Ch-3 - Angle Modulation

Angle modulation may be defined as the process in which the total phase angle of a carrier wave is varied in accordance with the instantaneous value of the modulating signal, while keeping the amplitude of the carrier constant.

Lets consider an unmodulated carrier signal, expressed as

$$c(t) = A \cos(\omega_c t + \theta_0)$$

where. A = Amplitude of the carrier.

ω_c = Carrier frequency

θ_0 = Some phase angle.

Substituting $\phi = \omega_c t + \theta_0$, we get

$$c(t) = A \cos \phi$$

where ϕ = total phase angle of the carrier signal.

Now we can vary this phase angle (ϕ) in two ways & there are two types of angle modulation.

Angle Modulation

- Frequency Modulation (FM)
- Phase Modulation (PM)

Frequency Modulation :-

In frequency modulation, frequency of carrier is varied in accordance to the instantaneous value of message signal.

Let ω_c = Carrier frequency

$x(t)$ = message signal

ω_i = instantaneous angular frequency value of modulated signal, Then

$$\boxed{\omega_i = \omega_c + K_f\, x(t)} \quad \text{——— (1)}$$

where K_f is proportionality constant and is known as frequency sensitivity of the modulator. This is expressed in $\left[\frac{rad/sec}{Volt} \text{ or } \frac{HZ}{Volt}\right]$

Note :- Instantaneous angular freq $(\omega_i) = \frac{d\phi_i}{dt}$

Let, the unmodulated carrier is given by

$$c(t) = A\cos\phi$$

where $\phi = \omega_c t + \theta_0$

Let ϕ_i be the instantaneous phase angle of the modulated signal.

So the expression for frequency modulated wave will be

$$s(t) = A\cos\phi_i \quad \text{——— (2)}$$

We know, $\phi = \omega_c t + \theta_0$

$$\Rightarrow \frac{d\phi}{dt} = \omega_c$$

$$\Rightarrow \quad \phi = \int \omega_c \, dt$$

$$\therefore \quad \phi_i = \int \omega_i \, dt$$

$$= \int \left[\omega_c + K_f \, x(t)\right] dt$$

$$\left[\because \omega_i = \omega_c + K_f \, x(t) \quad \text{— eq}^n \text{ (1)}\right]$$

$$\Rightarrow \quad \phi_i = \omega_c t + K_f \int x(t) \, dt \quad \text{———— (3)}$$

Putting eq^n (3), in eq^n (2), we get

$$S(t) = A \cos\left[\omega_c t + K_f \int x(t) \, dt\right]$$

Now, if phase angle of unmodulated carrier is taken at $t=0$, then the limit of integration will be 0 to t.

In this case, the expression for FM wave will be,

$$\boxed{S(t) = A \cos\left[\omega_c t + K_f \int_0^t x(t) \, dt\right]}$$

Which is the required general expression for FM wave.

Phase Modulation (PM)

In PM, the phase angle of the carrier is varied according to the message signal.

Mathematically,

Let
$$c(t) = A\cos(\omega_c t + \theta_0)$$
$$c(t) = A\cos\phi$$
where
$$\phi = \omega_c t + \theta_0$$

Neglecting θ_0, we get total phase angle of unmodulated carrier is
$$\phi = \omega_c t$$

Now, according to phase modulation, this phase angle ϕ is varied linearly with baseband or modulating signal $x(t)$.

Let the instantaneous value of phase angle be denoted by ϕ_i.

Therefore,
$$\boxed{\phi_i = \omega_c t + K_p\, x(t)} \quad \text{——} \; (1)$$

where K_p is the proportionality constant and is known as phase sensitivity of the modulator. This is expressed in radian/volt.

The expression for the phase modulated wave will be
$$s(t) = A\cos\phi_i \quad \text{——} \; (2)$$

Putting the value of ϕ_i from eq^n (1) in eq^n (2), we have

$$\boxed{s(t) = A \cos\left[\omega_c t + K_p\, x(t)\right]} \quad \text{—— (3)}$$

Which is the required mathematical expression for phase modulated wave.

Frequency deviation :-

We know that the instantaneous frequency of FM wave is given as,

~~$\omega_i = \omega_c + K_f\, x(t)$~~ $\boxed{\omega_i = \omega_c + K_f\, x(t)}$

The instantaneous frequency of FM signal varies with time around the carrier frequency ω_c.

So, the max^m change in instantaneous frequency from the average frequency (ω_c) is called frequency deviation.

The max^m change in (ω_i) from the average or carrier frequency (ω_c) depends on the magnitude and sign of $K_f\, x(t)$.

This means that frequency deviation would be either positive or -ve depending upon the sign of $K_f \cdot x(t)$. However, the amount of frequency deviation in both the cases is given by the maximum magnitude i'e $|K_f \, x(t)|_{max}$.

It is denoted by $\underline{\Delta W}$.

$\therefore$ $\boxed{\text{frequency deviation } (\Delta W) = |K_f \, x(t)|_{max}}$

$\underline{Ex}$:- If $x(t) = A_m \cos \omega_m t$.

$$\Delta W = |K_f \, x(t)|$$

$$\Rightarrow \boxed{\Delta W = K_f A_m}$$

BPUT

Relationship between phase modulation (PM) & frequency modulation (FM)

we know that, an angle modulated wave is given as

$$s(t) = A \cos \phi_i$$

where A = Amplitude of the Carrier
ϕ_i = Instantaneous total phase of the angle modulated wave.

For FM, the modulated wave is given by,

$$s(t) = A \cos \left[w_c t + K_f \int_0^t x(t) \, dt \right]$$

For PM,

$$s(t) = A \cos[w_c t + K_p x(t)]$$

It is observed that FM & PM are closely related to each other because in both the cases there is a variation in the total phase angle.

In PM, the phase angle varies linearly with the baseband signal $x(t)$ whereas in case of frequency modulation, the phase angle varies ~~with~~ linearly with integral of baseband signal $x(t)$. This mean FM may be obtained using PM and vice versa.

FM using PM :-

To get FM, by using PM, we first integrate the baseband signal and then apply to the phase modulator.

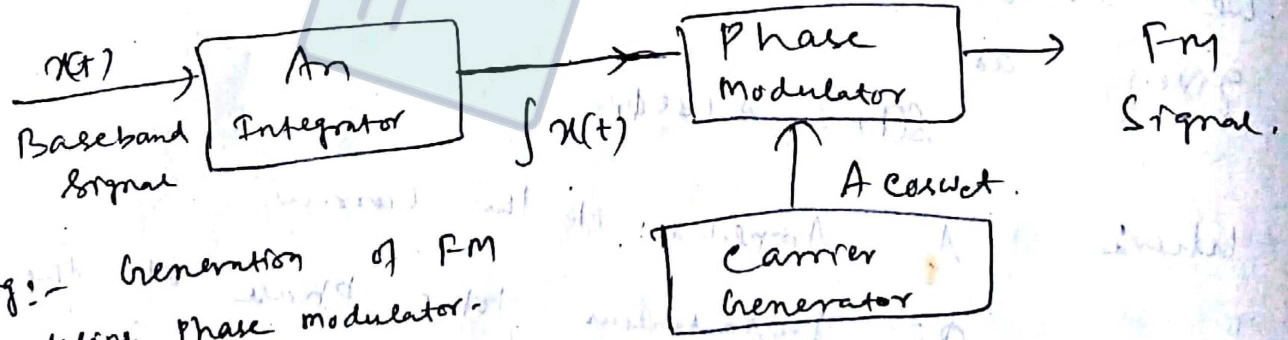


fig:- Generation of FM using phase modulator.

PM using FM :-

Similarly, PM wave may be generated by using frequency modulator by first differentiating modulating or base band signal $x(t)$ and then applying to the frequency modulator.

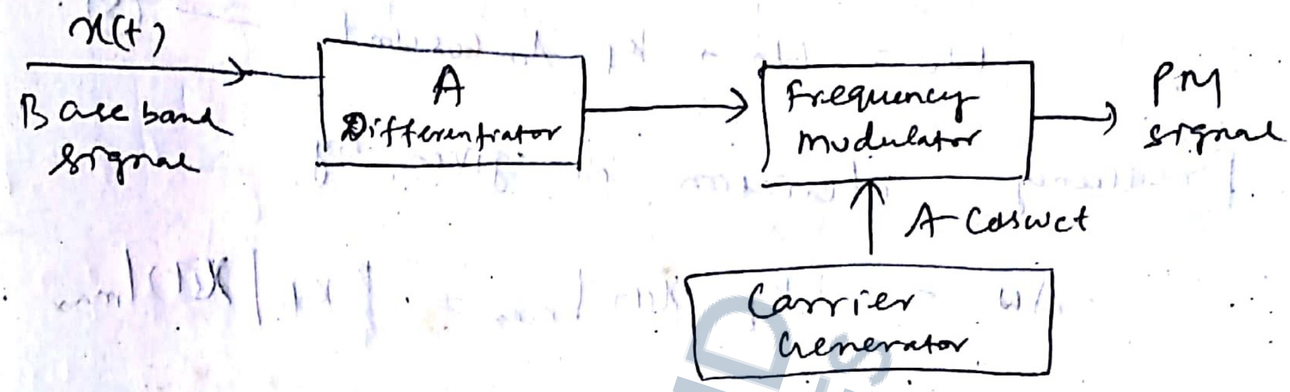


fig:- Generation of PM using frequency Modulator.

Single-tone frequency Modulation:-

(Assuming base band signal contain single frequency)

Let us consider a carrier signal as

$$c(t) = A_c \cos w_c t \quad \text{—— (1)}$$

Let modulating signal be

$$x(t) = A_m \cos w_m t \quad \text{—— (2)}$$

where A_m = Max^m amplitude of modulating signal.

w_m = frequency of modulating signal.

