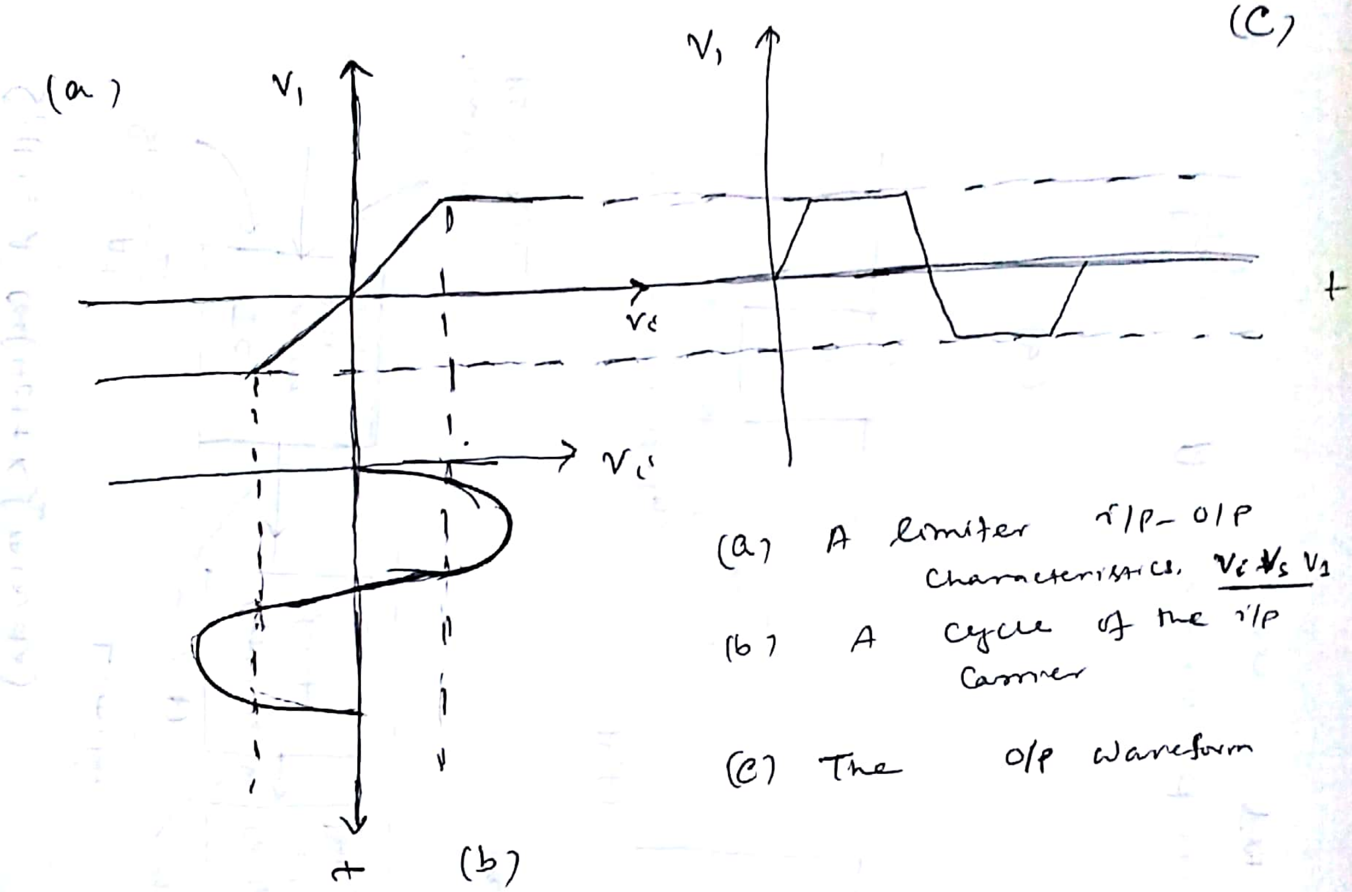


fig:- A limitor - discriminator used to demodulate an FM signal.

The limiter :-

In an FM system, the baseband signal varies only the frequency of the carrier. Hence any amplitude variation of carrier must be due to noise alone. The limiter is used to suppress such amplitude-variation noise.



(a) A limiter i/p – o/p characteristics, V_i Vs V_1

(b) A cycle of the i/p carrier

(c) The o/p waveform

→ In a limiter, diode, transistor or other devices are used to construct a circuit in which the o/p voltage V_1 is related to the i/p voltage V_i in the manner shown in fig (a).

→ The o/p follows the i/p only over a limited range. A cycle of carrier is shown in fig (b) and the o/p waveform in fig (c).

→ In limiter operation the carrier amplitude is very large in comparison with the limited range of the limiter.

As a consequence, the o/p waveform is a square wave. Thus, the o/p has a waveshape which is nearly entirely independent of modest changes in carrier amplitude.

→ The bandpass filter, following the limiter, selects the fundamental freq component of the square wave. Therefore, the filter o/p is again sinusoidal.

[Note:- Square wave = fundamental + 1st harmonic + 2nd harmonic + - - - -]

• It expressed in Fourier series.

Refer - Square wave testing - in AEC - Boylestad]

→ It has an amplitude which is very nearly independent of the input-carrier amplitude, but does, of course, have the same instantaneous freq as does the input carrier (FM).

→ In a physical circuit the limiter and filter generally form ~~as~~ an integral unit so that actually there is no point in the limiter-filter combination where square-wave waveform may be observed. [Directly we got ~~filter~~ sinusoidal after ~~filter~~ limiter - filter operation]

→ It is assumed that the discriminator is always preceded by an ideal hard limiter, i.e. that the limiter o/p is a perfect square wave, no matter how small $V_i(t)$ may be.

The discriminator :-

The discriminator also consist of 2 components parts. The first of these is a n/w which, over the range of excursion of the instantaneous freq, exhibits a transfer characteristic $H(f)$ such that $|H(f)|$ varies linearly with freq.

→ When the constant-amplitude FM signal passes through this n/w, it will appear at the o/p (i.e an envelope) with an amplitude variation which varies with time precisely as does the instantaneous freq of the carrier.

→ The baseband signal is now recovered by passing this amplitude-modulated waveform through an envelope demodulator such as diode detector.

→ The o/p to the envelope detector is freq-modulated as well as amplitude modulated, but the detector does not respond to the freq modulation.

Mathematical representation of the operation of the Limiter - Discriminator

The frequency-to-amplitude converter necessary to obtain freq demodulation have an $|H(f)|$ which varies linearly with 'ω' over only a limited range, and its slope may be +ve or -ve.

Mathematically $H(j\omega) = j\sigma\omega$ ——— (1)

where σ is a constant.

The advantage of such a selection of $H(j\omega)$ is that, the o/p of the converter $V_3(t)$ is related to the i/p $V_2(t)$ by the equation,

$$V_3(t) = \sigma \frac{d}{dt} V_2(t) \quad \text{——— (2)}$$

The eq^n (2) results from the fact that a multiplication by $j\omega$ in the freq domain is equivalent to differentiation in the time domain.

i.e

$$\sigma \frac{d}{dt} \leftrightarrow j\sigma\omega$$

Hints :-
$V_3(t) = \sigma \frac{d}{dt} V_2(t)$
$V_3(\omega) = \sigma.(j\omega).V_2(\omega)$
but
$V_3(\omega) = H(\omega).\ V_2(\omega)$
$\Rightarrow H(\omega) = \sigma j\omega$
Differentiation property of Fourier Transform

→ Now suppose that the voltage $V_2(t)$ is applied to the converter is

$$V_2(t) = A_L \cos[\omega_c t + \phi(t)] \quad \text{——— (3)}$$

Here A_L is the limited amplitude of the carrier so that A_L is fixed and independent of the i/p amplitude and $[\omega_c t + \phi(t)]$ is the instantaneous phase.

∴ Putting eq^n (3), in eq^n (2), we have

$$V_3(t) = \sigma \frac{d}{dt} A_L \cos[\omega_c t + \phi(t)]$$

$$= \sigma A_L \left\{ -\sin[\omega_c t + \phi(t)] \right\} \left[\omega_c + \frac{d}{dt}\phi(t)\right]$$

The o/p of envelope detector is, using $\alpha = \sigma A_L$

$$V_4(t) = \alpha\left[\omega_c + \frac{d}{dt}\phi(t)\right] = \alpha\omega_c + \alpha\frac{d}{dt}\phi(t) \quad (4)$$

$$= \sigma A_L \omega_c + \alpha\frac{d}{dt}\phi(t)$$

From eqn (4), it is seen that discriminator (?) o/p is proportional to the freq. of input. $\frac{d\phi}{dt}$ when passed through the baseband filter we get the message signal.

Note :-

$$\phi(t) = k \int_{-\infty}^{t} m(x)\, dx$$

$$\frac{d}{dt}\phi(t) = k\, m(t)$$

Through baseband filter we recover $m(t)$.

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Calculation of Signal to noise ratio (SNR)

Signal Power :-

Let the i/p to the IF carrier filter is

$$S_i(t) = A \cos\left[w_c t + k \int_{-\infty}^{t} m(x)\, dx \right] \quad \text{—} (5)$$

Where $m(t)$ = is the frequency-modulating baseband waveform.

→ It is assumed that the signal is embedded in additive white gaussian noise of PSD $\frac{\eta}{2}$.

→ The IF carrier filter has a BW,

$$B = 2(\Delta f + f_m) \quad [\text{Carson's rule}]$$

→ This filter passes the signal with negligible distortion and eliminates all noise outside the bandwidth B. The signal with its accompanying noise is ideally limited, discriminated and ~~after~~ after passing through the baseband filter, appears at the o/p as a signal $S_o(t)$ and a noise

Waveform $n_o(t)$

→ It can be shown that, if SNR is high, noise does not affect the o/p signal power. So on calculating o/p signal power, the noise is ignored.

→ When the i/p signal $S_i(t)$, arrives at the o/p of limiter, it is denoted by

$$S_2(t) = A_L \cos\left[w_c t + k\int_{-\alpha}^{t} m(x)\,dx\right]$$

From eqn (4),

The o/p of discriminator is,

$$S_4(t) = \alpha w_c + \alpha \frac{d}{dt}\phi(t) \quad \text{—— (6)}$$

But $\phi(t) = k\int_{-\alpha}^{t} m(x)\,dx$.

$$\therefore S_4(t) = \alpha w_c + \alpha \cdot k\, m(t) \quad \text{—— (7)}$$

The baseband filter rejects the d.c component and passes the signal component without distortion.

