Electromagnetic Waves

Text Books

1. M.N.O. Sadiku and S.V. Kulkarni, Principles of Electromagnetics, 6th edition, Oxford University Press, 2015.
2. E.C. Jordan and K.G. Balmain, Electromagnetic Waves and Radiating Systems, 2nd edition, Pearson Education, 2009.

Reference Books

1. W.H. Hayt and J. Buck, Engineering Electromagnetic, 7th edition, Tata McGraw Hill Publishing Company Ltd., 2006.
2. N.N. Rao, Fundamentals of Electromagnetics for Engineering, 1st edition, Pearson Education, New Delhi, 2009.

Preface

→ Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion.

→ It entails the analysis, synthesis, physical interpretation and application of electric and magnetic fields.

→ Thus Electromagnetics (EM) is a branch of Physics or electrical engineering in which electric and magnetic phenomena are studied.

→ EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conservation, radar meterology, and remote sensing.

EM fields are used in induction heater for melting, forging, annealing, surface hardening and soldering operations. EM devices include transformer, electric ~~dev~~ relays, radio/TV, telephones, electric motors, transmission lines, wave guides, antennas, optical fibers, radars and lasers. The design of these devices require through knowledge of the laws and principles of EM.

Chapter-'0' Recall of Vector Algebra (ii)

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended.

→ A quantity can be either a scalar or a vector.

→ A scalar is a quantity that has only magnitude.

(Ex:- Mass, time, distance, temperature, entropy, electric potential, populations, etc)

→ A vector is a quantity that has both magnitude and direction.

(Ex:- velocity, force, displacement, electric field intensity)

→ Vector are represented by a letter with an arrow on top of it or by a letter in Boldface type

ex:- $\vec{A}$, $\vec{B}$ or **A**, **B** (Bold)

→ A field is a function that specifies a particular quantity everywhere in a region.

→ If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. (iii)

<u>Examples of scalar field</u> are temperature distribution in a building, sound intensity in a theater, electric potential in a region, etc.

<u>Examples of vector field</u> are gravitational force on a body in space, velocity of rain drops in atmosphere, etc.

<u>Unit vectors:-</u>

A vector $\mathbf{A}$ (Bold) has both <u>magnitude & direction</u>. The magnitude of $\mathbf{A}$ (Bold) is a scalar written as A (Unbold) or $|\mathbf{A}|$ (Bold). A unit vector $\mathbf{a}_A$ (Bold) along $\mathbf{A}$ (Bold) is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along $\mathbf{A}$ (Bold), i.e.

$$\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

(Bold) (Bold) (A → Unbold)

or

$$a_A = \frac{\vec{A}}{|\vec{A}|} \qquad \text{(2)}$$

Since $|\mathbf{a}_A| = 1$, we may write

$$\mathbf{A} = A\,\mathbf{a}_A \qquad \text{(ii)}$$

which specifies $\mathbf{A}$ in terms of its magnitude A and its direction $\mathbf{a}_A$.

or $\vec{A} = |\vec{A}|\,\vec{a}_A$ —— (ii)

→ A vector A in Cartesian (or rectangular) coordinates may be represented as

(A_x, A_y, A_z) or $A_x a_x + A_y a_y + A_z a_z$ —— (iii)

where A_x, A_y, A_z are called the components of A along x, y and z directions respectively; a_x, a_y, a_z are unit vectors in the x, y, z directions, respectively. [Refer figure A.1]

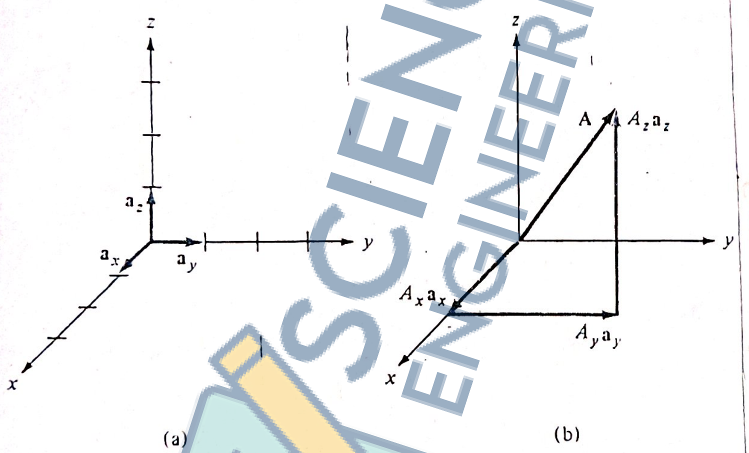


Figure A.1 (a) Unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, (b) components of **A** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