Let the expression for ~~AM~~ FM wave be,

$$s(t) = A_c \cos \phi_i \quad \text{—— (3)}$$

where ϕ_i is the instantaneous phase angle of the modulated wave,

We know, for FM,

$$w_i = w_c + K_f\, x(t) \quad \text{—— (4)}$$

Putting the value of $x(t)$ from eq^n (2), in eq^n (4), we have

$$w_i = w_c + K_f \, A_m \cos w_m t \quad \text{—— (5)}$$

Frequency deviation is given by

$$\Delta w = |K_f \, x(t)|_{max} = |K_f| \, |x(t)|_{max}$$

$$= K_f \cdot A_m$$

$\therefore$ frequency deviation $= \boxed{(\Delta w) = K_f A_m}$ —— (6)

Putting the value of $K_f A_m$ from eq^n (6) in eq^n (5), we have

$$w_i = w_c + \Delta w \cos w_m t \quad \text{—— (7)}$$

Total phase angle $\phi_i = \int w_i \, dt$

$$\therefore \phi_i = \int (w_c + \Delta w \cos w_m t) \, dt$$

$$\phi_i = w_c t + \Delta w \cdot \frac{\sin w_m t}{w_m}$$

$$\phi_i = w_c t + \left(\frac{\Delta w}{w_m}\right) \cdot \sin w_m t$$

Let $\frac{\Delta w}{w_m} = m_f =$ modulation index

$\therefore$ modulation index $(m_f) = \dfrac{\text{Frequency deviation}}{\text{modulating frequency}}$

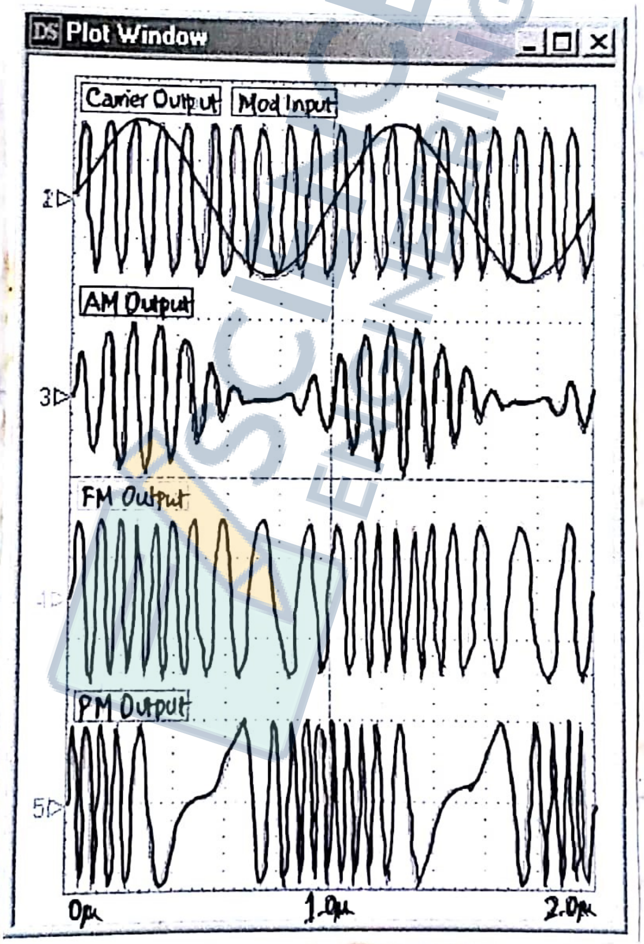
$$\therefore \quad \phi_i = \omega_c t + m_f \sin \omega_m t \quad \text{—— (8)}$$

Substituting eqn (8), in eqn (2), we have

$$\boxed{S(t) = A_c \cos\left[\omega_c t + m_f \sin \omega_m t\right]} \quad \text{—— (9)}$$

which is the required mathematical expression for single tone FM wave.

Figure :-



Prob:- 1) A single-tone FM is represented by a voltage eq^n as

$$V(t) = 12\cos[6\times10^8 t + 5\sin 1250t]$$

Determine the following:-

a) Carrier frequency (b) Modulating frequency

(c) The modulation index (d) Maximum deviation

(e) What Power will this FM wave dissipate in 10Ω resistor.

Ans:- $V(t) = 12\cos[6\times10^8 t + 5\sin 1250t]$

The standard eq^n for FM,

$$V(t) = A_c\cos[\omega_c t + m_f \sin\omega_m t]$$

Comparing,

$A_c = 12$, $\omega_c = 6\times10^8$, $m_f = 5$

$\omega_m = 1250$

(a) Carrier frequency, $\omega_c = 6\times10^8 \frac{rad}{sec}$

$\Rightarrow 2\pi f_c = 6\times10^8$

$\Rightarrow f_c = \frac{6\times10^8}{2\pi} = 95.5$ MHz

(b) Modulating frequency, $\omega_m = 1250 \frac{rad}{sec}$

$\Rightarrow 2\pi f_m = 1250$

$\Rightarrow f_m = \frac{1250}{2\pi} = 199$ Hz

(c) Modulation index $(m_f) = 5$

(d) Max^m deviation ($\Delta \omega$)

We know
$$m_f = \frac{\Delta \omega}{\omega_m} = \frac{\Delta f}{f_m}$$
$$\Rightarrow 5 = \frac{\Delta f}{199}$$
$$\Rightarrow \Delta f = 199 \times 5 = 995 \text{ Hz}$$

(e) The power dissipated is,

$$P = \frac{V_{rms}^2}{R} = \frac{\left(\frac{12}{\sqrt{2}}\right)^2}{10} = \frac{\frac{12^2}{2}}{10} = 7.2 \text{ watt}$$

Types of Frequency modulation (FM)

1) Narrow band FM :-

In this case, K_f is small. Hence BW of FM is narrow.

2) Wideband FM :-

In this case, K_f is large. Hence BW of FM is large (wide band)

1) Narrowband FM :-

$$X_{fm}(t) = A_c \cos[\underbrace{\omega_c t}_{\alpha} + \underbrace{m_f \sin \omega_m t}_{\beta}]$$

This is in a format, $A_c \cos(\alpha + \beta)$

$$= A_c [\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta]$$

$$X_{FM}(t) = A_c\left[\cos\omega_c t \cdot \cos(m_f \sin\omega_m t) - \sin\omega_c t \cdot \sin(m_f \sin\omega_m t)\right]$$

For narrow band FM, $K_f \to 0$, $m_f = \frac{\Delta\omega}{\omega_m} = \frac{K_f A_m}{\omega_m}$

$\therefore m_f \to 0$

We know when $\theta \to 0$, $\sin\theta \to \theta$

$$X_{FM}(t) = A_c\left[\cos\omega_c t \cdot \cos 0 - \sin\omega_c t \cdot (m_f \sin\omega_m t)\right]$$

$$= A_c\left[\cos\omega_c t - m_f \sin\omega_c t \cdot \sin\omega_m t\right]$$

$$= A_c \cos\omega_c t - \frac{m_f A_c}{2}\, 2 \sin\omega_c t \cdot \sin\omega_m t$$

$$X_{FM}(t) = A_c \cos\omega_c t - \frac{m_f A_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

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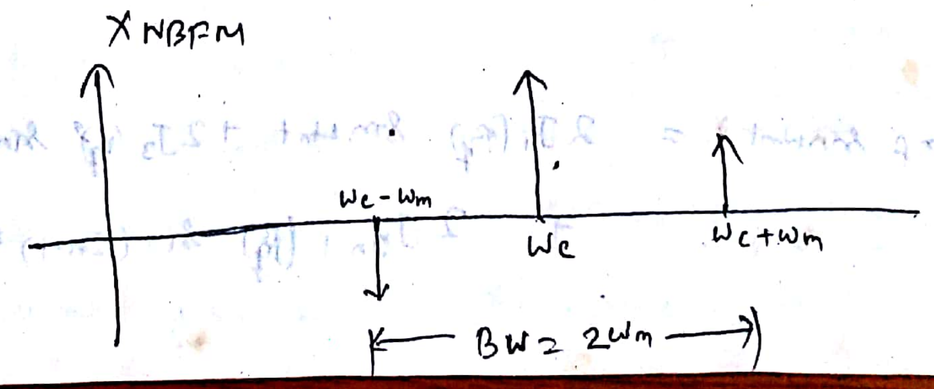
$$\boxed{X_{NBFM} = A_c \cos\omega_c t + \frac{m_f A_c}{2}\cos(\omega_c + \omega_m)t - \frac{m_f A_c}{2}\cos(\omega_c - \omega_m)t}$$

NBFM → Narrowband FM.

But

$$\boxed{X_{AM}(t) = A_c \cos\omega_c t + \frac{m A_c}{2}\cos(\omega_c + \omega_m)t + \frac{m A_c}{2}\cos(\omega_c - \omega_m)t}$$

So in narrowband FM, the lower sideband is 180° out of phase with the carrier and upper side band.