Thus, the o/p signal is $S_o(t) = \alpha k\, m(t)$

and the o/p signal power is,

$$\boxed{S_o = \alpha^2 k^2 \overline{m^2(t)}} \quad \text{—— (8)}$$

O/P noise power

Lets calculate the noise o/p of FM discriminator which results from the presence at the i/p of white ~~white~~ noise having PSD $\eta/2$. It can be

Can be shown that noise o/p approximately independent of m(t). For simplification, we set modulating signal m(t) = 0. The carrier & noise pass through the IF filter $H_c(\omega)$, which filters the noise. Thus the carrier and noise at the limiter i/p are

$$v_i(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t}_{\text{noise}}$$

[Note:- Refer eqn 7.79, Taub - Schilling, Quadrature component in noise [which is not in syllabus]. Quadrature means 2 components are 90° out of phase. $n_c(t)$ & $n_s(t)$ where c → Cosine, s → Sine in $n_c(t)$ & $n_s(t)$ are in quadrature [90°] phase.]

$$v_i(t) = \cancel{A\cos\omega_c t}\ [A + n_c(t)]\cos\omega_c t - n_s(t)\sin\omega_c t \quad (9)$$

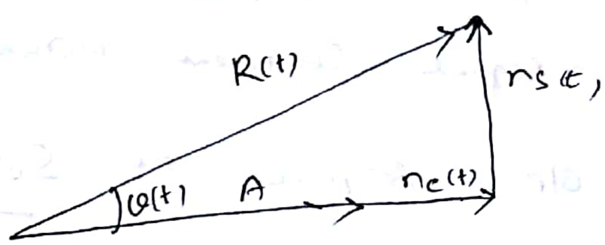


fig:- A phasor diagram of the term in eqn (9)

A phasor diagram of signal & noise is shown above. The phasor representing $n_c\cos\omega_c t$ is in phase with carrier phasor $A\cos\omega_c t$. The phasor representing $n_s(t)\sin\omega_c t$ has an amplitude $n_s(t)$ and is in phase-quadrature with other 2 terms.

The envelope $R(t)$ is given by

$$R(t) = \sqrt{[A + n_c(t)]^2 + [n_s(t)]^2} \quad \text{—— (10)}$$

Phase, $$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{A + n_c(t)}\right) \quad \text{—— (11)}$$

$\therefore$ $V_i(t)$ can be written as,

$$V_i(t) = R(t) \cos[w_c t + \theta(t)] \quad \text{—— (12)}$$

Ignoring the time-varying envelope $R(t)$, since all time variations are removed by the limiter. The o/p of the limiter - band pass filter is

$$V_2(t) = A_L \cos[w_c t + \theta(t)] \quad \text{—— (13)}$$

where A_L is determined by the limiter and is a constant.

→ Lets assume that it is operated on high i/p SNR. Then the noise power is much smaller than the carrier power. So we can assume

$$\left.\begin{aligned} |n_c(t)| &<< A \\ |n_s(t)| &<< A \end{aligned}\right\} \text{—— (14)}$$

For small θ, $\tan\theta \simeq \theta$ —— (15)

From eqn (11), $\tan\theta = \dfrac{n_s(t)}{A + n_c(t)}$

using eqn (14) & (15) $\theta(t) = \dfrac{n_s(t)}{A}$, $(\because n_c(t) << A)$ —— (16)

Now, from eqn (13)

$$V_2(t) = A_L \cos\left[w_c t + \frac{n_s(t)}{A}\right] \quad \text{—— (17)}$$

In the mathematical representation of operation of Limiter - discriminator, we have

$$V_2(t) = A_L \cos[w_c t + \phi(t)] \quad \text{—— (18)}$$

& $$V_4(t) = \alpha\left[w_c + \frac{d}{dt}\phi(t)\right] \quad \text{—— (19)}$$

Comparing (17) & (18)

$$\phi(t) = \frac{n_s(t)}{A}$$

and $$V_4(t) = \alpha\left[w_c + \frac{d}{dt}\frac{n_s(t)}{A}\right]$$

$$V_4(t) = \alpha\left[w_c + \frac{1}{A}\cdot\frac{d}{dt}n_s(t)\right] \quad \text{—— (20)}$$

Dropping the d.c component, $V_4(t) = \frac{\alpha}{A}\frac{d}{dt}n_s(t)$ —— (21)

The spectral density of $n_s(t)$ is $\underline{\eta}$ over the freq range $|f| < \frac{B}{2}$.

[See: Ex - 7.5, ~~417 page~~ 417 page, Taub-Schilling].
$G_{n_c}(f) = G_{n_s}(f) = \eta$

→ The differentiation is equivalent to passing $n_s(t)$ through a n/w whose transfer function is $\underline{H(jw) = jw}$.

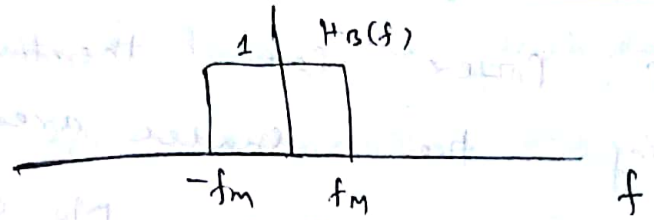
using time differentiation property of Fourier Transform

From eqn (21), $V_4(w) = \frac{\alpha}{A}\cdot(jw)\cdot n_s(w)$

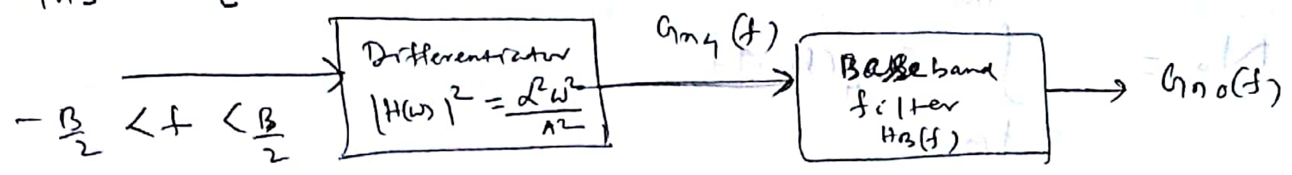
$$V_4(w) = H(jw)\cdot n_s(w).$$

$$\therefore H(jw) = \frac{\alpha jw}{A}$$

$$|H(jw)|^2 = \frac{\alpha^2 w^2}{A^2} \quad \text{—— (22)}$$



$G_{ns}(f) = \eta$



(a.)

fig :- (a) Indicating the operations performed by the discriminator and baseband filter on the noise o/p of the limiter.

∴ The spectral density of $n_4(t)$ is $G_{n4}(f)$ given by, ——— (23)

$$G_{n4}(f) = \frac{\alpha^2 \omega^2}{A^2} \cdot \eta \quad , \quad |f| < \frac{B}{2} \, ,$$

This spectral density is plotted below.

Hints :-
$x^2 = 4ay$
(Parabola)
$\Rightarrow y = \frac{x^2}{4a}$

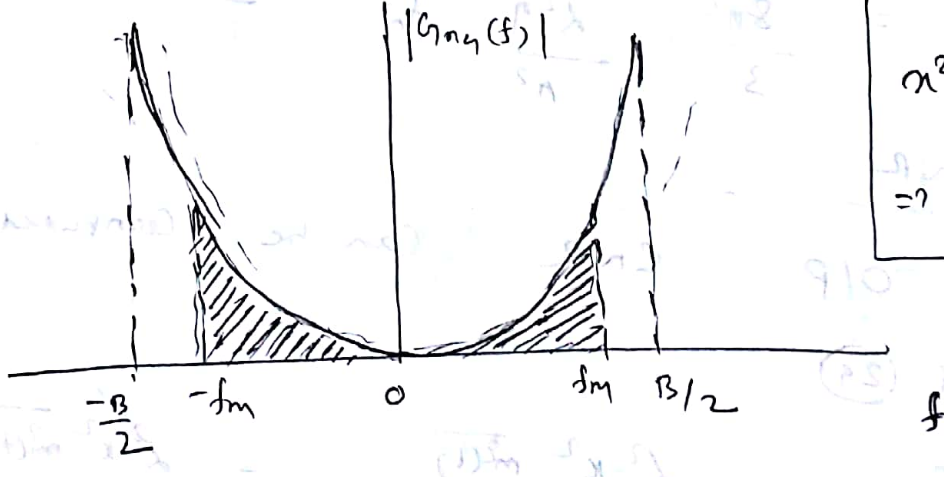


fig :- (b) Variation of PSD, ~~[illegible]~~ at the o/p of FM demodulator, with frequency.

Since the baseband filter passes frequencies just upto fm, only the shaded area in fig (b) contributes to the output noise power.

This O/P power can therefore be calculated by computing the shaded area.

The O/P noise power N_0 is

$$N_0 = \int_{-f_m}^{f_m} G_{n_4}(f)\, df$$

$$= \int_{-f_m}^{f_m} \frac{\alpha^2 \omega^2}{A^2} \cdot \eta \, df$$

$$= \frac{\alpha^2 \eta}{A^2} \cdot \int_{-f_m}^{f_m} 4\pi^2 f^2 df \qquad (\because \omega = 2\pi f)$$

(23-A)

$$= \frac{\alpha^2 \eta}{A^2} \cdot 4\pi^2 \times \left.\frac{f^3}{3}\right]_{-f_m}^{f_m}$$

$$= \frac{4\pi^2}{3} \cdot \frac{\alpha^2 \eta}{A^2} \cdot 2 f_m^3$$

$$N_0 = \frac{8\pi^2}{3} \cdot \frac{\alpha^2 \eta}{A^2} \cdot f_m^3 \qquad \text{—— (24)}$$

O/P SNR

The O/P SNR can be computed using eqⁿ (8) & (24)

$$\frac{S_0}{N_0} = \frac{\alpha^2 K^2 \overline{m^2(t)}}{\frac{8\pi^2}{3} \cdot \frac{\alpha^2 \eta}{A^2} \cdot f_m^3} = \frac{\cancel{\alpha^2} K^2 \overline{m^2(t)} \cdot 3A^2}{8\pi^2 \cancel{\alpha^2} \eta f_m^3}$$

$$\frac{S_0}{N_0} = \frac{3}{4\pi^2} \cdot \frac{K^2 \overline{m^2(t)}}{f_m^2} \cdot \frac{A^2/2}{\eta f_m} \qquad \text{—— (25)}$$

Let's consider that the modulating signal $m(t)$ is sinusoidal and produces freq deviation Δf, then eq^n (5) can be written as

$$S_i(t) = A \cos\left[w_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right] \quad \text{—(26)}$$

where f_m is modulating freq.

Comparing eq^n (5) & (26), we have.

$$K \int_{-\infty}^{t} m(\lambda)\, d\lambda = \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

Differentiating both the sides,

$$K\, m(t) = \frac{\Delta f}{\cancel{f_m}} \cdot (\cos 2\pi f_m t) \cdot (2\pi \cancel{f_m})$$

$$K\, m(t) = 2\pi \Delta f \cos 2\pi f_m t \quad \text{—(27)}$$

Hence,

$$K^2\, \overline{m^2(t)} = \frac{4\pi^2 (\Delta f)^2}{2} = 2\pi^2 (\Delta f)^2 \quad \text{—(28)}$$

~~Substituting~~ Substituting eq^n (28), in eq^n (25),

We have

$$\frac{S_o}{N_o} = \frac{3}{\cancel{4\pi^2}/2} \times \frac{\cancel{2\pi^2} (\Delta f)^2}{f_m^2} \cdot \frac{A^2/2}{\eta f_m}$$

$$\boxed{\frac{S_o}{N_o} = \frac{3}{2} \cdot \beta^2 \cdot \frac{S_i}{N_M}}$$

$\because \frac{\Delta f}{f_m} = \beta$ = modulation index,

$S_i = A^2/2$

$N_M = \eta f_m$

where β = Mod^n index

S_i = input-signal power

N_M = noise power at the i/p in the baseband bandwidth f_m

∴ Figure of merit of FM,

$$\gamma_{FM} = \frac{S_o/N_o}{S_i/N_m} = \frac{3}{2}\beta^2$$

$$\boxed{\gamma_{FM} = \frac{3}{2}\beta^2}$$

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Comparison of FM & AM in noisy environment :-

Lets consider the performance of FM & DSB-C (Conventional AM)

Figure of merit
$$\gamma_{FM} = \frac{3}{2}\beta^2 \quad \text{——} (1)$$

$$\gamma_{AM} = \frac{m^2}{2+m^2}, \quad \text{for sinusoidal amplitude modulation.}$$

$$= \frac{1}{2+1}, \quad \text{for } 100\% \text{ modulation}$$

$$\gamma_{AM} = \frac{1}{3} \quad \text{——} (2)$$

$$\therefore \frac{\gamma_{FM}}{\gamma_{AM}} = \frac{3}{2} \times \beta^2 \times \frac{3}{1} = \frac{9}{2}\beta^2 \quad \text{——} (3)$$

In both the cases, it is assumed equal

i/p noise-power density, $\eta/2$, equal baseband bandwidth

f_m, equal i/p-signal power S_i.

