Ch-6

Noise in Amplitude modulated System

The Superheterodyne receiver used in AM system is shown below.

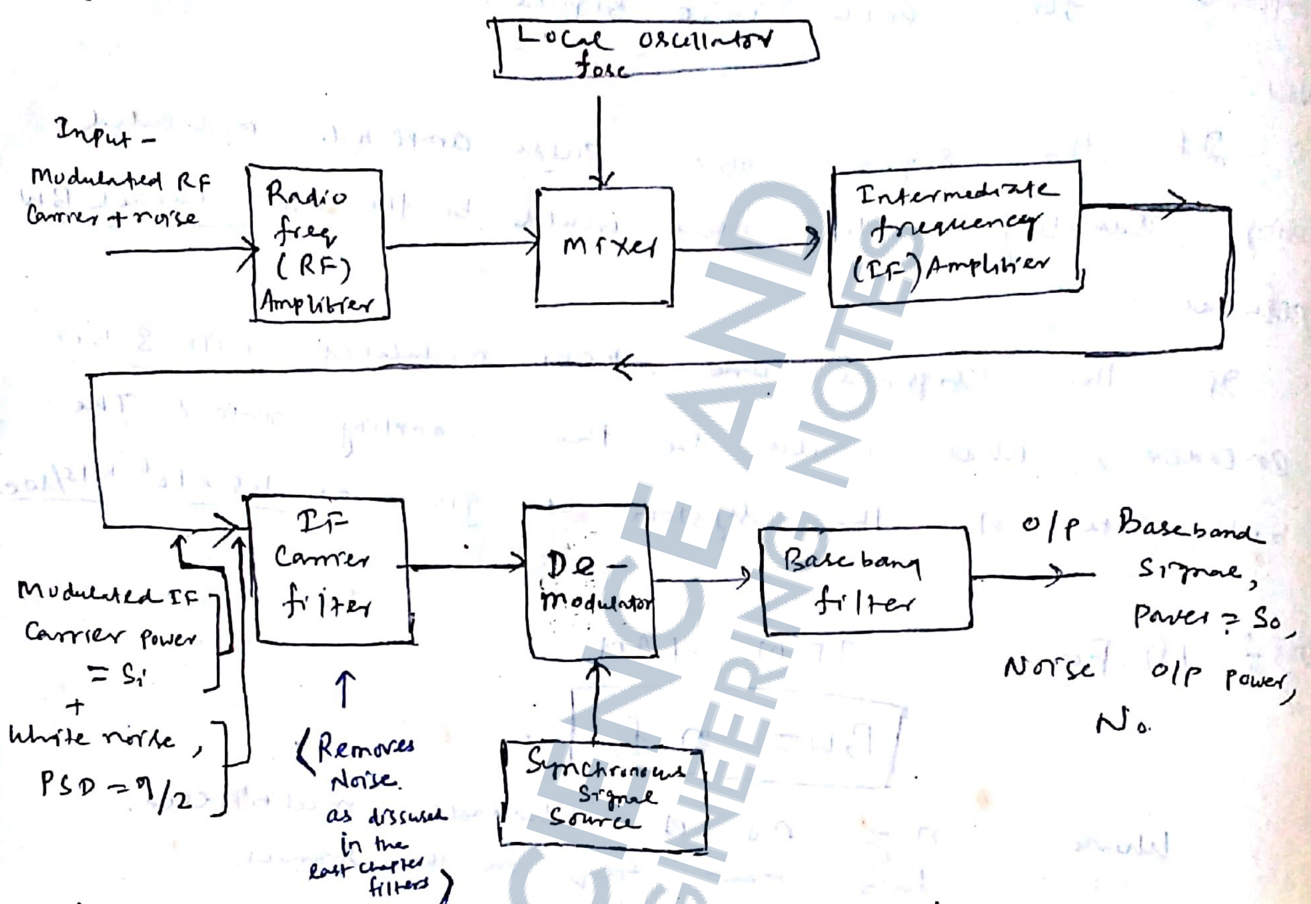


fig 1:- A receiving system for an amplitude-modulated signal.

→ This is suitable for reception and demodulation of all-type of amplitude-modulated signals. The only changes required for different type AM signal, are in demodulator & in the BW of the IF carrier filter.

→ The signal i/p to IF filter ~~Amplifier~~ is an amplitude-modulated IF Carrier [say $f_c = 455$ KHz]

→ The normalized Power [Power dissipated in 1Ω resistor]

If this signal is S_i. The signal arrives with noise. Added, is the noise generated in the RF amplifier and amplified in RF amplifier and IF amplifiers.

→ The IF amplifiers and mixer are also sources of noise, i.e. thermal noise, shot noise etc, but this noise, lacking the gain of the RF amplifier.

→ We assume that the noise is gaussian, white and of two-sided PSD $(\eta/2)$. The IF filter is assumed rectangular and of bandwidth no wider than is necessary to accomodate the signal.

→ The O/P baseband signal has a power S_o and is accompanied by noise of total power N_o.

BPUT - 2011, 2012

1) Single Sideband Suppressed Carrier (SSB-SC)

Calculation of Signal Power :-

With a single-sideband suppressed-carrier signal, the demodulator is a multiplier.

