Module-4 : Transmission Lines

<Chapter-7>

Our discussion on last chapter was essentially on wave propagation in unbounded media, media of infinite extent. Such wave propagation is said to be unguided in that the uniform plane wave exists through out all space, and EM energy associated with the wave spreads over a wide area. This type of wave propagation is used in radio or TV broadcasting, where the information being transmitted is meant for everyone who may be interested. Such means of wave propagation will not help in a situation like telephone conversation, where the information is received privately by one person.

Another means of transmitting power or information is by guided structures. Guided structures serve to guide the propagation of energy from source to the load. Typical example of such structures are transmission lines and wave guides.

Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). A transmission line basically consists of two or more parallel conductors used to connect a source to a load.

The source may be a hydroelectric generator, a transmitter, or an oscillator; a load may be a factory, an antenna or an oscilloscope. Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a microstrip line. These lines are portrayed in fig 7.1.

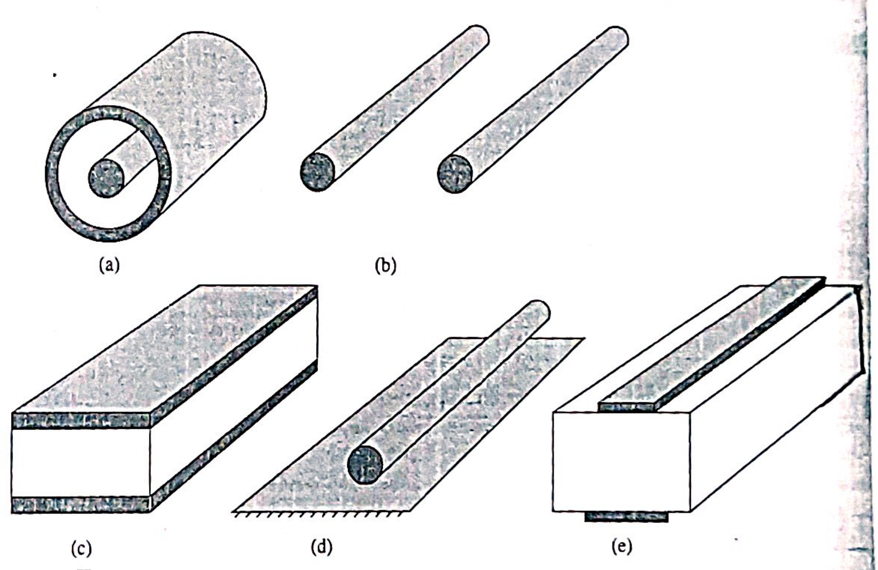


Figure 7.1 Typical transmission lines in cross-sectional view: (a) coaxial line, (b) two-wire line (c) planar line, (d) wire above conducting plane, (e) microstrip line.

- Notice that each of these lines consists of two conductor in parallel. Coaxial cables are routinely used in electrical laboratories and in connecting TV sets to antennas. Microstrip lines are particularly important in integrated circuits, where metallic strips connecting electronic elements are deposited on dielectric substrates.

Transmission line problems are usually solved by means of EM field theory and electric circuit theory. In this chapter, we use circuit theory because it is easier to deal with mathematically.

Transmission line parameters

It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length (R), inductance per unit length (L), conductance per unit length (G), and capacitance per unit length (C). Each of the lines shown in figure 7.1 has specific formulas for finding R, L, G, and C. For coaxial, two-wire, and planar lines, the formulas for calculating R, L, G, and C are provided in Table 7.1. The dimension of lines are shown in figure 7.2. It should be noted that

Table 7.1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\dfrac{1}{2\pi\delta\sigma_c}\left[\dfrac{1}{a}+\dfrac{1}{b}\right]$ $(\delta \ll a, c-b)$	$\dfrac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\dfrac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
L (H/m)	$\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$	$\dfrac{\mu}{\pi}\cosh^{-1}\dfrac{d}{2a}$	$\dfrac{\mu d}{w}$
G (S/m)	$\dfrac{2\pi\sigma}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\sigma}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\sigma w}{d}$
C (F/m)	$\dfrac{2\pi\varepsilon}{\ln\dfrac{b}{a}}$	$\dfrac{\pi\varepsilon}{\cosh^{-1}\dfrac{d}{2a}}$	$\dfrac{\varepsilon w}{d}$ $(w \gg d)$

$^*\delta = \dfrac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\dfrac{d}{2a} \simeq \ln\dfrac{d}{a}$ if $\left[\dfrac{d}{2a}\right]^2 \gg 1$.

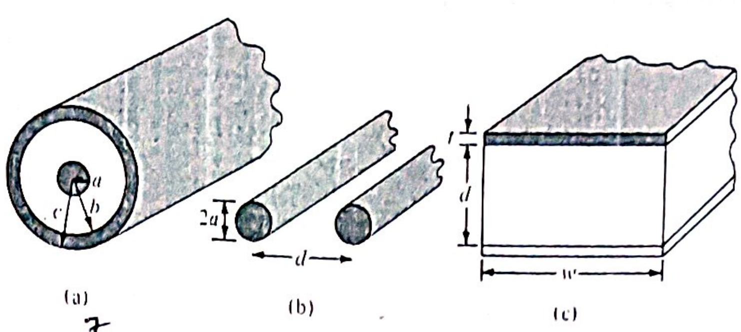


Figure 7.2 Common transmission lines: (a) coaxial line, (b) two-wire line, (c) planar line.

1) The line parameters R, L, G, and C are not discrete or lumped. Rather, they are distributed as shown on figure 7.3. By this we mean that parameters are uniformly distributed along the entire length of the line.

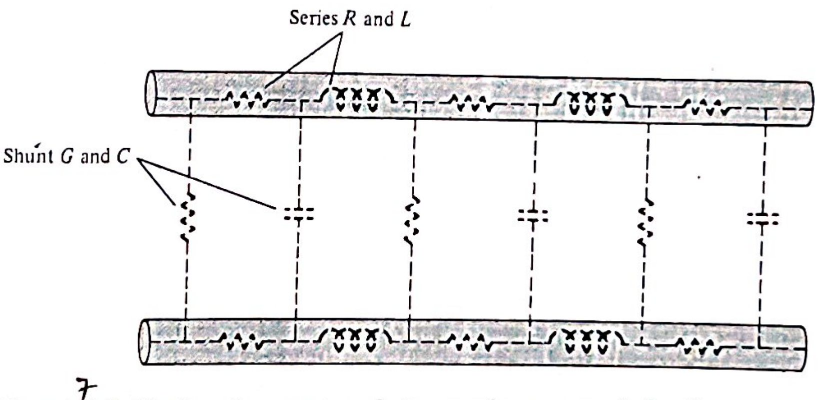


Figure 7.3 Distributed parameters of a two-conductor transmission line.

2) For each line, the conductors are characterized by σ_c, μ_c, $\epsilon_c = \epsilon_0$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ϵ.

3) $G \neq \frac{1}{R}$; R is the ac resistance per

unit length of the conductors comprising the line, and G is the conductance per unit length due to the dielectric medium seperating the conductors.

4. The value of L as shown in Table 7.1 is the external inductance ($= L_{ext}$) per unit length.

The effects of internal inductance ($= L_{in}$) are negligible at high frequencies at which most communication system operate.

$\because V = I . X_L$

$\Rightarrow X_L = \frac{V}{I} = R$

$\Rightarrow \omega L = R$

$\Rightarrow L = \frac{R}{\omega}$

$\Rightarrow L = \frac{R}{2\pi f}$

5. For each line, (Table 7.1)

$\boxed{LC = \mu \epsilon}$ and $\boxed{\frac{G}{C} = \frac{\sigma}{\epsilon}}$

Lets consider how an EM wave propagates through a two-conductor transmission line. For ex: Consider the coaxial line connecting the generator or source to the load as in figure 7.4 (a), when switch S is closed, the inner conductor is made +ve w.r.to outer one so that the E field is radially outward as in figure 7.4(b). According to Ampere's law, the H field encircles the current carrying conductor as in figure 7.4(b). The Poynting vector (E × H) points along the transmission line.

Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a non uniform plane wave and by means of it power is transmitted through the line.

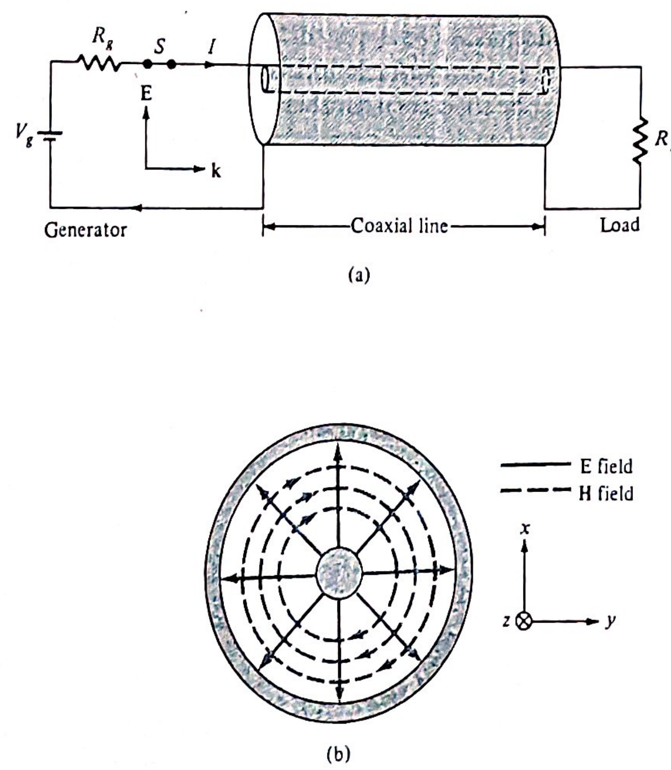


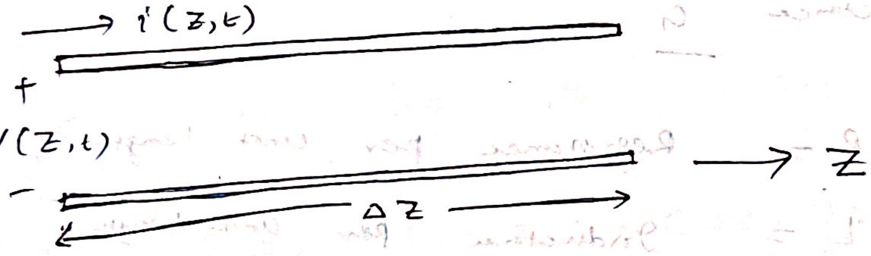
Figure 7.4 (a) Coaxial line connecting the generator to the load; (b) **E** and **H** fields on the coaxial line.

The Lumped - element circuit model for a transmission line

(compact) / ckt theory model of T.L

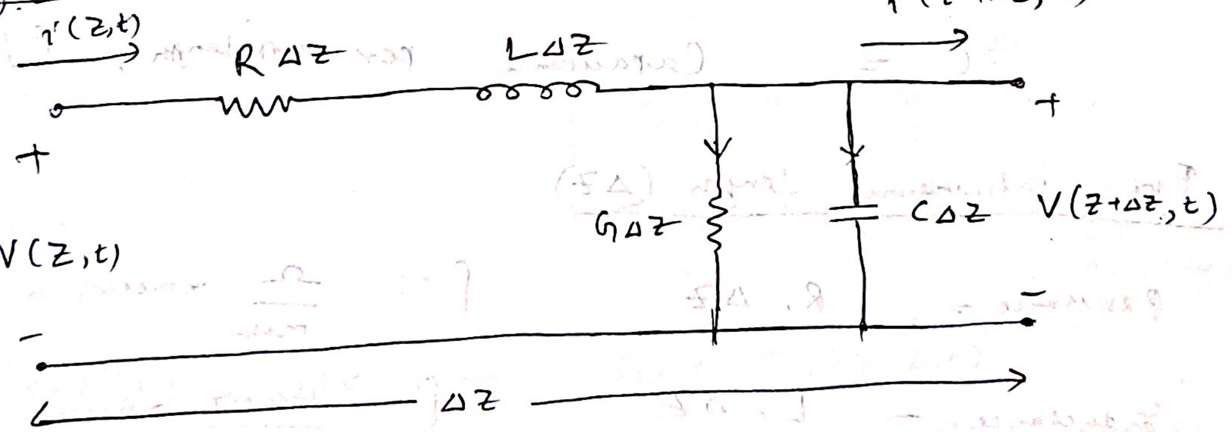
The key difference between circuit theory and transmission line theory is electrical size, circuit analysis assumes that the physical dimension of a n/w are much smaller than electrical wavelength (λ), while transmission line may be a considerable fraction of a wavelength many wavelengths, in size. Thus a transmission line is a distributed-parameter n/w, where voltages and current can vary in magnitude & phase over its length.

As shown in fig 3 (a), a transmission line is often schematically represented as a two-wire line. The piece of line of infinitesimal length Δz of fig 3 (a) can be modeled as a lumped-element circuit, as shown in fig 3 (b), where R, L, G, C are per unit length quantities. [distributed parameters]



(a)

~~fig 3 (a):~~



(b)

fig 3 :- Voltage and current definitions and equivalent circuit for an incremental length of transmission line. (a) Voltage and current definitions (b) Lumped-element equivalent circuit.

Since each conductor of the transmission line has finite length, diameter and finite area of cross section, they have resistance (R) and self inductance (L). As both the transmission lines are separated by dielectric with finite dielectric constant, they have a capacitance (C) between the two conductors.

Due to the presence of dielectric, there will be a leakage current through it which is represented by conductance 'G'.

$\therefore$ R = Resistance per unit length, Ω/meter

L = Inductance per unit length, H/meter.

G = Conductance per unit length, S/meter

C = Capacitance per unit length, F/meter.

For infinitesimal length (ΔZ)

Resistance = $R \cdot \Delta Z$ $\quad \left[\because \dfrac{\Omega}{\text{meter}} \times \text{meter} = \Omega\right]$

Inductance = $L \cdot \Delta Z$ $\quad \left[\dfrac{\text{Henery}}{\text{meter}} \times \text{meter} = \text{Henery}\right)$

Similarly

Conductance = $G \cdot \Delta Z$

Capacitance = $C \cdot \Delta Z$

Applying KVL in fig 3(b), we have

$$v(z,t) - i(z,t) \cdot R\Delta z - L\Delta z \cdot \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0$$

$\Rightarrow \quad v(z+\Delta z, t) - v(z,t) = -\left[i(z,t) \cdot R\Delta z + L\Delta z \cdot \frac{\partial i(z,t)}{\partial t} \right]$

Dividing both the sides by Δz, ~~we have~~ and taking the limit $\Delta z \to 0$, we have

$$\Rightarrow \lim_{\Delta z \to 0} \frac{V(z+\Delta z, t) - V(z,t)}{\Delta z} = -\left[i(z,t) \cdot R + L \frac{\partial}{\partial t} i(z,t) \right]$$

$$\Rightarrow \boxed{\frac{\partial}{\partial z} V(z,t) = -R \cdot i(z,t) - L \frac{\partial i(z,t)}{\partial t}} \quad (1)$$

$\left[\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right]$

Applying KCL in fig 3(b), we have

$$i(z,t) = G\Delta z \cdot v(z+\Delta z, t) + C\Delta z \cdot \frac{\partial V(z+\Delta z, t)}{\partial t} + i(z+\Delta z, t)$$

$\because \quad I_1 = \frac{V}{R} = V \cdot G$

$I_2 = C \frac{dv}{dt} \quad \left(\because Q = CV \quad I.t = CV \quad \Rightarrow I = C\frac{dv}{dt} \right)$

$$\Rightarrow i(z+\Delta z, t) - i(z,t) = -G\Delta z \cdot v(z+\Delta z, t) - C\Delta z \cdot \frac{\partial}{\partial t} v(z+\Delta z, t)$$

Dividing both the sides by Δz and taking the limit $\Delta z \to 0$, we have

$$\Rightarrow \lim_{\Delta z \to 0} \frac{i(z+\Delta z, t) - i(z,t)}{\Delta z} = -G v(z,t) - C \frac{\partial v(z,t)}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial z} i(z,t) = -G v(z,t) - C \frac{\partial}{\partial t} v(z,t)} \quad (2)$$

Equation ① & ② are the time-domain form of transmission line or $\boxed{\text{telegrapher, eq}^n\text{s.}}$

Instantaneous ~~term~~ line voltage can be expressed as, $v(z,t) = \text{Real}\left[V_s(z).e^{j\omega t}\right]$ — (3A)

$= \text{Real}\left[V_s(z).(\cos\omega t + j\sin\omega t)\right]$

$v(z,t) = V_s(z).\cos\omega t$ ——— ③

∴ Similarly, $i(z,t) = I_s(z).\cos\omega t$ ——— ④

[where $V_s(z)$ & $I_s(z)$] are phasor form whereas original source.

Putting eq^n ③ & ④ in eq^n ①, we have

$$\frac{\partial}{\partial z} V_s(z).\cos\omega t = -R.I_s(z).\cos\omega t - L.\frac{\partial}{\partial t} I_s(z).\cos\omega t$$

Taking $\frac{\partial}{\partial t} = j\omega$ as mathematical operator, we have

$$\Rightarrow \frac{\partial}{\partial z} V_s(z).\cancel{\cos\omega t} = -R\, I_s(z).\cancel{\cos\omega t} - L.(j\omega).I_s(z).\cancel{\cos\omega t}$$

Hints:-
$x = A.e^{j\omega t}$
$\frac{\partial x}{\partial t} = A.e^{j\omega t}.j\omega$
$\frac{\partial x}{\partial t} = (j\omega).x$
∴ $\frac{\partial}{\partial t} = j\omega$

$$\Rightarrow \frac{\partial}{\partial z} V_s(z) = -(R + j\omega L)\, I_s(z) \qquad \text{——— ⑤}$$

Similarly,
Putting eq^n ③ & ④ in eq^n ②, we have

$$\frac{\partial}{\partial z} I_s(z).\cos\omega t = -G. V_s(z).\cos\omega t - C.\frac{\partial}{\partial t}(V_s(z).\cos\omega t)$$

$$\Rightarrow \frac{\partial}{\partial z}.I_s(z).\cancel{\cos\omega t} = -G\, V_s(z).\cancel{\cos\omega t} - C.(j\omega).V_s(z).\cancel{\cos\omega t}$$

$$\left(\because \frac{\partial}{\partial t} = j\omega \right)$$

$$\Rightarrow \frac{\partial}{\partial z} I_s(z) = -(G + j\omega C)\, V_s(z) \quad \text{——} (6)$$

[For sinusoidal steady-state condition, with cosine-based phasors]

* Note:- Eq^n (5) & (6) are also called telegrapher eq^n.

Wave Propagation on a transmission line

The eq^ns (5) & (6) can be solved simultaneously to give wave equations for $V_s(z)$ & $I_s(z)$.

Differentiating eq^n (5) w.r.to z, we have

$$\frac{\partial^2}{\partial z^2} V_s(z) = -(R+j\omega L).\frac{\partial}{\partial z} I_s(z) \quad \text{——} (7)$$

Now, putting eq^n (6) in eq^n (7), we have

$$\frac{\partial^2}{\partial z^2} V_s(z) = -(R+j\omega L).\left[-(G+j\omega C).V_s(z)\right]$$

$$\therefore \frac{\partial^2}{\partial z^2} V_s(z) - (R+j\omega L)(G+j\omega C).V_s(z) = 0$$

Let $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ = Complex Propagation constant.

$$\therefore \boxed{\frac{\partial^2}{\partial z^2} V_s(z) - \gamma^2 V_s(z) = 0} \quad \text{——} (8)$$

Similarly,

Differentiating eqn 6, w.r.t z, we have

$$\frac{\partial^2}{\partial z^2} I_s(z) = -(G+j\omega C) \cdot \frac{\partial}{\partial z} V_s(z) \quad — 9$$

Putting eqn 5, in eqn 9, we have

$$\frac{\partial^2 I_s(z)}{\partial z^2} = -(G+j\omega C) \cdot \left[-(R+j\omega L)\, I_s(z)\right]$$

$$\Rightarrow \frac{\partial^2}{\partial z^2} I_s(z) - (R+j\omega L)(G+j\omega C)\, I_s(z) = 0$$

$$\cancel{\Rightarrow \frac{\partial^2}{\partial z^2} I_s(z)} \qquad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \text{Complex Propagation Constant}$$

$$\Rightarrow \boxed{\frac{\partial^2}{\partial z^2} I_s(z) - \gamma^2 I_s(z) = 0} \quad — 10$$

∴ Eqn 8 & 10 are similar to electric & magnetic ~~Eqtns the~~ wave equations, called final form of $\boxed{\text{transmission line eqn}}$.

$\gamma = \alpha + j\beta$, where α = Attenuation constant

β = ~~Propagation~~ Phase constant

Solution of transmission line equations

If the 2nd order differential eqn is

$$\frac{d^2 y}{dx^2} - k^2 y = 0$$

The soln of 17 is of the form

$y = C_1 \cdot e^{-Kx} + C_2 \, e^{Kx}$ where C_1 & C_2 are constants.