∴ For NBFM, $\boxed{\begin{array}{l} BW = 2\omega_m \ \frac{rad}{sec} \\ \text{or} \\ BW = 2 f_m \ HZ \end{array}}$

Appl^n of NBFM - Family Radio service, walkitalkies

2/ Wideband FM :- [Appl^n of WBFM :- FM broadcast - 88MHZ - 108MHZ
Ex:- Radio mirchi, Big Fm]

When the value of the modulation index (m_f) is quite large $\left[m_f = \frac{K_f A_m}{f_m} \right]$, then in FM, a large no. of sidebands are produced and hence the bandwidth of FM is sufficiently large. This type of FM system is known as wideband FM.

$$x_{FM}(t) = A_c \cos[\underbrace{\omega_c t}_{\alpha} + \underbrace{m_f \sin\omega_m t}_{\beta}]$$

$$= A_c \cos(\alpha + \beta)$$

$$= A_c \left[\cos\omega_c t \cdot \cos(m_f \sin\omega_m t) - \sin\omega_c t \cdot \sin(m_f \sin\omega_m t) \right] \quad \text{--- (1)}$$

This expression can be expressed in terms of Bessel's function $J_n(m_f)$,

$$\cos(m_f \sin\omega_m t) = J_0(m_f) + 2 J_2(m_f) \cos 2\omega_m t + 2 J_4(m_f) \cos 4\omega_m t + \cdots + 2 J_{2n}(m_f) \cos(2n\omega_m t) + \cdots \quad \text{--- (2)}$$

$$\sin(m_f \sin\omega_m t) = 2 J_1(m_f) \sin\omega_m t + 2 J_3(m_f) \sin 3\omega_m t + \cdots + 2 J_{2n-1}(m_f) \sin(2n-1)\omega_m t + \cdots \quad \text{--- (3)}$$

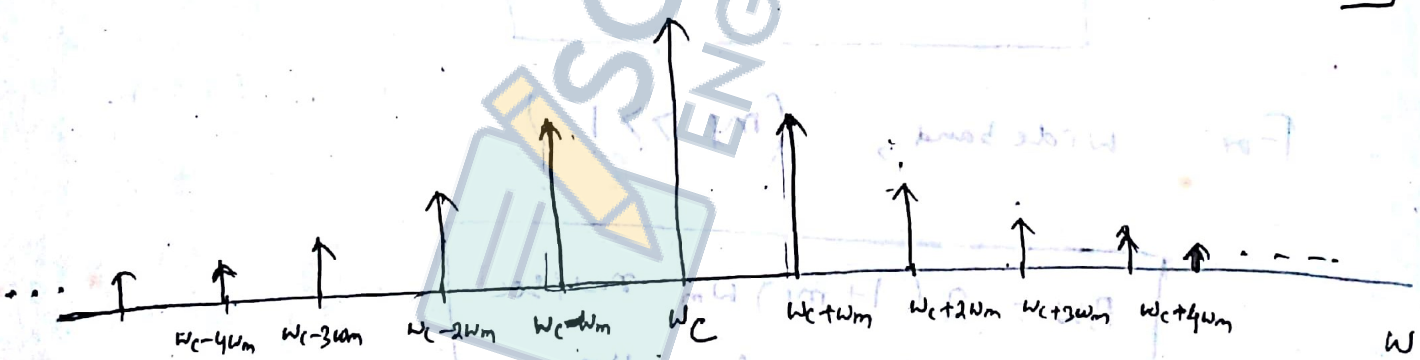
Using eqn (2) & (3) in eqn (1), and applying the formulas

$$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \cdot \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right],$$

we have,

$$X_{WBFM}(t) = A_c\Big[J_0(m_f)\cdot\cos\omega_c t$$
$$- J_1(m_f)\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right]$$
$$+ J_2(m_f)\left[\cos(\omega_c-2\omega_m)t + \cos(\omega_c+2\omega_m)t\right]$$
$$- J_3(m_f)\left[\cos(\omega_c-3\omega_m)t - \cos(\omega_c+3\omega_m)t\right]$$
$$+ J_4(m_f)\left[\cos(\omega_c-4\omega_m)t + \cos(\omega_c+4\omega_m)t\right]$$
$$+ \cdots\cdots\cdots\Big]$$



Due to the presence of infinite no. of sidebands in WBFM, theoretically transmission BW required is infinite. But higher sidebands have amplitude very small such that if these sidebands are not transmitted it doesn't affect the quality of transmission. Practical transmission BW in WBFM is given by <u>Carson's Rule</u>.

$$\boxed{\begin{aligned} BW &= 2(\Delta w + w_m) \ \frac{rad}{sec} \\ BW &= 2(\Delta f + f_m) \ Hz \end{aligned}}$$

But $m_f = \frac{\Delta w}{w_m} \Rightarrow \Delta w = m_f \cdot w_m$

$\therefore \quad BW = 2(m_f w_m + w_m) = 2(1+m_f) w_m$

If $m_f << 1$, i.e narrowband

$\therefore \quad BW = 2w_m$ or $2f_m$.

$\therefore$ For Narrowband, $(m_f << 1)$

$$\boxed{\begin{aligned} BW &= 2w_m \ rad/sec \\ BW &= 2f_m \ Hz \end{aligned}}$$

$\therefore$ For wideband, $(m_f >> 1)$

$$\boxed{\begin{aligned} BW &= 2(1+m_f) w_m \ rad/sec \\ BW &= 2(1+m_f) f_m \ Hz \end{aligned}}$$

<u>Ex:2</u> The max^m deviation allowed in an FM broadcast system is $\boxed{75 \text{ KHz}}$ (freq). If the modulating signal is single-tone sinusoid of 8KHz, determine the BW of FM signal. What will be the BW if modulating signal amplitude is doubled?

Ans :- Given, $\Delta f = 75\,KHz$.

$f_m = 8\,KHz$.

$BW = 2(\Delta f + f_m)$

$= 2(75+8)\,KHz$

$$\boxed{BW = 166\,KHz}$$

If amplitude is doubled, $\Delta f = K_f A_m$

Δf is doubled $(\because \Delta f = K_f A_m)$

$BW = 2(\Delta f + f_m)$

$= 2(150+8)$

$= 2 \times 158$

$= 316\,KHz$

$$\therefore \boxed{BW = 316\,KHz}$$

Phase Modulation:-

The total phase angle of PM is expressed as,

$$\phi_i = \omega_c t + K_p x(t) \quad \text{--- (1)}$$

Let the single-tone modulating signal be,

$x(t) = A_m \cos\omega_m t$,

$\therefore \phi_i = \omega_c t + K_p A_m \cos\omega_m t$

Phase deviation $= |K_p x(t)|_{max} = K_p A_m$

$$\boxed{\theta_d = K_p A_m}$$

$$(S(t))_{PM} = A_c \cos\left[w_c t + \theta_d \cos w_m t\right]$$

$$= A_c \cos \phi_i$$

Instantaneous frequency related to ϕ_i is expressed as,

$$w_i = \frac{d\phi_i}{dt} = \frac{d}{dt}\left[w_c t + \theta_d \cos w_m t\right]$$

$$= w_c + \theta_d (-w_m) \sin w_m t$$

$$\Rightarrow w_i = w_c - \theta_d w_m \sin w_m t$$

$$\Rightarrow w_i = w_c - K_p A_m w_m \sin w_m t$$

$\therefore$ Thus, max^m departure in the frequency from 'w_c' is $K_p A_m w_m$.

$$\therefore \boxed{\Delta W_{pm} = K_p A_m W_m}$$

Thus, the distinct feature of Phase ~~deviat~~ modulation is that the deviation in the carrier frequency (w_c) is linearly proportional to the baseband frequency (w_m). However, in FM, the deviation is independent of baseband frequency. ($\Delta W_{FM} = K_f A_m$)

$$\boxed{\begin{aligned} (S(t))_{PM} &= A_c \cos(w_c t + K_p A_m \cos w_m t) \\ &= A_c \cos(w_c t + m_p \cos w_m t) \\ \Delta W_{PM} &= K_p A_m W_m \end{aligned}}$$

Transmission BW of PM

$$(BW)_{PM} = 2(\Delta f + f_m) \text{ Hz}$$

$$= 2(\Delta\omega + \omega_m) \ \frac{rad}{sec}$$

OR

$$BW = 2 f_m \left(\frac{\Delta f}{f_m} + 1 \right)$$

$$BW = 2(1 + m_p) f_m \text{ Hz}$$

$$BW = 2(1 + m_p)\omega_m \ \frac{rad}{sec}$$

where

$$\boxed{\begin{aligned} \Delta\omega &= K_p A_m \omega_m \\ \Delta f &= K_p A_m f_m \end{aligned}}$$

Note:-

→ Modulation index $= \frac{\Delta f}{f_m} = \frac{K_p A_m f_m}{f_m} = K_p A_m.$

$$\boxed{m_p = K_p A_m}$$

Ex:3 (Taub - Schilling - 199 page - Ex-4.1)

Show that DSB-SC amplitude modulation is linear while phase modulation is not.

Soln:- Let $m_1(t)$ and $m_2(t)$ be two message signals and corresponding DSB-SC modulated signals are $m_{DSB_1}(t)$ and $m_{DSB_2}(t)$ while phase modulated signals are $m_{PM_1}(t)$ and $m_{PM_2}(t)$ respectively.

If ω_c = Carrier frequency then from defination of DSB-SC,

$$m_{DSB_1}(t) = m_1(t) \cdot \cos\omega_c t$$

$$m_{DSB_2}(t) = m_2(t) \cdot \cos\omega_c t$$

For a combined input $[m_1(t) + m_2(t)]$, DSB-SC signal

$$m_{DSB_{12}}(t) = [m_1(t) + m_2(t)] \cdot \cos\omega_c t$$

$$= m_1(t) \cdot \cos\omega_c t + m_2(t) \cdot \cos\omega_c t$$

$\therefore$ ~~MDSB~~ $m_{DSB_{12}}(t) = m_{DSB_1}(t) + m_{DSB_2}(t)$.

Hence DSB-SC is linear.

For angle modulated signal,

$$m_{PM_1}(t) = A_c \cos[w_c t + K_p m_1(t)]$$

$$m_{PM_2}(t) = A \cos[w_c t + K_p m_2(t)]$$

For combined input $[m_1(t) + m_2(t)]$

$$M_{PM_{12}}(t) = A \cos[w_c t + K_p \{m_1(t) + m_2(t)\}]$$

$$\neq m_{PM_1}(t) + m_{PM_2}(t)$$

$\therefore$ Phase modulation is not linear.

Note :- Similarly it can be shown that FM is also not linear.

Ex:-4 :- Consider an angle modulated signal,

$$x(t) = 3 \cos[2\pi \times 10^6 t + 2 \sin(2\pi 10^3 t)]$$

Find the

(a) Instantaneous frequency at time (i) t = 0.25 ms

(ii) t = 0.5 ms

(b) max^m phase deviation

(c) max^m frequency deviation.

Ans :-

$$x(t) = 3\cos\left[2\pi\times10^6 t + 2\sin(2\pi\times10^3 t)\right]$$

~~Comparing~~ $\phi_i = 2\pi\times10^6 t + 2\sin 2\pi\times10^3 t$.