IF carrier → $A\cos 2\pi f_c t$

For synchronous detection, locally generated carrier → $\cos 2\pi f_c t$

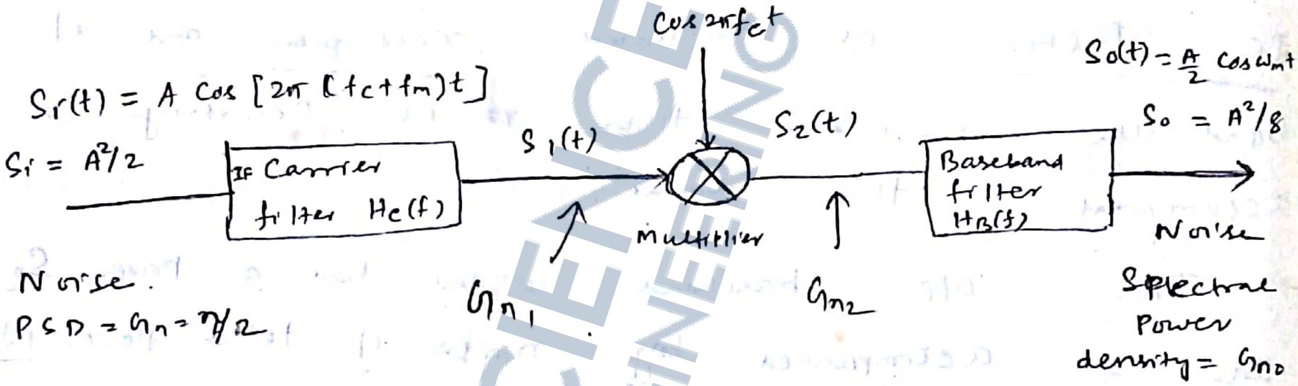
→ Assuming Upper side band (USB) used; Carrier filter is bandpass type, extends from f_c to $f_c + f_M$, f_M – baseband bandwidth.

→ BW of baseband filter 0 to f_M.

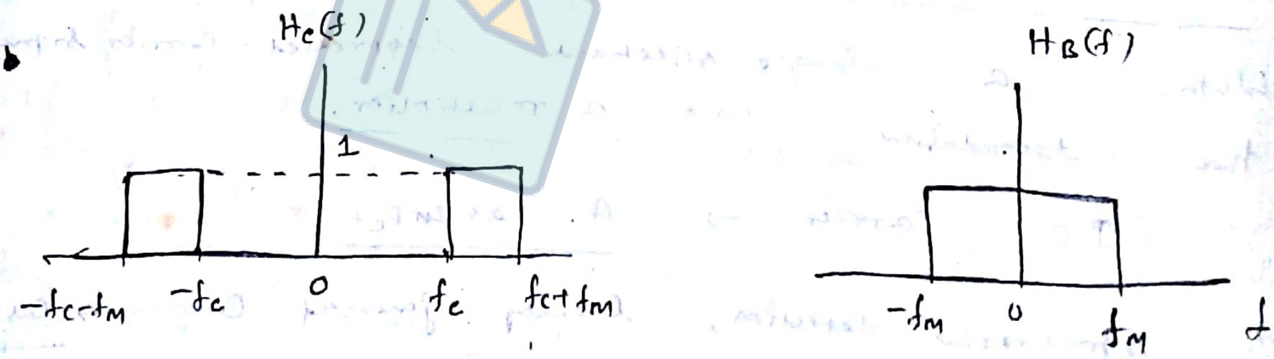
→ Let us assume the baseband signal is a sinusoid of angular freq 'f_m' ($f_m \le f_M$). The carrier freq is f_c [IF], since we have assumed that the USB being used, the received signal is

$$S_i(t) = A \cos\left[2\pi (f_c + f_m) t\right] \quad \text{--- (1)}$$

(Note ÷ Refer SSB-SC eqⁿ for USB]



(a) A synchronous demodulator operating on a single - sideband single tone signal.



(b) The bandpass range of the carrier filter

(c) The passband of the lowpass baseband filter.

The O/P of multiplier is

$$S_2(t) = S_1(t) \cdot \cos \omega_c t$$

$$S_2(t) = A \cos[2\pi(f_c+f_m)t] \cdot \cos 2\pi f_c t$$

$$= \frac{A}{2}\left[\cos(2\pi(2f_c+f_m)t + \cos 2\pi f_m t\right]$$

only the difference - freq term with pass [since LPF] through baseband filter. Therefore o/p signal is.

$$S_o(t) = \frac{A}{2} \cos 2\pi f_m t \quad \text{--- (2)}$$

which is the modulating signal amplified by $\frac{1}{2}$.

The i/p signal power,

$$S_i = \frac{A^2}{2} \quad \text{--- (3)}$$

o/p signal power,

$$S_o = \frac{\left(\frac{A}{2}\right)^2}{2} \qquad \left[\because \text{Power} = \frac{\text{Amplitude}^2}{2}\right]$$

$$S_o = \frac{A^2}{8} \quad \text{--- (4)}$$

$$\therefore \frac{S_o}{S_i} = \frac{\cancel{A^2}}{8} \times \frac{2}{\cancel{A^2}} = \frac{1}{4} \quad \text{--- (5)}$$

Calculation of Noise power

When a noise spectral component at a freq 'f' is multiplied by $\cos 2\pi f_c t$, the original noise component is replaced by 2 components, one at freq f_c+f and one at freq f_c-f. Each new component having $\frac{1}{4}$th the power of original. [Similar to $S_o/S_i = 1/4$]

The i/p noise is white & of spectral density $\eta/2$. The noise i/p to the multiplier has a spectral density G_{n1} as shown in fig below.