For $\frac{9}{2}\beta^2 = 1$

$$\Rightarrow \beta^2 = \frac{2}{9}$$

$$\Rightarrow \beta = \sqrt{\frac{2}{9}} \simeq 0.5$$

So for $\beta = 0.5$,

$$\gamma_{FM} = \gamma_{AM},$$

As β increases, ($\gamma_{FM} > \gamma_{AM}$) FM offers better SNR over AM. But this improvement is achieved at the expense of requiring greater BW.

By Carson's rule

$$B_{FM} = 2(\beta+1) f_m$$

$\because$ BW = $2(\Delta f + f_m)$

$2 f_m \left(\frac{\Delta f}{f_m} + 1\right)$

$2(\beta+1) f_m$

If β is large,

$$B_{FM} \simeq 2\beta f_m, \quad \text{—— (4)}$$

Bandwidth of AM, $B_{AM} = 2 f_m$. —— (5)

$$\frac{B_{FM}}{B_{AM}} = \beta \quad \text{—— (6)}$$

Substituting eqn (6), in eqn (3),

$$\frac{\gamma_{FM}}{\gamma_{AM}} = \frac{9}{2} \cdot \left(\frac{B_{FM}}{B_{AM}}\right)^2$$

So each increase in BW by a factor 2, increases $\frac{\gamma_{FM}}{\gamma_{AM}}$ by a factor 4.

→ Note:- For wideband FM 'β' is more than that of narrowband FM.

$$\gamma_{FM} = \frac{3}{2} \beta^2$$

So wideband FM has improved SNR than narrowband FM, [But BW requirement in wideband is more than that of narrow band FM]

→ * In all the derivation done, it is assumed noise power is small, comparison to signal power. When BW increases, noise power is not relatively small, the performance of FM system degrades ~~rap~~ rapidly i.e the system exhibits a threshold. It can be shown that when i/p noise power is not small in comparison with i/p - signal power, the system performance may be improved by restricting the BW, by reducing the mod^n index.

Ex - 1 - a) Find output SNR of an FM limiter-discriminator demodulator when i/p signal strength = 0.5 Watt, max^m freq deviation = 60 kHz, baseband signal cut-off freq = 15 kHz, received white Gaussian noise power spectral density = 10^{-9} Watt/Hz, and avg power of modulating signal = 0.1 watt.

(b) Find required transmitted power for above if channel has 20 dB loss and required o/p SNR greater than 40 dB.

Ans : Given, $S_i = 0.5$ watt, $\Delta f = 60$ kHz

$f_m = 15\,KHz$, $\eta = 10^{-9}\ \frac{Watt}{Hz}$. → 400 Refer eqn (23), page 394

$\overline{m^2(t)} = 0.1$ watt

$\eta_{ns}(f) = \eta$

(a)

$$\frac{S_o}{N_o} = \frac{3}{4\pi^2} \cdot \frac{K^2\,\overline{m^2(t)}}{f_m^2} \cdot \frac{A^2/2}{\eta f_m}$$

[Note:- for sinusoidal, $\frac{S_o}{N_o} = \frac{3}{2}\beta^2 \frac{S_i}{N_m}$]

$\Delta f = K_f A_m \simeq K_f$ since message $|m(t)| \le 1$.

$\therefore$ ~~Δf~~ $K = \Delta f$

$$\frac{S_o}{N_o} = \frac{3}{4\pi^2} \times \frac{(\Delta f)^2}{f_m^2} \times \overline{m^2(t)} \times \frac{S_i}{\eta f_m}$$ | $S_i = A^2/2$

$$= \frac{3}{4\pi^2} \times \left(\frac{60\times10^3}{15\times10^3}\right)^2 \times \frac{0.1\times 0.5}{10^{-9}\times 15\times10^3}$$

$\frac{S_o}{N_o} = 4052.8 = 36.08$ dB

(b) Required o/p SNR > 40 dB i.e. SNR $> 10^4$

For ~~Son~~ $\frac{S_o}{N_o} = 4052.8$, required $S_i = 0.5$ watt.

For $S_o/N_o = 1$, required $S_i = \frac{0.5}{4052.8}$ watt

For $\frac{S_o}{N_o} = 10^4$, required $S_i = \frac{0.5\times10^4}{4052.8}$

$S_i = 1.2337$ Watt

Since there is $20\,dB = 100$ loss in the channel, The transmitter power should be 100 times more than receiver power.

$\therefore$ Transmitter power should be $= 1.2337 \times 100$

$= 123.37$ watt.

(Ans)

Ex:-

2) Consider Limiter-Discriminator based FM demodulation with a sinusoidal modulation signal with maxm - freq deviation 60 KHz and baseband filter width 15KHz. Find the figure of merit. (a) Compare performance with an AM system if the I/p ~~power,~~ signal power, noise spectral density & baseband bandwidth are same. (b) How does the value change if BW of incoming FM is double that of AM signal compared to (a).

Ans:- Given, $\Delta f = 60KHz$, $f_m = 15KHz$.

Figure of merit, $\gamma_{FM} = \frac{3}{2} \cdot \beta^2$

$$= \frac{3}{2} \cdot \left(\frac{\Delta f}{f_m}\right)^2$$

$$= \frac{3}{2} \times \left(\frac{60KHz}{15KHz}\right)^2$$

$$= \frac{3}{2} \times 16$$

$$\boxed{\gamma_{FM} = 24}$$

(a) $$\frac{\gamma_{FM}}{\gamma_{AM}} = \frac{9}{2}\beta^2$$

$$\beta = \frac{\Delta f}{f_m} = \frac{60}{15} = 4$$

$$\gamma_{FM} = \frac{9}{2} \times 4^2 \times \gamma_{AM} = 72\,\gamma_{AM}.$$

$$\boxed{\gamma_{FM} = 72\,\gamma_{AM}} \Rightarrow \frac{\gamma_{FM}}{\gamma_{AM}} = 72 \quad -(1)$$

(b) If BW of FM is doubled,

We know,

$$\frac{\gamma_{FM}}{\gamma_{AM}} = \frac{9}{2} \times \left(\frac{B_{FM}}{B_{AM}}\right)^2 \quad -(2)$$

If BW of FM is doubled, as compared to (a),

$$\left(\frac{\gamma_{FM}}{\gamma_{AM}}\right)_{new} = \frac{9}{2} \times \frac{(2B_{FM})^2}{B_{AM}^2} \quad -(3).$$

Dividing eqn (2) by eqn (3).

$$\frac{(\gamma_{FM}/\gamma_{AM})}{(\gamma_{FM}/\gamma_{AM})_{new}} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{\gamma_{FM}}{\gamma_{AM}}\right)_{new} = 4 \times 72 = 288.$$

$$\Rightarrow \boxed{\left(\frac{\gamma_{FM}}{\gamma_{AM}}\right)_{new} = 288}$$

Pre-emphasis & De-emphasis :-

[For Short Note or Theoratical type question - write the note given below - written from Sanjay Sharma]

[For Mathematical analysis & Problems - Refer the note after this - written from Taub schilling]

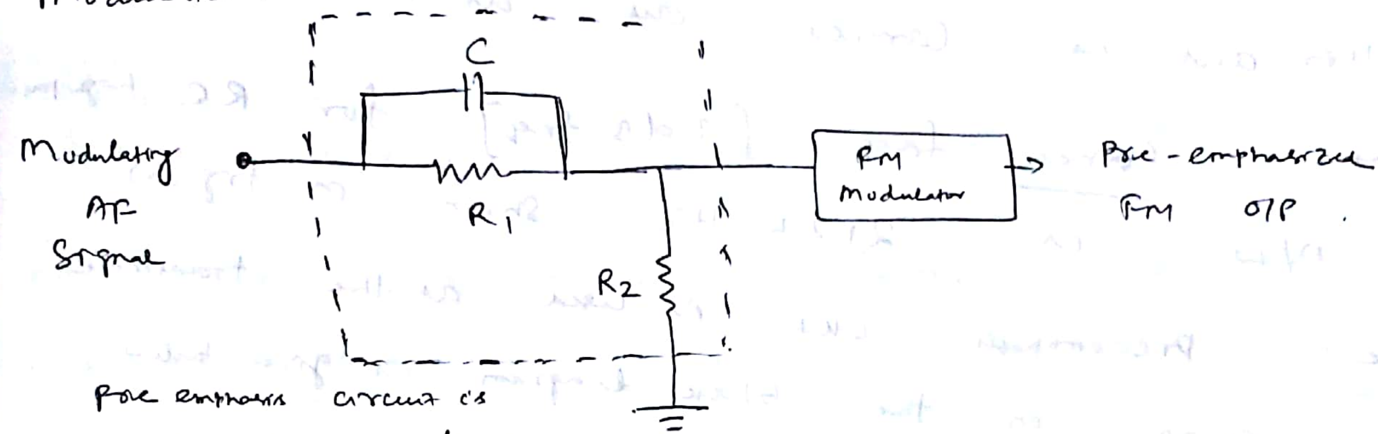
→ The noise power spectral density at the demodulator o/p increases parabolically with freq. This means this effect of noise increases with increase in freq. The SNR therefore becomes poor at high frequencies and the quality of FM reception degrades. The high freq components of the message are badly affected by the noise. This problem can be solved by using circuits called pre-emphasis & de-emphasis.

Pre-emphasis :-

The noise has a greater effect on the higher modulating frequencies. This effect can be reduced by increasing the value of modulation index (m_f or β) for higher modulating freq (f_m). This can be done by increasing the deviation (Δf) and Δf can be increased [$\because$ ~~$\Delta f \propto$~~ $m_f\uparrow = \frac{\Delta f\uparrow}{f_m}$] by increasing the amplitude of modulating signal at higher modulating frequencies [$\because$ $\Delta f\uparrow = K_f A_m\uparrow$)

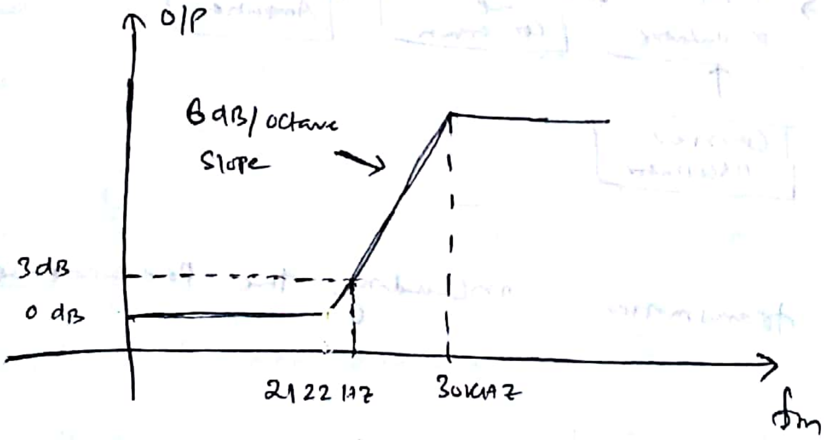
→ Thus if we boost the amplitude of higher freq modulating signals artificially then

It will be possible to improve the noise immunity at higher modulating frequencies. The artificial boosting of higher modulating frequencies is known as pre-emphasis. Boosting of higher frequency modulating signal is achieved by using the pre-emphasis circuit shown in fig below. The modulating AF (Audio Freq) signal is passed through a high pass RC filter, before applying it to the FM modulator.