~~So that~~ So solution to differential eq^n (8) & (10) are in the form,

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{--- (11)}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \text{--- (12)}$$

Here, $C_1 = V_0^+$ or I_0^+

$C_2 = V_0^-$ or I_0^-

$K = \gamma$

and V_0^+ & I_0^+ indicate complex amplitude in the +ve Z-direction, V_0^- & I_0^- signify complex amplitude in -ve Z-direction.

From eq^n (9A),

$$v(z,t) = \text{Real}\left[V_s(z) \cdot e^{j\omega t}\right]$$

$$= \text{Real}\left[\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) \cdot e^{j\omega t}\right]$$

$$= \text{Real}\left[V_0^+ e^{-(\alpha + j\beta)z} \cdot e^{j\omega t} + V_0^- e^{(\alpha+j\beta)z} \cdot e^{j\omega t}\right]$$

$$= \text{Real}\left[V_0^+ e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} + V_0^- e^{\alpha z} \cdot e^{j(\omega t + \beta z)}\right]$$

$$\Rightarrow \boxed{V(z,t) = \left[V_0^+ \cos(\omega t - \beta z) e^{-\alpha z} + V_0^- \cos(\omega t + \beta z) e^{\alpha z}\right]} \quad \text{--- (13)}$$

$\left(\because \text{Re}\left[e^{j(\omega t - \beta z)}\right] = \text{Re}\left[\cos(\omega t - \beta z) + j\sin(\omega t - \beta z)\right] = \cos(\omega t - \beta z)\right)$

Similarly,

$$\boxed{I(Z,t) = \left[I_0^+ \cos(\omega t - \beta Z) \cdot e^{-\alpha Z} + I_0^- \cos(\omega t + \beta Z)\, e^{\alpha Z} \right]} \quad (14)$$

Characteristic Impedance (Z_0)

Ratio of travelling voltage wave amplitude to current wave amplitude at any point on transmission line.

$$Z_0 = \frac{V_0^+}{I_0^+} \quad \text{[wave propagating in +ve Z direction]}$$

$$Z_0 = -\frac{V_0^-}{I_0^-} \quad \text{[wave propagating in -ve Z direction]}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Proof: From eqn (11),

$$V_s(Z) = V_0^+ e^{-\gamma Z} + V_0^- e^{\gamma Z}$$

$$\Rightarrow \frac{\partial V_s(Z)}{\partial Z} = V_0^+ \cdot (e^{-\gamma Z}) \cdot (-\gamma) + V_0^- \cdot e^{\gamma Z} \cdot (\gamma)$$

From eqn (5), $\dfrac{\partial V_s(Z)}{\partial Z} = -(R + j\omega L)\, I_s(Z)$

$\therefore$

$$-(R + j\omega L)\, I_s(Z) = -V_0^+ \gamma \cdot e^{-\gamma Z} + V_0^- \gamma \cdot e^{\gamma Z}$$

$$\Rightarrow I_s(Z) = \frac{V_0^+ \gamma}{R + j\omega L} \cdot e^{-\gamma Z} + \frac{V_0^- \gamma}{-(R + j\omega L)} \cdot e^{\gamma Z} \quad (15)$$

$$\Rightarrow \quad I_s(z) = \frac{V_0^+}{\left(\frac{R+j\omega L}{\gamma}\right)} \cdot e^{-\gamma z} + \frac{V_0^-}{\left(-\frac{R+j\omega L}{\gamma}\right)} \cdot e^{\gamma z} \quad \text{—(15)}$$

Comparing eqn (12) & (15),

$$I_0^+ = \frac{V_0^+}{\left(\frac{R+j\omega L}{\gamma}\right)}, \quad I_0^- = \frac{V_0^-}{-\left(\frac{R+j\omega L}{\gamma}\right)}$$

$$\Rightarrow \frac{V_0^+}{I_0^+} = \frac{R+j\omega L}{\gamma}, \quad -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma}$$

But we have defined, $\frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = Z_0$.

$$\therefore \quad Z_0 = \frac{R+j\omega L}{\gamma} \quad \text{—— 15(A)}$$

$$= \frac{(R+j\omega L)}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

(Proved)

Using eqn 15(A) in eqn (15)

$$\therefore \boxed{I_s(z) = \frac{V_0^+}{Z_0} \cdot e^{-\gamma z} - \frac{V_0^-}{Z_0} \cdot e^{\gamma z}} \quad \text{—— (16)}$$

The wavelength on the line, is

$$\boxed{\lambda = \frac{2\pi}{\beta}} \quad \text{—(17)}$$

[Refer - EMT - Plane wave propagation]

β = Phase Constant.

Phase velocity, $\boxed{v_p = \frac{\omega}{\beta} = f\lambda}$ —(18)

$\left[\because v = f\lambda = \frac{2\pi f \lambda}{2\pi} = \frac{(\omega)\lambda}{2\pi} = \frac{\omega}{\left(\frac{2\pi}{\lambda}\right)} = \frac{\omega}{\beta} \right.$

The above solution was for general transmission line, including loss effects and it was seen that propagation constant (γ) and characteristic impedance (Z_0) were complex. In many practical cases, however loss of line is very small and so can be neglected.

A. For lossless line,

[A transmission line is said to be lossless, if the conductor of line are perfect ($\sigma_c \simeq \infty$) and dielectric medium is lossless ($\sigma \simeq 0$)]

$R = 0$, $G = 0$.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{j\omega L \cdot j\omega C}$$

$$\therefore \boxed{\gamma = j\omega\sqrt{LC}} \quad \text{---} (19)$$

But $\gamma = \alpha + j\beta$

$$\Rightarrow j\omega\sqrt{LC} = \alpha + j\beta.$$

$$\therefore \boxed{\alpha = 0} \quad \text{---} (20)$$

$$\boxed{\beta = \omega\sqrt{LC}} \quad \text{---} (21)$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0 + j\omega L}{0 + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \quad \text{---} (22)$$

Since, $\alpha = 0$, $\gamma = j\beta$

From Eq^n (11) & (16), the general solution for voltage & current on lossless transmission line can be written as,

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \quad \text{——— (23)}$$

$$I_s(z) = \frac{V_0^+}{Z_0} \cdot e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z} \quad \text{——— (24)}$$

~~Wavelength~~ Wavelength, $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}}$ (From eq^n 21) ——— (25)

and Phase velocity, $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

$$\therefore \boxed{V_p = \frac{1}{\sqrt{LC}}} \quad \text{——— (26)}$$

B. Distortionless line

A signal normally consists of band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line because α is frequency dependent. Since, in general, the phase velocity of each frequency component $\left(v = \frac{\omega}{\beta}\right)$ is also frequency dependent, this will result in distortion.

A distortionless line is one in which the attenuation constant 'α' is frequency independent while the phase constant 'β' is linearly dependent on frequency.

Lets derive the condition for distortionless line.

We know

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{R\left(1 + \frac{j\omega L}{R}\right) G\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

If $\quad 1+\frac{jwL}{R} = 1+\frac{jwC}{G}$ — (1)

Then

$$\gamma = \sqrt{RG\left(1+\frac{jwL}{R}\right)^2}$$

$$\gamma = \sqrt{RG}\left(1+\frac{jwL}{R}\right)$$

$$\Rightarrow \alpha+j\beta = \sqrt{RG} + j\frac{wL}{R}\sqrt{RG}$$

$\therefore$

$$\alpha = \sqrt{RG}, \quad \beta = \frac{wL}{R}\sqrt{RG}$$

$$\therefore \boxed{\alpha = \sqrt{RG}, \quad \beta = wL\sqrt{\frac{G}{R}}} \quad (2)$$

Thus, α is frequency independent and β is linearly dependent on frequency.

$\therefore$ The condition for a line to be distortionless is

$$\cancel{1}+\frac{jwL}{R} = \cancel{1}+\frac{jwC}{G}$$

$$\Rightarrow \frac{\cancel{jw}L}{R} = \frac{\cancel{jw}C}{G}$$

$$\Rightarrow \frac{L}{R} = \frac{C}{G}$$

$$\Rightarrow \boxed{RC = LG} \text{ or } \boxed{\frac{R}{L} = \frac{G}{C}} \quad (3)$$

Also

$$Z_0 = \sqrt{\frac{R+jwL}{G+jwC}}$$ becomes,

$$= \sqrt{\frac{R\cancel{\left[1+\frac{jwL}{R}\right]}}{G\cancel{\left[1+\frac{jwC}{G}\right]}}}$$

$\because 1+\frac{jwL}{R} = 1+\frac{jwC}{G}$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{R}{G}}} = R_0 + jX_0 \quad -(4)$$

$$\therefore R_0 = \sqrt{\frac{R}{G}}, \quad X_0 = 0$$

from eqn (3)

$$\frac{R}{G} = \frac{L}{C}$$

$$\therefore \boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}} \quad -(5)$$

~~$U = \frac{\omega}{\beta} = \omega$~~

From eqn (2)

$$\beta = \omega L \sqrt{\frac{G}{R}} = \omega L\sqrt{\frac{C}{L}} = \omega\sqrt{LC}$$

$\because$ From eqn (2)

$$\beta = \omega L\sqrt{\frac{G}{R}}$$

From eqn (3)

$$\frac{G}{R} = \frac{C}{L}$$

$$\therefore \boxed{\beta = \omega\sqrt{LC}} \quad -(6)$$

$$U = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = f\lambda \quad -(7)$$

Note:

1) The Phase velocity $U = \frac{1}{\sqrt{LC}}$ i.e independent of frequency. If α is independent of freq, then there is no distortion of signals.

2) Both U (V_P) and Z_0 are same as lossless line.

3) A lossless line is also distortionless line, but a distortionless line is not necessarily lossless.

Table 7.2 Transmission line Characteristics

Case	Prop. Const. $\gamma = \alpha + j\beta$	Characteristics Impedance $Z_0 = R_0 + jX_0$
General	$\sqrt{(R+j\omega L)(G+j\omega C)}$	$\sqrt{\frac{R+j\omega L}{G+j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j.0$
Distortionless	$\sqrt{RG} + j\omega L\sqrt{\frac{G}{R}}$ or $\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j.0$

Example 1:- An air line has characteristic impedance of 70Ω and a phase constant of $3 \frac{rad}{m}$ at 100 MHz. Calculate the inductance per meter and capacitance per meter of the line.

Ans: Given

An airline can be regarded as a lossless line because $\sigma \simeq 0$, and $\sigma_c \to \infty$, Hence

$R = 0 = G$ and $\alpha = 0$

$Z_0 = R_0 = \sqrt{\frac{L}{C}}$, $\beta = \omega\sqrt{LC}$

$\therefore \frac{Z_0}{\beta} = \frac{\sqrt{\frac{L}{C}}}{\omega\sqrt{LC}} = \frac{\cancel{\sqrt{L}}}{\sqrt{C}} \times \frac{1}{\omega\cancel{\sqrt{L}}\sqrt{C}} = \frac{1}{\omega C}$

$\Rightarrow Z_0 \omega C = \beta$

$\Rightarrow C = \frac{\beta}{Z_0 \omega} = \frac{3}{70 \times 2\pi \times (100 \times 10^6)} = 68.2 \frac{pF}{m}$

$Z_0 = \sqrt{\frac{L}{C}}$

$\Rightarrow \frac{L}{C} = Z_0^2$ $\Rightarrow L = Z_0^2 \cdot C$

$\Rightarrow L = (70)^2 \times (68.2 \times 10^{-12}) = 334.2 \frac{nH}{m}$

$\therefore$ $\boxed{C = 68.2 \frac{pF}{m}, \quad L = 334.2 \frac{nH}{m}}$ (Ans)

2) A distortionless line has $Z_0 = 60\,\Omega$, $\alpha = 20\ m\frac{Np}{m}$, $U = 0.6\,C$, C = velocity of light in Vacuum

Find, R, L, G, C, and λ at $100\ MHz$.

Ans: For distortionless line

$$RC = LG \Rightarrow G = \frac{RC}{L} \quad \text{—(1)}$$

and $$Z_0 = \sqrt{\frac{L}{C}} \quad \text{—(2)}$$

$$\alpha = \sqrt{RG} = \sqrt{R} \times \sqrt{\frac{RC}{L}} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0} \text{ (2A)}$$

$$\therefore \boxed{R = \alpha Z_0} \quad \text{—(3)}$$

But
$$U = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad \text{—(4)}$$

$$\therefore \boxed{R = \alpha Z_0 = 20 \times 10^{-3} \times 60 = 1.2\ \frac{\Omega}{m}}$$

$\frac{\text{Eqn (2)}}{\text{Eqn (4)}}$ gives $\frac{Z_0}{U} = \frac{\sqrt{L}}{\cancel{\sqrt{C}}} \times \frac{\sqrt{L}\cancel{\sqrt{C}}}{1}$

$$\Rightarrow L = \frac{Z_0}{U}$$

$$\therefore \boxed{L = \frac{60}{0.6 \times 3 \times 10^8} = 333\ \frac{nH}{m}}$$

From eqn (2A),

$$\alpha^2 = RG$$

$$\Rightarrow \boxed{G = \frac{\alpha^2}{R} = \frac{(20 \times 10^{-3})^2}{1.2} = 333\ \frac{\mu S}{m}}$$

From eqn 1

$$C = \frac{LG}{R}$$

$$= \frac{337 \times 10^{-9} \times 337 \times 10^{-6}}{1.2}$$

$$= 92407.5 \times 10^{-15}$$

$$= 92.4075 \times 10^{-12}$$

$$\Rightarrow \boxed{C = 92.4075 \ \frac{PF}{m}}$$

$$\boxed{\lambda = \frac{v}{f} = \frac{0.6 \times 3 \times \cancel{10^8}}{\cancel{100} \times \cancel{10^6}} = 1.8 \text{ meter}}$$

But we know

$$\boxed{V_p = \frac{1}{\sqrt{\mu\epsilon}}}$$

[Refer EMT]

Case – I

for free space / air (contains no ferromagnetic material)

$$\boxed{V_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} = C = 3\times10^8 \text{ m/sec}} \quad (33)$$

Case – II

when dielectric of a lossy microwave transmission line is not air,

$V_p <$ velocity of light

$$\boxed{V_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}} \quad (34)$$

$\because \mu_r = \frac{\mu}{\mu_0}$

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$\frac{1}{\sqrt{\mu\epsilon}}$

$= \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$

$= \frac{1}{\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}}$

$= \frac{c}{\sqrt{\mu_r\epsilon_r}}$

Ex – 1) A transmission line has the following parameters

$R = 2\,\Omega/m$, $G = 0.5$ mmho/m, $f = 1$ GHz

$L = 8$ nH/m, $C = 0.23$ pF

Calculate (a) the characteristic impedance (b) propagation constant

Ans :- $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{2 + j\,.\,2\pi\,.\,10^9 \times 8\times10^{-9}}{0.5\times10^{-3} + j\; 2\,.\,\pi\,.\,10^9 \times 0.23\times10^{-12}}}$

$= \sqrt{\frac{2 + j\,16\pi}{10^{-3}(0.5 + j\,.\,0.46\pi)}} = \sqrt{\frac{50.31\,\angle 87.72^\circ}{15.29\times10^{-4}\,\angle 70.91^\circ}}$

[use Rectangular to Polar conversion in calculator]

$$\therefore \quad Z_0 = \sqrt{\frac{50.31 \times 10^4}{15.29}} \cdot \angle \left(\frac{87.72 - 70.91}{2} \right)$$

$$Z_0 = 181.39 \angle 8.40^\circ \quad \{\text{Polar form}\}$$

$$\boxed{Z_0 = 179.44 + j\, 26.50} \text{ ohm} \quad [\text{Rectangular form}]$$

(b) Propagation constant, (γ)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(50.31 \angle 87.72^\circ)(15.29 \times 10^{-4} \cdot \angle 70.91)}$$

$$= \sqrt{(50.31 \times 15.29 \times 10^{-4})(\angle 158.63^\circ)}$$

$$= 0.2774 \angle 79.31^\circ \quad \left(\because \frac{158.63}{2} \simeq 79.31^\circ \right)$$

$$\gamma = 0.051 + j\, 0.273$$

2009-BPUT - 2marks

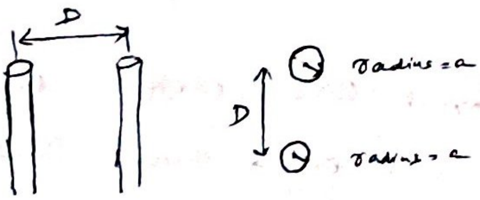
1) Write down 2 difference bet^n a parallel ~~wire~~ wire line and a Co-axial line.

Ans :

Parallel wire line

1) It contains 2-parallel conductors or wires without shielded.

2.



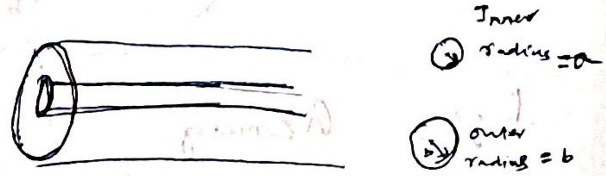
3. More power loss due to radiation.

4. It is less effective in unbalanced load condition

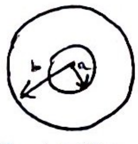
Co-axial line

1. It contains 2-parallel conductors in a shielded tube.

2.



i.e



3. Less radiation, power is lost

4. It is more effective in unbalanced load condition.

The terminated transmission line :-

In the analysis of the solutions of transmission-line eq's, the traveling wave along the line contains two components: one travelling in +ve Z-direction and other traveling the -ve Z direction. If the load impedance is equal to the line characteristic impedance, however, the reflected traveling wave doesn't exist.

Figure 6, shows a transmission line terminated in an impedance Z_L.

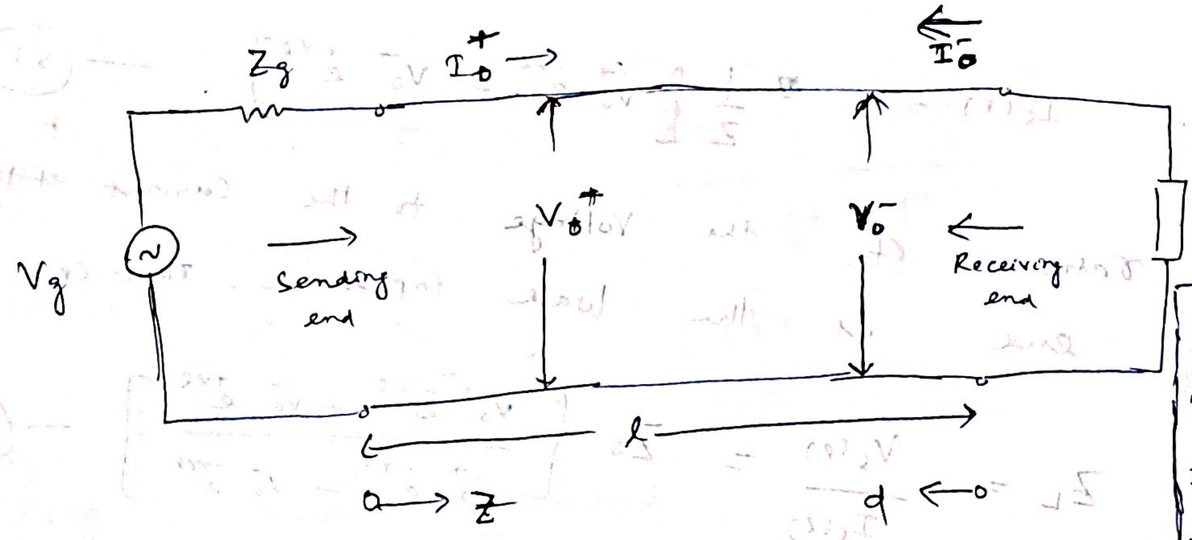


fig 6:- Transmission line terminated in a load impedance.