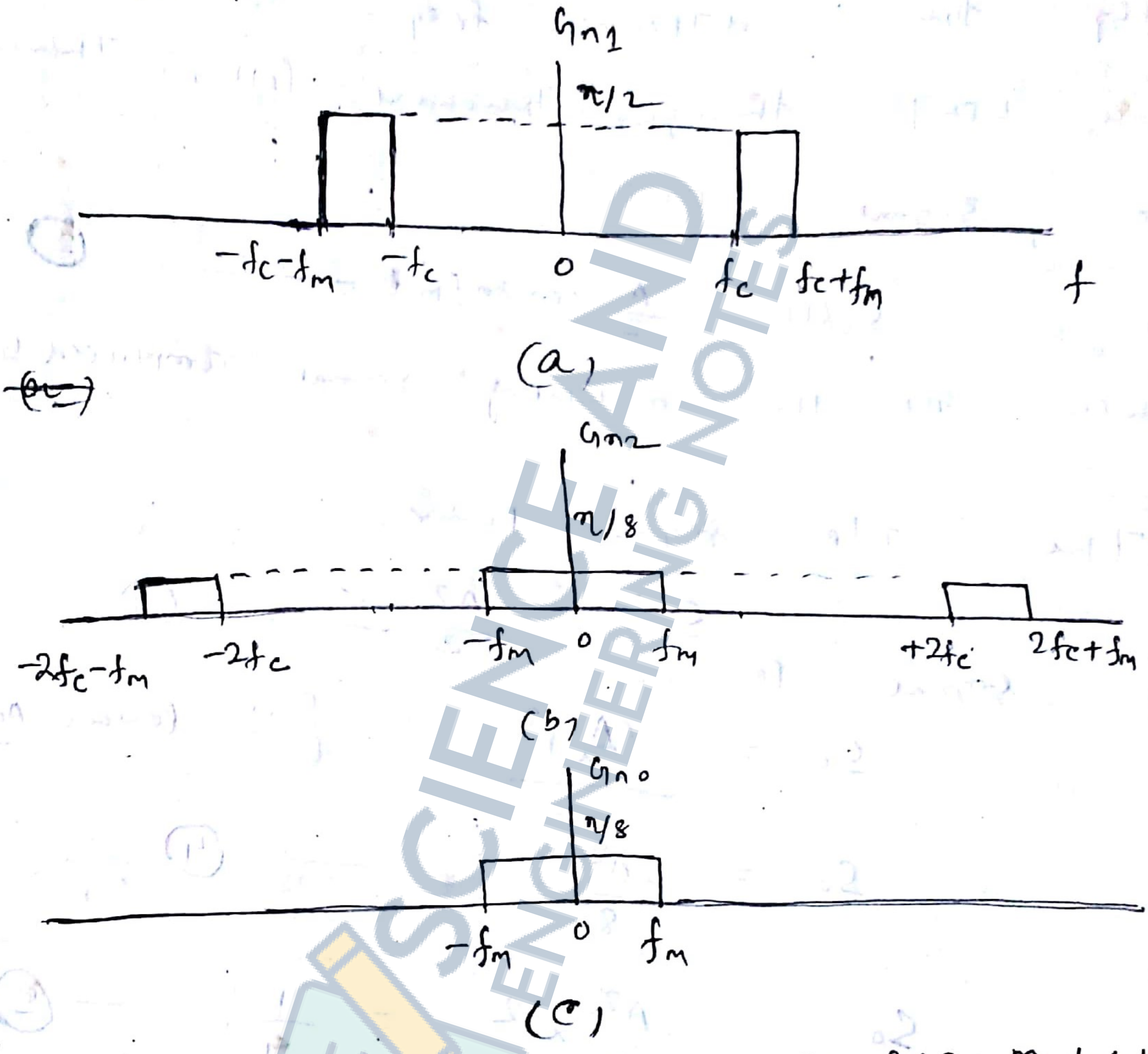


fig:- Spectral densities of noises in SSB Modulator.

(a) Density G_{n1} of noise i/p to the multiplier

(b) Density G_{n2} of noise o/p of multiplier

(c) Density G_{no} of noise output of baseband filter.

→ The density of noise after multiplication by $\cos 2\pi f_c t$ is G_{n2} [fig b] Finally the noise transmitted by the baseband filter is of density G_{no} [fig c]. The total noise o/p is the area under the plot. [fig c]

$$N_o = 2f_m \times \frac{\eta}{8} = \frac{\eta f_m}{4} \quad \text{— (6)}$$

SNR Calculation [Signal-to-noise ratio] Calculation

Using eqn (5) & (6)

$$S_o = S_i/4$$

$$N_o = \frac{\eta f_m}{4}$$

$$S_o/N_o = \frac{S_i/4}{\eta f_m/4} = \frac{S_i}{\eta f_m}$$

$$\boxed{\frac{S_o}{N_o} = \frac{S_i}{\eta f_m}} \quad \text{— (7)}$$

Defining N_M = Noise Power at i/p, measured on a freq band equal to the baseband freq,

$$\cancel{\frac{S_o}{N_o} = \frac{S_i}{N_M}}$$

~~where~~

$$N_M = \frac{\eta}{2} \times 2f_m$$

$$N_M = \eta f_m \quad \text{— (8)}$$

using eqn (8)

∴ Eqn (7), becomes,

$$\frac{S_o}{N_o} = \frac{S_i}{N_M} \quad \text{— (9)}$$

$\frac{S_i}{N_M}$ = referred to as input SNR

Note:- N_M is the noise Power transmitted through IF filter only when the IF filter BW is f_m

Defining, figure of merit $= \frac{\text{O/P SNR}}{\text{I/P SNR}}$

Figure of Merit $(\gamma) = \dfrac{S_o/N_o}{S_i/N_m}$

$= \dfrac{\cancel{(S_o/N_o)}}{\cancel{(S_o/N_o)}}$ Using eqn (9)

$\boxed{\gamma = 1}$, for SSB-SC System

(Proved)

2) **Double Sideband Suppressed Carrier (DSB-SC)**

When baseband signal of frequency range 'f_m' is transmitted over a DSB-SC system, the BW of the carrier filter must be $2f_m$ rather than f_m. Thus, i/p noise in the freq range $f_c - f_m$ to $f_c + f_m$ will contribute to the o/p noise.

Calculation of signal ~~noise~~ Power :-

We want to compare SSB-SC & DSB-SC. But they have different signal power. To keep the received power same as SSB-SC case, $S_i' = \frac{A^2}{2}$, we write DSB-SC

$$S_i(t) = \sqrt{2}A \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

$$= \frac{\sqrt{2}}{2}A \left[\cos 2\pi (f_c+f_m)t + \cos 2\pi (f_c-f_m)t\right]$$

$$= \frac{A}{\sqrt{2}} \cos 2\pi (f_c+f_m)t + \frac{A}{\sqrt{2}} \cos 2\pi (f_c-f_m)t$$