Pre emphasis circuit is basically a high pass filter

(a) Typical pre-emphasis circuit.



(b) Pre-emphasis characteristics

→ As f_m increases, reactance of C decreases $\left[X_C = \frac{1}{2\pi f_C}\right]$ and modulating voltage applied to FM modulator goes on increasing. The freq response characteristics of the RC high pass n/w is shown in fig (b)

The boosting is done according to this pre-arranged curve. The amount of pre-emphasis in US FM transmission & sound transmission in T.V has been standardized at 75 μsec.

$$\therefore \quad RC = 75\ \mu sec \qquad [\text{Time Const.}]$$

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 75 \times 10^{-6}} = 2{,}122\ Hz.$$

The pre-emphasis ckt is basically a high pass filter and is carried out at the transmitter.

The corner freq [3 dB freq] for RC high pass of n/w is 2,122 Hz shown in fig (b).

The pre-emphasis ckt is used at the transmitter as shown in the block diagram is given below

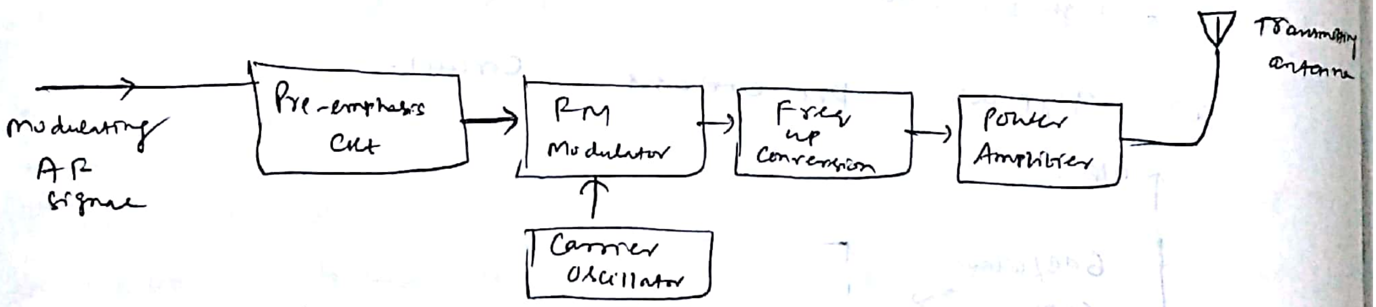
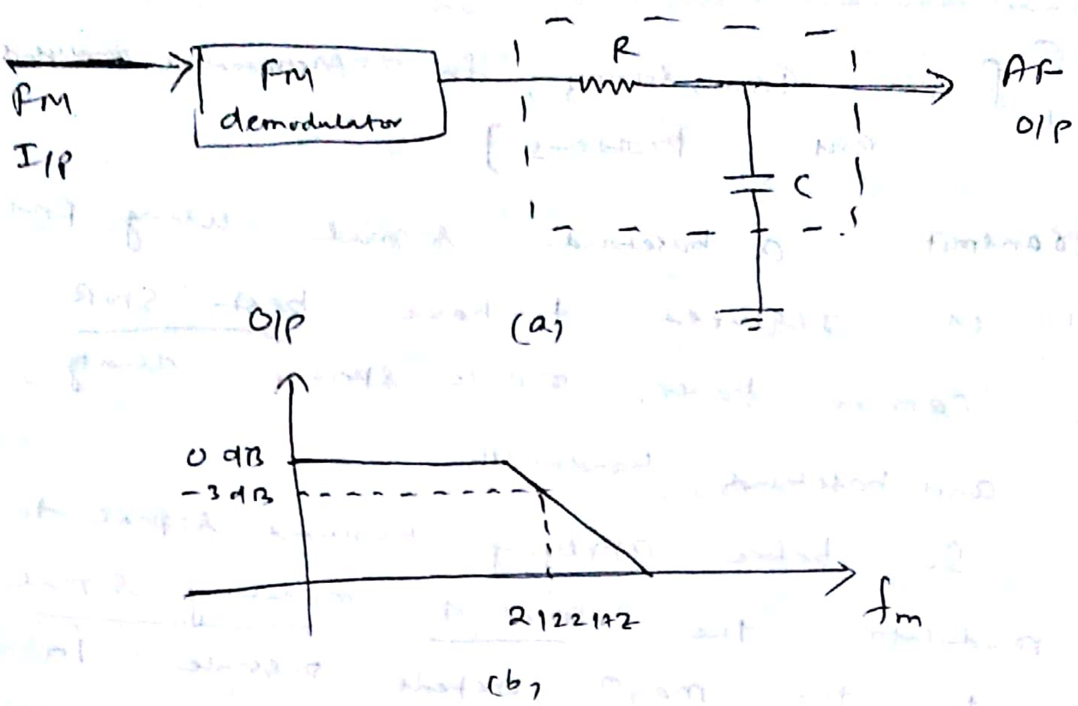


fig:- FM transmitter including the pre-emphasis

De-emphasis :-

The artificial boosting given to the higher modulating in the pre-emphasis is nullified or compensated at the receiver by a process known as De-emphasis. The high freq signals are brought back to their original amplitude by de-emphasis ckt.

De-emphasis ckt



fig! - Typical de-emphasis ckt & its characteristics.

→ The 75 μsec de-emphasis ckt is standard and is shown in fig above. It shows that it is a low pass filter. 75 μsec de-emphasis corresponds to a freq response curve that is 3 dB down at a freq whose RC time constant is 75 μsec.

i.e $f = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 75 \times 10^{-6}} = 2{,}122$ Hz.

The demodulated FM is applied to the De-emphasis circuit with increase in fm, reactance of C goes on decreasing and o/p of de-emphasis circuit will also reduce, so that artificial boosting is nullified and we get back the original baseband signal.

Pre emphasis & De emphasis and SNR improvement

[From Taub, Schilling, for mathematical analysis and problems]

→ To transmit a baseband signal using FM modulation, it is required to have best SNR for a given carrier power, noise spectral density, IF bandwidth and baseband bandwidth.

So before applying baseband signal to the FM modulator, the level of modulating signal is raised to the max^m extent possible in order to modulate the carrier as vigorously as possible.

An audio signal usually has the characteristic that its PSD is relatively high in the low-freq range & falls off rapidly at higher frequencies. Considering the characteristics of audio signal, in order to improve the performance of an FM system, the arrangement is shown below.

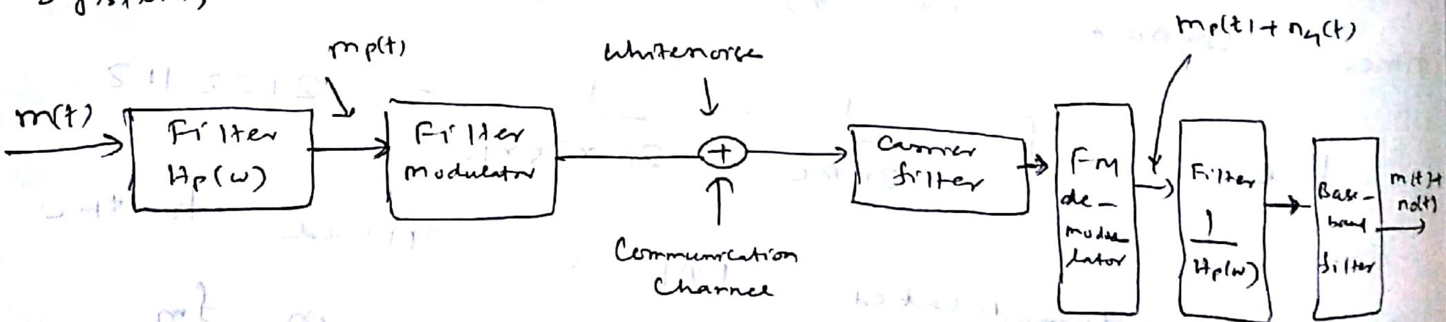


fig 1:- Pre emphasis & de emphasis in an FM System

→ At the transmitting end, the baseband signal m(t) is not applied directly to the FM modulator, but it is first passed through a filter of transfer characteristic $H_p(\omega)$, so that the modulating

Signal is $m_p(t)$.

The modulated carrier is transmitted across a communication channel, during which process, as usual, noise is added to the signal. The receiver is a conventional discriminator except that a filter has been introduced before the baseband filter. The transfer characteristic of this filter is reciprocal of the characteristics of the transmitter filter i.e $\frac{1}{H_p(\omega)}$.

Any modification introduced into the baseband signal by the first filter, prior to modn, is presently undone by the second filter which follows the discriminator. Hence the o/p signal at the receiver is exactly same as it would be if the filters had been omitted entirely.

The selection of the transfer characteristics $H_p(\omega)$ is based on following considerations. ~~The~~ At the o/p of demodulator the spectral density of the noise, given by $G_{n4}(f)$, increases with the square of the frequency.

[As derived earlier, $G_{n4}(f) = \frac{\alpha^2 \omega^2}{A^2} \eta$] Page-394 of the note.

Hence the receiver filter will be most effective in suppressing noise if the response of the filter falls off with increasing freq. In such a case, transmitter filter must exhibit a rising response with increasing freq.

Since higher freq components are more affected by noise, so the transmitter filter is designed such a way that it serves only to increase the spectral density of the higher-freq components of the signal m(t). Such a filter must necessarily increase the power in the modulating signal.

Thus the premodulation filtering in the transmitter, to raise the PSD of the baseband signal in its upper-freq range, is called preemphasis (or predistortion). The filtering at the receiver to undo the signal preemphasis and to suppress noise is called deemphasis.

SNR improvement using Pre-emphasis :-

→ BW occupied by the o/p of a FM demodulator is fixed if the normalized power of modulating signal is kept fixed.

