Amplitude - Modulation System :- Ch-2

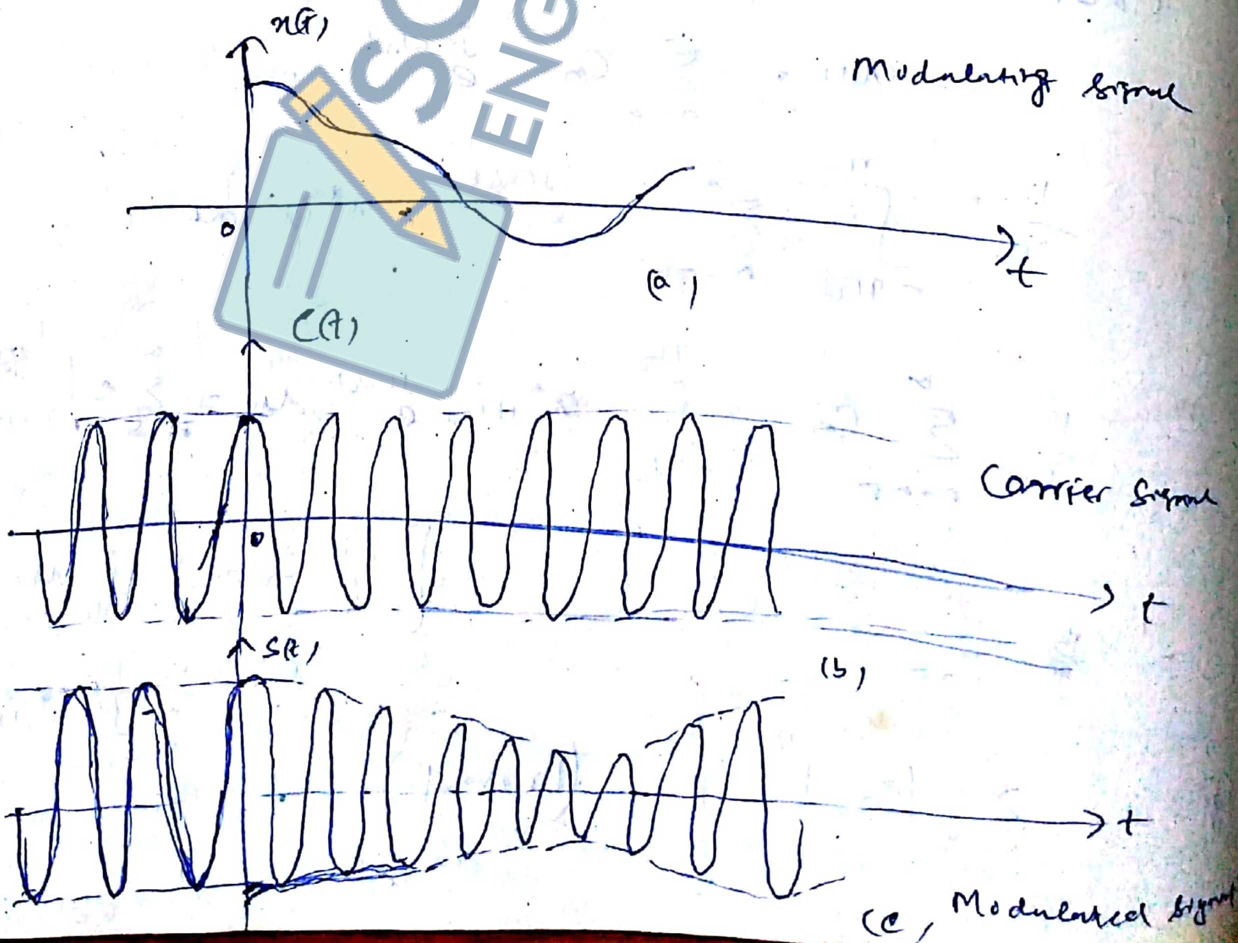
Modulate → Modify / Alter / Vary (Dictionary Meaning)

Modulation may be defined as a process by which some characteristics of a signal known as Carrier is varied according to the instantaneous value of another signal known as modulating signal.

→ The modulating signal contains information and are also called baseband signals.

→ The carrier freq. is greater than the modulating frequencies.

→ The signal which results from the process of modulation is known as Modulated signal.



Need for modⁿ or Need for freq. translation

1) **Practicability of Antennas**

If the channel is free space then antenna radiates & receives the signal. The antenna length depends on the wavelength of the signal. (Say $\frac{\lambda}{2}$).

For 1KHZ, message signal, antenna length will be.

$$c = f\lambda \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^3} = 3\times10^5 \text{ meter} = 300 \text{ km}.$$

Antenna length $= \frac{\lambda}{2} = 150$ km. (Practically impossible)

If freq translation is done;

Say $f = 300$ MHZ, $\lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1$ meter

antenna length $= \frac{\lambda}{2} = 0.5$ meter (Practically possible)

2) **Frequency multiplexing :-**

Suppose we want to transmit several different signals having same spectral range on a single communication channel and at the receiving end the signals can be separately recoverable and distinguishable from each other, then it is achieved by translating each one of the original signal to different freq. range. If the freq.

ranges don't overlap then the signals may be separated at the receiving end.

3) Narrow banding :-

Suppose the audio range extends from lowest freq to highest freq 50 to 10^4 Hz. The ratio of highest to lowest is 200 $\left(\because \frac{10,000}{50} = 200\right)$

Therefore an antenna used for this purpose ~~would be entirely too~~ for one end range would be entirely too short or too long for other end.

For $f = 50$, $\lambda = \frac{3\times10^8}{50} = \frac{300\times10^6}{50} = 6000$ km.

$f = 10^4$, $\lambda = \frac{3\times10^8}{10^4} = 3\times10^4 = 30$ km.

For $f = 50$, If use $\frac{\lambda}{2} = \frac{6000 \text{ km}}{2} = 3000$ km antenna, it is too large for freq $= 10^4$ Hz, because it requires only $\frac{30 \text{ km}}{2} = 15$ km antenna.

for $f = 10^4$ If we use 15 km antenna, it will be too short for $f = 50$, because it requires 3000 km antenna.

→ However, if audio spectrum were translated to that it occupied the range $10^6 + 50$ to $10^6 + 10^4$ Hz. Then highest to lowest freq $= 1.01$

$$\left(\because \frac{10^6 + 10^4}{10^6 + 50} = \frac{10^6 [1 + 10^{-2}]}{10^6 \left[1 + \frac{50}{10^6}\right]} \simeq \frac{\cancel{10^6} \times 1.02}{\cancel{10^6}} \right)$$

→ for $\underline{10^6 + 50}$ HZ will be

$$\lambda = \frac{3 \times 10^8}{10^6 + 50}$$

$$= \frac{3 \times \cancel{10^8}}{\cancel{10^6}\left[1 + \frac{50}{10^6}\right]}$$

$$\simeq 300 \text{ meter.}$$

→ for $\underline{10^6 + 10^4}$ HZ will be

$$\lambda = \frac{3 \times 10^8}{10^6 + 10^4}$$

$$= \frac{3 \times \cancel{10^8}\, 10^2}{\cancel{10^6}[1.02]}$$

$$\simeq 294 \text{ meter}$$

using freq translation

∴ a Same antenna can be used for both ranges $\underline{50}$ HZ and 10^4 HZ.

→ Thus, the process of frequency translation may be used to change a wide band signal into narrow band signal which may be conveniently processed.

4) Common processing:-

If we want to process a number of similar signals then it is necessary to adjust the freq range of our processing apparatus to corresponding to the freq range of the signal to be processed.

Then it is wiser to leave the processing apparatus to operate in some fixed frequency range and instead to translate the frequency range of each signal in turn to correspond to this fixed frequency.

Bandpass signal :-

If the modulated signal is transmitted over the channel it is known as bandpass / pass band signal.

e.g.: If $f_m = 1\,KHZ$ (modulating freq)

$f_c = 100\,KHZ$ (Carrier freq)

During AM, 2 sidebands will be generated.

Lower sideband will be at $(100-1)$ i.e $99\,KHZ$

and upper sideband will be at $(100+1)$ i.e $101\,KHZ$.

AM signal will have frequencies from $99\,KHZ$ to $101\,KHZ$. These frequencies are bandpass type.

A method of frequency translation:-

A signal may be translated to a new spectral range by multiplying the signal with an auxilliary sinusoidal signal.

Consider one sinusoidal signal,

$$V_m(t) = A_m \cos \omega_m t \quad \text{—— (1)}$$

$$V_m(t) = A_m \cos 2\pi f_m t$$

$$= \frac{A_m}{2} \left[e^{j2\pi f_m t} + e^{-j2\pi f_m t} \right] \quad \text{—(2)}$$

Here A_m = Const. Amplitude

$f_m = \frac{\omega_m}{2\pi}$ = frequency

Consider another sinusoidal signal,

$$V_c(t) = A_c \cos \omega_c t = A_c \cos 2\pi f_c t$$

$$V_c(t) = \frac{A_c}{2} \left[e^{j2\pi f_c t} + e^{-j2\pi f_c t} \right] \quad \text{—(3)}$$

A_c is Const. amplitude and f_c is the frequency.

Multiplying $V_m(t)$ and $V_c(t)$

$$V_m(t) \cdot V_c(t) = A_m \cos \omega_m t \cdot A_c \cos \omega_c t$$

$$= \frac{A_m A_c}{2} \, 2 \cos \omega_c t \cdot \cos \omega_m t$$

We know

$$2 \cos \alpha \cdot \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$V_m(t) \cdot V_c(t) = \frac{A_m A_c}{2} \left[\cos(\omega_c + \omega_m) t + \cos(\omega_c - \omega_m) t \right]$$

$$= \frac{A_m A_c}{2} \left[\frac{e^{j(\omega_c + \omega_m)t} + e^{-j(\omega_c + \omega_m)t}}{2} + \frac{e^{j(\omega_c - \omega_m)t} + e^{-j(\omega_c - \omega_m)t}}{2} \right]$$

$$= \frac{A_m A_c}{4} \left[e^{j(\omega_c + \omega_m)t} + e^{-j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t} + e^{-j(\omega_c - \omega_m)t} \right]$$

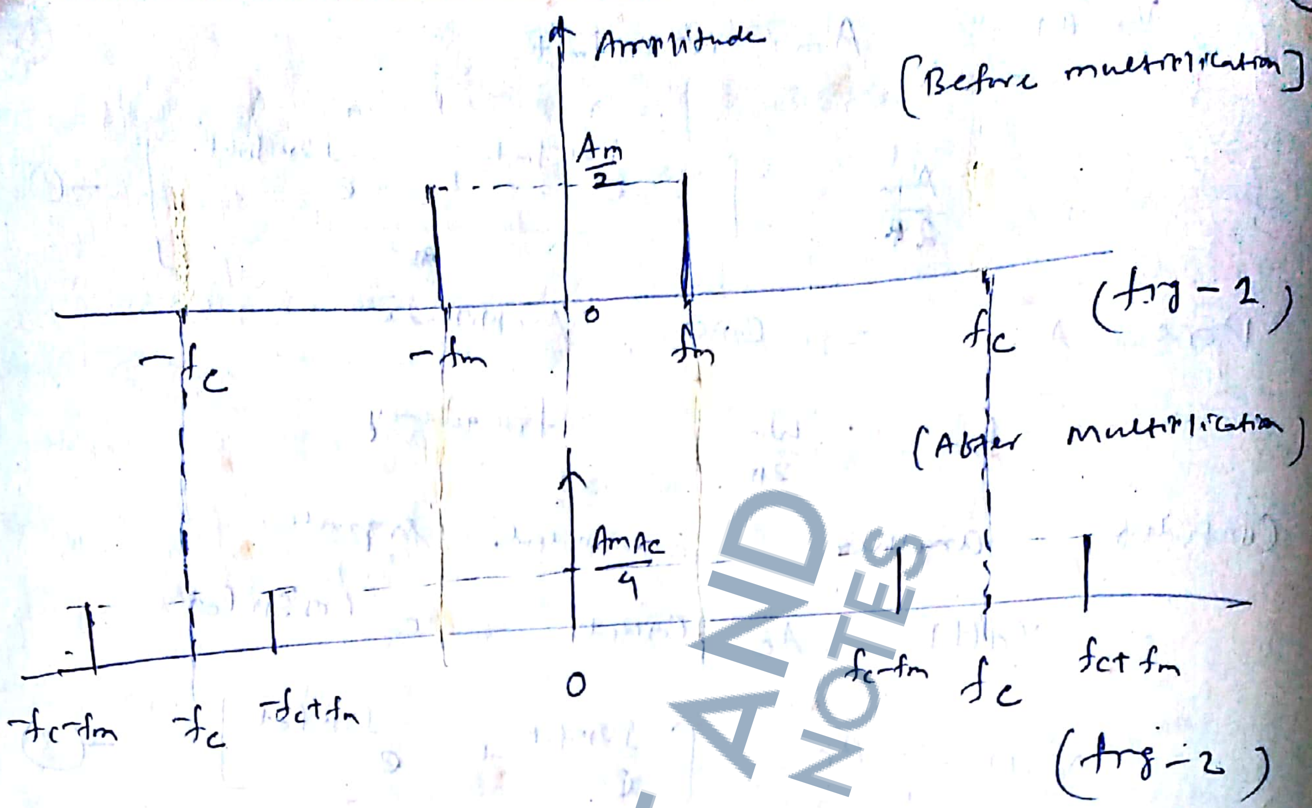


Fig (1), consists of 2 lines, each of amplitude $\frac{Am}{2}$, located at $f = f_m$ and $f = -f_m$.

The new spectral amplitude pattern as shown on figure 2. We see that the two original spectral lines have been translated both in the frequency direction by an amount f_c and on the -ve freq. direction by ~~an amount~~ $\cancel{-f_c}$. The same amount. That means there are now four spectral components f_c+f_m, f_c-f_m, $-f_c+f_m$, $-f_c-f_m$.

Note :- The product of 2 signals has 4 spectral components each of amplitude $\frac{AmAc}{4}$, there are only 2 frequencies i.e $[f_c+f_m$

and $f_c - f_m$] and amplitude of each sinusoidal component is $\frac{A_m A_c}{2}$.

Amplitude Modulation :- Double sideband with Carrier (DSB - C)

In AM, the amplitude of carrier signal is varied with instantaneous value of the modulating signal.

Lets consider ~~Am~~ a modulating signal

$$m(t) = A_m \cos \omega_m t$$

Let carrier signal be

$$c(t) = A_c \cos \omega_c t$$

Now AM will be,

$$s(t) = [A_c + x(t)] \cos \omega_c t$$

$$= [A_c + A_m \cos \omega_m t] \cos \omega_c t$$

$$= A_c \left[1 + \frac{A_m}{A_c} \cos \omega_m t\right] \cos \omega_c t$$

Lets define, $\frac{A_m}{A_c} = m =$ modulation index ~~as~~ for AM signal.

$$\Rightarrow s(t) = A_c [1 + m \cos \omega_m t] \cos \omega_c t$$

$$= A_c \cos \omega_c t + m A_c \cos \omega_c t \cdot \cos \omega_m t$$

$$s(t) = A_c \cos \omega_c t + \frac{m A_c}{2} \cdot 2 \cos \omega_c t \cdot \cos \omega_m t$$

$$\Rightarrow S(t) = A_c \cos w_c t + \frac{mA_c}{2}\left[\cos(w_c + w_m)t + \cos(w_c - w_m)t\right]$$

$$\therefore S(t) = A_c \cos w_c t + \frac{mA_c}{2}\cos(w_c + w_m)t + \frac{mA_c}{2}\cos(w_c - w_m)t$$

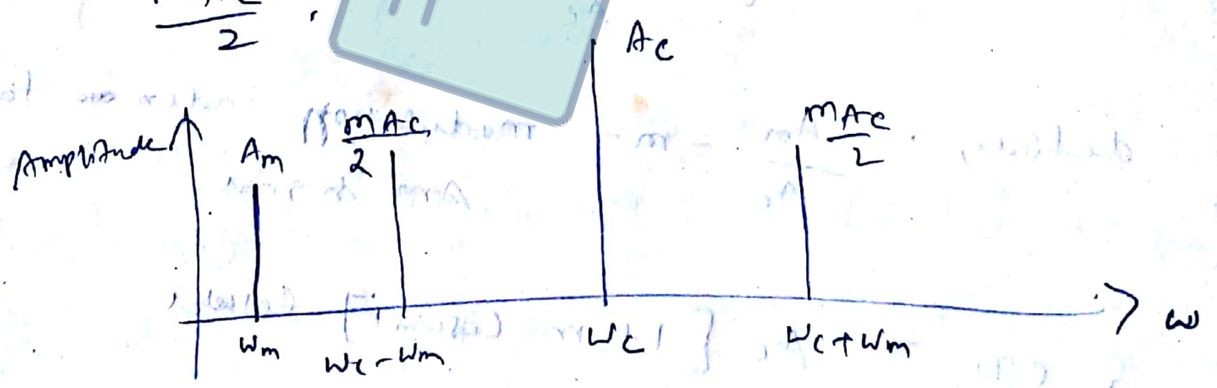
The process of generating such a waveform is called amplitude modulation and a communication system which employs such a method of freq. translation is called amplitude modulation system.

$\therefore$ There are 3 frequency components in Am signal.

(i) Carrier freq w_c having amplitude A_c.

(ii) upper side band freq $w_c + w_m$ having amplitude $\frac{mA_c}{2}$.

(iii) Lower side band freq $w_c - w_m$ having amplitude $\frac{mA_c}{2}$.



Note :- Power Content of a sinusoidal wave

Another approach on page 104

Lets consider a signal

$x(t) = A \cos\theta$

Power content of the signal is given by,

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \frac{1}{2\pi}\int_{0}^{2\pi} A^2\cos^2\theta \, d\theta$$

$$= \frac{1}{2\pi} \times A^2 \int_0^{2\pi}\left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{A^2}{4\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{A^2}{4\pi}\left[(2\pi + 0) - 0\right]$$

$$\boxed{P = \frac{A^2}{2}}$$

∴ Power content of a signal having amplitude A is $\frac{A^2}{2}$ i.e $\frac{(\text{Amplitude})^2}{2}$.

So Power Content in AM wave

= Power of Carrier + Power of upper side band + Power of lower side band

$$= \frac{(A_c)^2}{2} + \frac{\left(\frac{mA_c}{2}\right)^2}{2} + \frac{\left(\frac{mA_c}{2}\right)^2}{2}$$

$$\Rightarrow P_{total} = \frac{A_c^2}{2} + \frac{m^2 A_c^2}{8} + \frac{m^2 A_c^2}{8}$$

$$= \frac{A_c^2}{2} + \cancel{2} \cdot \frac{m^2 A_c^2}{\cancel{8}\,4}$$

$$= \frac{A_c^2}{2} + \frac{m^2 A_c^2}{4}$$

$$\Rightarrow \boxed{P_T = \frac{A_c^2}{2}\left(1 + \frac{m^2}{2}\right)}$$

or $\boxed{P_T = P_c\left(1 + \frac{m^2}{2}\right)}$ P_c = Power of Carrier signal $= \frac{A_c^2}{2}$.

Ex:-1) An AM Broadcast radio station radiates 10 kW of power. If modulation percentage is 60. Calculate how much is the carrier power.

Ans : $P_T = P_c\left(1 + \frac{m^2}{2}\right)$

$$\Rightarrow 10 \times 10^3 = P_c\left(1 + \frac{0.6^2}{2}\right)$$

$$= P_c\left(1 + \frac{0.36}{2}\right)$$

$$\Rightarrow 10 \times 10^3 = P_c\,(1 + 0.18)$$

$$\Rightarrow P_c = \frac{10 \times 10^3}{1.18} = 8.47\ \text{KW}$$

(Ans)

Transmission efficiency of AM signal :-

Transmission Efficiency $(\eta) = \frac{\text{Sideband Power}}{\text{Total power}} \times 100$.

Because out of total power P_t, the useful message or baseband power is carried by the sidebands (P_s). The large carrier power (P_c) is a waste from the transmission point of view because it does not carry any information or message.

$$\eta = \frac{P_s}{P_T} \times 100$$

$$P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

$$= P_c + \frac{P_c m^2}{2}$$

$$P_T = P_c + P_s$$

$$\eta = \frac{P_s}{P_T} = \frac{\cancel{P_c}\frac{m^2}{2}}{\cancel{P_c}\left(1+\frac{m^2}{2}\right)} = \frac{m^2}{\cancel{2}} \times \frac{\cancel{2}}{2+m^2} = \frac{m^2}{2+m^2}$$

$\eta = \frac{m^2}{2+m^2}$, for η_{max}, we have $m = 1$, $\left(m = \frac{A_m}{A_c}\right)$

$$\Rightarrow \eta_{max} = \frac{1}{2+1} = \frac{1}{3} = 33.33\%$$

This implies that only $\frac{1}{3}$ of the total power is carried by side ~~power~~ bands and rest $\frac{2}{3}$ is wasted.

Current Calculation of AM.

$$P_t = P_c\left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow I_t^2 \cdot \cancel{R} = I_c^2 \cancel{R}\left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow I_T^2 = I_c^2\left(1+\frac{m^2}{2}\right)$$

$$\Rightarrow \boxed{I_T = I_c\sqrt{1+\frac{m^2}{2}}}$$

Note :-

1) The baseband or modulating signal will be preserved in the envelope of AM signal only if

$$A_m \le A_c$$

$$\Rightarrow \frac{A_m}{A_c} \le 1$$

$$\Rightarrow \boxed{m \le 1}$$

4) If one carrier is modulated by 2 modulating signal, with modulation index m_1 and m_2. Then total modulation index,

$$m_T = \sqrt{m_1^2 + m_2^2}$$

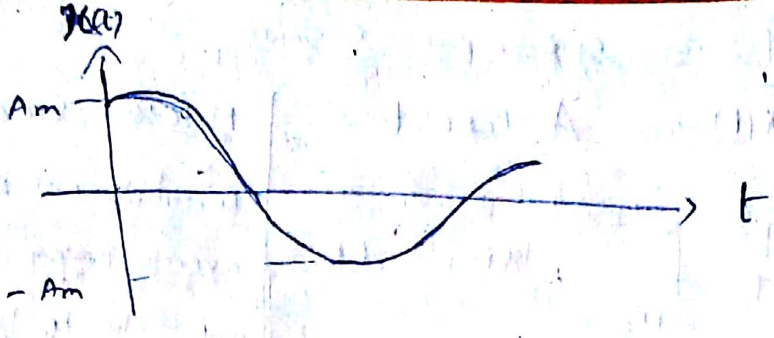
and $P_T = P_c\left(1+\frac{m_T^2}{2}\right)$

2) If $m > 1$, % of modulation is greater than 100 %. In this case the baseband signal recovered from the envelope will be distorted. This type of ~~modulation~~ is distortion is called envelope distortion and AM signal is called over modulated signal.