The received power is then

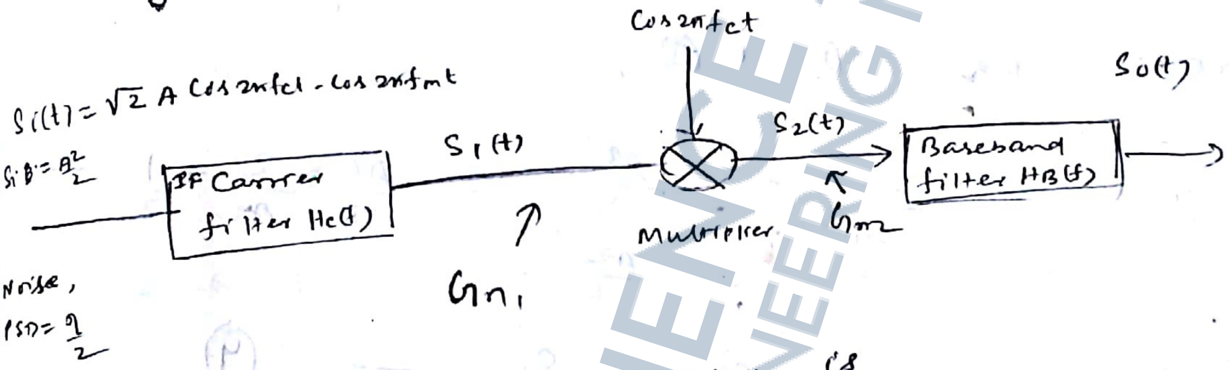
$$S_i = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{2} + \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{2}$$

$$= \frac{A^2}{4} + \frac{A^2}{4}$$

$$S_i = \frac{A^2}{2} \quad \text{——— (1)}$$

[So Now SSB-SC & DSB-SC have same received power]

→ In the demodulator, $S_i(t)$ is multiplied by $\cos \omega_c t$.



The o/p of multiplier is

$$S_2(t) = S_1(t) \cdot \cos \omega_c t$$

$$= \left[\sqrt{2}\, A \cos 2\pi f_c t \cdot \cos 2\pi f_m t\right]\left[\cos 2\pi f_c t\right]$$

$$= \sqrt{2}\, A \cdot \left[\cos^2 2\pi f_c t\right] \cos 2\pi f_m t$$

$$= \sqrt{2}\, A \left[\frac{1 + \cos 2\pi (2f_c) t}{2}\right] \cos 2\pi f_m t$$

$$= \frac{A}{\sqrt{2}} \left[1 + \cos 2\pi (2f_c) t\right] \cos 2\pi f_m t$$

$$= \frac{A}{\sqrt{2}} \cos 2\pi f_m t + \frac{A}{\sqrt{2}} \cdot \frac{1}{2} \cdot 2 \cos 2\pi (2f_c) t \cdot \cos 2\pi f_m t$$

$$= \frac{A}{\sqrt{2}} \cos 2\pi f_m t + \frac{A}{2\sqrt{2}} \left[\cos 2\pi (2f_c + f_m) t + \cos 2\pi (2f_c - f_m) t\right]$$

$$S_2(t) = \frac{A}{\sqrt{2}} \cos 2\pi f_m t + \frac{A}{2\sqrt{2}} \cos 2\pi (2f_c + f_m) t + \frac{A}{2\sqrt{2}} \cos 2\pi (2f_c - f_m) t \quad \text{--- (2)}$$

∴ The o/p of the baseband filter [since it is a LPF]

$S_o(t) = \frac{A}{\sqrt{2}} \cos 2\pi f_m t$, which has a

power $S_o = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{2} = \frac{A^2}{4}$ --- (3)

From eqn (1) & (3),

$$\frac{S_o}{S_i} = \frac{\frac{A^2}{4}}{\frac{A^2}{2}} = \frac{A^2}{4} \times \frac{2}{A^2} = \frac{1}{2}$$

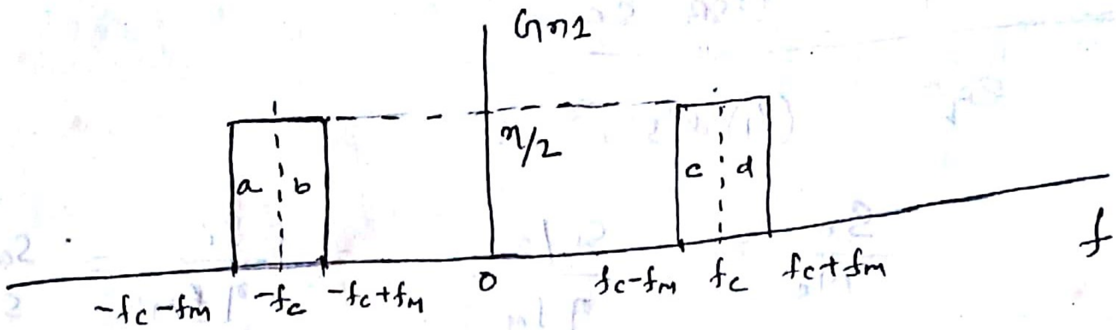
$$\Rightarrow S_o = \frac{S_i}{2} \quad \text{--- (4)}$$

Calculation of Noise Power

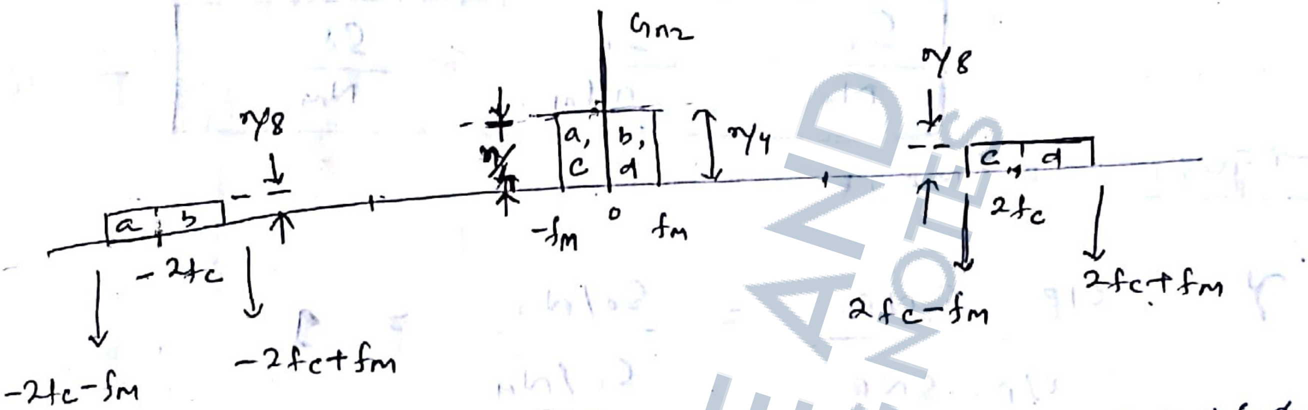
The spectral density $G_{n_1}(f)$ of the white i/p noise after the IF filter is shown in fig (a).