Note:-
$z \rightarrow$ distance of a point from source end.
$d \rightarrow$ distance from receiving end.
V_g = Voltage at generating end.
$Z_g \rightarrow$ Impedance of generator.
$V_0^+ \rightarrow$ Incident voltage Amplitude
$V_0^- \rightarrow$ Reflected voltage Amplitude
for Similarly Current.
and $l = z + d$

It is usually more convenient to start solving the transmission line problem from the receiving end rather than the sending ~~side~~ end, since the voltage-to-current relationship at the load point is fixed by the load impedance. The incident voltage and current waves traveling along the

The transmission line are given by

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \quad — (47)$$

$$I_s(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z} \quad — (48)$$

in which current wave can be expressed in terms of the voltage by

$$I_s(z) = \frac{V_o^+ e^{-\gamma z}}{Z_o} - \frac{V_o^-}{Z_o} . e^{+\gamma z} \quad — (49) \quad \left[\text{Refer eqn 16}\right]$$

If the line has a length 'l', the voltage and current at the receiving end become

$$V_s(l) = V_o^+ e^{-\gamma l} + V_o^- e^{+\gamma l} \quad —— (50)$$

$$I_s(l) = \frac{1}{Z_o}\left[V_o^+ e^{-\gamma l} - V_o^- e^{+\gamma l}\right] \quad —— (51)$$

The ratio of the voltage to the current at the receiving end is the load impedance. That is,

$$Z_L = \frac{V_s(l)}{I_s(l)} = Z_o\left[\frac{V_o^+ e^{-\gamma l} + V_o^- e^{+\gamma l}}{V_o^+ e^{-\gamma l} - V_o^- e^{+\gamma l}}\right] \quad — (52)$$

The reflection coefficient, which is designated by Γ (gamma), is defined as

Voltage reflection co-efficient $(\Gamma_v) = \dfrac{\text{Reflected voltage wave}}{\text{Incident voltage wave}}$

V. Imp

$$\boxed{\Gamma_v = \frac{V_o^- e^{+\gamma l}}{V_o^+ e^{-\gamma l}}} \quad —— (53)$$

Similarly Current reflection Coefficient $(\Gamma_i) = \dfrac{I_0^- e^{+\gamma l}}{I_0^+ e^{-\gamma l}}$ —— (54)

Generally reflection coefficient means voltage reflection coefficient, $\Gamma_v = \Gamma$.

$$\therefore \Gamma = \frac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}} \quad \text{—— (55)}$$

From eqn (52),

$$Z_L = \frac{Z_0 \cdot \cancel{V_0^+ e^{-\gamma l}}\left[1 + \dfrac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}}\right]}{\cancel{V_0^+ e^{-\gamma l}}\left[1 - \dfrac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}}\right]}$$

$$\Rightarrow Z_L = \frac{Z_0 (1+\Gamma)}{(1-\Gamma)}$$

$$\Rightarrow \frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

Using Componendo & dividendo,

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(1+\Gamma) - (1-\Gamma)}{(1+\Gamma) + (1-\Gamma)} = \frac{\cancel{1}+\Gamma \cancel{-1}+\Gamma}{1+\cancel{\Gamma}+1-\cancel{\Gamma}}$$

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\cancel{2}\Gamma}{\cancel{2}}$$

v. Imp

$$\Rightarrow \boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}} \quad \text{—— (56)}$$

If $Z_L = Z_0$ i.e. the terminating or load impedance is equal to the characteristic ~~load~~ impedance then

$$\Gamma = 0, \quad \text{[from equation (56)]}$$

and there will be no reflection from the receiving end. Thus a terminating impedance that differs from the characteristic impedance will create a reflected wave travelling toward the source from the termination.

Note:- $\Gamma = 0$

$\Rightarrow \dfrac{V_0^- e^{-\gamma l}}{V_0^+ e^{\gamma l}} = 0$

$\Rightarrow V_0^- e^{-\gamma l} = 0$

$\Rightarrow$ Reflected Voltage $= 0$

Current reflection Coefficient

$$I_s(l) = I_0^+ e^{-\gamma l} + I_0^- e^{+\gamma l}, \quad \text{from eq}^n \text{ (48)}$$

$$V_s(l) = I_0^+ Z_0 e^{-\gamma l} + (-I_0^- Z_0) \cdot e^{+\gamma l}, \quad \left[\because Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}\right]$$

from eq^n (47)

$$\frac{V_s(l)}{I_s(l)} = \frac{Z_0 \left[I_0^+ e^{-\gamma l} - I_0^- e^{+\gamma l}\right]}{\left[I_0^+ e^{-\gamma l} + I_0^- e^{+\gamma l}\right]}$$

$$\because \Gamma_i = \frac{I_0^- e^{+\gamma l}}{I_0^+ e^{-\gamma l}}$$

$$= \frac{Z_0 \cdot \cancel{I_0^+ e^{-\gamma l}} \left[1 - \dfrac{I_0^- e^{+\gamma l}}{I_0^+ e^{-\gamma l}}\right]}{\cancel{I_0^+ e^{-\gamma l}} \left[1 + \dfrac{I_0^- e^{+\gamma l}}{I_0^+ e^{-\gamma l}}\right]}$$

$$Z_L = \frac{Z_0 \cdot [1 - \Gamma_i]}{[1 + \Gamma_i]}$$

$$\Rightarrow \frac{Z_L}{Z_0} = \frac{1 - \Gamma_i}{1 + \Gamma_i}$$

Using componendo and dividendo,

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(1-\Gamma_i) - (1+\Gamma_i)}{(1-\Gamma_i) + (1+\Gamma_i)}$$

$$\Rightarrow \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\cancel{1} - \Gamma_i - \cancel{1} - \Gamma_i}{1 - \cancel{\Gamma_i} + 1 + \cancel{\Gamma_i}} = \frac{-\cancel{2}\Gamma_i}{\cancel{2}}$$

$$\Rightarrow \boxed{\Gamma_i = \frac{Z_0 - Z_L}{Z_0 + Z_L} = -\Gamma_v} \quad \text{—— (57)}$$

$\therefore$ Current reflection coefficient is -ve of voltage reflection coefficient.

Transmission Coefficient :-

A transmission line terminated in its characteristic impedance Z_0 is called a properly terminated line. Otherwise it is called an improperly terminated line. There is a reflection coefficient 'Γ' at any point along an improperly terminated line. According to the principle of conservation of energy, the incident power minus the reflected power must be equal to the power transmitted to the load.

Thus, Transmission coeffient 'T', is defined as

$$T = \frac{\text{Transmitted Voltage}}{\text{Incident voltage}} = \frac{\text{Transmitted Current}}{\text{Incident Current}}$$

or $$T = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}} \quad \text{—— (58)}$$

Consider a length 'l' of transmission line, terminated at load impedance (Z_L). Let the travelling wave at the receiving end be,

$$V_{tr}\, e^{-\gamma l} = V_0^{+} \cdot e^{-\gamma l} + V_0^{-} \cdot e^{+\gamma l} \quad \text{—— (59)}$$

$$I_{tr}\, e^{-\gamma l} = I_0^{+}\, e^{-\gamma l} + I_0^{-}\, e^{+\gamma l} \quad \text{—— (60)}$$

Dividing eq^n (59), by $V_0^{+} e^{-\gamma l}$, we have

$$\frac{V_{tr}}{V_0^{+}} = 1 + \frac{V_0^{-}\, e^{+\gamma l}}{V_0^{+}\, e^{-\gamma l}}$$

$$\Rightarrow \frac{V_{tr}}{V_0^{+}} = 1 + \Gamma_v$$

$$\Rightarrow \boxed{T = 1 + \Gamma_v} \quad \text{—— (61)}$$

$$\left[\because T = \frac{V_{tr}}{V_{inc}} = \frac{V_{tr}}{V_0^{+}} \right]$$

$$\Rightarrow T = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow T = \frac{Z_L + \cancel{Z_0} + Z_L - \cancel{Z_0}}{Z_L + Z_0}$$

Imp $\Rightarrow$

$$\boxed{T = \frac{2 Z_L}{Z_L + Z_0}} \quad \text{—— (62)}$$

Transmitted power = Incident power − Reflected power —— (63)

~~Incident~~

~~Power = V~~ Incident voltage, $= V_0^+ e^{-\gamma l}$

$= V_0^+ e^{-(\alpha + j\beta)l}$

~~Incident~~

~~Power = $\frac{V^2}{Z}$~~

$= V_0^+ e^{-\alpha l} \cdot e^{-j\beta l}$

$\left(V_{rms} = \frac{(V_m)\,\text{Real}}{\sqrt{2}}\right)$

$= V_0^+ \cdot e^{-\alpha l} \, [\cos\beta l - j\sin\beta l]$

In this case power,

$$P_{inc} = \frac{\left(\frac{V_0^+ e^{-\alpha l}}{\sqrt{2}}\right)^2}{Z_0} = \frac{\left(V_0^+ e^{-\alpha l}\right)^2}{2Z_0} \quad \text{— (64)}$$

Repeating the same for, transmitted and reflected power, eqn (63), becomes, $\overrightarrow{Z_0}$ $\overleftarrow{Z_0}$

(Power transmitted to the load) $\} Z_L$

$$\frac{\left(V_{tr}\, e^{-\alpha l}\right)^2}{\cancel{2} Z_L} = \frac{\left(V_0^+ e^{-\alpha l}\right)^2}{\cancel{2} Z_0} - \frac{\left(V_0^- e^{+\alpha l}\right)^2}{\cancel{2} Z_0}$$

$$\Rightarrow \frac{Z_0}{Z_L} \cdot \left(V_{tr} \cdot e^{-\alpha l}\right)^2 = \left(V_0^+ e^{-\alpha l}\right)^2 - \left(V_0^- e^{+\alpha l}\right)^2$$

$$\Rightarrow \frac{Z_0}{Z_L} \cdot \left(V_{tr} \cdot e^{-\alpha l}\right)^2 = \left(V_0^+ e^{-\alpha l}\right)^2 \left[1 - \left(\frac{V_0^- e^{+\alpha l}}{V_0^+ e^{-\alpha l}}\right)^2\right]$$

$$\Rightarrow \frac{Z_0}{Z_L} \cdot \left(\frac{V_{tr}\, e^{-\alpha l}}{V_0^+ e^{-\alpha l}}\right)^2 = 1 - \Gamma_v^2$$

$$\because \Gamma_v = \frac{V_0^- e^{+\alpha l}}{V_0^+ e^{-\alpha l}}$$

$$T = \frac{V_{tr} \cdot e^{-\alpha l}}{V_0^+ e^{-\alpha l}}$$

$$\Rightarrow \frac{Z_0}{Z_L} \cdot T^2 = 1 - \Gamma_v^2$$

$$\Rightarrow \boxed{T^2 = \frac{Z_L}{Z_0}\left(1 - \Gamma_v^2\right)}$$

or

Simply

$$\boxed{T^2 = \frac{Z_L}{Z_0}\left(1 - \Gamma^2\right)} \quad \text{— (65)}$$

Ex-2 :- A certain transmission line has a characteristic impedance of $75 + j\,0.01\,\Omega$ and is terminated in a load impedance of $70 + j\,50\,\Omega$. Compute (a) reflection coefficient; (b) the transmission coefficient (c) verify that $T^2 = \frac{Z_L}{Z_0}(1 - \Gamma^2)$ (eq^n 65) (d) verify that $T = 1 + \Gamma$ (eq^n 61)

Ans :- Given,

$$Z_0 = 75 + j\,0.01\,\Omega$$

$$Z_L = 70 + j\,50\,\Omega$$

(a) Reflection Coefficient $(\Gamma) = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{(70 + j50) - (75 + j0.01)}{(70 + j50) + (75 + j0.01)}$$

$$= \frac{-5 + j49.99}{145 + j50.01}$$

$$= \frac{50.24 \angle 95.71^\circ}{153.38 \angle 19.03^\circ}$$

$$= \frac{50.24}{153.38} \left(\angle (95.71) - (19.03) \right)$$

$$= 0.33 \angle 76.68^\circ$$

$$\Gamma = 0.076 + j\,0.32$$

(Ans)

(b) Transmission coefficient (T) $= \dfrac{2Z_L}{Z_L + Z_0}$

$$T = \frac{2 \times (70 + j50)}{(70 + j50) + (75 + j0.01)} = \frac{140 + j100}{153.38 \angle 19.03^\circ}$$

(As derived earlier)

$$= \frac{172.05 \angle 35.54^\circ}{153.38 \angle 19.03^\circ}$$

$$T = 1.12 \angle 16.51^\circ$$

$$\boxed{T = 1.08 + j0.32}$$

(c) L.H.S

$$T^2 = [1.12 \angle 16.51^\circ]^2$$

$$T^2 = 1.25 \angle 33.02$$

$\left(\because 2 \times 16.51 = 33.02 \right)$

R.H.S

$$\frac{Z_L}{Z_0}(1 - \Gamma^2)$$

$$= \frac{70 + j50}{75 + j0.01}\left[1 - (0.33 \angle 76.68^\circ)^2\right]$$

$$= \frac{86 \angle 35.54^\circ}{75 \angle 0.01^\circ}\left[(1) - (0.109 \angle 153.36)\right]$$

$\left(\because 2 \times 76.68 = 153.36 \right)$

$$= \frac{86 \angle 35.54}{75 \angle 0.01} \times \left[1 - (-0.097 + j0.049)\right]$$

$$= \frac{86 \angle 35.54}{75 \angle 0.01} \times (1.097 - j0.049)$$

$$\therefore \frac{Z_L}{Z_0}\left[1-\Gamma^2\right] = \frac{86 \angle 35.54}{75 \angle 0.01} \times 1.10 \angle -2.55$$

$$= \left(\frac{86 \times 1.10}{75}\right) \angle \left[35.54 - 0.01 - 2.55\right]$$

$$= 1.26 \angle 33^\circ \qquad \left(\underset{=}{L.H.S}\ R.H.S\right)$$

(d) <u>To verify</u>, $T = 1 + \Gamma$

$$T = 1.08 + j0.32 \qquad (L.H.S)$$

$$1 + \Gamma = 1 + (0.076 + j0.32)$$

$$1 + \Gamma = 1.076 + j0.32 \qquad (R.H.S)$$

$$L.H.S \simeq R.H.S$$

Time-Average power flow along the line

For a lossless transmission line, terminated by an arbitrary load impedance Z_L,

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \quad \text{--- (66)}$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot e^{-j\beta z} - \frac{V_0^-}{Z_0} \cdot e^{j\beta z} \quad \text{--- (67)}$$

$\left(\because \gamma = \alpha + j\beta\right.$ lossless line, $\alpha = 0$, $\gamma = j\beta$)

$$\Gamma = \left.\frac{V_0^-}{V_0^+}\right|_{at\ z=0} = \text{Reflection Coefficient}$$

$$\Rightarrow V_0^- = \Gamma V_0^+$$

Eq^n (66) becomes,

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma e^{j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{j\beta z} \right],$$

Time-avg power flow along the line at the point z;

$$P_{av} = \frac{1}{2} Re \left[V(z) . I^*(z) \right]$$

$$= \frac{1}{2} . Re \left\{ V_0^+ \left[e^{-j\beta z} + \Gamma e^{j\beta z} \right] . \frac{V_0^+}{Z_0} \left[e^{+j\beta z} - \Gamma^* e^{-j\beta z} \right] \right\}$$

$$= \frac{1}{2} \frac{|V_0^+|^2}{Z_0} Re \left\{ 1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{+2j\beta z} - |\Gamma|^2 \right\}$$

$$\boxed{P_{av} = \frac{1}{2} \frac{|V_0^+|^2}{Z_0} \left[1 - |\Gamma|^2 \right]} \leftarrow \left\{ \text{only Real part} \right]$$

Which shows that the avg. power flow is constant at any point on the line. and the total power delivered to the load (P_{av}) is equal to the incident power $\left(\frac{|V_0^+|^2}{2Z_0} \right)$, minus the reflected power $\left(\frac{1}{2Z_0} . |V_0^+|^2 . |\Gamma|^2 \right)$.

→ If $\Gamma = 0$, max^m power is delivered to the load.
If $\Gamma = 1$, no power is delivered.
(All reflected)

→ Imp Return loss:-

When the load is mismatched, not all of the available power from the generator is delivered to the load. This "loss" is called return loss (RL) and is defined (in dB) as

$$\boxed{RL = -20 \log |\Gamma| \ dB},$$

So, for a matched load ($\Gamma = 0$), $RL = \infty$ dB.

When total reflection takes place, $\Gamma = 1$, $RL = 0$ dB.

* (Derivation not required)

I/P Impedance of a transmission line (Z_{in}) or line impedance (Z_{in})

Real power flow on the line is a constant but that the voltage amplitude, at least for a mismatched line, is oscillatory with position on the line. Thus the impedance seen looking into the line varies with position.

The line impedance of a transmission line is the complex ratio of the voltage phasor at any point to the current phasor at that point. It is defined as,

$$Z = \frac{V_s(z)}{I_s(z)}$$

Lets consider a transmission line of length 'l' is terminated by load impedance Z_L. In general, the voltage and current along a line is sum of respective incident and reflected wave

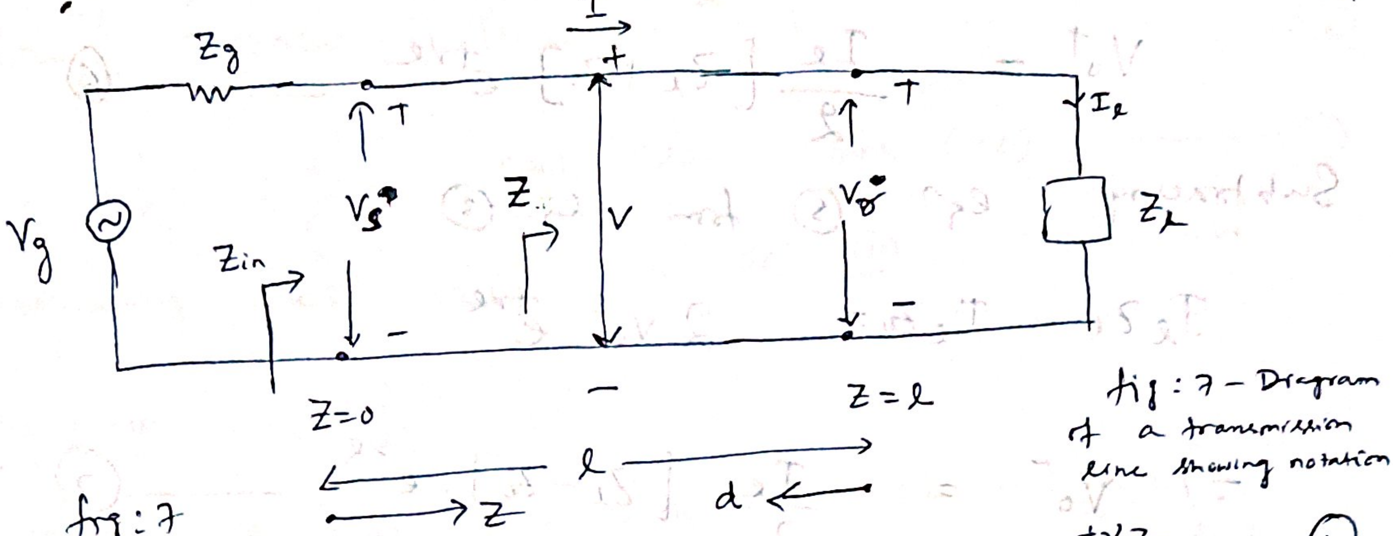


fig : 7

fig : 7 – Diagram of a transmission line showing notation

i.e
$$V_s(z) = V_{inc} + V_{ref} = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad \text{—} (1)$$

$$I_s(z) = I_{inc} + I_{ref} = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad \text{—} (2)$$

Line impedance can be expressed in terms of Z_L & Z_0. At $z = \ell$, $V_r = I_\ell . Z_\ell$.

