Ch-4. Pulse Modulation & Digital transmission of Analog Signal

~~When~~ If a signal is transmitted over a channel, then, when the signal arrives at its destination, it is greatly attenuated and also combined with the noise present in the channel ~~and~~ as well as noise present in the receiver.

An amplifier at the receiver will not help the above situation, since at this point both signal & noise level will be increased together.

But suppose that a repeater (repeater is the term used for an amplifier in a communication channel) is located at the midpoint of the long communications path. This repeater will raise the signal level; in addition it will raise the level of only the noise introduced in the first half of the communications path.

Hence, such a mid repeater, as contrasted with an amplifier at the receiver, has the advantage of improving the received signal-to-noise ratio. The midway repeater will relieve the burden imposed on transmitter & cable due to high power requirement, when repeater is not used.

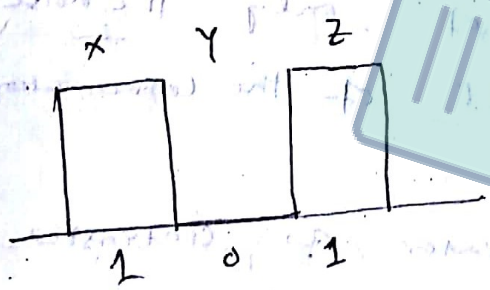
~~Amplifiers or repeaters~~ in analog Communication

Analog repeaters not only amplify noise along with signal but also generate noise internally, thus degrading the quality of communication.

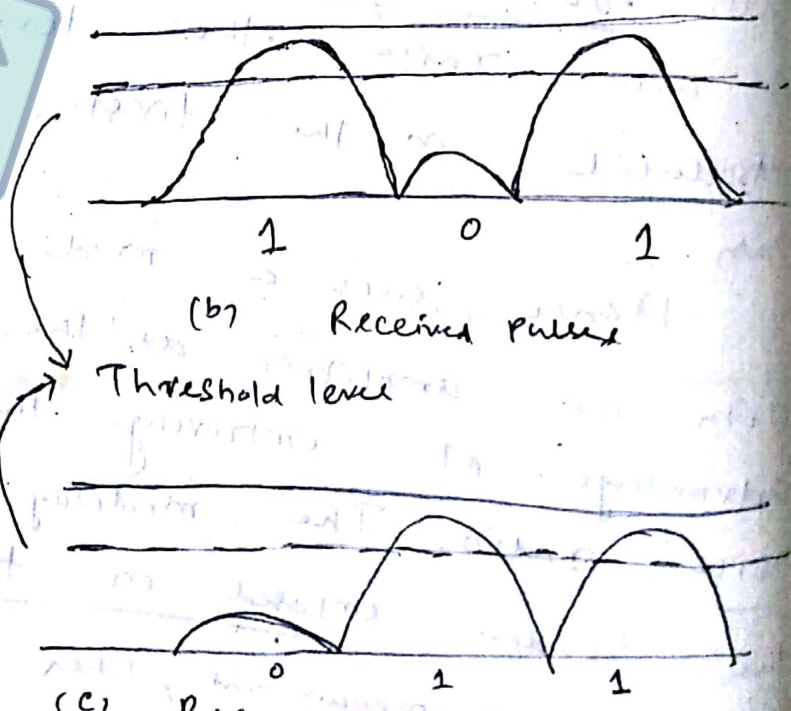
On the other hand, the repeaters needed in digital system (e.g. PCM*) require only regenerators which generate pulses in time slots according to the presence of 0 or 1, thus eliminating the effect of noise till that point.

*PCM → Pulse Code Modulation

Ex:- The major advantage of the digital transmission (e.g. using PCM) is that the information does n't lie in any property of pulse but it lies in the presence or absence of pulse. Thus, even if noise distorts the pulse it makes no difference as long as the decision regarding the presence or absence of pulse is correct.



(a) Transmitted pulse



(b) Received pulses

Threshold level

(c) Received pulses

Fig (b) Shows that although the received Pulses are distorted due to noise, there is no error of decision.

(Note:- When the voltage in a time slot crosses the threshold it is treated as 1 else it is treated as 0)

Transmitted → 101, received → 1.01 (No error)

In extreme noise condition, fig (c) the received pulse is 011, There is error in first and 2nd bit.

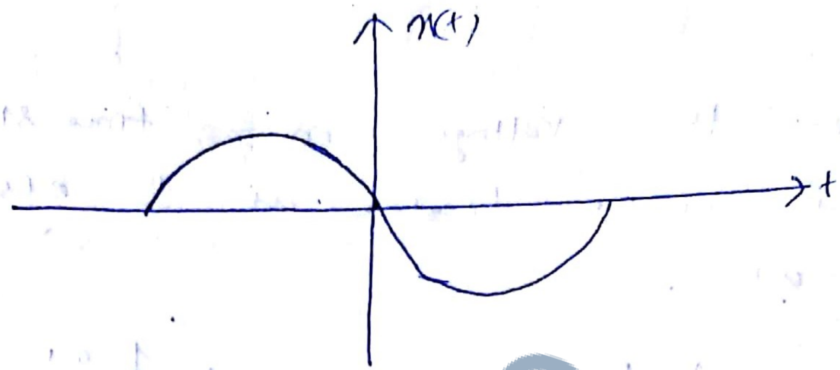
Sampling Theorem :-

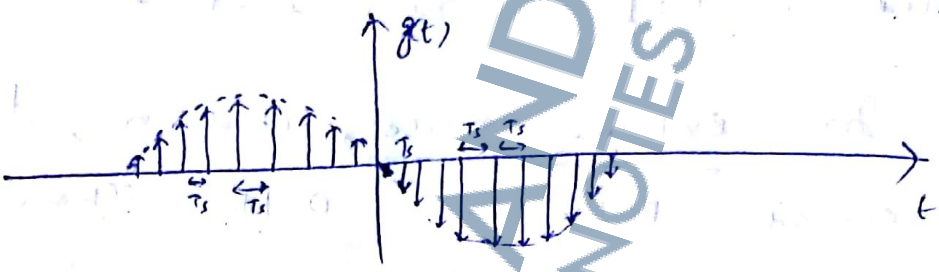
Broadly there are 2 types of signals, Continuous time signal & discrete-time signal. With the help of sampling theorem, a continuous-time signal may be completely represented and recovered from the knowledge of samples taken uniformly. This means that sampling theorem provides a mechanism for representing a continuous-time signal by a discrete time signal.

(A) Sampling of band limited signal :- (Low freq signals)

"A Continuous-time signal may be completely represented in its samples and recovered back if the sampling frequency is $f_s \geq 2f_m$, where 'f_s' is the sampling frequency & 'f_m'

is the maximum frequency present in the signal]





i.e uniform interval = T_s.

(i) A bandlimited signal of finite energy, which has no-frequency-component higher than 'f_m' Hz, is completely described by its sample values at uniform interval less than equal to $\frac{1}{2f_m}$ seconds apart

i.e $\boxed{T_s \leq \frac{1}{2f_m}}$

(ii) In other way, a bandlimited signal of finite energy, which has no frequency components higher than 'f_m' Hz, may be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

ie $\boxed{f_s \geq 2f_m}$

~~Proof :- To prove the sampling theorem, we assume that a signal has spectrum is bandlimited to 'fm' HZ. So it can be~~ we will show that

Proof :- To prove the sampling theorem, we will show that a signal whose spectrum is bandlimited to 'f_m' HZ, can be reconstructed exactly without any error, from its samples taken uniformly at a rate

$$f_s \geq 2f_m \text{ HZ}.$$

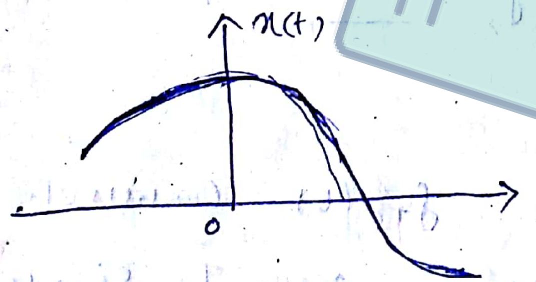
Let us consider a continuous time signal $x(t)$ whose spectrum is bandlimited to 'f_m' HZ, This means that the signal $x(t)$ has no frequency components beyond 'f_m' HZ. Therefore, $X(\omega)$ is zero for $|\omega| > \omega_m$

i.e

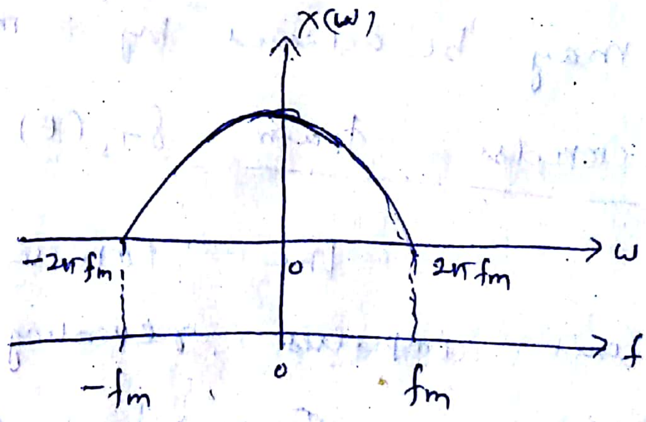
$$X(\omega) = 0, \text{ for } |\omega| > \omega_m$$

where

$$\omega_m = 2\pi f_m$$



(a)



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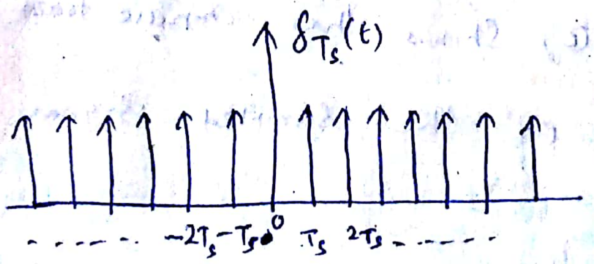
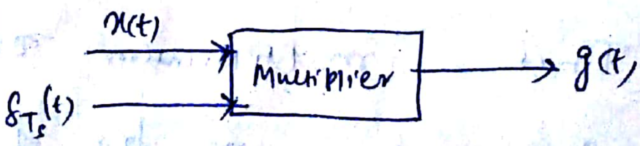
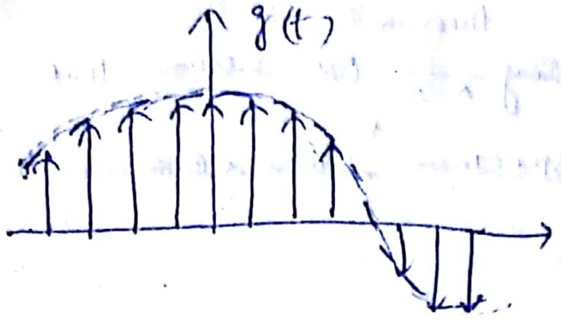


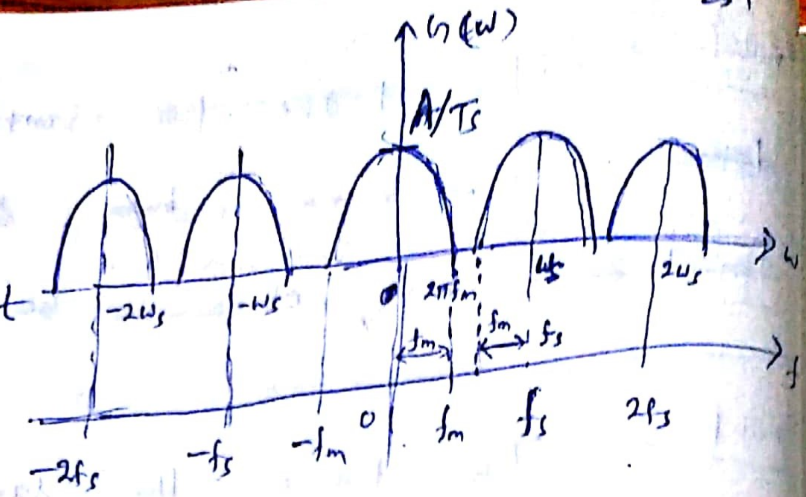
(c)



(d)



(e)



(f)

fig 1 : (a) A Continuous-time signal

(b) Spectrum of Continuous-time signal

(c) Impulse train as sampling function

(d) Multiplier

(e) Sampled signal

(f) Spectrum of sampled signal

Fig 1 (a) shows a continuous-time signal $x(t)$. Let $X(\omega)$ represents its Fourier transform or frequency spectrum as shown in fig 1 (b). Sampling of $x(t)$ at a rate of f_s Hz (Samples per second) may be achieved by multiplying $x(t)$ by an impulse train $\delta_{T_s}(t)$.

The impulse train $\delta_{T_s}(t)$ consists of unit impulses repeating periodically every T_s seconds, Where $T_s = \frac{1}{f_s}$. Fig 1 (c) shows this impulse train. This multiplication results in the sampled signal $g(t)$ shown in fig 1 (e).

This sampled signal consists of impulses spaced every T_s seconds (the sampling interval). The resulting or sampled signal may be written as,

$$g(t) = x(t) \cdot \delta_{T_s}(t) \quad \text{--- (1)}$$

Again since the impulse train $\delta_{T_s}(t)$ is a periodic signal of period T_s, it may be expressed as Fourier series. The trigonometric Fourier series expansion of impulse-train $\delta_{T_s}(t)$ is expressed as,

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \ldots\right] \quad \text{--- (2)}$$

Here $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$

Putting the values of $\delta_{T_s}(t)$ from eqn (2), in eqn (1), the sampled signal is

$$g(t) = \frac{1}{T_s}\left[x(t) + 2\,x(t)\cdot\cos\omega_s t + 2\,x(t)\cos 2\omega_s t + 2x(t)\cos 3\omega_s t + \ldots\right].$$

Now to obtain $G(\omega)$, the fourier transform of $g(t)$, we will have to take F.T of right hand side.

$$F[x(t)] \rightarrow X(\omega)$$

$$F[2x(t)\cdot\cos\omega_s t] \rightarrow [X(\omega-\omega_s) + X(\omega+\omega_s)]$$

$$F[2x(t)\cdot\cos 2\omega_s t] \rightarrow [X(\omega-2\omega_s) + X(\omega+2\omega_s)]$$ and so on.

$$\therefore \; G(\omega) = \frac{1}{T_s}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + X(\omega-3\omega_s) + X(\omega+3\omega_s) + \cdots \right] \quad \text{—} (3)$$

$$\text{or} \quad G(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \quad \text{—} (4)$$

From eqⁿ (3) & (4), it is clear that the spectrum $G(\omega)$ consists of $X(\omega)$ repeating periodically with period $\omega_s = \frac{2\pi}{T_s}$ rad/sec or $f_s = \frac{1}{T_s}$ HZ, as shown in fig 1(f).

Now if we have to reconstruct $x(t)$ from $g(t)$; we must be able to recover $X(\omega)$ from $G(\omega)$. This is possible if there is no overlap between successive cycles of $G(\omega)$. Fig 1.(f) shows that this requires,

$$\boxed{f_s \geq 2f_m}$$

$$\text{or} \quad \boxed{T_s \leq \frac{1}{2f_m}}$$

Therefore, as long as sampling freq f_s is greater than or equal to twice the maxm signal freq (f_m), $G(\omega)$ will consist of non-overlapping repetitions of $X(\omega)$. If this is true, fig 1.(f) shows $x(t)$, can be recovered from its samples $g(t)$, by passing ~~...~~ $g(t)$ through an ideal LPF of band width f_m HZ. This proves the sampling theorem.

Nyquist rate & Nyquist interval :-

When the sampling rate becomes exactly equal to '$2f_m$' samples per second, then it is called Nyquist rate. Nyquist rate is also called minm sampling rate.

$$\boxed{f_s = 2f_m}$$

Similarly, maxm sampling interval is called the Nyquist interval.

Nyquist interval, $\boxed{T_s = \frac{1}{2f_m}}$

Ex:- 1) Find Nyquist rate

$x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$

$f_{m_1} = \frac{50\pi}{2\pi} = 25\ Hz$

$f_{m_2} = \frac{300\pi}{2\pi} = 150\ Hz$

$f_{m_3} = \frac{100\pi}{2\pi} = 50\ Hz.$

($\cancel{\omega t} = 50\pi \cancel{t}$

$2\pi f = 50\pi$

$f = \frac{50\pi}{2\pi} = 25$

$f_s = 2\,(f_m)_{max}$

$= 2\times 150$

$$\boxed{f_s = 300\ Hz}$$

2) $x(t) = \frac{1}{2\pi}\cos(4000\pi t).\cos(1000\pi t)$

$x(t) = \frac{1}{4\pi}.\ 2\cos 4000\pi t\cdot\cos(1000\pi t)$

$$2\cos\alpha \cdot \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$\therefore x(t) = \frac{1}{4\pi}\left[\cos(5000\pi t) + \cos(3000\pi t)\right]$$

$\omega_1 = 5000\pi$, $\quad f_{m1} = \frac{5000\pi}{2\pi} = 2500$.