– The magnitude of A (→ Bold) is given by

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad \text{—— (iv)}$$

$$\text{or } |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad \text{—— (iv)}$$

and the unit vector along A is given by

$$a_A = \frac{A_x a_x + A_y a_y + A_z a_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \quad \text{—— (V)}$$

Vector Addition and Subtraction

Two vectors $\vec{A}$ & $\vec{B}$ can be added together to give another vector $\vec{C}$ i.e

$$\vec{C} = \vec{A} + \vec{B} \quad \text{—— (VI)}$$

or

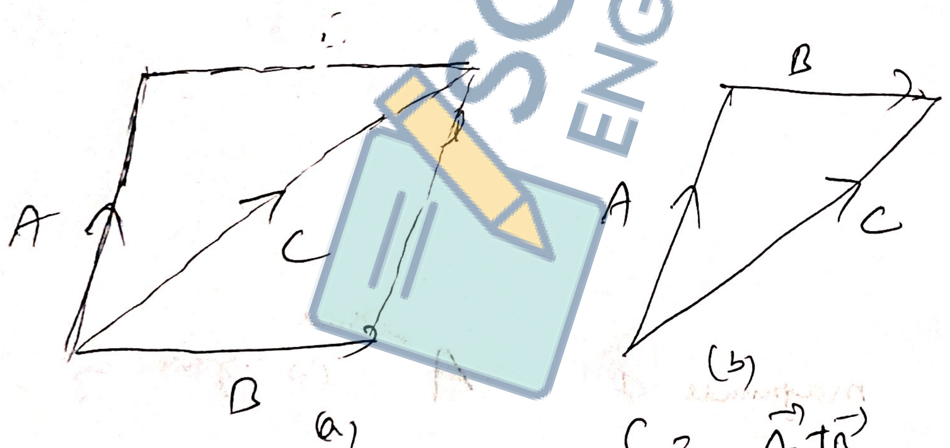
$$\vec{C} = (A_x + B_x)\, a_x + (A_y + B_y)\, a_y + (A_z + B_z)\, a_z \quad \text{—— (VII)}$$

Similarly,

$$\vec{D} = \vec{A} - \vec{B} \quad \text{—— (VIII)}$$

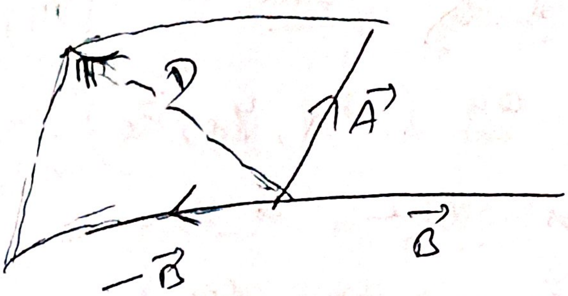
or

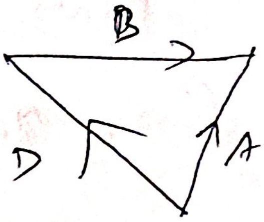
$$\vec{D} = (A_x - B_x)\, a_x + (A_y - B_y)\, a_y + (A_z - B_z)\, a_z \quad \text{—— (IX)}$$



$C = \vec{A} + \vec{B}$ (a) $C = \vec{A} + \vec{B}$ (b)

Fig A.2 vector addition $\vec{C} = \vec{A} + \vec{B}$ (a) Parallelogram rule (b) head-to-tail rule





$\vec{D} + \vec{B} = \vec{A}$

$\Rightarrow \vec{D} = \vec{A} - \vec{B}$

Fig A.3 Vector subtraction $D = \vec{A} - \vec{B}$!