∴ The noise is multiplied by $\cos\omega_c t$. The multiplication ~~by $\cos 2\pi f_c t$~~ results in frequency shift $\pm f_c$ and redn of power in the PSD of noise by a factor 4. $\left[\frac{A^2}{4} \rightarrow \frac{A^2}{16}\right]$ $\left[\because \left(\frac{A}{\sqrt{2}}\right)^2/2 \rightarrow \frac{A^2}{4},\ \left(\frac{A}{2\sqrt{2}}\right)^2/2 \rightarrow \frac{A^2}{16}\right]$

Note that the noise-power spectral density in the region $-f_m$ to f_m is $\eta/4$, while the noise density in SSB case is $\eta/8$

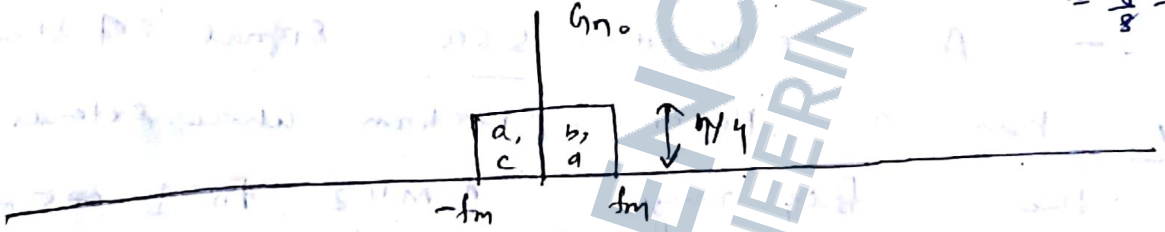


(a)



(b)

→ $G_{no} = a,b + c,d$
$= \frac{\eta}{8} + \frac{\eta}{8} = \frac{\eta}{4}$



(c)

fig:- Spectral densities of noise in DSB-SC Modulation

(a) Density G_{n1} of noise at o/p of RF filter

(b) Density G_{n2} of noise at o/p of multiplier

(c) Density G_{no} of noise at o/p of baseband filter.

→ Hence the o/p noise power is twice as large as o/p noise power of SSB.

→ The o/p noise for DSB-SC after baseband filtering is therefore

$$N_o = \frac{\eta}{4} \times 2f_m = \frac{\eta f_m}{2} \quad \text{—} \text{⑤}$$

SNR of DSB-SC

From eqⁿ (4) & (5),

$$\frac{S_o}{N_o} = \frac{S_i/2}{\frac{\eta f_M}{2}} = \frac{S_i}{\eta f_M} = \text{Same as SSB-SC}$$

$$\boxed{\frac{S_o}{N_o} = \frac{S_i}{\eta f_M} = \frac{S_i}{N_M}} \quad \text{— (6)}$$

Figure of Merit

$$\gamma = \frac{\text{O/P SNR}}{\text{I/P SNR}} = \frac{S_o/N_o}{S_i/N_M} = 1.$$

Ex-1 :- A received SSB signal of strength 1 mW has a power spectrum which extends over the freq range 1 MHz (f_c) to 1.5 MHz (f_c+f_m). The accompanied noise has uniform PSD 10^{-9} (η) W/Hz. This is multiplied by a local oscillator of freq 1 MHz and then followed by a baseband filter of cut-off freq f_m to get message signal. What is message bandwidth? Find signal & noise energy at the O/P of the baseband filter & calculate the SNR there. How does the SNR change if the BW of the message signal is reduced by 25 %.

Ans : SSB of message is of width,

$f_m = 1.5 - 1.0 = 0.5$ MHz.

$f_m \le f_M$

$f_M \simeq f_m = 500$ KHz

Signal strength $= (S_o) = \frac{S_i}{4} = \frac{1\,mW}{4} = 0.25$ mW

Noise strength $= (N_o) = \frac{\eta f_m}{4} = \frac{10^{-9} \times 500 \times 10^3}{4}$

$= 125\ \mu W$

$= 0.125$ mW

$SNR = \frac{0.25}{0.125} = 2$

$(SNR)_{dB} = 10 \log 2 = 3$ dB

Note:-
If 'f_m' change ~~no~~ no change in N_o or SNR
If 'f_M' changes then there is change in N_o or SNR

→ If baseband filter width (f_M) remains same as message signal is reduced, SNR does not change, if BW 4 500 KHz, then SNR does not change [$\because N_o \propto f_M$, $N_o \not\propto f_m$]

→ If baseband filter cutoff freq is also reduced in-line with message signal,

~~$N_o = \eta$~~, i.e. $f_M \to 0.75 f_M$ (Reduced by 25 %)

$N_o = \frac{\eta \times 0.75 \times f_M}{4} = \frac{\eta f_M}{4} \times 0.75$

$= 0.125 \times 0.75$

$= 0.0938$

~~Change in SNR~~ $N_o = .25$

Changed SNR $= \frac{0.25}{.0938} = 2.667 = 4.2597$ dB.

$10 \log (2.667)$

Ex:-2 Repeat ... for DSB-SC.

Given:- $f_c - f_m = 0.5\,MHz$

$f_c + f_m = 1.5\,MHz$.

Ans:

$$2f_m = 1.5 - 0.5 = 1\,MHz$$

$$\Rightarrow f_m = 0.5\,MHz = 500\,KHz.$$

$$S_o = \frac{S_i}{2} = \frac{1\,mW}{2} = 0.5\,mW$$

$$N_o = \frac{\eta \times f_m}{2} = \frac{10^{-9} \times 500 \times 10^3}{2} = 250 \times 10^{-6}$$

$N_o = 0.25\,mW$

$$SNR = \frac{0.5}{0.25} = 2 = 3.01\,dB.$$

Similar to SSB, if BW of baseband signal reduced, and base band filter BW = 500 KHz, then no change on SNR.