$\omega_2 = 3000\pi$, $\quad f_{m2} = \frac{3000\pi}{2\pi} = 1500$.

$$f_s = 2(f_m)_{max}$$

$$f_s = 2 \times 2500$$

$$\Rightarrow f_s = 5000\ Hz = 5\ KHz \quad \text{(Ans)}$$

Nyquist rate $(f_s) = 5KHz$

Nyquist interval $= \frac{1}{f_s} = \frac{1}{5\times10^3} = 0.2\ msec$

$$\boxed{\begin{aligned} f_s &= 5KHz \\ T_s &= 0.2\ msec \end{aligned}}$$

Aliasing effect :-

When a continuous-time band limited signal is sampled at a rate lower than Nyquist rate i.e. $f_s < 2f_m$, the successive cycle of the spectrum $G(\omega)$ of sample signal gets overlap with each other. This type of distortion that results from under sampling is known as aliasing error or aliasing distortion.

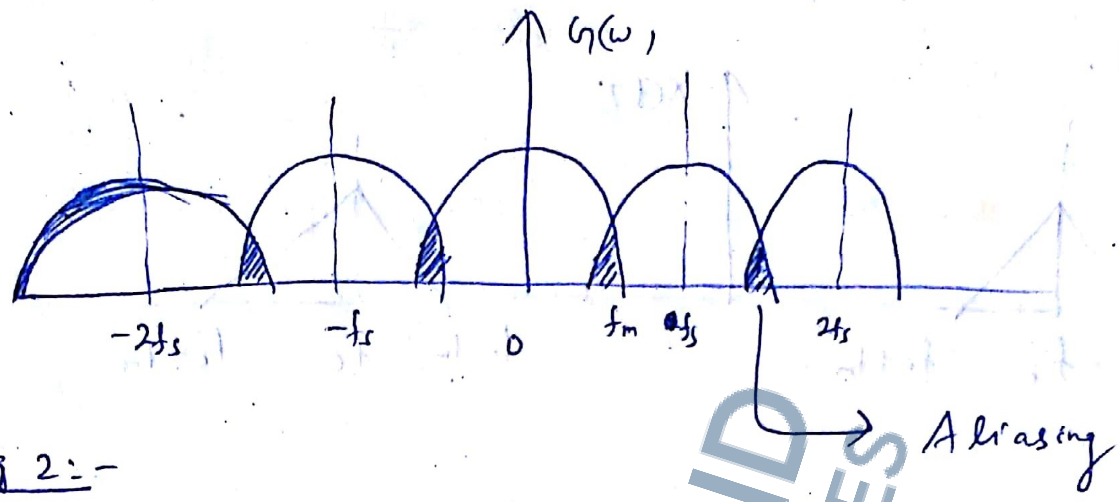


fig 2:-

Spectrum of sampled signal for the case $\boxed{f_s < 2f_m}$

To avoid aliasing:-

1) Prealias filter must be used to limit band of frequency of signal to f_m Hz.

2) Sampling freq 'f_s' must be selected such that $f_s > 2f_m$.

Sampling of band pass signal:-

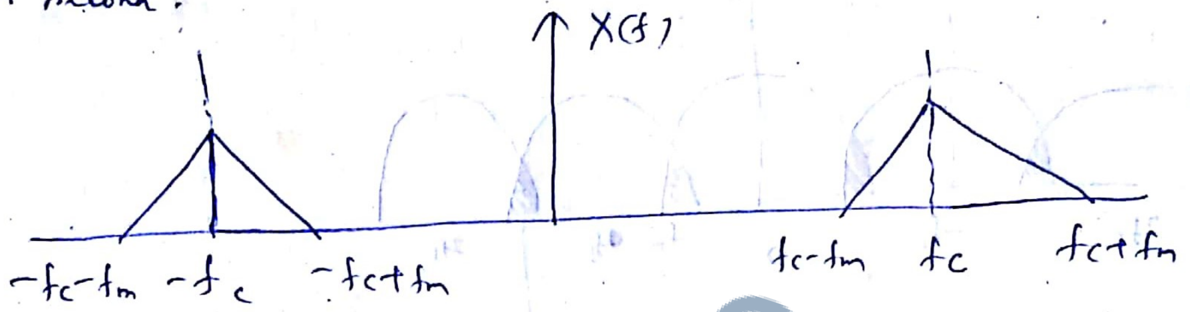
(which has a lower & upper cutoff freq)

→ one particular band of freq.

The bandpass signal $x(t)$ whose max^m bandwidth is '$2f_m$' can be completely represented into & recovered from its samples if it is sampled at the min^m rate of twice of the bandwidth.

Here, f_m = max^m freq. component present in the signal.

Hence, if the BW is $2f_m$, max^m sampling rate for bandpass signal must be $4f_m$ samples per second.



Min^m sampling rate = Twice of BW

$$= 2 \times 2 f_m$$

$$= 4 f_m$$

$$\boxed{f_s = 4 f_m \text{ samples / second}}$$

ex — 3) The spectral range of function extends from 10.0 to 10.2 MHZ. Find the min^m sampling rate & max^m sampling time.

Ans :

$$f_s = 2 \times BW = 2 \times (f_H - f_L)$$

$$= 2 \times (10.2 - 10.0) \times 10^6$$

$$= 2 \times 0.2 \times 10^6$$

$$\boxed{f_s = 0.4 \text{ MHZ}}$$

$$T_s = \frac{1}{f_s} = \frac{1}{0.4 \times 10^6} = 2.5 \ \mu s.$$

(Ans)

Pulse Modulation System :-

In pulse modulation systems the Carriers are no longer Continuous in nature but Consist of several pulse train. In pulse modulation, the parameters of pulse are varied in accordance with instantaneous values of modulating signal.

Pulse modln are of 2 types

→ Pulse Amplitude modulation (PAM)

→ Pulse Time Modulation (PTM)

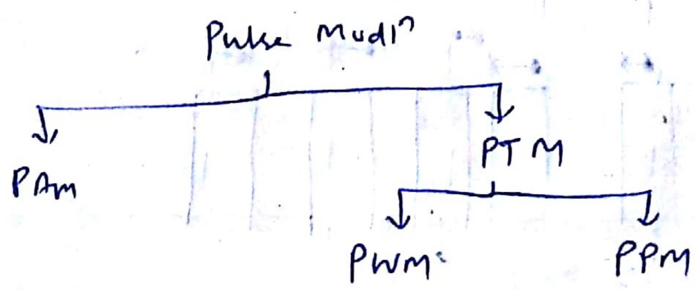
PAM :- In PAM Scheme, the amplitude of the pulses of Carrier signal is varied according to the modulating signal.

PTM :- In PTM Scheme the timing of the pulses of the carrier signal are varied according to the modulating signal.

Again PTM Scheme are of 2 types

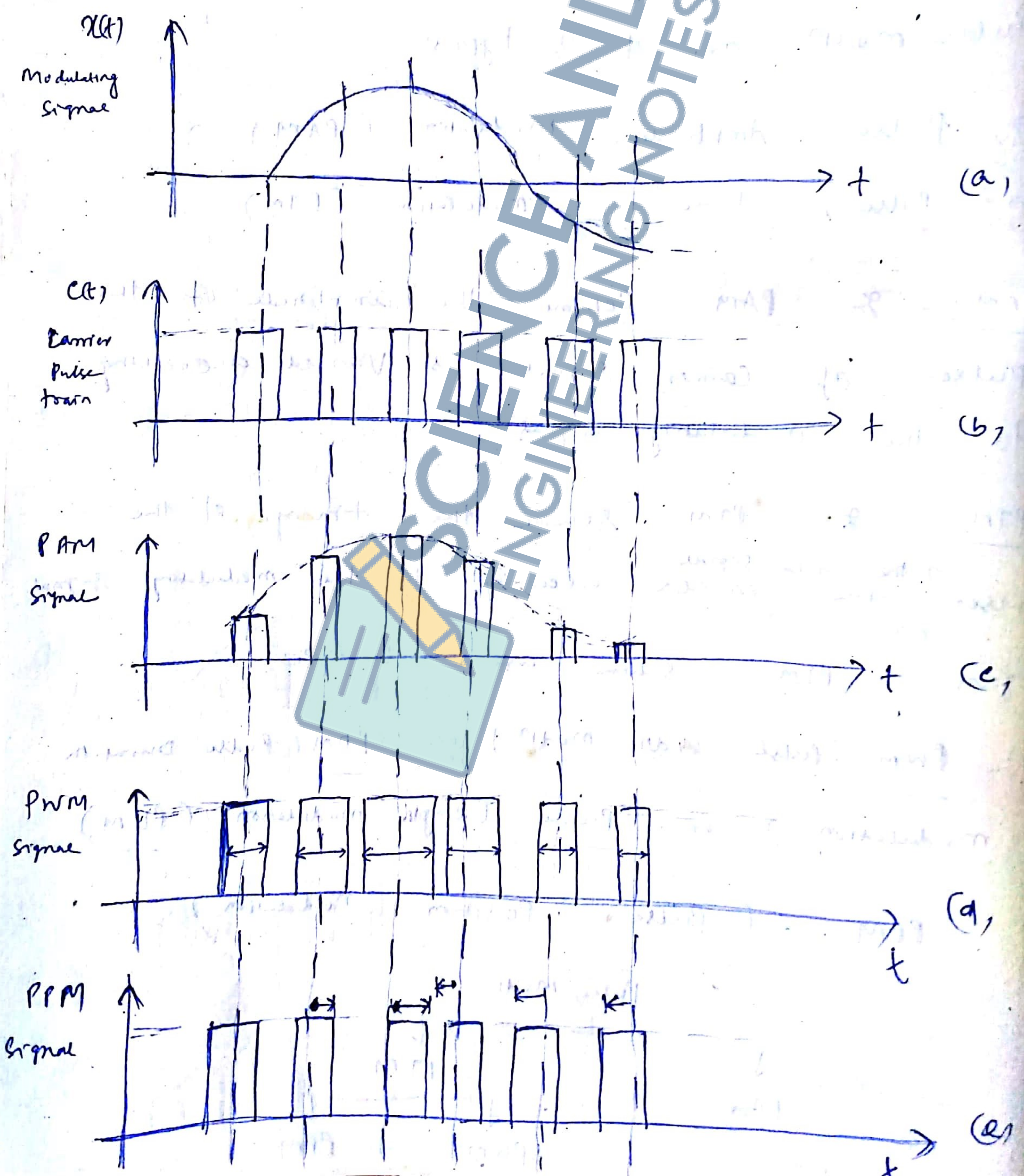
(a) PWM (Pulse Width modln) or PDM (Pulse Duration modulation) or Pulse Length Modulation (PLM)

(b) PPM (Pulse Position Modulation)



PWM :- In PWM, the width of the pulses of the carrier pulse train is varied in accordance with the modulating signal

PPM :- In PPM, the position of the pulses of the carrier pulse train is varied in accordance with the modulating signal.



Q) <u>Nyquist rate</u>

(i) $\operatorname{Sinc}(500t)$ (ii) $Sa(1500\pi t)$

<u>Ans</u> :- (i) $\operatorname{Sinc}(x) = \dfrac{\sin \pi x}{\pi x} = \dfrac{\sin \pi \cdot 500t}{\pi \cdot 500t}$.

~~$2\pi f_m$~~ $\omega_m \cancel{t} = 500\pi \cancel{t}$

$\Rightarrow 2\cancel{\pi} f_m = 500 \cancel{\pi}$

$\Rightarrow 2f_m = 500$.

$\Rightarrow \boxed{f_s = 500 \text{ Hz}}$

(ii) $Sa(x) = \dfrac{\sin x}{x}$

$Sa(1500\pi t) = \dfrac{\sin 1500\pi t}{1500\pi t}$.

$\omega_m \cancel{t} = 1500\pi \cancel{t}$

$\Rightarrow 2\cancel{\pi} f_m = 1500 \cancel{\pi}$

$\Rightarrow 2f_m = 1500$

$\Rightarrow \boxed{f_s = 1500 \text{ Hz}}$

* <u>Note</u> :- In some books, $\operatorname{Sin\cdot c}(x) = Sa(x) = \dfrac{\sin x}{x}$.

So you first check the question,

If $\operatorname{Sinc}(500t)$ given, apply $\dfrac{\sin \pi(500t)}{\pi \cdot (500t)}$ i.e $\boxed{\dfrac{\sin \pi x}{\pi x}}$.

If $\operatorname{Sinc}(500\pi t)$ given apply, $\dfrac{\sin(\pi\, 500t)}{(\pi \cdot 500t)}$, i.e $\boxed{\dfrac{\sin x}{x}}$.

Sampling Techniques :-

Basically, there are 3 types of sampling techniques

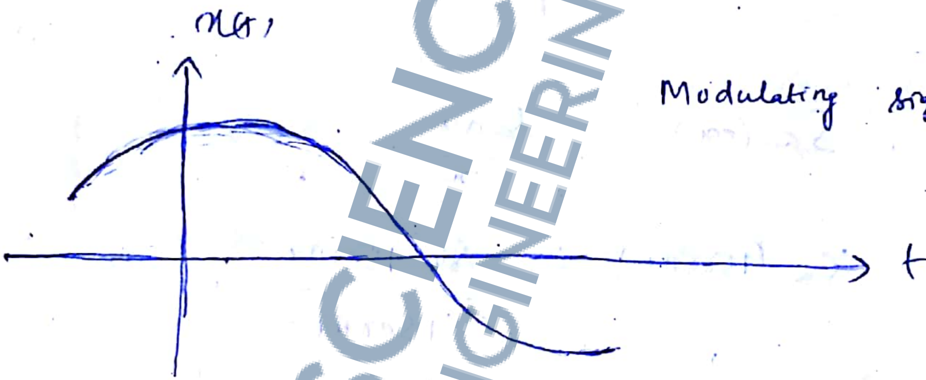
(i) Instantaneous sampling

(ii) Naturne sampling

(iii) Flat-top sampling.

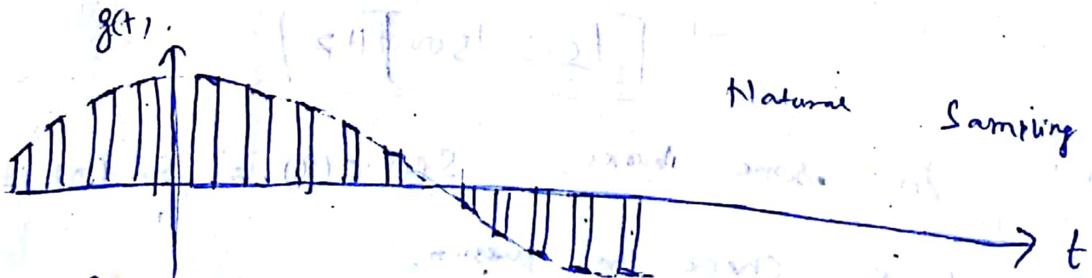
Out of these three, instantaneous sampling is called ideal sampling where as natural sampling and flat-top sampling are called practical sampling methods.

~~Instant~~

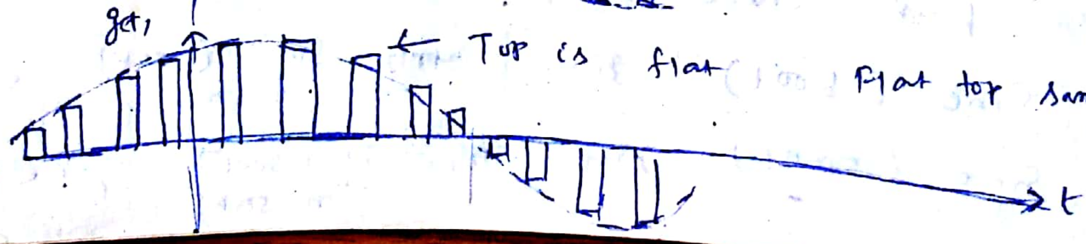


Modulating signal.