So, it is required that,

normalized power of baseband signal m(t) = Normalized Power of Preemphasized signal $m_p(t)$ —(1)

~~$G_m(f)$ = PSD of m(t)~~,

$$m(t) \longrightarrow \boxed{\begin{matrix}\text{filter}\\ H_p(f)\end{matrix}} \longrightarrow m_p(t)$$

We know that O/P PSD, related to input PSD, as $G_{no}(f) = |H(f)|^2 G_{ni}(f)$

[In Mathematical representation of noise chapter]

If PSD of $m(t) = G_m(f)$, then
PSD of $m_p(t) = |H_p(f)|^2 \, G_m(f)$

From eqn ①,

$$P_m = \int_{-f_m}^{f_m} G_m(f)\, df = \int_{-f_m}^{f_m} |H_p(f)|^2 \, G_m(f)\, df \quad \text{--- ②}$$

where f_m is the maxm freq of the modulating signal. [$\because$ Power $= \int$ PSD.df]

In the absence of deemphasis, o/p noise power

$$N_o = \frac{\alpha^2 \eta}{A^2} \cdot 4\pi^2 \int_{-f_m}^{f_m} f^2\, df$$

[eqn 23-A Page 395 of note] is ③

with deemphasis filter, o/p noise

$$N_{oD} = \frac{\alpha^2 \cdot \eta}{A^2} \cdot 4\pi^2 \int_{-f_m}^{f_m} f^2 \cdot \left|\frac{1}{H_p(f)}\right|^2 df \quad \text{--- ④}$$

Dividing eqn ③ by eqn ④, we have

$$\frac{N_o}{N_D} = \frac{\int_{-f_m}^{f_m} f^2\, df}{\int_{-f_m}^{f_m} \left[f^2/|H_p(f)|^2\right] df} = \frac{\left.\frac{f_m^3}{3}\right]_{-f_m}^{f_m}}{2 \times \int_0^{f_m} f^2\, df/|H_p(f)|^2}$$

$$\Rightarrow R = \frac{\frac{1}{3} \times \cancel{2} f_m^3}{\cancel{2} \times \int_0^{f_m} f^2\, df/|H_p(f)|^2} = \frac{f_m^3/3}{\int_0^{f_m} f^2\, df/|H_p(f)|^2} \quad \text{--- ⑤}$$

where R = ratio of N_o & N_D

'R' is the ratio by which pre emphasis - de-emphasis improves SNR. [The value of 'R' for FM broadcasting will be discussed in next section]

Pre emphasis & De emphasis in Commercial FM broadcasting

We know that, the spectral density of the noise at the O/P of an FM demodulator increases with square of the freq. Hence a deemphasis n/w at the receiver will be most effective in suppressing noise if its response falls with increasing freq.

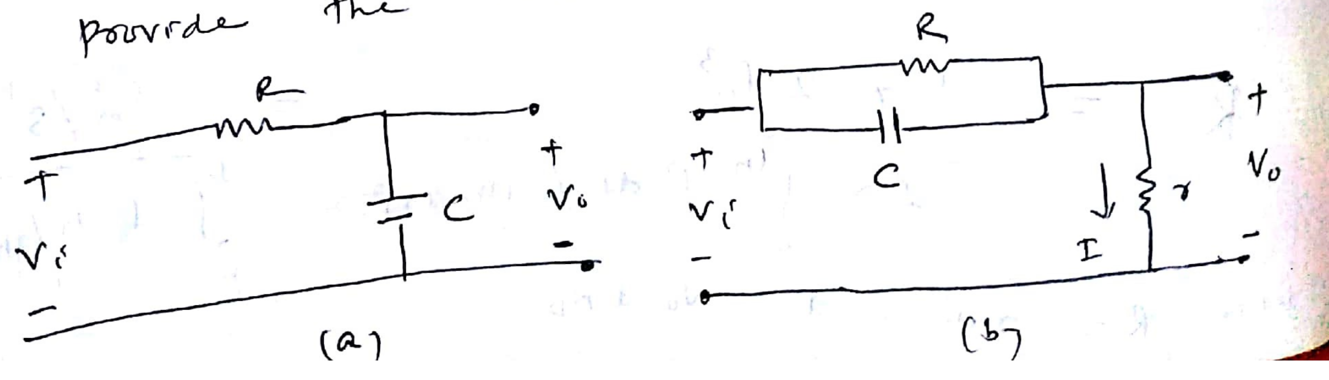
In commercial FM the deemphasis is performed by the simple low-pass resistance - capacitance n/w fig (a). This n/w has a transfer function $H_d(f)$ given by

$$H_d(f) = \frac{1}{1 + j\frac{f}{f_1}} \quad \text{--- (1)}$$

[Refer Boylestan for derivation]

where $f_1 = \frac{1}{2\pi RC}$

At the transmitter we require an inverse n/w. A simple n/w which may be adjusted to provide the required response is shown in fig b.



(a) De emphasis n/w (b) Pre emphasis n/w used in Commercial radio.

→ Lets assume $r << R$, and r is also small in comparison with reactance of capacitor C.

From fig (b), current $I(f)$ can be calculated as

$$I(f) = \frac{V_i}{Z_i}$$

$$= V_i \times \frac{1}{Z_i}$$

$$= V_i \times \left[\frac{1}{R} + \frac{1}{X_c}\right]$$

$$= V_i \times \left[\frac{1}{R} + \frac{1}{1/j\omega C}\right]$$

$Z_i = R // X_c + r$

r is small

$Z_i \simeq R // X_c$

$\frac{1}{Z_i} = \frac{1}{R} + \frac{1}{X_c}$

$$I(f) = V_i(f)\left[\frac{1}{R} + j\omega C\right] \quad \text{——— (2)}$$

The o/p voltage,

$$V_o(f) = I(f) \cdot r$$

$$V_o(f) = V_i(f) \cdot \left[\frac{1}{R} + j\omega C\right] \cdot r \quad \left[\text{using eq}^n \text{ (2)}\right]$$

$$\Rightarrow \frac{V_o(f)}{V_i(f)} = r\left[\frac{1}{R} + j\omega C\right]$$

$$\Rightarrow H_p(f) = r\left[\frac{1+j\omega CR}{R}\right]$$

$$\Rightarrow H_p(f) = \frac{r}{R}(1 + j\omega CR)$$

$$\Rightarrow H_p(f) = \frac{r}{R}(1 + j\,2\pi f CR)$$

$$\Rightarrow H_p(f) = \frac{r}{R}\left(1 + j\frac{f}{\frac{1}{2\pi RC}}\right)$$

$$\Rightarrow H_p(f) = \frac{r}{R}\left(1 + j\frac{f}{f_1}\right) \quad \text{——— (3)}$$

where $f_1 = \frac{1}{2\pi RC}$

Hence $H_p(f)$ has a freq dependence inverse to $H_d(f)$ as required, in order that no net distortion be introduced into the signal.

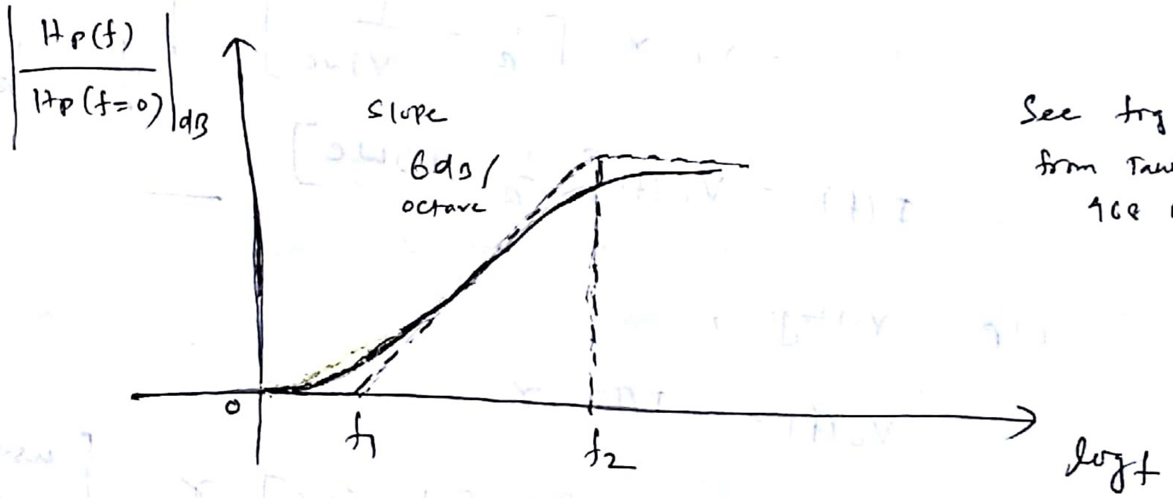
From eqn ① & ③, we have

$$H_p(f) \cdot H_d(f) = \frac{1}{\cancel{\left(1+j\frac{f}{f_1}\right)}} \cdot \frac{r}{R}\cancel{\left(1+j\frac{f}{f_1}\right)}$$

$$H_p(f) \cdot H_d(f) = \frac{r}{R} = \text{Constant}$$

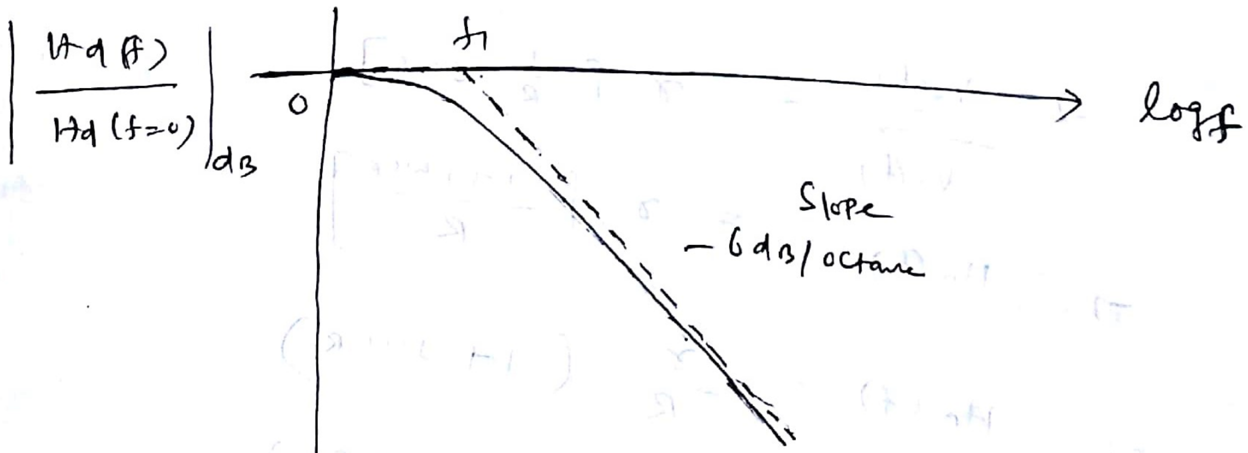
The normalized logarithmic plots of H_p & H_d are shown below.

Normalized



See fig:- 9.7 from Taub, Schilling 408 page

(a)



(b)

fig:- Normalized logarithmic plots of freq characteristics of (a) preemphasis n/w (b) deemphasis n/w

The improvement in SNR which results from pre-emphasis, depends on the frequency dependence of, PSD of baseband signal. Let us assume that the spectral density of a typical audio signal, say music, may reasonably represented as having a freq dependence given by

$$G_m(f) = \begin{cases} G_0 \dfrac{1}{1+\left(\frac{f}{f_1}\right)^2}, & |f| \le f_M \\ 0, & \text{elsewhere} \end{cases} \quad \text{———} (4)$$

where G_0 is the spectral density at low frequencies, while f' is the freq at which $G(f)$ has fallen by 3dB from its low freq value. It is assumed that pre emphasis n/w is adjusted such that

$$f_1 = f'.$$

The baseband signal to be preemphasized is transmitted through a n/w of fig (a), and also through an adjustable gain amplifier. Hence, the baseband signal m(t) passes through a n/w whose transfer function is

$$H_P(f) = K\left(1+j\frac{f}{f_1}\right) \quad \text{———} (5)$$

where K is the product of amplifier gain & the ratio $\frac{r}{R}$ (See eqn (3)).