(a) Parallelogram rule (b) head-to-tail rule

Law	Addition	Multiplication
Commutative	$\vec{A} + \vec{B} = \vec{B} + \vec{A}$	$K\vec{A} = \vec{A}K$
Associative	$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$	$K(l\vec{A}) = Kl(\vec{A})$
Distributive	$K(\vec{A} + \vec{B}) = K\vec{A} + K\vec{B}$	

Where K & l are scalars and $\vec{A}, \vec{B}, \vec{C}$ are vectors.

Position and Distance Vectors

A point P in Cartesian coordinates may be represented by (x, y, z). The position vector r_p (or radius vector) of point P is defined as the directed distance from the origin O to P, that is

$$r_p = OP = x a_x + y a_y + z a_z \quad \text{——} \quad (x)$$

[Refer fig A.4]

The distance vector is the displacement from one point to another.

If two points P & Q are given by (x_P, y_P, z_P) and (x_Q, y_Q, z_Q), the distance vector is the displacement from P to Q as shown in fig A-5 (vii)

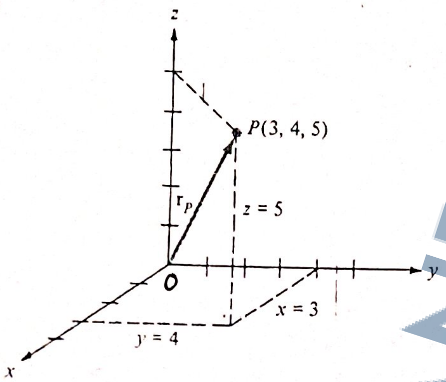


Fig A.4 Illustration of position vector

$r_P = 3a_x + 4a_y + 5a_z$

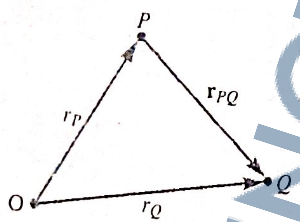


Fig A.5: Distance vector r_{PQ}

$$r_{PQ} = r_Q - r_P$$

$$r_{PQ} = (x_Q - x_P)a_x + (y_Q - y_P)a_y + (z_Q - z_P)a_z \quad \text{(xi)}$$

→ A vector may depend on point P.

e.g. $\vec{A} = 2xy\, a_x + y^2 a_y - xz^2 a_z$, P (2, -1, 4)

so, $\vec{A} = -4a_x + a_y - 32a_z$

But a constant or uniform vector does not contain space variables x, y, z. e.g

$$\vec{B} = 3a_x - 2a_y + 10a_z$$

Vector Multiplication

When two vectors are multiplied, the result is either a scalar or a vector depending on how they are multiplied.

1. Scaler (or dot) Product : $A \cdot B$
2. Vector (or Cross) Product : $A \times B$
3. Scalar triple product : $A \cdot (B \times C)$
4. Vector triple product : $A \times B \times C$

1. Dot Product

The dot product of two vectors A and B, written as $A \cdot B$, is defined geometrically as the product of the magnitudes of A and B and the cosine of the angle between them.

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta_{AB} \qquad \text{(xii)}$$

or If $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \qquad \text{(xiii)}$$

→ If $\vec{A} \cdot \vec{B} = 0$, then two vectors $\vec{A}$ & $\vec{B}$ are said to be orthogonal

$(\because \vec{A} \cdot \vec{B} = AB \cos\theta$, If $\theta = 90^\circ$, then $\vec{A} \cdot \vec{B} = 0)$

The dot product obeys the following rules

(i) <u>Commutative law:</u>

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \quad \text{—— (XIV)}$$

(ii) <u>Distributive law</u>

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C} \quad \text{—(XV)}$$

(iii)

$$\vec{A}\cdot\vec{A} = |\vec{A}|^2 = A^2 \quad \text{—— (XVI)}$$

<u>Also</u>

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0 \quad \text{—(XVII)}$$

(∵ Unit vectors are ⊥ , $\cos 90° = 0$)

$$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1 \quad \text{—(XVIII)}$$

(∵ $\cos 0° = 1$)

2. Cross Product

The cross product of two vectors A and B, written as $\vec{A}\times\vec{B}$, is a vector quantity whose <u>magnitude</u> is the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ (See fig A.7) and is in the direction of advance of right-handed screw as $\vec{A}$ is turned into $\vec{B}$.