→ But if BW of filter is reduced by 25%, then

$$N_o = 0.75 \times [0.25] = 0.1875$$

$$\rightarrow SNR = \frac{0.5}{.1875} = 2.667 = ~~4.2597\,dB~~.$$

$$= 4.2597\,dB$$

Double Sideband with Carrier (DSB-C)

In this case, carrier accompanies the double-sideband signal. Demodulation is achieved synchronously as in SSB-SC & DSB-SC. The carrier increase the total input-signal power but makes no contribution to the output-signal power. The SNR equation for DSB-SC applies directly to this,

$$\frac{S_o}{N_o} = \frac{S_i^{(SB)}}{\eta f_M} \quad \text{where} \quad \text{—— (1)}$$

[If filter allows only the sidebands]

$S_i^{(SB)} \rightarrow$ Power in the sidebands alone.

Suppose that the received signal is

$$S_i(t) = A[1+m(t)]\cos 2\pi f_c t$$

$$= A\cos 2\pi f_c t + A\,m(t)\cos 2\pi f_c t \quad \text{—— (2)}$$

where $m(t)$ is the baseband signal which amplitude-modulates the carrier $A\cos 2\pi f_c t$. The carrier power is $\frac{A^2}{2}$. The sidebands are contained in the term $\underline{A\,m(t)\cos 2\pi f_c t}$.

The power associated with this term is

$\frac{A^2}{2}\overline{m^2(t)}$, where $\overline{m^2(t)}$ is the time average of square of the modulating waveform.

$\therefore$ Total I/P Power S_i is given by,

$$S_i = \frac{A^2}{2} + S_i^{(SB)} = \frac{A^2}{2} + \frac{A^2}{2}\overline{m^2(t)} \quad \text{—— (3)}$$

$$S_i = \frac{A^2}{2}\left[1+\overline{m^2(t)}\right] \quad \text{—— (4)}$$

But $$S_i^{(SB)} = \frac{A^2}{2}\overline{m^2(t)} \quad \text{—— (5)}$$

Dividing eq^n (5) by eq^n (4),

$$\frac{S_i^{(SB)}}{S_i} = \frac{\overline{m^2(t)}}{1+\overline{m^2(t)}} \quad \text{—— (6)}$$

$\therefore$ Using eqⁿ ⑥, in eqⁿ ①, we have

$$\boxed{\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{1+\overline{m^2(t)}} \cdot \frac{S_i}{\eta f_m}} \quad \text{--- (7)}$$

To express S_o/N_o,
In terms of carrier power $\left[P_c = \frac{A^2}{2}\right]$,

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{\cancel{(1+\overline{m^2(t)})}} \cdot \frac{\frac{A^2}{2}\left[\cancel{1+\overline{m^2(t)}}\right]}{\eta f_m} \quad \left[\text{using eq}^n\ 4\right]$$

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)} \cdot P_c}{\eta f_m} \quad \text{--- (8)}$$

If the modulation is sinusoidal, with

$m(t) = m \cos 2\pi f_m t \quad [m = \text{a const}]$,

then

$S_r(t) = A(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t$.

In this case, $\overline{m^2(t)} = \frac{m^2}{2}$ --- (9)

Putting eqⁿ (9), in eqⁿ (7), we have

$$\frac{S_o}{N_o} = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} \cdot \frac{S_i}{\eta f_m}$$

$$\frac{S_o}{N_o} = \frac{m^2}{2+m^2} \cdot \frac{S_i}{\eta f_m} \quad \text{--- (10)}$$

$$\frac{S_o}{N_o} = \frac{m^2}{2+m^2} \cdot \frac{S_i}{N_m}$$

Figure of merit

$$\boxed{\gamma = \frac{S_0/N_0}{S_i/N_m} = \frac{\overline{m^2(t)}}{1+\overline{m^2(t)}}}$$

[from eqn (7) ∵ $\eta f_m = N_m$]

If modulation is sinusoidal

$$\boxed{\gamma = \frac{S_0/N_0}{S_i/N_m} = \frac{m^2}{2+m^2}}$$

[from eqn 8 (10)]

Square-law demodulator & its threshold

A double-sideband signal with carrier (DSB-C) may be demodulated by passing the signal through a n/w whose i/p-o/p characteristic is not linear. Such a non-linear demodulation has the advantage, over the linear synchronous demodulation methods, that a synchronous local carrier need n't be obtained. This eliminates the rather costly synchronizing ckts.

Such a non linear demodulator ~~which~~ uses a n/w whose o/p signal y (voltage or current) is related to the i/p signal x (voltage or current) by

$$y = \lambda x^2 \qquad \text{(1)}$$

in which λ is a constant. As shown in fig '1', this non linear n/w, which constitutes the demodulator, is preceded by a bandpass IF filter of BW $2f_m$ and is followed by a baseband low-pass filter of BW f_m.