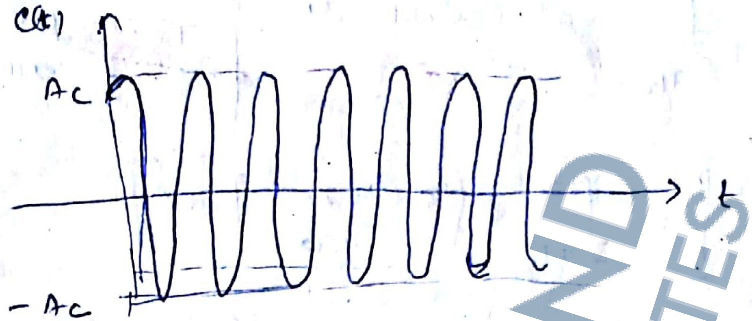
3) H.W Prove that Modulation index $(m) = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

A_{max} = Envelope max $= A_c + A_m$

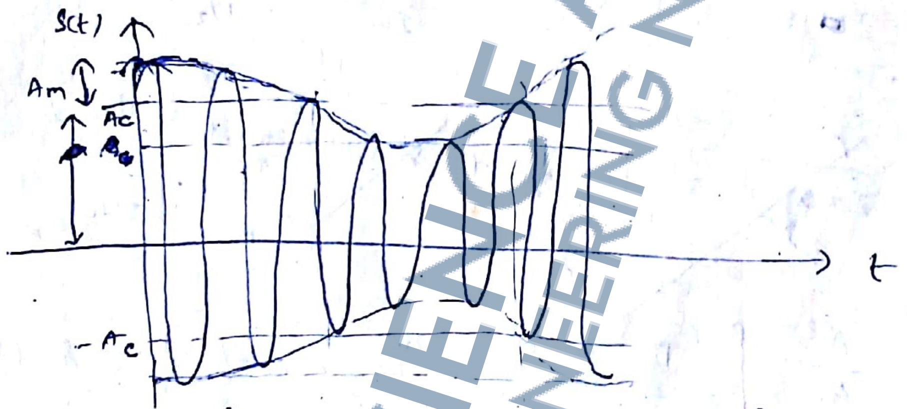
A_{min} = Envelope min ~~[illegible]~~ $= A_c - A_m$



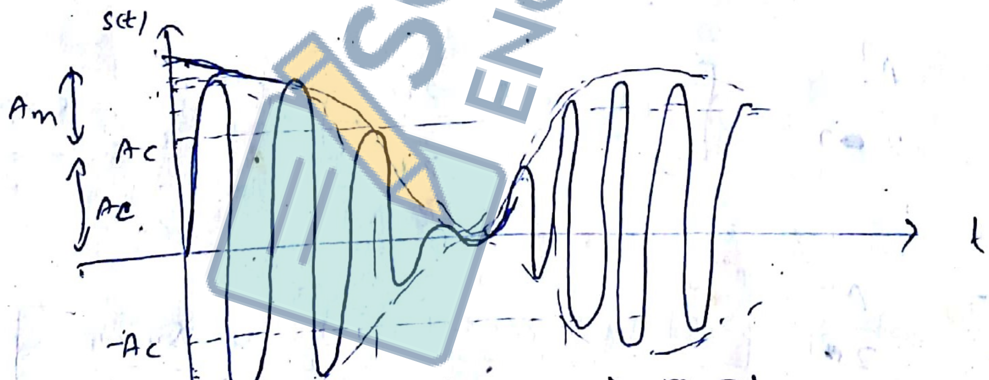
(a) Modulating signal.



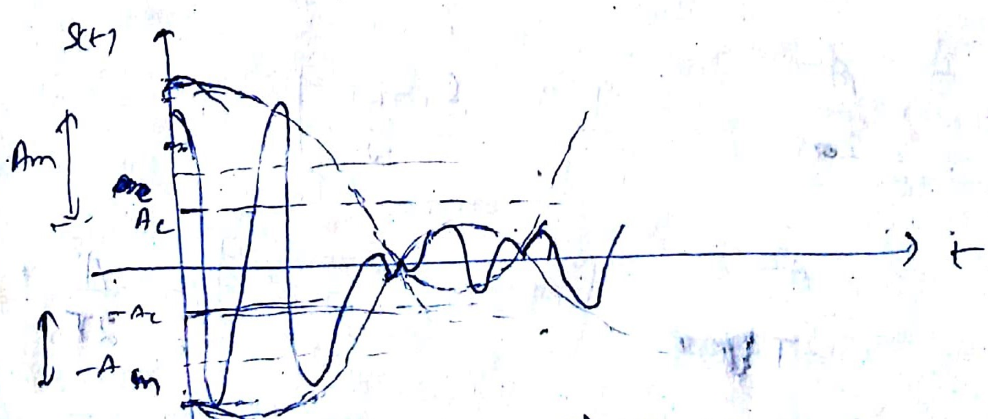
(b) Carrier signal



(c) for $A_m < A_c \Rightarrow \frac{A_m}{A_c} < 1 \Rightarrow \boxed{m < 1}$



(d) $A_m = A_c \Rightarrow m = 1$



(e) $A_m > A_c \Rightarrow m > 1$ (over modulated)

(*) Power of a sinusoidal signal.

$$x(t) = A\cos\omega t$$

Note:- Since, $x(t)$ is periodic, i.e repetitive, average power over $-\infty$ to ∞ and $-\frac{T_0}{2}$ to $\frac{T_0}{2}$ will give same result, where T_0 is the time period of $x(t)$.

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A^2 \cdot \cos^2\omega t \, dt$$

$$= \frac{A^2}{T}\left[\int_{-T/2}^{T/2}\left(\frac{1+\cos 2\omega t}{2}\right) dt\right]$$

$$= \frac{A^2}{2T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_{-T/2}^{T/2}$$

$$= \cancel{\frac{A^2}{2\times \frac{2\pi}{T}}\left[\frac{t}{2} + \frac{\sin 2\times\frac{2\pi}{T}\cdot t}{2\times\frac{2\pi}{T}}\right]_{-T/2}^{+T/2}}$$

$$= \cancel{\frac{A^2}{4\pi}\left[\left(\frac{T}{2} + \frac{\sin 2\times\frac{2\pi}{T}\cdot\frac{T}{2}}{2\times\frac{2\pi}{T}}\right) - \left(-\frac{T}{2} + \frac{\sin\left(2\times\frac{2\pi}{T}\cdot\left(-\frac{T}{2}\right)\right)}{2\times\frac{2\pi}{T}}\right)\right]}$$

$$= \frac{A^2}{2T}\left[\left(\frac{T}{2} + \frac{\sin \cancel{2}\omega\cdot\frac{T}{\cancel{2}}}{2\omega}\right) - \left(-\frac{T}{2} - \frac{\sin\cancel{2}\omega\frac{T}{\cancel{2}}}{2\omega}\right)\right]$$

$$= \frac{A^2}{2T}\left[\frac{T}{2} + \frac{\sin\omega T}{2\omega} + \frac{T}{2} + \frac{\sin\omega T}{2\omega}\right]$$

$$= \frac{A^2}{2T}\left[T + \frac{\sin\omega T}{\omega}\right]$$

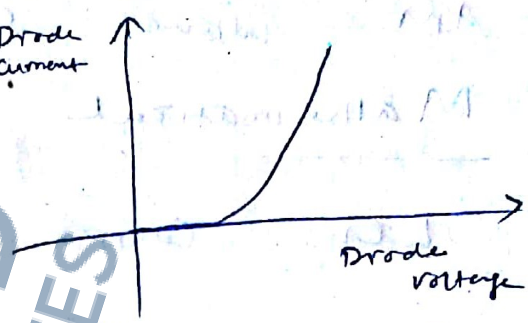
$$= \frac{A^2}{2T}\left[T + \frac{\sin\frac{2\pi}{\cancel{T}}\cdot\cancel{T}}{\omega}\right] = \frac{A^2}{2\cancel{T}}\left[\cancel{T} + 0\right]$$

$$= \frac{A^2}{2}$$

Generation of AM (Double sideband with carrier) (D.S.B-C)

Square law diode modulator

Square law diode modulation ckt make use of non linear current - voltage characteristics of diode



This method is suited at low voltage levels because of the fact that current - voltage characteristics of a diode is highly non linear particularly in the low voltage region as shown in above figure.

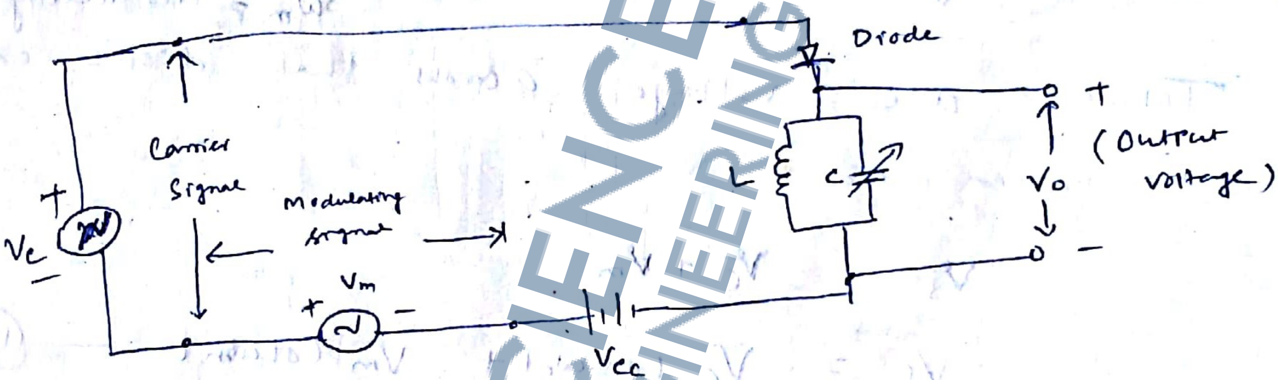


fig:- Square law diode modulation.

In the above figure, carrier and modulating signals are applied across the diode. A d.c battery Vcc is connected across the diode to get a fixed operating point on the V-i characteristics of diode.

When carrier and modulating frequencies are applied at the i/p of diode, then different frequency terms appear at the output of the diode. These different frequency terms are applied across a tuned ckt which is tuned to the carrier freq and has a narrow

bandwidth just to pass 2 side bands along (118)
with carrier and reject other frequencies.
Hence at the o/p of the tuned ckt, carrier and 2 side bands are obtained i.e AM wave is produced.

Mathematical Analysis:-

Let carrier voltage is expressed as,

$$v_c = V_c \cos \omega_c t \quad , \omega_c = \text{Carrier freq.}$$

Modulating voltage, $v_m = V_m \cos \omega_m t$

ω_m = modulating freq.

Total a.c voltage across the diode is given as

$$v_s = v_c + v_m$$

$$v_s = V_c \cos \omega_c t + V_m \cos \omega_m t \quad \text{--- (1)}$$

The non-linear relationship between voltage and current for diode is expressed as

$$i = a + b v_s + c v_s^2 \quad \text{--- (2)}$$

where a, b, c are constants

i = current through the diode

v_s = voltage across the diode

Putting the value of v_s from eqn (1), in eqn (2), we have

$$i = a + b\left(V_c \cos \omega_c t + V_m \cos \omega_m t\right) + c\left(V_c \cos \omega_c t + V_m \cos \omega_m t\right)^2$$

$$\Rightarrow i = a + bV_c \cos\omega_c t + bV_m \cos\omega_m t + c\left[V_c^2 \cos^2\omega_c t + V_m^2 \cos^2\omega_m t + 2.V_c.V_m.\cos\omega_c t.\cos\omega_m t\right].$$

$$\Rightarrow i = a + bV_c \cos\omega_c t + bV_m \cos\omega_m t + \frac{cV_c^2}{2}.2\cos^2\omega_c t + \frac{cV_m^2}{2}.2\cos^2\omega_m t + c\left[V_c V_m\right].\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$\Rightarrow i = a + bV_c \cos\omega_c t + bV_m \cos\omega_m t + \frac{cV_c^2}{2}\left[1+\cos 2\omega_c t\right] + \frac{cV_m^2}{2}\left[1+\cos 2\omega_m t\right] + cV_c V_m\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$\Rightarrow i = \underbrace{\left[a + \frac{cV_c^2}{2} + \frac{cV_m^2}{2}\right]}_{(1)} + \underbrace{bV_c \cos\omega_c t}_{(2)} + \underbrace{bV_m \cos\omega_m t}_{(3)} + \underbrace{\left(\frac{1}{2}cV_c^2 \cos 2\omega_c t + \frac{1}{2}cV_m^2 \cos 2\omega_m t\right)}_{(4)} + \underbrace{cV_cV_m \cos(\omega_c+\omega_m)t}_{(5)} + \underbrace{cV_cV_m \cos(\omega_c-\omega_m)t}_{(6)} \quad \text{--- (3)}$$

The eqn (3), consist of 6 terms.

term (1) is the d.c term

term (2) is carrier signal

term (3) is modulating signal

term (4) consists of harmonics of carrier and modulating signals

term (5) represents upper sideband

term (6) represents lower sideband.

In this diode modulation ckt, the load impedance is a tuned circuit which is turned to the carrier frequency w_c. Therefore, this tuned ckt responds to a narrowband of frequencies centered about the carrier frequency w_c. Thus the frequency components which are actually developed in the output are terms of frequency w_c, $w_c + w_m$, $w_c - w_m$. The rest of frequency components are rejected by the tuned ckt.

Therefore, the required expression of o/p current will be

$$i_0 = bV_c \cos w_c t + CV_cV_m \left[\cos(w_c + w_m)t + \cos(w_c - w_m)t\right]$$

$$= bV_c \cos w_c t + \frac{CV_cV_m}{2}\left[2 \cos w_c t \cdot \cos w_m t\right]$$

$$\left(\because \cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha \cdot \cos\beta\right)$$

$$= bV_c \cos w_c t \left[1 + \frac{2C}{b} V_m \cos w_m t\right].$$

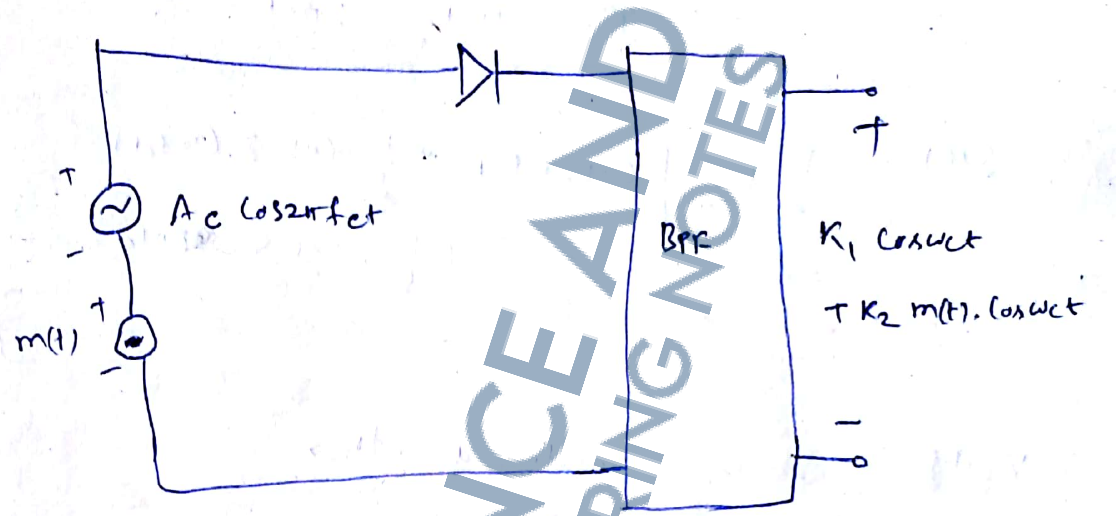
$$i_0 = bV_c \left[1 + m_a \cos w_m t\right] \cos w_c t \quad \text{——(4)}$$

where $m_a = \frac{2CV_m}{b}$ is the modulation index.

Eqn (4) is the required expression for AM current.

Switching Modulator: -

Another method for generating AM modulated signal is by means of a switching modulator. Such a modulator can be implemented by the system in the figure given below.

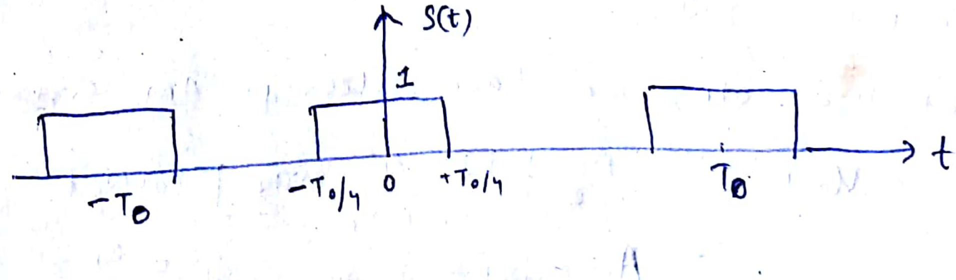


The sum of the message signal & the carrier, i.e $V_i(t) = m(t) + A_c \cos 2\pi f_c t$, are applied to the diode.

The switching operation of the diode may be viewed mathematically as a multiplication of input $V_i(t)$ with switching function $S(t)$ i.e

$$V_o(t) = [m(t) + A_c \cos 2\pi f_c t] \cdot S(t)$$

where $S(t)$ is given below



Hence

$$V_o(t) = [m(t) + A_c \cos 2\pi f_c t] \cdot S(t)$$

Since $S(t)$ is a periodic function it is represented in the Fourier Series, as

$$S(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$$

$$S(t) = \frac{1}{2} + \frac{2}{\pi}\Big[\cos 2\pi f_c t - \frac{1}{3}\cos 3.(2\pi f_c t) + \frac{1}{5}\cos 5.(2\pi f_c t) - \cdots\Big]$$

$V_o(t)$ can be reduced to,

$$[m(t) + A_c \cos 2\pi f_c t]\Big\{\frac{1}{2} + \frac{2}{\pi}\Big[\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\Big]\Big\}$$

$$= \frac{m(t)}{2} + \frac{A_c \cos \omega_c t}{2} + \frac{2}{\pi} m(t)\cos \omega_c t + \frac{2A_c}{\pi}\cos^2 \omega_c t - \cdots$$

The desired AM Modulated Signal is obtained by passing $V_o(t)$ through a band pass filter with center freq ~~$f = f_c$~~ $\omega = \omega_c$ and B.W $2\omega_m$.

At the O/P, we have desired AM Signal

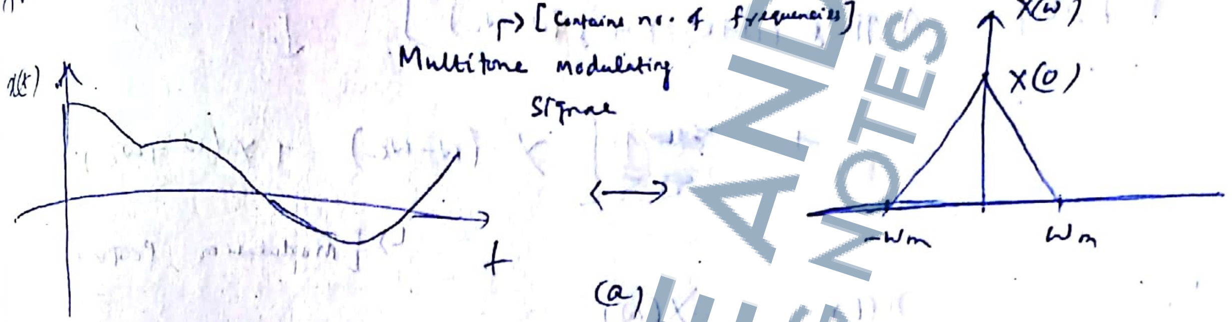
$$v_o(t) = \frac{A_c}{2}\Big[1 + \frac{4}{\pi A_c} m(t)\Big]\cos \omega_c t$$

$$= \frac{A_c}{2}\cos \omega_c t + \frac{2}{\pi} m(t)\cos \omega_c t = K_1 \cos \omega_c t + K_2\, m(t)\cos \omega_c t$$

$v_o(t)$ = Carrier + Side bands.

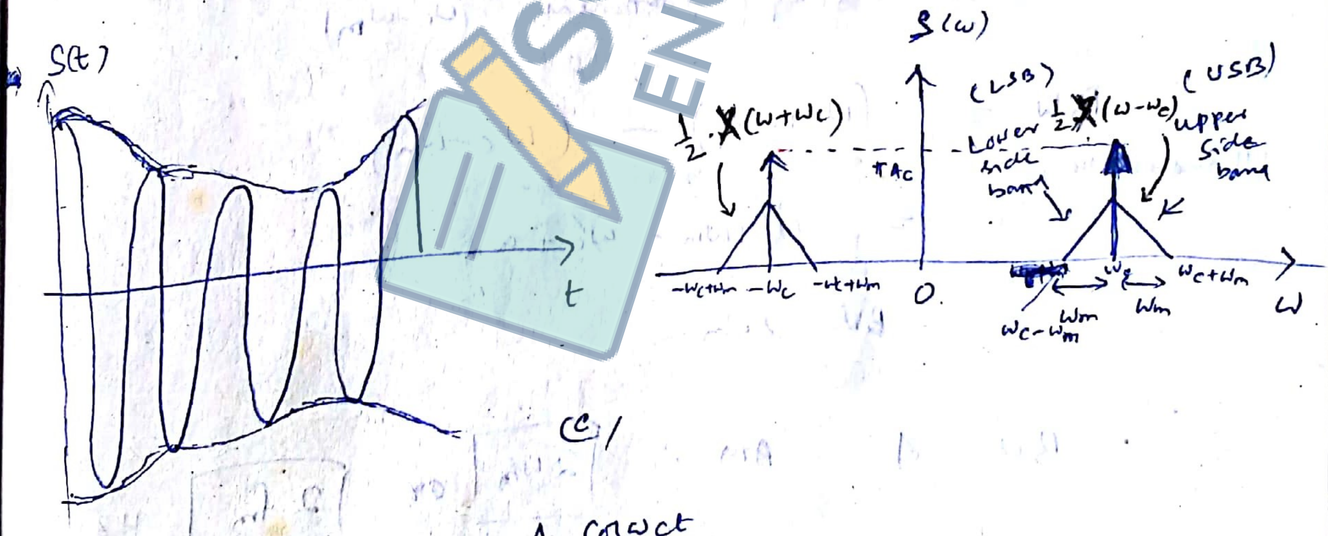
Freq - domain Representation or Spectrum of AM

If we want to know the frequency description frequencies present in AM wave, we will have to find its spectrum or frequency domain representation. For this purpose, we have to take the Fourier transform of AM wave.



(a)

(b)



(c)

Since $c(t) = A_c \cos\omega_c t$

$$F[c(t)] = A_c . \pi \ [\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$

Impulse fn at $\omega=\omega_c$ & $\omega=-\omega_c$.

So 2 impulse in fig (b).

In AM wave

$$s(t) = [A_c + x(t)] \cos\omega_c t$$

$$= A_c \cos\omega_c t + x(t) \cdot \cos\omega_c t$$

$$= A_c \cos\omega_c t + x(t) \cdot \left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right]$$

$$F[s(t)] = A_c \pi [\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$

$$+ \frac{1}{2}[X(\omega-\omega_c) + X(\omega+\omega_c)]$$

↳ [Modulation property]

(∵ $x(t) = X(\omega)$

$e^{j\omega_c t} x(t) = X(\omega - \omega_c)$ freq shifting property)

→ For +ve side, highest freq component present in the spectrum of AM is $(\omega_c + \omega_m)$ and lowest freq component $(\omega_c - \omega_m)$

$$BW = (\omega_c + \omega_m) - (\omega_c - \omega_m)$$

(Bandwidth)

$$= \cancel{\omega_c} + \omega_m - \cancel{\omega_c} + \omega_m$$

$$BW = 2\omega_m$$

∴ BW of AM = $\boxed{2\omega_m}$ or $\boxed{2f_m}$ Hz

↳ rad/sec

Note :- In AM wave, carrier ~~amplitude~~ strength remains same i.e πA_c but the base band signal strength is halved. i.e $\frac{1}{2}[X(\omega-\omega_c) + X(\omega+\omega_c)]$

Demodulation of AM waves :- (DSB-C)

The process of extracting a modulating or baseband signal from the modulated signal is called demodulation or detection. The devices used for demodulation or detection are called demodulators or detectors.