Thus

$$\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\,\sin\theta_{AB}\ \vec{a}_n \quad \text{—(XIX)}$$

<u>Where</u> $\vec{a}_n$ is a unit vector normal to the plane containing $\vec{A}$ and $\vec{B}$.

(X)

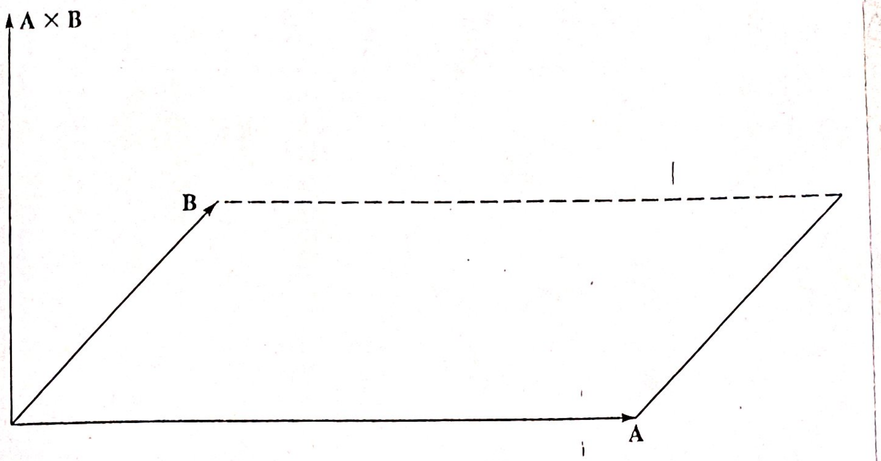


Figure A.7 The cross product of **A** and **B** is a vector with magnitude equal to the area of the parallelogram and direction, as indicated.

— p. 7

The direction of a_n is ~~the~~ taken as the direction of the right ~~p~~ thumb when the fingers of the right hand rotate from A to B as shown in figure A.8 (a).

Alternatively, the direction of a_n is taken as that of the advance of a right-handed screw as $\vec{A}$ is turned into $\vec{B}$ as shown in figure A.8 (b).

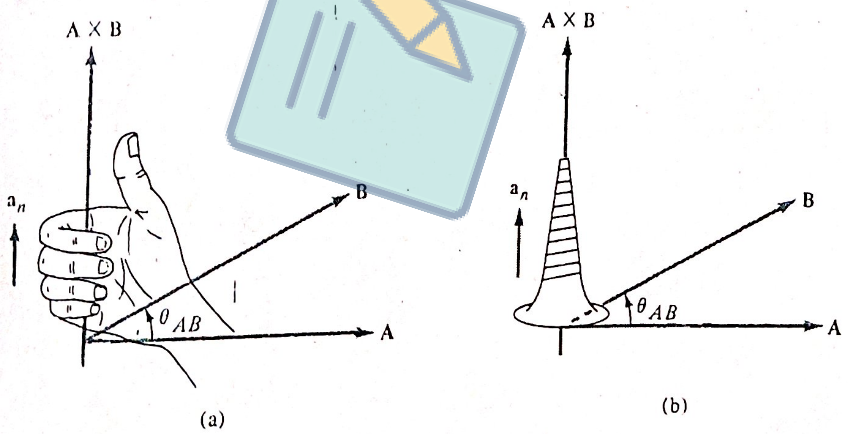


Figure A.8 Direction of **A** × **B** and $\mathbf{a}_n$ using **(a)** the right-hand rule and **(b)** the right-handed-screw rule.

A.8.

If $\vec{A} = A_x a_x + A_y a_y + A_z a_z$

$\vec{B} = B_x a_x + B_y a_y + B_z a_z$

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \text{— (XX)}$$

$= (A_y B_z - A_z B_y) a_x + (A_z B_x - A_x B_z) a_y + (A_x B_y - A_y B_x) a_z$

which is obtained by "crossing" terms in cyclic permutation, hence the name "Cross Product".