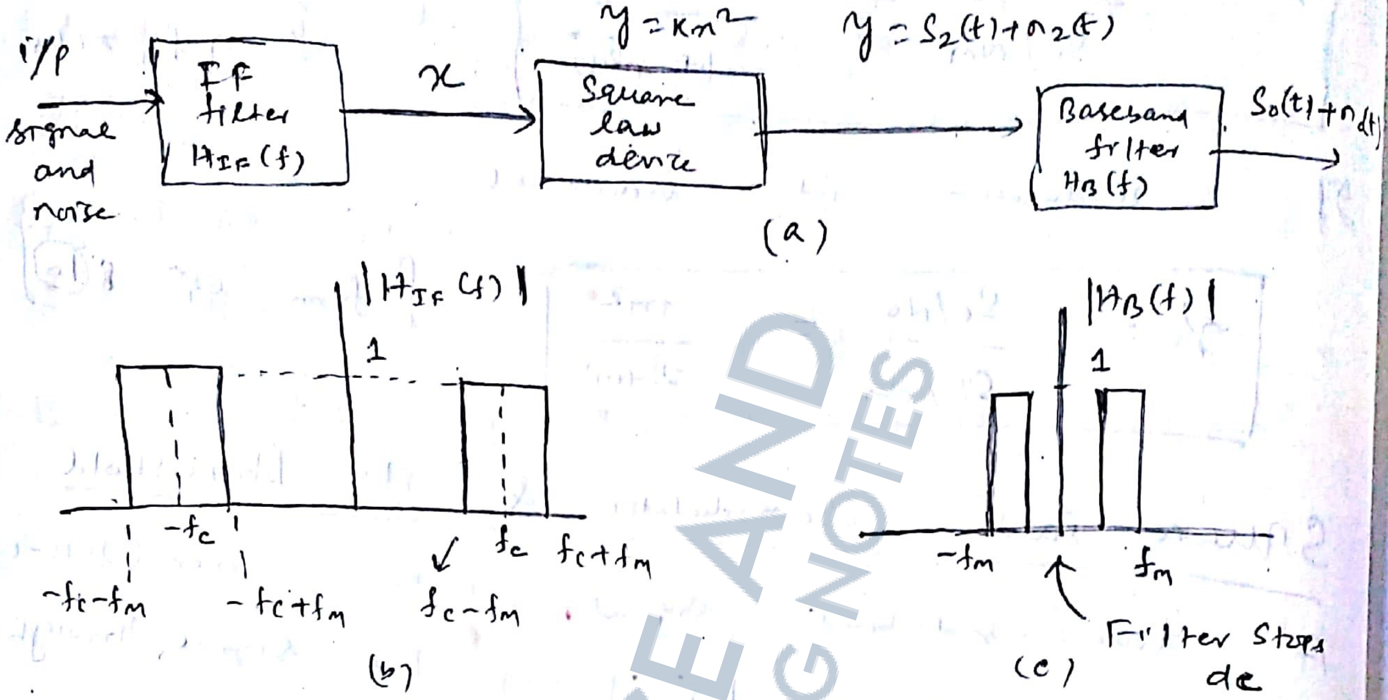


fig 1 :- Square law AM demodulator

The quantity, $\gamma = \dfrac{S_o/N_o}{S_i/N_m}$, figure of merit, defines the performance of demodulator in the presence of noise. γ is a const [AM demod. SSB-SC = 1, DSB-SC = 2 etc]

→ Therefore, if the i/p S_i/N_m decrease, say by a factor 'α', the o/p S_o/N_o will also decrease by 'α'.

→ The nonlinear demodulator also has a range where the figure of merit 'γ' is independent of S_i/N_m.

→ However, as S_i/N_m decreases, there is a point, a threshold, at which the o/p S_o/N_o decreases more rapidly than does the i/p S_i/N_m. This threshold often marks the limit of usefulness of the demodulator.

The baseband filter is designed to pass the modulation & suppress as much noise as possible. Thus the maxm freq pass is f_m.

On the low-freq end, the filter extends only low enough to pass the minm signal freq, often 100 to 300 HZ. However, we assume for simplicity that the filter passes all low-freq components with the exception of d.c (zero freq). Thus we stop d.c terms.
[Shown in fig c]

Envelope Demodulator and its threshold

Consider an AM signal with modulation $|m(t)| < 1$ ~~the~~ a signal, To demodulate this DSB-C signal, a n/w is used which accepts the modulated carrier and provides an o/p which follows the waveform of the envelope of the carrier.

The diode demodulator is a physical ckt which performs the required operation to a good approximation. The demodulator, is preceded by a band pass filter with center freq 'f_c' and BW $2 f_m$, is followed by a low-pass ~~filter~~ baseband filter of BW f_m.

Like the square-law demodulator, the

envelope demodulator exhibits a threshold. As the i/p signal to noise ratio decreases, a point is reached where the signal-to-noise ratio at the o/p decrease more rapidly than at the i/p.

Q - BPUT - 2011

1) What is meant by threshold in DSB-C reception? Between square-law demodulator & envelope demodulator, which one does show lower threshold? [3 mark]

Ans:- In DSB-C reception, if the i/p SNR (S_i/N_m) decreases, say, by a factor 'α', the o/p S_o/N_o will also decrease by 'α'. However, there is point, a threshold, at which, as the S_i/N_m decreases, the o/p S_o/N_o decreases more rapidly than the i/p S_i/N_m. This threshold often marks the limit of usefulness of the demodulator.

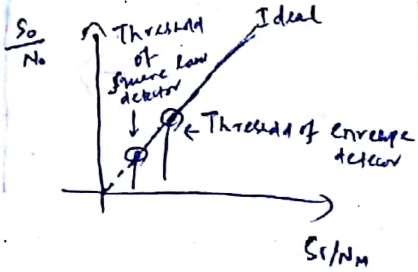
The comparison betn square-law & envelope detector, indicates that below threshold the square-law demodulator performs better than envelope detector. The threshold in square-law demodulation is lower than threshold in envelope demodulation. Therefore, a square-law demodulator will operate above threshold on a weaker signal than will an envelope demodulator. 378

Summary :- Above threshold the synchronous demodulator, the square-law demodulator and the envelope demodulator all perform equally well, provided $\overline{m^2(t)} << 1$.

→ On strong signals all demodulator work equally well except that the square-law demodulator requires $\overline{m^2(t)} << 1$ to avoid base-band signal distortion.

→ On weak-signals, a synchronous demodulation does best since it exhibits no threshold.

→ When synchronous demodulation is not feasible, square-law demodulation does better than envelope demodulation.



For Problems :-

1) Square law demodulator

$$\frac{S_o}{N_o} = \overline{m^2(t)}\ \frac{P_c}{N_m}\ \frac{1}{1+\frac{3}{4}\left(N_m / P_c\right)} \quad (1)$$

P_c → Carrier Power.

$\overline{m^2(t)}$ → time avg of square of modulating waveform.