For AM, detectors are catagorized as

(a) Square-law detectors
(b) Envelope detectors.

→ AM signal with large carriers are detected by using envelope detector. The envelope detector uses the ckt which extract the envelope of AM wave and the envelope of AM wave is the baseband signal.

→ But a low-level amplitude modulated signal can only be detected by using square-law detectors in which a device operating in non-linear region is used to detect the modulating signal.

(a) Square-law Detector :-

The square law detector ckt is used for detecting modulating signal of small magnitude (i.e below 1 volt) so that the operating region may be restricted to the portion of the V-I characteristics of the non-linear device. The fig 1: shows the ckt. of a square-law detector. It may be observed that the ckt. is very similar to the square-law modulator. The only difference lies in the filter ckt. In a

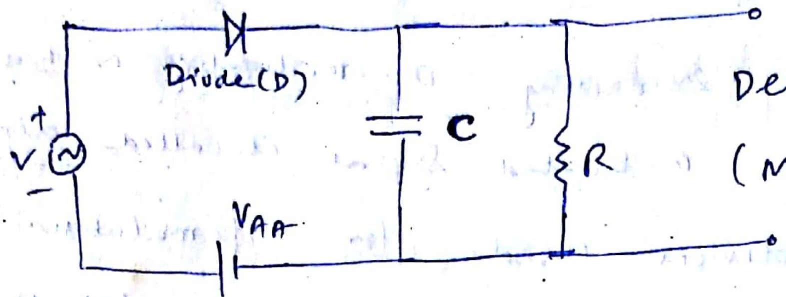


fig 1:- Basic ckt of square law diode detector

Square law modulator, the filter used is a bandpass filter where as in a square law detector, a low-pass filter is used.

In the ckt, the dc supply voltage V_{AA} is used to get the fixed operating point in the non-linear portion of the diode V-I characteristics. The O/P diode current is expressed by non-linear V-i relationship (i.e square law) as

$$i = av + bv^2 \quad \text{——} ①$$

Here, v is the i/p modulated voltage.

AM wave can be expressed as,

$$v = A_c(1+m\cos\omega_m t)\cos\omega_c t \quad \text{——} ②$$

Substituting eqn ② in eqn ①, we have,

$$i = a\left[A_c(1+m\cos\omega_m t)\cos\omega_c t\right] + b\left[A_c(1+m\cos\omega_m t)\cos\omega_c t\right]^2$$

Now, if the above expression is expanded, then we may observe the presence of terms of frequencies like $2\omega_c$, $2(\omega_c \pm \omega_m)$, ω_m and $2\omega_m$

besides the input frequency terms.

Hence, this diode current 'i' containing all these frequency terms is passed through a low pass filter which allows to pass the frequencies below or upto the modulating frequency 'ω_m' and rejects the other higher frequency components. Therefore, the modulating or baseband signal with frequency 'ω_m' is recovered from the i/p modulated signal.

$$(i = av + bv^2)$$

Note:- $i = a[A_c(1+m\cos\omega_m t)\cos\omega_c t]$

$$+ b\left[A_c^2\cos^2\omega_c t\,(1+m^2\cos^2\omega_m t + 2m\cos\omega_m t)\right]$$

$$\Rightarrow i = a[A_c(1+m\cos\omega_m t)\cos\omega_c t]$$

$$+ b\left[A_c^2\cos^2\omega_c t + A_c^2 m^2\cos^2\omega_c t\cdot\cos^2\omega_m t + 2m\cos\omega_m t\cdot A_c^2\cos^2\omega_c t\right]$$

$$= a\left[A_c\cos\omega_c t + mA_c\cdot\cos\omega_c t\cdot\cos\omega_m t\right]$$

$$+ b\left[\frac{A_c^2}{2}\,2\cos^2\omega_c t + \frac{m^2A_c^2}{4}\,2\cos^2\omega_c t\cdot 2\cos^2\omega_m t + m A_c^2\cos\omega_m t\,(2\cos^2\omega_c t)\right]$$

$$= aA_c\cos\omega_c t + \frac{m\cdot a\cdot A_c}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$+ \frac{bA_c^2}{2}(1+\cos 2\omega_c t) + \frac{b\,m^2A_c^2}{4}(1+\cos 2\omega_c t)(1+\cos 2\omega_m t)$$

$$+ b\,m\,A_c^2\cos\omega_m t\,(1+\cos 2\omega_c t)$$

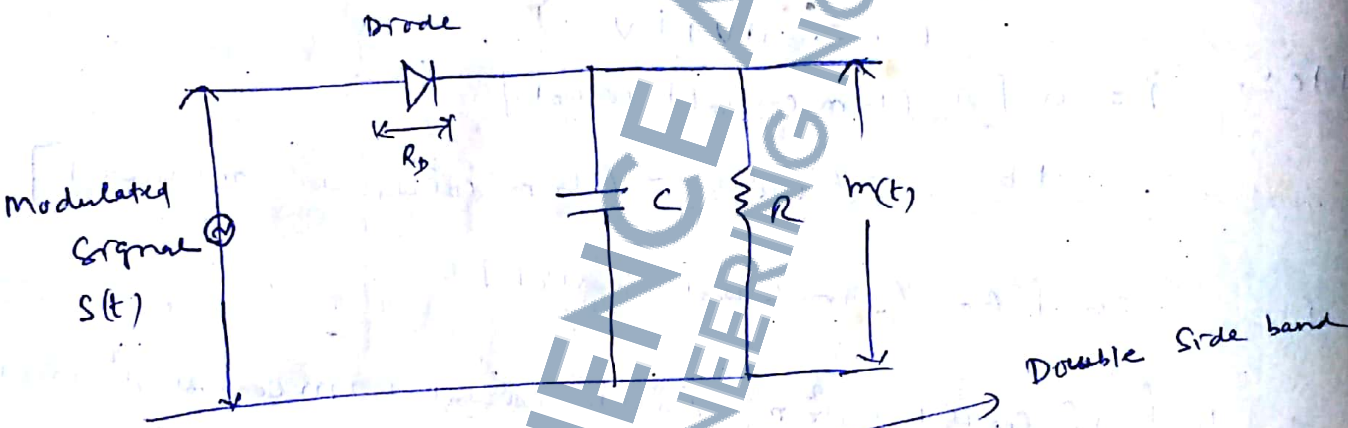
Through LPF (Low pass filter) only freq $\leq \omega_m$ is filtered i.e. $b\,m\,A_c^2\cos\omega_m t = b\cdot\left(\frac{A_m}{A_c}\right)\cdot A_c^2\cos\omega_m t$

$\therefore$ o/p is $= b A_m A_c \cos\omega_m t$ which contains modulating or baseband signal frequency $\underline{\underline{\omega_m}}$.

$$\boxed{o/p = K \cos\omega_m t} \quad (K = \text{Const.})$$

Envelope detector :- (Linear diode)

The ckt diagram for an envelope detector is shown below :



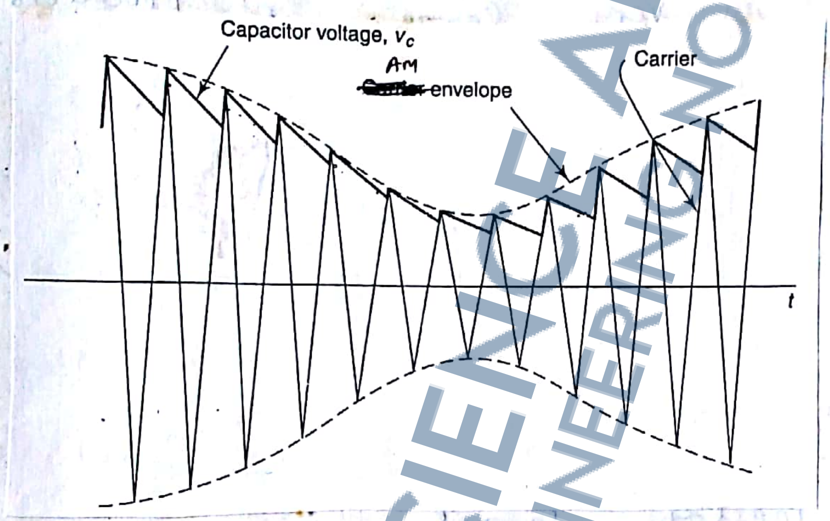
→ The conventional DSB AM signals are easily demodulated by means of an envelope detector. It consist of a diode and a RC ckt, which is basically a simple low pass filter.

→ During the +ve half cycle of the i/p signal, the diode is conducting and the capacitor charges up to the peak value of the i/p signal.

→ When the i/p falls below the voltage on the capacitor, the diode becomes reverse biased and the i/p becomes disconnected from the o/p.

→ During this period, the Capacitor discharge slowly through the load resistor R. On the next cycle of the Carrier, the diode conducts again when the i/p signal exceeds the voltage across the Capacitor.

→ The Capacitor Charges up again to the peak value of the i/p signal and the process is repeated again.



→ The time constant (RC) must be selected so as to follow the variation in the envelope of the Carrier-modulated signal.

The Charging time constant i.e [$R_D \cdot C$] where R_D is the internal resistance of diode, should be very small, so that the Capacitor Charges instantaneously to the peak value of the signal.

→ ~~But~~ The Capacitor discharges through the load resistor 'R'. So the discharging time constant RC should be more to ensure that capacitor discharge slowly through load resistance R between peak to peak point of carrier. So the condition is

$$\boxed{R_D \cdot C << \frac{1}{f_c} << RC << \frac{1}{f_m}} \quad \text{—— (1)}$$

If 'RC' is very large, it produces another problem known as <u>diagonal clipping</u>. So we can n't increase RC beyond certain limit. $\left(RC << \frac{1}{f_m}\right)$. Thus, if cond^n (1) is satisfied, the capacitor discharges slowly through resistor (R) and o/p of envelope detector closely follow the message signal.

→ Here the diode operates in <u>linear region</u> of V-I characteristics.

→ Envelope detector is most popular in commercial receiver ckt since it is very simple and is not expensive, also at the same time, it gives satisfactory performance for the reception of broadcasting programmes.

<u>Note:-</u> <u>Ex- 3.3 · Taub/Schilling</u> :- Find the max^m value of time const. RC of detector that can always follow the message envelope?

Ans : $\boxed{RC \le \frac{1}{\omega_m}\left(\frac{\sqrt{1-m^2}}{m}\right)}$ · [or condition to avoid diagonal clipping]

Double-Sideband – Supressed Carrier (DSB-SC) System:-

The general form of a single tone modulation is

$$s(t) = A_c \cos\omega_c t + \frac{mA_c}{2}\cos(\omega_c + \omega_m)t + \frac{mA_c}{2}\cos(\omega_c - \omega_m)t$$

$\uparrow$

We see, the carrier component in AM wave remains constant in amplitude and frequency.

The total power content of AM wave (P_T), $P_T = P_c\left(1 + \frac{m^2}{2}\right)$

P_c = carrier power

$$\frac{P_T}{P_c} = 1 + \frac{m^2}{2} = \frac{2 + m^2}{2}$$

$$\Rightarrow \frac{P_c}{P_T} = \frac{2}{2 + m^2}$$

for $m = 1$, $\frac{P_c}{P_T} = \frac{2}{2+1} = \frac{2}{3} = 67\%$.

$\therefore$ For 100% modulation ($m = 1$), about 67% of total power is required for transmitting the carrier which doesn't contain any information. Hence the carrier can be suppressed, only side band can remain. This type of suppression of carrier doesn't affect the base band signal. The resulting signal obtained by suppressing the carrier from the modulated wave is called Double Side Band Suppressed Carrier Signal or DSBSC signal.

Power Content of DSB-SC Signal:-

$$P_T = P_c\left(1+\frac{m^2}{2}\right)$$

$$= P_c + \frac{P_c m^2}{2}$$

$$= P_{carrier} + P_{Double\ sideband\ -SC}$$

$$P_{Double-sideband,SC} = P_c \frac{m^2}{2}$$

$$= \frac{A_c^2}{2}\cdot\frac{m^2}{2} \qquad \left(\because P_c = \frac{A_c^2}{2}\right)$$

$$\boxed{P_{DSBSC} = \frac{m^2 A_c^2}{4}}$$

Expression for DSBSC

The eqn of DSB AM,

$$s(t) = [A_c + \cancel{m}x(t)]\cos w_c t$$

$$= A_c \cos w_c t + \cancel{m}x(t)\cdot\cos w_c t$$

$$= \text{Carrier} + \text{~~Double~~ DSB-SC}$$

$$\therefore \boxed{\text{Eq}^n \text{ of DSB-SC} = \cancel{m}x(t)\cdot\cos w_c t.}$$

$$x(t) \leftrightarrow X(w)$$

$$s(t) = \cos w_c t \cdot x(t) = \left(\frac{e^{jw_c t} + e^{-jw_c t}}{2}\right)x(t)$$

$$\boxed{S(w) = \frac{1}{2}\left[X(w-w_c) + X(w+w_c)\right]}$$

$$\left(\because \begin{array}{l} x(t) \leftrightarrow X(w) \\ e^{jw_c t}x(t) \leftrightarrow X(w-w_c) \quad \text{freq. shifting property} \end{array}\right)$$

Notes:-

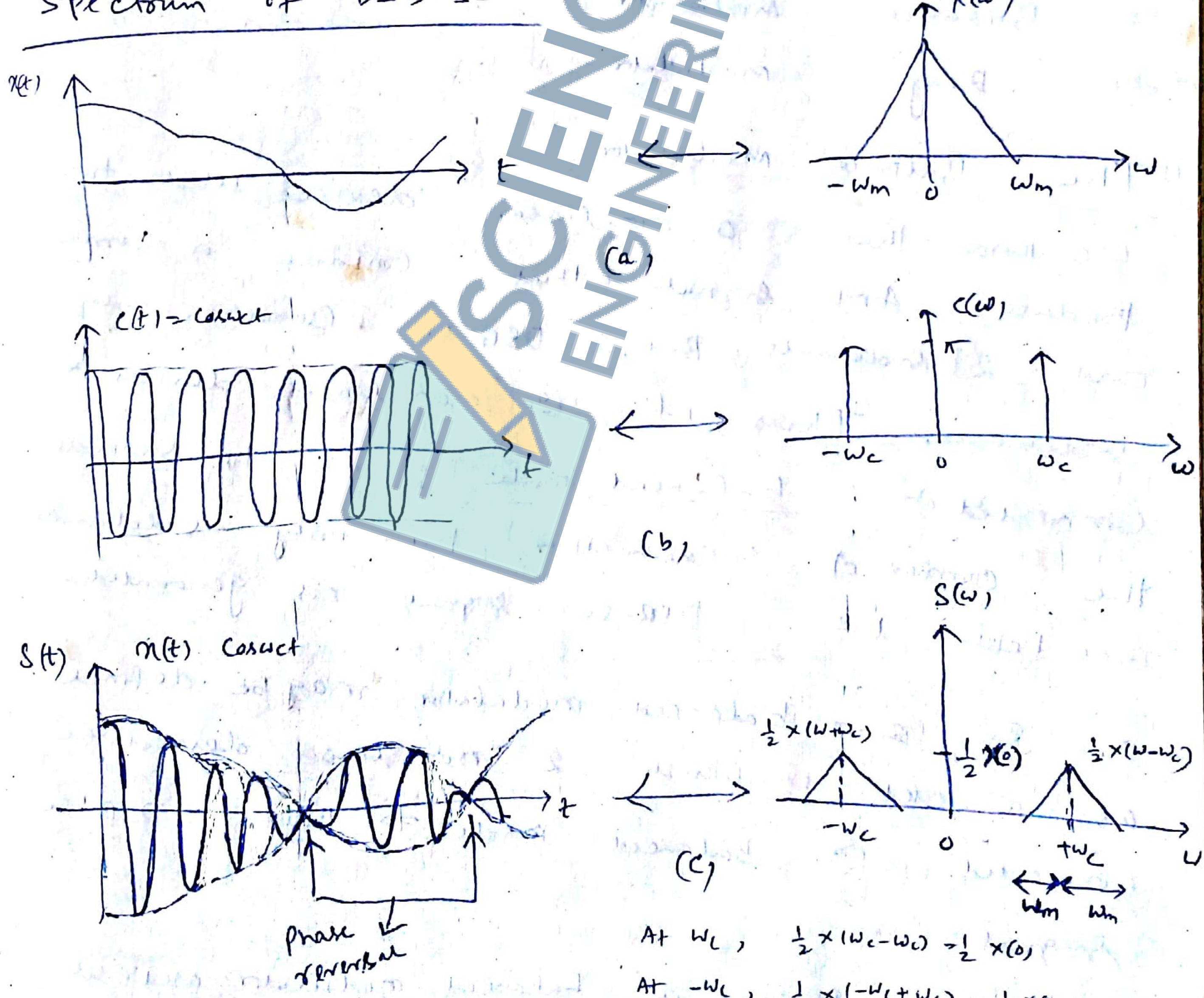
1) DSB-SC signal exhibits phase-reversal at zero crossings i.e whenever the baseband $x(t)$ crosses zero.

2) Impulse at $\pm W_c$ are missing which means that carrier term is suppressed in the spectrum and only 2 sideband terms, USB and LSB are left. (fig - c) - given below

3) Bandwidth = $(W_c + W_m) - (W_c - W_m) = 2W_m$ or $2f_m$

= same as that of AM wave.

Spectrum of DSB-SC



At W_c, $\frac{1}{2}X(W_c - W_c) \rightarrow \frac{1}{2}X(0)$

At $-W_c$, $\frac{1}{2}X(-W_c + W_c) \rightarrow \frac{1}{2}X(0)$

Generation of DSB-SC Signal :-

The expression for DSB-SC signal is given as

$$s(t) = x(t) \cdot \cos w_c t$$

where $x(t)$ = Baseband signal.

$\cos w_c t$ = Carrier signal.

$\therefore$ DSB-SC signal is basically the product of modulating or baseband signal with the Carrier signal.

A ckt to achieve the generation of DSB-SC signal is called a product modulator.

2 type of Porduct modulator

1) Balance modulator
2) Ring modulator.

1) The Balance Modulator:-

We know that a non-linear device used to produce AM signal that contains a carrier and 2 sidebands. But, DSB-SC contains only 2 sidebands. Thus, if 2 non-linear devices are connected in balanced mode so as to suppress the carrier of each other, then only sidebands are left i.e DSB-SC signal is generated.

→ So, a balanced modulator may be defined as a ckt in which 2 non-linear devices are connected in balanced mode to produce DSB-SC signal.

→ Fig (1) shows a balanced modulator ckt using

2 diodes. A modulating signal $x(t)$ is applied to the 2 diodes through a center-tapped transformer with the carrier signal $\cos w_c t$.

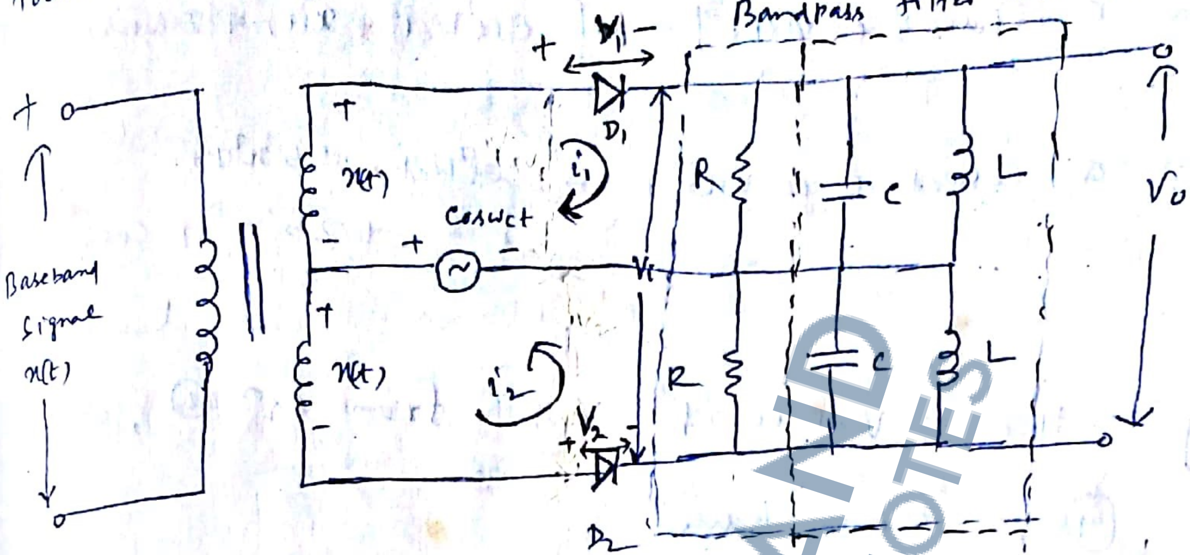


fig 1:- Balanced modulator using diodes.

A non-linear V-i relationship is given as,

$$i = av + bv^2$$

From the above figure, we write the two i/p voltage V_1 & V_2 to ~~across~~ the 2 diodes as,

$$V_1 = \cos w_c t + x(t) \quad \text{——— (1)}$$

$$V_2 = \cos w_c t - x(t) \quad \text{——— (2)}$$

For diode D_1, the non-linear V-i relationship becomes

$$i_1 = aV_1 + bV_1^2 \quad \text{——— (3)}$$

Similarly, for diode D_2,

$$i_2 = aV_2 + bV_2^2 \quad \text{——— (4)}$$

Putting the value of V_1, from eqn (1) in eqn (3), we have

$$i_1 = a[\cos w_c t + x(t)] + b[\cos w_c t + x(t)]^2$$

$$\Rightarrow i_1 = a[\cos w_c t + x(t)] + b[\cos^2 w_c t + x^2(t) + 2\cos w_c t \cdot x(t)]$$

$$\Rightarrow i_1 = a\cos w_c t + a\,x(t) + b\cos^2 w_c t + b\,x^2(t) + 2b\,x(t)\cdot\cos w_c t \quad \text{--- (5)}$$

Putting the value of v_2, from eqn (2), in eqn (4), we have

$$i_2 = a[\cos w_c t - x(t)] + b[\cos w_c t - x(t)]^2$$

$$\Rightarrow i_2 = a[\cos w_c t - x(t)] + b[\cos^2 w_c t + x^2(t) - 2\,x(t)\cdot\cos w_c t]$$

$$= a\cos w_c t - a\,x(t) + b\cos^2 w_c t + b\,x^2(t) - 2b\,x(t)\cdot\cos w_c t \quad \text{--- (6)}$$

Due to current i_1 and i_2, the net voltage v_i at the input of bandpass filter is expressed as,

$$v_i = i_1 R - i_2 R$$

$$= R(i_1 - i_2)$$

$$= R[2a\,x(t) + 4b\,x(t)\cos w_c t] \quad \text{--- using eq}^n \text{ (5) \& (6)}$$

$$v_i = 2R[a\,x(t) + 2b\,x(t)\cos w_c t]$$

This voltage v_i is the i/p to the Bandpass filter centered around $\pm w_c$.

So the BPF will pass a narrowband frequencies centered at $\pm\omega_c$ with a small Bandwidth of $2\omega_m$ to preserve the sidebands.

Therefore, the o/p of BPF centered around $\pm\omega_c$, is given by

$$V_0 = 2R \cdot 2b\, x(t) \cos\omega_c t$$

$$= 4Rb\, x(t) \cos\omega_c t$$

$$\boxed{V_0 = K\, x(t) \cdot \cos\omega_c t}$$

which is the expression for a DSB-SC signal.