→ Eqn (1) becomes,

$$I_\ell . Z_\ell = V_0^+ e^{-\gamma \ell} + V_0^- e^{+\gamma \ell} \quad \text{—} (3)$$

Eqn (2), can be written as

$$I_s(z) = \frac{V_0^+}{Z_0} . e^{-\gamma z} - \frac{V_0^-}{Z_0} . e^{+\gamma z}$$

~~At z=ℓ,~~ $\Rightarrow I_s(z) . Z_0 = V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}$ — (4)

At $z=\ell$, eqn (4) becomes,

$$I_\ell . Z_0 = V_0^+ e^{-\gamma \ell} - V_0^- e^{+\gamma \ell} \quad \text{—} (5)$$

Solving eqn (3) & (5),

Adding eqn (3) + (5)

$$I_\ell Z_\ell + I_\ell Z_0 = 2 V_0^+ e^{-\gamma \ell}$$

$$V_0^{+} = \frac{I_l}{2}\left[Z_l + Z_0\right] e^{+\gamma l} \quad \text{——— (6)}$$

Subtracting eq^n (5) from eq^n (3)

$$I_l Z_l - I_l Z_0 = 2 V_0^{-} e^{+\gamma l}$$

$$\Rightarrow V_0^{-} = \frac{I_l}{2}\left[Z_l - Z_0\right] . e^{-\gamma l} \quad \text{——— (7)}$$

Substituting eq^n (6), (7) in eq^n (1), ~~we have~~ and letting $z = l - d$, we obtain

$$V_s(z) = \frac{I_l}{2}\left[Z_l + Z_0\right] . e^{+\gamma l} . e^{-\gamma(l-d)} + \frac{I_l}{2}(Z_l - Z_0) . e^{-\gamma l} . e^{+\gamma(l-d)}$$

$$V_s(z) = \frac{I_l}{2}\left[(Z_l + Z_0) . e^{\gamma d} + (Z_l - Z_0) . e^{-\gamma d}\right] \quad \text{——— (8)}$$

Substituting eq^n (7), in eq^n (4), we have and letting $z = l - d$,

$$I_s(z) = \frac{1}{Z_0}\left[\frac{I_l}{2}(Z_l + Z_0) . e^{+\gamma l} . e^{-\gamma(l-d)} - \frac{I_l}{2}(Z_l - Z_0) . e^{-\gamma l} . e^{+\gamma(l-d)}\right]$$

$$I_s(z) = \frac{I_l}{2 Z_0}\left[(Z_l + Z_0)\, e^{\gamma d} - (Z_l - Z_0) . e^{-\gamma d}\right] \quad \text{——— (9)}$$

$$Z_{in} = \frac{V_s(z)}{I_s(z)}$$, dividing eq^n (8) by eq^n (9), we have

$$Z = Z_0 \left[\frac{(Z_l + Z_0) . e^{\gamma d} + (Z_l - Z_0) . e^{-\gamma d}}{(Z_l + Z_0) . e^{\gamma d} - (Z_l - Z_0) . e^{-\gamma d}}\right] \quad \text{——— (10)}$$

The above eqn can be simplified using hyperbolic function

$$e^{\pm \gamma z} = \cosh(\gamma z) \pm \sinh(\gamma z) \quad \text{——— (11)}$$

Substituting eqn (11), in eqn (10), we have

Numerator

$$(Z_L + Z_0) \cdot (\cosh \gamma d + \sinh \gamma d) + (Z_L - Z_0)\left[\cosh \gamma d - \sinh \gamma d\right]$$

$$= Z_L \cosh \gamma d + Z_L \cancel{\sinh \gamma d} + Z_0 \cancel{\cosh \gamma d} + Z_0 \sinh \gamma d$$
$$+ Z_L \cosh \gamma d - Z_L \cancel{\sinh \gamma d} - Z_0 \cancel{\cosh \gamma d} + Z_0 \sinh \gamma d$$

$$= 2\left[Z_L \cosh \gamma d + Z_0 \sinh \gamma d\right]$$

Denominator

$$(Z_L + Z_0) \cdot (\cosh \gamma d + \sinh \gamma d) - (Z_L - Z_0) \cdot \left[\cosh \gamma d - \sinh \gamma d\right]$$

$$= Z_L \cancel{\cosh \gamma d} + Z_L \sinh \gamma d + Z_0 \cosh \gamma d + Z_0 \cancel{\sinh \gamma d}$$
$$- Z_L \cancel{\cosh \gamma d} + Z_L \sinh \gamma d + Z_0 \cosh \gamma d - Z_0 \cancel{\sinh \gamma d}$$

$$= 2\left[Z_0 \cosh \gamma d + Z_L \sinh \gamma d\right]$$

$\therefore$ Eqn (10), becomes,

$$Z = Z_0 \cdot \left[\frac{Z_L \cosh \gamma d + Z_0 \sinh \gamma d}{Z_0 \cosh \gamma d + Z_L \sinh \gamma d}\right] \quad \text{— (12)}$$

For lossless line, $\gamma = j\beta$ $\quad (\because \alpha = 0)$

$$\sinh \gamma z = \sinh (j\beta z) = j \sin\beta z \quad \text{--- (13)}$$

$$\cosh \gamma z = \cosh (j\beta z) = J \cos\beta z \quad \text{--- (14)}$$

Putting eqn (13) & (14) in eqn (12), we have

$$Z = Z_0 \left[\frac{Z_\ell (j\cos\beta d) + Z_0 (j \sin\beta d)}{Z_0 (j \cos\beta d) + Z_\ell (j \sin\beta d)} \right]$$

$$\Rightarrow Z = Z_0 \left[\frac{Z_\ell \cos\beta d + j Z_0 \sin\beta d}{Z_0 \cos\beta d + j Z_\ell \sin\beta d} \right]$$

v. imp

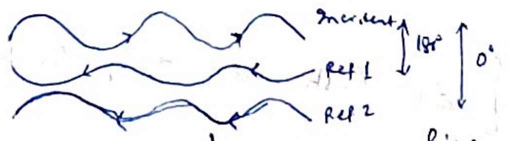
$$\boxed{Z = Z_0 \left[\frac{Z_\ell + j Z_0 \tan\beta d}{Z_0 + j Z_\ell \tan\beta d} \right]} \quad \text{--- (15)}$$

~~Case-I~~ When $d = \ell$,

v.v. imp

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_\ell + j Z_0 \tan\beta \ell}{Z_0 + j Z_\ell \tan\beta \ell} \right]} \quad \text{--- (16)}$$

Standing wave :-



In case of improperly terminated line i.e. when line terminated with $Z_L \neq Z_0$ [e.g. 1) $Z_L = 0$, (short circuit) or 2) $Z_L = \infty$ (open circuit) etc.] there will be reflection from the load end resulting in the reflected wave from the load end to generator end i.e. in a direction

Opposite to that of travelling wave. So, In a mismatched terminated line, the incident & reflected wave interfere to produce a standing wave pattern along the line. These two oppositely directed travelling wave interfere at some points constructively and at some other points destructively gives rise to maximum and minimum voltage and results standing wave.

The general solutions of the transmission-line consist of 2 waves travelling in opposite directions with unequal amplitude is given by,

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$= V_0^+ e^{-(\alpha+j\beta)z} + V_0^- e^{(\alpha+j\beta)z}$$

$$= V_0^+ \cdot e^{-\alpha z} \cdot e^{-j\beta z} + V_0^- e^{\alpha z} \cdot e^{+j\beta z}$$

$$= V_0^+ e^{-\alpha z}\left[\cos\beta z - j\sin\beta z\right] + V_0^- e^{\alpha z}\left[\cos\beta z + j\sin\beta z\right]$$

$$V_s(z) = \left[V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}\right]\cos\beta z - j\left[V_0^+ e^{-\alpha z} - V_0^- e^{\alpha z}\right]\sin\beta z$$

1. The maximum amplitude is

$$V_{max} = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}$$

$$\boxed{V_{max} = V_0^+ e^{-\alpha z}\left[1 + |\Gamma_v|\right]} \quad (17) \qquad \left(\because |\Gamma_v| = \frac{V_0^- e^{\alpha z}}{V_0^+ e^{-\alpha z}}\right)$$

This occurs at $\beta z = n\pi$, where $n = 0, \pm 1, \pm 2$ ---

and $\sin\beta z = 0$ and $V_s(z) = \left[V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}\right]\cos\beta z$

2) The min^m amplitude is,

$$V_{min} = V_0^+ e^{-\alpha z} - V_0^- e^{+\alpha z}$$

This occurs at $\beta z = (2n-1)\frac{\pi}{2}$ where

$n = 0, \pm 1, \pm 2 \ldots$

$$\therefore V_{min} = V_0^+ e^{-\alpha z} \left[1 - |\Gamma_v| \right] \quad \text{—— (18)}$$

3) Distance between two successive maxima or minima is one-half of wavelength. $\left(\frac{\lambda}{2}\right)$.

[e.g.:- $n = 1$, $\beta z = \frac{\pi}{2}$; $n = 2$, $\beta z = \frac{3\pi}{2}$ } Betn 2 successive minima, Phase diff $= \frac{3\pi}{2} - \frac{\pi}{2} = \pi$

for 2π phase difference $\rightarrow \lambda$

π " " $\rightarrow \frac{\lambda}{2}$

or $\beta z = n\pi$
$\Rightarrow \frac{2\pi}{\lambda} \cdot z = n\pi$
$\Rightarrow z = n \cdot \frac{\lambda}{2}$
$n = 0, \pm 1, \pm 2 \ldots$]

4) Similarly,

$$I_{max} = I_0^+ e^{-\alpha z} (1 + |\Gamma_i|) \quad \text{—— (19)}$$

$$I_{min} = I_0^+ e^{-\alpha z} (1 - |\Gamma_i|) \quad \text{—— (20)}$$

The standing wave pattern of 2 oppositely traveling waves with unequal amplitude in lossy or lossless line are shown in fig (8) and (9).

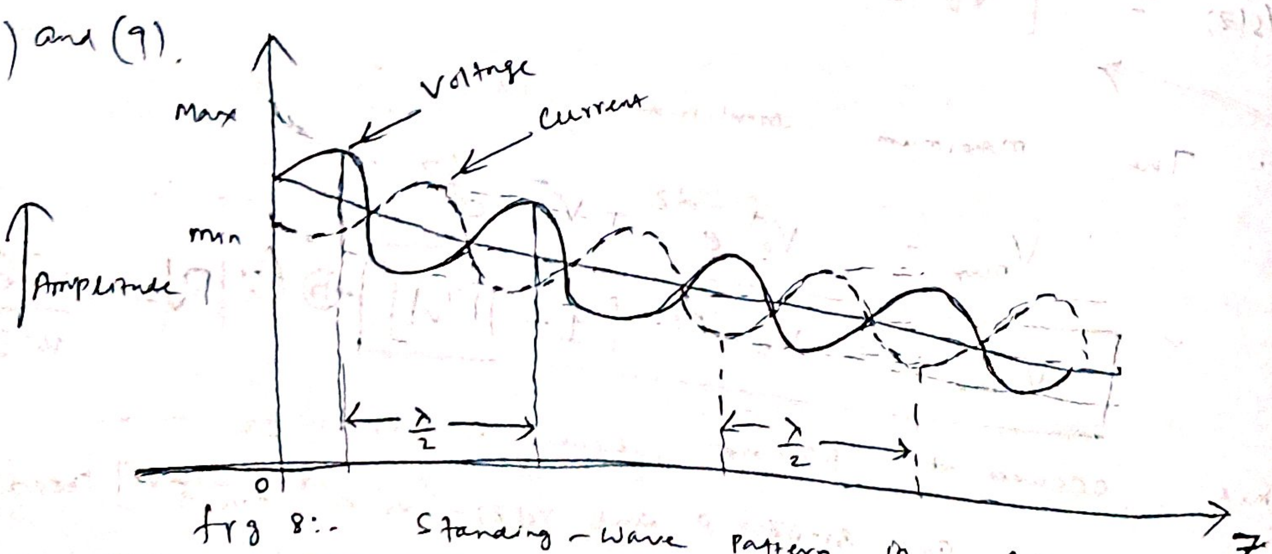


fig 8:- Standing-wave pattern in a lossy line.

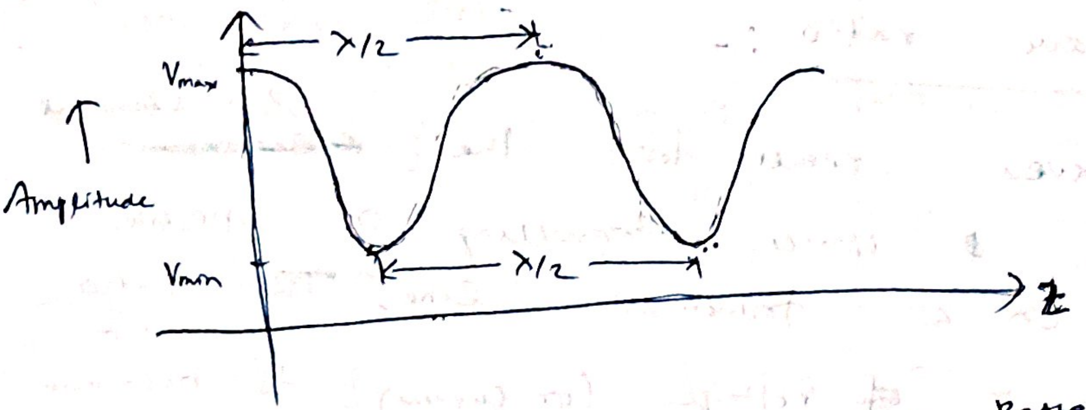


fig 9:- Voltage Standing wave pattern in a loss less line.

5) $V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

when $V_0^+ \neq 0$ and $V_0^- = 0$, then ~~the standing wave~~ ~~becomes~~,

$$\boxed{V_s(z) = V_0^+ e^{-\gamma z}}$$ (Pure travelling wave) — (21)

when $V_0^+ = 0$, $V_0^- \neq 0$

$$\boxed{V_s(z) = V_0^- e^{\gamma z}}$$ (Pure reflecting wave) — (22)

when $\boxed{V_0^+ = V_0^- = V_0}$

$$V_s(z) = V_0 \left(e^{-\gamma z} + e^{\gamma z}\right)$$

$$\boxed{V_s(z) = 2 V_0 \cosh(\gamma z)} \quad (23)$$

$\cosh(\gamma z) = \frac{e^{-\gamma z} + e^{\gamma z}}{2}$

$$V_s(z) = 2 V_0 \cosh(\alpha + j\beta) z$$

For lossless line $\alpha = 0$, $V_s(z) = 2 V_0 \cosh(j\beta z)$

$$\boxed{V_s(z) = 2 V_0 \cos(\beta z)} \quad (24)$$

$(\because \cosh(j\beta z) = \cos \beta z)$

(Pure Standing wave)

Standing-wave ratio :-

Standing waves result from the simultaneous ~~[illegible]~~ presence of waves travelling in opposite directions on a transmission line. The ratio of the max^m voltage (or current) to minimum voltage (or current) is defined as standing-wave ratio.

→ Interferes constructively → V_{max} occurs
→ " des " → V_{min} "

$$VSWR = \frac{V_{max}}{V_{min}} \quad (25) \qquad , \quad ISWR = \frac{I_{max}}{I_{min}} \quad (26)$$

(Voltage Standing wave ratio) (Current Standing wave ratio)

$$VSWR(S) = \frac{V_{max}}{V_{min}} = \frac{\cancel{V_0^+ e^{-\alpha z}}(1+|\Gamma|)}{\cancel{V_0^+ e^{-\alpha z}}(1-|\Gamma|)} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

V. imp

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}} \quad (27)$$

$0 \leq |\Gamma| \leq 1$ (Magnitude)

$$\boxed{\begin{matrix} -1 \leq \Gamma \leq 1 \\ 1 \leq S < \infty \end{matrix}}$$

when $\Gamma = 0$, $S = 1$
$\Gamma = \pm 1$, $S = \infty$

If $Z_L = 0$, Short circuit, $\Gamma = -1$
If $Z_L = \infty$, open circuit, $\Gamma = 1$
If $Z_L = Z_0$, Matched load, $\Gamma = 0$

Note:- $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$, $Z_L \to \infty$ $\Gamma = \lim_{Z_L \to \infty} \left(\frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}} \right)$

$\Gamma = 1 \rightarrow V_0^+ = V_0^-$

1) For pure travelling wave, $V_0^- = 0$, $|\Gamma| = 0$, $S = 1$ (CAO)

2) For pure standing wave, $V_0^- = V_0^+$, $|\Gamma| = 1$, $S = \infty$

3) VSWR (S) = 1, means No reflection, Matching load ($Z_L = Z_0$) ($\Gamma = 0$)

From eqⁿ (27)

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Using Componendo & dividendo,

$$\frac{\rho-1}{\rho+1} = \frac{\cancel{1}+|\Gamma|-\cancel{1}+|\Gamma|}{1+\cancel{|\Gamma|}+1-\cancel{|\Gamma|}} = \frac{\cancel{2}|\Gamma|}{\cancel{2}}$$

$$\Rightarrow \boxed{|\Gamma| = \frac{\rho-1}{\rho+1}} \quad \text{— (28)}$$

② Note:-

1) $\Gamma = 0$, No reflected wave.

2) $\Gamma = 1$, Incident & reflected wave are in same phase. 100% reflection.

3) $\Gamma = -1$, Incident & reflected wave are 180° out of phase. 100% reflection.

① → $0 \leq |\Gamma| \leq 1$ (Reflection Coefficient magnitude lies betn 0 to 1)

→ Standing wave ratio only defined for lossless line. [on lossy line VSWR changes from position to position]

Input Impedance in different Cases

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] \quad \left[\text{from eq}^n\ 16\right]$$

<u>Case-I</u> (Line is terminated in a <u>short ckt</u>, $Z_L = 0$)

$$Z_{in} = Z_0 \left[\frac{j\cancel{Z_0}\tan\beta l}{\cancel{Z_0}}\right]$$

$$\boxed{\begin{aligned} Z_{in} &= jZ_0 \tan\beta l \\ Z_{sc} &= |Z_{in}|_{Z_L=0} = jZ_0 \tan\beta l \end{aligned}} \quad \text{— (29)}$$

So the i/p impedance at short ckt line is pure <u>reactive</u>, It may be <u>capacitive</u> or <u>Inductive</u> depending upon the value of 'l'.

Note:-
$Z_{in} = +jx$ (Inductive)
$= -jx$ (Capacitive)

$$\Gamma_v = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1.$$

Voltage reflection coefficient $\Gamma_v = -1$.

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$

Case - II :- Open circuited line $(Z_L = \infty)$

$$Z_{oc} = \lim_{Z_L \to \infty} Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$= \lim_{Z_L \to \infty} Z_0 \left[\frac{1 + j\frac{Z_0}{Z_L}\tan\beta l}{\frac{Z_0}{Z_L} + j\tan\beta l} \right]$$

$$= Z_0 \left[\frac{1+0}{0 + j\tan\beta l} \right] \qquad \because \frac{1}{j} = \frac{j}{j^2} = -j$$

$$\boxed{Z_{oc} = -jZ_0 \cot\beta l} \quad — (30)$$

So i/p impedance for open circuited line is also reactive. It may be capacitive or inductive depending upon the value of 'l'.

Note:-
$Z_{in} = +jx$ (Inductive)
$= -jx$ (Capacitive)

reflection coefficient

$$\Gamma_v = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad (\Gamma_v)_{oc} = \lim_{Z_L \to \infty} \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$(\Gamma_v)_{oc} = \lim_{Z_L\to\infty} \frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}} = \frac{1-0}{1+0} = 1.$$

$$\boxed{(\Gamma_v)_{oc} = 1}$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty.$$

$$\therefore Z_{sc} = j\tan\beta l \,.\, Z_0$$

$$Z_{oc} = -j Z_0 \,.\, \cot\beta l$$

$$Z_{sc}\,.\,Z_{oc} = j\,\cancel{\tan\beta l}\,.\,Z_0\,.\,(-j)\,.\,Z_0\,.\,\cancel{\cot\beta l}$$

$$\boxed{Z_{sc}\,.\,Z_{oc} = Z_0^2} \quad \text{(31)}$$

Z_{sc} = Short circuited Impedance
Z_{oc} = Open circuited Impedance
Z_0 = Characteristic Impedance

Case-3 (Terminated by Matched load $Z_L = Z_0$)

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

$$= Z_0 \left[\frac{(\cancel{Z_0} + j\cancel{Z_0}\tan\beta l)}{(\cancel{Z_0} + j\cancel{Z_0}\tan\beta l)}\right]$$

$$\boxed{Z_{in} = Z_0} \text{ or } \boxed{Z_{in} = Z_L} \quad \text{(32)} \quad (\because Z_0 = Z_L)$$

Reflection coefficient $(\Gamma) = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0}{1-0} = 1.$$

$\Gamma = 0$, No reflection.

$\therefore$ Max^m power is transferred during this matched load condition.

Special Cases

(i) $l = \frac{\lambda}{2}$,
or
(l = multiple of $\frac{\lambda}{2}$)

$\tan\beta l = \tan \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{2} = \tan\pi = 0$

$\tan\beta l = 0$.

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$= \cancel{Z_0} \left[\frac{Z_L}{\cancel{Z_0}} \right] \qquad (\because \tan\beta l = 0)$$

$$\Rightarrow \boxed{Z_{in} = Z_L} \quad — (33)$$

$\therefore$ A half-wavelength line (or any multiple of $\lambda/2$) does not alter or transform the load impedance, regardless of the characteristic impedance. Z_{in} is purely resistive if Z_L is purely resistive.

(ii) $l = \frac{\lambda}{4}$

$\tan\beta l = \tan \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{4_2} = \tan\frac{\pi}{2} = \infty$

$$\therefore \quad Z_{in} = \lim_{\tan\beta l \to \infty} Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$= \lim_{\tan\beta l \to \infty} Z_0 \left[\frac{\frac{Z_L}{\tan\beta l} + jZ_0}{\frac{Z_0}{\tan\beta l} + jZ_L} \right]$$

$$Z_{in} = \frac{Z_0 \times \cancel{j} Z_0}{\cancel{j} Z_L} = \frac{Z_0^2}{Z_L}$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}} \quad \text{—— (34)}$$

From eqn (31) & (34)

$$\boxed{Z_0^2 = Z_{sc} \cdot Z_{oc}} \rightarrow \text{[True for any length '} l \text{']}$$

and $\boxed{Z_0^2 = Z_{in} \cdot Z_L} \rightarrow$ [True only for $l = \frac{\lambda}{4}$]

Thus, $l = \frac{\lambda}{4}$, line is known as quater-wave transformer because it has the effect of transforming the load impedance, in an inverse manner, $\left(Z_{in} = \frac{Z_0^2}{Z_L} \right)$, depending on the characteristic impedance of the line.