Case - I , if $\frac{P_c}{N_m} >> 1$,

$$\Rightarrow P_c >> N_m$$

$\frac{N_m}{P_c} \simeq 0$, eqⁿ (1) becomes

$$\boxed{\frac{S_o}{N_o} = \overline{m^2(t)} \cdot \frac{P_c}{N_m}} \qquad \text{--- (2)}$$

Case - II , When $\frac{P_c}{N_m} << 1$, $\Rightarrow \frac{N_m}{P_c} >> 1$

eqⁿ (1) becomes,

$$\frac{S_o}{N_o} = \overline{m^2(t)} \times \frac{P_c}{N_m} \times \frac{4}{3} \times \frac{P_c}{N_m}$$

$$\Rightarrow \boxed{\frac{S_o}{N_o} = \frac{4}{3} \overline{m^2(t)} \left(\frac{P_c}{N_m}\right)^2} \qquad \text{--- (3)}$$

2) For envelope detector

$$\gamma = \frac{S_o/N_o}{S_i/N_m} = \frac{\overline{m^2(t)}}{1 + \overline{m^2(t)}}$$

3) Threshold point in squarelaw modulator,

$$\boxed{P_c = 2.9\ N_m}$$

Ex:-3A) – An audio signal of 4 KHZ BW is to be transmitted through a channel that introduces 30 dB loss and white noise of PSD 10^{-9} $\frac{W}{Hz}$. Calculate required minm transmitter power if the message is sent by SSB, DSB-SC, DSB-C modulation. The received output SNR should be more than 40 dB. For DSB-C, energy in sideband is considered.

(i) For SSB

Ans ÷ Require SNR = 40 dB

$$\Rightarrow \frac{S_i}{\eta f_m} = 10^4$$

$\therefore 10\log_{10} x = 40 \Rightarrow x = 10^4$

$$\Rightarrow \frac{S_i}{2\times10^{-9}\times4000} = 10^4$$

given, white noise $\frac{\eta}{2} = 10^{-9}$ $\Rightarrow \eta = 2\times10^{-9}$

$$\Rightarrow S_i = 8\times10^{-2} = 0.08 \text{ Watt.}$$

∴ Required minm signal strength i/p at receiver = 0.08 Watt.

⇒ Required minm signal strength at the transmitter

$= S_i \times 30\text{ dB}$ [Since 30dB loss]

$= S_r \times 10^3$ [$30\text{dB} = 10^3$]

Tx power should be 10^3 time Rx power.

$= 0.08\times10^3$

$= 80$ Watt (Ans)

(ii) For DSB-SC

$$\frac{S_o}{N_o} = \frac{S_i}{\eta f_m}.$$ [Same as SSB]

So Tx Power is same = 84 watt. (Ans)

(iii) DSB-C

Useful energy in the side band = $\frac{S_i}{3}$

$\left[\because \text{DSB-C}, \quad P_T = P_c\left(1+\frac{m^2}{2}\right) = P_c + \frac{P_c m^2}{2} \right.$

~~Useful power~~ $= P_c + P_{sideband}$

$$\frac{P_{sideband}}{P_{Total}} = \frac{\cancel{P_c}\frac{m^2}{2}}{\cancel{P_c}\left(1+\frac{m^2}{2}\right)} = \frac{m^2}{2+m^2}, \quad \text{for } m=1,$$

$$\frac{P_{sideband}}{P_{Total}} = \frac{1}{2+1} = \frac{1}{3}$$

$$\left. \Rightarrow \quad P_{sideband} = \frac{P_{Total}}{3} \right]$$

~~or~~

$$\frac{S_o/N_o}{S_i/N_m} = \frac{m^2}{2+m^2} \quad [\text{For DSB-C}]$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{1}{3} \times \frac{S_i}{N_m}, \quad \left[\frac{m^2}{2+m^2} = \frac{1}{3}, \text{ for } m=1 \right]$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{1}{3} \times \frac{S_i}{\eta f_m}$$

$$\frac{1}{3} \frac{S_i}{2\times 10^{-9} \times 4000} = 40\text{dB} = 10{,}000$$

$$\Rightarrow S_i = 24 \times 10^{-2} = 0.24 \text{ watt}$$

Since there is loss of $30\text{dB} = 10^3$ in the Channel, Tx power should be 10^3 times received power.

$$Tx \text{ power} = 0.24 \times 10^3$$
$$= 240 \text{ watt.} \quad \text{(Ans)}$$

Ex-4 The time avg of the square of a modulating signal of 60 KHZ bandwidth is calculated as 0.1 watt. The signal is used in DSB-C modulation with carrier power 10W. If additive white noise power spectral density is 10^{-6} W/HZ, find o/p SNR for a square demodulator. Also find o/p SNR if carrier power is reduced by 100 times and noise power increased by 10 times.

Ans :- Given $\frac{\eta}{2} = 10^{-6}$

$$\Rightarrow \eta = 2 \times 10^{-6} \frac{\text{watt}}{\text{HZ}}.$$

$$N_M = \eta f_M = 2 \times 10^{-6} \times 60 \times 10^3 = 0.12 \text{ W}$$

Threshold ~~at~~ occures at carrier power

$$P_c = 2.9 N_m = 2.9 \times 0.12 = 0.348.$$

→ For carrier power, 10 watt, noise power 0.12 watt.

$$P_c \gg N_m$$

$$\frac{S_o}{N_o} = \overline{m^2(t)} \cdot \frac{P_c}{N_m}$$

Given time-avg. of square of a modulating signal

$$\overline{m^2(t)} = 0.1$$

$$\frac{S_o}{N_o} = 0.1 \times \frac{10}{0.12}$$

$$\Rightarrow \frac{1}{0.12}$$

$$\Rightarrow \boxed{\frac{S_o}{N_o} = 9.21 \text{ dB}}$$

→ If $P_c \to \frac{P_c}{100}$, $N_m \to \frac{N_m \times 10}{}$

$P_c = \frac{10}{100}$, $N_m = 0.12 \times 10 = 1.2$

$P_c = 0.1$, $N_m = 1.2$

$$\frac{P_c}{N_m} < < 1$$

$$\frac{S_o}{N_o} = \frac{4}{3} \cdot \overline{m^2(t)} \cdot \left(\frac{P_c}{N_m}\right)^2$$

$$= \frac{4}{3} \times 0.1 \times \left(\frac{0.1}{1.2}\right)^2$$

$$S_o/N_o = 9.25 \times 10^{-4}$$

$$(S_o/N_o)_{dB} = -30.33 \text{ dB}$$

—o—