2) Ring Modulator :-

Ring Modulator is another product modulator, which is used to generate DSB-SC signal. Fig (2), shows the ckt diagram of ring modulator. In ring modulator ckt, 4 diodes are connected in the form of ring in which all 4 diodes Point in the same manner. All the four diodes in ring are controlled by a square wave carrier signal c(t) of frequency f_c applied through a center-tapped transformer.

In case, when diodes are ideal and transformers are perfectly balanced, the 2 outer diodes are switched on if the carrier signal is +ve, where as the 2 inner diodes are switched off and thus presenting very high impedance as shown in fig 3 (a). Under this condition, the

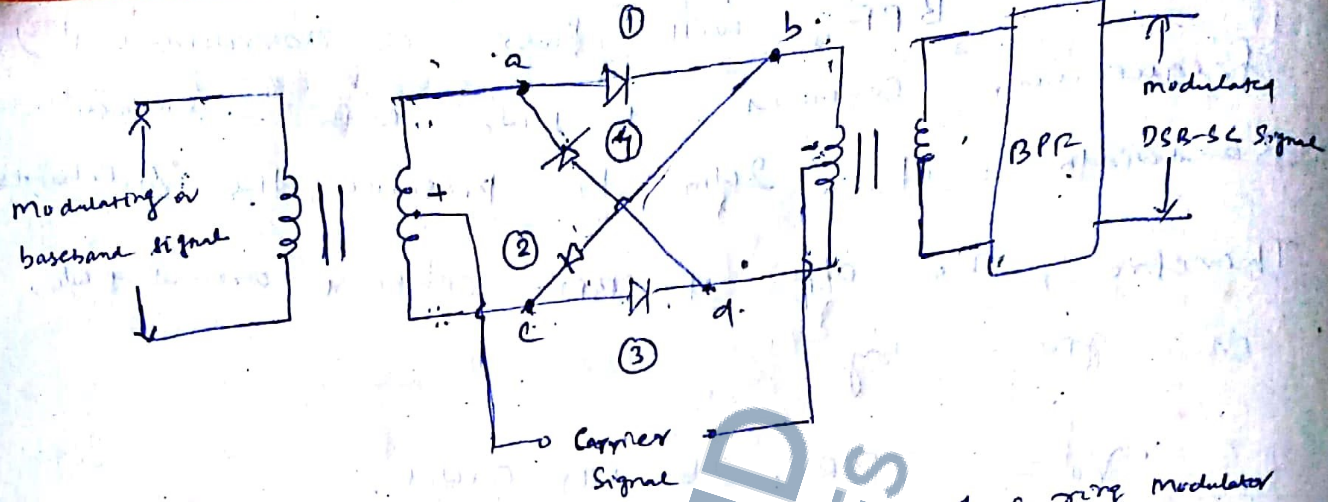


Fig 2:- Ckt diagram of a ring modulator

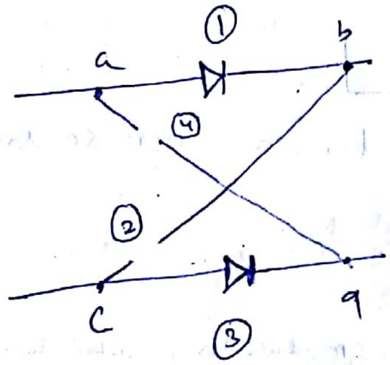


fig 3:(a)

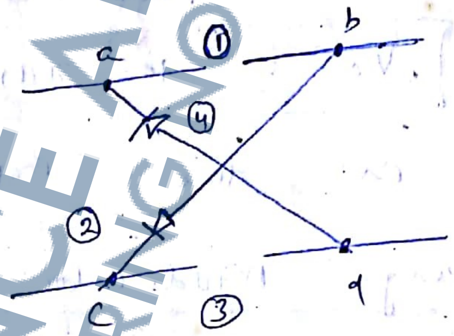


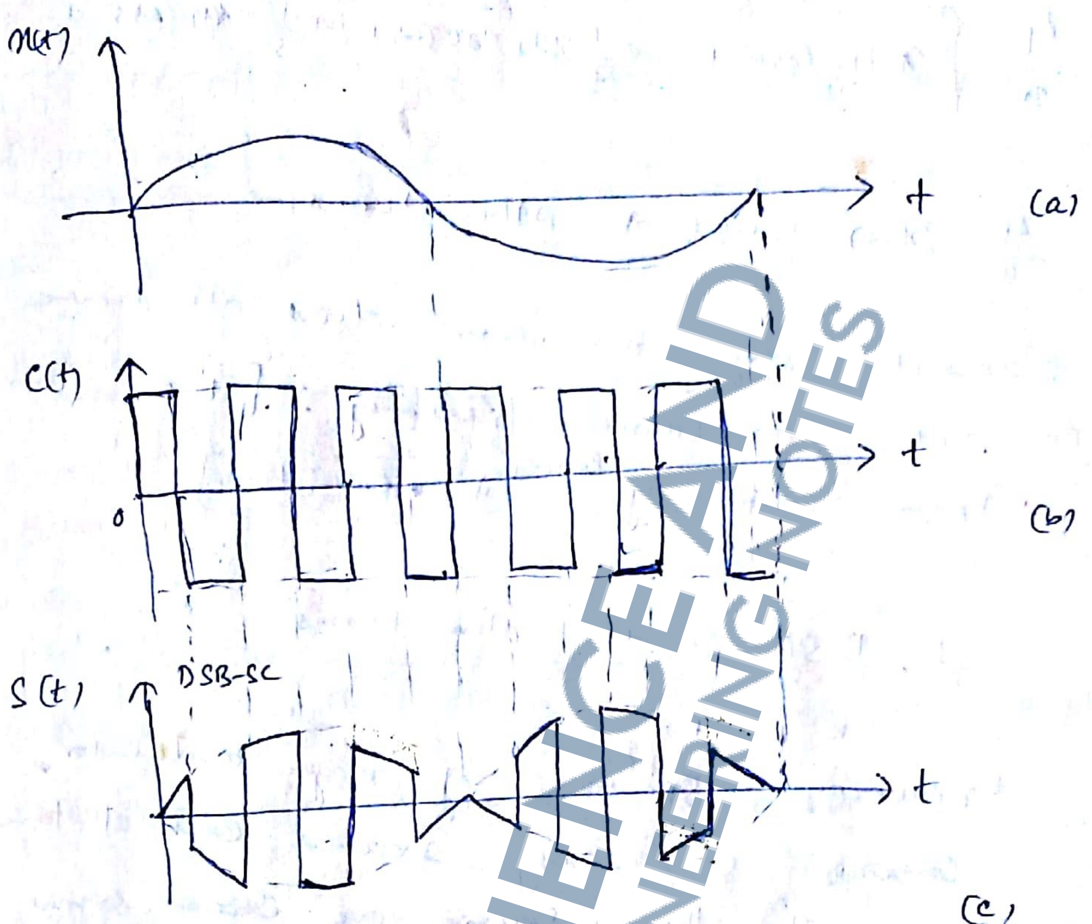
fig 3:(b)

(a) Illustration of condition when diodes ~~(1)&(3)~~ (1) & (3) are switched ON and diodes (2) & (4) are switched off.

(b) Illustration of condition when diodes (2) & (4) are switched ON and diodes (1) & (3) are switched, off.

modulator multiplies the modulating signal x(t) by +1. Now in case, when carrier signal is -ve, the situation becomes reversed as shown in fig 3(b). In this case; the modulator multiplies the modulating signal by -1.

Hence, the ring modulator is a product modulator for a square wave carrier and modulating signal.



(a) Sinusoidal modulating wave

(b) Square-wave carrier

(c) Modulated wave (DSB-SC) output of Ring modulator.

The square-wave carrier may be represented in Fourier series as,

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} \left\{ \cos\left[2\pi f_c t\,(2n-1)\right] \right\}$$

$$c(t) = \frac{4}{\pi} \left[\cos\omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t \; - - - - \right]$$

$s(t) = x(t) \cdot c(t)$, substituting the value of $c(t)$

$$s(t) = x(t) \cdot \frac{4}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t \ \cdots\right]$$

$$= \frac{4}{\pi}\left[x(t)\cdot\cos\omega_c t - \frac{1}{3}x(t)\cos 3\omega_c t + \frac{1}{5}\cdot x(t)\cos 5\omega_c t \ \cdots\right]$$

$$s(t) = \frac{4}{\pi}\, x(t)\cos\omega_c t + \text{other terms.}$$

So output of ring modulator does n't have any component at carrier frequency. [i.e there is no term containing purely $A_c \cdot \cos\omega_c t$]

$$s(t) = K \cdot DSB\text{-}SC + \text{other terms.}$$

K = const.

The frequency spectrum of ring modulator output contains sidebands around each of the odd harmonics of the square wave carrier signal as shown below.

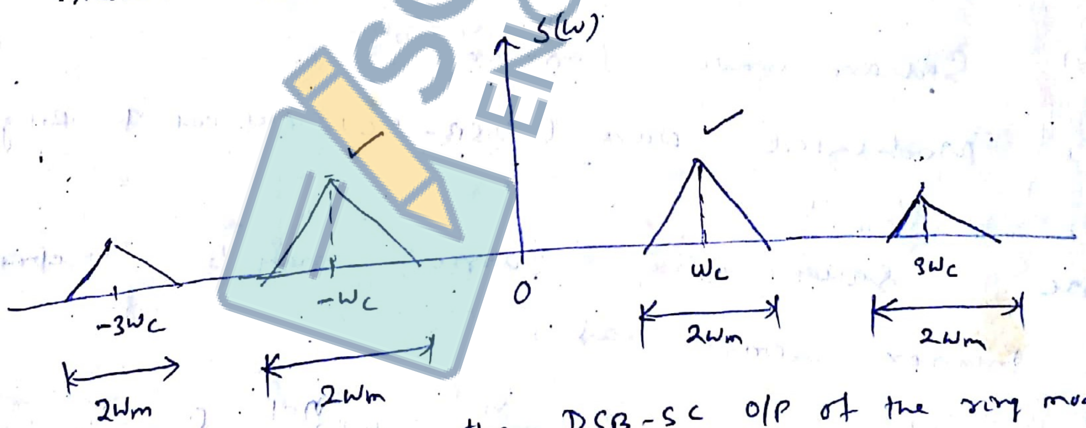


fig. - Spectrum of the DSB-SC o/p of the ring modulator.

We assumed modulating signal band limited to $-\omega_m < \omega < \omega_m$.

The desired sideband around the carrier ω_c may be selected by using a band pass filter (BPF) having center frequency $\boxed{\omega_c}$ & BW $\boxed{2\omega_m}$.

Note:- To avoid overlaping of sidebands we must have $\boxed{\omega_c > \omega_m}$

Demodulation of DSB-SC Signal

(i) Synchronous detection method:-

The demodulation of DSB-SC AM signal requires a synchronous demodulator. Fig (1) shows the block diagram of synchronous detection method.

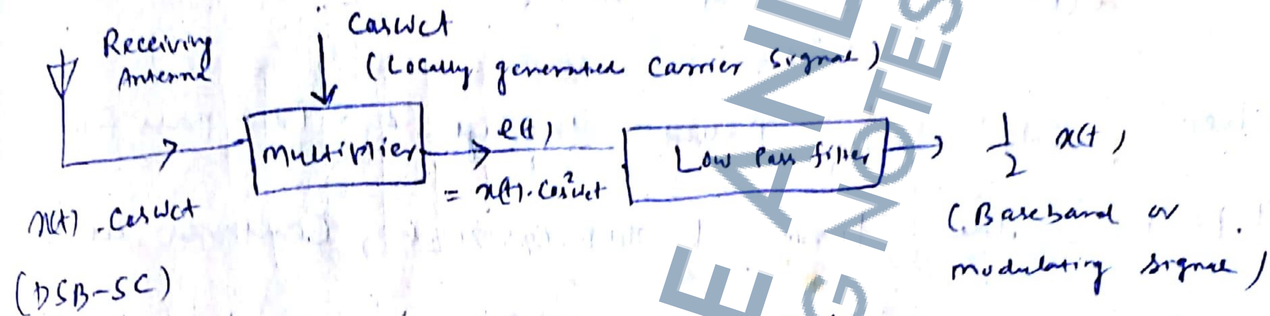


Fig 1:- Synchronous detection method

→ Here the frequency & phase of ~~[illegible]~~ carrier signal used in generation of DSB-SC ($\cos\omega_c t$) must be identical to the frequency & phase of locally generated carrier signal ($\cos\omega_c t$). That is why this method is called Synchronous detection method.

Working principle:-

In synchronous detection method, the received signal or DSB-SC signal is first multiplied with locally generated carrier signal $\cos\omega_c t$. Here the multiplier may be a product modulator (e.g. Balance modulator).

Mathematically:-

$$e(t) = \underbrace{x(t)\cos\omega_c t}_{\text{DSB-SC}} \cdot \underbrace{\cos\omega_c t}_{\text{Locally generated carrier signal}}$$

$$\Rightarrow e(t) = x(t) \times \cos^2 \omega_c t$$

$$= x(t) \times \left[\frac{1 + \cos 2\omega_c t}{2}\right]$$

$$= x(t) \times \left[\frac{1}{2} + \frac{\cos 2\omega_c t}{2}\right]$$

$$e(t) = \frac{x(t)}{2} + \frac{1}{2} x(t) \cdot \cos 2\omega_c t$$

When $e(t)$ is passed through a Low pass filter (LPF), the term $\frac{1}{2} x(t) \cdot \cos 2\omega_c t$, centered at $\pm 2\omega_c$ is suppressed by LPF and thus the o/p of LPF, ~~the~~ original modulating signal $\frac{1}{2} x(t)$ is obtained.

Limitation :-

1) If the local oscillator signal is not exactly synchronized or coherent with carrier signal at the transmitter, both in frequency & phase, the detected signal will be get distorted.

2) So, this synchronous detection requires an additional system at the receiver to ensure that the locally generated carrier is synchronized with the transmitter carrier, making the receiver complex and costly.

Effect of Phase & frequency error in Synchronous detection

The frequency & Phase of local oscillator signal in coherent detection method at the receiver end, must be identical to the ~~transmitted~~ carrier signal used at the transmitter. Any kind of discrepancy in freq or phase produces a distortion in detected o/p at the receiver end.

Lets assume that modulated signal reaching at the receiver is denoted by $x(t).\cos w_ct$. Considering a locally generated signal with frequency & phase error equal to Δw and ϕ respectively. So o/p of product modulator is,

$$e_d(t) = [x(t)\cos w_ct]\cdot[\cos[(w_c+\Delta w)t+\phi]]$$

$$= \frac{1}{2}x(t)\cdot\{\cos[\Delta wt+\phi]+\cos[2w_ct+\Delta wt+\phi]\}$$

$$[\because 2\cos\alpha\cdot\cos\beta = \cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

After passing through LPF, having cut-off freq w_m,

$$e_o(t) = \frac{1}{2}x(t).\cos[(\Delta w)t+\phi]$$

$\therefore$ The base band signal is multiplied by a slow-time varying function, $\cos[(\Delta w)t+\phi]$ which distorts the message signal $x(t)$.

Case-I :-

(i) When freq error (Δw) & Phase error (ϕ) are both

Zero

$$e_0(t) = \frac{1}{2} x(t)$$

This means that there is no distortion in detected o/p signal

Case-II :- When there is only phase error

i.e $\phi \neq 0$, $\Delta\omega = 0$

$$e_0(t) = \frac{1}{2} x(t) \cdot \cos\phi$$

This shows the o/p signal is multiplied by $\cos\phi$. When ϕ is time-independent, there is no distortion, rather there is attenuation. The o/p will be maximum when $\phi = 0$ [$\cos\phi = 1$], and minimum when $\phi = 90°$. But, in general ϕ randomly varies w.r.to time due to random variation of propagation media (i.e ionosphere). This causes undesirable distortion in the detected o/p.

It may be noted that the detected o/p is zero when $\phi = 90°$. This is known as quadrature null effect since the signal is zero when the local oscillator carrier is in-phase quadrature with carrier used at the transmitted side.

Case-III :- When there is only freq error,

i.e. $\Delta\omega \neq 0$, $\phi = 0$

$\therefore \quad e_0(t) = \frac{1}{2} x(t) \cos(\Delta w) t$

Here, the multiplying factor $\cos(\Delta w) t$ is a time-dependent & produces distortion on the o/p signal.

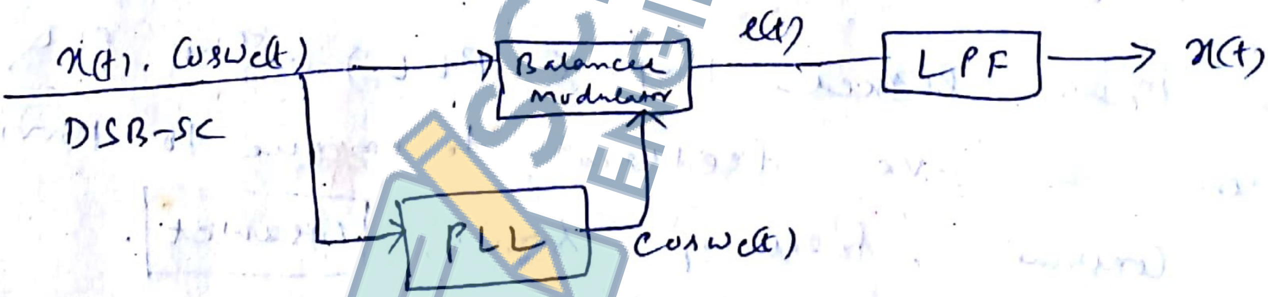
Case-IV :- When Both errors are non zero

$$\Delta w \neq 0, \quad \phi \neq 0$$

$$e_0(t) = \frac{1}{2} x(t) \cos(\Delta w t + \phi).$$

So constant phase error provides attenuation & frequency error produces distortion on the detected o/p signal.

So to ensure coherent detection, a PLL (Phase Locked Loop) is used to generate the local carrier signal i.e synchronous with the carrier signal used at the transmitter.



(b) Squaring Loop method of detection :-

In this method, the received signal is squared by a squaring ckt. The o/p of squarer will be given as $[x(t). \cos w_c t]^2$

If $x(t) = A \cos w_m t$, then o/p of squarer becomes,

$$[A \cos\omega_m t \cdot \cos\omega_c t]^2 = A^2 \cos^2\omega_m t \cdot \cos^2\omega_c t$$

$$= \frac{A^2}{4} \cdot 2\cos^2\omega_m t \cdot 2\cos^2\omega_c t$$

$$= \frac{A^2}{4} \cdot [1 + \cos 2\omega_m t][1 + \cos 2\omega_c t]$$

$$= \frac{A^2}{4} [1 + \cos 2\omega_c t + \cos 2\omega_m t + \cos 2\omega_c t \cdot \cos 2\omega_m t]$$

The term $\underline{\cos 2\omega_c t}$ can be obtained by using a narrow band filter centered at $\pm 2\omega_c$. This frequency $\pm 2\omega_c$ is kept constant by tracking through

fig 2:- A squaring ckt for synchronization

a phase locked loop (PLL). The PLL uses a -ve feedback technique to provide a constant frequency signal, $\boxed{\cos 2\omega_c t}$.

The PLL o/p is divided by 2, to yield a synchronized local carrier of frequency $\boxed{\omega_c}$. This local carrier signal is used in synchronous detector. The frequency division can be accomplished by using a bistable multivibrator. Now this $\underline{\cos\omega_c t}$, will be ~~used for~~ multiplied with DSB-SC and passed through LPF to get baseband signal.

SSB-SC (Single Side Band - Suppressed Carrier) AM

A DSB-SC AM signal requires a channel $BW = 2W_m$ for transmission, where W_m is the bandwidth of baseband signal. However, the two sidebands are redundant. So, the transmission of either sideband is sufficient to reconstruct the message signal $x(t)$ at the receiver. Thus, we reduce the BW required for transmission from $\boxed{2W_m \text{ to } W_m}$

The general form of AM signal is,

$$s(t) = A_c \cos w_c t + \frac{mA_c}{2} \cos(w_c + w_m)t + \frac{mA_c}{2} \cos(w_c - w_m)t.$$

$$\text{Power of SSB-SC} = \frac{\left(\frac{mA_c}{2}\right)^2}{2} = \frac{m^2 A_c^2}{8}$$

$$\% \text{ of power saving} = \frac{\text{power saved}}{\text{Total Power}}$$

Carrier

Lower sideband / upper sideband

$$= \frac{\frac{A_c^2}{2} + \frac{\left(\frac{mA_c}{2}\right)^2}{2}}{P_c\left(1 + \frac{m^2}{2}\right)}$$

$$= \frac{\frac{A_c^2}{2} + \frac{m^2 A_c^2}{8}}{\frac{A_c^2}{2}\left(1 + \frac{m^2}{2}\right)} = \frac{\cancel{\frac{A_c^2}{2}}\left(1 + \frac{m^2}{4}\right)}{\cancel{\frac{A_c^2}{2}}\left(1 + \frac{m^2}{2}\right)}$$

$$= \frac{4 + m^2}{\cancel{4}} \times \frac{\cancel{2}}{2 + m^2} = \frac{4 + m^2}{4 + 2m^2}$$

For $m = 1$, $\%$ of power saving $= \frac{5}{6} \times 100 = 83.33\%$.

∴ Through SSB-SC transmission 83.33 % power is saved.