Properties of Cross Product

(i) **Not Commutative**

$$A \times B \neq B \times A \quad \text{—— (XXI)}$$

Anti commutative

$$(A \times B) = -(B \times A) \quad \text{— (XXII)}$$

(ii) **Not associative**

$$A \times (B \times C) \neq (A \times B) \times C$$

(iii) **Distributive**

$$A \times (B + C) = A \times B + A \times C$$

(iv) $A \times A = 0 \quad (\because \sin 0 = 0)$

(xiii)

$$\left.\begin{aligned} a_x \times a_y &= a_z \\ a_y \times a_z &= a_x \\ a_z \times a_x &= a_y \end{aligned}\right\} \quad \text{—} \quad (xxiii)$$

which are obtained in cyclic permutation and illustrated in fig A-9.

$$\therefore \left.\begin{aligned} a_y \times a_x &= -a_z \\ a_z \times a_y &= -a_x \\ a_x \times a_z &= -a_y \end{aligned}\right\} \quad \text{—} \quad (xxiv)$$

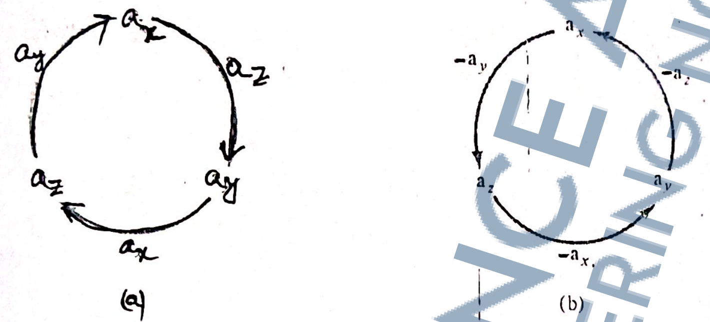


Figure A.9 Cross product using cyclic permutation **(a)** Moving clockwise leads to positive results. **(b)** Moving counterclockwise leads to negative results.

A.9

Scalar triple Product

$$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B) \quad \text{—} \quad (xxv)$$

→ $A \cdot (B \times C)$ is the volume of a parallelogram having A, B, and C as edges.

$$A \cdot (B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \quad \text{—} \quad (xxvi)$$

Since the result of this vector multiplication is scalar eq.^{n} (XXV) and eq.^{n} (XXVI) are called scalar triple product. (XXVII)

Vector triple product

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \quad \text{— (XXVII)}$$

(Recall BAC - CAB rule)

Note that

$$(A \cdot B) C \neq A (B \cdot C)$$

but

$$(A \cdot B) C = C (A \cdot B)$$

Component of a vector

Given a vector $\vec{A}$, we define the scalar component A_B of $\vec{A}$ along vector $\vec{B}$ as

$$A_B = A \cos\theta_{AB} = |\vec{A}| \, |\vec{a}_B| \cos\theta_{AB}$$

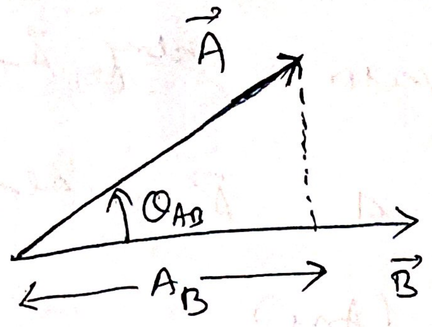
$$\Rightarrow \boxed{A_B = \vec{A} \cdot \vec{a}_B} \quad \text{— (XXVIII)}$$

i.e. dot product of $\vec{A}$ and unit vector along $\vec{B}$.

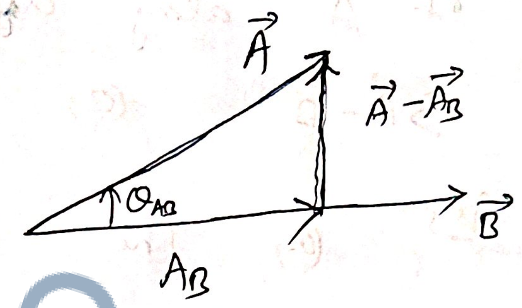
The vector component $\vec{A_B}$ is simply the scalar component in eq.^{n} (XXVIII) multiplied by a Unit vector along $\vec{B}$. (xiv)

$$\vec{A_B} = A_B \vec{a}_B = (\vec{A} \cdot \vec{a}_B)\, \vec{a}_B \quad \text{—(xxix)}$$

(from eqn xxviii)



(a)