Instantaneous freq,

$$\boxed{w_i = \frac{d\phi_i}{dt} = 2\pi\times10^6 + 2\cos(2\pi\times10^3)t\cdot(2\pi\times10^3)}$$

$$w_i\Big]_{t=0.25\,ms} = 2\pi\times10^6 + 4\pi\cos\left[(2\pi\times\cancel{10^3})\times0.25\times\cancel{10^{-3}}\right]\times10^3$$

$$= 2\pi\times10^6 + 4\pi\times10^3\cdot\cancel{\cos\frac{\pi}{2}}^{\,0}$$

$$= \frac{2\pi\times10^6}{\cancel{2\pi}}\ \frac{rad}{sec}$$

$$w_i\Big|_{t=0.25ms} = 10^6\ Hz$$

$$w_i\Big|_{t=0.5ms}, = 2\pi\times10^6 + 4\pi\times10^3\cdot\cos(2\pi\times\cancel{10^3}\times0.5\times\cancel{10^{-3}})$$

$$= 2\pi\times10^6 - 4\pi\times10^3 \quad (\because \cos\pi = -1)$$

$$= 2\pi\times10^3\left[10^3 - 2\right]$$

$$= 2\pi\times10^3\times998\ \frac{rad}{sec} \simeq 2\pi\times10^6\ \frac{rad}{sec}$$

$$= 998\ KHZ \simeq 1\ MHZ.$$

(b) Max^m phase deviation, θ_d

Comparing with standard PM,

~~$S(t) = A_c\cos(w_c t + \theta_d\cos w_m t)$~~

$$S(t) = A_c\cos\left[w_c t + K_p\, x(t)\right]$$

Here $x(t) = 3\cos\left[2\pi\times10^6 t + 2\sin(2\pi\times10^3 t)\right]$

$$\theta_d = \left|K_p\, x(t)\right|_{max} = \left|2\sin(2\pi\times10^3 t)\right|_{max} = 2\ rad.$$

(C) For Max^m frequency deviation,

~~$\omega_i = \omega_c + K_f\, x(t)$~~

$$\Delta\omega = \underline{K_p \cdot A_m}\, \omega_m$$

$$= \theta_d \cdot \omega_m$$

$$= 2 \times [2\pi \times 10^3]$$

$$\boxed{\Delta\omega = 4\pi \times 10^3 \ \frac{rad}{sec}}$$

$$\boxed{\Delta f = 2000 \text{ Hz}}$$

genl or

$\theta_d = |K_p\, x(t)|_{max}$

$\Delta\omega = \left\{ \frac{d}{dt} |K_p\, x(t)| \right\}_m$

Generalized approach, whether modulating signal is $A_m \sin\omega_m t$ or $A_m \cos\omega_m t$, doesn't matter. The above formula works for all.

$(\because \Delta f = \frac{\Delta\omega}{2\pi})$

~~Ex:-5~~ ~~Note:- Phase deviation = ω_m x frequency deviation~~

~~$\theta_d = \omega_m \times \omega_m$~~

Note:-

$\boxed{\text{frequency deviation} = \text{Phase deviation} \times \omega_m}$

$$\Delta\omega = \theta_d \times \omega_m$$

$$\Rightarrow \boxed{\Delta\omega = K_p A_m \times \omega_m}$$

Ex:- 5 A baseband signal, $x(t) = 5\cos 2\pi \times 15 \times 10^3 t$ Angle modulates a carrier signal $A\cos\omega_c t$

(i) Determine the modulation index and BW for (a) FM system (b) PM System.

(ii) Find the change in BW and modulation index for both FM & PM if f_m reduced to 5 KHz

Assume $K_f = K_p = 15 \ \frac{KHz}{Volt}$

Ans :- $x(t) = 5 \cos 2\pi \times 15 \times 10^3 t$

$f_m = 15$ KHz

$A_m = 5$ volt.

(i) (a) FM

$$m_f = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} = \frac{15 \times 10^3 \times 5}{15 \times 10^3} = 5.$$

$$BW = 2(1+m_f) f_m$$

$$= 2(1+5) \times 15 \times 10^3$$

$$= 180 \times 10^3$$

$$\boxed{BW = 180 \text{ KHz}}$$

(b) For PM

$$m_p = \frac{\Delta f}{f_m} = \frac{K_p A_m \cancel{f_m}}{\cancel{f_m}} = 15 \times 10^3 \times 5 = 75 \text{ KHz}.$$

$$BW = 2(1+m_p) f_m = 2(1+75 \times 10^3) \cdot 15 \times 10^3$$

$$\simeq 2 \times 75 \times 10^3 \times 15 \times 10^3$$

$$\boxed{BW = 2250 \text{ MHz}}$$

(ii) If f_m reduced to 5 KHz

(a) FM

$$m_f = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} = \frac{15 \times \cancel{10^3} \times \cancel{5}}{\cancel{5} \times \cancel{10^3}} = 15$$

$$BW = 2(1+m_f) f_m = 2(1+15) \times 5 \times 10^3 = 160 \text{ KHz}$$

(b) PM

$$m_p = \frac{K_p A_m \cancel{f_m}}{\cancel{f_m}} = K_f A_m = 15 \times 10^3 \times 5 = 75\ \text{kHz}$$

$\left(\frac{\Delta f}{f_m}\right)$

(Same as that previous one)

$$BW = 2(1 + m_p) f_m$$

$$= 2(1 + 75 \times 10^3) \times 5 \times 10^3$$

$$\simeq 2 \times 75 \times 5 \times 10^6$$

$$\boxed{BW = 750\ \text{MHz}}$$

FM	AM
1) Amplitude of FM wave is constant.	1) Amplitude of AM wave will change with modulating voltage.
2) Hence transmitted power remains constant.	2) Transmitted power depends on modulation index.
3) All transmitted power is useful.	3) Carrier power and sideband power are useless.
4) FM receiver are immune to noise	4) AM receiver are not immune to noise.
5) $BW = 2(\Delta f + f_m)$	5) $BW = 2f_m$
6) BW is large, wide channel required.	6) BW is much less than FM.

7) FM transmission & reception equipments are more complex.	7) AM equipments are less complex.
8) The number of sidebands having significant amplitude depends on modulation index (m_f)	8) Number of sidebands in AM will be constant and equal to 2.
9) The information is contained in the frequency variation of the carrier	9) The information is contained in the amplitude variation of the carrier.

FM	PM
1) If baseband signal is $A_m \cos w_m t$ and carrier is $A_c \cos w_c t$ then FM signal is given by $S(t) = A_c \cos[w_c t + m_f \sin w_m t]$	1) If baseband signal is $A_m \cos w_m t$ and carrier is $A_c \cos w_c t$, then PM signal is given by $S(t) = A_c \cos[w_c t + m_p \cos w_m t]$
2) $\Delta f = K_f A_m$ Frequency deviation is independent of modulating frequency.	2) $\Delta f = K_p A_m f_m$ Frequency deviation depends on the modulating frequency.
3) Modulation index $m_f = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$	3) Modulation index $m_p = \frac{\Delta f}{f_m} = K_p A_m$
4) It is possible to receive FM on a PM receiver.	4) It is possible to receive PM on a FM receiver.

5) Noise immunity is better than AM & PM	5) Noise immunity is better than AM but less than FM.
6) Amplitude of FM is constant.	6) Amplitude of PM is also constant.
7) Signal to noise ratio (SNR) is better than PM	7) SNR is inferior to FM
8) FM is widely used.	8) PM is used in some mobile system.

FM Generation :-

Assignment

The FM Modulator ckt. used for generating FM signal may be put into two categories

(i) The direct method or Parameter Variation method.

(ii) Indirect method or Armstrong method.

(i) The direct method or Parameter Variation method :-

In direct method or Parameter Variation method, the baseband or modulating signal directly modulates the carrier. The carrier signal is generated with the help of an oscillator circuit. This oscillator ckt uses a parallel tuned L-C circuit. Thus frequency of oscillation of carrier generation is governed by the expression, $\boxed{w_c = \frac{1}{\sqrt{LC}}}$

Now, we can make the carrier frequency ω_c to vary in accordance with the baseband or modulating signal $x(t)$ if L or C is varied according to $x(t)$.

An oscillator circuit whose frequency is controlled by a modulating ~~signal~~ voltage is called Voltage controlled oscillator (VCO). The frequency of VCO is varied according to the modulating signal simply by putting a shunt voltage variable capacitor with its tuned ckt. This voltage variable capacitor is called Varactor (variable + capacitor) or Varicap. This type of property is exhibited by reverse biased semiconductor diodes. Also the capacitance of BJT and FET (FET is varied by the Miller-effect. This miller capacitance may be utilized for frequency modulation).

The inductance L of the tuned circuit may be varied according to the baseband signal $x(t)$. The FM ckt using such inductors is called saturable reactor modulator.

Varactor Diode Method for FM Generation

The varactor diode is a semiconductor diode whose junction capacitance changes with d.c bias voltage. This varactor diode is connected in shunt with the tuned circuit of the carrier oscillator. The arrangement is shown in figure 1.