Spectrum of SSB-SC modulation

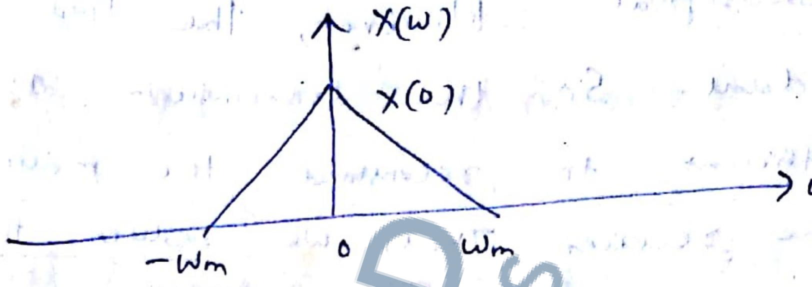


fig (a)

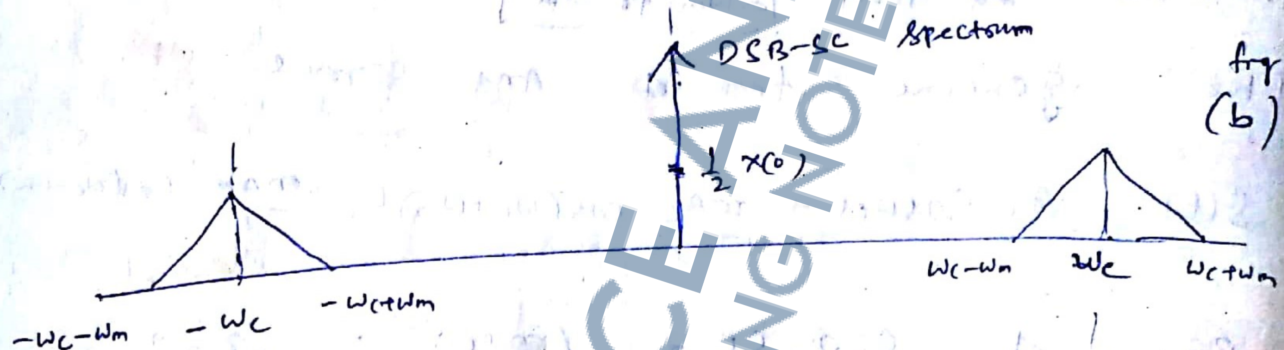


fig (b)

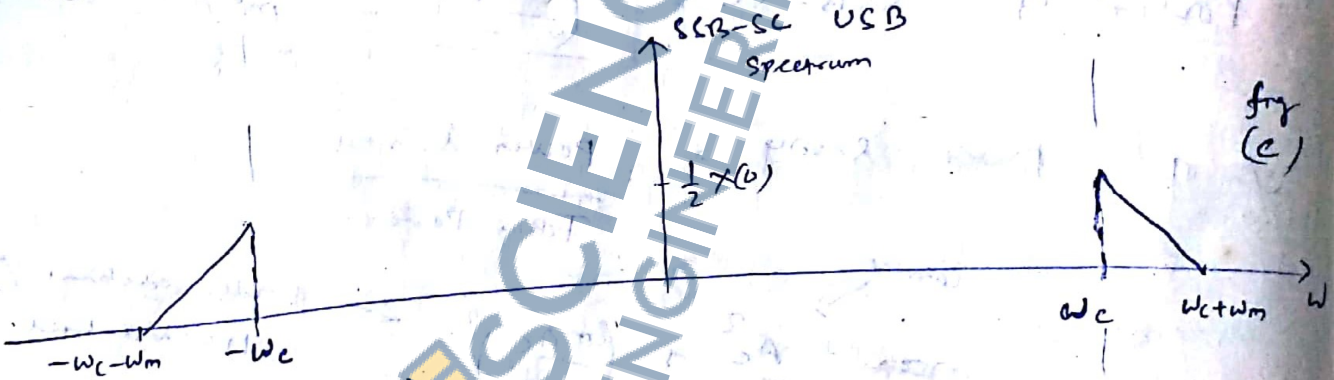


fig (c)

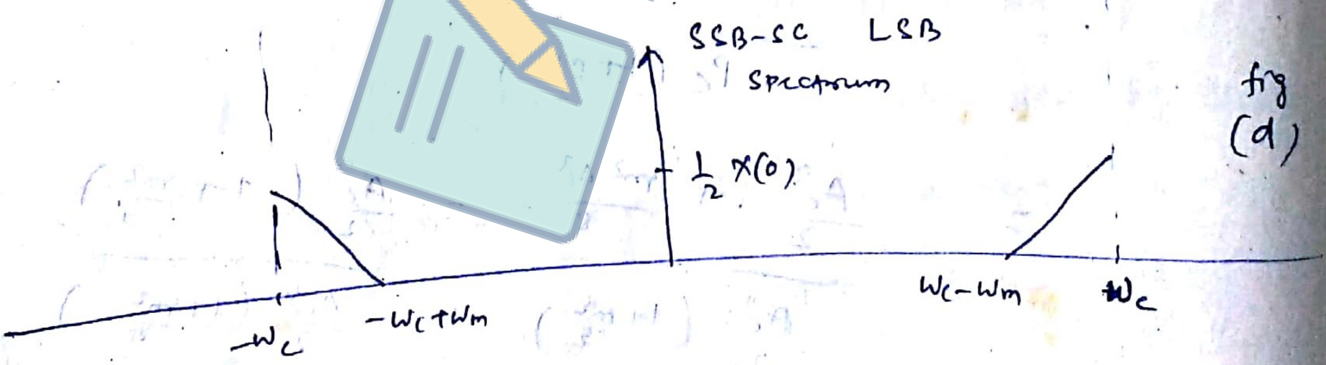


fig (d)

(a) Spectrum of baseband signal

(b) Spectrum of DSB-SC wave

(c) Spectrum of SSB-SC wave with upper sideband transmitted.

(d) Spectrum of SSB-SC wave with lower sideband transmitted.

Time-domain Description of SSB-SC wave

Single tone modulating signal: - (only one freq 'W_m')

Let

$x(t) = A_m \cos W_m t$

$c(t) = A_c \cos w_c t$.

A_m

$= [A_c + x(t).] \cos w_c t$

$= A_c \cos w_c t + x(t) . \cos w_c t$

$=$ Carrier + DSB-SC

$DSBSC = A_m \cos w_c t \cdot \cos W_m t$

$(\because x(t) . \cos w_c t)$

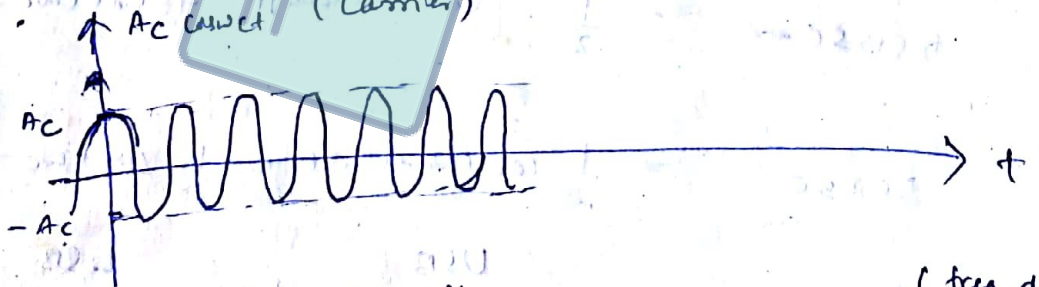
$$DSBSC = \frac{A_m}{2} . [\cos(w_c + w_m)t + \cos(w_c - w_m)t]$$

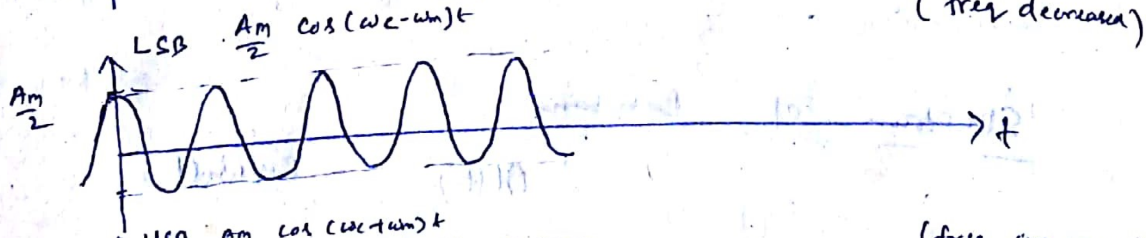
$$(SSB\text{-}SC)_{LSB} = \frac{A_m}{2} \cos(w_c - w_m)t = \text{Lower side band}$$

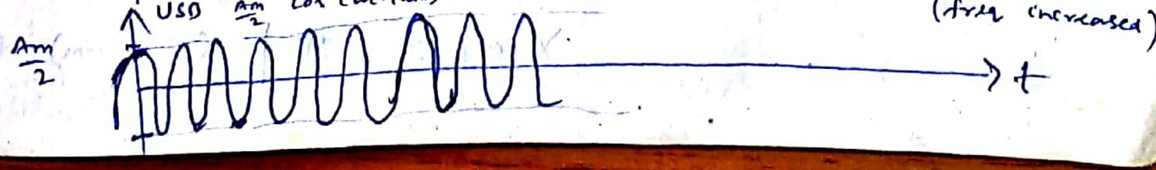
$$(SSB\text{-}SC)_{USB} = \frac{A_m}{2} \cos(w_c + w_m)t = \text{upper side band}$$

So in $(SSB\text{-}SC)_{LSB}$, no amplitude fluctuation, only frequency of carrier decreased from w_c to $w_c - w_m$.

In $(SSB\text{-}SC)_{USB}$, no amplitude fluctuation, only frequency of carrier increases from w_c to $w_c + w_m$.



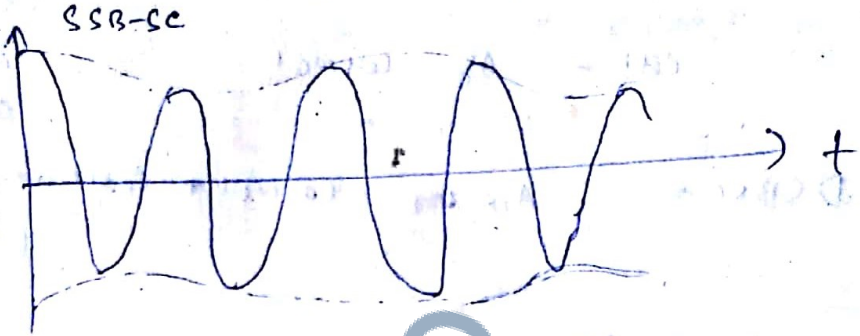




Two tone baseband signal

$$x(t) = E_1 \cdot \cos w_1 t + E_2 \cos w_2 t$$

SSB-SC



So a multitone SSB-SC signal shows amplitude variation.

Mathematical Analysis & Spectrum of Single tone SSB-SC

For simplicity, lets consider a single-tone modulating signal as
(only one freq. 'wm')

$$x(t) = \cos w_m t \quad \text{--- (1)}$$

Let the carrier $c(t) = \cos w_c t$.

$$DSB\text{-}SC = x(t) \cdot \cos w_c t$$

$$= \cos w_m t \cdot \cos w_c t$$

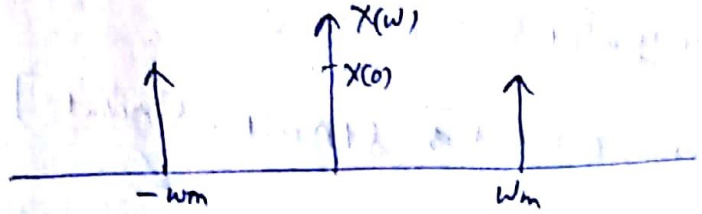
$$DSB\text{-}SC = \frac{1}{2} \cdot \left[\cos(w_c + w_m)t + \cos(w_c - w_m)t\right] \quad \text{--- (2)}$$

$$DSBSC = \underbrace{\frac{1}{2}\cos(w_c + w_m)t}_{USB} + \underbrace{\frac{1}{2}\cos(w_c - w_m)t}_{LSB}$$

Spectrum of Baseband

$$x(t) = \cos w_m t,$$

$$X(w) = \pi\left[\delta(w - w_m) + \delta(w + w_m)\right]$$



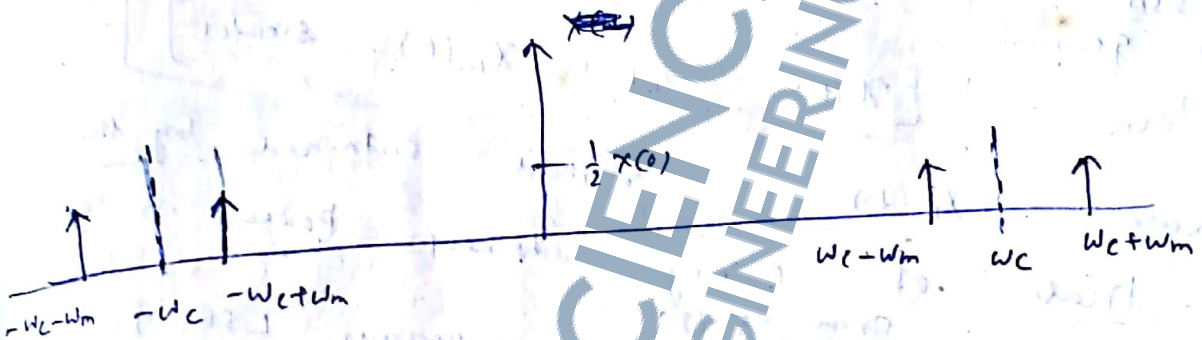
Spectrum of DSB-SC

$$DSB\text{-}SC = \frac{1}{2}\left[\cos(W_c+W_m)t\right] + \frac{1}{2}\cos(W_c-W_m)t\,]$$

~~freq~~ Fourier Transform

$$\frac{1}{2}\left[\pi\,\delta(w-(W_c+W_m)) + \pi\,\delta(w+(W_c+W_m))\right]$$

$$+\frac{1}{2}\left[\pi\,\delta(w-(W_c-W_m)) + \pi\,\delta(w+(W_c-W_m))\right]$$

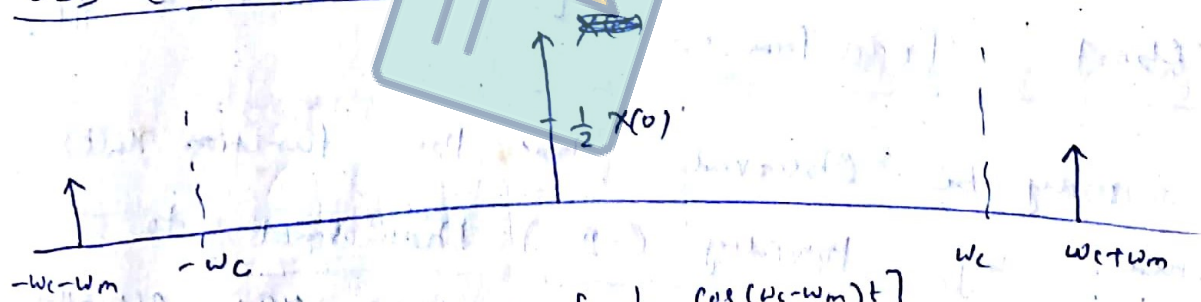


Spectrum of SSB-SC :

$$F\left[\frac{1}{2}\cos(W_c+W_m)t\right]$$

$$= \frac{\pi}{2}\left[\delta(w-(W_c+W_m)) + \delta(w+(W_c+W_m))\right]$$

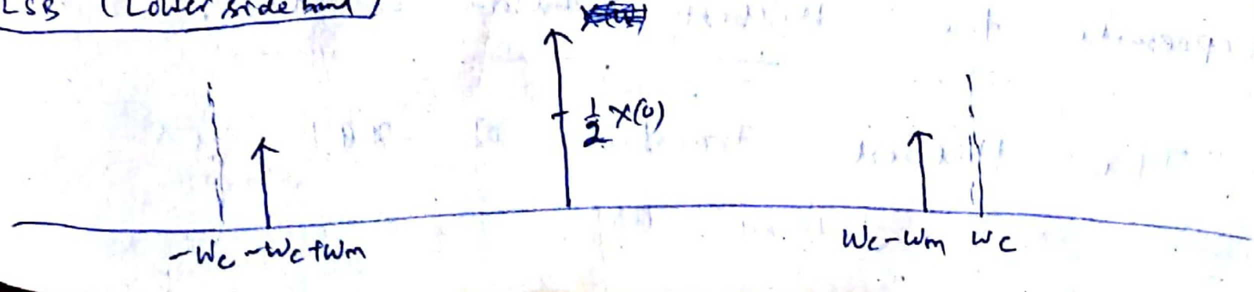
USB (upper side band) :-



$$F\left[\frac{1}{2}\cos(W_c-W_m)t\right]$$

$$= \frac{\pi}{2}\left[\delta(w-(W_c-W_m)) + \delta(w+(W_c-W_m))\right]$$

LSB (Lower side band)



$USB = \frac{1}{2}\cos(\omega_c + \omega_m)t$

$USB = \frac{1}{2}[\cos\omega_c t \cdot \cos\omega_m t \mp \sin\omega_c t \cdot \sin\omega_m t]$

$USB = \frac{1}{2}[\cos\omega_m t \cdot \cos\omega_c t \mp \cos(\omega_m t - \frac{\pi}{2}) \cdot \sin\omega_c t]$ —(1)

$LSB = \frac{1}{2}\cos(\omega_c - \omega_m)t$

$= \frac{1}{2}[\cos\omega_m t \cdot \cos\omega_c t + \sin\omega_m t \cdot \sin\omega_c t]$

$LSB = \frac{1}{2}[\cos\omega_m t \cdot \cos\omega_c t + \cos(\omega_m t - \frac{\pi}{2}) \cdot \sin\omega_c t]$ —(2)

In general,

$S(t)_{SSB} = \frac{1}{2}[\cos\omega_m t \cdot \cos\omega_c t \pm \sin\omega_m t \cdot \sin\omega_c t]$

$S(t)_{SSB} = \frac{1}{2}[\cos\omega_m t \cdot \cos\omega_c t \pm \cos(\omega_m t - \frac{\pi}{2}) \cdot \sin\omega_c t]$

In, general

$$\boxed{S(t)_{SSB} = \frac{1}{2}[x(t) \cdot \cos\omega_c t \pm x_h(t) \cdot \sin\omega_c t]}$$ —(3)

where $x_h(t)$ is signal obtained by shifting the phase of every component present in $x(t)$ by $(-\frac{\pi}{2})$. From eqn (3), (+) signal represents LSB & (−) sign corresponds to the USB.

[H.W. – Prove that $IFT[sgn(\omega)] = \frac{j}{\pi t}$]

Hilbert Transform :-

It may be observed that the function $x_h(t)$ obtained by providing $(-\frac{\pi}{2})$ Phase shift to every frequency component present in $x(t)$, actually represents the Hilbert transform of $x(t)$.

The Hilbert transform of $x(t)$, is defined as imp

$$\boxed{x_h(t) = \frac{1}{\pi} x(t) \circledast \frac{1}{t} = \frac{1}{\pi} \int_{-\infty}^{\infty} x(\tau) \cdot \frac{1}{(t-\tau)} \, d\tau}$$

Inverse, Hilbert transform is defined as

$$\boxed{x(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x_h(\tau)}{t-\tau} \, d\tau}$$

Ex-1) Show that if every frequency component of a signal $x(t)$ is shifted by an amount $(-\frac{\pi}{2})$ the resultant signal $x_h(t)$ is the Hilbert transform of $x(t)$.

Ans ÷ The given situation may be considered as though the signal $x(t)$ is passed through a phase shifting system having transfer function $H(\omega)$ and o/p is $x_h(t)$ as shown below

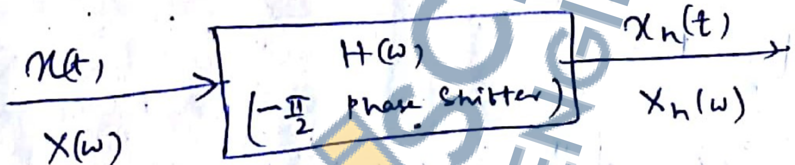


fig :- A Phase Shifting System.

The Characteristics of the system is defined as;

(i) The magnitude of the frequency components present in $x(t)$ remains unchanged when it is passed through the system i.e $|H(\omega)| = 1$.

(ii) The phase of the +ve frequency components is shifted by $-\frac{\pi}{2}$. Now, since the phase spectrum of $\theta(\omega)$ has odd symmetry, the

Phase of −ve frequency components is shifted by $+\frac{\pi}{2}$. The spectrums $H(\omega)$ & $Q(\omega)$ is shown below

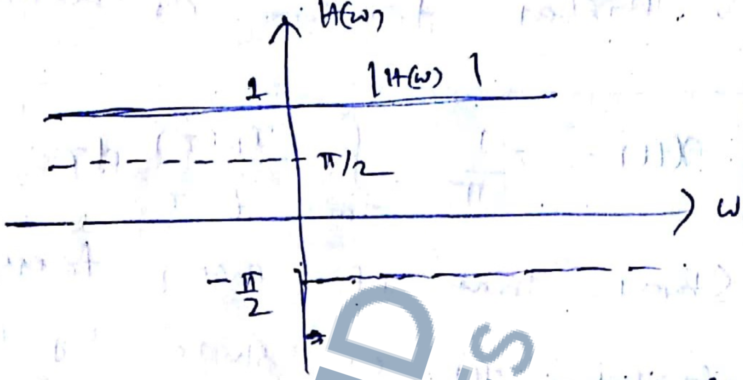


fig:- Transfer function of $\frac{\pi}{2}$ phase shifter.

The transfer function is expressed as;

$$H(\omega) = |H(\omega)| \cdot e^{j\,Q(\omega)}$$

but $Q(\omega) = \begin{cases} +\frac{\pi}{2}, & \omega < 0 \quad \text{(i.e −ve frequencies)} \\ -\frac{\pi}{2}, & \omega > 0 \quad \text{(i.e +ve frequencies)} \end{cases}$

$$|H(\omega)| = 1.$$

$\therefore$ $H(\omega) = \begin{cases} e^{j\frac{\pi}{2}}, & \omega < 0 \\ e^{-j\frac{\pi}{2}}, & \omega > 0 \end{cases}$

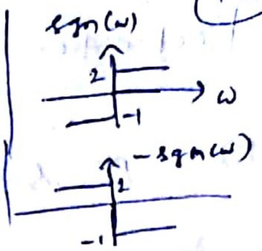
$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = 0 + j = j$$

$$e^{-j\frac{\pi}{2}} = \cos(-\tfrac{\pi}{2}) - j\sin\frac{\pi}{2} = 0 - j = -j$$

$$H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

$$\Rightarrow \frac{H(\omega)}{j} = \begin{cases} 1, & \omega < 0 \\ -1, & \omega > 0 \end{cases}$$

$$\Rightarrow \frac{H(\omega)}{j} = -\,sgn(\omega) \quad \text{--- (1)}$$



The response $X_h(\omega)$ of phase shifting system is related to i/p $X(\omega)$ as,

$$X_h(\omega) = X(\omega) \cdot H(\omega)$$

where
$$X(\omega) = F[x(t)]$$
$$X_h(\omega) = F[x_h(t)]$$

$$\Rightarrow X_h(\omega) = X(\omega)\,(-j\,sgn(\omega)) \quad \text{using eq}^n \text{ (1)}$$

We know that

$$F\left[\frac{1}{\pi t}\right] = -j\,sgn(\omega)$$

$$\Rightarrow X_h(\omega) = X(\omega) \cdot F\left[\frac{1}{\pi t}\right]$$

~~Taking Inverse Fourier Transform both the sides.~~

$$\Rightarrow F[x_h(t)] = F\left[x(t) \otimes \left(\frac{1}{\pi t}\right)\right]$$

$$\left(\because F[x_1(t) \otimes x_2(t)] = X_1(\omega) \cdot X_2(\omega)\right)$$

Taking Inverse F.T both the sides,

$$x_h(t) = x(t) \otimes \frac{1}{\pi t} = \frac{1}{\pi}\, x(t) \otimes \frac{1}{t}$$

$\therefore x_h(t) =$ Hilbert transform of $x(t)$

(proved).

Properties of Hilbert transform :-

1) A signal $x(t)$ & its Hilbert transform $x_h(t)$ have same energy density spectrum or ESD.

2) $x(t)$ & $x_h(t)$ have same Auto Correlation function. (ACF)

3) $x(t)$ & $x_h(t)$ are mutually orthogonal

i-e $$\int_{-\infty}^{\infty} x(t) \cdot x_h(t)\, dt = 0$$

Note :- 2-mark imp BPUT :- **Define orthogonal signals.**

Ans :- Lets consider a set of functions $g_1(x), g_2(x), \ldots g_n(x)$, defined over the interval $x_1 \le x \le x_2$ and which are related to one another in the very special way that any 2 different ones of sets satisfy the condition

$$\boxed{\int_{x_1}^{x_2} g_i(x)\, g_j(x)\, dx = 0}$$

That is, when we multiply 2 different f^ns and then integrate over the interval x_1 & x_2 the result is zero. A set of functions which has this property is described as being orthogonal over the interval from x_1 to x_2.

Ex :-

$$g_1(x) = \sin x$$
$$g_2(x) = \cos x$$
$$x_1 = 0, \quad x_2 = 2\pi$$

$$\int_0^{2\pi} \sin x \cdot \cos x \, dx$$

$$= \frac{1}{2} \int_0^{2\pi} 2 \sin x \cdot \cos x \, dx$$

$$= \frac{1}{2} \int_0^{2\pi} \sin 2x \, dx$$

$$= \frac{1}{2} \left[-\frac{\cos 2x}{2} \right]_0^{2\pi}$$

$$= -\frac{1}{4} \left[\cos 4\pi - \cos 0 \right]$$

$$= 0$$

$\therefore$ $\sin x$ & $\cos x$ are mutually orthogonal signals.