The Coefficient K is adjusted so that the constraint of eqn (2), - Page - 410, is satisfied.

$$\left[P_M = \int_{-f_M}^{f_M} G_m(f)\, df = \int_{-f_M}^{f_M} |H_P(f)|^2 G_m(f)\, df \right] \text{——} (6)$$

$$\therefore \int_{-f_M}^{f_M} G_m(f)\, df = \int_{-f_M}^{f_M} G_0 \frac{1}{1+\left(\frac{f}{f_1}\right)^2}\, df \quad \text{———} (7)$$

using $f' = f_1$

$$\int_{-f_m}^{f_m} |H_p(f)|^2 \, G_m(f)\, df = \int_{-f_m}^{f_m} K^2 \cancel{\left[1+\left(\frac{f}{f_1}\right)^2\right]} \cdot G_0 \times \frac{1}{\cancel{\left[1+\left(\frac{f}{f_1}\right)^2\right]}}\, df$$

$\because H_p(f) = K\left(1+j\frac{f}{f_1}\right)$

$|H_p(f)|^2 = K^2\left[1+\left(\frac{f}{f_1}\right)^2\right]$

$$\int_{-f_m}^{f_m} |H_p(f)|^2 \, G_m(f)\, df = \int_{-f_m}^{f_m} K^2 \, G_0 \, df \qquad \text{——— (8)}$$

Equating eqn (7) & (8), by eqn (6)

$$\int_{-f_m}^{f_m} \frac{\cancel{G_0}\, df}{1+\left(\frac{f}{f_1}\right)^2} = \int_{-f_m}^{f_m} K^2 \cancel{G_0}\, df$$

Put $\frac{f}{f_1} = x$, $df = f_1\, dx$

$f \to -f_m$, $x \to -\frac{f_m}{f_1}$

$f \to f_m$, $x \to \frac{f_m}{f_1}$

$$\Rightarrow \int_{-f_m/f_1}^{+f_m/f_1} \frac{f_1\, dx}{1+x^2} = K^2 \cdot [f]_{-f_m}^{f_m}$$

$$\Rightarrow f_1 \times \tan^{-1}(x)\Big]_{-\frac{f_m}{f_1}}^{+f_m/f_1} = K^2\,[2 f_m]$$

$$\Rightarrow f_1 \times \cancel{2} \tan^{-1}\left(\frac{f_m}{f_1}\right) = K^2\, \cancel{2}\,[f_m]$$

$$\Rightarrow K^2 = \frac{f_1}{f_m} \tan^{-1}\left(\frac{f_m}{f_1}\right) \qquad \text{——— (9)}$$

Substituting eqn (9), in eqn (5)

$$H_p(f) = \sqrt{\frac{f_1}{f_m}\tan^{-1}\left(\frac{f_m}{f_1}\right)} \cdot \left(1+j\frac{f}{f_1}\right) \quad \text{— (10)}$$

We know,

$$R = \frac{N_o}{N_D} = \frac{f_m^3/3}{\int_0^{f_m} \frac{f^2\,df}{|H_p(f)|^2}} \quad \text{———— (11)}$$

[see eq^n (5) in page 410]

∴ using eq^n (10), in eq^n (11), we have

$$R = \frac{f_m^3/3}{\int_0^{f_m} \frac{f^2\,df}{\frac{f_1}{f_m}\tan^{-1}\left(\frac{f_m}{f_1}\right) \times \left[1+\left(\frac{f}{f_1}\right)^2\right]}} \quad \text{———— (12)}$$

Consider Denominator $= \int_0^{f_m} \frac{f^2\,df}{\frac{f_1}{f_m}\tan^{-1}\left(\frac{f_m}{f_1}\right).\left[1+\left(\frac{f}{f_1}\right)^2\right]}$

$$= \frac{f_m}{f_1} \times \frac{1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \int_0^{f_m} \frac{f^2}{1+\left(\frac{f}{f_1}\right)^2}\,df$$

$$= \frac{f_m}{\cancel{f_1}\tan^{-1}\left(\frac{f_m}{f_1}\right)} \cdot \int_0^{f_m} \frac{f^2 \times f_1^{\cancel{2}}}{f_1^2+f^2}\,df$$

$$= \frac{f_m f_1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \cdot \int_0^{f_m} \frac{f^2}{f_1^2+f^2}\,df$$

$$= \frac{f_m f_1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \cdot \int_0^{f_m} \frac{f^2+f_1^2-f_1^2}{f_1^2+f^2}\,df$$

$$= \frac{f_m f_1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \int_0^{f_m} \left[1 - \frac{f_1^2}{f_1^2+f^2}\right] df$$

$$\therefore \quad = \frac{f_m f_1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \cdot \left[f - f_1^2 \times \frac{1}{f_1} \tan^{-1}\left(\frac{f}{f_1}\right) \right]_0^{f_m}$$

$$\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \right.$$

Denominator.

$$= \frac{f_m f_1}{\tan^{-1}\left(\frac{f_m}{f_1}\right)} \left[f_m - f_1 \tan^{-1}\left(\frac{f_m}{f_1}\right) \right]$$

$$R = \frac{f_m^3/3}{\text{denominator}}$$

$$= \frac{f_m^3}{3} \times \frac{\tan^{-1}\left(\frac{f_m}{f_1}\right)}{f_m f_1 \left[f_m - f_1 \tan^{-1}\left(\frac{f_m}{f_1}\right) \right]}$$

$$R = \frac{\cancel{f_m^3}^{\,f_m} \times \tan^{-1}\left(\frac{f_m}{f_1}\right)}{3 \times \cancel{f_m} \times f_1 \times f_m \left[1 - \frac{f_1}{f_m} \tan^{-1}\left(\frac{f_m}{f_1}\right) \right]}$$

$$R = \frac{\tan^{-1}(f_m/f_1)}{3(f_1/f_m)\left[1 - (f_1/f_m)\tan^{-1}(f_m/f_1)\right]} \quad \text{--- (13)}$$

If the power in the baseband signal is principally confined to the lower frequencies, so that $\frac{f_m}{f_1} >> 1$,

$$\therefore \tan^{-1}\left(\frac{f_m}{f_1}\right) \simeq \frac{\pi}{2}$$

$\therefore$ Eqn (13), becomes,

$$R \simeq \frac{\cancel{\Phi}\ \frac{\pi}{2}}{3\frac{f_1}{f_m}\left[1 - \text{small quantity} \cdot \frac{\pi}{2}\right]}$$

$$R \simeq \frac{\frac{\pi}{2}}{3\frac{f_1}{f_m}}$$

$$R \simeq \frac{\pi}{2} \times \frac{f_m}{3f_1}$$

$$R \simeq \frac{\pi}{6} \cdot \frac{f_m}{f_1} \quad \text{—— (14)}$$

In commercial FM broadcasting, $f_1 = 2.1$ KHZ, while ~~f_m~~ f_m may reasonably be taken as $f_m = 15$KHZ

$$\therefore \frac{f_m}{f_1} = 7.5$$

$$R = \frac{\pi}{6} \times 7.5 = 3.92$$

Using eq^n (13), $\frac{f_m}{f_1} = 7.5$

$R \simeq 4.4$, take $\tan^{-1}(f_m/f_1)$ in radian.

See ex-3, next page

But in Taub, Schilling, $R \simeq 4.7 = 6.7$dB

$\therefore$ 6.7 dB improvement in o/p SNR using pre emphasis. Since o/p SNR are typically 40 to 50 dB, this represents an improvement of approximately 15%, a significant improvement.

$$\left[\begin{array}{l} 45 - 50 \simeq 45 \\ 45 \times \frac{15}{100} = 6.7 \end{array}\right]$$

Note :-

1) Pre emphasis is particularly effective in FM Systems which are used for transmission of audio signals. The advantage of using pre-emphasis

is less pronounced in an AM system. For in AM the noise spectral density is constant & does not increase with freq.

Ex :- 3) An RC filter based pre emphasis - de emphasis is employed in FM system. The deemphasis first order RC filter has $R = 1 k\Omega$, $C = 0.1 \mu F$, Find the gain in dB in an FM broadcasting system which has baseband BW 15 kHz. How is the improvement if baseband BW is increased to 30 kHz.

Ans :- The break point for RC filter,

$$f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 1 \times 10^3 \times 0.1 \times 10^{-6}} = 1591.55 \text{ Hz.}$$

$$f_m = 15 \text{ kHz.}$$

$$\therefore \frac{f_1}{f_m} = 0.1061, \quad \frac{f_m}{f_1} = 9.4248$$

$$R = \text{Gain} = \frac{\tan^{-1}(f_m/f_1)}{3 \times (f_1/f_m)\left[1 - (f_1/f_m)\tan^{-1}(f_m/f_1)\right]}$$

$$= \frac{\tan^{-1}(9.4248)}{3 \times 0.1061 \times \left[1 - 0.1061 \times \tan^{-1}(9.4248)\right]}$$

$$\text{Gain} = \frac{1.4651}{3 \times .1061 \times 0.8445}$$ → in radian [If we calculate in degree, multiply $\frac{\pi}{180}$ to convert into radian] Otherwise in calculator set a radian mode]

Gain $= 5.4504$, In dB, $10\log(5.4504)$

Gain $= 7.364$ dB.

If $f_m = 30$ KHZ,

$$\frac{f_1}{f_m} = \frac{1591.55}{30{,}000} = 0.0531$$

$$\frac{f_m}{f_1} = 18.8495$$

$$\text{Gain} = \frac{\tan^{-1}(18.8495)}{3\times(0.0531)\times\left[1 - 0.0531\times\tan^{-1}(18.8495)\right]}$$

$$= \frac{1.5178}{3\times 0.0531\times 0.9194}$$

Gain $= 10.363$

Gain]$_{dB}$ $= 10\log(10.363) = 10.155$ dB.

Threshold in FM (Short Note - BPUT) - 2009, 2012

In communication system in which the modulation is linear and demodulation is accomplished by coherent detection (ex.. SSB, DSB-SC), we have

$$\frac{S_o}{N_o} = \frac{S_i}{N_m} \quad \text{——— (1)}$$

or

$$10\log\frac{S_o}{N_o} = 10\log\frac{S_i}{N_m}$$

$$\Rightarrow \left[\frac{S_o}{N_o}\right]_{dB} = \left(\frac{S_i}{N_m}\right)_{dB} \quad \text{——— (2)}$$

In FM, we have

$$\frac{S_o}{N_o} = \frac{3}{2} \cdot \beta^2 \frac{S_i}{N_m}$$

$$\Rightarrow \left(\frac{S_o}{N_o}\right)_{dB} = 10 \log \left[\frac{3}{2} \beta^2 \cdot \frac{S_i}{N_m}\right]$$

$$\Rightarrow \left(\frac{S_o}{N_o}\right)_{dB} = 10 \log \frac{3}{2} \beta^2 + \left(\frac{S_i}{N_m}\right)_{dB} \quad \text{--- (3)}$$

Plotting eq^n (2), in a co-ordinate system in which co-ordinate axes are marked off in dB, we have a straight line passing through the origin, shown in fig below.