Instantaneous sampling



Natural Sampling



Flat top sampling

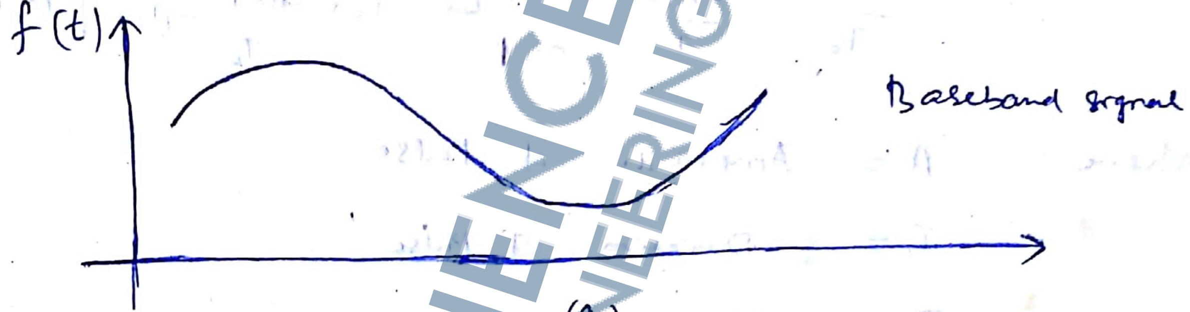
PAM can be obtained in 2 ways

(a) Natural Sampling

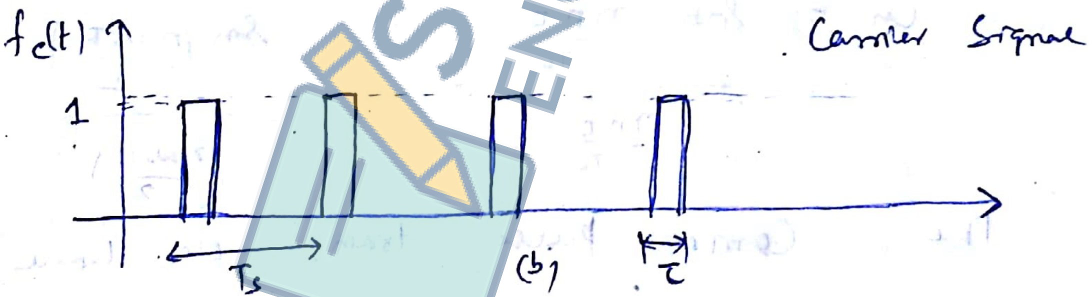
(b) Flat-top Sampling

(a) Natural Sampling :-

The 'name' natural sampling is given because the top of the PAM signal are not flat but they follow the natural waveform of the modulating signal f(t), shown in fig (a).



(a)



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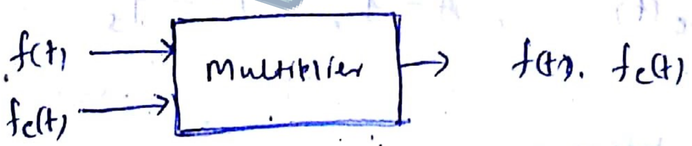

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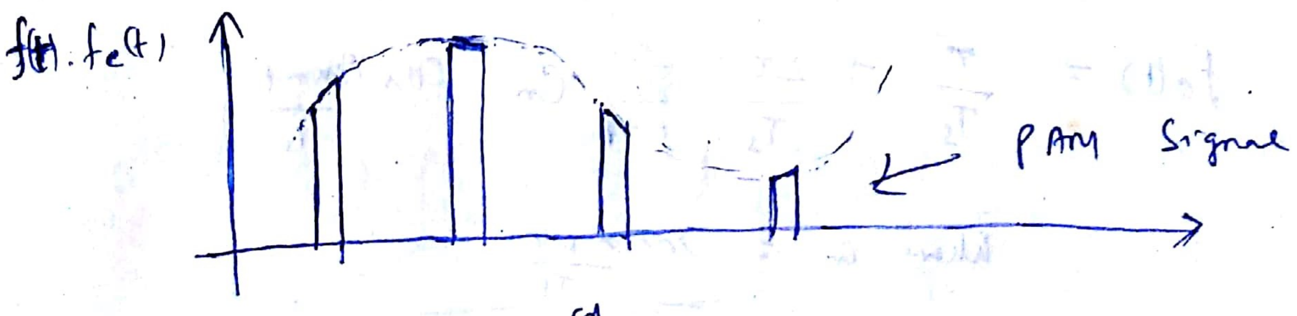


(d)

Consider fig(b), Here we have assumed the amplitude of the carrier signal is 1, having duration τ and separated by T_s (Time period of pulse

By multiplying $f(t)$ & $f_c(t)$ of fig(b), we can get PAM signal $f(t) \cdot f_c(t)$.

Mathematical Analysis :-

We know the Fourier series of a periodic pulse train is

$$v(t) = \frac{A\tau}{T_0} + \frac{2A\tau}{T_0} \sum_{n=1}^{\infty} C_n \cos \frac{2\pi n t}{T_0} \quad -(1)$$ [* Trigonometric form]

Where A = Amplitude of pulse

τ = Duration of pulse

T_0 = Time period of pulse

$$C_n = \frac{\sin \frac{n\pi\tau}{T_0}}{\frac{n\pi\tau}{T_0}} = \frac{\sin\left(\frac{n\omega_0\tau}{2}\right)}{\left(\frac{n\omega_0\tau}{2}\right)}$$

For the carrier pulse train, we have

$v(t) = f_c(t)$, $A = 1$, $T_0 = T_s$

$\therefore$ Eqn (1), becomes,

$$f_c(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s} \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi n t}{T_s}\right)$$

where $C_n = \dfrac{\sin\left(\frac{n\pi\tau}{T_s}\right)}{(n\pi\tau/T_s)}$

$$f_c(t) = \frac{\tau}{T_s} + \frac{2\tau}{T_s}\left[C_1 \cos\left(\frac{2\pi t}{T_s}\right) + C_2 \cos\left(\frac{2 \times 2\pi \times t}{T_s}\right) + \cdots\right]$$

The o/p of the multiplier is,

$$f(t) \cdot f_c(t) = \frac{\tau}{T_s} \cdot f(t) + \frac{2\tau}{T_s}\left[f(t) \cdot C_1 \cos\left(\frac{2\pi t}{T_s}\right) + f(t) \cdot C_2 \cdot \cos\left(\frac{2 \times 2\pi t}{T_s}\right) + \cdots\right] \quad \text{--- (2)}$$

By using sampling thm, we have

$$T_s = \frac{1}{2f_m} \qquad \left[\because f_s = 2f_m \Rightarrow T_s = \frac{1}{2f_m}\right]$$

f_m = max^m frequency component in $f(t)$.

Substituting $T_s = \frac{1}{2f_m}$ in eqn (2), we have

$$f(t) \cdot f_c(t) = \underbrace{\tau \cdot 2f_m\, f(t)}_{\text{first term}} + \underbrace{2\tau \cdot 2f_m\left[f(t) \cdot C_1 \cos(2\pi t \cdot 2f_m) + \cancel{f(t) \cdot C_2 \cos(2\pi t} + f(t) \cdot C_2 \cos(2 \times 2\pi t \times 2f_m) + \cdots\right]}_{\text{2nd term}} \quad \text{--- (3)}$$

→ Neglecting the multiplication factor $(\tau \cdot 2f_m)$, in the first term, then the first term in eq^n (3) is the baseband $f(t)$ itself.

→ The first component of second term is product of $f(t)$ and a sinusoidal frequency component $2f_m$.

i.e. $\{ f(t) \cdot \cos(2\pi \cdot (2f_m) t)\}$

If $f(t) = A_m \cos 2\pi f_m t$

then $f(t) \cdot \cos 2\pi (2f_m)t$ will gives

$$= \underline{A_m \cos 2\pi f_m t} \cdot \cos 2\pi (2f_m) t$$

$$= \frac{A_m}{2} \cdot 2 \cos \underbrace{2\pi (2f_m) t}_{\alpha} \cdot \cos \underbrace{2\pi (f_m) t}_{\beta}$$

$(2 \cos\alpha \cdot \cos\beta)$

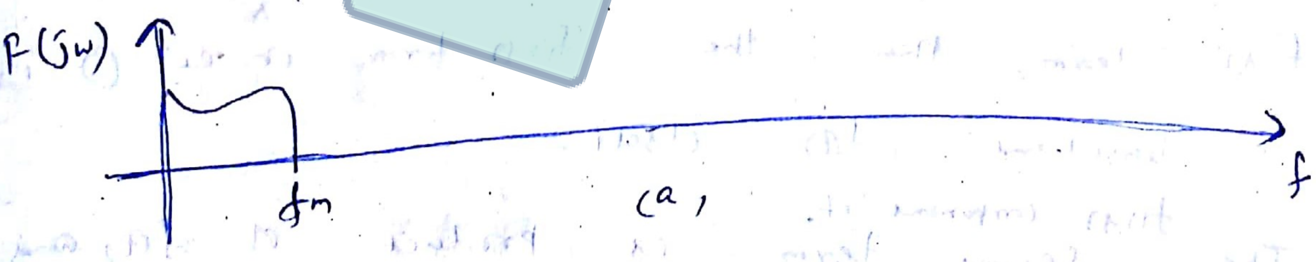
$$= \frac{A_m}{2} \cdot [\cos 2\pi (2f_m + f_m)t] + \cos [2\pi (2f_m - f_m)t]$$

$$= \frac{A_m}{2} \cdot [\cos (2\pi \, 3f_m t) + \cos (2\pi f_m t)]$$

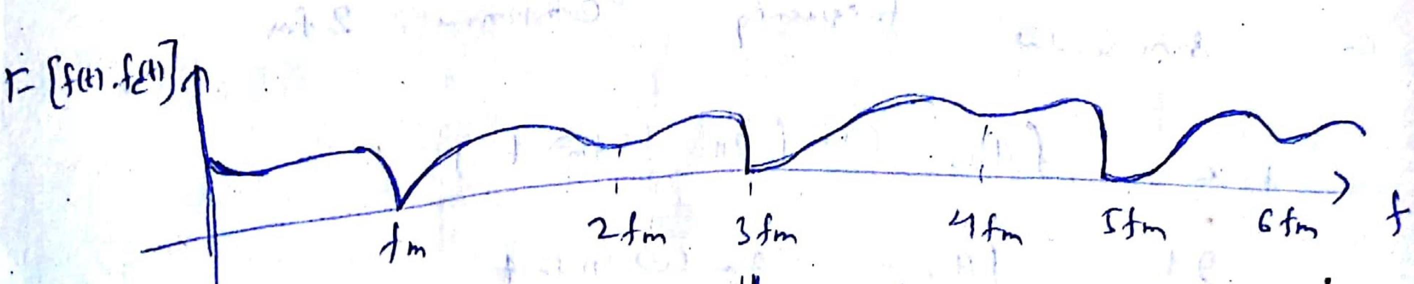
Thus the spectrum of first component of second term is from $\underline{f_m}$ to $\underline{3f_m}$.

Similarly the 2nd component of second term is from $(4f_m + f_m)$ to $(4f_m - f_m)$ i.e. $5f_m$ to $3f_m$.

and so on.



(a)



(b)

fig (a) - Magnitude plot of spectral density of $f(t)$;

(b) - magnitude plot of spectral density of $f_s(t)$

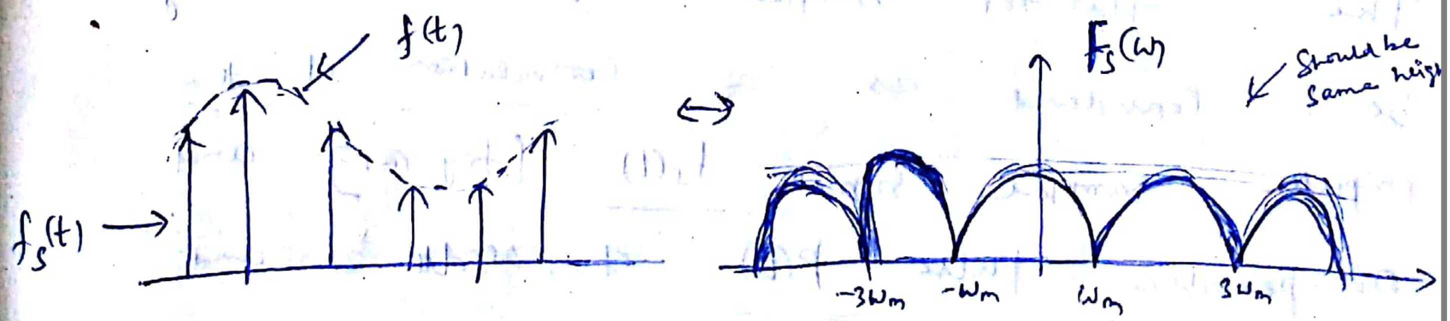
$f_s = f(t) \cdot f_c(t)$.

Flat top Sampling :-

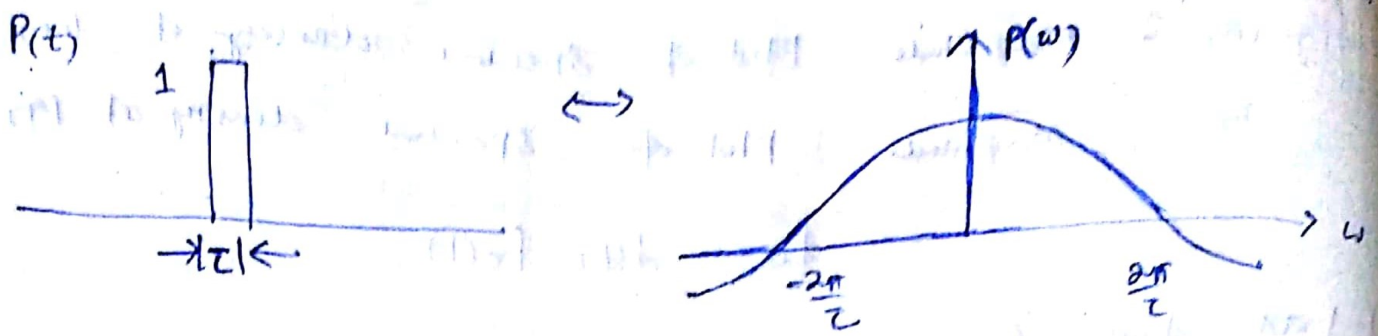
The electronic circuitry needed to perform natural sampling is somewhat complicated because the the pulse-top shape is to be maintained. These complications are reduced by flat-top sampling.

During transmission, PAM signal is contaminated by noise. In case of naturally sampled signal, it becomes quite difficult to determine the shape of the top of the pulse and thus amplitude detection of pulse is not exact.

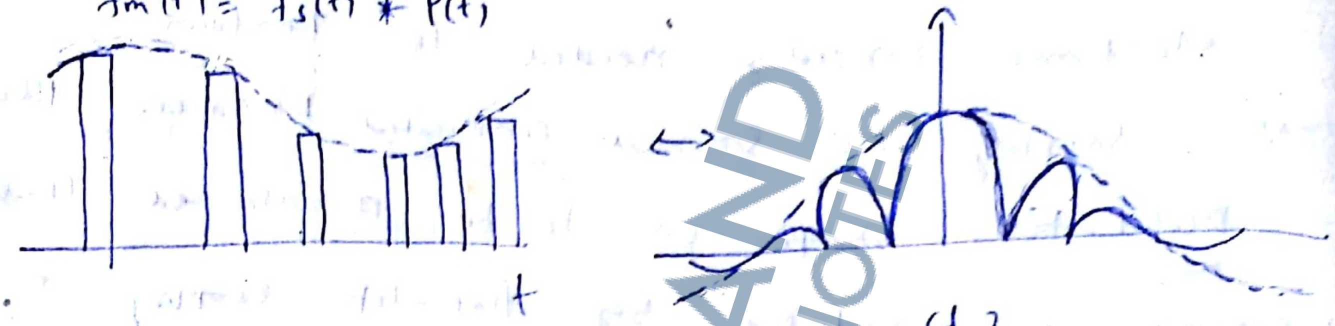
Therefore, flat top sampled PAM is widely used. In this flat-top sampling, the top of modulated pulses are flat. Thus the pulse have a constant amplitude within pulse interval.



(a) (b)



$f_m(t) = f_s(t) * p(t)$



(e) (f)

fig:

(a) Impulse sampled signal $f_s(t)$

(b) Spectrum of $f_s(t)$

(c) Non periodic pulse $p_c(t)$ of width τ & height 1

(d) Spectrum of $P(t)$

(e) Flat-top sampled PAM signal $f_m(t)$

(f) Spectrum of $f_m(t)$

The flat-top sampled signal $f_m(t)$ may be considered as a convolution of the impulse sampled signal $f_s(t)$ [fig (a)] and non-periodic pulse $p(t)$ of width 'τ' and height 1 . [fig (c)] .