4) If $x_h(t)$ is Hilbert transform of $x(t)$ then, Hilbert transform of $x_h(t)$ is $-x(t)$.

i.e If $H[x(t)] = x_h(t)$

$$H[x_h(t)] = -x(t)$$

'H' denotes Hilbert transform.

Generation of SSB-SC Signal :-

SSB-SC signal may be generated by 2 methods

(i) Frequency discrimination method or filter method.

(ii) Phase discrimination method or Phase-shift method.

(ii) Frequency Discrimination Method:-

In frequency discrimination method, firstly, a DSB-SC signal is generated simply by using an ordinary product modulator or a balanced modulator. After this, from the DSB-SC, signal one of the two sidebands is filtered by a suitable bandpass filter (BPF.). The schematic diagram for this method is shown below.

BPF → USB → ω_c to $\omega_c+\omega_m$
LSB = $\omega_c-\omega_m$ to ω_c

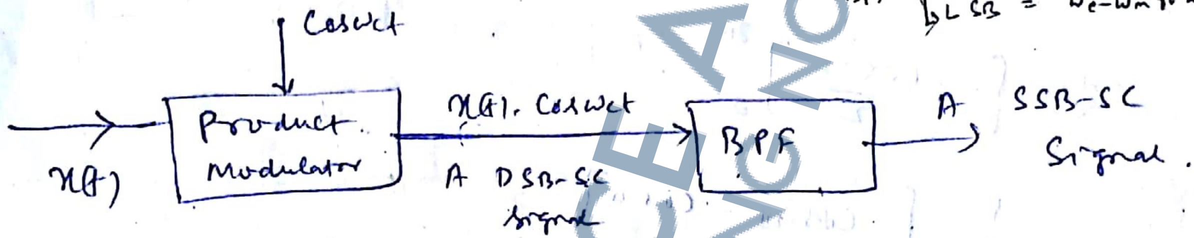


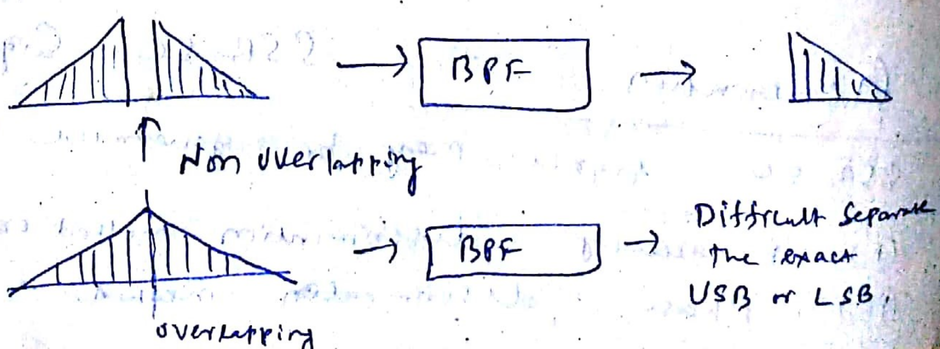
fig:- Frequency - discrimination method for SSB-SC generation.

→ The design of BPF is quite critical, thus puts same limitation on the modulating and carrier frequencies.

Limitation :-

1) The frequency - discrimination method is useful only if the baseband signal is restricted at its lower edge due to which the upper and lower sidebands are non-overlapping.

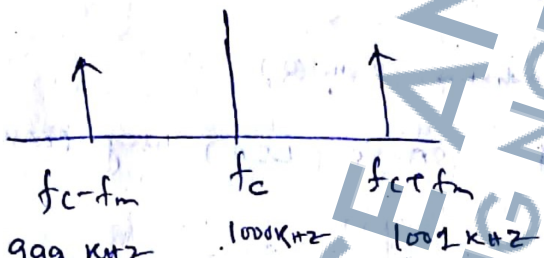
Ex:-



2) Another restriction of frequency discrimination method is that baseband signal must be appropriately related to carrier frequency. In fact, the design of bandpass filter becomes difficult if carrier freq is ~~long~~ quite higher than the bandwidth of base band signal.

Ex :- $f_c = 1\,MHz = 10^3\,KHZ = 1000\,KHZ$

$f_m = 1\,KHZ$, so $f_c >> f_m$



So it is very difficult to design a BPF with very sharp lower & upper cutoff frequency.

(ii) Phase-Shift method :-

Figure shown below, shows the method for generating SSB-SC signal.

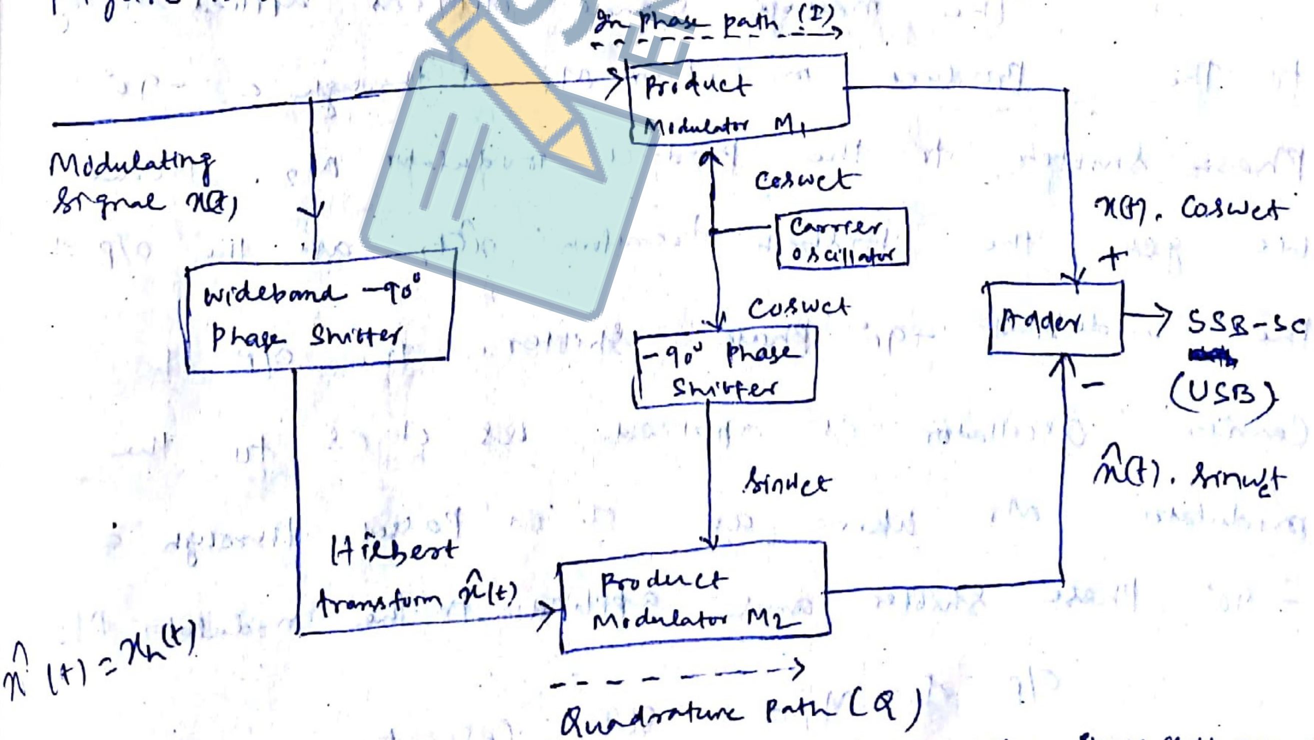


Fig 1 :- Generation of SSB-SC (USB) using Phase Shift Method.

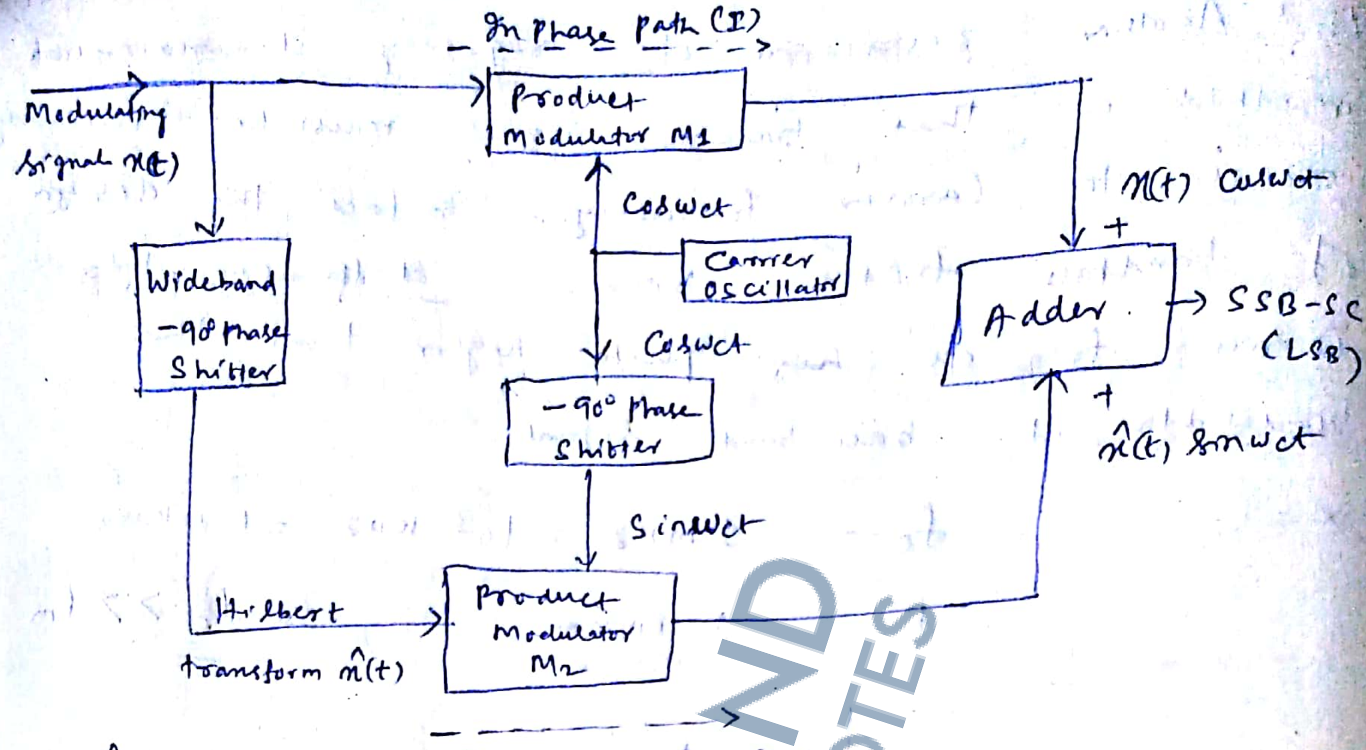


fig : 2 Quadrature Path (Q)

Generation of SSB-SC (LSB) using phase-shift method

Working operation :-

~~The message signal x(t) is applied directly to the product modulator~~. This system is used for suppression of sideband. It uses 2 product modulators M_1 & M_2 and two $-90°$ phase shifting networks as shown in fig 1 & 2.

The message signal $x(t)$ is applied directly to the product modulator M_1 & through a $-90°$ phase shifter to the product modulator M_2. Hence we get the Hilbert transform $\hat{x}(t)$ at the o/p of the wideband $-90°$ phase shifter. The o/p of carrier oscillator is applied as it is to the modulator M_1 where as it is passed through a $-90°$ phase shifter and applied to the modulator M_2.

$$\text{o/p of } M_1 = x(t) \cdot \cos\omega_c t$$

O/P of $M_2 = \hat{x}(t) \cdot \sin\omega_c t$

The O/P of M_1 & M_2 are applied to adder.

If we want ~~only~~ SSB-SC with only USB, then we have to substract the O/P of M_2 from ~~from~~ the O/P of M1. (as shown on fig 1)

So Adder O/P

$$= x(t) \cdot \cos\omega_c t - \hat{x}(t) \sin\omega_c t$$

$$= \text{USB}.$$

If we want ~~only~~ SSB-SC with only LSB, then we have to add O/P of M_2 with O/P of M1. (As shown on fig 2)

So Adder O/P

$$= x(t) \cdot \cos\omega_c t + \hat{x}(t) \sin\omega_c t$$

$$= \text{LSB}$$

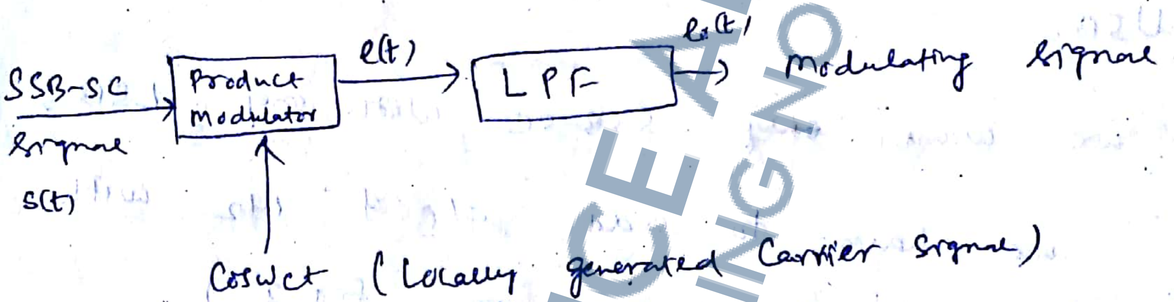
→ So for getting USB, −ve sign on quadrature path & +ve sign on in phase path.

→ And for getting LSB, adder polarities for in phase & quadrature path are both +ve.

Demodulation of SSB-SC wave:

Coherent SSB-SC demodulation:-

The Product modulator is a type of Coherent SSB demodulator. To recover the modulating signal from SSB-SC signal, we require a phase & frequency Coherent or synchronous demodulator.



The received signal is first multiplied with locally generated carrier signal. The locally generated carrier signal should have exactly the same frequency & phase as that of suppressed carrier. The product modulator multiplies the 2 signals at its i/p end and the product signal is passed through a low-pass filter with a bandwidth of wm. At the o/p of the filter, we get the modulating signal back.

Mathematically,

Let SSB-SC wave at the i/p is given by

$$s(t) = \frac{1}{2}\left[x(t) \cdot \cos w_c t \pm \hat{x}_h(t) \cdot \sin w_c t\right]$$

The locally generated carrier is $\underline{\cos\omega_c t}$.

The o/p of product modulator is given by,

$$e(t) = s(t) \cdot \cos\omega_c t$$

$$= \frac{1}{2}\left[x(t) \cdot \cos\omega_c t \pm x_h(t) \sin\omega_c t\right] \cdot \cos\omega_c t$$

$$= \frac{1}{2}\left(x(t) \cdot \cos^2\omega_c t \pm x_h(t) \cdot \sin\omega_c t \cdot \cos\omega_c t\right)$$

$$= \frac{1}{2}\left\{\frac{x(t)}{2}\left[2\cos^2\omega_c t\right] \pm \frac{x_h(t)}{2}\left[2\sin\omega_c t \cdot \cos\omega_c t\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{x(t)}{2}\left[1 + \cos 2\omega_c t\right] \pm \frac{x_h(t)}{2}\left[\sin 2\omega_c t\right]\right\}$$

$$= \underbrace{\frac{x(t)}{4}}_{\text{Scaled message signal}} + \underbrace{\frac{\cos 2\omega_c t \cdot x(t)}{4} \pm \frac{x_h(t)}{4} \cdot \sin 2\omega_c t}_{\text{Unwanted terms}}$$

When $e(t)$ is passed through a LPF, ~~of with~~ with cutoff frequency $\underline{\omega_m}$, it will allow only the first term to pass through, & will reject all other unwanted terms.

$\therefore$ At the o/p of filter we get the modulating signal.

Effect of Phase & freq error on Synchronous detection

The locally generated carrier signal should have exactly the same freq & phase as that of suppressed carrier. Any kind of discrepancy in freq a phase produces a distortion in detected O/P at the receiver end.

Let the SSB-SC signal reaching at the receiver is

$$s(t) = \frac{1}{2}\left[x(t) \cdot \cos\omega_c t + x_h(t)\sin\omega_c t\right]$$

Considering the locally generated signal with freq & phase error equal to $\Delta\omega$ & ϕ respectively. So the O/P of product modulator is

$$e(t) = \frac{1}{2}\left[x(t)\cos\omega_c t + x_h(t)\sin\omega_c t\right]\left[\cos[(\omega_c+\Delta\omega)t+\phi]\right]$$

$$= \frac{1}{2}\left[x(t) \cdot \cos\omega_c t \cdot \cos[(\omega_c+\Delta\omega)t+\phi] + x_h(t) \cdot \sin\omega_c t \cdot \cos[(\omega_c+\Delta\omega)t+\phi]\right]$$

$$= \frac{1}{2}\left[\frac{x(t)}{2} \cdot \left\{\cos(2\omega_c t+\Delta\omega t+\phi) + \cos(\Delta\omega t+\phi)\right\} + \frac{x_h(t)}{2}\left\{\sin(2\omega_c t+\Delta\omega t+\phi) + \sin(\Delta\omega t+\phi)\right\}\right]$$

$(\because \sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha \cdot \cos\beta)$

After passing through LPF,

$$= \frac{1}{4}\left[x(t) \cdot \cos(\Delta\omega t+\phi) + x_h(t) \cdot \sin(\Delta\omega t+\phi)\right]$$

The baseband signal is multiplied by slow time varying function ~~due to~~ which distorts the message signal. Also due to presence of Hilbert transform $x_h(t)$, the detector O/P suffers from phase distortion.

Case-I : when phase & freq error both zero

$(\phi = 0, \ \Delta\omega = 0)$

o/p of LPF

$$e_0(t) = \frac{1}{4}\left[x(t) \cdot \cos 0 + x_h(t) \cdot \sin 0\right]$$

$$= \frac{1}{4} x(t).$$

= Modulating signal.

(No distortion on detected o/p)

Case-II : when $\phi \neq 0$, $\Delta\omega = 0$

$$e_0(t) = \frac{1}{4}\left[x(t) \cdot \cos\phi + x_h(t) \cdot \sin\phi\right]$$

Due to presence of Hilbert transform, there will be phase distortion. If $\phi = 90°$, message will be completely absent. (Known as quadrature null effect)

Case-III : when $\phi = 0$, $\Delta\omega \neq 0$

$$e_0(t) = \frac{1}{4}\left[x(t) \cdot \cos\Delta\omega t + x_h(t) \cdot \sin\Delta\omega t\right]$$

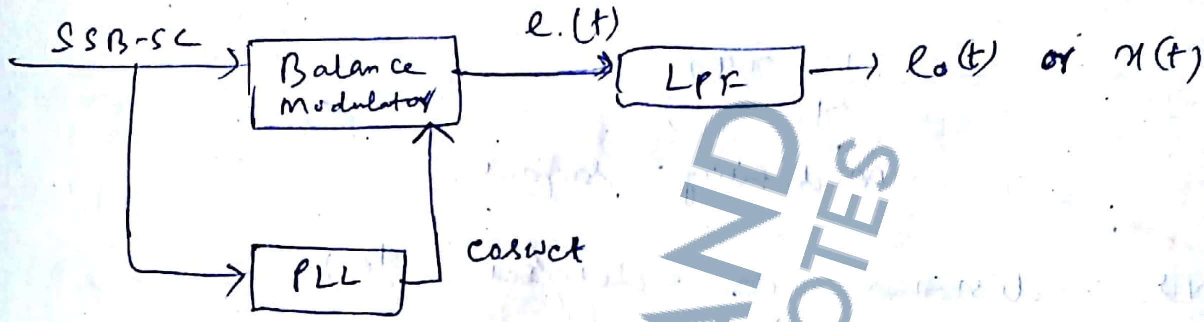
The o/p suffers from both freq & phase distortion.

Case-IV : $\phi \neq 0$, $\Delta\omega \neq 0$

$$e_0(t) = \frac{1}{4}\left[x(t) \cdot \cos(\Delta\omega t + \phi) + x_h(t) \cdot \sin(\Delta\omega t + \phi)\right]$$

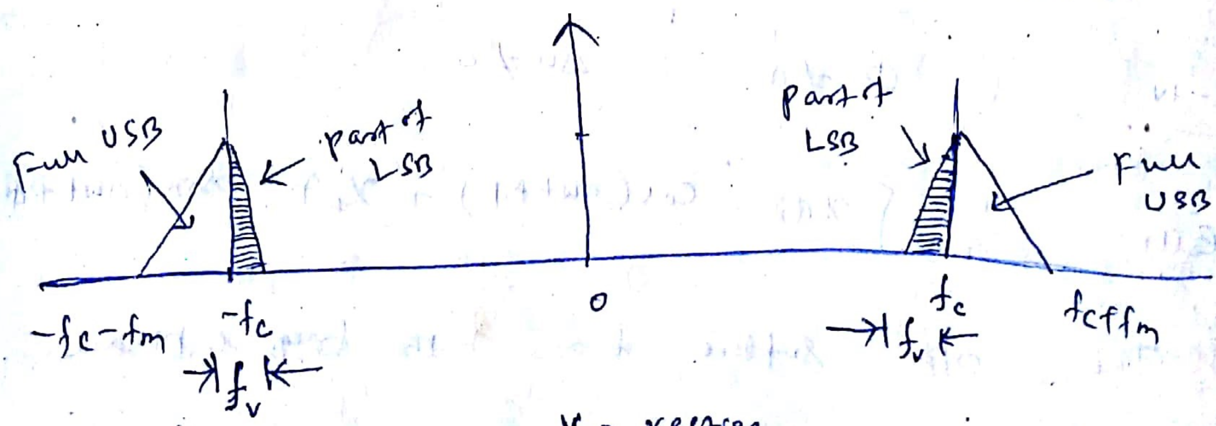
→ The o/p suffers from both freq & phase distortion.

So to ensure coherent detection, a PLL (Phase locked loop) is used to generate the local carrier signal i.e synchronous with the suppressed carrier signal.



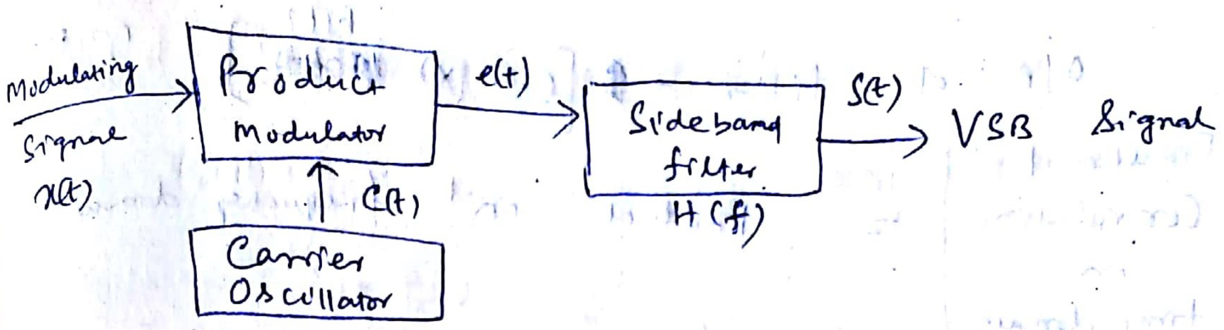
Vestigial Sideband transmission (VSB)

The stringent frequency-response requirements on the sideband filter in a SSB AM system can be relaxed by allowing a part, called a vestige, of unwanted sideband to appear at the o/p of the modulator. Thus, we simplify the design of the sideband filter at the cost of a modest increase in the channel Bandwidth required to transmit the signal. The resulting signal is called vestigial-sideband (VSB) AM.