(b)

Fig A.10: Components of A along B: (a) Scalar Component A_B (b) Vector Component A_B

Problems

1) Given vectors $\vec{A} = 3a_x + 4a_y + a_z$ and $\vec{B} = 2a_y - 5a_z$. Find the angle between $\vec{A}$ & $\vec{B}$

Ans :- $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta_{AB}$

$$\Rightarrow \cos\theta_{AB} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{(3\times 0) + (4\times 2) + (1\times -5)}{\sqrt{3^2+4^2+1^2}\ \sqrt{0^2+2^2+25}}$$

$$\Rightarrow \cos\theta_{AB} = \frac{3}{\sqrt{26}\sqrt{29}} = 0.1092$$

$$\Rightarrow \theta_{AB} = \cos^{-1}(0.1092) = 83.73^\circ.$$

2) If $\vec{A} = 10a_x - 4a_y + 6a_z$ and $\vec{B} = 2a_x + a_y$. Find (a) Component of $\vec{A}$ along $\vec{a}_y$ (b) the magnitude of $3\vec{A} - \vec{B}$ (c) Unit vector along $\vec{A} + 2\vec{B}$

Ans:- (a) The Component of $\vec{A}$ along

$$a_y = -4 \text{ (Ans)}$$

(b) $3\vec{A} - \vec{B} = 3(10a_x - 4a_y + 6a_z) - (2a_x + a_y)$

$$= 28a_x - 13a_y + 18a_z$$

$$|3\vec{A} - \vec{B}| = \sqrt{28^2 + 13^2 + 18^2} = \sqrt{1277}$$

$$|3\vec{A} - \vec{B}| = 35.74 \text{ (Ans)}$$

(c) Let $\vec{A} + 2\vec{B} = \vec{C}$

$$\therefore \vec{C} = (10a_x - 4a_y + 6a_z) + 2(2a_x + a_y)$$

$$\vec{C} = 14a_x - 2a_y + 6a_z$$

Unit vector $\vec{a}_c = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{14a_x - 2a_y + 6a_z}{\sqrt{14^2 + 2^2 + 6^2}}$

$$\Rightarrow \vec{a}_c = 0.9114 - 0.1302a_y + 0.3906a_z$$

Note:- $|\vec{a}_c|$ is 1, as expected.

3) Points P & Q are located at (xvi)
(0, 2, 4) and (−3, 1, 5). Calculate
(a) The position vector P
(b) The distance vector P to Q
(c) The distance between P & Q
(d) A vector parallel to PQ with magnitude 10.

Ans:

(a) Position vector $\hat{r}_P = 0 \cdot a_x + 2a_y + 4a_z = 2a_y + 4a_z$

(b) r_{PQ} = is the distance vector P to Q

$= r_Q - r_P$

$= (-3, 1, 5) - (0, 2, 4)$

$= (-3, -1, 1)$

or $\vec{r}_{PQ} = -3a_x - a_y + a_z$

(C) Since r_{PQ} is the distance vector from P to Q, the distance between P and Q is the magnitude of this vector; i.e

$$|\vec{r}_{PQ}| = \sqrt{9+1+1} = \sqrt{11} = 3.317$$

or

By Coordinate geometry, distance betn 2 points

$$d = \sqrt{(0-3)^2 + (2-1)^2 + (4-5)^2}$$

$$= \sqrt{9+1+1}$$

$$= \sqrt{11}$$

$$= 3.317$$

(d) Let the required vector be $\vec{A}$.

$$\vec{A} = |\vec{A}| \, \vec{a}_A$$

Where $|\vec{A}| = 10$ is the magnitude of $\vec{A}$.