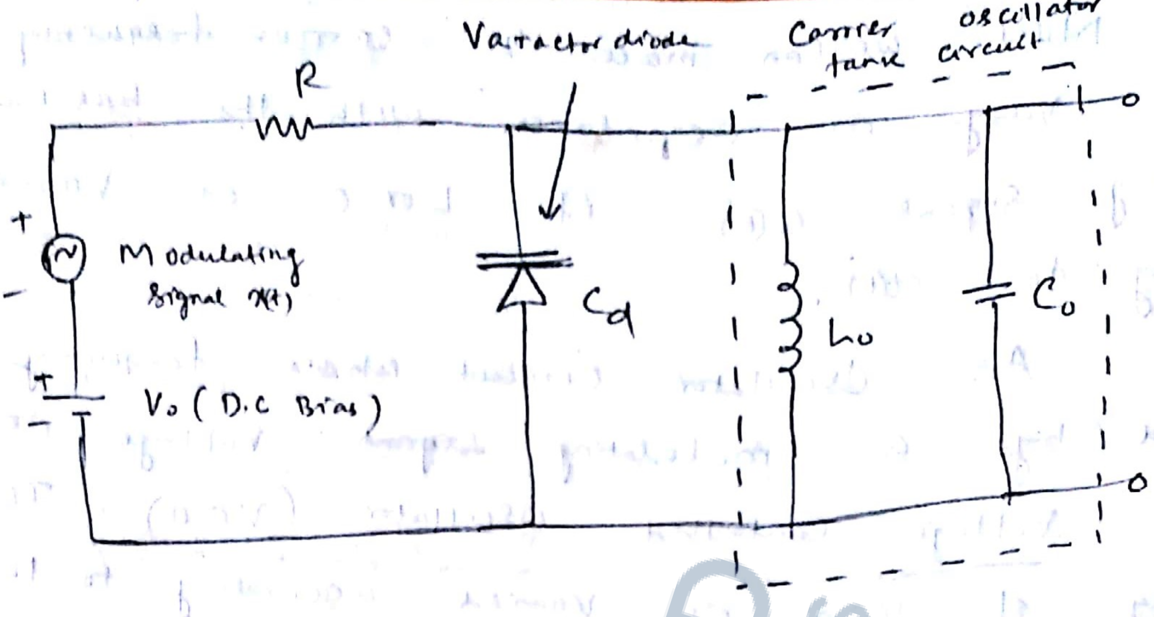


fig 1:- Varactor diode method of FM generation

Mathematical Analysis :-

The capacitance Cd of the varactor diode is expressed as,

$$C_d = \frac{K}{\sqrt{V_D}} = K \, V_D^{-1/2} \quad \text{---} \; (1)$$

K = Proportionality constant.

Here, V_D is the total instantaneous voltage across the varactor diode and is given by

$$V_D = V_0 + x(t) \quad \text{---} \; (2)$$

The oscillation frequency is given by,

$$W_c = \frac{1}{\sqrt{LC}} \quad \text{---} \; (3)$$

Now, the total capacitance of the oscillator tank circuit will be $C_0 + C_d$ and thus the instantaneous frequency of oscillation $\underline{W_i}$ is expressed as,

$$\omega_i = \frac{1}{\sqrt{L_0 (C_0 + C_d)}} \quad \text{--- (4)}$$

Putting the value of Cd, from eqn (1), in eqn (4), we get

$$\omega_i = \frac{1}{\sqrt{L_0 (C_0 + K V_D^{-1/2})}}$$

$$\boxed{\omega_i = \frac{1}{\sqrt{L_0 \{C_0 + K (V_0 + x(t))^{-1/2}\}}}} \quad \text{--- (5)}$$

$(\because \; V_D = V_0 + x(t)$, from eqn (1))

Thus, the instantaneous oscillator frequency ω_i depends upon the baseband or modulating signal $x(t)$ and hence frequency modulated signal is generated.

Drawbacks of Direct method :-

1) In this method, it is not easy to get high order stability in carrier frequency. This is due to the fact that generation of carrier signal is directly affected by the baseband or modulating signal.

2) The non-linearity of the varactor diode produces a frequency variation due to harmonics of the modulating

or base band signal and therefore FM signal is distorted.

Note :-

1) The solution to the drawback (1), is the indirect method i.e Armstrong method of FM generation. In fact, in this method, the carrier oscillator does not respond to the modulating signal directly rather the generation of the carrier is made isolated from other parts of the circuit. Thus stable crystal oscillator may be used to generate carrier signal.

Indirect or Armstrong method of FM Generation :-

In Armstrong method of FM ~~the~~ generation, we can get very high frequency stability since in this case the crystal oscillator may be used as carrier frequency generator. The working principle of Armstrong method is to generate a narrowband FM (NBFM) indirectly by utilizing the phase-modulation technique and then changing this narrowband FM into a wideband FM. This phase modulation technique is preferred because its generation is easy. (Since modulation index (m_f) is small in NBFM, distortion is low in NBFM.) As shown in fig 1, the multiplier ckt apart from multiplying the carrier frequency also increases the frequency deviation and hence the narrow band FM (with small freq deviation) is converted into wideband FM (with large freq deviation).

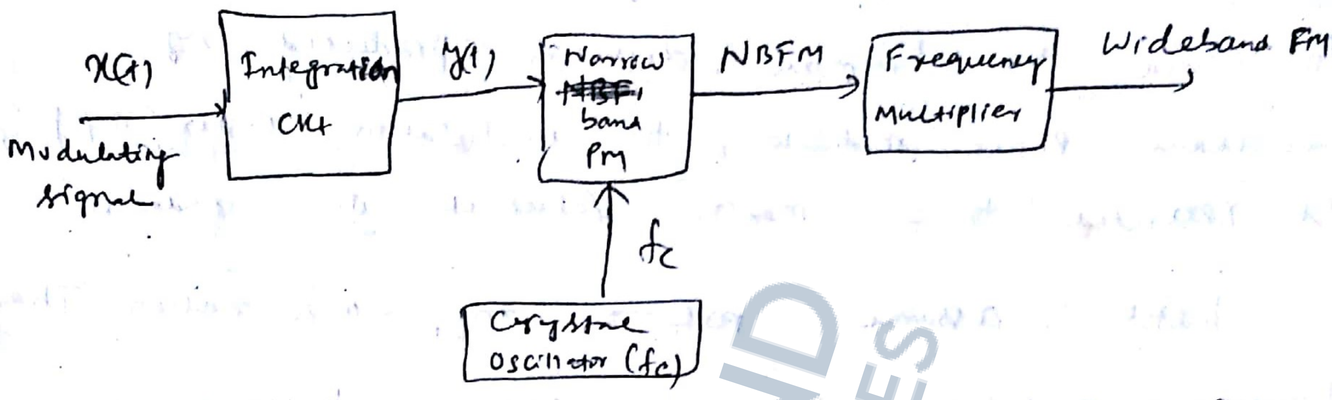


fig 1 :- Armstrong method for FM generation

Practical Armstrong method for FM generation :-

Fig 2, Shows a simplified block diagram of a Commercial FM generation system using Armstrong method.

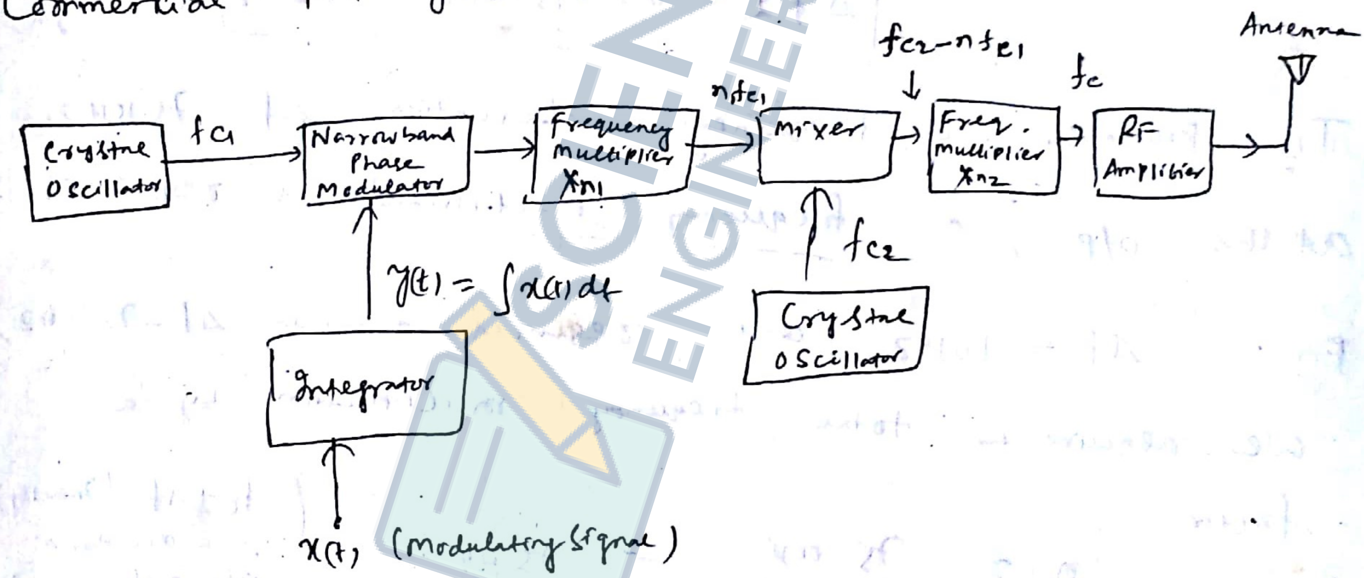


fig 2:- Commercial FM generation system using Armstrong method.

→ In Commercial use, we require to transmit audio signal consisting frequencies in the range 50 Hz to 15 KHz and the value of $\Delta f = 75\ \text{KHz}$.

Let us assume that the final carrier frequency of the FM required is $\underline{f_c = 100\ MHz}$.

Lets begin with ~~the~~ NBFM having a carrier frequency $\underline{f_{c1} = 100\ KHz}$ generated by a crystal oscillator. To check the harmonic distortion produced by narrowband phase modulator, the modulation index (m_f) is restricted to a max^m value of $\underline{0.3}$ radians.

Lets assume ~~m_{f1}~~ $m_{f1} = 0.2$ radian. The lowest modulating frequency $\underline{50\ Hz}$ produces a frequency deviation $\boxed{\Delta f_1 = 0.2 \times 50 = 10\ Hz}$ at the narrowband phase modulator o/p where as largest modulating frequency 15KHz produces a frequency deviation of

$$\boxed{\Delta f_2 = 0.2 \times 15\ KHz = 3\ KHz}. \left(\because m_f = \frac{\Delta f}{f_m}\right)$$

To produce a frequency deviation $\Delta f = 75\ KHz$, at the o/p, a $\underline{\text{frequency multiplication}}$ is required

For $\Delta f_1 = 10\ Hz$, and required deviation $\Delta f = 75\ KHz$, we require a total frequency multiplication by a factor

$$n = \frac{75{,}0\cancel{0}0}{1\cancel{0}} = 7500.$$

($f_c \pm \Delta f$ (Initially) with a multiplier 'n' $nf_c \pm n\Delta f$)

However, a straight forward frequency multiplication equal to this value leads to a very high value of carrier frequency than the required 100MHz. So to get the frequency-deviation and carrier frequency we use a two-stage frequency multiplier as shown in previous page (fig-2)

This arrangement makes use of two multipliers & a mixer. With the help of mixer, the carrier freq is translated suitably without altering frequency deviation Δf_1. The final stage multiplier provides the required carrier frequency and deviation.