V → vestige

Generation of VSB Modulated wave :-



The modulating signal $x(t)$ is applied to a product modulator. The o/p of the carrier oscillator is also applied to the other i/p of the product modulator. o/p of the product modulator, $e(t)$.

$$\therefore \; e(t) = x(t) \cdot c(t)$$

$$e(t) = x(t) \cdot \cos 2\pi f_c t$$

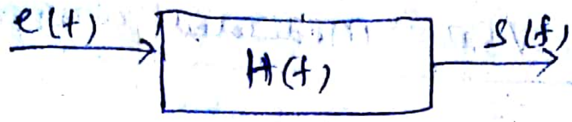
This represents DSB-SC modulated wave. This DSB-SC signal is then applied to a sideband shaping filter. The design of this filter depends on the desired spectrum of the VSB modulated signal. This filter will pass the wanted sideband and vestige (part) of the unwanted sideband.

Let the transfer function of the filter be $H(f)$. Hence, the spectrum of the VSB modulated signal is given by

$$\boxed{S(f) = \frac{1}{2}\left[X(f-f_c) + X(f+f_c)\right] \cdot H(f)}$$

$H(f)$ can be found out, by determining necessary conditions for the coherent detection output to provide an undistorted version of original modulating signal $x(t)$. (For explanation P.T.O)

$(\because$



$\therefore$ O/P of filter $= [e(t) \circledast h(t)]$ in time domain

[F. Transform of Convolution in time domain] = Product in frequency domain.

$$\Rightarrow S(f) = E(f) \cdot H(f)$$

$$= F[e(t)] \cdot H(f)$$

$$= F[x(t) \cos 2\pi f_c t] \cdot H(f)$$

$$= F\left[x(t) \left\{ \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right\} \right] \cdot H(f)$$

$$\Rightarrow S(f) = \frac{1}{2}\left[X(f - f_c) + X(f + f_c) \right] \cdot H(f) \quad \text{--- (1)}$$

(By frequency shifting property) (or) - Modulation property

Demodulation of VSB wave:-

The synchronous detector for the detection of VSB of VSB modulated wave is shown in figure 2.

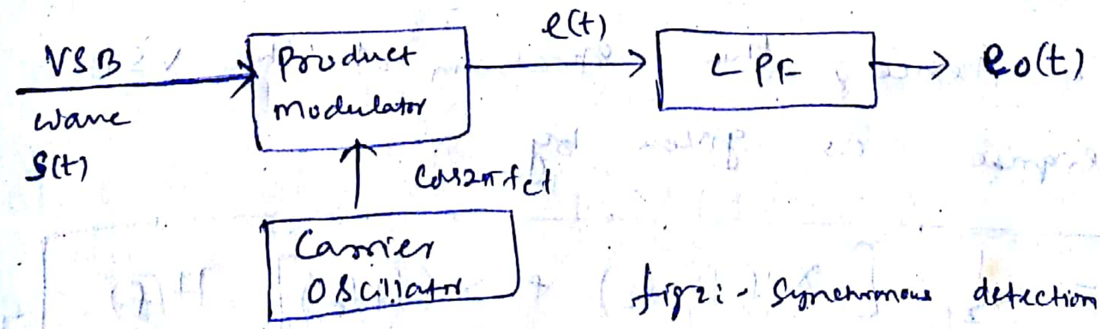


fig 2:- Synchronous detection of VSB

Mathematically, the O/P of the product modulator is given by,

$$e(t) = s(t) \cdot c(t)$$

$$e(t) = s(t) \cdot \cos 2\pi f_c t$$

Taking Fourier Transform, both the sides,

$$E(f) = F\left[s(t) \cdot \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right)\right]$$

$$E(f) = \frac{1}{2}\left[S(f - f_c) + S(f + f_c)\right] \quad \text{—— (2)}$$

[By freq. shifting property]

But $S(f) = \frac{1}{2}\left[X(f - f_c) + X(f + f_c)\right] \cdot H(f)$ [From eqn (1)]

$$\therefore S(f - f_c) = \frac{1}{2}\left[X(f - f_c - f_c) + X(f - \cancel{f_c} + \cancel{f_c})\right] H(f - f_c)$$

(Replacing $f \to f - f_c$ in eqn (1))

$$S(f - f_c) = \frac{1}{2}\left[X(f - 2f_c) + X(f)\right] \cdot H(f - f_c) \quad \text{—— (3)}$$

$$S(f + f_c) = \frac{1}{2}\left[X(f + \cancel{f_c} - \cancel{f_c}) + X(f + f_c + f_c)\right] \cdot H(f + f_c)$$

($\because$ Replacing $f \to f + f_c$ in eqn (1))

$$S(f + f_c) = \frac{1}{2}\left[X(f) + X(2f + 2f_c)\right] \quad \text{—— (4)}$$

Putting eqn (3) & (4) in eqn (2)

$$E(f) = \frac{1}{2}\left[\frac{1}{2}\{X(f) + X(f - 2f_c)\} \cdot H(f - f_c) + \frac{1}{2}\{X(f) + X(f + 2f_c)\} \cdot H(f + f_c)\right]$$

$$= \frac{1}{4}\left[X(f)\right]\left[H(f - f_c) + H(f + f_c)\right] + \ldots$$

$$\Rightarrow E(f) = \overbrace{\frac{1}{4} X(f) \left[H(f-f_c) + H(f+f_c) \right]}^{\text{1st term}} +$$

$$\frac{1}{4} \left[X(f+2f_c) \cdot H(f+f_c) + X(f-2f_c) \cdot H(f-f_c) \right]$$

The first term represents the spectrum of demodulated VSB o/p, the LPF will pass this term & eliminate the second term.

$\therefore$ The frequency spectrum of the signal $e_0(t)$ at the o/p of LPF is given by

$$E_0(f) = \frac{1}{4} X(f) \left[H(f-f_c) + H(f+f_c) \right]$$

For distortionless reception by the VSB modulation, scheme, it is necessary that the transfer function of the filter $H(f)$ must satisfy the condition given as under,

$$H(f-f_c) + H(f+f_c) = 2H(f_c)\text{, where}$$

$H(f_c)$ is the transfer function of ~~LPF at~~ sideband filter (at transmitter side) at carrier frequency f_c. It is a constant value.

$$\therefore E_0(f) = K \cdot X(f)$$

$$e_0(t) = K\, x(t).$$

$\therefore$ Modulating signal can be obtained at the o/p of LPF of the synchronous detector.

~~Now~~ As we have discussed that H(f) can be found out by determing the necessary cond^n for coherent detection o/p to provide undistorted version of modulating signal x(t).

∴ The condn is,

$$H(f-f_c) + H(f+f_c) = 2H(f_c). \quad \text{(5)}$$

(H(f) is the sideband filter used at transmitter side)

This requirement (eq^n 5), can be fulfilled by using a filter having a normalized amplitude response as shown below.

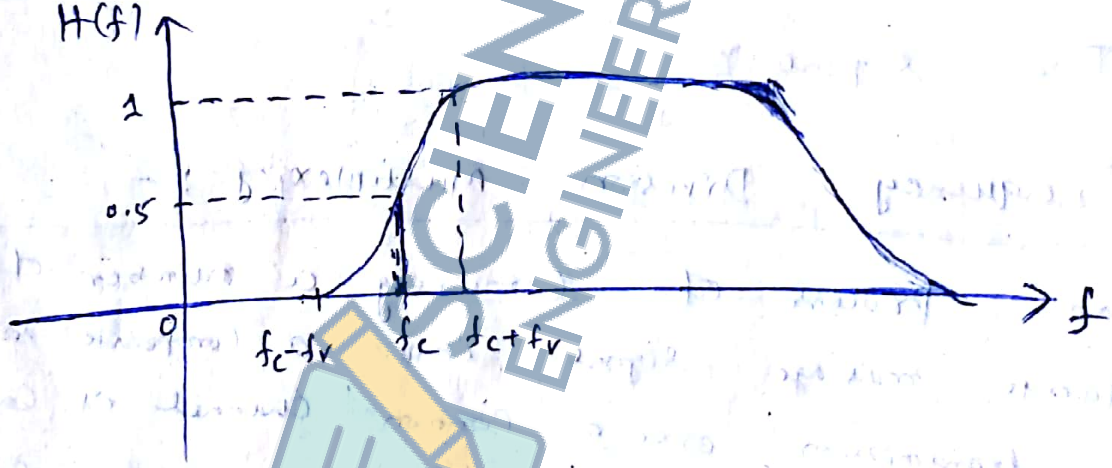


fig 3:- Illustration of amplitude response of VSB filter.

→ The design of such a filter is much more simpler than the design of filters needed in generation of SSB signals.

Appln :- VSB Modulation has become standard for transmission of T.V signals.

Advantage :-

1) Here the difficulty of SSB generation i.e requirement of a sharp cut-off filter or phase shifter is avoided.

2) The BW requirement is not high as DSB-SC ($2f_m$). Additional spectrum required is usually less than $\frac{1}{4}$ of SSB requirement.

$$BW = f_m + f_v = f_m + \frac{f_m}{4} = 1.25 f_m.$$

3) Makes the transmission of low frequency components possible (ex:- d.c components in T.V signal)

Frequency Division Multiplexing :-

The process of combining a number of separate message signal into a composite signal for transmission over a common channel is called multiplexing. Two methods commonly used for signal multiplexing

1) Frequency-division-multiplexing (FDM)
2) Time-division-multiplexing (TDM)

TDM is usually used in the transmission of digital information.
FDM can be used both in analog & digital communication.

FDM:- The system used for FDM is shown below. The individual baseband signals $m_1(t), m_2(t), \ldots m_\lambda(t)$, each band limited to f_M, are applied to individual modulators, each modulator being supplied as well with carrier waveforms of frequency $f_1, f_2, \ldots f_\lambda$.

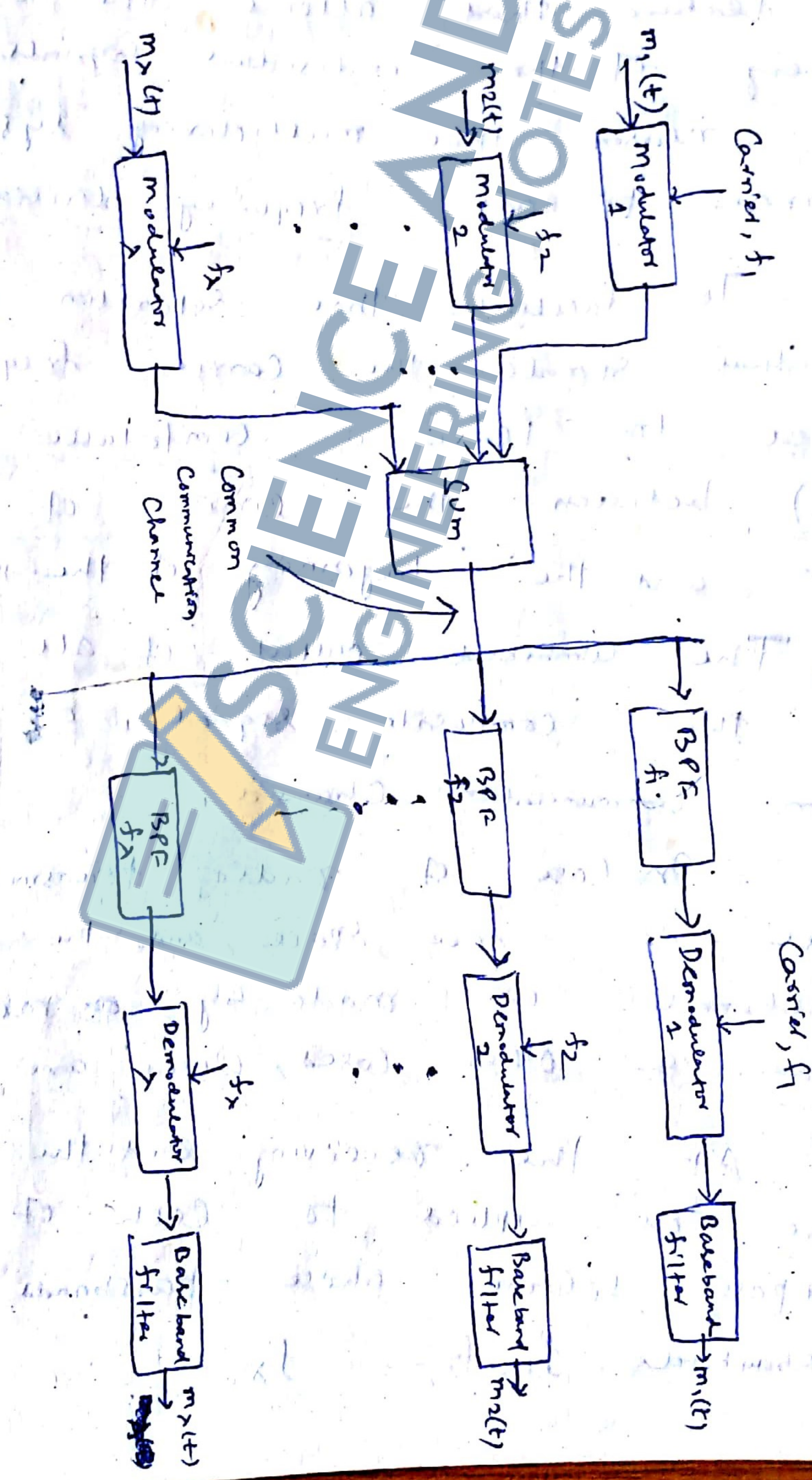


fig:- Multiplexing many baseband signals over a single communication channel.

The individual modulator-output (i.e. AM) → f_c+f_m to f_c-f_m signals extend over a limited range in the neighbourhood of the individual carrier frequencies.

Most importantly the carrier frequencies are selected, so that the spectral ranges of the modulator-output signals don't overlap.

This separation in frequencies is precisely the feature that allows the eventual recovery of the individual signals, and for this reason this multiplexing system is referred to as frequency division multiplexing.

To facilitate this separation of the individual signals, the carrier frequencies are selected to leave a comfortable margin (guard band) between the limit of one frequency range and the beginning of the next.

The combined output of all the modulators, i.e. the composite signal, is applied to a common communication channel.

In case of radio transmission, the channel is free space, and the coupling to the channel is made by means of an antenna. In other cases, wires are used.

At the receiving end the composite signal is applied to each of a group of bandpass filters whose passbands are in the neighbourhood $f_1, f_2, \ldots f_n$.

T~~he filter f_1~~ is

The bandpass filter f_1, passes only the spectral range of the output of modulator 1 and similarly for the other bandpass filters.

The signals have thus been separated. They are then applied to individual demodulators (e.g. AM ") which extract the baseband signal from the carrier. The carrier i/ps to the demodulators are required only for synchronous demodulation and are not used otherwise.

The final step consists in passing the demodulator o/p through a baseband filter. The baseband filter is a low-pass filter with cutoff at the frequency f_m to which the baseband signal is limited. This baseband filter will pass, without modification, the baseband signal. Thus the original baseband signals are recovered.

Radio transmitter & Receiver :-

Radio Transmitter :-

In this, the baseband signal is translated to a radio frequency by a modulation process which is then

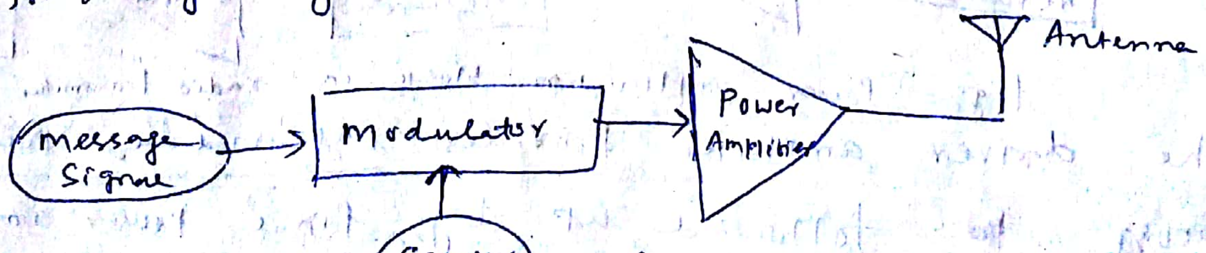


fig:- A simple block diagram of radio transmitter.

amplified by a power amplifier and radiated through Antenna assembly. A simple block diagram shown in fig, shows these steps.

An AM broadcast transmitter occupies 10kHz bandwidth i.e if the Carrier signal is of 600KHz frequency then it occupies frequency range of ~~595~~ 595 — 605 KHZ. The oscillator used for carrier is usually made up from $\boxed{\text{crystal}}$ as LC oscillators tend to drift with time and not as stable.

The oscillator output passes through a buffer a buffer amplifier which increases its power level but more importantly serves as an isolator that prevents variation of load affecting frequency of the oscillator.

Power Amplifier :-

A typical modulator output has few mW of power while a commercial AM broadcast transmitter radiates several KW of power. This is usually a two step process as shown below.

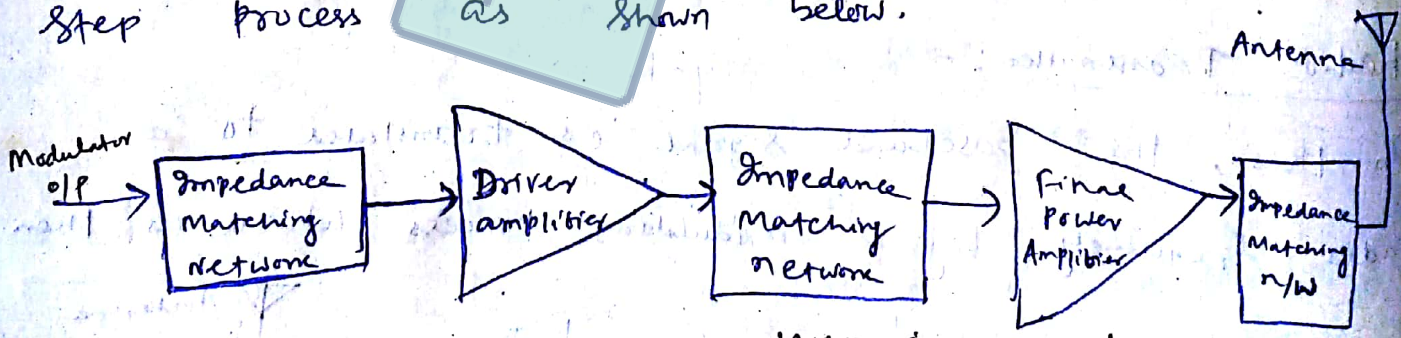
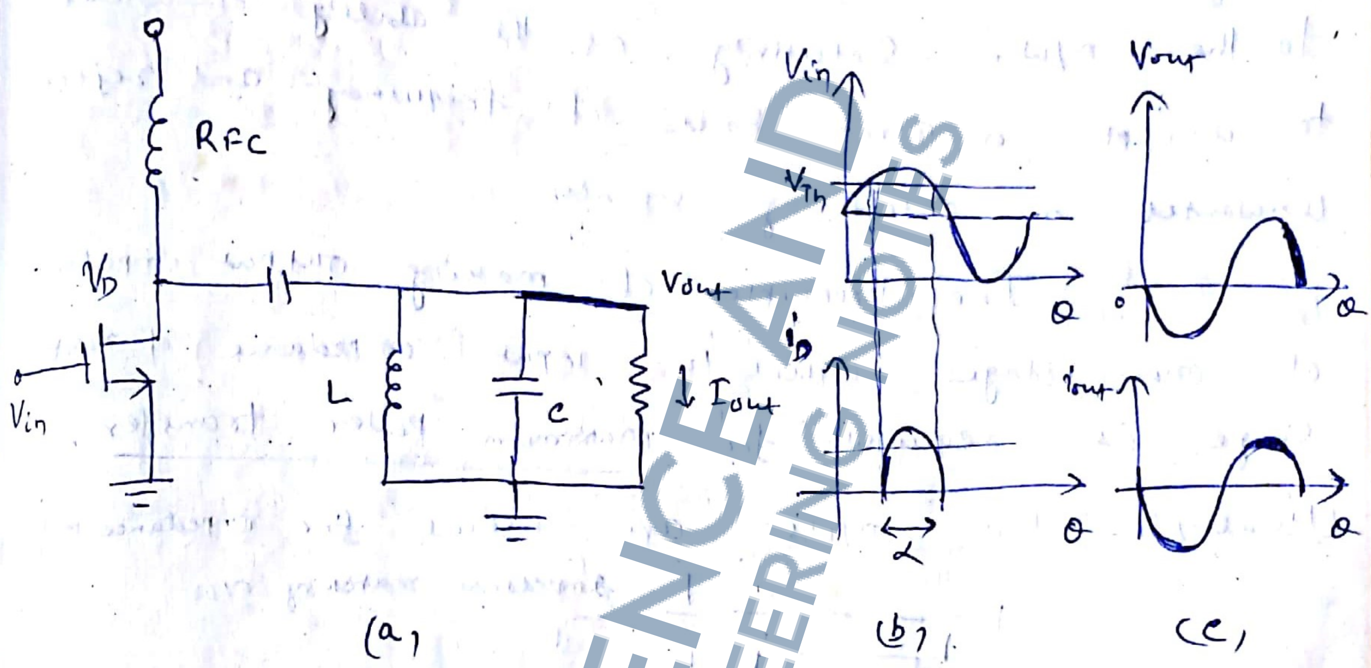


fig:- Power amplification block in radio transmitter

The driver amplifier provides intermediate power boost followed up by final power amplifier.

A Class C amplifier is usually used for power amplification. These amplifiers conducts less than 50% of the i/p signal and distortion at the o/p is high, but up to 90% of power efficiency can be achieved. In RF transmitters this distortion can vastly be reduced by using tuned loads.



(a) A Class C amplifier with tuned Output

b) Amplification $\alpha < 180°$

c) Output of tuned section.

Fig (a) shows a MOSFET based Class C amplifier which conducts when $V_{in} > V_{Th}$, the threshold voltage. The corresponding current flows i_D shown in fig (b). So i_D is available only for $\alpha < 180°$ in a full cycle because $V_{Th} > 0$. The LC resonator is tuned to fundamental and exchange of energy between L & C provides oscillation that is sent to antenna via RF coil or choke and radiated from there.

RF → Radio Frequency.

Impedance Matching Network:-

Impedance matching n/w in radio transmitter facilitates efficient power coupling between two stages. They also do filtering job and provides selectivity to the n/w. Selectivity is the ability of a ckt to accept a given band of frequency and reject unwanted or interfering signals.

The principle of making output impedance of one stage equal to input impedance of next stage is required for maximum power transfer. Usually LC n/ws are useful for impedance matching.