Since $\vec{A}$ is // to PQ, it must have the same unit vector as $\vec{r}_{PQ}$ or $\vec{r}_{QP}$

Hence $\vec{a}_A = \pm \dfrac{\vec{r}_{PQ}}{|\vec{r}_{PQ}|} = \pm \dfrac{-3a_x - a_y + a_z}{\sqrt{11}}$

and

$$\vec{A} = 10 \times \vec{a}_A = 10 \times \left(\pm \frac{-3a_x - a_y + a_z}{3.317} \right)$$

$$\Rightarrow \vec{A} = \pm (-9.045 a_x - 3.015 a_y + 3.015 a_z)$$

(Ans)

4) Homework

Three field quantities are given by

$\vec{P} = 2a_x - a_z$, $\vec{Q} = 2a_x - a_y + 2a_z$

$\vec{R} = 2a_x - 3a_y + a_z$

Determine

(a) $(P+Q) \times (P-Q)$

(b) $Q \cdot R \times P$ (i.e. $Q \cdot (R \times P)$)

(c) $P \cdot Q \times R$

(d) $\sin \theta_{QR}$

(e) $P \times (Q \times R)$

(f) A unit vector perpendicular to both Q & R

(g) The component of P along Q

Ans ÷ (a) $(P+Q) \times (P-Q)$

$= P\times(P-Q) + Q\times(P-Q)$

$= P\times P - P\times Q + Q\times P - Q\times Q$

$= 0 + Q\times P + Q\times P - 0$

$= 2\,(Q\times P)$

$= 2 \begin{vmatrix} a_x & a_y & a_z \\ 2 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix}$

$= 2\left[(1-0)a_x - a_y(-2-4) + a_z(0+2)\right]$

$= 2\left[a_x + 6a_y + 2a_z\right]$ (Ans)

$= 2a_x + 12a_y + 4a_z$

(b) $Q.(R\times P)$

$= Q. \begin{vmatrix} a_x & a_y & a_z \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix}$

$= (2,-1,2).(3,4,6)$

i.e. $= (2a_x - a_y + 2a_z).(3a_x + 4a_y + 6a_z)$

$= 6 - 4 + 12$

$= 14$

(X IX)

or By scalar triple product formula

$$Q\cdot(R\times P) = A\cdot(B\times C) = \begin{vmatrix} Q_x & Q_y & Q_z \\ R_x & R_y & R_z \\ P_x & P_y & P_z \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -1 & 2 \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= 2(3-0) + 1(-2-2) + 2(0+6)$$

$$= 6 + (-4) + 12$$

$$= 14$$

(c) $P\cdot(Q\times R) = Q\cdot(R\times P) = 14$

or $P\cdot(Q\times R) = (2,0,-1)\cdot(5,2,-4)$

$$= 10+0+4$$

$$= 14$$

(d) $$\sin\theta_{QR} = \frac{|\vec{Q}\times\vec{R}|}{|\vec{Q}||\vec{R}|} = \frac{|(5,2,-4)|}{\sqrt{9}\,\sqrt{14}}$$

$$= \frac{\sqrt{45}}{3\sqrt{14}} = \frac{\sqrt{5}\times\cancel{\sqrt{9}}}{\cancel{3}\times\sqrt{14}} = 0.5976$$

(e) $P\times(Q\times R) = (2,0,-1)\times(5,2,-4)$

$$= (2,3,4)$$

(XX)

or Using bac-cab rule,

$$Q(P\cdot R) - R(P\cdot Q)$$

$$= (2,-1,2)(4-0-1) - (2,-3,1)(4+0-2)$$

$$= 3(2,-1,2) - 2(2,-3,1)$$

$$= (6,-3,6) - (4,-6,2)$$

$$= (2,3,4)$$

$$= 2a_x + 3a_y + 4a_z$$

(f) A Unit vector $\perp$ to both Q & R is

$$\vec{a} = \frac{\pm Q\times R}{|Q\times R|} = \frac{\pm(5,2,-4)}{\sqrt{45}}$$

$$\vec{a} = \pm(0.745, 0.298, -0.596)$$

(g) The component of P along Q (vector component)

$$\vec{P}_Q = P_Q \vec{a}_Q$$

$$= P\cos\theta\, \vec{a}_Q$$

$$= (P\cdot a_Q)\cdot\frac{\vec{Q}}{|Q|}$$

$$= \frac{(\vec{P}\cdot\vec{Q})}{|Q||Q|}\cdot\vec{Q} = \frac{(\vec{P}\cdot\vec{Q})\vec{Q}}{|Q|^2}$$

$$= \frac{(4+0-2)(2,-1,2)}{4+1+4}$$

$$\vec{P}_Q = \frac{2}{9}(2,-1,2) = 0.444a_x - 0.222a_y + 0.444a_z$$

(Ans)