A PAM Modulator Circuit :-

The Flat top PAM is most popular and is widely used. The reason for using flat top PAM is that during the transmission, the noise interferes with the top of the transmitted pulses and this noise can be ~~its~~ easily removed if the PAM pulse has flat top.

However, in case of natural samples PAM signal, the pulse has varying top in accordance with signal variation. Now when such type of pulse is received at the receiver, it is always contaminated by noise. Then it becomes quite difficult to determine the shape of the top of the pulse and thus amplitude detection of pulse is not exact. Due to this, errors are introduced in the received signal. Therefore, flat top sampled PAM is widely used.

Fig 1, shows the sample & hold circuit to produce flat top sampled PAM and the waveform for flat top sampled PAM.

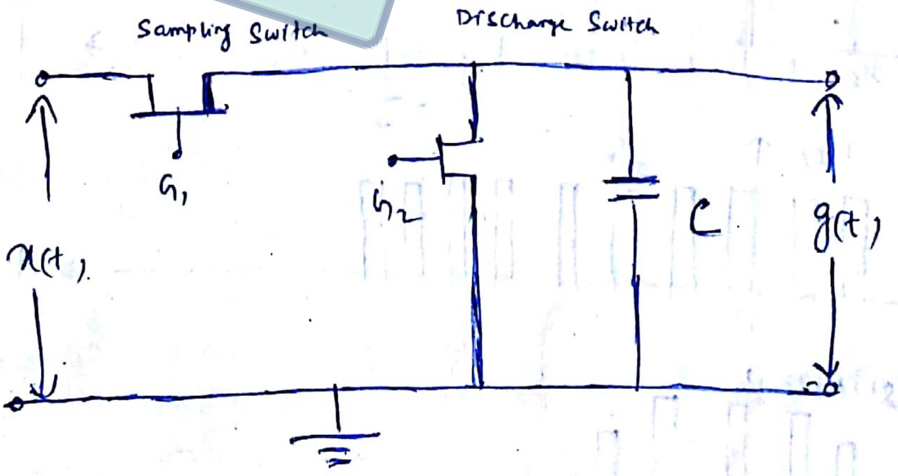
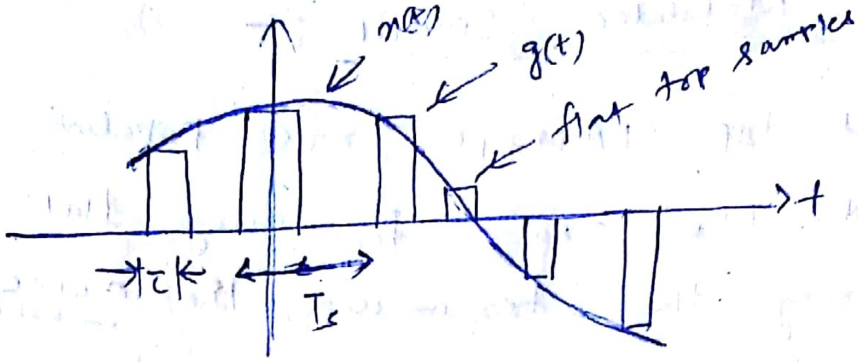


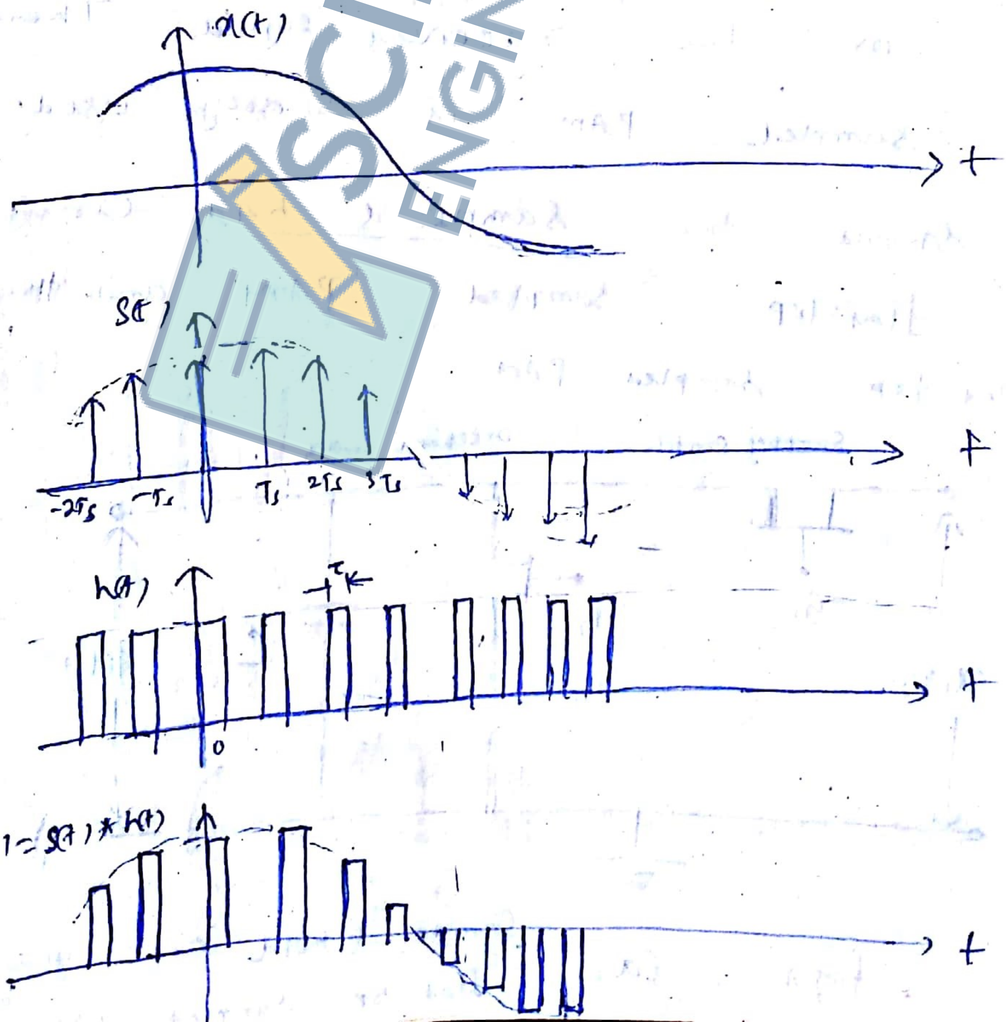
fig 1: - (a) Sample & hold ckt generating flat top sampled PAM.



(b) Waveform of flat top sampled PAM.

Working principle :-

A sample & hold circuit shown in fig 1 (a) is used to produce flat top sampled PAM. The working principle of this circuit is quite simple. The sample & hold (S/H) circuit consists of two field effect transistor (FETs) switches & a capacitor.



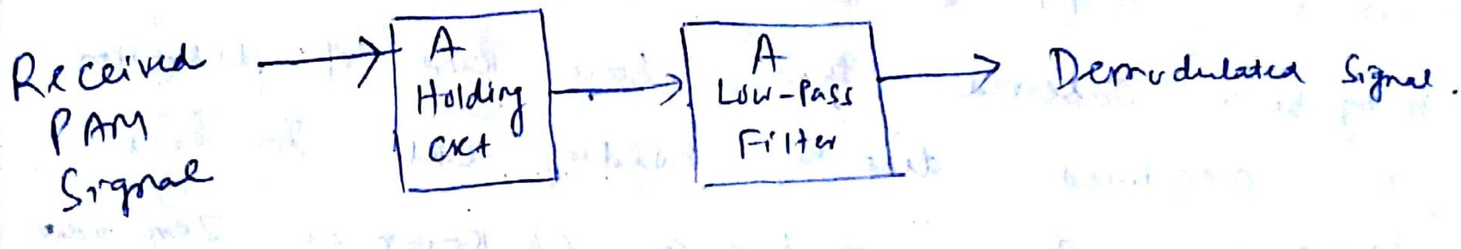
The sampling switch is closed for a short duration by a short pulse applied to the gate G_1 of the transistor. During this period, the capacitor 'C' is quickly charged up to a voltage equal to the instantaneous sample value of the incoming signal $x(t)$.

Now, the sampling switch is opened and the capacitor 'C' holds the charge. The discharge switch is then closed by a pulse applied to gate G_2 of the other transistor. Due to this, the capacitor 'C' is discharged to zero volts. The discharge switch is then opened and thus the capacitor has no voltage.

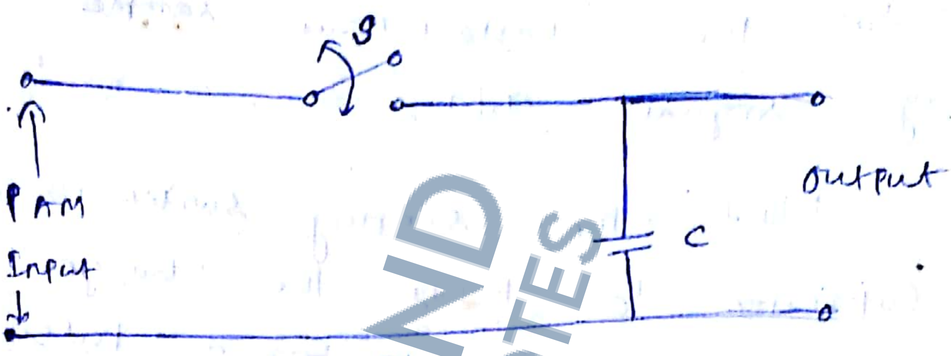
Hence, the o/p of the sample and hold ckt consists of sequence of flat top samples as shown in fig (b).

Demodulation of PAM Signal:-

For pulse Amplitude Modulated (PAM) signals, the demodulation is done using a holding ckt. Fig shown below, shows the block diagram of a PAM demodulator.

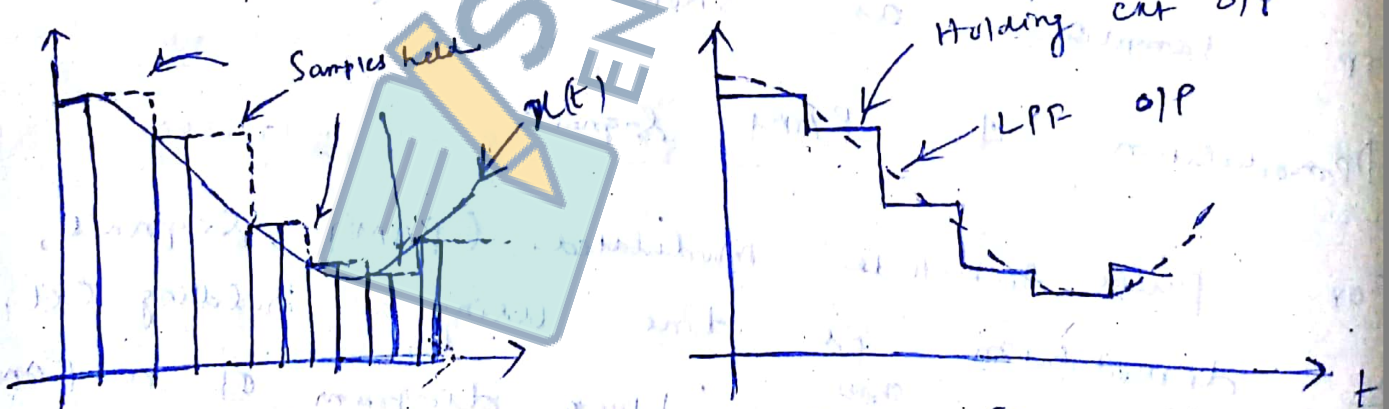


In this method, the received PAM signal is allowed to pass through a holding ckt, and a low-pass-filter as shown in the figure below.
A simple holding ckt is shown



(a)

Here the switch 'S' is closed after the arrival of the pulse and is opened at the end of the pulse. In this way, the capacitor 'C' is charged to the pulse amplitude value and it holds this value during the interval between the two pulses. Hence, the sampled values are held as shown in figure below. (b)



(b) (c)

After this holding ckt o/p is smoothened in a LPF. & as shown in fig (c). It may be observed that some kind of distortion is introduced due to holding ckt. In fact the ckt, shown in fig. (a) is known as zero-order holding ckt. The Zero-order holding ckt considers only the previous samples to decide the value between the two pulses.

Note :- It may be noted that the first order hold ckt considers the previous two samples where as a second order holding ckt considers the previous previous three samples & so on. However, as the order of the holding ckt. increases, the distortion decreases at the cost of the ckt complexity.

Ex:- 1) For a PAM transmission of voice signal having max^m freq. equal to $f_m = 3$ KHZ, Calculate the transmission BW. It is given that sampling freq $f_s = 8$ KHZ and pulse duration $\tau = 0.1\,T_s$.

Ans :

$$T_s = \frac{1}{f_s} = \frac{1}{8\times10^{3}} = 0.125 \text{ ms}$$

$$\tau = 0.1\,T_s = (0.1).\ (0.125)\times10^{-3}$$
$$= 12.5\ \mu s$$

For PAM,

$$BW \geq \frac{1}{2\tau}$$

$$BW \geq \frac{1}{2\times12.5\times10^{-6}}$$

$$BW \geq \frac{10^2\times10^4}{25}$$

$$\boxed{BW \geq 40 \text{ KHZ}}$$

$\because \tau << T_s$

Max^m $T_s = 2\tau\ (\tau_{on} + \tau_{off})$

Min^m $f_s = \frac{1}{2\tau}$.

$BW|_{min} = \frac{1}{2\tau}$

$BW \geq \frac{1}{2\tau}$

TDM (Time division multiplexed) / PAM system

In a PAM system the pulse duration (τ) is less than time period of the pulse (T_s). i.e $\tau << T_s$ (shown in fig (a)). Due to this no information is transmitted through the system for most of time.

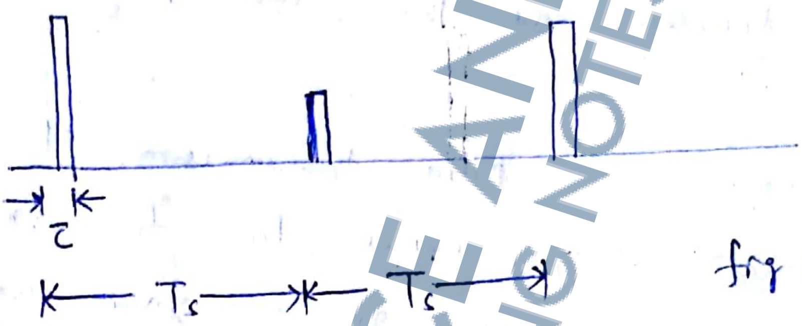


fig (a)

→ The remaining time ($T_s - \tau$) can be utilized to transmit the information from other signals.