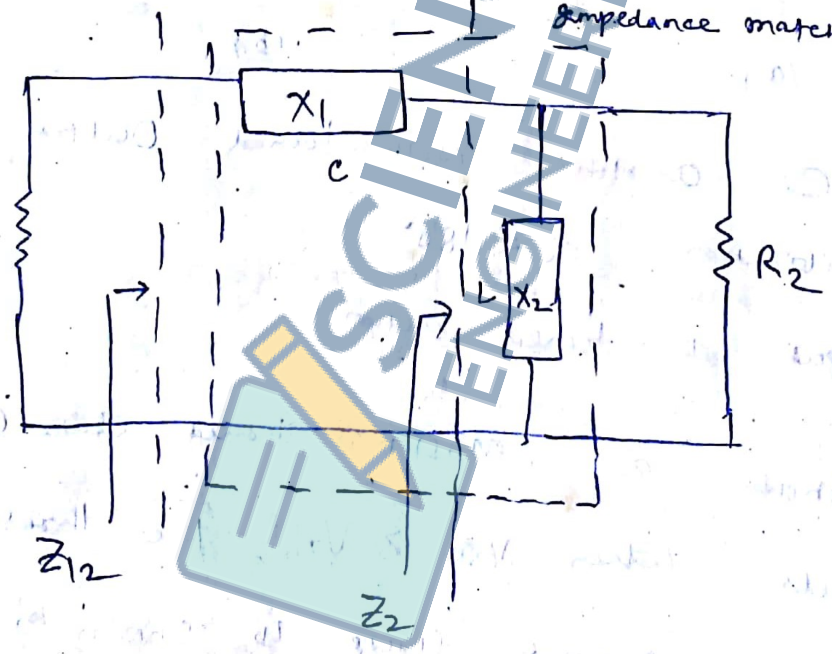


fig:- Impedance matching by L - network ($R_1 > R_2$)

Consider an LC n/w, our objective is to match two purely resistive loads R_1, (output impedance of previous stage) & R_2 (input impedance of next stage)

$$Z_2 = R_2 \parallel jX_2 = \frac{jR_2X_2}{R_2 + jX_2}$$

$$= \frac{jR_2X_2(R_2 - jX_2)}{R_2^2 + X_2^2}$$

$$Z_2 = \frac{R_2X_2^2}{R_2^2+X_2^2} + j\frac{R_2^2X_2}{R_2^2+X_2^2}$$

This makes Z_2 & X_1 in series & their combined impedance can be written as,

$$Z_{12} = Z_2 + jX_1 = \frac{R_2X_2^2}{R_2^2+X_2^2} + j\frac{R_2^2X_2}{R_2^2+X_2^2} + jX_1$$

For impedance matching, we want $R_1 = Z_{12}$

$$R_1 + 0.j = \frac{R_2X_2^2}{R_2^2+X_2^2} + j\left(X_1 + \frac{R_2^2X_2}{R_2^2+X_2^2}\right)$$

$$\boxed{R_1 = \frac{R_2X_2^2}{R_2^2+X_2^2}} \qquad X_1 + \frac{R_2^2X_2}{R_2^2+X_2^2} = 0$$

$$\Rightarrow \boxed{X_1 = -\frac{R_2^2X_2}{R_2^2+X_2^2}}$$

In usual cases, X_1 is a capacitive element (C) & X_2 is an inductive element (L)

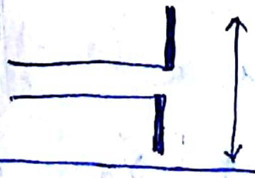
Note:- Transformer & specially designed balun can also be for impedance matching.

Antenna :- Def^n :- A means for radiating or receiving radio wave.

Antenna is the final subsystem of a radio transmitter. When current enters antenna, an electrical conductor, it creates a magnetic field around it. The magnetic field creates an electric field in the surrounding space which in turn induces another magnetic field; which again induces another ~~mag~~ electric field and so on. These way electric & magnetic fields (together electromagnetic fields) induce each other and move with speed of light in a direction away from the antenna.

If electric field remains horizontal to the earth, produced by a horizontal antenna it is called horizontal polarization & if vertical known as vertical polarization.

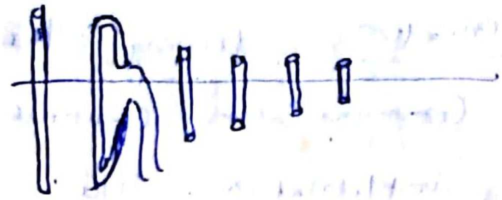
The shape & size of an antenna can have a wide variety but always depends on the wavelength which it operates. (Say length of antenna = $\frac{\lambda}{10}$). The simplest of them is a half wave dipole which is a simple wire piece of length half the wavelength it can radiate.



Length $l = \frac{\lambda}{2}$.

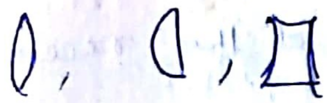
*Extra: Notes:-

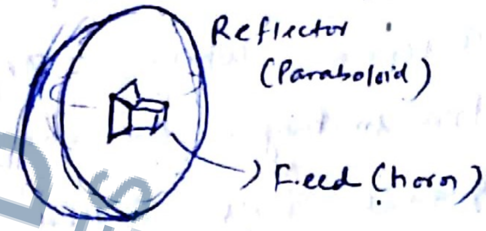
Different types of antenna:- Dipole, Circular, Helix, (⟶) Conical, horn, pyramidal horn, Rectangular wave guide, Microstrip antenna (In Mobile phones), Array antenna

(Yagi-Uda array →  (In T.V reception)

→ Microstrip Patch array), Parabolic reflector antenna

Lense antenna





Like in - (Dish T.V)

V.V. Imp

Radio Receiver : / AM receiver

The receiver most commonly used in A.M radio broadcast is called 'Superheterodyne receiver'. The superheterodyne principle is used in all type of receiver like television, receiver, radar receiver etc.

It consist of a radio frequency (RF) tuned amplifier, a mixer, a local oscillator, an intermediate frequency (IF) amplifier, a detector, an audio frequency amplifier, a power amplifier and a loud speaker.

$f_i = f_o - f_s$

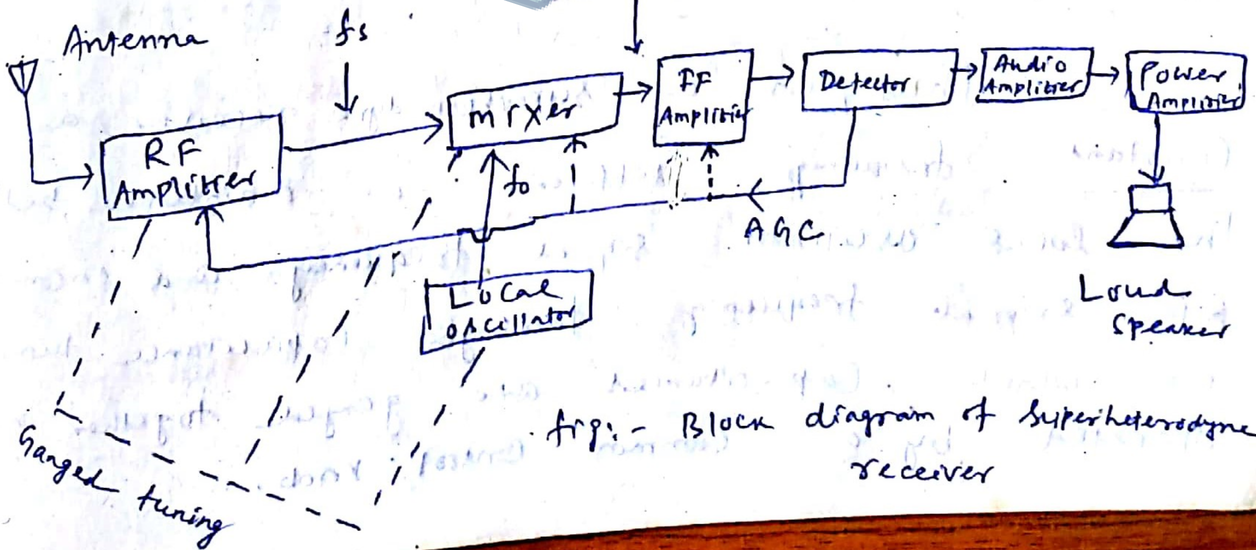


fig:- Block diagram of Superheterodyne receiver

We assume that the signal has suffered great attenuation during the course of transmission over the communication channel and hence is in need of amplification. The i/p to the system might be a signal furnished by a receiving antenna which receives its signal from a transmitting antenna. The carrier of the received signal is called Radio-frequency (RF) carrier, and its frequency is the radio ~~...~~ frequency. The i/p signal is amplified in a RF amplifier. and then passed on to a mixer.

In the mixer, the incoming signal frequency and local oscillator freq are mixed. That is why it is called superheterodyne receiver, as heterodyne stands for 'mixing'.

After mixing, the RF signal ~~[illegible]~~ is converted into a signal of lower fixed frequency. This lower fixed frequency is known as Intermediate frequency (IF).

Thus, in a superheterodyne receiver, a constant frequency difference is maintained betn the local oscillator signal frequency and incoming RF signal frequency through capacitance tuning in which capacitances are ganged together and operated by a common control knob.

In A.M radio broadcast, the frequency band is 535-1605 KHZ. The Carrier frequency has band of 540 - 1600 KHZ with 10 KHZ spacing. The baseband message m(t) is limited to a bandwidth of approximately 5 KHZ.

In superheterodyne receiver, every A-M radio signal is converted to a common IF frequency

$$\boxed{f_i = 455 \text{ KHZ}}$$

The frequency conversion to IF is performed by the combination of RF amplifier and Local oscillator frequency through mixer.

$$\therefore \boxed{f_o = f_s + f_i}$$

f_o = Local oscillator freq.

$f_s \rightarrow$ incoming signal freq.

$f_i \rightarrow$ IF freq.

Then the mixer o/p is passed to the IF amplifier. It generally contains a number of transformers each consisting of a pair of mutually coupled tuned ckts. Thus, with this large number of double-tuned ckts, operating at a specially chosen freq, the IF amplifier provides most of gain (i.e sensitivity) and BW requirement (Selectivity) of the receiver.

The IF amplifier is designed to have BW of 10 KHZ, which matches the bandwidth of the transmitted signal.

Also, since the characteristics of the IF amplifier are independent of the incoming signal frequency to which the receiver is tuned, the selectivity and sensitivity of the superheterodyne receivers are quite uniform throughout its tuning range.

Since IF amplifier works at a fixed IF frequency, the design of the system is quite easy to provide high gain & constant BW.

After the IF Amplifier, the signal is applied at the i/p of the detector which extracts the original modulating signal. This audio signal is amplified by an audio amplifier to get a particular voltage level. This amplified audio signal is further amplified by a power amplifier to get a specified power level so that it may activate loud speaker.

The loud speaker is a transducer which converts this audio electrical signal into audio sound signal & thus the original signal is reproduced.

When the ~~carrier~~ RF carrier signal has less amplitude, the loudspeaker output becomes inaudible and gets mixed in noise. On the other hand, ~~at~~ when carrier amplitude is high, the output of the the loudspeaker becomes intolerably large. So a AGC (Automatic Gain Control) is provided which adjust the gain of the system based on the power level of the RF signal. ~~at~~

Advantage of superheterodyning :-

1) No variation in BW, BW remains constant over the entire operating range.

2) High sensitivity (Gain) and selectivity (BW)

3) High adjacent channel rejection.

4) Suitable in majority radio receiver appl^n like AM, FM, communication, single-sideband, T.V and even radar receiver.

Limitation :- (Image frequency)

The superheterodyne receiver suffers from a major drawback known as 'Image frequency' problem. This problem of Image frequency arises because of superheterodyne principle.

We know

$f_o = f_s + f_i$ where

f_o = Local oscillator freq.

$f_s \rightarrow$ Incoming RF signal freq

$f_i \rightarrow$ IF freq.

$\Rightarrow f_i = f_o - f_s$.

If a frequency f_{si}, manages to reach mixer such that

$\underline{f_{si} = f_o + f_i}$, then IF will be

$$IF = f_o - (f_{si})$$
$$= f_o - (f_o + f_i)$$
$$= \cancel{f_o} - \cancel{f_o} - f_i$$
$$= -f_i$$

Thus f_{si} would produce f_i $(-f_i)$ when mixed with f_o. This undesirable or spurious IF signal will also be amplified by I.F. stage and thus would cause interference. This has a effect of 2 sources or stations being received simultaneously. This situation is obviously undesirable.

The rejection of an image freq. signal by a single tuned ckt (in RF stage), may be defined as the ratio of the gain at the signal frequency to the gain at the image freq.

This is given as,

$$\alpha = \sqrt{1 + Q^2 \rho^2}$$

where Q = Quality factor of the tuned ckt.

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$\therefore$ Image freq, f_{si} is given by

$$f_{si} = f_o + f_i$$

$$= \underbrace{f_s + f_i}_{} + f_i$$

$$f_{si} = f_s + 2f_i$$

$$\therefore \boxed{f_{si} = f_s + 2 \times IF}$$

<u>Ex</u> :- Given $Q = 100$, $f_i = 455$ KHz = IF

$f_s = 1000$ KHz, find <u>'α'</u>

<u>Ans</u> :- $f_{si} = f_s + 2 \times IF$

$= 1000 + 2 \times 455 = 1910$ KHz.

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910}{1000} - \frac{1000}{1910} = 1.91 - 0.524$$

$$= 1.386.$$

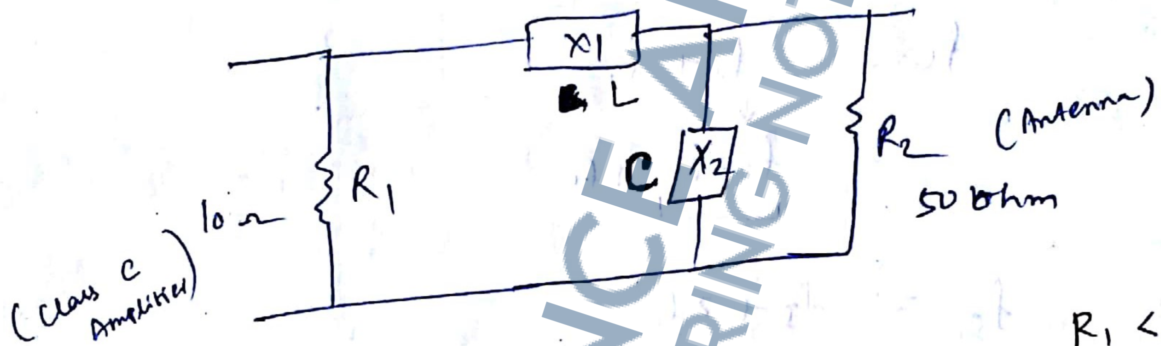
$$\alpha = \sqrt{1 + Q^2 \rho^2} = \sqrt{1 + 100^2 \times 1.386^2} = 138.6.$$

* More the value of 'α', better is the Image freq. rejection.

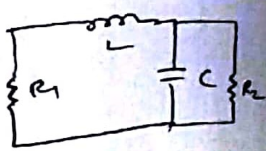
Ex-2 :- (Taub-Schilling - 3.7 - 185 page) (Regarding impedance matching in Transmitter)

A Class C power amplifier has o/p load 10 ohm. This is to be matched with an antenna of impedance 50 ohm by an L-n/w. Find value of L & C, if carrier freq to be used is 500 KHZ.

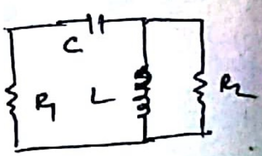
Ans :-



$R_1 < R_2$



$R_1 > R_2$



$$R_1 = \frac{R_2 X_2^2}{R_2^2 + X_2^2}$$

$$\Rightarrow 10 = \frac{50 X_2^2}{50^2 + X_2^2}$$

$$\Rightarrow 25000 + 10X_2^2 = 50 X_2^2$$

$$\Rightarrow 40 X_2^2 = 25000$$

$$\Rightarrow X_2^2 = \frac{2500}{4} = 625$$

$$\Rightarrow \boxed{X_2 = 25 \text{ ohm}}$$

$$X_1 = -\frac{R_2^2 X_2}{R_2^2 + X_2^2} = -\frac{50^2 \times 25}{50^2 + 25^2} = \frac{2500 \times 25}{2500 + 625}$$

$X_1 = -20 \text{ ohm}$

$|X_1| = |j\omega L| = \omega L = 20$

$|X_2| = \left|\frac{1}{j\omega C}\right| = \frac{1}{\omega C} = 25$

$$\cancel{\omega} L \times \frac{1}{\cancel{\omega} C} = 20 \times 25$$

$$\Rightarrow \frac{L}{C} = 500 \text{ ——— } \cancel{1}$$

$$\Rightarrow 500 = \frac{L}{C} \text{ ——— } (1)$$

The operating range of ω needs to be near resonant frequency for necessary freq. selectivity,

$$\omega_c^2 = \frac{1}{LC}$$

$$(2\pi \times (500 \times 10^3))^2 = \frac{1}{LC} \text{ ——— } (2)$$

$(\because \omega_c = 2\pi f_c)$

Multiplying (1) & (2)

$$500 \times 4\pi^2 \times \times 500^2 \times 10^6 = \frac{1}{C^2}$$

$$\Rightarrow C^2 = \frac{1}{4\pi^2 \times 5 \times 25 \times 10^{12}} ~~= 1.4 \text{ nF}~~$$

$$\Rightarrow \boxed{C = 14 \text{ nF}}$$

& $L = 500 \times C$

$= 500 \times 14 \text{ nF}$

$$\boxed{L = 7\ \mu H}$$

Extra:- Problems on Fourier transform

1) Fourier transform of Gaussian pulse

$$x(t) = e^{-b^2t^2}.$$

Ans:- $F[x(t)]$

$$= \int_{-\infty}^{\infty} e^{-b^2t^2} \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-(b^2t^2 + j\omega t)}\, dt$$

Put $b^2t^2 + j\omega t = (bt)^2 + 2 \cdot bt \cdot \frac{j\omega t}{2bt}$

$$= (bt)^2 + 2 \cdot bt \cdot \left(\frac{j\omega t}{2bt}\right) + \left(\frac{j\omega t}{2bt}\right)^2 - \left(\frac{j\omega t}{2bt}\right)^2$$

$$= \left(bt + \frac{j\omega \cancel{t}}{2b\cancel{t}}\right)^2 - \left(\frac{j\omega \cancel{t}}{2b\cancel{t}}\right)^2$$

$$= \left(bt + \frac{j\omega}{2b}\right)^2 - \left(\frac{j\omega}{2b}\right)^2$$

$$= \left(bt + \frac{j\omega}{2b}\right)^2 + \frac{\omega^2}{4b^2}$$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} e^{-\left(bt+\frac{j\omega}{2b}\right)^2} \cdot e^{-\frac{\omega^2}{4b^2}}\, dt$$

$$= e^{-\frac{\omega^2}{4b^2}} \int_{-\infty}^{\infty} e^{-\left(bt+\frac{j\omega}{2b}\right)^2}\, dt$$

Put $bt + \frac{j\omega}{2b} = x$

$\Rightarrow b\frac{dt}{dx} = 1$

$t \to \infty,\ x \to \infty$

$t \to -\infty,\ x \to -\infty$

$\Rightarrow \quad dt = \frac{dx}{b}$

$\Rightarrow \quad X(\omega) = e^{-\omega^2/4b^2} \cdot \int_{-\infty}^{\infty} e^{-x^2} \frac{dx}{b}$

$= \frac{e^{-\frac{\omega^2}{4b^2}}}{b} \cdot \left[2 \times \int_{0}^{\infty} e^{-x^2} dx \right]$

$= \frac{e^{-\frac{\omega^2}{4b^2}}}{b} \cancel{2} \left(\frac{\sqrt{\pi}}{\cancel{2}} \right) \quad \left(\because \int_{0}^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \right)$

$\Rightarrow \quad X(\omega) = \frac{\sqrt{\pi}}{b} \cdot e^{-\frac{\omega^2}{4b^2}}$

$\Rightarrow \quad \boxed{X(\omega) = \frac{\sqrt{\pi}}{b} \cdot e^{-\left(\frac{\omega}{2b}\right)^2}}$

4) Inverse Fourier transform of $\boxed{sgn(\omega)}$.

Let $x(t) = sgn(t)$ —— ①

$X(\omega) = \frac{2}{j\omega}$ —— ②

$X(t) = \frac{2}{jt}$ —— ③

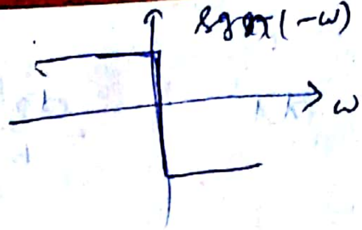
Duality Property; say's.

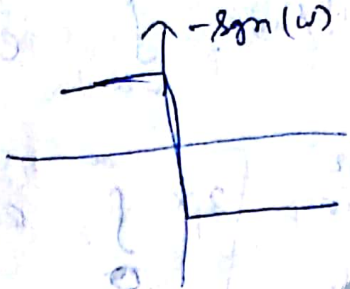
$F[X(t)] = 2\pi \, x(-\omega)$

$F\left[\frac{2}{jt}\right] = 2\pi \, sgn(-\omega)$ (using eqn ③ & ①)

($\because$ $x(t) = sgn(t)$, $x(-\omega) = sgn(-\omega)$)

But $sgn(-\omega) = -sgn(\omega)$





$$F\left[\frac{2}{jt}\right] = -2\pi \, \text{sgn}(w)$$

$$\Rightarrow \frac{\cancel{2}}{j} \times F\left(\frac{1}{t}\right) = -\cancel{2}\pi \, \text{sgn}(w)$$

$$\Rightarrow \frac{1}{j} F\left[\frac{1}{t}\right] = -\pi \, \text{sgn}(w)$$

$$\Rightarrow F\left[\frac{1}{t}\right] = -j\pi \, \text{sgn}(w)$$

Taking Inverse Fourier transform, both the sides.

$$\frac{1}{t} = -j\pi \; \text{I.F.T}\left[\text{sgn}(w)\right]$$

$$\Rightarrow \boxed{\text{I.F.T}\left[\text{sgn}(w)\right] = -\frac{1}{j\pi t} = \frac{j}{\pi t}}$$

$$\Rightarrow F\left[\frac{1}{\pi t}\right] = -j\,\text{sgn}(w)$$

Note :-

$F[u(t)] \neq u(w)$

$F[\text{sgn}(t)] \neq \text{sgn}(w)$

But

$F[u(t)] = \frac{1}{jw} + \pi\delta(w)$

$F[\text{sgn}(t)] = \frac{2}{jw}$

3) I.F.T of $u(\omega)$

Ans :- $u(t) = \frac{1}{2} sgn(t) + \frac{1}{2}$

$u(\omega) = \frac{1}{2} sgn(\omega) + \frac{1}{2}$

$I.F.T[u(\omega)] = \frac{1}{2} I.F.T(sgn\,\omega) + \frac{1}{2} I.F.T[1]$. — (1)

Lets find out

I.F.T [1].

We know $F[1] = 2\pi\,\delta(\omega)$

Let $x(t) = 1$.

$X(\omega) = 2\pi\,\delta(\omega)$

By Duality property,

$F[X(t)] = 2\pi\, x(-\omega)$

$\Rightarrow F[2\pi\,\delta(t)] = 2\pi \cdot 1$ $\left[\because \begin{matrix} x(t) = 1 \\ x(-\omega) = 1 \end{matrix}\right]$

$\Rightarrow \cancel{2\pi}\, F[\delta(t)] = \cancel{2\pi}$

Taking I.F.T both the sides,

$\delta(t) = F^{-1}[1]$

$\therefore$ $\boxed{IFT[1] = \delta(t)}$ — (2)

$I.F.T[u(\omega)] = \frac{1}{2} \times \left[\frac{j}{\pi t}\right] + \frac{1}{2} \cdot \delta(t)$

$$\boxed{F^{-1}[u(\omega)] = \frac{j}{2\pi t} + \frac{\delta(t)}{2}}$$