Let us assume that n_1 & n_2 are the freq. multiplication factors for 2 multipliers, so that

$$n = n_1 . n_2 \quad \text{——— (1)}$$

$$= \frac{\Delta f}{f_1} = \frac{75,000}{10} = 7500.$$

The carrier freq at the o/p of the first frequency multiplier is translated downwards to frequency $(f_{c2} - n_1 f_{c1})$ by mixing it with carrier wave of frequency f_{c2} which is produced by another oscillator.

Let the carrier freq at the o/p of the second multiplier is $\boxed{\frac{f_c}{n_2}}$. [∵ So o/p of second multiplier $= \frac{f_c}{n_2} . n_2 = f_c$]

∴ $$\frac{f_{c2} - n_1 f_{c1}}{\cancel{f_2 - n f_1}} = \frac{f_c}{n_2}$$

With $f_{c1} = 0.1$ MHz, $f_{c2} = 8.5$ MHz, we have

$$8.5 - 0.1\, n_1 = \frac{100}{n_2} \quad \text{—(2)} \qquad (∵ f_c = 100 \text{ MHz required})$$

$$\Rightarrow 8.5 n_2 - \frac{n_1 n_2}{10} = 100.$$

$$\Rightarrow 8.5 n_2 - \frac{750\cancel{0}}{\cancel{10}} = 100 \qquad (∵ n_1 . n_2 = 7500 \text{ using eq}^n (1))$$

$$\Rightarrow 8.5 n_2 = 100 + 750 = 850$$

$\therefore \quad n_2 = 100$

$$n_1 = \frac{7500}{100} = 75$$

$\therefore$ Thus the frequency multipliers can be designed accordingly to get the desired frequency multiplication.

<u>Note</u> :- A frequency multiplier is a combination of a $\underbrace{\text{nonlinear element}}$ and a bandpass filter.

(Ex- diode, transistor etc)

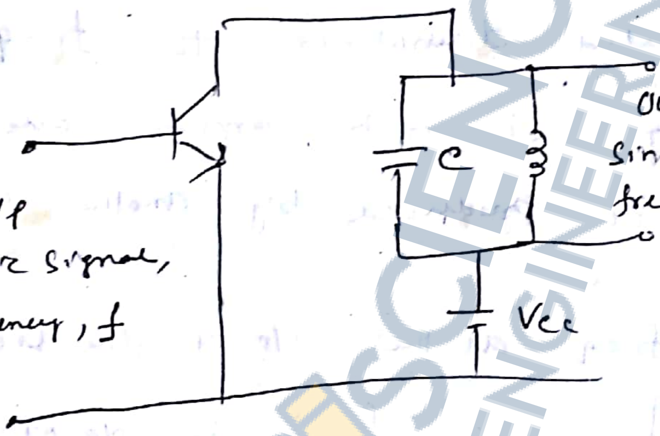


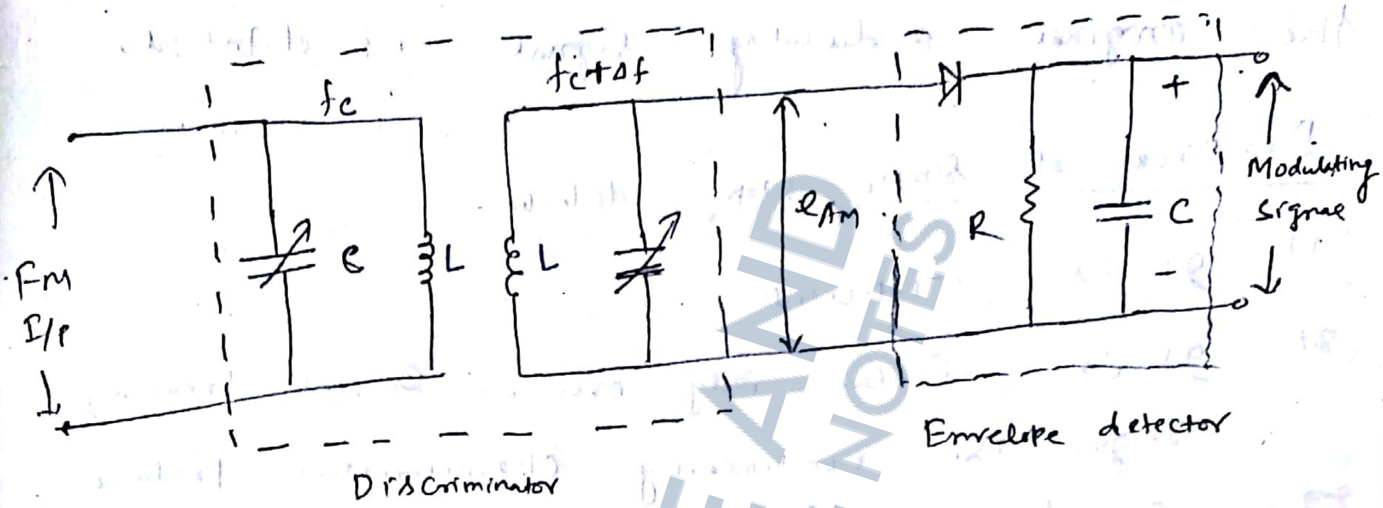
fig:- A frequency multiplier ckt.

<u>FM demodulators</u> :-

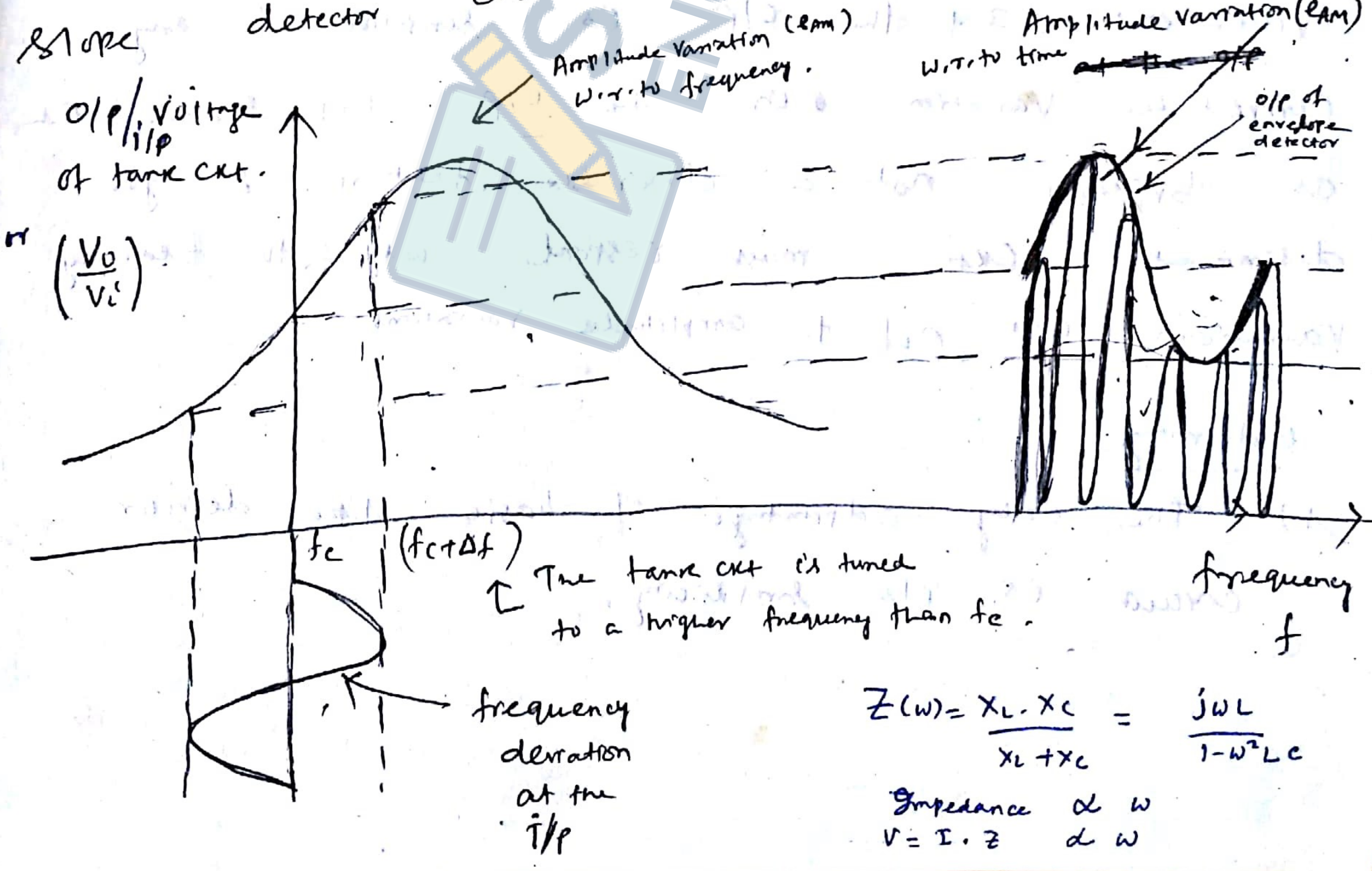
<u>1) Simple Slope detector (Frequency discriminator)</u>

The circuit consist of <u>2 tuned ckts</u> which are tuned to two different frequencies. First one is tuned to the ~~A fc~~ incoming FM carrier frequency fc where as second one is tuned to.

frequency slightly different from the carrier frequency ($f_c + \Delta f$). Therefore, this portion of the circuit which contains two tuned circuits tuned to different frequencies, is called discriminator.