→ For example :- Here, the time period T_s is equally divided between the 4 signals. So allocating $\frac{T_s}{4}$ time slot for each signal (fig. b)

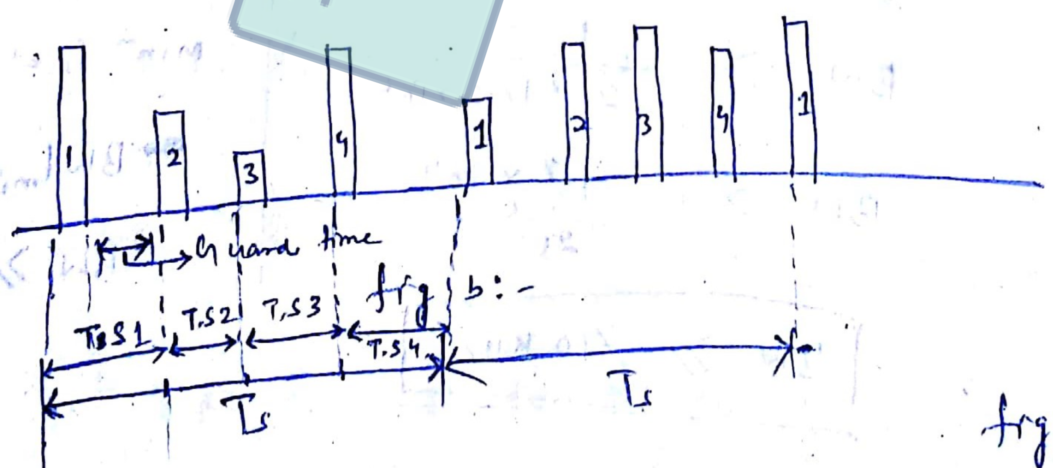


fig b:-

T.S → Time slot

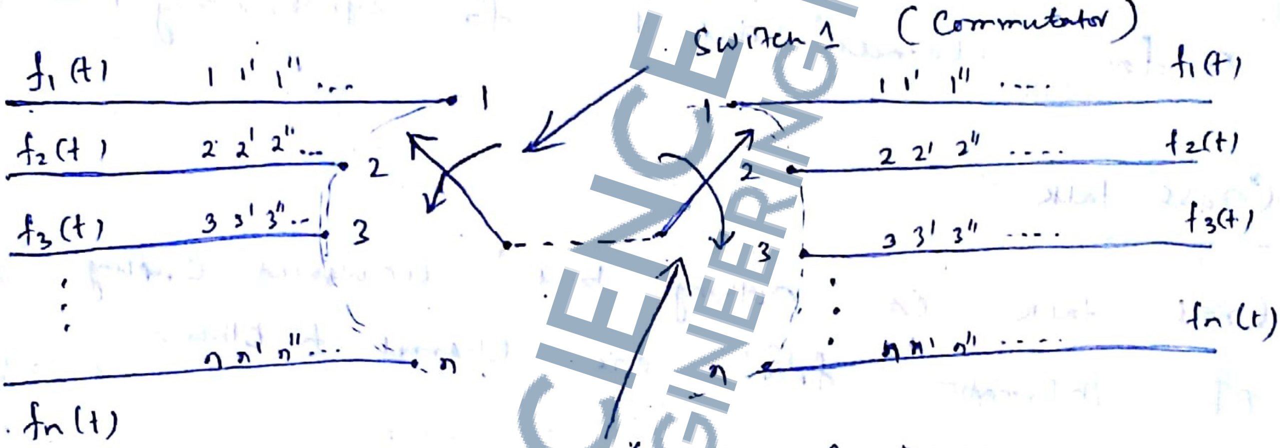
T_s → Time period of pulse.

$T.S\,1 = T.S\,2 = T.S\,3 = T.S\,4 = \frac{T_s}{4}$

→ The duration of each slot is such that $\frac{T_s}{4} > \tau$.

→ The duration $\frac{T_s}{4} - \tau$ is called guard time between all successive sampling pulses.

→ The arrangement by which the information from more than one signal is transmitted in this manner is called Time Division multiplexing (TDM) shown in figure below.



Switch 2 (De commutator)

fig:- A TDM PAM System

→ The above ckt is used to transmit information from n signals. The switch 1 & switch 2 are known as commutator & de commutator respectively. These 2 switches rotates at the same speed $2f_m$ rotation per second.

→ The commutator samples and combines the samples, while decommutator separates the samples belonging to individual signals.

→ To provide synchronization a synchronizing pulse is transmitted in every frame (time interval between 2 successive samples of the same signal i.e T_s)

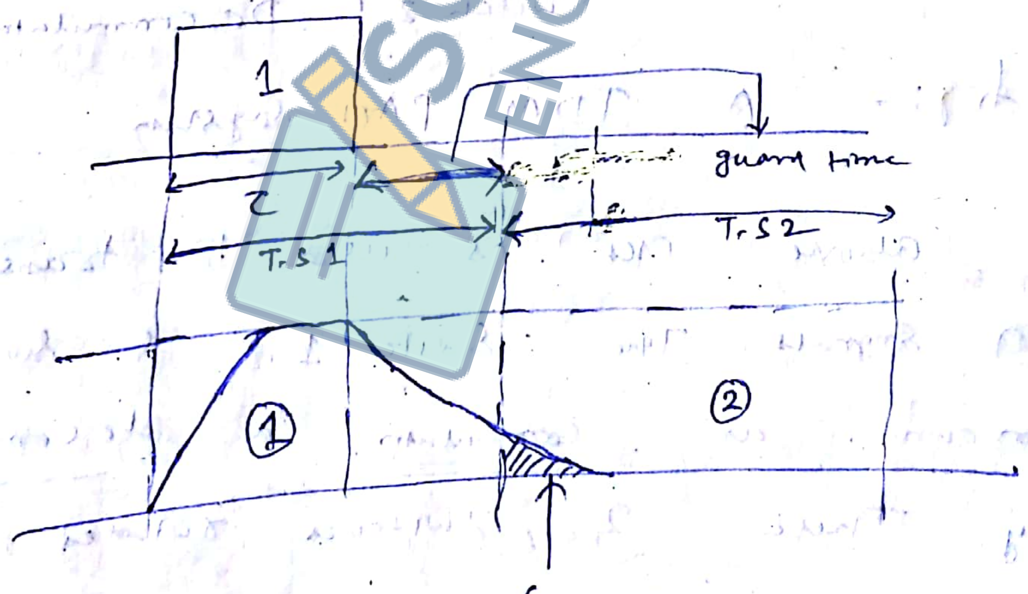
→ Ex:- 1 2 3 4 $\boxed{S}$ 1 2 3 4 $\boxed{S}$

Synchronizing pulse.

→ Thus to multiplex n channels, (n+1) time slots are provided in a frame i.e n for channels and 1 for synchronizing pulse.

Cross talk :-

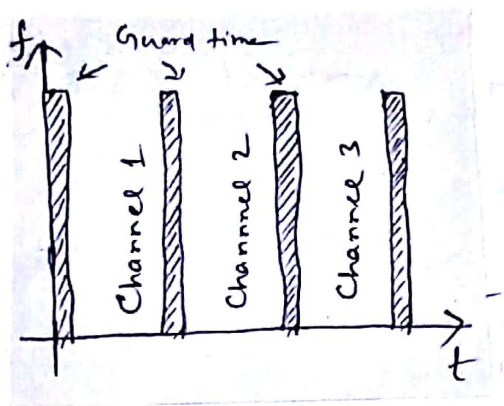
Cross talk is nothing but unwanted coupling of information from one channel to other.



The cross talk is due to overlapping of pulse from ~~TS~~ time slot 1 into the time slot 2.

TDM

1) It is a technique for transmitting several messages in one channel by dividing the time domain slots for one message.

2) 

3) It requires commutator at the transmitter end a decommutator working in a perfect synchronization with commutator at the receiving end.

4) Perfect synchronization between transmitter & receiver is required.

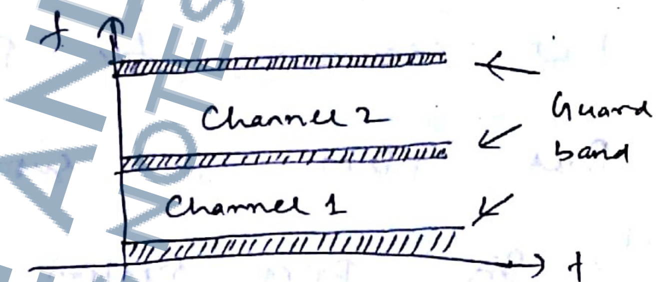
5) Cross talk problem is not severe in TDM

6) It is usually preferred for digital signal transmission.

7) It does n't require complex circuitry.

FDM

1) In this technique, to transmit several messages in one channel, message signals are distributed in freq domain such that they don't overlap.

2) 

3) FDM requires modulator, filters and demodulator.

4) Synchronization between transmitter & receiver is not required.

5) FDM suffers from cross talk problem due to imperfect band pass filter.

6) It is usually preferred for analog transmission.

7) It requires complex circuitry for transmission & reception.

8) BW requirement | BW requirement

AM/SSB → } nf_m | AM-SSB } nf_m
PAM → | PAM

n → no. of signals to be transmitted simultaneously

AM/DSB } $2nf_m$ | AM/DSB } $2nf_m$
AM | AM

BW requirement for FDM/TDM are same.

But TDM is superior to FDM in following ways

(i) In FDM system, different carriers are to be generated for different channels. Also each channel occupies a different freq band, different band pass filter are required.

On the other hand, in TDM system, all the channels require identical ckts, consisting of simple synchronous switches, gates & LPF. The circuitry needed in TDM system is much simpler than the one needed in FDM system.

(ii) The non-linearities in the various amplifiers of an FDM system produce harmonic distortion and hence, they introduce interference within the channels.

But in TDM, the signal from different channels are allotted different time slots and they are not applied to the system simultaneously. Thus, the TDM is relatively immune to the interference within the channels as compared to FDM system.

Comparision ~~be~~ among PAM | PWM | PPM

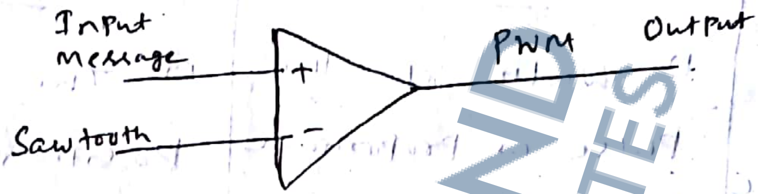
PAM	PWM	PPM
1) Amplitude of Pulse is proportional to the amplitude of modulating signal.	1) Width of the Pulse is proportional to the amplitude of the modulating signal.	1) The relative position of the pulse is proportional to the amplitude of modulating signal.
2) BW of transmission channel depends on the width of the Pulse.	2) BW of transmission channel depends on rise time of the pulse.	2) BW of transmission channel depends on rise time of pulse
3) The instantaneous Power of the transmitter varies.	3) Instantaneous power of the transmitter varies	3) Instantaneous power of the transmitter remains constant.
4) Noise interference high	4) Noise interference is medium	4) Noise interference is minimum.
5) Similar to AM	5) Similar to FM	5) Similar to PM
6) SNR is low	6) SNR is medium.	6) SNR is high.

SNR → signal to noise ratio.

More the SNR, better is the system.

~~Modulation~~ Generation of PWM & PPM

For Generation of PWM, a comparator is used. One i/p to the comparator is input message signal, other is a sawtooth signal which operates at carrier frequency.



The maximum of the input signal (considering both ±ve side) should be less than that of sawtooth signal. If that is so, we will get PWM signal at the o/p of comparator. Note that, PWM pulses occur at regular interval, its rising edge coinciding with the falling edge of sawtooth signal.

Because in this case +ve edge of comparator (input signal) is at higher potential than its −ve edge (sawtooth signal). Hence, comparator output is +ve.

Next, when sawtooth signal rises with a fixed slope & crosses input signal, the −ve input of comparator is at higher potential and comparator output will be −ve.

The duration, for which the comparator stays at high is thus dependent on input signal magnitude and this decides the width of pulse generated. Thus message information gets reflected on the time during which comparator is at high (+ve) or width of the pulse generated at its o/p which is directly proportional to the amplitude of the message signal at that instant.

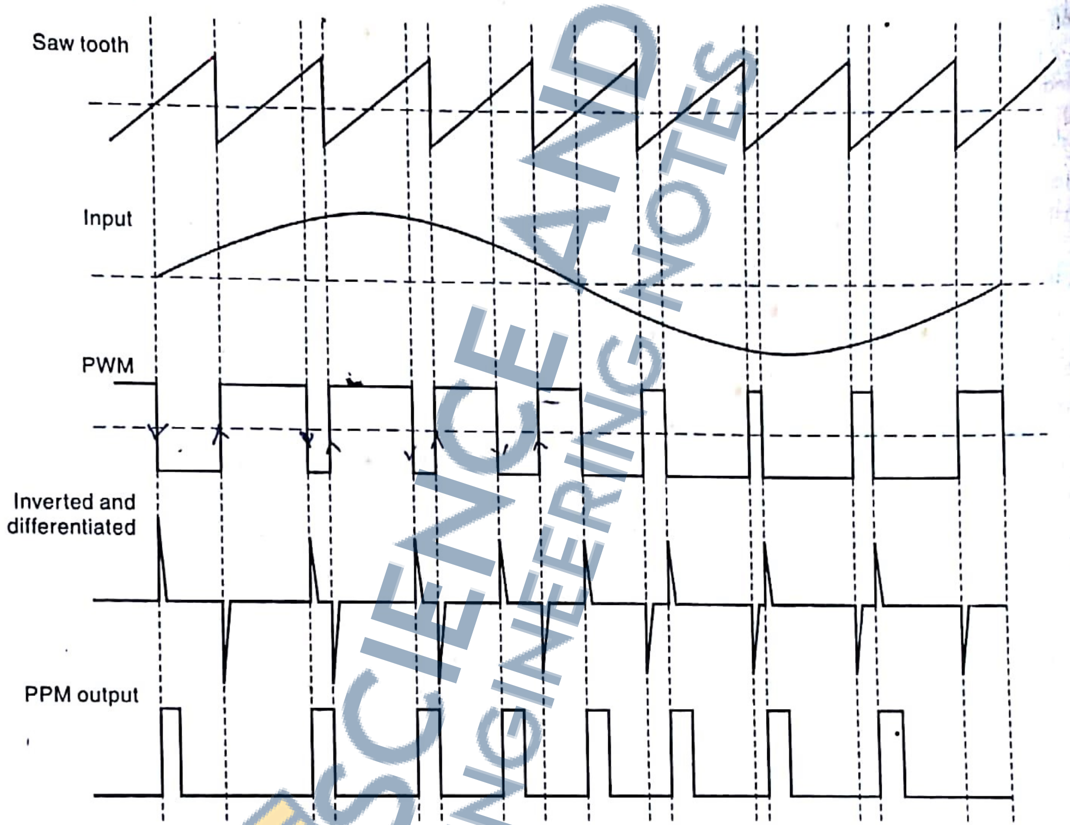
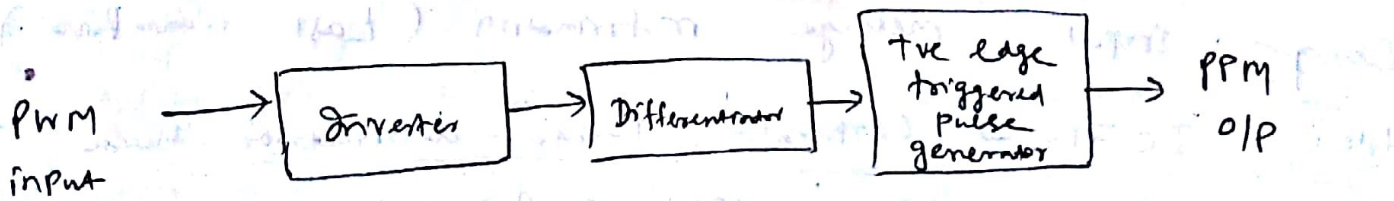


Fig. 5.18 Principle of PWM and PPM generation.

PPM generation :-

PWM signal generated, is sent to an inverter which reverses polarity of pulses. If it is followed by a differentiator we will have +ve spikes where



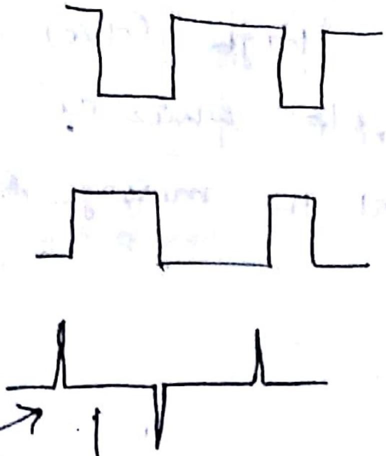
Original PWM signal pulse was going from High to low and -ve spikes where Low to High.