$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L} = \frac{1}{\left(\frac{Z_L}{Z_0} \right)}$$

$\Rightarrow$ Normalized i/p impedance $= \dfrac{1}{\text{Normalized load impedance}}$

Insertion Loss :-

$$\boxed{\text{Insertion loss} = -20\log|T| \ dB}$$ — (35)

where T = Transmission Coefficient $\left[T = 1+\Gamma\right]$

$$T = \frac{2Z_L}{Z_L+Z_0}$$ (As discussed earlier)

Decibel / Neper / dBm

* 1) The ratio of two power levels, P_1 & P_2, in microwave system is expressed as,

$$10\log\frac{P_1}{P_2}, \ dB.$$ — (36)

→ voltage, $$20\log\frac{V_1}{V_2}, \ dB.$$ — (37)

The ratio of 2 voltages across equal load resistance can expressed in terms of

→ 2) Neper as, $$\boxed{\ln\frac{V_1}{V_2} \ Np}$$ — (38)

Corresponding expression for power,

$$\boxed{\frac{1}{2}\ln\frac{P_1}{P_2} \ Np}$$, (39) (∵ $P = \frac{V^2}{R}$ → $V = \sqrt{P.R}$

$$\boxed{1 \ Np = 8.686 \ dB}$$ — (40)

Not required

Proof :- Power Gain

~~Power~~, in dB

$$G = 10 \log_{10}\left(\frac{P_1}{P_2}\right) \text{ dB}$$

$$= 10 \cancel{\log} \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln 10} \text{ dB} \quad \left(\because \log_{10} x = \frac{\ln x}{\ln 10}\right)$$

$$= \left(\frac{10}{\ln 10}\right) \cdot \left(\ln \frac{P_1}{P_2}\right) \text{ dB}$$

$$G = \left(\frac{10}{\ln 10}\right) 2 \times \left[\frac{1}{2} \ln \frac{P_1}{P_2}\right] \text{ dB} \quad \text{—— (41)}$$

$$\cancel{\frac{20}{\ln 10} \text{ Np}}$$

$$\boxed{\cancel{1 \text{ dB} = 8.686 \text{ Np}}}$$

Power gain in Neper,

$$G = \left[\frac{1}{2} \ln \frac{P_1}{P_2}\right], \text{ Np.} \quad \text{—— (42)}$$

Dividing eqn (41), by 42

$$1 = \frac{20}{\ln 10} \frac{\text{dB}}{\text{Np}}$$

$$1 = 8.686 \frac{\text{dB}}{\text{Np}}$$

$$\Rightarrow \boxed{1 \text{ Np} = 8.686 \text{ dB}} \quad \text{—— (43)}$$

3) If reference power $P_2 = 1 \text{ mW}$, power gain (G)

~~1 dBm~~ $G = 10 \log \frac{P_1}{1 \text{ mW}}$, dBm,

If $P_1 =$ 1 Watt, $G = 10 \log 10^3 = 30$ dBm.

Ex-3 A transmission line has a characteristic impedance of $50 + j0.01\,\Omega$ is terminated in a load impedance $73 - j42.5\,\Omega$.

Calculate

(a) The reflection coefficient

(b) The standing-wave ratio

Ans:

Given,

$$Z_0 = 50 + j0.01\,\Omega$$

$$Z_L = 73 - j42.5\,\Omega$$

(a) Reflection coefficient $(\Gamma) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

~~$\Gamma = (50 + j0.01)$~~

$$\Gamma = \frac{(73 - j42.5) - (50 + j0.01)}{(73 - j42.5) + (50 + j0.01)}$$

$$= \frac{23 - j42.51}{123 - j42.49}$$

$$= \frac{48.33 \angle -61.58}{130.13 \angle -19.05}$$

$$= 0.371 \angle(-61.58 + 19.05)$$

$$\Gamma = 0.371 \angle -42.53^\circ$$

(b) $$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= \frac{1+0.377}{1-0.377}$$

$$VSWR = 2.17$$

Ex-4: BPUT-2010 A certain transmission line has a characteristic impedance $75 + j0.01\,\Omega$ and is terminated in a load impedance of $70 + j50\,\Omega$. Calculate (i) (a) Reflection Coefficient
(b) SWR
(c) Transmission Coefficient
(d) Insertion loss.

Ans :- (a) Reflection Coefficient

$$Z_0 = 75 + j0.01\,\Omega$$

$$Z_L = 70 + j50\,\Omega.$$

[As derived in Ex-2 Page - 34]

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.076 + j0.32\,\Omega$$

$$= 0.33 \angle 76.68^\circ$$

(b) VSWR

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.33}{1-0.33} = 1.98$$

(c) Transmission Coefficient

$$T = 1+\Gamma = 1 + (0.076 + j0.32) = 1.076 + j0.32\,\Omega$$

(a) Insertion loss

$$IL = -20 \log |T| \text{ dB}$$

$$= -20 \log |(1.076 + j0.32)|$$

$$= -20 \log 1.12$$

$$\boxed{IL = -0.98 \text{ dB}}$$

(ii) A lossless Co-axial cable is used to delay a pulse by 10ns. The Inductance and Capacitance of the Cable are 0.20 μH/m and 60 PF/m respectively. Determine the length of Cable?

Ans:-

$$L = 0.20 \frac{\mu H}{m}$$

$$C = 60 \text{ PF/m}$$

Phase velocity, $V_P = \dfrac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{0.20 \times 10^{-6} \times 60 \times 10^{-12}}}$$

$$= \frac{1}{\sqrt{12 \times 10^{-18}}}$$

$$= \frac{1}{3.46 \times 10^{-9}}$$

$$V_P = 2.88 \times 10^{8} \text{ m/sec.}$$

time taken = 100 ns

$$\Rightarrow \frac{l}{V_P} = 100 \times 10^{-9}$$

$$\Rightarrow l = 100 \times 10^{-9} \times 2.88 \times 10^{8}$$

$$= 288 \times 10^{-1}$$

$$\boxed{l = 28.8 \text{ meter}}$$

(iii) A 25 m long lossless transmission line is terminated with a load having an equivalent impedance of $40 + j30\,\Omega$ at 10 MHz. The inductance and capacitance of the line are 310.4 nH/m and 38.28 pF/m respectively. Calculate,

(a) Characteristic impedance

(b) Phase Constant

(c) Input impedance at the sending end and

(d) Input impedance at the midpoint of line.

Ans :- $l = 25\,m$, $Z_L = 40 + j30\,\Omega$, $f = 10\,MHz$

$L = 310.4\,\frac{nH}{m}$, $C = 38.28\,\frac{pF}{m}$.

(a) Characteristic Impedance

For lossless line [Eqn (22), page 18]

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{310.4 \times 10^{-9}}{38.28 \times 10^{-12}}}$$

$$= \sqrt{\frac{3104 \times 10^{2}}{38.28}} = 90.04\,\Omega$$

$$Z_0 = 90.04\,\Omega$$

(b) Phase Constant

$$\cancel{\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}}$$

$c = f\lambda$

$\Rightarrow \lambda = \frac{c}{f}$

$c = V_p$

$$\cancel{= \frac{2\pi \times 10 \times 10^6}{3 \times 10^8} = \frac{2\pi}{30}}$$

$$\cancel{\beta = 0.209}$$

$$\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC}$$

$$\beta = 2\times\pi\times 10\times10^6 \sqrt{310.4\times10^{-9}\times 38.28\times10^{-12}}$$

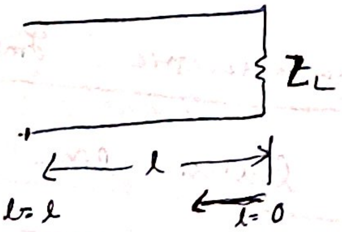
$$= 2\times\pi\times10\times10^6\times 34.47\times10^{-10}$$

$$\beta = 0.216\ \frac{rad}{m}$$

(c) I/P impedance at the sending end.

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

At the sending end



l = length of the line

$l = 25\,m$

~~$\tan\beta l = \tan(0.216\times25) =$~~

$$\beta l = 0.216\ \frac{rad}{m}\times 25\ m = 5.4\ rad.$$

$\tan\beta l = -1.21$, $Z_0 = 90.04\,\Omega$, $Z_L = 40 + j30\,\Omega$

$$Z_{in} = 90.04 \left[\frac{(40+j30\Omega) + j.(90.04).(-1.21)}{90.04 + j(40+j30).(-1.21)} \right]$$

$$= 90.04 \left[\frac{40 - j78.94}{126.34 \cancel{+} j48.4} \right]$$

$$= \frac{90.04 \times 88.49 \angle -63.12^\circ}{135.29 \angle \cancel{30} -20.96^\circ}$$

$$= 58.89 \angle \cancel{30.48} \angle -42.15 \quad \left(-63.12 + 20.96 = \cancel{-1.4} - 42.15 \right)$$

$$\boxed{(Z_{in})_{\text{sending end}} = \cancel{43.66 \angle -39.51} \; \cancel{19.73 - j59.32} \; 43.66 - j39.51}$$

(iv) Z_{in} at the midpoint of the line

$$l = \frac{25}{2} = 12.5 \text{ meter.}$$

$$\tan\beta l = \tan(0.216 \times 12.5) = \tan(2.7\,\text{rad}) = -0.472$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$= 90.04 \left[\frac{(40+j30) + j(90.04)(-0.472)}{90.04 + j(40+j30)(-0.472)} \right]$$

$$= 90.04 \left[\frac{40 - j12.5}{\cancel{9}104.2 - j18.88} \right]$$

$$= \frac{90.04 \times 41.90 \angle -17.35}{105.89 \angle -10.27} = 35.62 \angle -7.08 = 35.34 - j4.39$$

$$\therefore \boxed{(Z_{in})_{\substack{\text{At mid point}\\ \text{of transmission line}}} = 35.34 - j\,4.39\,\Omega}$$

Ex - r BPUT 2010

The Characteristic impedance of 10m long lossless less Co-axial cable is 50Ω. The dielectric ~~Constant~~ material between the inner and outer Conductor of the cable has $\epsilon_r = 3.5$ and $\mu_r = 1$. If the radius of the inner Conductor is 1 mm,

* (a) What should be the outer radius of the Cable?

(b) Find R, L, C, G parameters of Co-axial transmission line. [f = 10 kHz]

Ans :-

$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right) \quad \left[\because Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{2\pi}\ln(b/a)}{\frac{2\pi\epsilon}{\ln(b/a)}}} = \sqrt{\frac{\mu \ln^2(\frac{b}{a})}{2\pi \cdot 2\pi\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{2\pi} \cdot \ln\frac{b}{a} = \frac{\eta}{2\pi}\ln\frac{b}{a}\right]$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\mu_r}{\epsilon_r}}$

(Intrinsic wave Impedance)

$$\eta = 120\pi \times \sqrt{\frac{1}{3.5}} = 201.51$$

$$\left(\because \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad \mu_0 = 4\pi\times10^{-7}\ \frac{H}{m}, \quad \epsilon_0 = \frac{10^{-9}}{36\pi}\ \frac{F}{m}, \quad \sqrt{\frac{4\pi\times10^{-7}\times36\pi}{10^{-9}}} = \sqrt{144\times\pi^2\times10^2} = 120\pi\right)$$

$$Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow 50 = \frac{201.51}{2\pi}\ln\left(\frac{b}{1}\right)$$

$$\Rightarrow \frac{100\pi}{201.51} = \ln_e b$$

$$\Rightarrow b = e^{1.559} = 4.754\text{ mm} \quad \text{(Ans)}$$

(b) $R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

$= \frac{50}{2\pi}\left[\frac{1}{1\times10^{-3}} + \frac{1}{4.754\times10^{-3}}\right]$

$= \frac{50}{2\pi}\left[1.21\times10^{3}\right]$

$R = 9.63\ k\Omega$

$L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$

$= \frac{\mu_0 \cdot \mu_r}{2\pi}\ln\left(\frac{4.754}{1}\right)$

$= \frac{\cancel{4\pi}^{2}\times10^{-7}\times1}{\cancel{2\pi}}\ln(4.754)$

$= \frac{3.11\times10^{-7}\times10^{-2}}{10^{-2}}$

$\boxed{L = 311\ \frac{nH}{m}}$

$C = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi\ \epsilon_0\cdot\epsilon_r}{\ln(4.754)}$ $\quad 2\cancel{\pi}\times\left(\frac{10^{-9}}{36\cancel{\pi}}\right)_{18}\times3.5\times\frac{1}{1.558}$

$= 0.1248\times10^{-9}$

$\boxed{C = 124.8\ pF/m}$

$G = \frac{2\pi\omega\epsilon'}{\ln(b/a)} = 2\pi\times(2\pi\times100\times10^{3})\times\left(\frac{10^{-9}}{36\pi}\right)\times3.5\times\frac{1}{1.558}$

$\left(\because \omega = 2\pi f = 2\pi\times100\,KHZ\right),\ \epsilon_0 = \frac{-10^{-9}}{36\pi}$

$= 0.784\times10^{-4} = 78.4\ \frac{\mu S}{m}.$

$\boxed{G = 78.4\ \frac{\mu S}{m}}$

Smith Chart

Many of the computations required to solve transmission-line problems involves the use of rather complicated equations. The solution of such problems is tedious and difficult because the accurate manipulation of numerous equation is necessary. To simplify their solution, we need a graphic method of arriving at a quick answer.

The Smith Chart, developed by Phillip H. Smith, is a a very useful tool, for solving transmission line problems.

→ ~~In Smith Chart, we plot~~

→ The Smith Chart consists of a plot of the normalized impedance or admittance with angle and magnitude of a generalized complex reflection coefficient in a unity circle.

→ The chart is applicable to the analysis of a lossless line as well as lossy line.

→ It is used to find out

(a) Unknown impedance at any distance 'l' on the transmission line.

(b) VSWR and reflection coefficient

(c) Return loss and transmission coefficient

(d) Quarter wave matching and stub matching

→ A Smith chart is a graph that enables any

Complex impedance to be plotted

→ It can use any value of resistance from 0 to ∞ and reactance from $-j\infty$ to $+j\infty$.

→ By using Smith Chart we can get a graphical solution very quickly with accuracy.

* Mathematical Analysis *

[Derivation not required for exam point of view. For knowledge you can refer how these const resistance & reactance circle came in Smith Chart]

Smith Chart consist of 2 part

(i) Normalized Resistance

(ii) Normalized Reactance

We know reflection coefficient, (Γ)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow \frac{\Gamma}{1} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Using Componendo & dividendo

$$\frac{\Gamma - 1}{\Gamma + 1} = \frac{(Z_L - Z_0) - (Z_L + Z_0)}{(Z_L - Z_0) + (Z_L + Z_0)}$$

$$\Rightarrow \frac{\Gamma - 1}{\Gamma + 1} = \frac{\cancel{Z_L} - Z_0 - \cancel{Z_L} - Z_0}{Z_L - \cancel{Z_0} + Z_L + \cancel{Z_0}} = -\frac{\cancel{2}Z_0}{\cancel{2}Z_L}$$

$$\Rightarrow \frac{\Gamma - 1}{\Gamma + 1} = -\frac{Z_0}{Z_L}$$

$$\Rightarrow \frac{Z_0}{Z_L} = \frac{1 - \Gamma}{1 + \Gamma}$$

$$\Rightarrow \boxed{\frac{Z_L}{Z_0} = \frac{1 + \Gamma}{1 - \Gamma}} \quad \text{—— (51)}$$

Normalized Impedance, defined as, $\dfrac{\text{Impedance}}{Z_0 \text{ (Characteristic Impedance)}}$

$\therefore$ Z = Normalized load Impedance = $\dfrac{Z_L}{Z_0}$.

$$Z = \frac{1+\Gamma}{1-\Gamma} \quad \text{——— (52)}$$

Let Γ consist of real & imaginary part.

Z consist of real & imaginary part.

$$\Gamma = \Gamma_r + j\Gamma_i \quad \text{——— (53)}$$

$$Z = r + jx \quad \text{——— (54)}$$

Using eqn (53) & (54) in eqn (52), we have

$$r + jx = \frac{1 + (\Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)} = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$= \frac{\{(1+\Gamma_r) + j\Gamma_i\}\{(1-\Gamma_r) + j\Gamma_i\}}{\{(1-\Gamma_r)^2 + \Gamma_i^2\}}$$

[Multiply and divide by $(1-\Gamma_r) + j\Gamma_i$]

$$= \frac{1 - \Gamma_r^2 + j\Gamma_i(1+\Gamma_r) + j\Gamma_i(1-\Gamma_r) - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Equating the real & imaginary part, we have

$$r = \left\{ \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \right\} \quad \text{——— (55)}$$

$$jx = \frac{+j\Gamma_i + j\Gamma_i\cancel{\Gamma_r} + j\Gamma_i - j\cancel{\Gamma_i}\Gamma_r}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2} \quad \text{—— (56)}$$

[Rearranging eqn 55, i.e

$$r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$\Rightarrow \quad r(1-\Gamma_r)^2 + r\cdot\Gamma_i^2 = 1-\Gamma_r^2-\Gamma_i^2$$

$$\Rightarrow \quad r(1+\Gamma_r^2-2\Gamma_r) + (r+1)\Gamma_i^2 + \Gamma_r^2 - 1 = 0$$

$$\Rightarrow \quad r + r\Gamma_r^2 - 2r\Gamma_r + (1+r)\Gamma_i^2 + \Gamma_r^2 - 1 = 0$$

$$\Rightarrow \quad (1+r)\Gamma_r^2 - 2r\Gamma_r + (1+r)\Gamma_i^2 = 1-r$$

$$\Rightarrow \quad \Gamma_r^2 - 2\cdot\Gamma_r\cdot\frac{r}{1+r} + \Gamma_i^2 = \frac{1-r}{1+r}$$

$$\Rightarrow \quad \Gamma_r^2 - 2\cdot\Gamma_r\cdot\frac{r}{1+r} + \left(\frac{r}{1+r}\right)^2 - \left(\frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r}$$

$$\Rightarrow \quad \left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{1+r}\right)^2$$

$$\Rightarrow \quad \left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{(1-r)(1+r)+r^2}{(1+r)^2} = \frac{1-\cancel{r^2}+\cancel{r^2}}{(1+r)^2}$$

$$\Rightarrow \quad \left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \quad \text{—— (57)}$$

Eqⁿ (57) represents a <u>family of circles</u> in which each circle has a constant resistance 'r'. The radius of any circle is $\frac{1}{1+r}$ and center of any circle is $\frac{r}{1+r}$ along the real axis in the unity circle, where r varies from zero to ∞. All constant resistance circles are plotted in fig 17, according to eqⁿ (57).

Center $\left(\frac{r}{1+r}, 0\right)$, radius $= \frac{1}{1+r}$

→ If $r = 0$, then $C(0,0)$ and $R = 1$

→ If $r = 1$, then $C(0.5, 0)$ and $R = \frac{1}{2}$

→ If $r = 2$, $C\left(\frac{2}{3}, 0\right)$ and $R = \frac{1}{3}$

→ If $r = \infty$ $C(1, 0)$ and $R = 0$

$\left[\because C = \left(\frac{r}{1+r}, 0\right) = \left(\frac{1}{\frac{1}{r}+1}, 0\right)\right.$
$r \to \infty, \; C = (1, 0)$

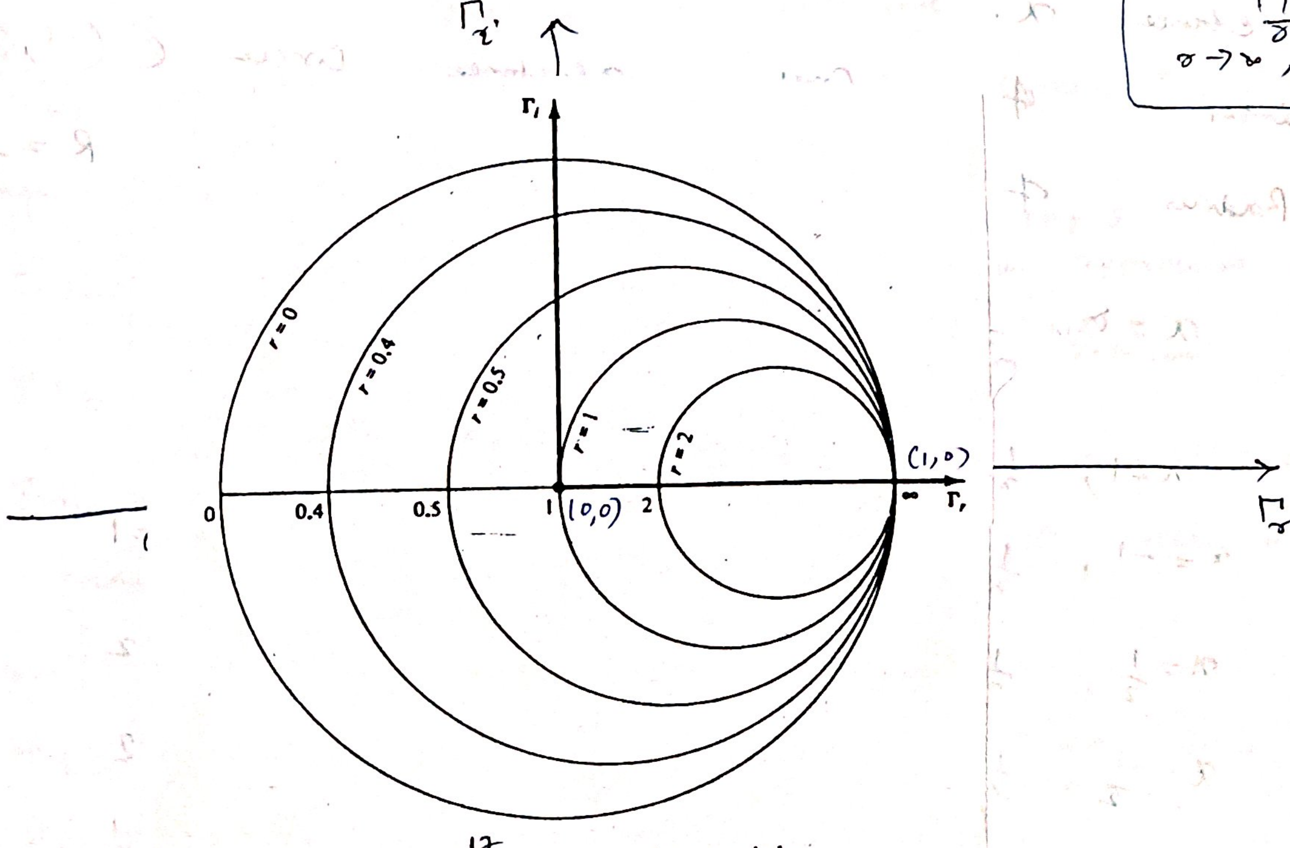


Figure ~~3-5-2~~ 17 Constant resistance r circles.