The amplitude of the o/p voltage of the tank circuit depends on the frequency deviation of the i/p FM signal. Thus, this ckt converts FM signal into AM signal as shown ~~bel~~ in slope detector characteristic curve.



$$Z(\omega) = \frac{X_L \cdot X_C}{X_L + X_C} = \frac{j\omega L}{1-\omega^2 LC}$$

Impedance $\propto \omega$

$V = I \cdot Z \propto \omega$

The other portion of the circuit is envelope detector. The AM signal from the O/P of the discriminator is applied as the I/P of the envelope detector. At the O/P of the envelope detector the original modulating signal is obtained.

Drawbacks of simple slope detector

1) It is inefficient.
2) It is linear only over a limited frequency range. The non-linearity characteristics produces a harmonic distortion.
3) It is difficult to adjust as the primary and secondary windings of the transformer must be tuned to slightly different frequencies.
4) The ckt doesn't eliminate the amplitude variations and the O/P is sensitive to any amplitude variation on the I/P FM signal which is obviously not a desirable feature. A good discriminator ckt must respond only to frequency variations and not to amplitude variations.

Advantage :-

1) The only advantage of basic slope detector circuit is its simplicity.

2) Balanced Slope detector / Stagger-tuned detector

Fig 1:- Shows the circuit diagram of the balanced slope detector. The circuit diagram shows that the balanced slope detector consists of 2 slope detector circuits. The i/p transformer has a center tapped secondary. Hence, the i/p voltages to the two slope detectors are 180° out of phase.

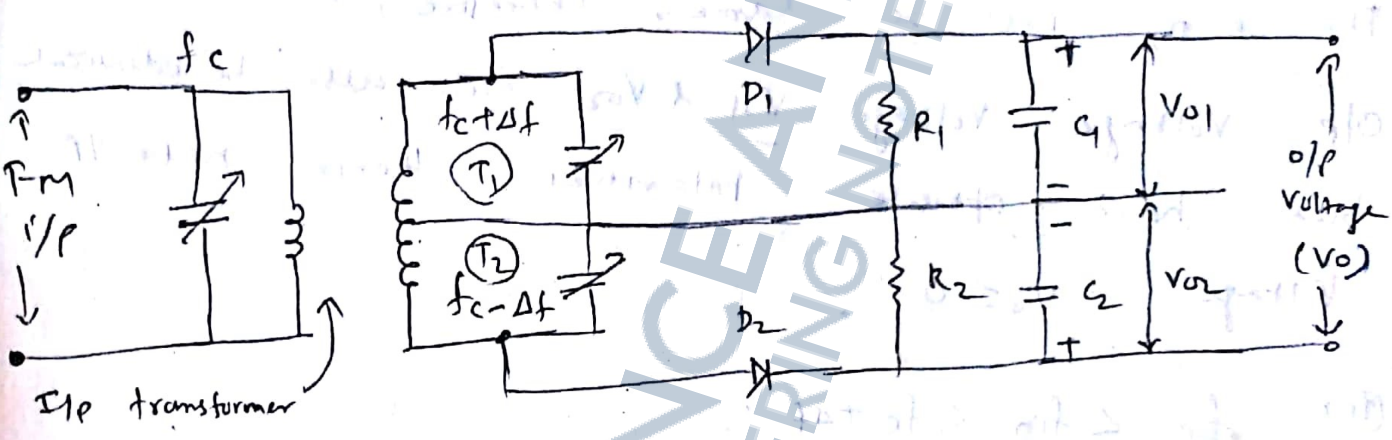


fig 1:- Balanced Slope detector.

→ There are 3 tuned ckts. Out of them, the ~~tt~~ Primary is tuned to f_c. The upper tuned ckt of the secondary (T_1) is tuned above f_c by Δf i.e its resonant freq $f_c+\Delta f$

→ The lower tuned circuit of the secondary is tuned below f_c by Δf i.e at $f_c-\Delta f$. V_{01} and V_{02} are the o/p voltages of the 2 slope detectors. The final o/p voltage V_0 is obtained by taking the subtraction of individual voltages V_{01} & V_{02}. i.e

$$\boxed{V_0 = V_{01} - V_{02}}$$

Working operation of the ckt

(i) $f_{in} = f_c$:-

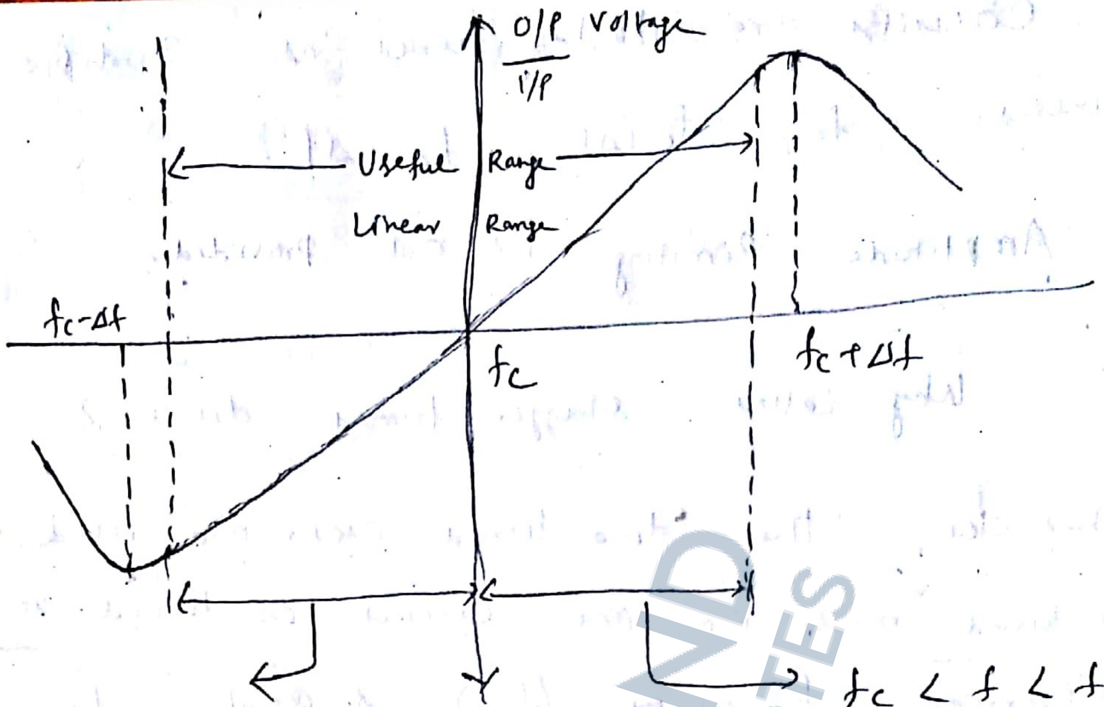
When i/p frequency is instantaneously equal to f_c, the induced voltage in T_1 winding of secondary is exactly equal to that induced in the winding T_2. Thus, the i/p voltages to both the diodes D_1 & D_2 will be same. Therefore, their D.C o/p voltage voltage V_{01} & V_{02} will also be identical but have opposite polarities. Hence, net o/p voltage $V_0 = 0$.

(ii) $f_c < f_{in} < f_c + \Delta f$:-

In this range of i/p frequency, the induced voltage in the winding T_1 is higher than that induced in T_2. Therefore the i/p to D_1 is higher than D_2. Hence the +ve o/p V_{01} of D_1 is higher than the -ve o/p V_{02} of D_2. Therefore, the o/p voltage V_0 is +ve. As the frequency increases towards $f_c + \Delta f$, the +ve o/p voltage increases as shown in figure 2.

Similarly as frequency decreases towards $f_c - \Delta f$, the -ve o/p voltage increases.

→ If the o/p frequency goes outside the range $f_c - \Delta f$ to $f_c + \Delta f$, the o/p voltage



$f_c - \Delta f < f < f_c$

Input to D_1 is less than input to D_2

↓

V_{01} is less than V_{02}

↓

$V_0 = V_{01} - V_{02}$

$V_0 \Rightarrow$ is negative

$f = f_c$

input to D_1 is equal to input to D_2.

$V_{01} = V_{02}$

$\Rightarrow V_0 = V_{01} - V_{02}$

$V_0 = 0$

V_0 is zero

$f_c < f < f_c + \Delta f$

Input to D_1 is higher than input to D_2.

$V_{01} > V_{02}$

$V_{01} - V_{02} > 0$

$\Rightarrow V_0 > 0$

V_0 is +ve

fig 2 :- Characteristics of the balanced slope detector.

will fall due to reduction in tuned circuit response.

Advantage :-

1) This ckt is more efficient than simple slope detector

2) It has better linearity than simple slope detector. So ~~the~~ no harmonic distortion is caused when operation is restricted to linear region.

Drawbacks :-

1) Even though linearity is good, it is not good enough.

2) This ckt is difficult to tune since three

tuned circuits are to be tuned at 3 different frequencies f_c, $f_c+\Delta f$, $f_c-\Delta f$.

3) Amplitude limiting is not provided.

Note:- Why called Stagger-tuned detector?

In this ckt, the two tuned ckts are used in stagger-tuned mode i.e one circuit is tuned above the carrier frequency (f_c) & another tuned ckt is tuned below f_c.

Discuss about internal Question:- [AM receiver / superheterodyne receiver]

3) * PLL (Phase-Locked Loop) - FM demodulator :-

A phase locked loop (PLL) is primarily used in tracking the phase & frequency of the carrier component of an incoming FM signal. PLL is also useful for synchronous demodulation of of AM-SC (Amplitude Modulation with suppressed carrier)

Further, PLL is ~~also~~ useful for demodulatory FM signals in presence of large noise and low signal power.