[Shown in figure last page] 4th waveform

[Note :- PWM (original Pwm)

Inverted (Inverted Pwm)

Differentiated



At transition in i/p, derivative in spike,

derivative of const. i/p, has slope zero]

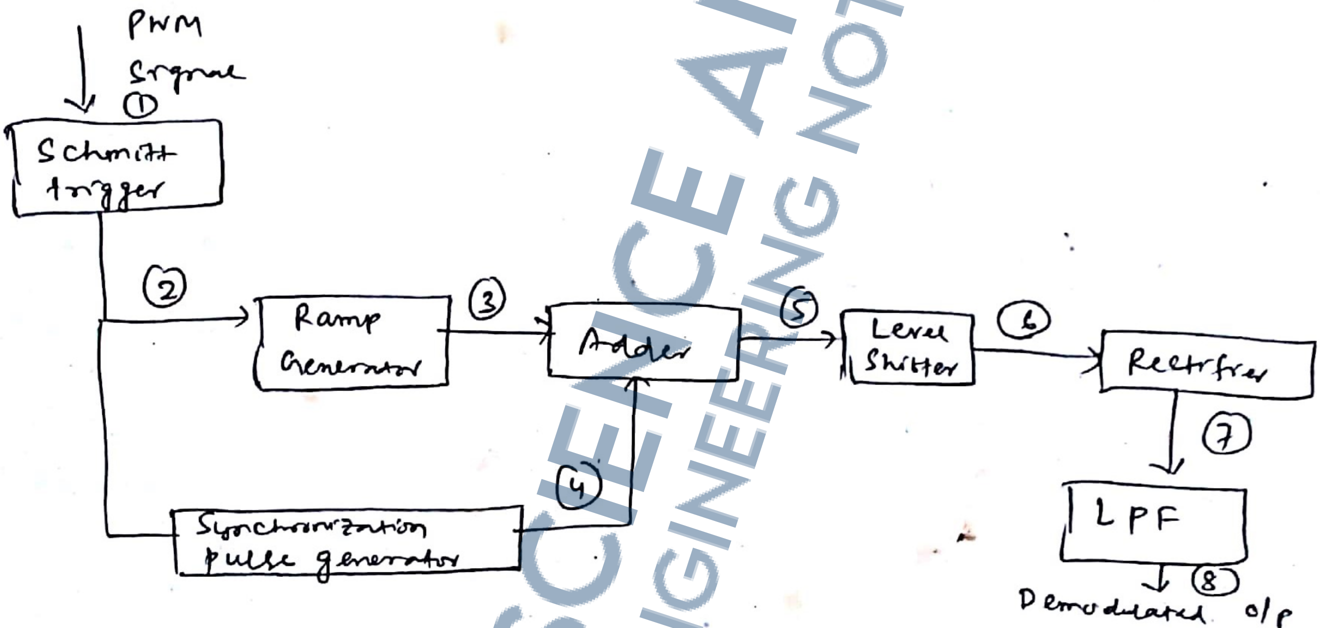
→ These spikes are then fed to a +ve edge triggered fixed width pulse generator which generates pulses of fixed width when +ve spike appears, coinciding with the falling edge of original PWM signal. Note that, the occurance of these falling edges were dependent (proportional to the amplitude of message) on i/p messages and hence the delay in occurance of these fixed width pulses are proportional to the amplitude of the i/p message at that instant. These are the PPM outputs when positions of the pulses in a sample period carry input message information (Last waveform)

Note :- IC 710 → Comparator, Inverter - differentiator block can be designed using - OPAmp & RC component

fixed width pulse generator → IC 74121 or $\boxed{\text{IC 555}}$

Demodulation of PWM :-

Fig, shown below shows the block diagram of PWM detector. The received PWM signal is applied to Schmitt trigger circuit. This Schmitt trigger circuit removes the noise in the PWM waveform. The regenerated PWM is then applied to the ramp generator and the synchronization pulse detector.



The ramp generator produces ramps for the duration of pulses such that height of ramps are proportional to the width of PWM pulses. The maximum ramp voltage is retained till the next pulse.

On the other hand, synchronous pulse detector produces reference pulses with constant amplitude and pulse width. These pulses are delayed by specific amount of delay as shown in fig. The delayed reference pulses and the o/p of ramp generator is added with help of adder. The o/p of adder is given to the level shifter. Here, negative offset shifts the waveform as shown on fig (b). ⑥. Then the -ve part of the waveform is clipped by rectifier. Finally, the o/p of rectifier is passed through low-pass filter to recover the modulating signal.

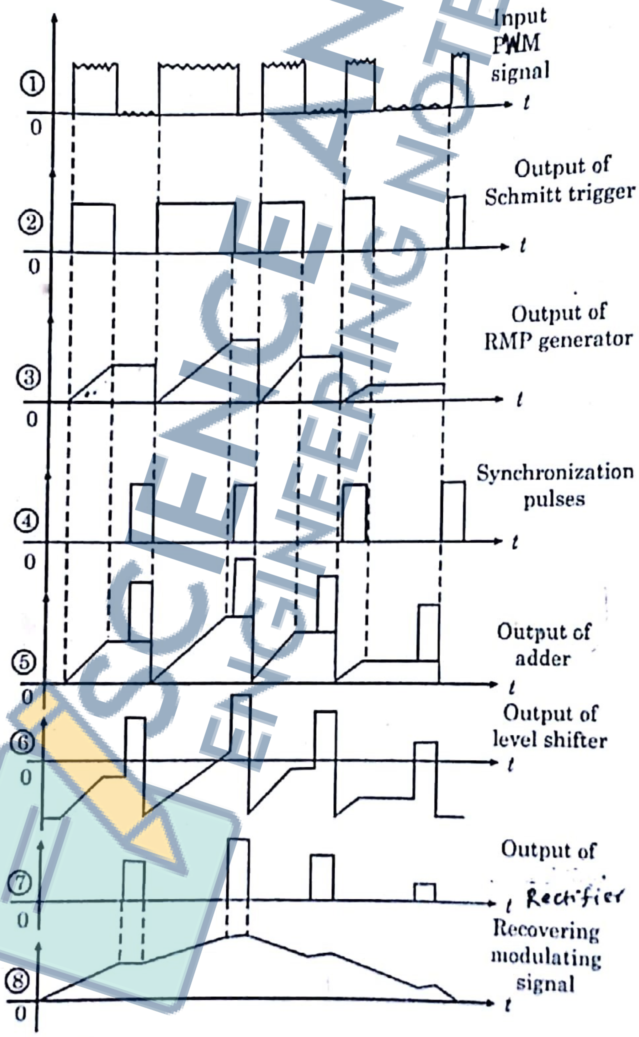
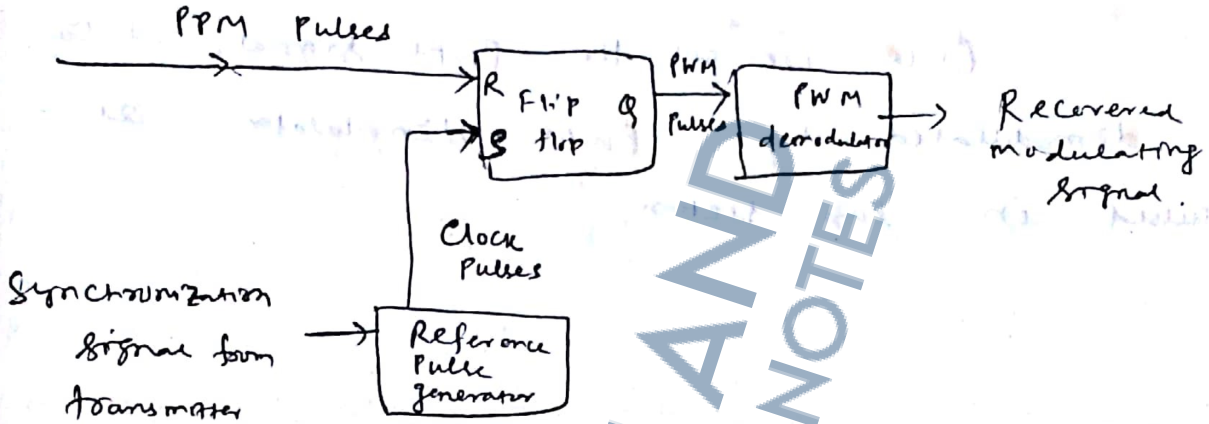


Fig. 9.32. *(b) Waveforms for PWM detection circuit.*

Demodulation of PPM: -

In the demodulation of PPM, first PPM is converted to PWM and then PWM demodulator can be used to get the message signal back.

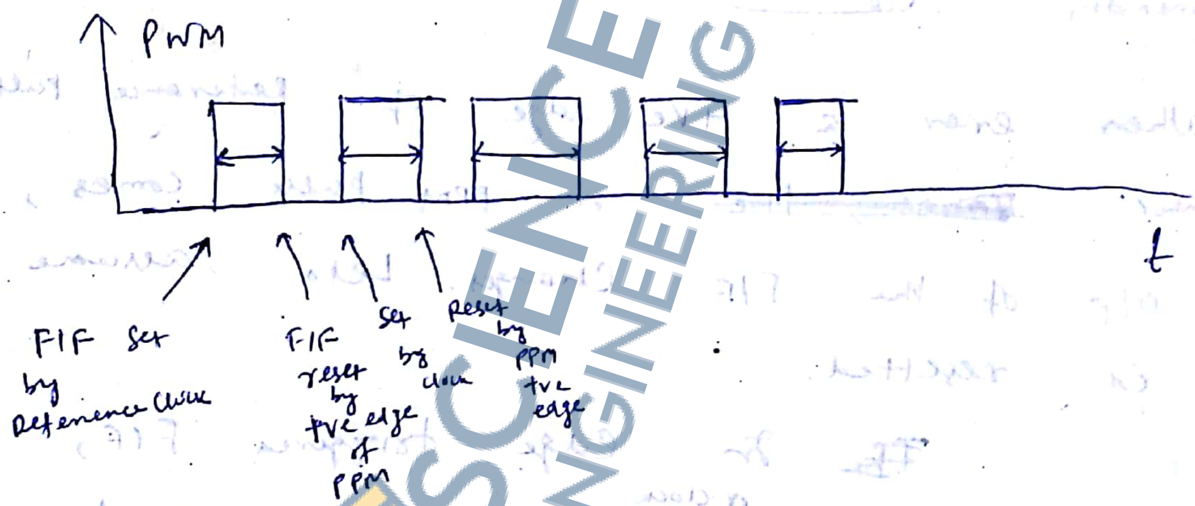


The SR edge triggered flip-flop is set (S=1) by a +ve edge of the clock (Reference pulse). These reference pulses is generated by reference pulse generator of the receiver with the synchronization signal from the transmitter.

The F/F circuit is reset (R=1) at the +ve edge from PPM pulse. So o/p of F/F remains high during this period. (i.e S=1 to R=1 period). [Fig:- Next Page]

The more the delay in arrival, the longer the duration Q remains high. It is again set on the next clock period by rising edge of the clock pulse. Thus the o/p of F/F is a train of pulses, the width of which is decided by how late PPM pulses arrive in a particular clock period, in which again the message information is contained. Thus we get a PWM o/p at the F/F o/p, the width of which ~~~~ ~~~~ in cycle is proportional to the amplitude of the message signal.

Once we got the PWM signal, it can be demodulated by PWM demodulator as discussed in last section.



Digital Representation of Analog Signal :-

Pulse modulation systems are not completely digital. e.g PAM signal is a discrete time signal where signal on time axis is discrete and on amplitude axis it is continuous. [Because amplitude can take any value]

→ To get a digital signal from this signal, the signal on amplitude axis should also be made discrete.

→ The process of converting a continuous amplitude & (discrete/continuous) time signal into discrete amplitude & (discrete/continuous) time signal is called quantization.

Ex :- Consider a PAM signal (Case-1)

discrete time, continuous amplitude to be converted → discrete time discrete amplitude

→ Lets consider a PAM signal whose amplitude varies from −0.5 V to 7.5 V.

→ This range is divided into 8 (2^3) levels known as quantization levels.

→ The pulses having values −0.5 V to 0.5 V are approximated (quantized) to a value 0V. Then pulses having values from 0.5 to 1.5 V are approximated to value 1V & so on.

→ It is assumed that, as per probability theory exact values 0.5, 1.5 etc, will never occure.

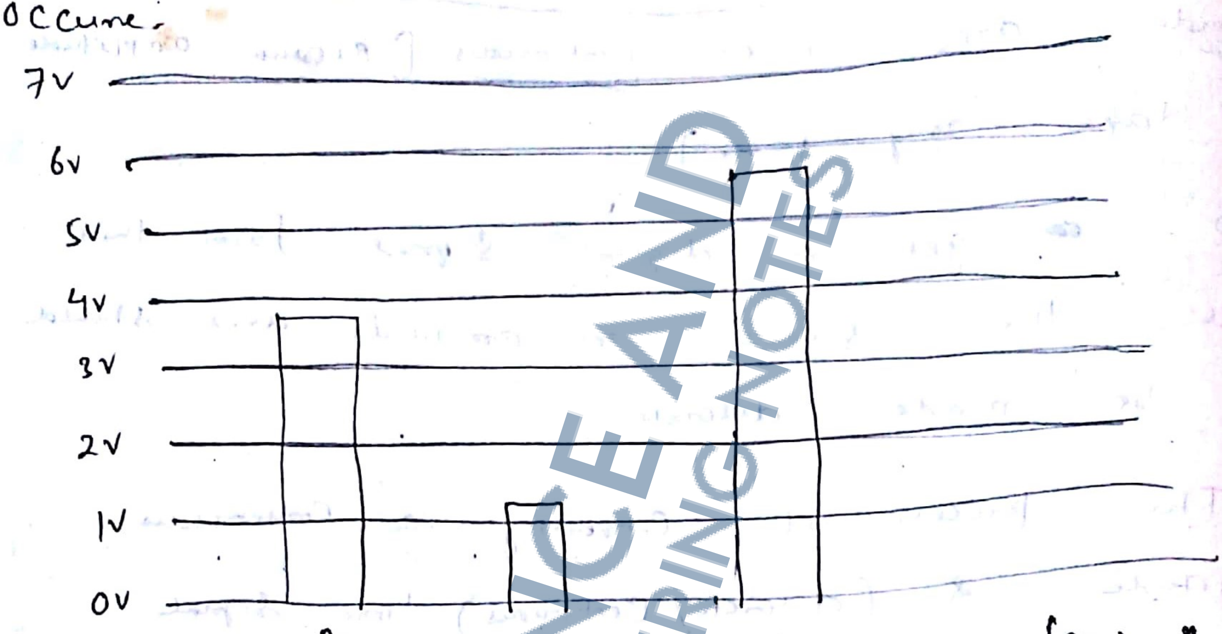


fig :- 1

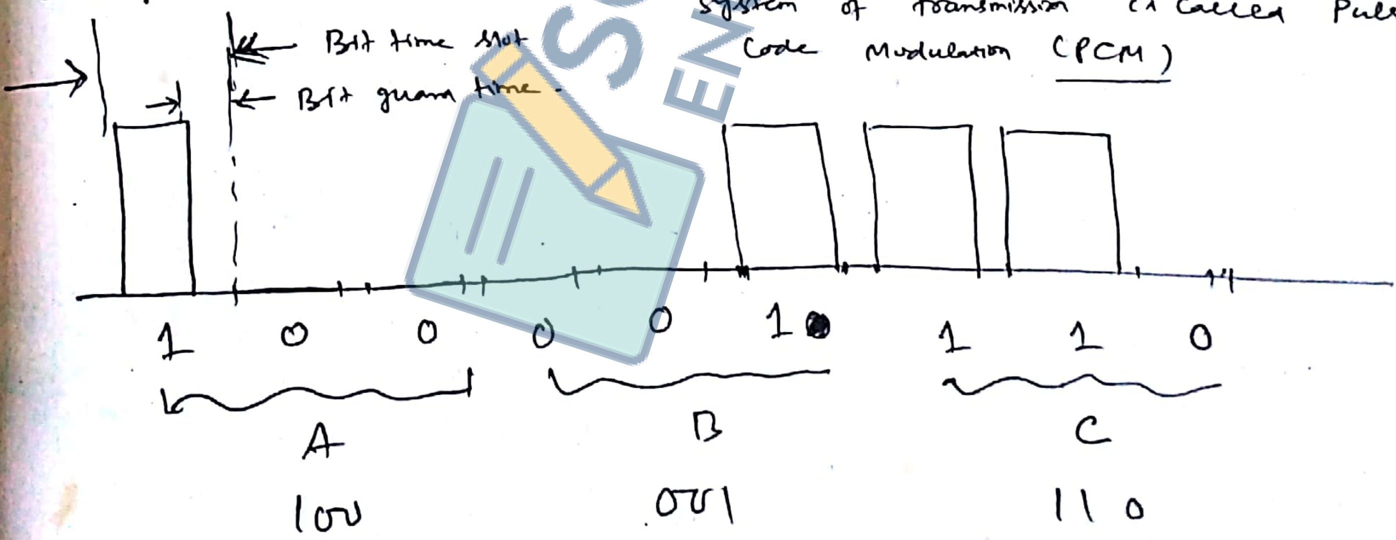
Thus, any pulse can be approximated to one of the values of quantization levels 0V, 1V etc.

Voltage Range (in volt)	Quantization level	Binary Code
−0.5 to 0.5	0	000
0.5 to 1.5	1	001
1.5 to 2.5	2	010
2.5 to 3.5	3	011
3.5 to 4.5	4	100
4.5 to 5.5	5	101
5.5 to 6.5	6	110
6.5 to 7.5	7	111

In fig 1, pulses A, B, C have amplitude 3.8 V, 1.2 V & 5.7 V respectively, they will be approximated to 4V, 1V & 6V respectively.

→ These are known as quantization pulses. ~~So~~ These quantized pulses are then coded. So three quantized pulses having values 4V, 1V & 6V will be encoded as 100, 001 & 110 respectively.