Rearranging eqn (56), i.e

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\Rightarrow \quad x(1-\Gamma_r)^2 + x(\Gamma_i^2) = 2\Gamma_i$$

$$\Rightarrow \quad (1-\Gamma_r)^2 + \Gamma_i^2 = \frac{2}{x}\Gamma_i$$

$$\Rightarrow \quad (\Gamma_r - 1)^2 + \Gamma_i^2 - 2 \cdot \Gamma_i \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 - \frac{1}{x^2} = 0$$

$$\Rightarrow \quad (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad \text{--- (58)}$$

This equation describes a family of circles, but each circle specifies a constant reactance x.

Center of constant reactance circle $C\left(1, \frac{1}{x}\right)$.

Radius of constant reactance circle $R = \frac{1}{x}$.

If $x = \infty$, $\frac{1}{x} = 0$, then $C(1,0)$ and, $R = 0$

If $x = 1$, $\frac{1}{x} = 1$, then $C(1,1)$, and $R = 1$

If $x = -1$, $\frac{1}{x} = -1$, then $C(1,-1)$, and $R = 1$

If $x = \frac{1}{2}$, $\frac{1}{x} = 2$, then $C[1, 2]$, $R = 2$

If $x = -\frac{1}{2}$, $\frac{1}{x} = -2$, then $C[1, -2]$, $R = 2$

If $x = 2$, $\frac{1}{x} = \frac{1}{2}$ then $C[1, \frac{1}{2}]$, $R = \frac{1}{2}$]

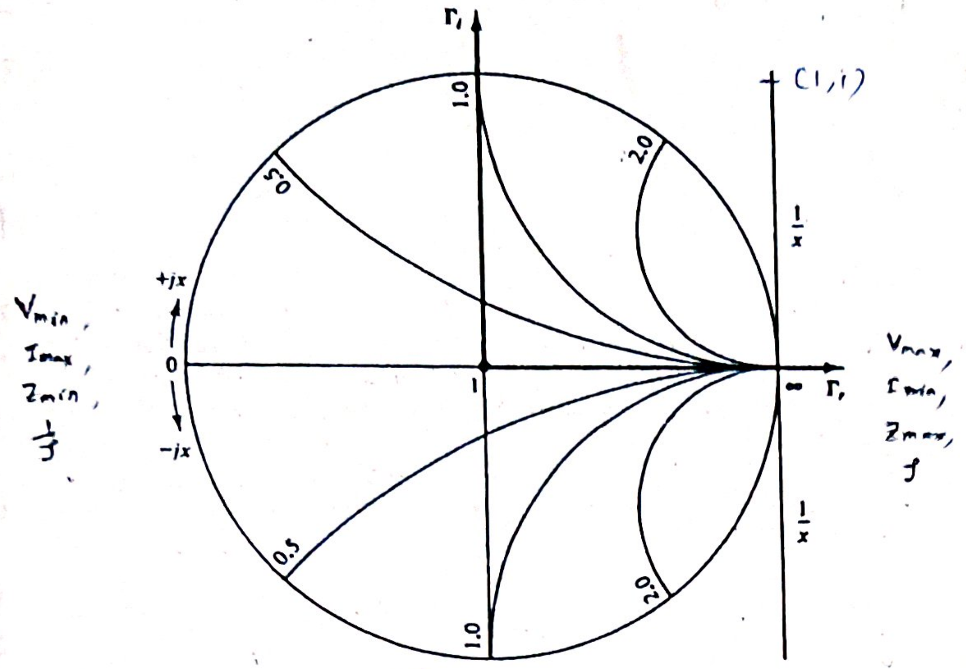


Figure ~~3-5-3~~ 18 Constant reactance x circles.

Characteristics of Smith Chart :-

1) The constant 'r' (resistance) and constant 'x' (reactance) loci form two families of orthogonal circle in the chart.

2) The constant 'r' and constant 'x' circle all pass through the point ($\Gamma_r = 1$, $\Gamma_i = 0$).

3) The upper half of the diagram (fig 18) represents $+jx$.

4) The lower half of the diagram (fig 18) represents $-jx$.

5) For admittance [or upper or lower half of Real axis] the constant 'r' circles become constant 'g' (conductance ~~admittance~~) circles and the constant 'x' circle become constant 'b' (susceptance) circles.

6) The distance around the Smith chart once is one-half wavelength ($\lambda/2$)

7) At a point of $Z_{min} = 1/S$, there is V_{min} on

the line. [Note: ρ = VSWR (Voltage Standing wave Ratio)]

8) At a point $Z_{max} = \rho$, there is V_{max} on the line.

9) The horizontal radius to the right of the Chart Center corresponds to V_{max}, I_{min}, Z_{max} and ~~ρ~~ ρ. ↳ [Refer fig 18]

10) The horizontal radius to the left of the chart Center corresponds to V_{min}, I_{max}, Z_{min} and $\frac{1}{\rho}$.

11) Since the normalized admittance 'y' is a reciprocal of the normalized impedance 'z', the corresponding quantities in the admittance chart are 180° out of phase with those in the impedance chart.

12) The normalized impedance or admittance is repeated for every half wavelength of distance.

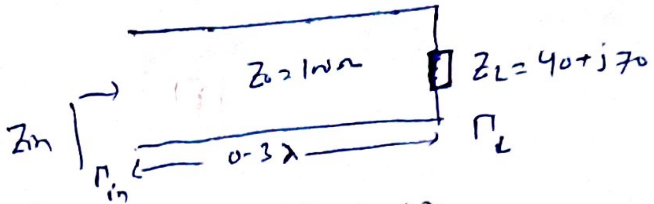
13) The distances are given in wavelengths toward the generator and also toward the load.

Note: (Derived earlier) The magnitude of reflection coefficient is related to standing-wave ratio by the following relation

$$|\Gamma| = \frac{\rho - 1}{\rho + 1}.$$

EX-9:- (Pozar book, ex-2.2, page 67)

A load impedance of $40 + j70\,\Omega$ terminates a $100\,\Omega$ transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the i/p to the line, the input impedance, the SWR on the line and the return loss.



Soln ÷ Steps (Refer the Smith chart attached)

1) First find the normalized impedance. (Because the Smith Chart uses normalized impedance)

Given $Z_0 = 100\Omega$

$Z_L = 40+j70\Omega$

$$z = \frac{Z_L}{Z_0} = \frac{40+j70}{100} = 0.4+j0.7$$

2) Then locate the $0.4+j0.7$ point i.e Find the intersection of 0.4 resistance circle and 0.7 ~~inductance~~ inductive reactance (+jx) circle.

3) Then draw a circle, whose center is the center of Smith Chart (written (1.0)), and radius is distance betn center of Smith Chart and the point $0.4+j0.7$. [use compass to measure the radius]

4) The location intersected by the circle, at the right portion of real axis indicates the SWR. [In this example it is found 3.87]

5) To find reflection coefficient (at the load), use the

Example-9

Join the center with [0.4+j0.7] point, which gives reference of Load position on the wavelength-towards generator scale.

(Step-6) 0.106λ

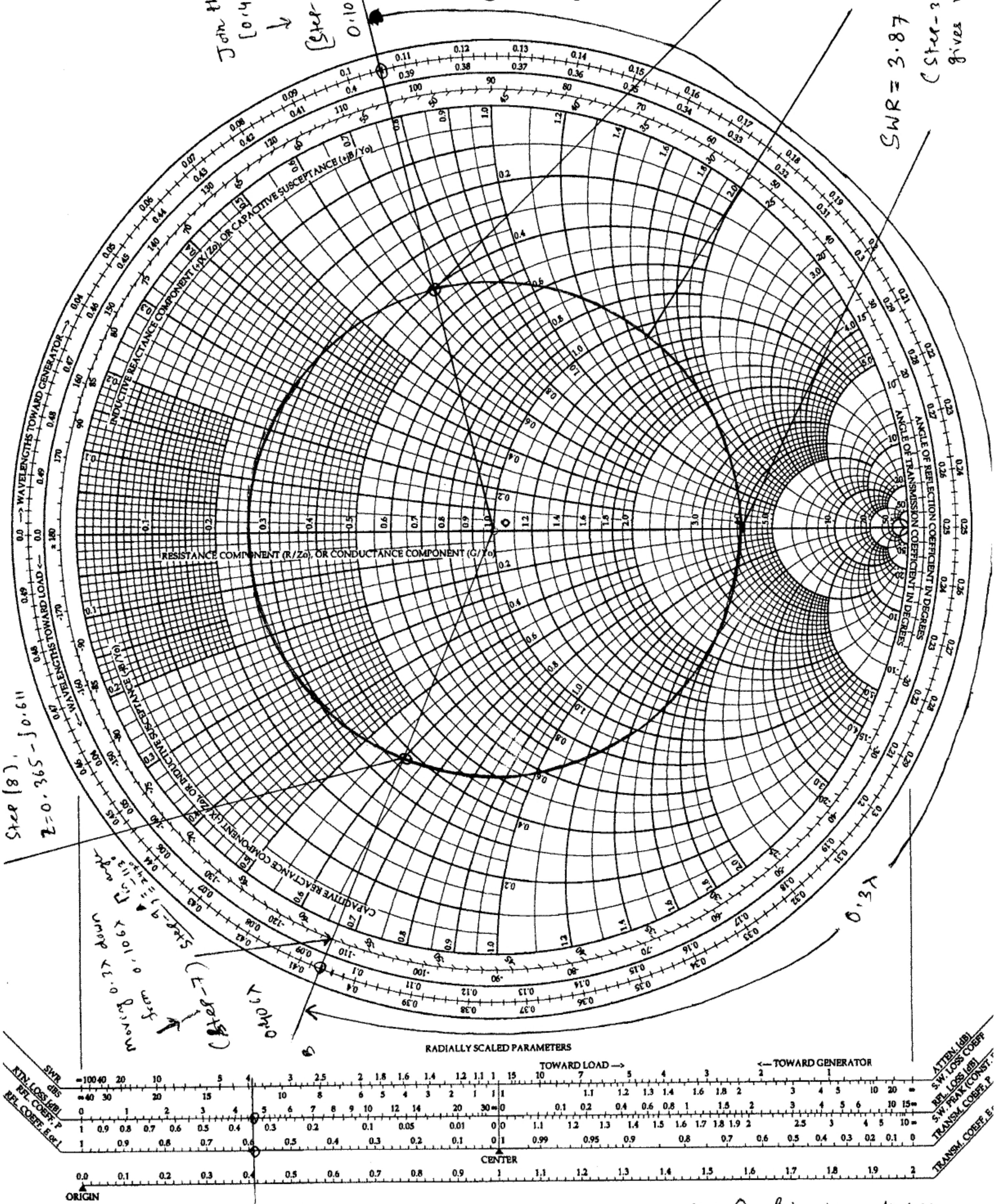
The Complete Smith Chart

Black Magic Design

0.4+j0.7 (Step-1) Locate the Point

(Step-2 - Draw the circle)

SWR = 3.87 (Step-3: Intersection Point gives VSWR)



Step (8), Z = 0.365 - j0.611

(Step-7) Moving 0.3λ down from 0.106λ

0.406λ

0.3λ

Step (4) Γ = 0.59 (Reflection Coefficient E or I), Step (5), Return Loss = 4.6 dB formula $\cancel{|\Gamma| = \frac{S-1}{1+S}}$

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3.87-1}{3.87+1} \simeq 0.59$$

$$|\Gamma| = 0.59$$

or

In the footer of Smith Chart, See the radially Scaled parameters. For SWR 3.87, Check the Corresponding reflection Co-efficient (E or I) i.e Voltage reflection Coefficient or Current reflection Coefficient. [It is found $|\Gamma| = 0.59$.]

6) Return loss (dB) = 4.6 dB. [From the Scaled parameter]

7) Draw the line between, Center of Smith Chart and load impedance point $[0.4+j0.7]$.

on Extension of this line It gives, → reflection Coefficient angle $104°$. $[\Gamma_L = 0.59\angle 104°]$

→ Wavelength towards generator = 0.106λ.

[i.e reference position of the load on the wavelength - towards - generator scale]

8) Since the transmission line has length (0.3λ), moving 0.3λ towards the generator brings us to 0.406λ $[.106+0.3]$ on wavelength towards generator (WTG) scale.

9) Now join with a straight line 0.406λ point and center of Smith Chart point. The intersection of this line with the circle gives normalized i/p impedance. From the chart it is found to be 0.365 (Resistance component) −j0.611 (capacitive reactance) −ve

$$Z = 0.365 - j0.611$$

$$\Rightarrow \frac{Z_{in}}{Z_0} = 0.365 - j0.611$$

$$\Rightarrow Z_{in} = Z_0 (0.365 - j0.611)$$

$$\Rightarrow Z_{in} = 100 (0.365 - j0.611)$$

$$\Rightarrow \boxed{Z_{in} = 36.5 - j61.1} \ \Omega$$

10) The reflection coefficient at the i/p still has magnitude $|\Gamma| = 0.59$. The phase angle can be calculated as follows

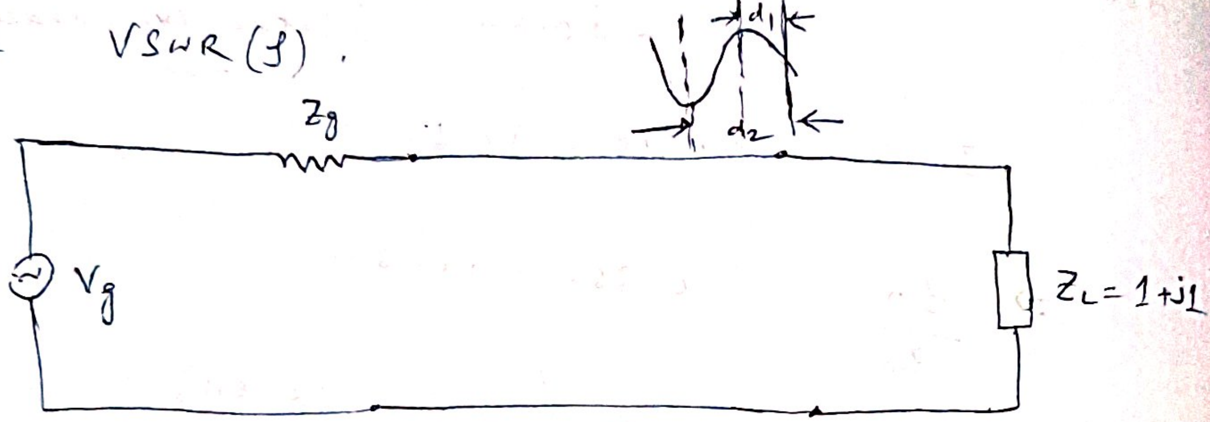
(i) Find the intersection of line OB [line joining center of Smith Chart and 0.406λ] with 'angle of ~~[illegible]~~ reflection coefficient in degree' scale. It gives angle −113°

i.e. $360° - 113° = 247°$.

$$\Gamma_{in} = 0.59 \angle 247°$$

Ex-10 (Liao) ex- 3.5.1 : Location determination of voltage maximum and minimum from load.

Given the normalized load impedance $Z_\ell = 1 + j1$ and the operating wavelength $\lambda = 5$ cm, determine the first V_{max}, first V_{min} from the load, and the VSWR (ρ).



Ans :- 1) Locate $1 + j1$ point on the Smith chart

2) Draw the line between center of Smith Chart and load impedance point $(1 + j1)$. Extension of this line gives, wavelength towards generator $= 0.162\lambda$

3) We know, V_{max} occures at 0.25λ

∴ Distance of V_{max} from load end,

$$d_1 = 0.25\lambda - 0.162\lambda = .088\lambda$$

$$\Rightarrow d_1 = 0.088 (5 \text{ cm})$$

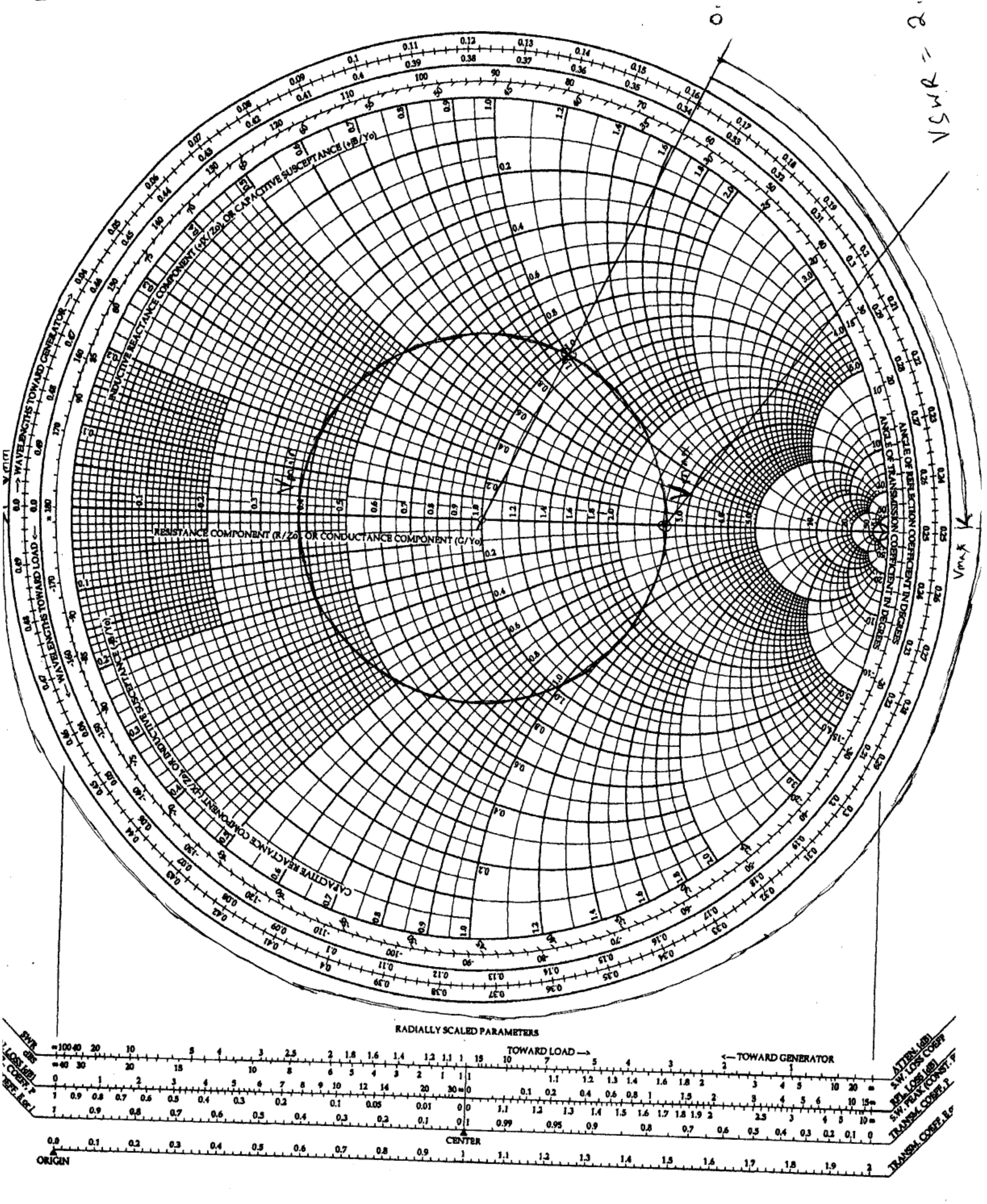
$$\Rightarrow \boxed{d_1 = 0.44 \text{ cm}}$$

4) V_{min} occures at 0.5λ,

Example-10

The Complete Smith Chart

Black Magic Design



distance of V_{min} from the load,

$$d_2 = (0.50 - 0.162)\lambda$$

$$d_2 = 0.338 \times (5\text{ cm})$$

$$\Rightarrow \boxed{d_2 = 1.69\text{ cm}}$$

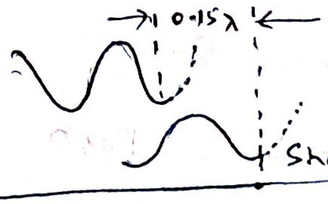
5) VSWR can be found out, by the drawing the circle, center at center of Smith Chart and radius is the distance between center and point $1 + j1$.

6) As derived in earlier problem, the SWR found to be 2.6, from the Smith Chart.