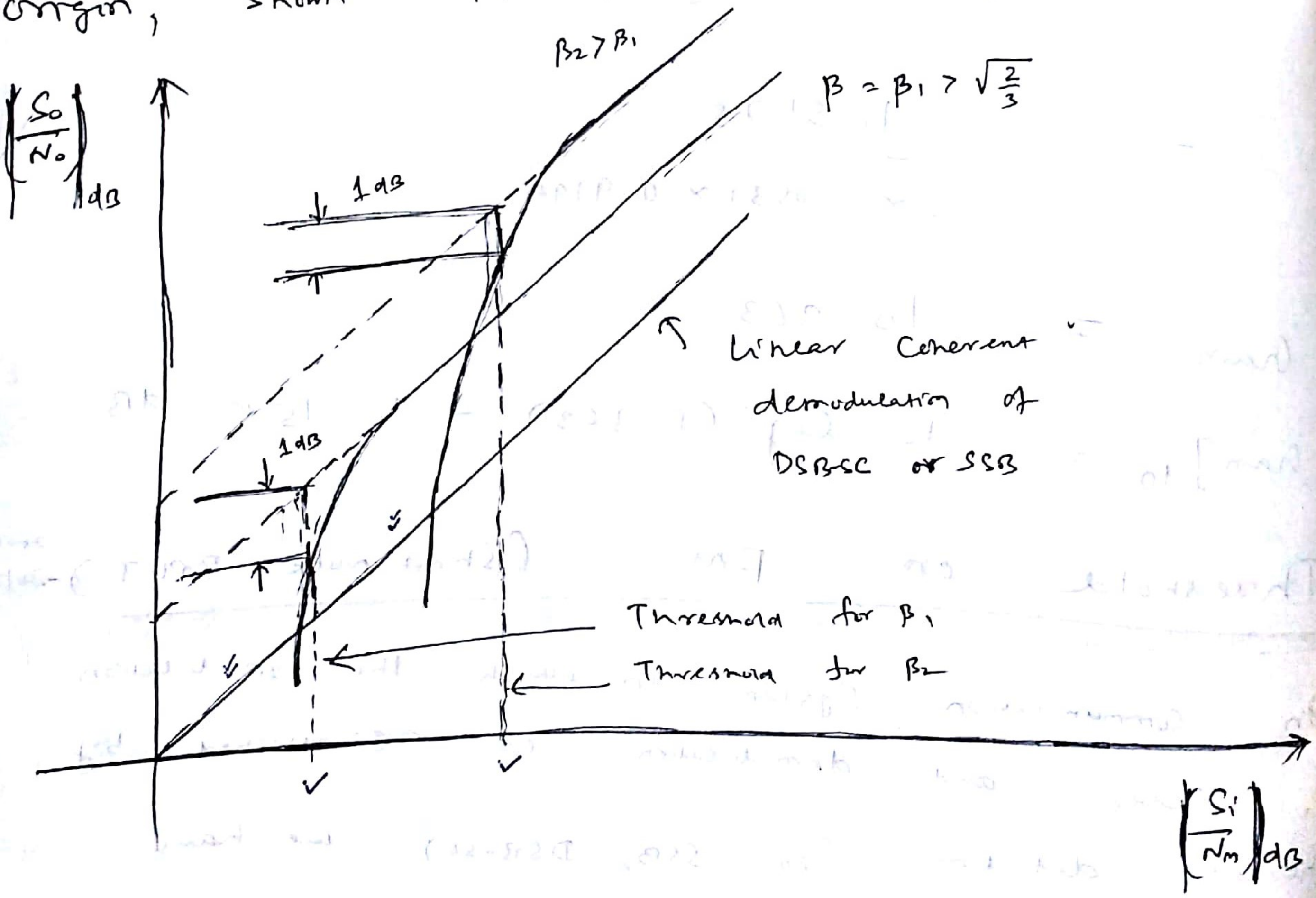


fig :- Plots of O/P SNR against I/P SNR for linear modulation & demodulation and also for an FM system. Illustrating the phenomenon of threshold in FM.

Eqn (3) is also plotted, for 2 different values of β, $\beta = \beta_1$ & $\beta = \beta_2$. $\beta_1 > \sqrt{\frac{2}{3}}$ & $\beta_2 > \beta_1$.

These plots, as indicated by the dashed extensions, are also straight lines. In the FM case a plot for a value of 'β' is raised by an amount $10 \log \frac{3\beta^2}{2}$ above the plot for a linear coherent modulation system. The quantity $10 \log \left(\frac{3\beta^2}{2}\right)$ expresses in dB, precisely the improvement afforded by the FM system in return for a sacrifice of BW.

Experimentally it is determined, however, that the FM system exhibits a threshold. Thus as indicated by the solid-line plots, for each value of β, as S_i/N_M decreases, a point is reached where S_o/N_o falls off much more sharply than S_i/N_M.

The threshold value of S_i/N_M is arbitrarily taken to be the value at which S_o/N_o falls 1 dB below the dashed extension. From the plot, it is observed that for larger β, the threshold of S_i/N_M is also higher.

→ Suppose, then, that we are operating with a modulation index above the threshold for β_1 but below the threshold for β_2. Suppose, further at this I/P S_i/N_M, we increase β from β_1 to β_2 hoping thereby to improve the O/P SNR by a sacrifice of BW.

But such a sacrifice of BW actually decrease the O/P SNR. Similarly, if we are operating sufficiently below threshold for any value of β, we do better with linear coherent system than with an FM system. [As shown in the plot] (E

The onset threshold may be observed by examining the noise O/P of an FM discriminator on a CRO. At High I/P SNR the noise displays the usual random-variation characteristics of thermal noise generally, shown in fig below.

[CRO]

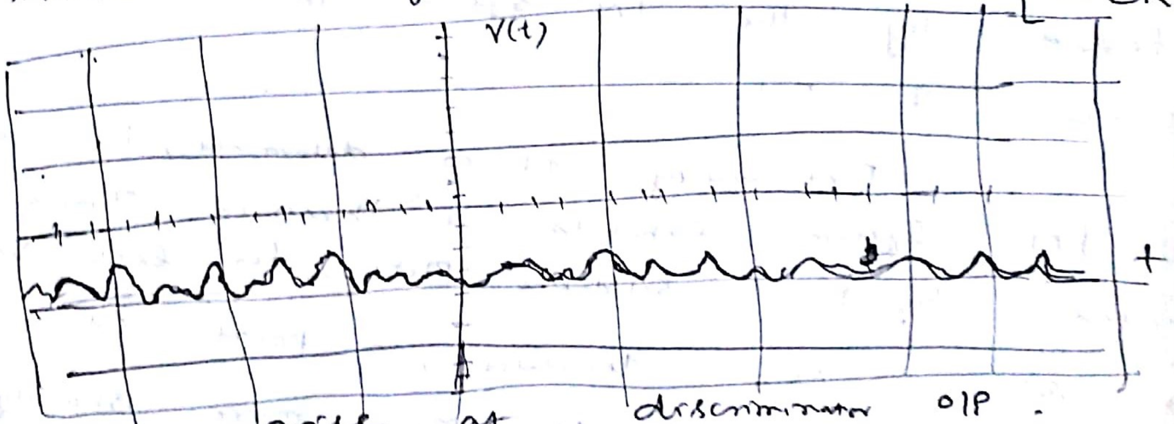


fig 1: Thermal noise at discriminator O/P.

[CRO]

fig: 2:- A Spike superimposed on a background of smooth (thermal) noise

The Instantaneous noise amplitude has a gaussian probability density. As Si/Nm decreases, a point is reached where the character of noise waveform changes markedly. The noise now has appearance in fig 2. A pulse type wave superimposed on the background thermal-type (smooth noise) noise. Because of its appearance, this new component of noise is often referred to as spike noise or impulse noise.

If we were to listen to the discriminator, we would hear a clicking sound on each occasion ~~of~~ that we observed a spike.

The appearance of these spikes denotes the onset of threshold. Although the freq of occurance of a spike is small, the noise energy associated with a spike is very large compared with energy of the smooth noise occuring during a comparable time interval. Hence the spike noise greatly increases the total noise o/p and thereby causes a threshold.

The FM demodulator using Feedback (FMFB)

An FM demodulator using feedback (FMFB) is shown in fig 3. ~~As~~ Here, a voltage controlled oscillator or VCO is freq modulated by the o/p FMFB demodulator.

The input signal $V_i(t)$ of the $V_i(t)$ of carrier freq f_c is multiplied with output $V_{osc}(t)$ of VCO. The freq of VCO is offset from f_c by an amount f_0.

The FMFB includes in its forward transmission path

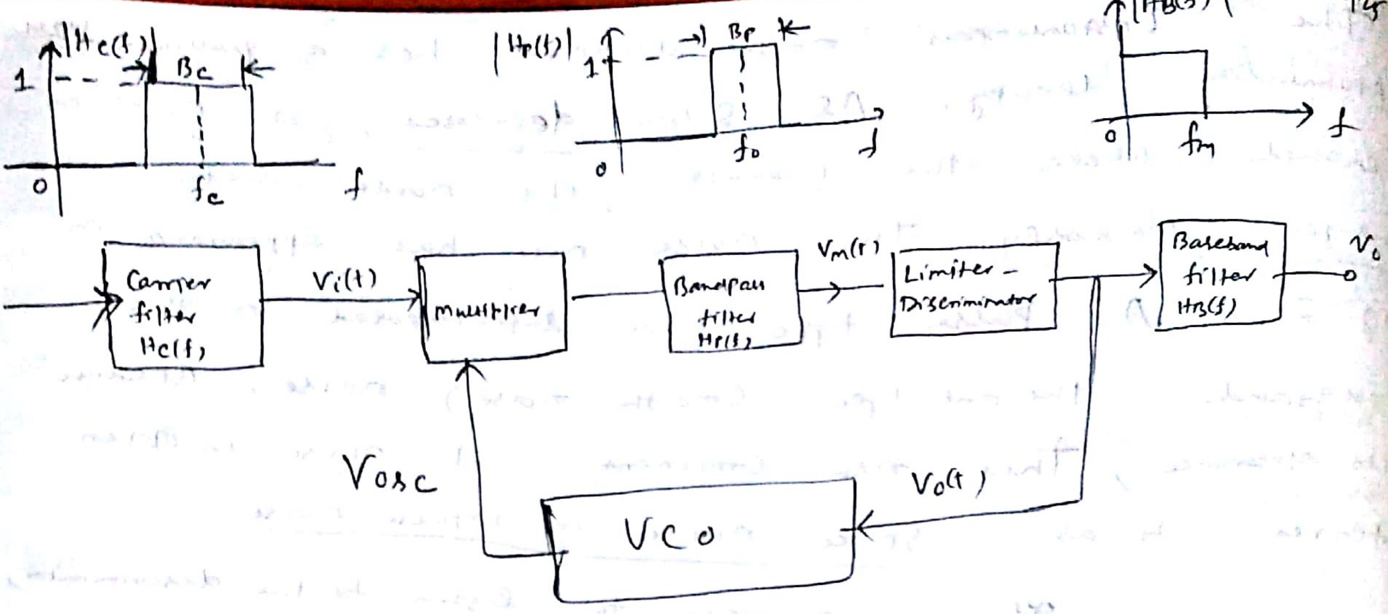


fig 3:- The FM demodulator using feedback

a bandpass filter and also a limiter - discriminator.

The bandpass filter, following multiplier, is centered at the offset freq 'f_0'. and hence passes the difference - freq o/p of the multiplier.

The FMFB recovers the baseband signal from an FM modulated carrier and when operating above threshold yield the same SNR o/p as does a simple limiter - discriminator.