→ The presence of pulse may be represented by 1 and its absence by a 0. The transmitted signal for pulse A, B, C, will be as shown in fig 2. Note that there is a time slot allotted to each bit, a portion of which is guard time. Since the digits of the binary representation of the code number are transmitted as pulses, so the system of transmission is called Pulse Code Modulation (PCM)



→ There is a possibility of error, due to quantization, as there is no way to know whether a 4V signal is result of 3.8 V signal or

4.3 V signal. However the quantization error can be minimized by reducing the step size i.e increasing the number of quantization levels.

Case-II : Continuous time, ~~Continuous~~ Continuous amplitude $\rightarrow$ Continuous time, discrete in amplitude.

Let $m(t)$ be the original signal, whose excursion is confined to V_L to V_H. We divide the total range into M equal intervals each of size S. Accordingly S, called step size is

$$S = \frac{V_H - V_L}{M}$$

or $V_H - V_L = M.S$

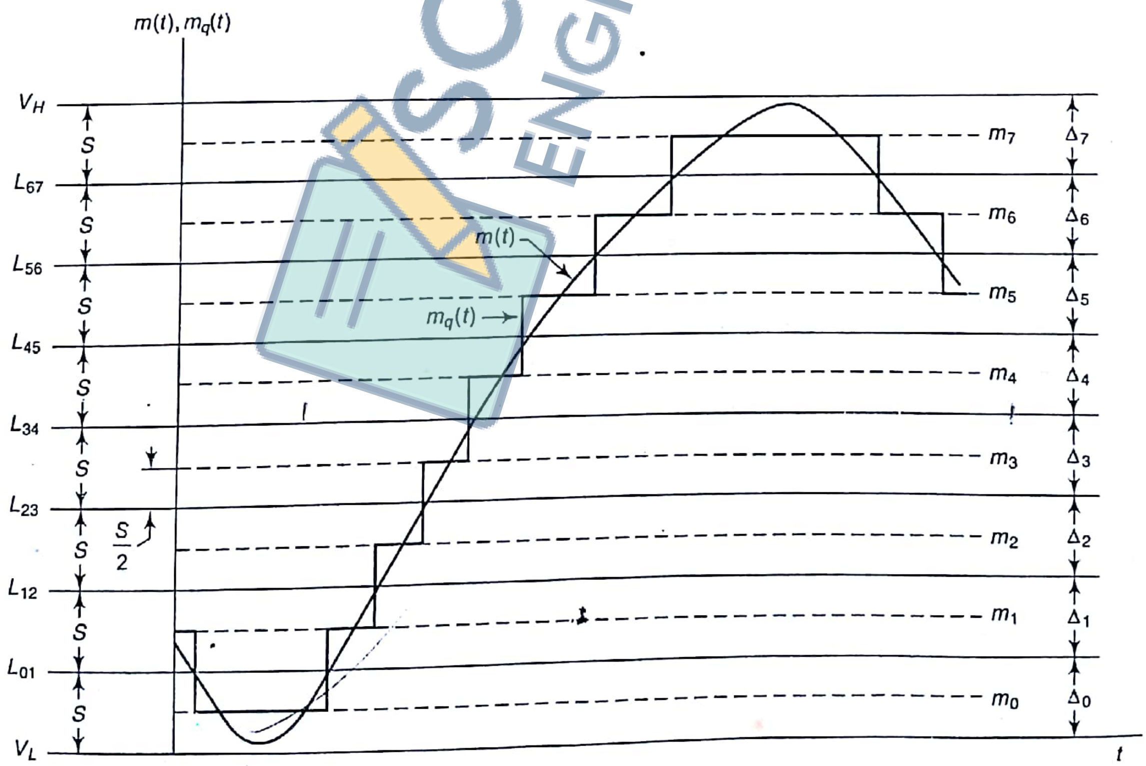


Fig. 5.21 The operation of quantization.

Here, M is taken as '8'. In the center of each of these steps we locate quantization levels $m_0, m_1, \ldots, m_7$.

The quantized signal $m_q(t)$ is generated in the following way;

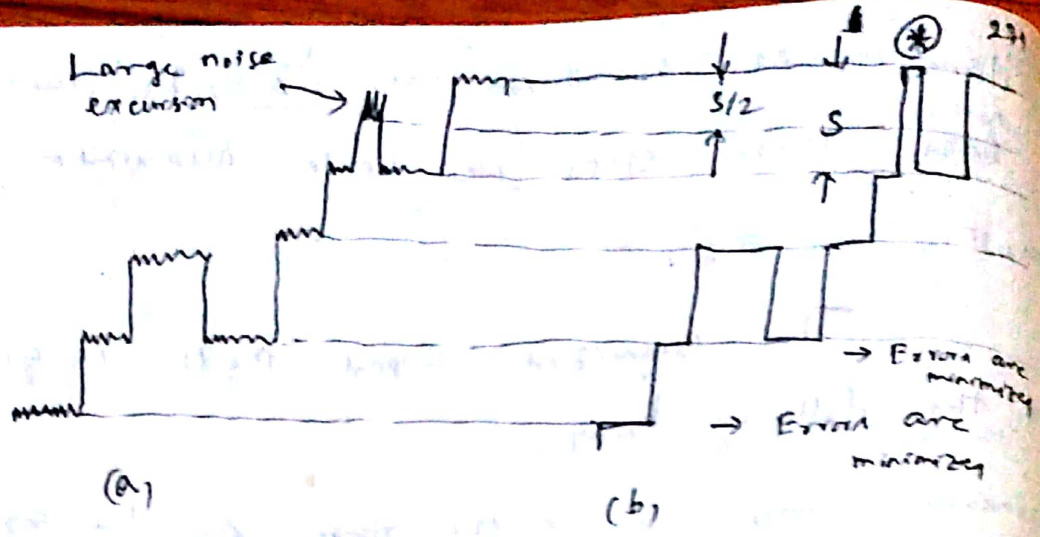
Whenever $m(t)$ is in the range Δ_0, the signal $m_q(t)$ maintains the constant level m_0; whenever $m(t)$ is in the range Δ_1, $m_q(t)$ maintains a constant level m_1; so on. The transition in $m_q(t)$ from $m_q(t) = m_0$ to $m_q(t) = m_1$ is made abruptly when $m(t)$ passes the transition level L_{01} which is midway between m_0 & m_1 and so on.

At every instant of time, the quantization error $(m(t) - m_q(t))$ has a magnitude which is equal to or less than $\frac{S}{2}$.

→ Increasing No of quantization level, i.e reducing step size, quantization error decreases.

→ For practical system, $\boxed{256}$ levels can be used to obtain $\boxed{\text{quality}}$ of commercial color T.V, while 64 levels gives fairly good color T.V performance.

→ The quantization is used to reduce the effects of noise.



(a) (b)

(*) Due to large noise excursion, quantizer is not able to remove the error.

* → Indicates Error in level.

→ When noise is small, ~~it is~~ quantization process remove the noise.

<u>Pulse Code Modulation</u>:-

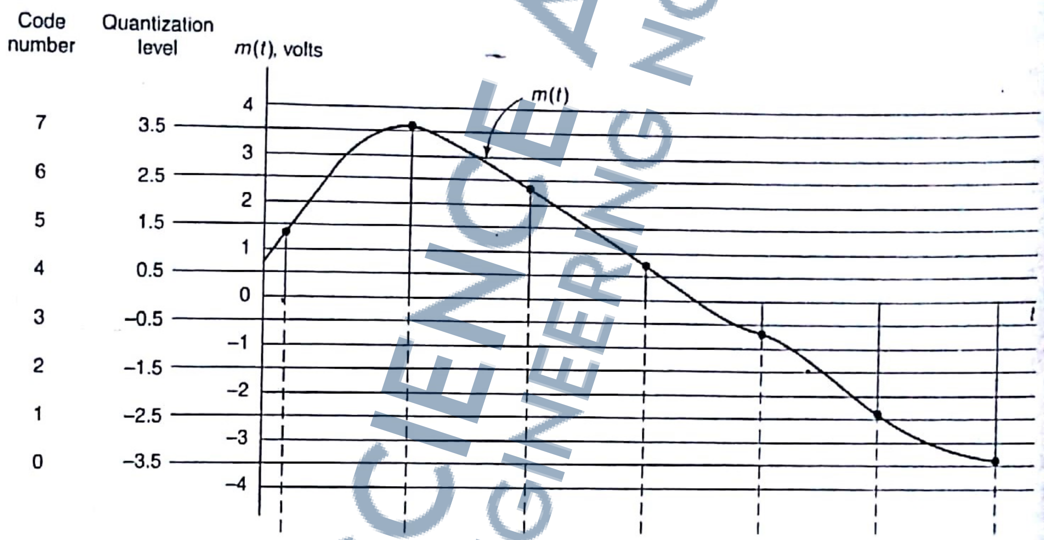
< Discussed in Case-I, PAM Signal >

The quantization is used to reduce the effect of noise and sampling allows us to time-division multiplexing a number of messages. The Combined operation of sampling and quantization generates a quantized PAM waveform, that is a train of pulses

[Note:-
Sampling → Continuous time to discrete time
Quantization → Continuous amplitude to discrete amplitude]

whose amplitude are restricted to a number of discrete magnitudes.

We may transmit these quantized samples value directly, alternatively we may represent each quantized level by a code number and transmit the code number rather than the sample value itself. Most frequently the code number is converted, before transmission, into its representation in binary arithmetic i.e base-2 arithmatic. The digits of binary representation of the code number are transmitted as pulses. Hence, the system of transmission is called (binary) Pulse Code Modulation (PCM).



Sample value	1.3	3.6	2.3	0.7	−0.7	−2.4	−3.4
Nearest quantization level	1.5	3.5	2.5	0.5	−0.5	−2.5	−3.5
Code number	5	7	6	4	3	1	0
Binary representation	101	111	110	100	011	001	000

Fig. 5.24 A message signal is regularly sampled. Quantization levels are indicated. For each sample the quantized value is given and its binary representation is indicated.

PCM System :-

The block diagram of a PCM system is shown in fig 3.(a)

The fig 3(a) shows a PCM transmitter. The baseband signal is sampled at Nyquist rate by a sampler. The sampled pulses are then quantized in the quantizer.

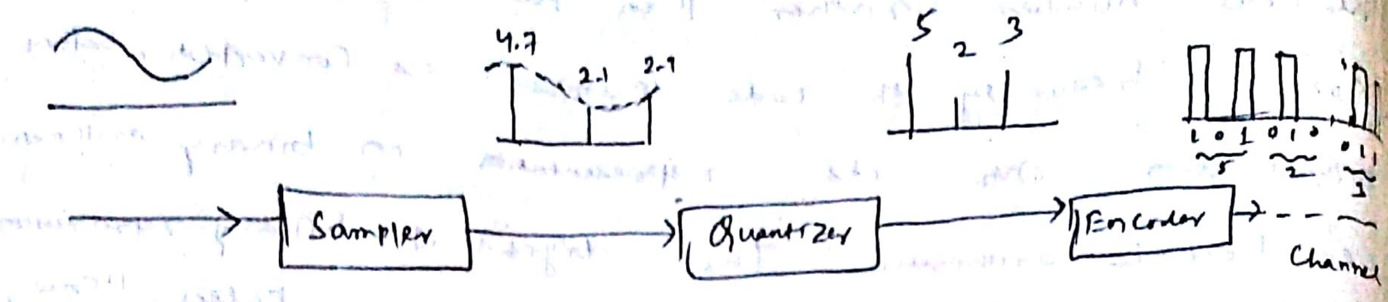


fig 3:- (a) Transmitter

fig 3: (b) Receiver

The encoder (an A/D convertor) encodes these quantized pulse into bits which are transmitted over the channel.

Fig 3(b) shows a PCM receiver. The first block is again a quantizer. But this quantizer is different from the transmitter quantizer because it has to take a decision about the presence (1) or absence (0) of a pulse.

The o/p of ~~a~~ the quantizer goes to the decoder which is D/A converter, performs the inverse operation of ~~decod~~ encoder. The decoder o/p is a sequence of quantized pulses. The original baseband signal is reconstructed in the holding ckt and LPF.

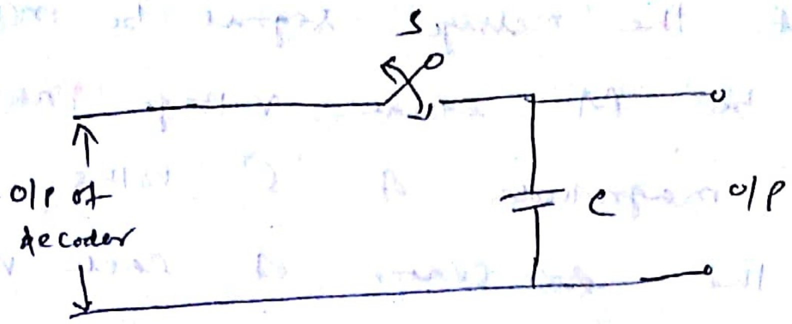


fig:- Holding ckt.

The switch 'S' is closed after the arrival of pulse and opened at the end of the pulse. Hence capacitor charges to the pulse amplitude value & holds this value during the interval betn 2 pulses. Hence, the sampled values are held as shown in fig 3 (b). The LPF is used to get a smooth modulating signal.

Noise in PCM System:-

There are 2 major sources of noise in a PCM system.

(a) Transmission noise introduced outside the transmitter.

(b) Quantization noise introduced in the transmitter.

Imp
Quantization Noise:- [BPUT - 2011]

In the PCM transmitter, a quantized value of the sample is encoded, instead of actual value. Hence, an error occures. As the difference betn actual value & quantized value of the sample is random, this difference or error may be

Viewed as noise due to quantization.

Let the message signal be m(t) and let there be M equal voltage intervals, each having magnitude of 'S' volts.

At the ~~...~~ center of each voltage interval, there are quantization levels $m_1, m_2 \ldots m_M$ as shown in figure below. The dashed level represents the actual sample value of the message signal m(t) at time t.

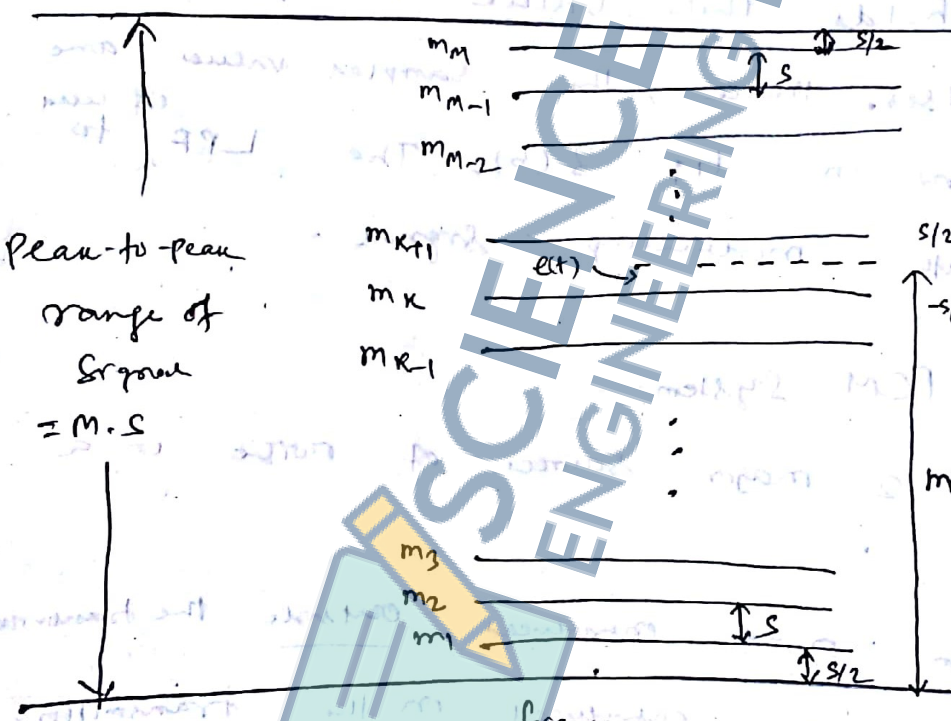


fig (a)

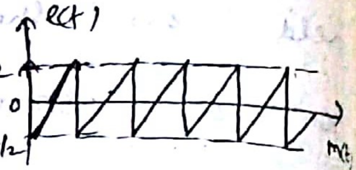


fig (b)

Let m(t) be closest to the quantization level m_K. Then the quantized o/p will be m_K.

The quantization error is then

$$e = m(t) - m_K$$

Fig (b) gives the error voltage e(t) as function of the instantaneous value of m(t).