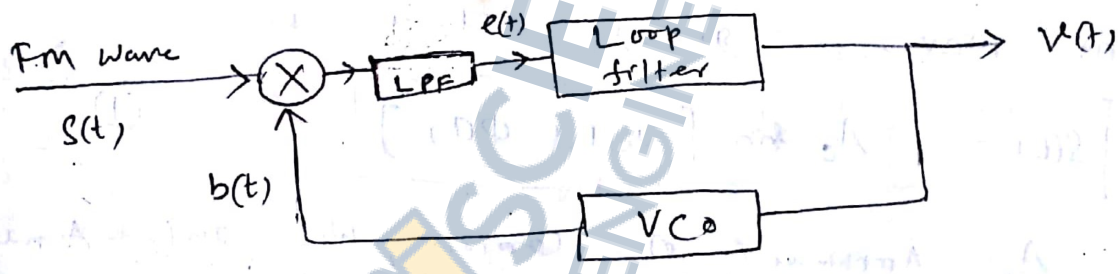
→ Suitable for space vehicle-to earth ~~ele~~ data link, commercial FM receivers.

→ A PLL is basically a -ve feedback system. It consist of ~~three~~ four major components. These components are multiplier, LPF, a loop filter &

a Voltage controlled oscillator (VCO). A VCO is a sine wave generator whose frequency is determined by the voltage applied to it from an external source. It means any frequency modulator can work as a VCO.

Operation

The operation of a PLL is similar to any other feedback system. In any feedback system, the feedback signal tends to follow the i/p signal. If the signal feedback is not equal to the i/p signal, the error signal will change the value of the feedback signal until it is equal to the i/p signal. The difference signal betn



s(t) & b(t) is called an error signal. A PLL operates on a similar principle except for the fact that the quantity feedback is not the amplitude but a generalized phase $\phi(t)$.

The error signal or difference signal e(t) is utilized to adjust the VCO frequency in such a way that the instantaneous phase angle comes close to the angle of the incoming signal s(t). At this point, the two signal s(t) and $b(t)$ are in synchronism and the PLL is locked to the incoming signal $s(t)$.

Here, we have assumed that the VCO is adjusted initially so that when the control voltage comes to zero, the following 2 conditions are satisfied.

(i) The frequency of the VCO is precisely set at the unmodulated carrier frequency f_c and

(ii) The VCO output has $90°$ phase shift w.r.to the unmodulated carrier wave.

Mathematical Analysis :-

Let the i/p signal applied to the PLL be an FM wave. It is defined as

$$\boxed{s(t) = A_c \sin[w_c t + \phi(t)]} \quad \text{--- (1)}$$

where A_c = Amplitude of carrier, $w_c = 2\pi f_c$ = Angular carrier frequency, $\boxed{\phi(t) = 2\pi K_f \int_0^t m(t)\, dt} \quad \text{--- (2)}$

$[\because w_i = 2\pi f_c + 2\pi K_f\, m(t)$, $\Rightarrow \phi_i = 2\pi f_c t + 2\pi K_f \int_0^t m(t)dt]$ $[\because \phi_i = \int w_i\, dt]$

where $m(t)$ = message or base band signal.

K_f = frequency sensitivity of freq. modulator.

Let the VCO o/p be defined by

$$b(t) = A_v \cos[w_c t + \phi_v(t)] \quad \text{--- (3)}$$

where A_v = Amplitude of VCO o/p when the control voltage applied to VCO is defined by $v(t)$, then we have

$$\phi_v(t) = 2\pi K_v \int_0^t v(t)\,dt \quad \text{——— (4)}$$

It may be observed from eqn (1) & (3), that VCO output and incoming signals are 90° out of phase, [sin θ and cos θ] i.e. s(t) & b(t) while the VCO frequency (ω_c) in the absence of $v(t)$ is precisely equal to the unmodulated frequency (ω_c) of the FM signal. The incoming FM have $s(t)$ & the VCO output $b(t)$ are applied to a multiplier.

$$s(t) \cdot b(t)$$

$$\{A_c \sin[\omega_c t + \phi(t)]\}\{A_v \cos[\omega_c t + \phi_v(t)]\}$$

Note:- $2\sin\alpha \cdot \cos\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

∴ The O/P of multiplier has the following components

(i) A high frequency component represented by

$$\frac{A_c \cdot A_v}{2} \sin[2\omega_c t + \phi(t) + \phi_v(t)]$$

(ii) A low frequency component represented by

$$\frac{A_c \cdot A_v}{2} \sin[\phi(t) - \phi_v(t)] \quad \text{——— (5)}$$

The high frequency component can be eliminated by an LPF. Hence the o/p of LPF is

$$e(t) = \frac{A_c A_v}{2} \cdot \sin[\phi(t) - \phi_v(t)] \quad \text{——— (6)}$$

Where the difference $\phi(t) - \phi_v(t) = \phi_e(t)$, constitutes the phase error.

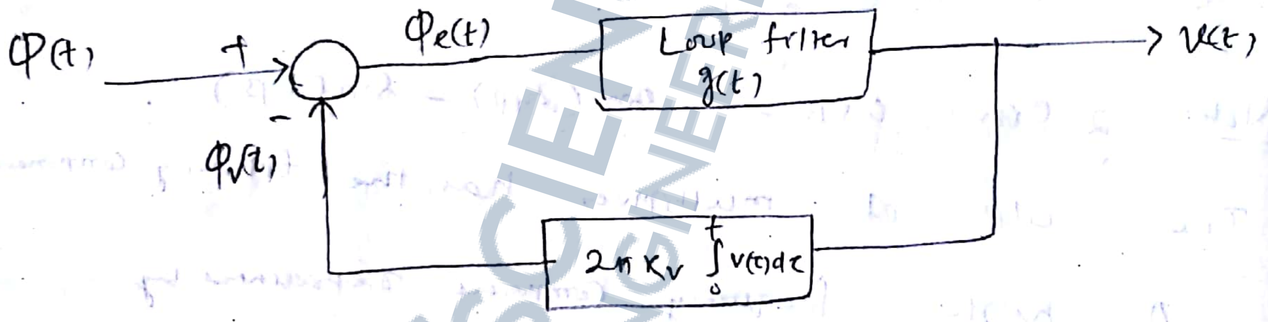
$$\therefore \quad e(t) = \frac{A_c A_v}{2} \sin \phi_e(t) \qquad \text{——— (7)}$$

The signal $e(t)$ is the i/p to the loop filter. The loop filter operates on error signal $e(t)$ to produce the o/p $v(t)$.

Lets assume the PLL is in lock, so that the phase error is small, then

$$\sin \phi_e(t) \simeq \phi_e(t)$$

Under this condⁿ, we may deal with linearized model of PLL,



ϕ_e may be expressed as,

$$\phi_e(t) = \phi(t) - 2\pi K_v \int_0^t v(\tau)\, d\tau \qquad \text{——— (8)}$$

Differentiating,

$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi(t) - 2\pi K_v\, v(t)$$

$$\Rightarrow \frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi(t) - 2\pi K_v \int_0^\infty \phi_e(\tau) \cdot g(t-\tau)\, d\tau$$

$$(\because \; v(t) = \phi_e(t) * g(t)) \qquad \text{——— (9)}$$

Taking Fourier transform both the sides,

$$\Rightarrow j \cancel{2\pi} f \, \Phi_e(f) = j \cancel{2\pi} f \, \Phi(f) - \cancel{2\pi} K_v \cdot [\Phi_e(f) \cdot G(f)]$$

$$\left(\because F\left[\frac{d}{dt} x(t)\right] = j\omega X(\omega) = j 2\pi f X(f) \right.$$

$$\left. F[\phi_e(t) * g(t)] = \Phi_e(f) \cdot G(f) \right)$$

$$\Rightarrow jf \, \Phi_e(f) + K_v \Phi_e(f) G(f) = jf \, \Phi(f)$$

$$\Rightarrow \Phi_e(f) [jf + K_v G(f)] = jf \, \Phi(f)$$

$$\Rightarrow \Phi_e(f) = \frac{jf \, \Phi(f)}{jf + K_v G(f)} = \frac{\cancel{jf} \, \Phi(f)}{\cancel{jf}\left[1 + \frac{K_v G(f)}{jf}\right]}$$

$$\Rightarrow \Phi_e(f) = \frac{\Phi(f)}{1 + \frac{K_v}{jf} \cdot G(f)}$$

We design $G(f)$ such that, $\left| \frac{K_v}{jf} \cdot G(f) \right| >> 1$.

$$\therefore \Phi_e(f) = \frac{\Phi(f)}{\frac{K_v}{jf} G(f)} \quad \text{——— (10)}$$

From eqn (9), [∵ F.T[Convolution in time domain] = multiplication in freq. domain]

$$V(f) = \Phi_e(f) \cdot G(f) \quad \text{——— (11)}$$

Putting eqn (10), in eqn (11), we have

$$V(f) = \frac{\Phi(f)}{\frac{K_v}{jf} \cdot \cancel{H(f)}} \cancel{H(f)}$$

$$V(f) = \frac{\Phi(f) \cdot (jf)}{K_v}$$

$$= \frac{j 2\pi f \; \Phi(f)}{2\pi K_v}$$ (Multiplying 2π in n.r & d.r)

Taking I.F.T,

$$v(t) = \frac{1}{2\pi K_v} \cdot IFT\left[j 2\pi f \; \Phi(f)\right]$$

$$v(t) = \frac{1}{2\pi K_v} \cdot \frac{d}{dt} \phi(t)$$ $\left[\because F\left[\frac{d}{dt} x(t)\right] = j2\pi f \; X(f)\right]$

$$= \frac{1}{2\pi K_v} \cdot \frac{d}{dt}\left[2\pi K_f \int^t m(t)\, dt\right]$$

(from eqn (2))

$$= \frac{1}{\cancel{2\pi} K_v} \cdot \cancel{2\pi} K_f \cdot m(t)$$

$$v(t) = \frac{K_f}{K_v} \cdot m(t) = K \; m(t)$$

Since the control voltage of VCO is proportional to message signal, $v(t)$ is the desired demodulated signal.

Note :- Advantage of using feedback :- SNR (Signal to noise ratio) of signal improved.