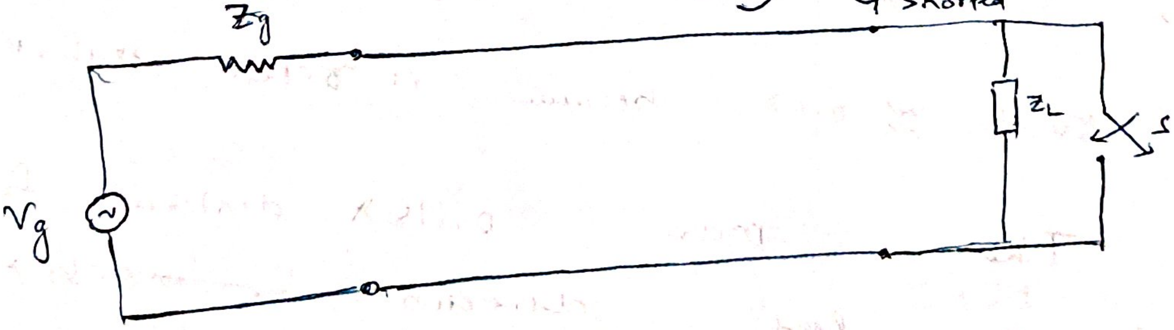
(Liao)

<u>Ex-11</u> :- Determination of unknown Impedance

Given that characteristic impedance of line (Z_0) is $50\,\Omega$, SWR $\rho = 2$, when the line is loaded. When the load is shorted, the minima shift 0.15λ towards the load. Determine the load impedance



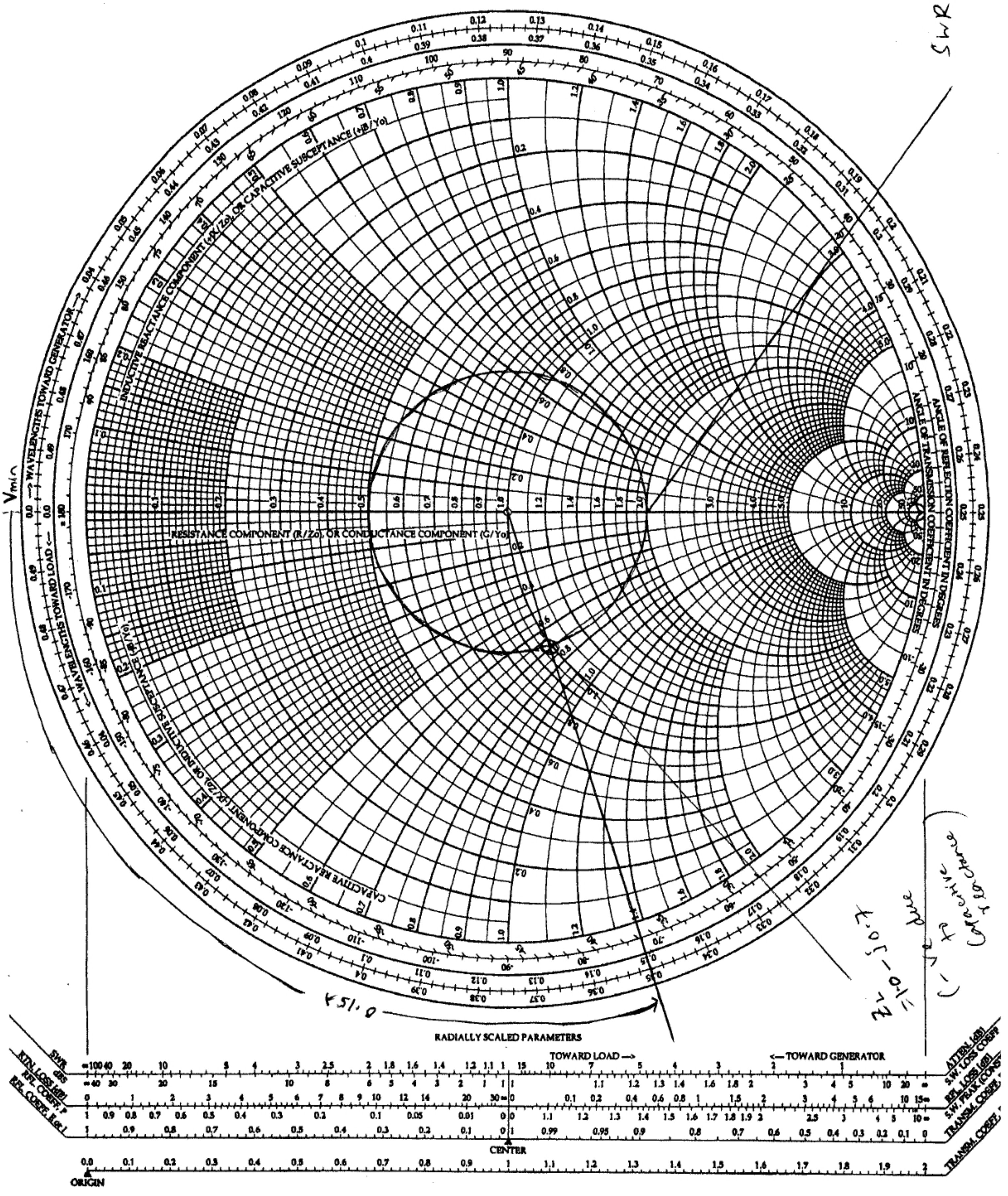
Ans :



Example – 11

The Complete Smith Chart

Black Magic Design



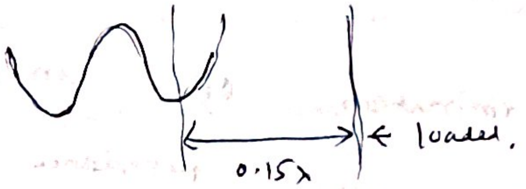
Ans ÷ 1) We know, when the line is shorted, first voltage minimum occures at the place of load.

2) When the line is loaded, the voltage minimum occures at $\underline{0.15\lambda}$ from the load.

3) In the last question, load position was known, we have to go to the voltage minimum position. (Towards generator).

4) Now ~~Voltage minm~~ to find the

6) unknown load position,

7) 

on Ext So we have to move 0.15λ towards the load to get the unknown impedance.

5) So in Smith Chart, first locate the Vmin posn (i.e 0.50λ or 0.0λ)

8) (.50 λ ≃ 0.0 λ because it repeat itself at 0.5λ]

6) Then move 0.15λ distance from Vmin towards load direction. ~~i.e 0.35λ~~

7) Join this point $(0.15\lambda$ point on wavelength towards the load scale) with center of the Smith Chart.

8) Then the draw the circle, corresponds to $VSWR = 2$, from the center of the Smith chart.

9) The intersection of circle & the line gives the unknown impedance value.

10) From the Smith Chart, it is found to be

$$z = 1.0 - j0.7$$

$$\Rightarrow \frac{Z_L}{Z_0} = 1.0 - j0.7$$

$$\Rightarrow Z_L = 50(1.0 - j0.7) = 50 - j35\,\Omega$$

$$\therefore \boxed{Z_L = 50 - j35\,\Omega}$$

Ex-12 (Pozar) Determination of admittance.

A load of $Z_L = 100 + j50\Omega$ terminates a 50Ω line. What are the load admittance and i/p admittance if the line is 0.15λ long.

Ans: 1) $Z_L = 100 + j50\Omega$

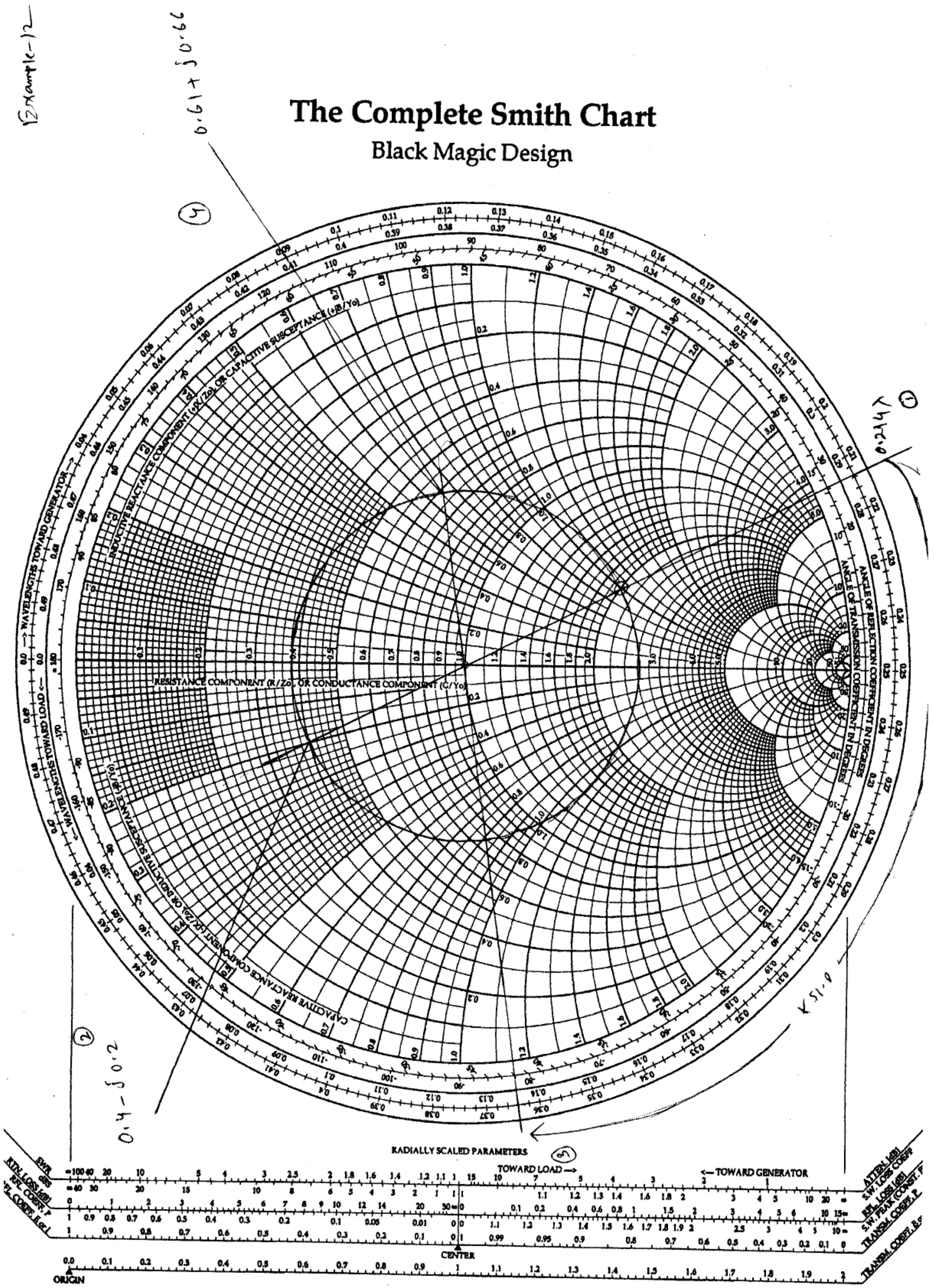
$Z_0 = 50$

Normalized impedance $z = \frac{Z_L}{Z_0} = 2 + j1\,\Omega$.

Locate $(2 + j1)$ point on the Smith Chart.

The Complete Smith Chart

Black Magic Design



2) Draw a circle, with center at center of Smith chart & radius is the distance between center and (2+j×1) point.

3) Draw a straight line between center of Smith chart and (2+j.1) point, extend it in both the direction, so that it intersect the circle at another point. This point is exactly opposite of impedance point, and this is the admittance point. From the Smith chart, it was found

6 $y = 0.4 - j0.2$ (-ve due to capacitive reactance)

7 $\Rightarrow \frac{Y_L}{Y_o} = 0.4 - j0.2$

$\Rightarrow Y_L = Y_o (0.4 - j0.2)$

$\Rightarrow Y_L = \frac{1}{Z_o} \cdot (0.4 - j0.2)$

$= \frac{0.4 - j0.2}{50}$

$$\boxed{Y_L = .008 - j0.004 \text{ Siemen}}$$

8

4) To determine i/p admittance, first find i/p impedance. The line joining center & load (2+j1) gives the

location of load = $\underline{0.214\lambda}$.

Since the line is 0.15λ, moving 0.15λ towards generator gives the i/p impedance.

$$0.214\lambda + 0.15\lambda = 0.364\lambda.$$

∴ Joining 0.364λ, with center ~~and~~ ~~extend~~ intersect the circle at a point. This point ~~as~~ gives i/p impedance. Extending the line further intersect the ~~β~~ circle on another point. This point gives the i/p admittance.

5) From the Smith chart it gives,

$$y_{in} = 0.61 + j0.66$$

$$\Rightarrow \frac{Y_{in}}{Y_0} = 0.61 + j0.66$$

$$\Rightarrow Y_{in} = Y_0 (0.61 + j0.66)$$

$$= \frac{0.61 + j0.66}{Z_0}$$

$$= \frac{0.61 + j0.66}{50}$$

$$\Rightarrow \boxed{Y_{in} = 0.0122 + j0.0132 \text{ Simen}}$$

BPUT

Example - 13 - 6th sem Special exam - 2012

A 50Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j60)\,\Omega$.
The wavelength = 5 cm; find the following

(i) Reflection Coefficient at the load

(ii) Standing wave ratio on the line.

(iii) The position of voltage max^m nearest the load.

(iv) The position of current max^m nearest the load.

Ans: Given

$Z_0 = 50\Omega$, $Z_L = 30 - j60$, $\lambda = 5\,cm$.

(a) $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30-j60)-(50)}{(30-j60)+50}$$

$$\Gamma_L = \frac{-20-j60}{80-j60} = \frac{63.24\angle -108.43}{100\angle -36.86}$$

$$\boxed{\begin{aligned}\Gamma_L &= .6324\angle -71.57 \\ \Gamma_L &= 0.199 - j0.59\end{aligned}}$$

$\therefore |\Gamma_L| = 0.6324$

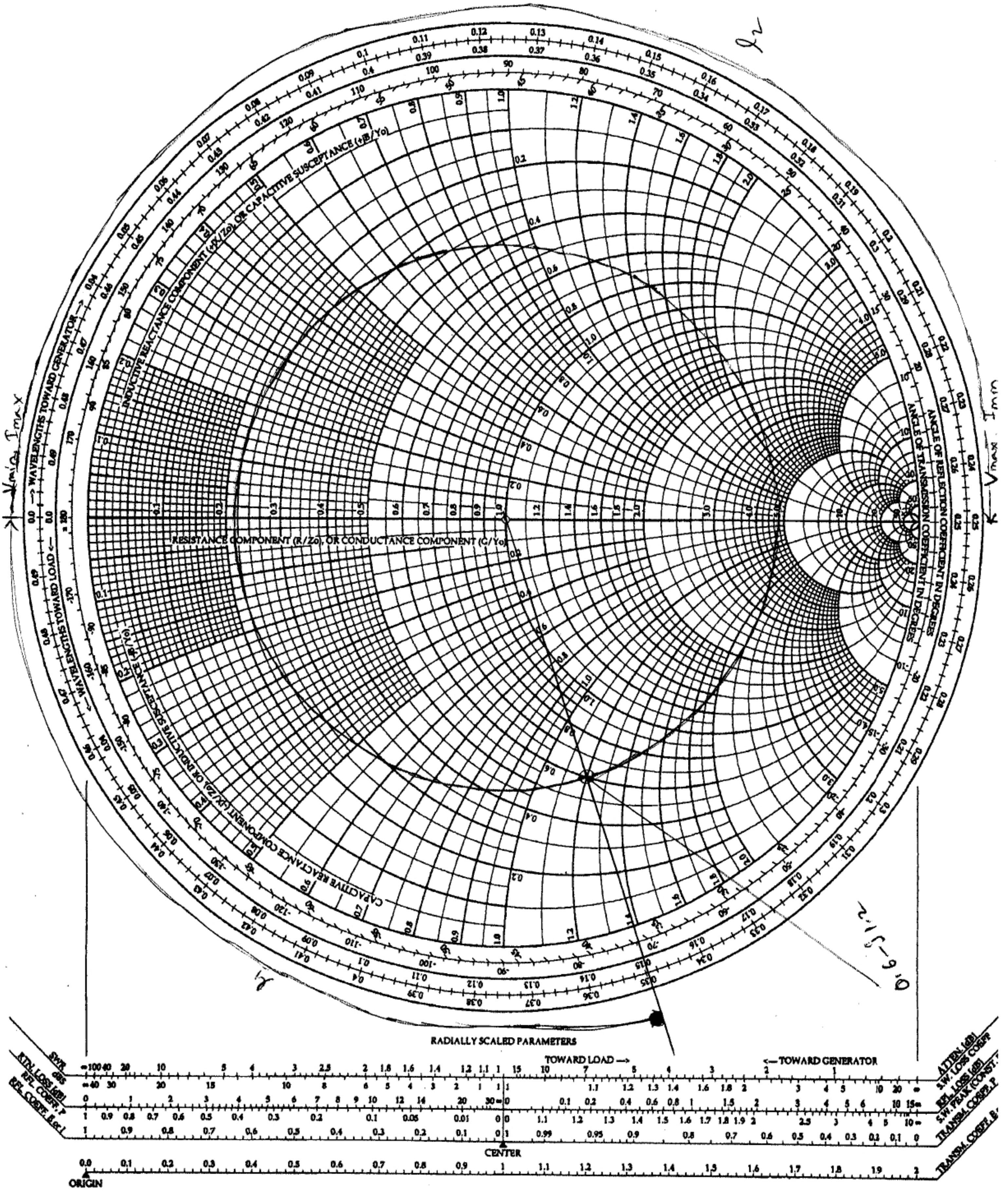
(b) $$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.6324}{1-0.6324} = 4.44$$

(C) Normalized impedance $= \frac{Z_L}{Z_0} = \frac{30-j60}{50} = (0.6 - j1.2)$

Example 13

The Complete Smith Chart

Black Magic Design



→ Plot the point $0.6 - j1.2$ in the Smith Chart.

→ Join this point with center of Smith Chart and extend the line, so that we can get the location of load. It is found to be 0.35λ.

→ V_{min} or I_{max} occures at 0.5λ.

$l_1 = 0.5\lambda - 0.35\lambda = 0.15\lambda = 0.15 \times 5$

$\left[l_1 = .75 \text{ cm from the load} \right]$

(a) ∴ I_{max} occures at 0.75 cm from the load.

→ (c) V_{max} or I_{min} occures at $\underline{0.25\lambda}$

$l_2 = 0.15\lambda + 0.25\lambda = 0.40\lambda = .4 \times 5 = 2\text{cm}.$

∴ V_{max} occures at 2 cm from the load.

__Ex-14__ - BPUT - 2012 - 8th sem.

A low loss co-axial cable of characteristic impedance 50Ω is terminated on a resistive load of 75Ω. The peak voltage across the load is found to be $+30V$. Calculate

(i) Reflection coefficient of the load.

(ii) The amplitude of the forward and reflected

Voltage waves.

(iii) Amplitude of the forward and reflected current waves.

(iv) The VSWR

Ans : Given $Z_0 = 50\Omega$, $Z_L = 75\Omega$, $V_0^+ = 30V$

(i) $\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75-50}{75+50} = \frac{25}{125} = \frac{1}{5} = 0.2$

(ii) Amplitude of forward wave = 30 V.

$$|\Gamma_l| = \frac{V_0^-}{V_0^+}$$

$$\Rightarrow 0.2 = \frac{V_0^-}{V_0^+}$$

$$\Rightarrow V_0^- = V_0^+ \times 0.2 = 30 \times 0.2 = 6V$$

∴ Reflected wave amplitude 6V.

(iii) $I_{incident} = \frac{V_{incident}}{Z_0} = \frac{V_0^+}{Z_0} = \frac{30}{50} = 0.6A$

$\Gamma_i = -\Gamma_v$ $|\Gamma_i| = |\Gamma_v| = 0.2$

So, $\Gamma_i = \frac{I_{ref}}{I_{inc}} = \frac{I_{ref}}{0.6}$

$\Rightarrow \frac{0.6}{I_{inc}}$

$$\Rightarrow \quad 0.2 = \frac{I_{ref}}{0.6}$$

$$\Rightarrow \quad I_{ref} = 0.2 \times 0.6 = .12 \text{ Amp.}$$

(iv) $$VSWR\ (\rho) = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.2}{1-0.2} = \frac{1.2}{0.8}$$

$$\rho = \frac{12}{8} = \frac{3}{2} = 1.5 \quad \text{(Ans).}$$

Impedance Matching :-

When $Z_0 \neq Z_L$, we say that the load is mismatched and a reflected wave exists on the line. Thus impedance matching is required

(i) To transfer maximum power from source to load.

(ii) To avoid reflected signal in the main line.

(iii) To avoid ~~loading~~ the loading effect.

A line terminated in the characteristic impedance $(Z_L = Z_0)$ has a standing wave ratio (ρ) unity and transmits a given power without reflection. $[\Gamma_V = 0, \rho = 1]$

There are 2 types of impedance matching

(a) Quarter-wave transformer ($\lambda/4$) matching

(b) Stub Matching

Stub Matching are again 2 types

(i) Single stub matching

(ii) Double stub matching

(a) Quarter ~~Quarter~~ - wave transformer matching :-

A transmission line having length $\frac{1}{4}^{th}$ (quarter) of wavelength (λ), of the wave ~~($\lambda/4$)~~, propagating through it is called quarter wave transformer.