Let the i/p signal & noise $V_i(t)$ be

$$V_i(t) = R(t) \sin[\omega_c t + \phi_s(t) + \phi_n(t)] \quad -①$$

where $R(t)$ is the envelope of the carrier signal and noise, $\phi_s(t)$ is the angular modulation due to the signal and $\phi_n(t)$ is due to the noise.

If $V_i(t)$ were the i/p to a limiter - discriminator whose baseband o/p was α times the departure of the instantaneous angular freq from the carrier freq ω_c, $\left[\frac{d}{dt}(\phi_s(t) + \phi_n(t))\right]$ the o/p voltage of the discriminator

$V_0(t)$ would be

$$V_0(t) = \alpha \frac{d}{dt}\left[\phi_s(t) + \phi_n(t)\right] \quad \text{--- (2)}$$

$(\because$ As discussed earlier, discriminator is a differentiator. 384 page - Note, 338

Lets represent VCO O/P as,

$$V_{osc}(t) = B \cos\left[(w_c - w_0)t + G_0 \int_{-\infty}^{t} V_0(\lambda)\, d\lambda\right] \quad \text{--- (3)}$$

Where B is the VCO amplitude & G_0 is the change in angular freq of the VCO per unit change in $V_0(t)$. We neglect temporarily the effect of bandpass filter following the multiplier except to take account of the fact that it passes only the difference-freq component of the product signal $V_m(t)$ of multiplier. On this basis, The low-pass component of which is equal to the [multiplying eqn (1) & (3)]

$V_i \cdot V_{osc}$ is

$$= R(t) \sin\left[\underbrace{w_c t + \phi_s(t) + \phi_n(t)}_{\alpha}\right] \cdot B \cos\left[\underbrace{w_c t - w_0 t + G_0 \int_{-\infty}^{t} V_0(\lambda)\, d\lambda}_{\beta}\right]$$

(A)

$$= \frac{AB}{2} \; 2 \sin\alpha \cdot \cos\beta$$

$$= \frac{AB}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

Low-pass component

$$V_m(t) = \frac{AB}{2} \sin(\alpha-\beta)$$

$$V_m(t) = \frac{AB}{2} \sin\left[w_0 t + \phi_s(t) + \phi_n(t) - G_0 \int_{-\infty}^{t} V_0(\lambda)\, d\lambda\right] \quad \text{--- (4)}$$

The product signal is applied to the limiter-discriminator. The O/P of the discriminator $V_0(t)$ is then,

$$V_0(t) = \alpha\left[\frac{d}{dt}\phi_s(t) + \frac{d}{dt}\phi_n(t) - G_0 V_0(t)\right] \quad \text{--- (5)}$$

$\left[\because \frac{d}{dt}\int_{-\infty}^{t} V_0(\lambda)\, d\lambda = V_0(t)\right]$

$\Rightarrow V_0(t)\left[1+\alpha G_0\right] = \alpha \frac{d}{dt}\left[\phi_s + \phi_n\right]$

$$\Rightarrow V_0(t) = \frac{\alpha}{1+\alpha G_0} \frac{d}{dt}(\phi_s + \phi_n) \quad \text{--- (6)}$$

Comparing (2) & (6), we observe that FMFB ~~since~~ indeed demodulates. Only difference betn o/p of discriminator and FMFB is that the amplitude of o/p of FMFB is smaller by the factor $(1+\alpha G_0)$.

Since both signal & noise have been reduced by the same factor $\frac{1}{1+\alpha G_0}$, the SNR is same for FMFB & FM discriminator.

→ Consider the Band pass filter, we will check how narrow it can be made, to pass the signal without undue distortion.

Putting eqn (6), in eqn (4), we have

$$V_m(t) = \frac{AB}{2} \sin\left[\omega_0 t + \phi_s + \phi_n - G_0 \int_{-\infty}^{t} \frac{\alpha}{1+\alpha G_0} \frac{d}{dt}(\phi_s + \phi_n)\, dt\right]$$

$$= \frac{AB}{2} \sin\left[\omega_0 t + \phi_s + \phi_n - G_0 \frac{\alpha}{1+\alpha G_0}(\phi_s + \phi_n)\right]$$

$$= \frac{AB}{2} \sin\left[\omega_0 t + (\phi_s + \phi_n)\left(1 - \frac{\alpha G_0}{1+\alpha G_0}\right)\right]$$

$$V_m(t) = \frac{AB}{2} \sin\left[\omega_0 t + (\phi_s + \phi_n) \cdot \frac{1}{1+\alpha G_0}\right] \quad \text{--- (7)}$$

Comparing eqn (1) & (7), we observe that feedback has suppressed the freq deviation, produced by the signal ϕ_s, by the factor $\frac{1}{1+\alpha G_0}$. If the modulation is sinusoidal,

$\phi_s(t) = \beta \sin \omega_m t$ and the phase of the signal present on the multiplied signal is

$$\frac{\phi_s(t)}{1+\alpha G_0} = \frac{\beta \sin \omega_m t}{1+\alpha G_0}.$$

Hence ~~the ...~~ if the BW of the carrier filter is $\underline{B_c}$ and BW of the bandpass filter preceding the discriminator is $\underline{B_p}$ then,

$$B_p = 2\left(\frac{\beta}{1+\alpha G_0} + 1\right) f_m$$

$\because$ BW $= 2(\beta+1) f_m$
$= 2(\Delta f + f_m)$

and $B_c = 2(\beta + 1) f_m$.

$$\Rightarrow \frac{B_p}{B_c} = \frac{\frac{\beta}{1+\alpha G_0} + 1}{\beta + 1}$$

$$= \cancel{\beta + 1 + \alpha G_0}$$

$$\Rightarrow B_p = \frac{[\beta/(1+\alpha G_0)] + 1}{\beta + 1} \cdot B_c \quad \text{--- (8)}$$

If $\beta = 9$, $\alpha G_0 = 8$, $\boxed{B_p = \frac{1}{5} B_c}$

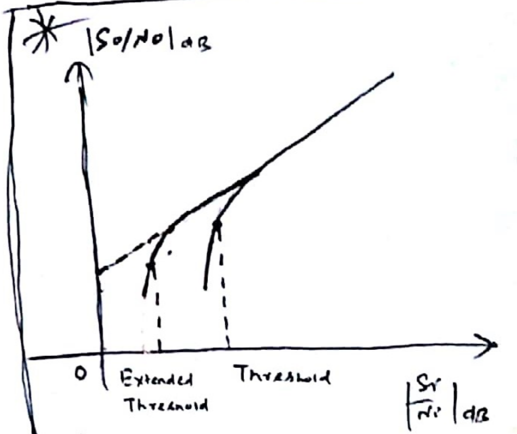


fig 2:-

*Note :- ~~If N is total number of spikes (noise)~~

) If total avg number of spikes per second is 'N', then $\boxed{N_{FMFB} < N_{discriminator}}$ i.e noise is reduced.

So threshold is extended using FMFB. [Refer fig 2:- Above]

Problems

1) A modulating signal $10 \sin(2\pi \times 10^3 t)$ is used to modulate a carrier signal $20 \sin(2\pi \times 10^4 t)$. Determine the modulation index, % of modulation, frequencies of the sideband components and their amplitudes. What will be the BW of modulated signal.

Ans :

$V_m = 10 \sin(2\pi \times 10^3 t) \Rightarrow V_m = 10$ (Amplitude of modulating signal)

$V_c = 20 \sin(2\pi \times 10^4 t) \Rightarrow V_c = 20$ (Amp. of carrier)

$f_m = 10^3$, $f_c = 10^4$

(i) Modulation index $= m = \frac{V_m}{V_c} = \frac{10}{20} = \frac{1}{2} = 0.5$

% of modulation $= 50\%$.

(ii) $f_{LSB} = f_c - f_m = 10^4 - 10^3 = 10K - 1K = 9\,KHz$

$f_{USB} = f_c + f_m = 10^4 + 10^3 = 10K + 1K = 11\,KHz$

(iii) Amplitude of sideband $\frac{mV_c}{2} = \frac{0.5 \times \cancel{20}^{10}}{\cancel{2}} = 5\,V$

(iv) Bandwidth $= 2f_m = 2 \times 10^3 = 2KHz$.

2) An AM broadcast radio transfer radiates 10K watts of power if modulation % is 60. Calculate how much of this is the carrier power?

Ans ,

$P_T = P_c\left(1 + \frac{m^2}{2}\right)$

$\Rightarrow 10K = P_c\left(1 + \frac{0.6^2}{2}\right) = P_c(1 + 0.18) = P_c(1.18)$

$\Rightarrow P_c = \frac{10}{1.18}$ KWatt $= 8.47$ KWatt.

3)

$$P_t = P_c \left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow I_t^2 . \cancel{R} = I_c^2 . \cancel{R} \left(1+\frac{m^2}{2}\right) \quad (\because P = I^2R)$$

$$\Rightarrow \boxed{I_t = I_c\sqrt{\left(1+\frac{m^2}{2}\right)}}$$

t = total

I_c = Carrier current

4) The antenna current of an AM transmitter is 8 Amp if only the carrier is sent, but increases to 8.93 Amp if the carrier is modulated by a single sinusoidal wave. Determine the %. of modulation and also find the antenna current if % of modulation changes to 0.8.

Ans :- (a)

$$I_T = I_c\sqrt{\left(1+\frac{m^2}{2}\right)}$$

$$\Rightarrow 8.93 = 8\sqrt{\left(1+\frac{m^2}{2}\right)}$$

$$\Rightarrow \sqrt{1+\frac{m^2}{2}} = \frac{8.93}{8} \quad \Rightarrow 1+\frac{m^2}{2} = \left(\frac{8.93}{8}\right)^2$$

$$\Rightarrow \frac{m^2}{2} = \left(\frac{8.93}{8}\right)^2 - 1$$

$$\Rightarrow m^2 = 2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]$$

$$\Rightarrow m = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]} = 0.701 = 70.1\%$$

(b)

$$I_T = I_c\sqrt{\left(1+\frac{m^2}{2}\right)} = 8\sqrt{\left(1+\frac{0.8^2}{2}\right)} = 8\sqrt{1+\frac{0.64}{2}}$$

$$= 8\sqrt{1.32} = 8 \times 1.149$$

$$= 9.19 \text{ Amp}$$

5) If the carrier amplitude after AM varies bet^n 4V to 1V. Calculate depth of modulation.

Ans :

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{4-1}{4+1} = \frac{3}{5} = 0.6 \text{ or } 60\%$$

6) A certain transmitter radiates 10kW with carrier unmodulated and 12 kW when the carrier is sinusoidally modulated. Calculate the modulation index. If another sinewave corresponding to 50% modulation ~~index~~ is transmitted simultaneously, determine the total radiated power.

Ans : (a) $P_c = 10kW$, $P_t = 12kW$

$$P_t = P_c\left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow 12 = 10\left(1+\frac{m^2}{2}\right) \Rightarrow \boxed{m_1 = 0.6324}$$

(b) Total modulation index $m_t = \sqrt{m_1^2 + m_2^2}$

Given $m_2 = 0.5$

$$\Rightarrow m_t = \sqrt{(0.6324)^2 + (0.5)^2} = 0.8$$

(c) Now

$$P_T = P_c\left(1+\frac{m_t^2}{2}\right) = 10\left(1+\frac{0.8^2}{2}\right) = 13.2 \text{ kwatt}$$