Let $f(m)\,dm$ be the probability that $m(t)$ lies in the voltage range $\left(m - \frac{dm}{2}\right)$ to $\left(m + \frac{dm}{2}\right)$

Then mean square quantization error or quantization noise is,

[Note :- Mean square value of a random variable, X is

$$\overline{X^2} = E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx \quad , f(x) = \text{Probability density function (PDF)}$$]

$$N_q = \int_{m_1 - S/2}^{m_1 + S/2} f(m)\,(m-m_1)^2\, dm + \int_{m_2 - S/2}^{m_2 + S/2} f(m)\,(m-m_2)^2\, dm + \cdots$$

$$\cdots + \int_{m_M - S/2}^{m_M + S/2} f(m)\,(m-m_M)^2\, dm \quad \text{——— (1)}$$

If the number of quantization level M is large (as is the usual case), it can be assumed that the probability density function $f(m)$ is constant within each quantization range, because in this case the step size 'S' is very small as compared to the peak to peak range $M.S$ of the message signal.

Let $f(m) = f^{(1)}$, in the first term of eq^n (1)

$f(m) = f^{(2)}$, in the 2nd term of eq^n (1)

$\vdots$

$f(m) = f^{(M)}$, in the last term of eq^n (1), So eq^n (1) becomes,

$$N_q = f^{(1)} \int_{m_1 - S/2}^{m_1 + S/2} (m-m_1)^2 dm + f^{(2)} \int_{m_2 - S/2}^{m_2+S/2} (m-m_2)^2 dm + \cdots$$

$$\cdots \; f^{(M)} \int_{m_M - S/2}^{m_M + S/2} (m-m_M)^2 dm \quad -(2)$$

Let $x = m - m_x$, $\Rightarrow dx = dm$ and the range of integration all term in eq(2) becomes $-S/2$ to $S/2$,

$\therefore$ Eq^n (2), becomes,

$$N_q = f^{(1)} \int_{-S/2}^{S/2} x^2 dx + f^{(2)} \int_{-S/2}^{S/2} x^2 dx \; --- + f^{(M)} \int_{-S/2}^{+S/2} x^2 dx$$

$$= \left[\int_{-S/2}^{S/2} x^2 dx \right] \left[f^{(1)} + f^{(2)} - - - + f^{(M)} \right]$$

$$= \left. \frac{x^3}{3} \right]_{-S/2}^{S/2} \left[f^{(1)} + f^{(2)} --- + f^{(M)} \right]$$

$$= \frac{1}{3} \left[\frac{\cancel{2} S^3}{\cancel{8}_4} \right] \left[f^{(1)} + f^{(2)} --- + f^{(M)} \right]$$

$$= \frac{S^3}{12} \left[f^{(1)} + f^{(2)} - —— + f^{(M)} \right]$$

$$N_q = \frac{S^2}{12} \left[f^{(1)} \cdot S + f^{(2)} \cdot S —— - + f^{(M)} \cdot S \right]$$

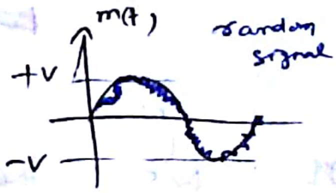
Now $f^{(1)}.S$ is the probability that m lies in the first quantization range, $f^{(2)}.S$ is the probability that m lies in the 2nd quantization range & so on. Hence the terms in the bracket of eqn (3), is the probability that m lies in the entire range of the signal. Hence,

$$f^{(1)}.S + f^{(2)}.S + f^{(3)}.S \; --- + f^{(M)}.S = 1 .$$

∴ eqn (3) becomes,

$$\boxed{N_q = \frac{S^2}{12}}$$

To Calculate Signal power:



Let the signal m(t) is in the range ±V volt. So it is characterized by the uniform probability density. ~~function~~ $\left(\frac{1}{2V}\right)$.

The normalized Avg. signal power of the i/p signal, is

$$S_i = \overline{m^2(t)} = \int_{-V}^{V} m^2(t)\,\frac{1}{2V}\,dm$$

$$= \frac{1}{2V}\cdot\frac{m^3}{3}\Big]_{-V}^{V} = \frac{1}{6V}\times 2V^3 = \frac{V^2}{3}.$$

$$\therefore S_i = \frac{V^2}{3} \quad \text{——— (1)}$$

The quantization noise is given by,

$$N_q = \frac{S^2}{12} \quad \text{—— (2)}$$

If the number of quantization level is M,

then $M \cdot S = 2V$

$$\Rightarrow V = \frac{MS}{2} \quad \text{—— (3)}$$

Putting eqn (3), in eqn (1),

$$S_i = \frac{\left(\frac{MS}{2}\right)^2}{3} = \frac{M^2 S^2}{12} \quad \text{—— (4)}$$

Dividing eqn (4) by eqn (2), we have

$$\frac{S_i}{N_q} = M^2 \quad \text{—— (5)}$$

If we have a useful communication system, then presumably the effect of quantization noise is not much. In such case o/p power S_o may be same as i/p power

$S_o \simeq S_i$, so that eqn (5) becomes,

$$\frac{S_o}{N_q} = M^2 \quad \text{—— (6)}$$

If we want to encode in 'N' bits,

No of quantization levels $= 2^N$

$$\Rightarrow M = 2^N$$

(Ex → $N = 3$, we have $2^3 = 8$ levels)

∴ eqn ⑥ becomes,

$$\frac{S_o}{N_q} = (2^N)^2 = 2^{2N}$$

In dB,

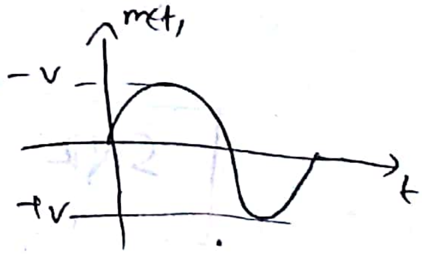
$$\left(\frac{S_o}{N_q}\right)_{dB} = 10 \log \frac{S_o}{N_q} = 10 \log (2^{2N})$$

$$= 20N \, (\cdot 3010)$$

$$\boxed{\left(\frac{S_o}{N_q}\right)_{dB} = 6N \; dB}$$

Ex-1 :- Derive an expression for a signal to quantization noise ratio for a PCM system which employs linear (uniform) quantization technique. Given that i/p to PCM system is a ~~sa~~ sinusoidal signal.

Ans :-



$V_H - V_L = 2V$

M = no. of quantization levels.

$M \cdot S = 2V$

$\Rightarrow S = \frac{2V}{M}$ — ① S = Step Size.

$$N_q = \frac{S^2}{12} = \frac{\left(\frac{2V}{M}\right)^2}{12} = \frac{4V^2}{M^2 \times 12} = \frac{V^2}{3M^2} \quad \text{②}$$

For a sinusoidal signal,

$$S_o = \frac{V^2}{2} \quad \text{--- (3)}$$

Dividing eqn (3) by eqn (2)

$$\frac{S_o}{N_q} = \frac{\cancel{V^2}}{2} \times \frac{3M^2}{\cancel{V^2}} = \frac{3}{2} M^2 ,$$

$M = 2^N$ (N = No of bits.
M = No. of quantization level)

$$\frac{S_o}{N_q} = \frac{3}{2} \cdot (2^N)^2 = \frac{3}{2} \cdot 2^{2N}$$

$$(SNqR)_{in\ dB} = 10 \log \left(\frac{3}{2} \cdot 2^{2N} \right)$$

$$= 10 \left[\log \frac{3}{2} + \log 2^{2N} \right]$$

$$= 10 \left[0.1761 + 0.602 N \right]$$

$$\boxed{SNR = (1.761 + 6.02 N)\, dB}$$

Ex:- 2 A T.V signal having BW of 4.2 MHz is transmitted using binary PCM, Given that number of quantization level is 512. Determine

(a) Codeword length (b) Tx BW (c) Final bit rate

(d) O/P signal to quantization noise ratio

Ans: (i) No of quantization level = 512

$$2^N = 512$$

$$N = 9$$

$\therefore$ Code length = 9 bits

(ii) Transmission BW = $N f_m$

T.V signal has BW = 4.2 MHz, This means highest freq component will have freq 4.2 MHz.

$$\therefore f_m = 4.2 \text{ MHz}$$

Transmission BW $\geq N f_m$

Tx BW $\geq 9 \times 4.2$

$$\boxed{\text{Tx BW} \geq 37.8 \text{ MHz}}$$

(iii) bit rate $= N f_s$ bits/sec

$= N \times 2 f_m$ $f_s \to$ Sampling freq

$= 2 \times 9 \times 4.2 \times 10^6$

$= 75.6 \times 10^6$ bits/sec

(iv) $SN_qR = (1.761 + 6N)$ dB

$\simeq 1.8 + 6N$ dB

$= 1.8 + 6 \times 9$

$$\boxed{SN_qR = 55.8 \text{ dB}}$$

Ex-3 :- [BPUT - 2010] The discrete samples of an analog signal is to be uniformly quantized for PCM system. If the max^m value of the analog sample is to be represented within 0.1 % accuracy, find the min^m number of binary digits required.

Ans :- Let A be the max^m value of discrete sample. Error tolerated is 0.1 %.

i.e $\frac{.1}{100}$ i.e. .001 A.

If Δ is ~~the~~ the step size,

max^m error = $\frac{\Delta}{2}$

$$\frac{\Delta}{2} = .001 A$$

$$\Rightarrow \Delta = .002 A$$

$$\Rightarrow \frac{A}{\Delta} = \frac{1}{.002} = \frac{1000}{2} = 500.$$

$\therefore$ Thus No. of levels required = 500.

$$M = 500$$

$$2^N = 500$$

$$\Rightarrow N \log 2 = \log(500)$$

$$\Rightarrow N = 8.96$$

$$\therefore \boxed{N = 9 \text{ bits}}$$

Companding :- ~~short note~~

Quantization error depends upon the step size ($N_q = \frac{S^2}{12}$). When the step size are uniform in size, the small amplitude signal will have poorer signal to quantization noise ratio than large amplitude signals, because in both the cases denominator (quantization noise) is the same; where as numerator is small for small amplitude and large for large amplitude

$$\left[\because \frac{S_o}{N_q} \begin{array}{l} \rightarrow \text{Small for small amplitude, large for large amplitude} \\ \leftarrow \text{remains same. Signal power} \propto \text{Amplitude}^2 \end{array} \right]$$

To improve, SNqR, ~~for~~ step size should be small ($N_q = \frac{S^2}{12}$) for small amplitude signals. So step size should be varied according to signal level to keep ~~to~~ the signal to noise ratio adequately high. This is called non-uniform quantization.

In practice, it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal level So a method is used, in which small amplitude signals are amplified or enhanced more than high amplitude signals. Thus, the low amplitude signal will carry more quantization levels than the undistorted signal (~~solid~~ dashed line) in fig 2.

This is achieved through a process called Companding.

Companding = Compressing + expanding.

fig 1 :- Companding Model

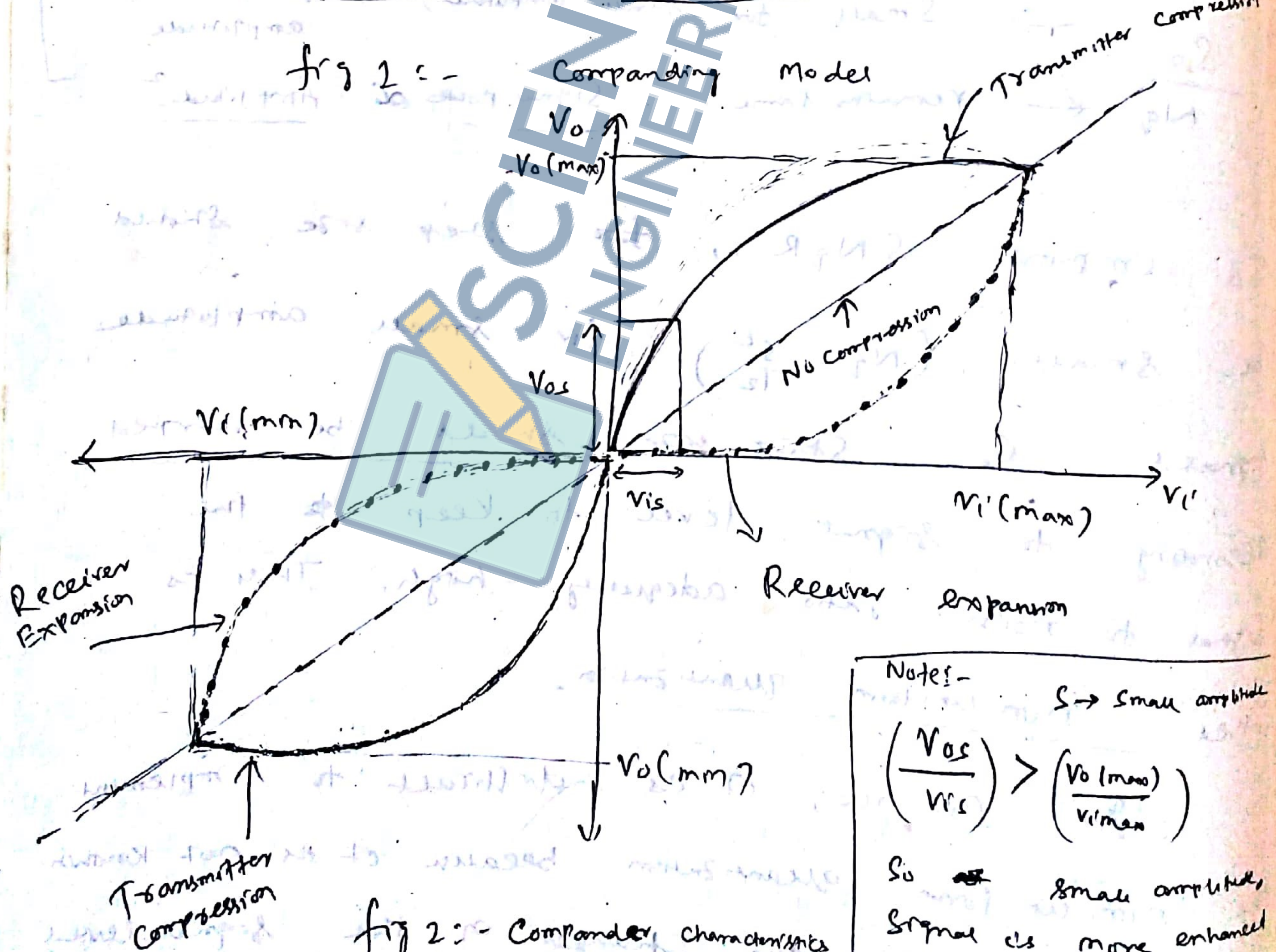


fig 2 :- Compander characteristics

Characteristics Above Vi axis → For +ve i/p signal
" Below Vi axis → For -ve i/p signal

Notes:-

S → Small amplitude

$$\left(\frac{V_{os}}{V_{is}}\right) > \left(\frac{V_{o(max)}}{V_{i(max)}}\right)$$

So small amplitude signal is more enhanced than large amplitude signal.

The signal transmitted through a non-linear n/w with the characteristics shown by solid ~~line~~ curve which has extremities compressed. Hence such a n/w is called a compressor.

At the ~~rec~~ receiver end, an inverse operation is to be performed to recover the original signal. This is achieved by an expander connected betn the decoder and holding ckt, whose characteristics are given by dotted line.

→ The combination of compressor & expander is known as compander, which performs the companding operation.

Note :- * (Not required for 'ACT', Just for knowledge)

In US, Canada, Japan μ-law compander is used.

A μ-law ~~compander~~ compressor is ~~given as~~ defined as,

$$|y| = \frac{\log_{10}(1+\mu|x|)}{\log(1+\mu)}$$

where y, x are normalized o/p & i/p respectively.

Rest of the world, A-law compander is used. The A-law ~~compander~~ compressor is defined as,

$$|y| = \begin{cases} \dfrac{A|x|}{1+\log_{10} A}, & 0 \le |x| \le \frac{1}{A} \\[2ex] \dfrac{1+\log_{10} A|x|}{1+\log_{10} A}, & \frac{1}{A} \le |x| \le 1 \end{cases}$$

where y, x are normalized o/p & i/p respectively.

→ For a given value of A the reciprocal slope of compression curve is given by derivative ~~$|y|$ w.r.to $|x|$~~ $|x|$ w.r.t $|y|$

$$\cancel{\frac{d|y|}{d|x|}} \quad \cancel{\text{i.e. } dy} \quad \text{i.e. } \frac{d|x|}{d|y|}$$