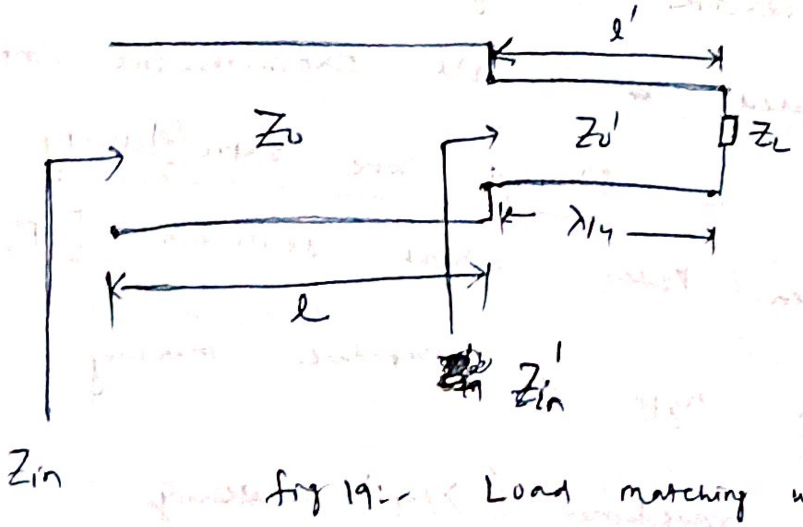
→ It has a effect of transforming the load impedance in such a manner that the ~~so that~~ load impedance & characteristic impedance match with each other.

→ Consider a transmission line having length 'l'. & It has a characteristic impedance $= Z_0$.

6 Consider a quarter wave transmission line, length $\boxed{l' = \frac{\lambda}{4}}$

7 Connected in series with the main line.

on Ex



8 fig 19:- Load matching using $\lambda/4$ transformer

We know, Input impedance of a Transmission line is given by

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan\beta l}{Z_0 + j Z_L \tan\beta l} \right]$$

For $\frac{\lambda}{4}$, line,

$$Z_{in}' = Z_0' \left[\frac{Z_L + j Z_0' \tan\beta l}{Z_0' + j Z_L \tan\beta l} \right]$$

$$\tan\beta l = \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \tan\frac{\pi}{2} = \infty$$

$$Z_{in}' = Z_0' \left[\frac{\frac{Z_L}{\tan\beta l} + j Z_0'}{\frac{Z_0'}{\tan\beta l} + j Z_L} \right]$$

$$= Z_0' \left[\frac{\cancel{j} Z_0'}{\cancel{j} Z_L} \right]$$

$$\Rightarrow Z_{in}' = \frac{(Z_0')^2}{Z_L}$$

$$\Rightarrow (Z_0')^2 = Z_{in}' Z_L$$

$$\Rightarrow Z_0' = \sqrt{Z_{in}' \cdot Z_L} \quad \text{——— (59)}$$

For impedance matching, characteristic impedance of main transmission line (Z_0) should be equal to input impedance (Z_{in}') of $\frac{\lambda}{4}$ line.

$$\therefore \quad Z_0 = Z_{in}' \quad \text{——— (60)}$$

Putting eqn (60), in eqn (59), we have

V. imp $\boxed{Z_0' = \sqrt{Z_0 \cdot Z_L}}$ — (61)

$$\therefore \quad Z_0' = \sqrt{Z_0 \cdot Z_L}$$

Where Z_0', Z_0, Z_L all are real. If, for example, a 120Ω load is to be matched to a 75Ω line the quarter-wave transformer must have characteristic impedance $\sqrt{75 \times 120} \simeq 95\Omega$. Thus 95Ω quarter-wave transformer will also match a 75Ω ~~line~~ load to a 120Ω line. (Because both the cases product (75×120) or (120×75) is equal)

Disadvantage :-

→ The reflected wave is eliminated only at the desired wavelength (or freq 'f').

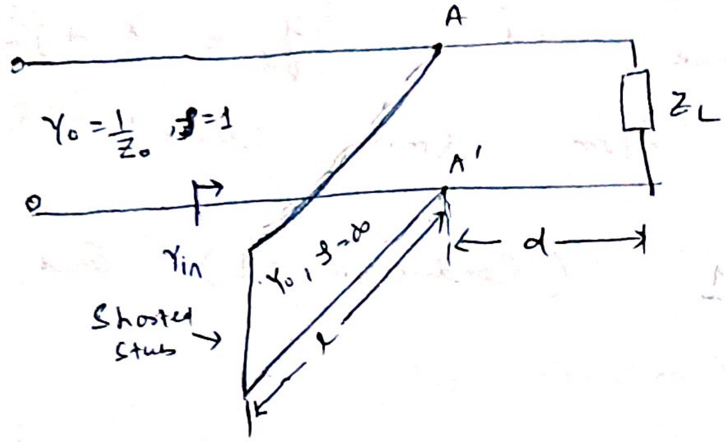
→ There will be reflection at a slightly different wavelength. $[\lambda \to \lambda', \frac{\lambda}{4}$ line won't work] we need $\lambda'/4$ line.

→ Thus, the main disadvantage of the quarter-wave transformer is that it's a narrow-band or freq. sensitive device.

(b) Stub Matching (Short note)

The major drawback of using ~~quarter-wave~~ quarter-wave transformer as a line matching device is eliminated using stub-matching. A stub consists of an open or shorted section of transmission line.

of length 'l' connected in parallel with main line at some distance 'd' from the load as shown in figure 20.



$Y_L = g_L + jb_L$

Y_L = Admittance

g_L = Conductance

b_L = Susceptance

fig 20:- Matching with Single-Stub tuner.

Notice that the stub has same characteristic impedance as the main line, although stubs may be designed with different values of Z_0. It is more difficult to use a series stub although it is theoretically feasible. An open-circuited stub radiates some energy at high frequencies. Consequently, $\boxed{\text{Shunt}}$ $\boxed{\text{Short-circuited}}$ parallel stubs are preferred. (2 mark BPUT)

(i) Single - Stub Matching.

Consider a transmission line having characteristic impedance (Z_0) is terminated by a load (Z_L) of admittance $Y_L = g_L + jb_L$. (Fig 20: above)
(capital letter 'L')

g_L = Conductance, b_L = Susceptance of the load

→ The stub of length (l) which is at a distance 'd' from the load.

→ Lets consider a point on T. line where admittance is $y_l = 1 + jb_l$ (which is located from Smith chart) (Small letter 'l')

where $g_L = 1$ (Conductance = 1) and susceptance is b_L.

→ We have to design a stub which offers a susceptance of $y_s = -jb_l$, so that total admittance will be $(y_s + y_l)$

$$y_{in} = (-jb_l) + (1 + jb_l) = 1 \quad \text{(only conductance part)}$$

— (62)

∴ Normalized input admittance = 1, which can be visualised in Smith chart.

$$\Rightarrow z_{in} = 1$$

$$\Rightarrow \frac{Z_{in}}{Z_0} = 1$$

$$\Rightarrow Z_{in} = Z_0$$

We have derived earlier [page 49 - eqn 32] that At matched load, $Z_L = Z_0$, $Z_{in} = Z_0$

∴ Here $Z_L = Z_0 \Rightarrow$ (Matched)

→ So in this way the characteristic admittance and load admittance will be matched to get conductance value (one).

→ Since $Z_L = Z_0$, Γ (Reflection coefficient) is zero.

~~Ex-15~~ Note:- For Smith Chart Problems

$|Z_{in}|_{max}$ (Maxm i/p impedance)

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = \frac{I_{max} \cdot Z_0}{I_{min}} = (S) \cdot Z_0 .$$

Similarly,

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{I_{min} \cdot Z_0}{I_{max}} = \frac{Z_0}{\left(\frac{I_{max}}{I_{min}}\right)} = \frac{Z_0}{S}$$

Ex-15 :- (Problem on Stub matching)

An antenna (Load) with an impedance of $40 + j30\ \Omega$ to be matched to a $100\ \Omega$ lossless line with a shorted stub. Determine

(a) The required stub admittance

(b) The distance between stub and the antenna

(c) The stub length

(d) The standing wave ratio on each segment of the system.

Solution :- (a) Normalized impedance

$$z = \frac{Z_L}{Z_0} = \frac{40+j30}{100} = 0.4+j0.3$$

1) Locate z on the Smith Chart. Then draw a circle with center at center of Smith Chart & radius = distance betn center & 0.4+j0.3 point.

2) Joins center & 0.4+j0.3 point & extend the line in the opposite of 0.4+j0.3 point, so that it intersect the circle, to get the admittance. It was found to be 1.6 − j1.2

Otherwise

$$y = \frac{1}{z} = \frac{Z_0}{Z_L} = \frac{100}{40+j30} = \frac{100\times(40-j30)}{1600+900} = \frac{1}{25}(40-j30)$$

$$y = \frac{40}{25} - j\frac{30}{25} = 1.6 - j1.2$$

3) Locate point A & B where the constant resistance circle / conductance circle $(r=1/g=1)$ intersects the circle we have drawn.

4) At A, $y_A = 1+j1.04$, B, $y_B = 1-j1.04$,

Stub required

$y_s = -j1.04$ $(\because y_A+y_s=1)$

$y_s = +j1.04$ $(\because y_B+y_s=1)$

∴ The required stub admittance

$$y_s = \frac{Y_s}{Y_0} \Rightarrow Y_s = y_s \cdot Y_0 = \pm j1.04\times\left(\frac{1}{100}\right) = \pm j10.4\,mS$$

Example-15

Step 1 Locate $0.4+j0.3$

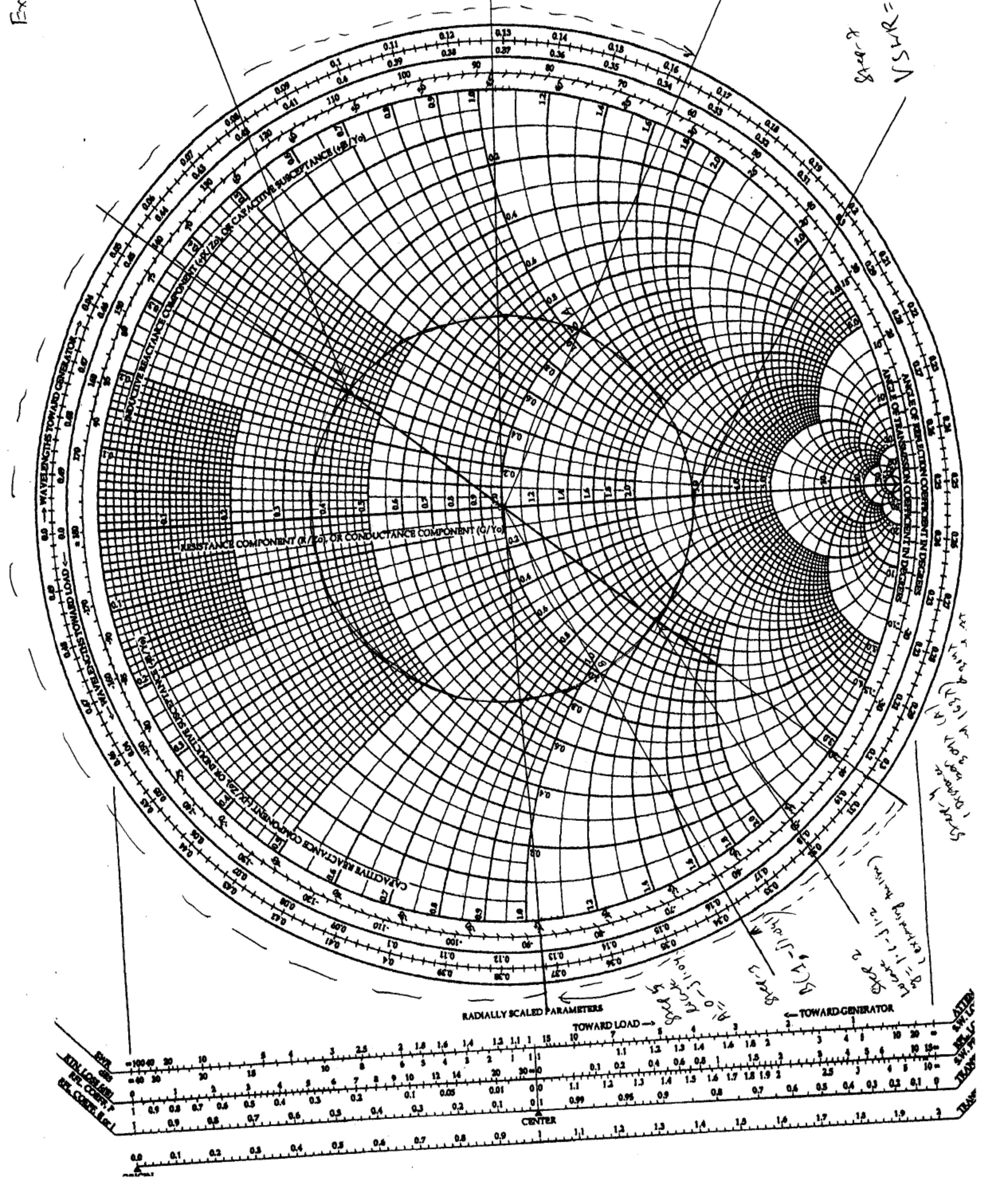
Step 6, locate $b_1 = 0+j1.04$

The Complete Smith Chart

Black Magic Design

Step 3 $A(1+j1.04)$

Step 7 VSWR = 2.8



5) 2 Stubs can be possible,

at A, $\rightarrow -j10.4$ ms

B, $\rightarrow +j10.4$ ms

b) <u>To determine Stub position</u>

<u>If we choose stub at A</u>

~~We have to locate $-j1.04$ i.e. $0-j1.04$~~
~~(Normalized value)~~

~~Position of Stub $1+j1.04$ = 0.163λ (Admittance)~~ (wavelength towards generator scale)

~~Position of load = (Admittance)~~

Reference Position of A = 0.163λ (Admittance $1+j1.04$) in (wavelength towards generator scale)

Reference position of load $\left[\begin{matrix}\text{Admittance} \\ 1.6-j1.2\end{matrix}\right] = 0.304\lambda$

$$\text{Distance} = \left[(0.5-0.304)+.163\right]\lambda$$

$$= (0.196+.163)\lambda$$

$$= 0.359\lambda$$

$$\simeq 0.36\lambda$$

<u>If we choose stub at B</u>

Reference posn of B = 0.337λ

" " " Load = 0.304λ

Distance = 0.033λ

∴ If choose stub at A, stub posⁿ is 0.36λ from load.

If choose stub at B, stub posⁿ is 0.033λ from load.

(c) <u>To find stub length</u>

Locate A′ & B′ corresponding to stub admittance $-j1.04$ & $+j1.04$.

Determine stub length (Distance from 0.25λ point or $y=\infty$ point or short ckt point)

$$\text{Stub length } \cancel{A'} \text{ at } A = \text{Location of } A' \, (-j1.04) - 0.25\lambda$$

$$= 0.372 - 0.25$$

$$= 0.122\lambda$$

$$\text{Stub length at } B = \cancel{\text{Length of}} \text{ Location of } B' \, (+j1.04) \text{ from } 0.25\lambda \text{ point}$$

$$= \left[(0.5 - 0.25) + 0.128\right]\lambda$$

$$= [0.25 + .128]\lambda$$

Stub length at B = 0.378λ

(d) Standing wave ratio betⁿ load to stub = 2.8 (As shown in chart)

" " " from stub towards generator = 1

(Because matching is done)

and VSWR = ∞ along the stub because stub is short circuited.

Ex-16 (Home work) [Here stub admittance & line admittance are not equal $= Y_0$]

(Refer the Smith chart attached)

A T-L having $Z_L = 75 - j150\,\Omega$ and $Z_0 = 50\,\Omega$ is matched with stub having $Z_0' = 100\,\Omega$.

Find the location & length of stub for matching.

Ans :-

1) $$Z = \frac{Z_L}{Z_0} = \frac{75 - j150}{50} = 1.5 - j3$$

$$y = \frac{1}{1.5 - j3} = \frac{1.5 + j3}{(1.5)^2 + 3^2} = \frac{1.5 + j3}{2.25 + 9} = 0.13 + j0.26$$

or first locate Z, then extend the line in opposite direction to get 'y'.

2) Draw the circle with the center at center of Smith chart & radius = Distance betn center & Z point or y point.

3) Check where the constant circle $r = 1$ / $g = 1$ intersect the circle we have drawn. It will intersect at two point A & B. Lets take the point A. [$1 + j\,2.5$]

4) Location of stub = $0.196\lambda - 0.042\lambda = 0.154\lambda$ from load.

Example - 16

The Complete Smith Chart

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Step-1 y = 0.13 + j0.26

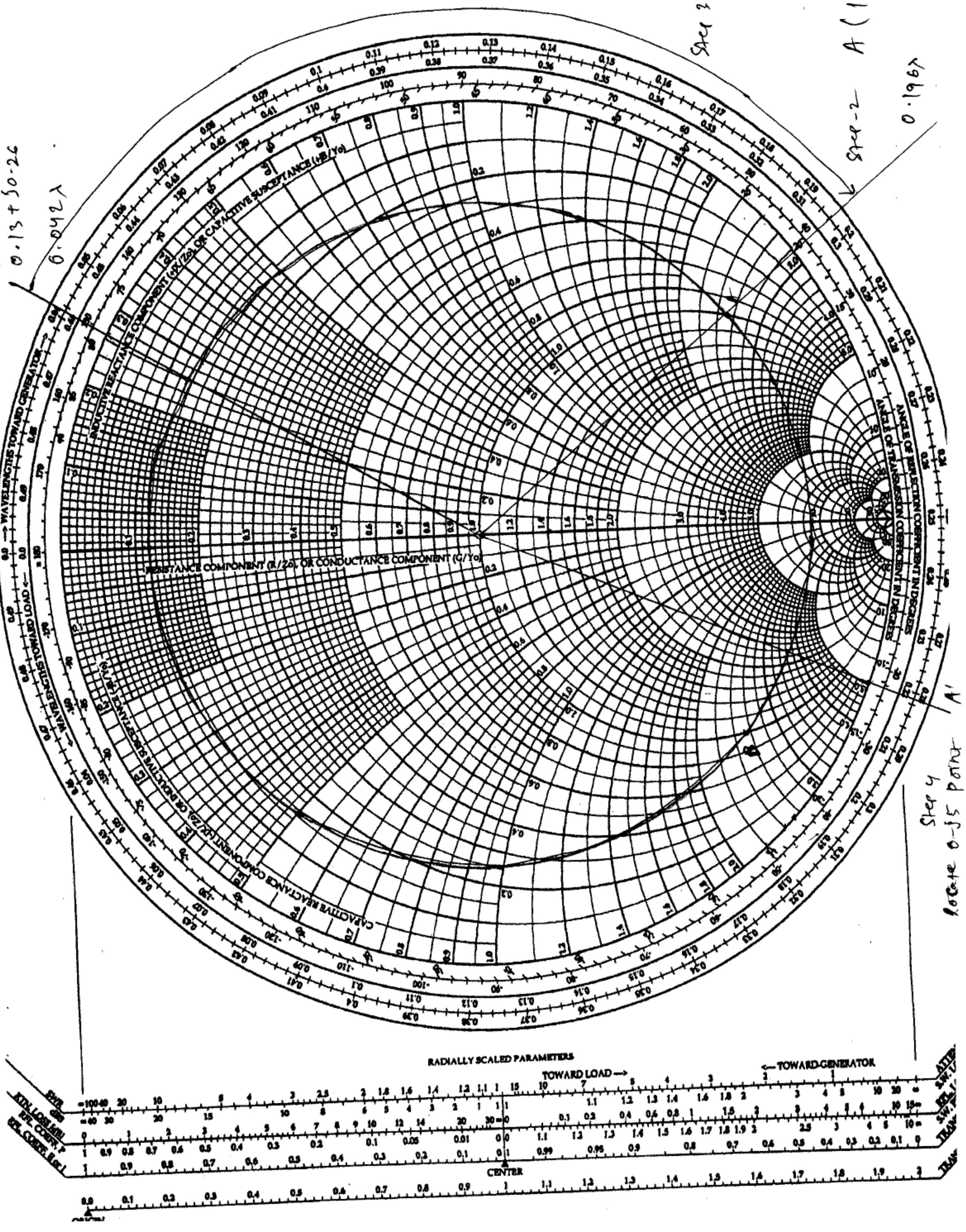
0.042λ

Step-2

0.196λ

Step 3 - Location of Stub (0.196-0.042)λ

A (1+j2.5)



A'

Step 4 Rotate 0.35 point

5) Stub should have admittance = ~~j0.25~~

$y_s = -j2.5$

So that net Admittance, $= (1+j0.25) - j2.5$

$= 1.$

6) ~~Locate 0 - j2.5~~

Since characteristic impedance of T.L $\neq$ Characteristic impedance of Stub, $(Z_0 \neq Z_s)$

$$y_s = -j2.5 \times \frac{Z_s}{Z_0} = -j2.5 \times \frac{100}{50} = -j5.0.$$

[Hints :- ~~for 50~~ for $Z_0 = 50$, $y_s = -j2.5$

$Z_0 = 1$ $y_s = \frac{-j2.5}{50}$

$Z_0 = 100$, & $y_s = \frac{-j2.5}{50} \times 100$]

7) Now locate 0 - j5 point.

8) Find the length of the stub

= Distance from 0.25λ point [Short cut point] $y = \infty$

$= 0.281 - 0.25$

$= 0.031\lambda$

9) ∴ Stub is at 0.154λ distance from load having length 0.031λ.